Final Project

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ECE 371
July 2, 2025

Contents

1 Overview

The goal of this project is to design impedance matching networks using both lumped and distributed elements, as described in Part I and Part II, respectively. The system under consideration consists of a lossless transmission line with characteristic impedance $Z_0 = 50\,\Omega$ and a load impedance of $Z_L = 60 - j80\,\Omega$.

The design objective is to match the load to the transmission line such that the input impedance seen at the beginning of the line equals Z_0 at an operating frequency of 2 GHz. This ensures maximum power transfer and minimal reflection at the interface between the line and the load.

Microwave Office (MWO) will be used to simulate both circuits and verify that the networks are properly mattached at 2 GHz.

2 Part I: Lumped-Element Matching Network

2.1 Theory and Design Equations

An L-section network will be employed to match the specified load impedance, Z_L, to the system's characteristic impedance. Figure 2.1 illustrates the two configurations of L-section matching networks.

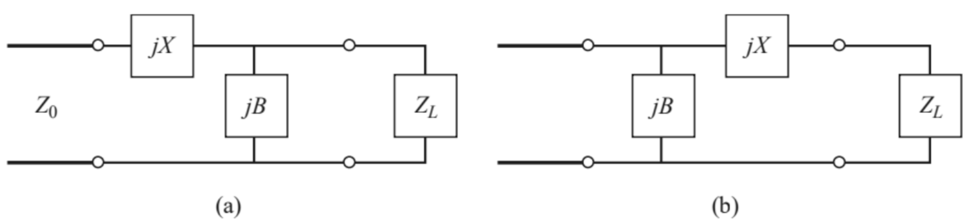

Figure 2.1: L-section Network Topology

If the normalized load impedance,

$$\bar{Z}_L = \frac{Z_L}{Z_0} \tag{2.1}$$

is inside the $1 + jx$ circle on the Smith chart, then the circuit of Figure 2.1a should be used. If the normalized load impedance is outside the $1 + jx$ circle on the Smith chart, the circuit of Figure 2.1b should be used. In this case,

$$\bar{Z}_L = \frac{Z_L}{Z_0} = \frac{60 - j80}{50} = 1.2 - j1.6, \tag{2.2}$$

which lies **inside** the $1+jx$ circle as shown in Figure 2.2. Therefore, the circuit of Figure 2.1a is the appropriate choice for impedance matching.

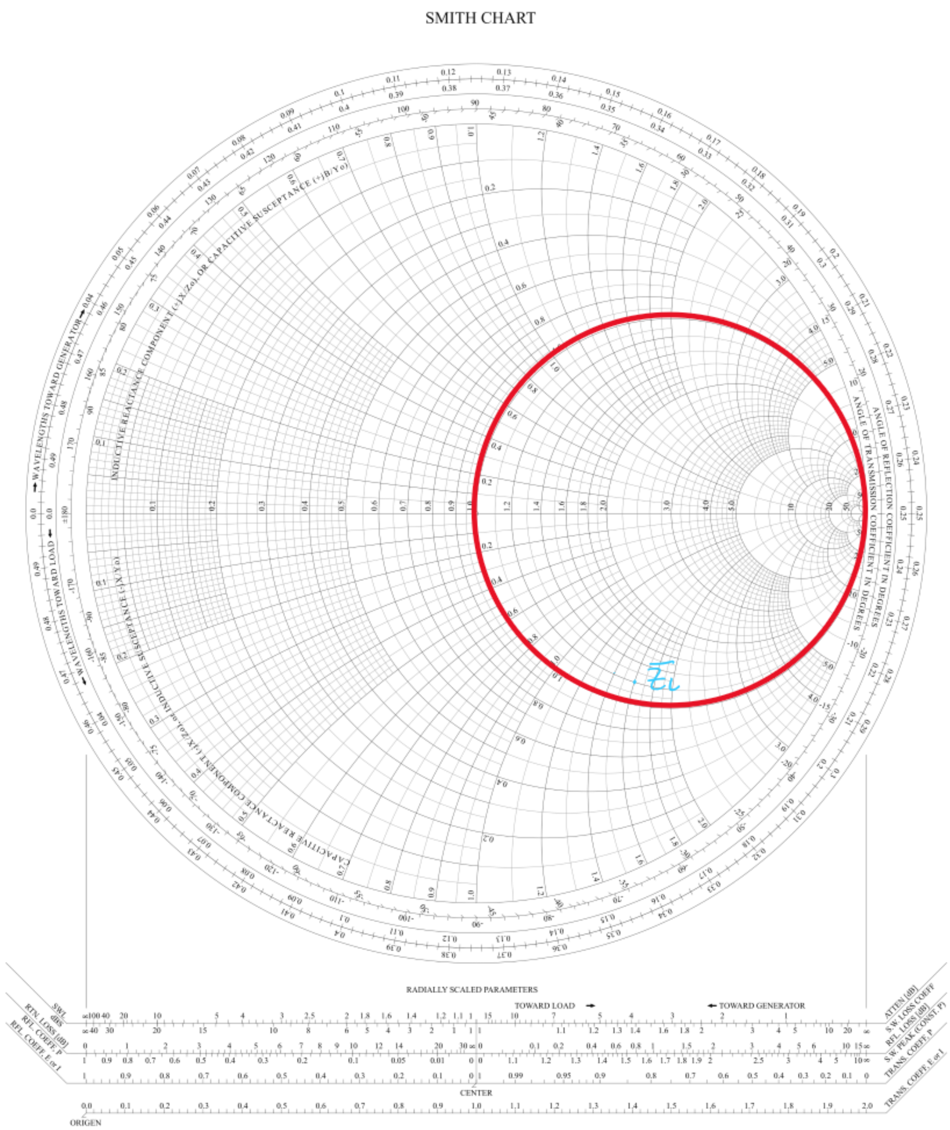

Figure 2.2: z_L plotted on the Smith Chart

Therefore, a series inductor and shunt capacitor will be used. To find the appropriate values for L and C, we must first find the appropriate Reactance and Susceptance values from the following equations:

$$X = \frac{1}{B} + \frac{X_L Z_0}{R_L} - \frac{Z_0}{B R_L} \tag{2.3}$$

$$B = \frac{X_L \pm \sqrt{\frac{R_L}{Z_0}}\sqrt{R_L^2 + X_L^2 - Z_0 R_L}}{R_L^2 + X_L^2} \tag{2.4}$$

Then, use those values to find the L and C values of the inductor and capacitor respectively. This is done with the following two equations:

$$L = \frac{X}{2\pi f} \tag{2.5}$$

$$C = \frac{B}{2\pi f} \tag{2.6}$$

2.2 Calculation of X and B; Conversion to L and C

These calcuations can be done in MWO, but are shown here as reference:

$$Z_0 = 50 \tag{2.7}$$
$$R_L = 60 \tag{2.8}$$
$$X_L = -80 \tag{2.9}$$
$$f = 2 \cdot 10^9 \text{ Hz} \tag{2.10}$$

$$B = \frac{-80 \pm \sqrt{\frac{60}{50}}\sqrt{60^2 + (-80)^2 - (50)(60)}}{60^2 + (-80)^2} = 0.001165 \tag{2.11}$$

$$X = \frac{1}{0.001165} + \frac{(-80)(50)}{60} - \frac{50}{0.001165 \cdot 60} = 76.376 \tag{2.12}$$

$$L = \frac{76.376}{2\pi(2 \cdot 10^9)} = 6.0778nH \tag{2.13}$$

$$C = \frac{0.001165}{2\pi(2 \cdot 10^9)} = 92.72fF \tag{2.14}$$

Thus an inductor with a value of 6.0778nH and a capacitor with a value of 92.72fF will be used in the simulation.

2.3 MWO Simulation for Lumped-Element

The schematic in Figure 2.3 illustrates the implementation of a lumped-element L-section matching network. Notice how the values of the indctor and capactior are parameterized such that the values are calculated in MWO.

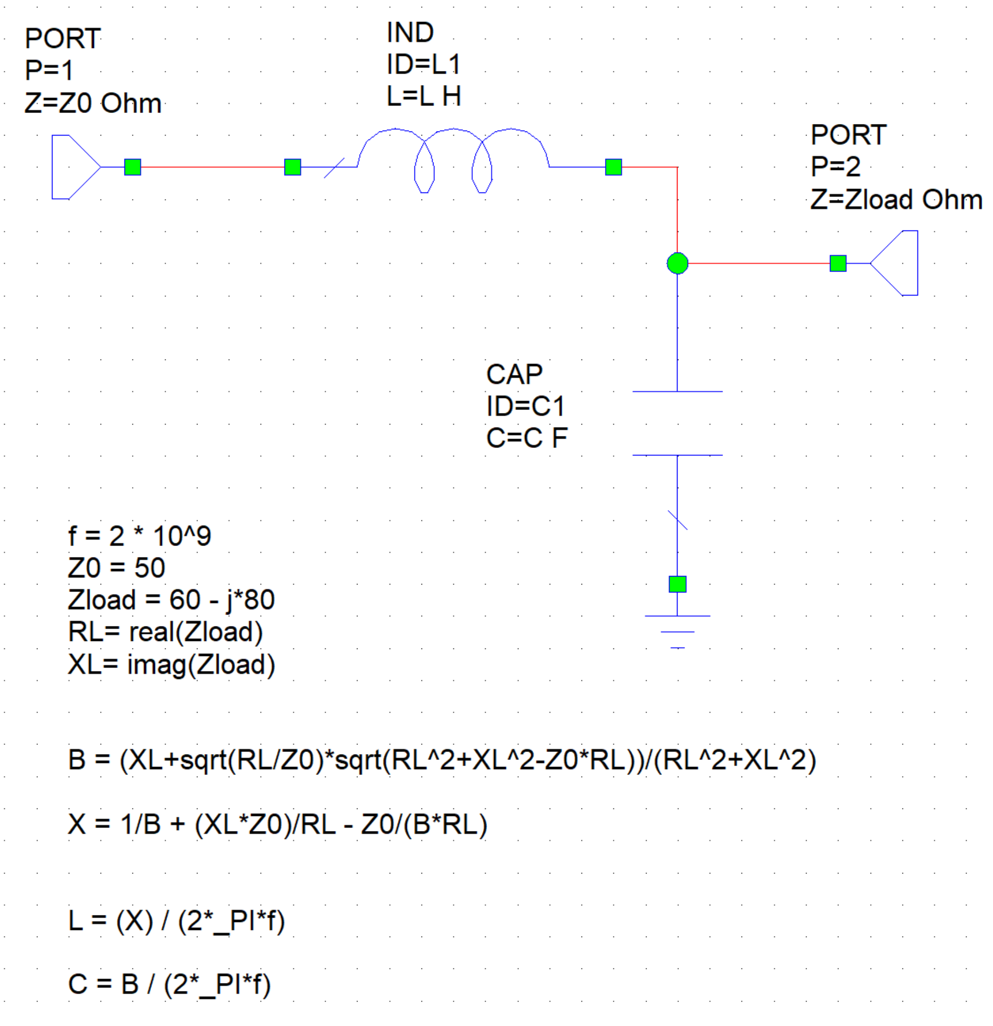

Figure 2.3: MWO Schematic of Lumped-Element Circuit

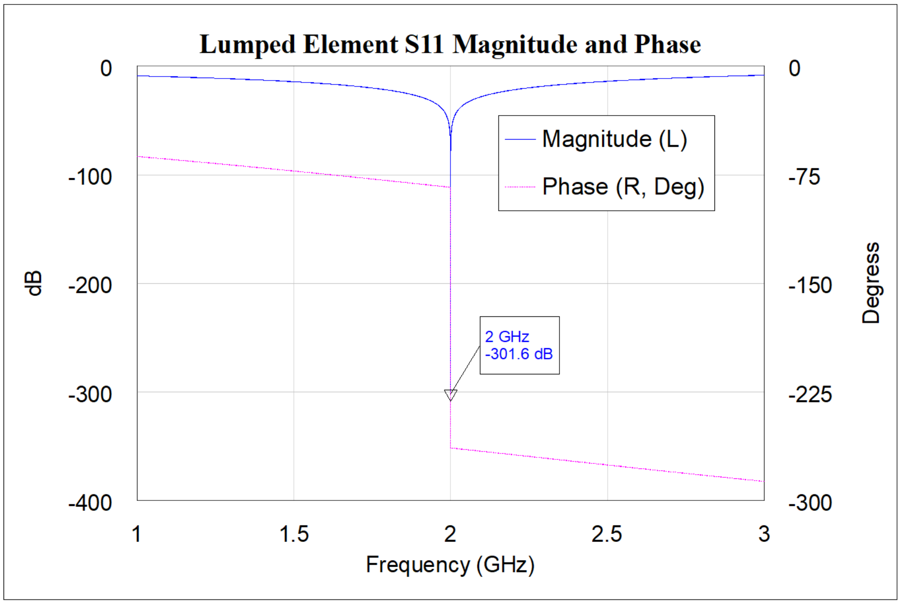

Figure 2.4: Lumped-Element Plot for 1-3 GHz

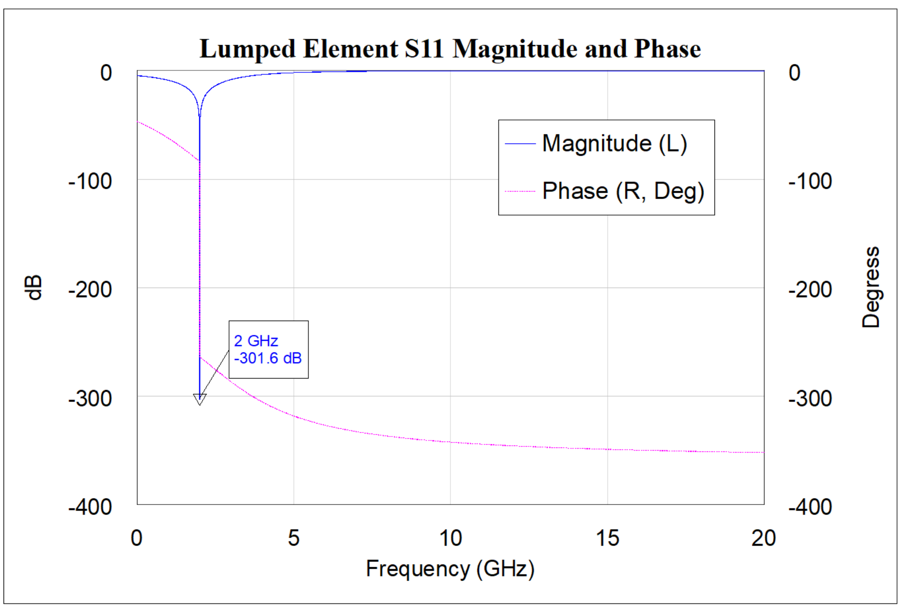

Figure 2.5: Lumped-Element Plot for 0-20 GHz

2.4 Analysis of Results

The two plots in Figures 2.4 and 2.5 show the magnitude and phase of the reflection coefficient S_{11} for the lumped-element matching network.

At exactly 2 GHz, the magnitude of S_{11} reaches a minimum value of approximately -301.6 dB, indicating nearly perfect impedance matching and thus maximum power transfer to the load. This value is effectively zero, demonstrating that all incident power is delivered to the load at this frequency, with negligible reflection.

However, the narrow bandwidth around 2 GHz also reveals a key limitation of this design. As the frequency deviates even slightly from 2 GHz, the magnitude of S_{11} rapidly increases toward 0 dB, signifying complete signal reflection and thus poor power transfer.

This response characterizes a highly frequency-selective, narrowband matching network. It is highly effective at the target frequency but unsuitable for wideband applications, where consistent performance across a range of frequencies is required.

3 Part II: Distributed-Element Matching Network

3.1 Determination of Transmission and Stub Lengths

The matching network should consist of a transmission line and a short-circuit shunt stub as shown in Figure 3.1. The general procedure to matching a networking using transmission lines and stubs is as follows:

1. Plot the normalized load impedance $\bar{Z}_L$ on the Smith chart and draw a line from the origin through $\bar{Z}_L$.
2. Draw the corresponding Standing Wave Ratio (SWR) circle and the $1 + jx$ circle on the Smith chart.
3. Rotate $\bar{Z}_L$ along the SWR circle toward the generator until it intersects the $1 + jx$ circle. This angular movement represents the required length of the transmission line.
4. Determine the stub length by finding the distance between the imaginary part of the intersection point and short-circuit/open-ciruit.

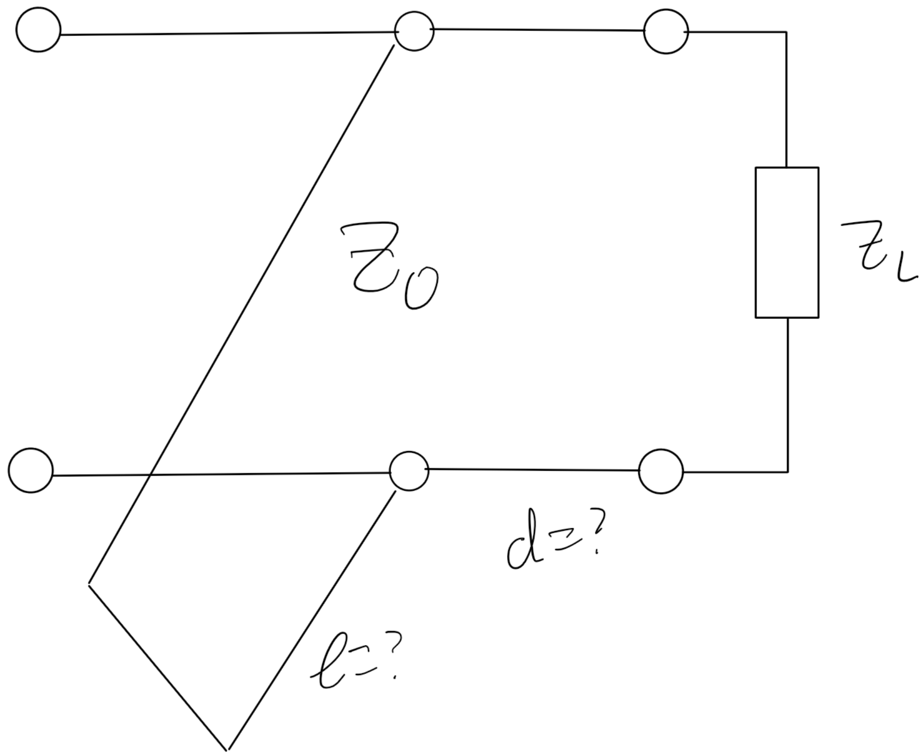

Figure 3.1: Transmission Line and Short-Circuit Shunt Stub Circuit

To determine the length d of the 50-Ω transmission line, first plot the normalized $\bar{Z}_L$ onto the Smith Chart as shown in Figure 3.1.

$$\bar{Z}_L = 1.2 - j1.6 \tag{3.1}$$

Draw the SWR circle corresponding to the normalized load impedance $\bar{Z}_L$, indicated by the blue circle. The red circle denotes the $1 + jx$ constant-resistance circle. To convert the impedance to admittance, draw a line from the center of the Smith chart through the point $\bar{Z}_L$. This reflection maps $\bar{Z}_L$ to the corresponding normalized admittance $\bar{Y}_L$, which simplifies the design process when using shunt (parallel) components.

$$\bar{Y}_L = 0.3 + j0.41 \tag{3.2}$$

Then rotating until the intersection point between the SWR and $1 + jx$ circles we find

$$\bar{Y}_L' = 1 + j1.45, \tag{3.3}$$

with total length rotated from $\bar{Y}_L$ to $\bar{Y}_L'$ of

$$d = 0.175\lambda - 0.066\lambda = 0.109\lambda, \tag{3.4}$$

resulting in an electrical length of

$$\theta = \beta d = \left(\frac{180}{\pi}\right)\left(\frac{2\pi}{\lambda}\right) \cdot 0.109\lambda = 39.24^\circ. \tag{3.5}$$

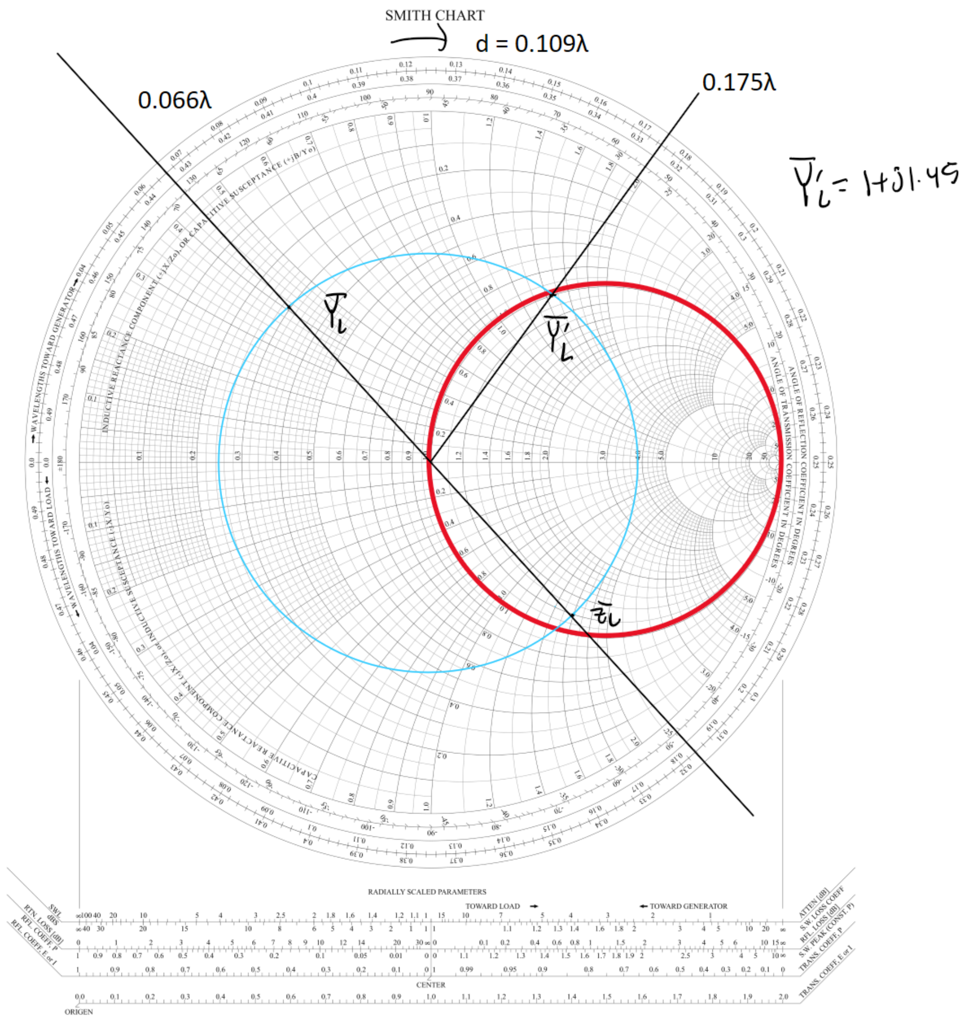

Figure 3.2: Smith Chart for 50-Ω Transmission Line

To find the stub length, draw a line through the origin and the imaginary part of $\bar{Y}_L'$ as shown in Figure 3.3

$$\text{Imag}(\bar{Y}_L') = -j1.45. \tag{3.6}$$

Rotating from short-circuit in admittance towards the imaginary part yields,

$$\ell = 0.346\lambda - 0.25\lambda = 0.096\lambda, \tag{3.7}$$

resulting in an electrical length of

$$\theta = \beta\ell = \left(\frac{180}{\pi}\right)\left(\frac{2\pi}{\lambda}\right) \cdot 0.096\lambda = 34.56°. \tag{3.8}$$

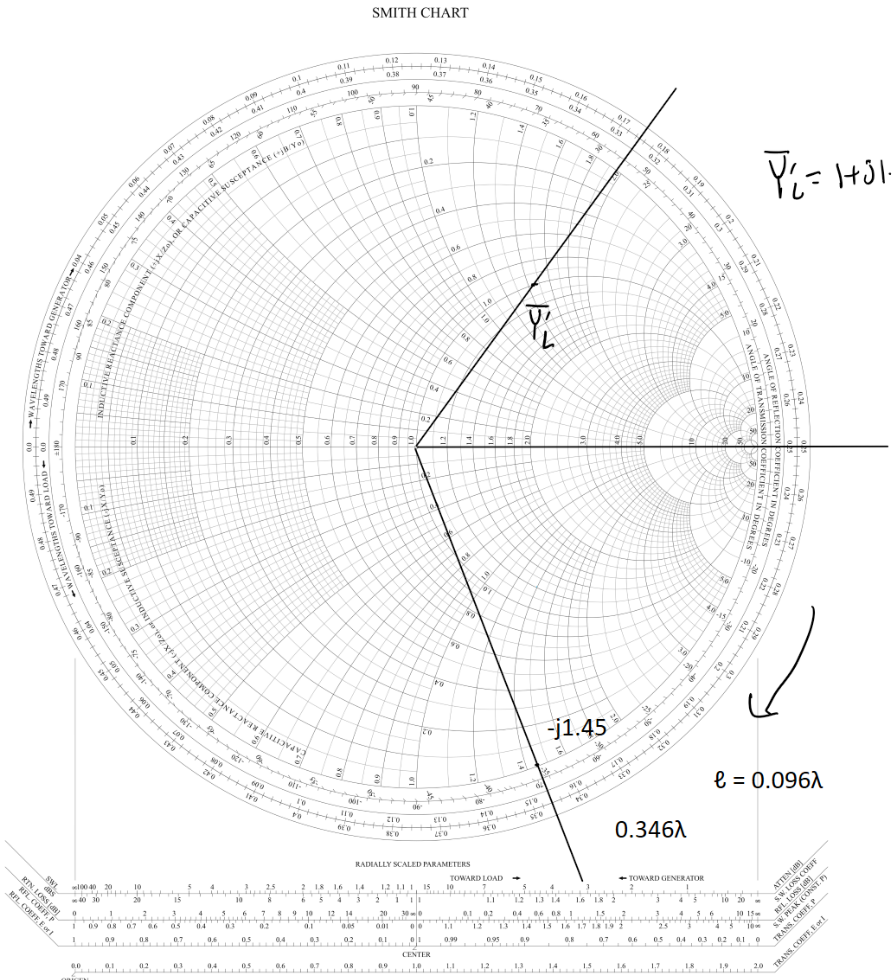

Figure 3.3: Smith Chart for Short-Circut Stub

3.2 MWO Simulation for Distributed Elements

The schematic in Figure 3.4 illustrates the implementation of a distributed-element matching network.

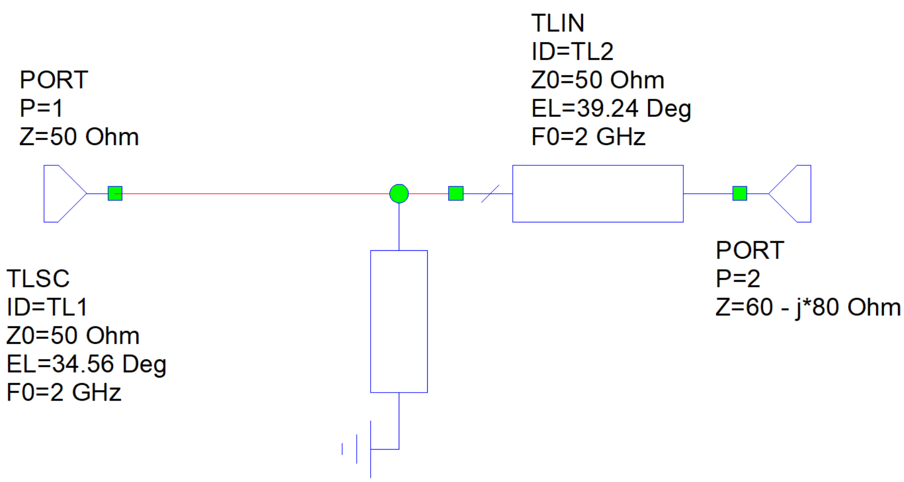

Figure 3.4: MWO Schematic of Lumped-Element Circuit

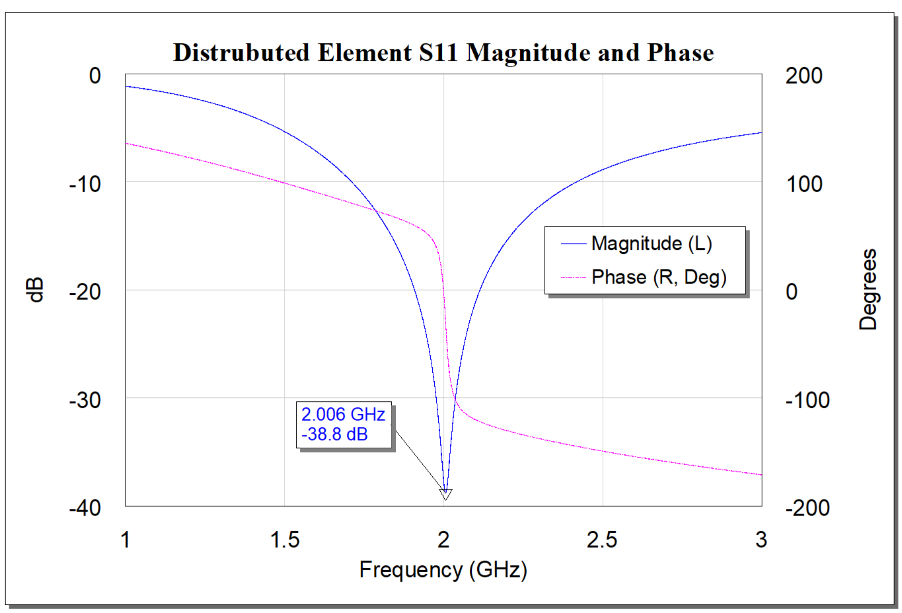

Figure 3.5: Distributed-Element Plot for 1-3 GHz

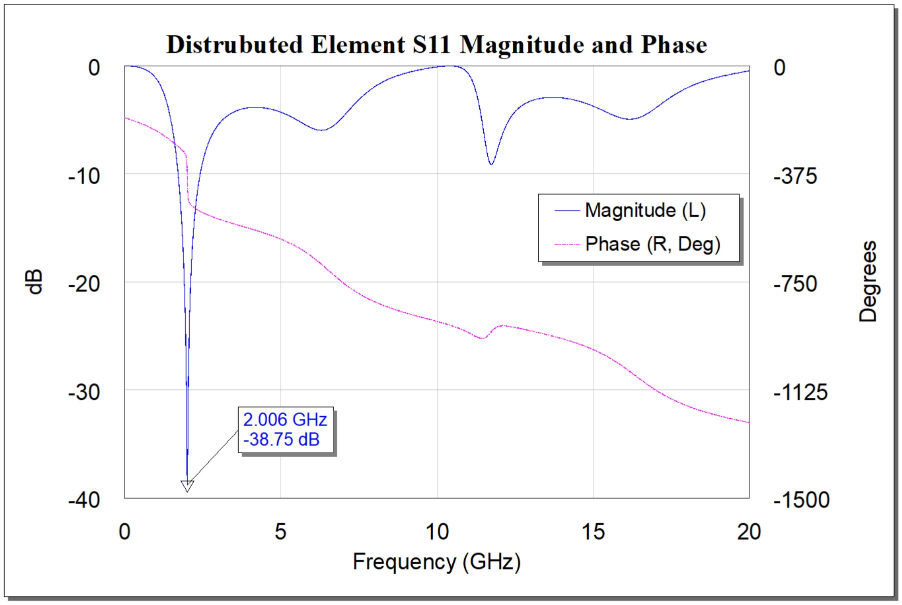

Figure 3.6: Distributed-Element Plot for 0-20 GHz

3.3 Analysis of Results

Figures 3.5 and 3.6 display the magnitude and phase of the reflection coefficient S_{11} for the distributed element matching network across a narrow view around the operating frequency and a wide range of frequencies.

At approximately 2.006 GHz, the magnitude of S_{11} reaches a minimum value of about -38.8 dB. This indicates good, though not perfect, impedance matching. While the reflection is not as low as the lumped-element design (which reached -301.6 dB), it is still sufficient to ensure that most of the incident power is transferred to the load at this frequency.

Compared to the lumped-element case, the distributed network provides a wider bandwidth. This is evident from the broader range of frequencies over which S_{11} remains below -10 dB, which is a common criterion for acceptable matching in RF systems.

The phase response also changes more gradually across the band, suggesting smoother impedance variation with frequency. This characteristic makes distributed matching networks more suitable for broadband applications where consistent performance is needed over a wider frequency range.

4 Part III: Comparison and Conclusions

4.1 Pros and Cons of Lumped vs. Distributed Networks

Lumped-element and distributed-element matching networks each offer distinct advantages and limitations depending on the application. Lumped networks, composed of discrete inductors and capacitors, are compact and straightforward to design, making them ideal for low-frequency or narrowband applications where physical size is constrained. They can achieve extremely precise matching, as demonstrated by the reflection coefficient S_{11} reaching values as low as $-301.6\,\mathrm{dB}$. However, their performance is highly sensitive to frequency, resulting in a narrow operational bandwidth and limited usefulness in broadband systems.

In contrast, distributed-element networks rely on transmission line sections and are inherently better suited for high-frequency and wideband applications. These networks exhibit a broader bandwidth, as seen by a more gradual variation in S_{11} with frequency and acceptable reflection levels over a wider range. While they typically cannot achieve the same ultra-low S_{11} values as lumped networks, their less reactive response and physical realizability at microwave frequencies make them advantageous in RF and microwave circuit design. However, distributed designs require more physical space and are generally more complex to implement in compact systems.

In summary, lumped-element networks excel in precision and size efficiency but are narrowband, whereas distributed-element networks offer broader bandwidth and are preferable at higher frequencies, albeit with larger physical footprints.

5 Extra Credit

While the schematic in Figure 3.4 is electrically valid and functions well in simulation, it poses challenges in real-world PCB implementation. Specifically, the junction where the shunt stub connects to the main transmission line also directly connects to both ports. Physically, this requires three microstrip lines to converge at a single point, which complicates routing and may introduce layout-induced discontinuities or impedance mismatches.

Additionally, in practice, the load (represented as Port 2) would often be a physical component or antenna that cannot be seamlessly integrated into the end of a simulated transmission line segment. To address these issues, a short feedline segment is typically introduced to provide routing space and mechanical access to the load. This modification ensures a cleaner, manufacturable layout while preserving the designed impedance characteristics.