Strategic Trading, Liquidity, and Information Acquisition

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We study endogenous liquidity trading in a market with long-lived asymmetric information. We distinguish between *public* information, *tractable* information that can be acquired, and *intractable* information that cannot be acquired. Besides information asymmetry and noise, the adverse-selection spread depends on the diffusion of intractable information and on the interest rate. With endogenous liquidity trading, efficiency is lower than that implied by noise-trading models. Liquidity traders benefit from the information released through the insider's trades in spite of their monetary losses. We study factors that affect the insider's information acquisition decision, including the amount of intractable information, observability, and information acquisition costs.

Providing liquidity to investors who want to exchange financial assets for cash or vice versa without possessing superior information is a key role of financial markets [Amihud and Mendelson (1986), Grossman and Miller (1988)]. The choice problem faced by such liquidity traders and its effects on market performance are often overlooked in dynamic models of asymmetric information in financial markets. In this article, liquidity traders endogenously determine the quantities they trade, taking information asymmetry and asset riskiness into account. We study the effect of endogenous liquidity trading on market performance and on the informed trader's incentives to acquire information and to release it to the market.

We propose a tractable model of endogenous liquidity trading and derive its equilibrium in closed form. The results enable us to make qualitative and quantitative predictions on the effects of different types of information on liquidity, efficiency, and welfare. We also formulate testable implications regarding the determinants of the spread, the nature of the price-adjustment process, and the effects of changes in the investor

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base and in accounting rules on market liquidity, efficiency, information acquisition, and research.

Our starting point is a multiperiod discrete-time model based on Kyle (1985), with an informed trader who obtains a noisy signal about the value of a risky asset. Information about the security's value also diffuses to the market as an independent process. We model liquidity (uninformed) traders as risk-averse agents with idiosyncratic liquidity preferences. Each liquidity trader chooses her order size, and competitive market makers price the security at its expected value given all publicly available information. In equilibrium, the risk-neutral informed trader maximizes his expected profit and risk-averse liquidity traders maximize their expected utilities.

We find that endogenous liquidity trading leads to qualitatively different equilibrium characteristics compared to dynamic models with pure noise trading. Our analysis shows that models with exogenous liquidity trading are likely to overestimate the speed of information dissemination. We also find that both liquidity and efficiency are increasing functions of agents' discount rates and decreasing functions of the liquidity traders' risk aversion.

We distinguish between *intractable*, *tractable*, and *public* information. Intractable information cannot be acquired: its release is entirely exogenous, and it is unavailable to the informed trader, to the market makers, and to the liquidity traders. In contrast, tractable information can be acquired by the informed trader, creating informational asymmetry. We find that intractable information increases the adverse selection spread, although it does not involve informational asymmetry. Further, intractable information increases the sensitivity of the spread to asymmetric information and to the liquidity trading interest.

Making information asymmetry endogenous reduces market efficiency. When the informed trader chooses how much information to acquire, he takes into account the effect of his choice on liquidity traders' behavior, which creates an incentive to limit the information asymmetry. We find that even at zero information acquisition cost, he may still choose not to acquire all the tractable information he could. This is in contrast to traditional noise trading models, where he would acquire all available information when it is free. Further, there is an interaction between the amount of intractable information and the informed trader's information acquisition strategy: other things being equal, he will acquire less tractable information when there is more intractable information.

We show that insider trading can increase the welfare of liquidity traders by reducing the risk they face. Thus the informed trader provides a valuable service, which is paid for by his trading profits. However, he may have an incentive to acquire more information than the amount that maximizes liquidity traders' welfare. These results can contribute to the debate regarding the costs and benefits of insider trading [cf. Manne (1966), Glosten (1989), Fishman and Hagerty (1992)] and provide a framework for structuring and quantifying the trade-off between them.

When the informed trader is allowed to release some of his private information to the market, his strategy is characterized by two regions in the parameter space: For small amounts of intractable information, he chooses to acquire all available information and release most of it. In this case, voluntary information disclosure may improve market efficiency. On the other hand, when the amount of intractable information is large, the informed trader acquires a fraction of the tractable information without releasing any of it. Hence our results suggest that intractable information reduces efficiency by curtailing both the acquisition and release of information.

A number of studies consider information acquisition and release in financial markets [cf. Grossman and Stiglitz (1980), Kim and Verrecchia (1991), Demski and Feltham (1994), Verrecchia (2001), and references therein]. Huddart, Hughes, and Levine (2001) study the effects of information release when the insider is required to disclose the quantity he had traded at the end of each trading round. Under this requirement, the insider uses a "dissimulation" strategy that adds random noise to his trades, so the market makers cannot perfectly infer his information. They find that this form of disclosure accelerates price discovery and lowers insider profits compared to Kyle (1985). Unlike Huddart, Hughes, and Levine (2001), where noise trading is exogenous, in our model, information release actually makes the informed trader better off, as it induces liquidity traders to increase their trades.

The information acquisition decision depends on whether its outcome can be observed by the market. When the informed trader's investment is unobserved, if the information acquisition costs are low, he acquires more information compared to the case where his investment is observed. In contrast, with high information acquisition costs, he may choose to acquire less information than in the observable case. In all cases, however, his profits decline as a result of unobservability. Thus the informed trader has an incentive to hire an independent auditor to monitor his information acquisition investment and report it to the market. We show that hiring an outside auditor benefits not only the informed trader, but the liquidity traders as well.

A number of studies, building on Kyle (1985), examine the effect of long-lived private information with inelastic noise trading [cf. Holden and Subrahmanyam (1992), Foster and Viswanathan (1996), Back and Pedersen (1998), Back, Cao, and Willard (2000), Baruch (2002)]. Several papers examine the choices made by uninformed traders in static or short-lived private information environments [cf. Admati and Pfleiderer (1988), Foster and Viswanathan (1990)]. Foucault (1999) models reservation prices as the sum of asset value and a trader-specific component; Glosten and Milgrom (1985) use different discount factors as drivers of different valuations.

Glosten (1989) was the first to introduce liquidity traders with hedging motivations in a market with information asymmetry. Using uninformed traders who trade to hedge their initial positions, Spiegel and Subrahmanyam (1992) build on the one-period Kyle (1985) model to show that each uninformed trader trades so as to decrease the absolute value of her position in the risky security. While in both Spiegel and Subrahmanyam (1992) and our model, uninformed traders' decisions are endogenous, the trading motivations are different: our liquidity traders satisfy disparate liquidity preferences, whereas in Spiegel and Subrahmanyam (1992), they hedge their positions in the risky security.[1]

The rest of the article is organized as follows. Section 1 presents our discrete-time model and derives the equilibrium. Section 2 introduces our continuous-time model and derives its equilibrium characteristics. Section 3 extends the model to the cases where information acquisition is endogenous and disclosure of private information is allowed. Our concluding remarks are in Section 4. All proofs are in the appendix.

1. The Model

1.1 Model and notation

Our model is based on Kyle (1985). The market for a risky security operates for T periods. Trading takes place between times 0 and 1 at points t_τ, $1 \leq \tau \leq T$, where $0 = t_0 < t_1 < t_2 < \cdots < t_T = 1$. We define $\Delta t_\tau = t_\tau - t_{\tau-1}$, $1 \leq \tau \leq T$, and call the period that ends at t_τ "period τ." A dollar paid at the end of period τ is discounted by $1 + r_\tau \Delta t_\tau$.

There are three types of market participants: liquidity traders, an informed trader, and market makers. In our model, liquidity traders *decide* what quantities they will buy or sell by trading off their desire to effect an execution against their expected losses to the informed.

1.1.1 Liquidity traders. Bagehot (1971) viewed the market maker as a conduit between liquidity traders, who trade because they want to exchange securities into cash or vice versa without delay, and insiders, who possess superior information which they can monetize through trading.[2] Formally, Kyle (1985) studied the equilibrium among an informed

[1] This difference in the motivation for liquidity trading has a major impact on the results (see the concluding remarks). Other studies with hedging-motivated trading include Hillion (1994), Spiegel and Subramanyam (1995), Vayanos (1999), and Massoud and Bernhardt (1999).

[2] A third type is "transactors acting on information which they believe has not yet been fully discounted in the market price but which in fact has" [Bagehot (1971, p. 13)].

trader, noise traders, and competitive market makers. Kyle modeled noise trading as being exogenous, that is, the quantity traded by the liquidity traders is not sensitive to their expected losses to the insider.

Liquidity traders are willing to pay a fee (buy each share at a premium or sell each share at a discount, the difference between which is the bid-ask spread) in return for immediacy of execution. A liquidity trader's willingness to pay for immediacy reflects the urgency of her willingness to trade, which in turn is determined by idiosyncratic, private factors. Examples include cash needs in response to an emergency or an unexpected opportunity, the desire to balance a suboptimal portfolio, the desire to minimize the tracking error of an investment that was designed to meet a specific target, or the need to reduce a short position. Such liquidity trades are characterized by an idiosyncratic opportunity cost of failing to trade at a given point in time, which reflects the liquidity trader's particular circumstances as opposed to information about the future value of the security.

Similar to earlier market microstructure models that focused on liquidity trading [e.g., Garman (1976), Cohen et al. (1978, 1981), Amihud and Mendelson (1980), Ho and Stoll (1981), Mendelson (1982, 1985, 1987)], we assume that liquidity traders are heterogeneous in their willingness to pay for immediacy. Thus, although all liquidity traders have the same expectations regarding the security's final value, each has a different willingness to pay for immediate execution, which reflects her private liquidity preference. In Garman's (1976) classical model and related work, heterogeneous liquidity preferences create a downward-sloping demand curve and an upward-sloping supply curve that reflect the arrivals of buyers and sellers to the market, respectively. Thus if the market's mean valuation of the asset is p, then liquidity trader i's private valuation of the asset is $p+\tilde{u}_i$, where $E[\tilde{u}]=0$.

Our model handles both liquidity trading and information asymmetries, combining the early market-microstructure approach with Kyle's. In our model, there is a continuum of liquidity traders. Each liquidity trader has a mean-variance utility function $U(w)$, where w is the wealth generated by her trade.[3] For a liquidity trader arriving at time t with immediacy value u, the utility from buying z shares is given by $E[W(v,p,u,z)]-a_\tau \mathrm{var}[W(v,p,u,z)]$, where $W(v,p,u,z)=z(v\cdot\prod_{k=\tau+1}^{T}(1+r_k\Delta t_k)+u-p(t))$. A negative value for z means that the trader actually sells z shares of the asset.

1.1.2 The informed trader, market makers, and sequencing of events. A risk-neutral informed trader obtains a signal ξ about the

[3] This is equivalent to assuming that liquidity traders have a CARA (i.e., negative exponential) utility function with preferences specified as above, when combined with the normal distribution of the asset's value.

liquidation value, v, of the asset before trading starts. We assume that $v=\xi+\epsilon+\eta$, where ξ, ϵ and η are ex ante independently normally distributed with expected values p_0, 0, and 0, respectively. The variance of ξ is Σ_0 and $\text{var}[\epsilon]+\text{var}[\eta]=\Omega$. Moreover, $\eta=\sum_{\tau=1}^{T-1}\Delta\eta_\tau$, where $\Delta\eta_\tau$ are independently normally distributed with mean 0 and variance $\omega_\tau\Delta t_\tau$; η represents the publicly released information independent of the informed trader's private information (this information may include, e.g., macroeconomic data or news concerning relevant industry segments). After trading takes place at the end of period τ, $\Delta\eta_\tau$ is revealed to the market. We define $\eta_\tau=\sum_{k=1}^{\tau-1}\Delta\eta_k$.

At each period τ, the informed trader submits an order to buy or sell a quantity of the security without observing the liquidity traders' orders. Competitive market makers, who cannot distinguish between liquidity-motivated and informed order flow, price the asset at its expected present value given the information available to the market at that point. The security's price at the beginning of the first period is p_0. We denote the end-of-period τ trading price of the security (before $\Delta\eta_\tau$ is revealed) by p_τ. At the end of period T (i.e., at $t=1$), ξ is publicly disclosed. On the other hand, ϵ can be fully disclosed, partly disclosed, or not disclosed at all.

The market-makers' pricing function at τ is $p_\tau=P_\tau(\Delta V_\tau, p_{\tau-1})$, where ΔV_τ is the observed net order flow. Let $x_\tau(\xi, p_{\tau-1})$ denote the informed trader's order size function in period τ and $z_{j\tau}=Z_{j\tau}(u_{j\tau})$ denote the order size function of liquidity trader j at τ. Let X_τ be the security position of the informed trader, accumulated from $t=0$ through the end of period τ.

1.1.3 Definition of an equilibrium. Define $\mathcal{F}_\tau=\sigma(x_1+\sum_j z_{js}, p_1,\ldots, x_\tau+\sum_j z_{js}, p_\tau, \eta_\tau)$, that is, $\mathcal{F}_\tau$ is the public information set at the end of period τ. The discounted future profit as of the beginning of period τ for the informed trader by following the strategy $\{x_\tau\}$, $\tau\geq 0$ is given by

$$\pi_\tau^{inf}(x)=\sum_{k=\tau}^{T}x_k\left(v\prod_{l=\tau}^{T}(1+r_l\Delta t_l)-p_\tau\prod_{l=\tau}^{k}(1+r_l\Delta t_l)\right). \tag{1}$$

An *equilibrium* satisfies the following requirements:

(*a*) *In each period* τ, *each liquidity trader maximizes her utility given available information:* For a liquidity trader arriving at period τ, $1\leq\tau\leq T$ and indexed by j:

$$\begin{aligned}U(W_{j\tau}^{liq}\mid u_{j\tau},\mathcal{F}_\tau)&=U\left(Z_{j\tau}(u_{j\tau})(v\prod_{l=\tau}^{T}(1+r_l\Delta t_l)+u_{j\tau}-p_\tau)\,\middle|\,u_{j\tau},\mathcal{F}_\tau\right)\\&\geq U\left(y\left(v\prod_{l=\tau}^{T}(1+r_l\Delta t_l)+u_{j\tau}-p_\tau\right)\,\middle|\,u_{j\tau},\mathcal{F}_\tau\right),\\&\qquad\text{for all } y\in\mathbb{R}.\end{aligned} \tag{2}$$

(*b*) *The informed trader maximizes his expected profits:*
For $1 \leq \tau \leq T$:

$$E[\pi_\tau^{inf}(x) \,|\, \xi, \mathcal{F}_\tau] \;\geq\; E[\pi_\tau^{inf}(y) \,|\, \xi, \mathcal{F}_\tau], \quad \text{for all adapted } \{y_\tau\} \in \mathbb{R}^T. \tag{3}$$

(*c*) The market makers set prices efficiently, that is, the market pricing function satisfies:

$$P_\tau(\Delta V_\tau, p_{\tau-1}) = E\left[v \prod_{k=\tau}^{T} (1 + r_k \Delta t_k) \,|\, \mathcal{F}_\tau \right]. \tag{4}$$

Define $\hat{p}_\tau = E[\xi_\tau \,|\, \mathcal{F}_\tau]$, where $\xi_\tau = \xi \cdot \prod_{k=\tau+1}^{T}(1 + r_k \Delta t_k)$. Analogous to Kyle (1985) and subsequent studies, we concentrate on equilibria in linear strategies. That is, the quantities placed by the informed trader satisfy

$$x_\tau^i = \beta_\tau^i (\xi - \hat{p}_{\tau-1}) \Delta t_\tau + \phi_\tau (E[\eta_T \,|\, \mathcal{F}_\tau] - (p_{\tau-1} - \hat{p}_{\tau-1})),$$
$$\text{for some } \{\beta_\tau^i\} \in \mathbb{R}^n, \quad 1 \leq \tau \leq T. \tag{5}$$

The market-makers' prices satisfy

$$p_\tau = p_{\tau-1}(1 + r_\tau \Delta t_\tau) + \lambda_\tau \left(\sum_{i=1}^{n} x_\tau^i + \sum_j z_{j\tau} \right) + \lambda_{\eta\tau}(\eta_\tau - \eta_{\tau-1})$$
$$\text{for some } \{\lambda_\tau\} \in \mathbb{R}^T. \tag{6}$$

The liquidity traders' order quantities satisfy $z_{j\tau} = \gamma_{j\tau} u_{j\tau}$, for some $\{\gamma_{j\tau}\}$ $1 \leq \tau \leq T$. A *linear equilibrium* is an equilibrium where all participants play linear strategies. A *symmetric equilibrium* is an equilibrium in which all the liquidity traders arriving in the same period play the same function $(Z(u, p_0))$ as their respective strategies. Note that each liquidity trader has her own immediacy premium $u_{j\tau}$, so the liquidity traders end up buying or selling different quantities, although they play the same strategy in a symmetric equilibrium. We define the market uncertainty about the informed trader's signal at the end of period τ as $\sum_\tau = \mathrm{var}[\xi_\tau \,|\, \mathcal{F}_\tau]$. Also define $\Omega_\tau = (\Omega - \sum_{k=1}^{\tau-1} \omega_k \Delta t_k) \cdot \prod_{k=\tau}^{T}(1 + r_k \Delta t_k)^{-2}$.

1.2 Equilibrium outcome: discrete-time model

Proposition 1. *Assume that*

$$\Sigma_0 < \frac{\sigma_T^2 m_T \Delta t_T}{\prod_{\tau=1}^{T}(1 + r_\tau \Delta t_\tau)^2}. \tag{7}$$

There exists a unique symmetric linear equilibrium in which $\{\pi_\tau\}$ *satisfies*

$$E[\pi_\tau \,|\, p_1, \ldots, p_{\tau-1}, \xi] = \alpha_{\tau-1}(\xi_{\tau-1} - p_{\tau-1})^2 + \delta_{\tau-1} \tag{8}$$

for some $\{\alpha_\tau\}$ *and* $\{\delta_\tau\}$.

Given Σ_0, for $1 \le \tau \le T$, β_τ, γ_τ, λ_τ, $\lambda_{\eta\tau}$, α_τ, δ_τ, and Σ_τ are the solutions to the difference equation system:

$$\alpha_{\tau-1} = \frac{(1+r_\tau \Delta t_\tau)}{4\lambda_\tau(1-\alpha_\tau\lambda_\tau)} \tag{9}$$

$$\delta_{\tau-1} = (1+r_\tau\Delta t_\tau)^{-1}(\alpha_\tau\lambda_\tau^2\gamma_\tau^2 m_\tau \sigma_\tau^2 \Delta t_\tau + \delta_\tau) \tag{10}$$

$$\beta_\tau \Delta t_\tau = \frac{(1+r_\tau\Delta t_\tau)(1-2\alpha_\tau\lambda_\tau)}{2\lambda_\tau(1-\alpha_\tau\lambda_\tau)} \tag{11}$$

$$\gamma_\tau = (2\lambda_\tau + a_\tau\Omega_\tau + a_\tau\Sigma_\tau((1+r_\tau\Delta t_\tau) - \lambda_\tau\beta_\tau\Delta t_\tau)^2 \tag{12}$$

$$+ a_\tau\lambda_\tau^2 m_\tau \gamma_\tau^2 \sigma_\tau^2 \Delta t_\tau)^{-1} \tag{13}$$

$$\lambda_\tau = \frac{\beta_\tau\Sigma_\tau}{m_\tau\gamma_\tau^2\sigma_\tau^2(1+r_\tau\Delta t_\tau)} \tag{14}$$

$$\lambda_{\eta\tau} = \prod_{k=\tau}^{T}(1+r_k\Delta t_k) \tag{15}$$

$$\Sigma_\tau = ((1+r_\tau\Delta t_\tau) - \beta_\tau\lambda_\tau\Delta t_\tau)(1+r_\tau\Delta t_\tau)\Sigma_{\tau-1} \tag{16}$$

with second-order conditions $\lambda_\tau(1-\alpha_\tau\lambda_\tau) > 0$ and boundary conditions $\alpha_T = \delta_T = 0$. In equilibrium, ϕ_τ is irrelevant.

The irrelevance of ϕ_τ is a direct result of semistrong form efficiency. Since, in equilibrium, the price reflects all publicly released information, the price deviation resulting from a public announcement is always zero. Since the informed trader cannot act on this information, the associated coefficient is irrelevant.

The equilibrium outcome for different clearing frequencies, as well as its convergence to the continuous-time equilibrium, which we derive next, is depicted in Figure 1. Equation (7) ensures there is sufficient liquidity trading interest relative to the information asymmetry so an equilibrium exists. Clearly this assumption is satisfied for any given parameter set if the number of liquidity traders is large enough.

2. Continuous-Time Model

The dynamics of trading and information dissemination are best studied in a continuous-time model, where traders' actions are not constrained by one or more discrete trading epochs. Further, the continuous-time analysis allows us to obtain key closed-form results.

2.1 Model and equilibrium outcome

Here the market operates in continuous time over the interval [0, 1]. Again, the security's true value v is realized at $t = 1$. The ex ante distribution of v is normal with mean p_0 and variance $\hat{\Lambda} = \Sigma_0 + \Omega$. The instantaneous

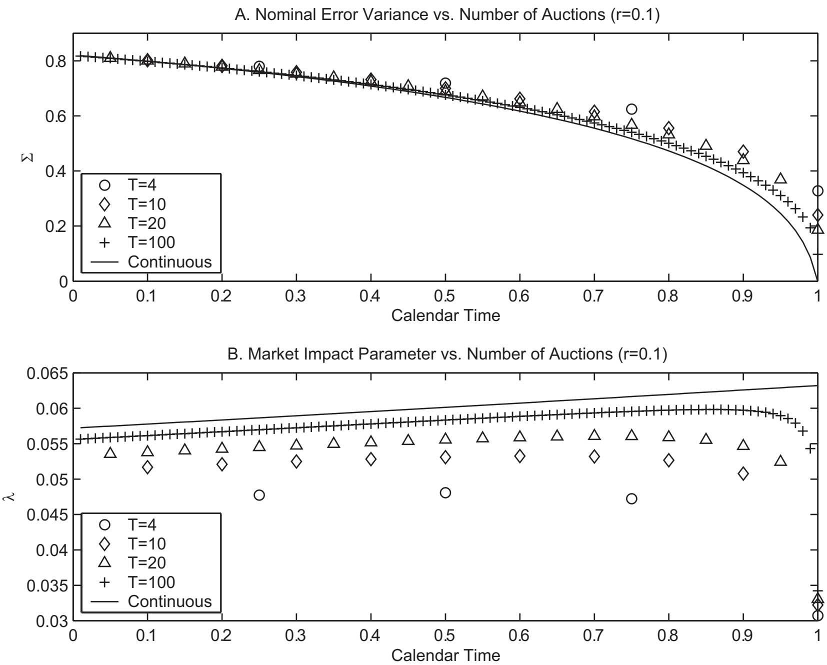

Figure 1
Convergence of the nominal error variance of the informed trader's signal Σ_τ (panel A) and the liquidity parameter λ_τ (panel B) to the continuous-time equilibrium as the number of trades (auctions) increases. The time intervals between trades are equal, with the number of trades increasing from 4 to 100. The parameter values are $r_\tau = 0.1$, $m_\tau \sigma_\tau^2 = 100$ (constants for all τ), $\Omega = 0$, and $a = 1$.

discount rate for all agents at time t is r_t, where $\int_0^t r_s ds$ exists for all t in $[0, 1]$. Liquidity traders enter the market at rate μ_t at time $t \in [0, 1]$, with a finite total mass $\int_0^1 \mu_t \, dt$. As in the discrete-time model, each liquidity trader places a market order specifying the quantity to be bought or sold and departs after her order is executed. A liquidity trader with immediacy value u arriving at t values the asset, in time t dollars, at $E[v \cdot e^{-\int_t^1 r_s ds}] + u$, where u is normally distributed with mean 0 and variance σ_t^2 and is independent across time and liquidity traders. Each liquidity trader has a mean-variance utility function $U(w)$, where w is the wealth generated by her trade. For a liquidity trader with immediacy value u who arrives at t, the utility from buying z shares is given by $E[W(v,p,u,z)] - a_t \mathrm{var}[W(v,p,u,z)]$, where $W(v,p,u,z) = z(PV_{t,1}(v) + u - p(t))$ and for any payment q at time $s > t$, $PV_{t,s}(q) = q \cdot e^{-\int_t^s r_u du}$. We impose two regularity conditions on the liquidity trader arrival process: $\int_0^t \mu_s \sigma_s^2 \, ds$ exists for all $t \in [0, 1]$; and $\mu_t {\sigma_t}^2$ is strictly positive a.e. on $[0, 1]$.[4]

[4] The first condition prevents (potential) liquidity trading from being unstable, and the second prevents degeneracy.

We assume that $a:[0,1]\rightarrow\mathbb{R}^+$ is strictly positive and bounded and that r, ω, a, and $\mu\sigma^2$ are continuous at $t=0$.[5]

The informed trader obtains a signal ξ about the liquidation value, v, of the asset before trading starts, and ϵ and η are defined analogously to the discrete-time model. The market makers observe the total net order flow at each trading point and set the market price accordingly. Public information about η diffuses to the market with rate ω_t at time t (i.e., as a Brownian motion with instantaneous variance ω_t). At time t, the accumulated released signal about v is $\eta(t)=\int_0^t\sqrt{\omega_s}\,dW_\eta(s)$, where $W_\eta(t)$ is a standard Brownian motion, independent of ξ, ϵ, and the liquidity traders' immediacy values.

At each trading instant $t\in[0,1]$, the informed trader submits an order to buy or sell the security without observing the liquidity traders' orders. Market prices are set by competitive market makers, who cannot distinguish between informed and liquidity traders and price the asset (semi strongly) efficiently. The rest of the definitions are analogous to their discrete-time counterparts (see Appendix A). The following proposition provides the equilibrium outcome of this model.

Proposition 2. *In the continuous-time market setting described above, there exists a unique linear equilibrium if and only if*

$$\int_0^1 \mu_t\sigma_t^2 e^{2\int_t^1 r_s ds}\,dt \geq 4\Sigma_0. \tag{17}$$

If an equilibrium exists, $\breve{\Sigma}(t)$ *solves the differential equation*

$$\frac{d}{dt}\breve{\Sigma}(t)=2r_t\breve{\Sigma}(t)-\frac{\lambda_0^2 e^{2\int_0^t r_s ds}\mu_t\sigma_t^2}{(2\lambda_0 e^{\int_0^t r_s ds}+a_t(\breve{\Sigma}(t)+\Omega_t))^2} \tag{18}$$

with $\breve{\Sigma}(0)=\Sigma_0 e^{-2\int_0^1 r_s ds}$, *where* λ_0 *is a constant chosen to satisfy* $\breve{\Sigma}(1)=0$. *The equilibrium strategies are given by*

$$\lambda(t)=\lambda_0 e^{\int_0^t r_s ds}, \tag{19}$$

$$\gamma(t)=(2\lambda_0 e^{\int_0^t r_s ds}+a_t(\breve{\Sigma}(t)+\Omega_t))^{-1}, \tag{20}$$

and

$$\beta(t)=\frac{\lambda_0 e^{\int_0^t r_s ds}\mu_t\sigma_t^2}{\breve{\Sigma}(t)(2\lambda_0 e^{\int_0^t r_s ds}+a_t(\breve{\Sigma}(t)+\Omega_t))^2}. \tag{21}$$

In equilibrium, $\lambda_\eta(t)=e^{-\int_t^1 r_s ds}$ *and* ϕ *is irrelevant.*

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[5] In order to align the continuous-time model parameter-wise with the discrete-time model perfectly, for given μ, σ^2, a, and r, functions on [0, 1], set $m_\tau=\int_{t_{\tau-1}}^{T_\tau}\mu_s ds$ and $\sigma_\tau^2=\frac{\int_{t_{\tau-1}}^{t_\tau}\mu_s\sigma_s^2 ds}{\int_{t_{\tau-1}}^{t_\tau}\mu_s ds}$ besides letting $\{a_\tau\}\rightarrow a$ and $\{r_\tau\}\rightarrow r$ as $\Delta t_\tau\rightarrow 0$.

Figure 1 demonstrates the convergence of the sequence of discrete-time equilibria to the continuous-time equilibrium.

With no discounting and constant liquidity trader risk aversion, we have the equilibrium outcome in closed form. We define $M=\int_0^1 \mu_s\sigma_s^2 ds$ as the total liquidity trading interest, and $M(t)=\int_0^t \mu_s\sigma_s^2 ds$. When μ and σ^2 are constant, $M(t)=\mu\sigma^2 t$.

Corollary 1. *With $r_t=\omega_t=0$ and $a_t=a$ for a constant $a>0$ for all $t\in[0,1]$, a unique equilibrium exists if and only if $\int_0^1 \mu_t\sigma_t^2 dt > 4\Sigma_0$. If an equilibrium exists, the market liquidity parameter is constant and given by*

$$\lambda_0=a\Sigma_0\frac{3(2\hat{\Lambda}-\Sigma_0)+\Sigma_0(3(M((\Sigma_0^2+3(\hat{\Lambda}-\Sigma_0)\hat{\Lambda})/\Sigma_0^3)-1))^{\frac{1}{2}}}{3(M-4\Sigma_0)}. \tag{22}$$

The equilibrium error variance is

$$\Sigma(t)=\frac{1}{a}((2\lambda_0+a\hat{\Lambda})^3-3a\lambda_0^2M(t))^{\frac{1}{3}}-(2\lambda_0+a(\hat{\Lambda}-\Sigma_0)). \tag{23}$$

The equilibrium strategies for the liquidity traders and the informed trader are

$$\gamma(t)=((2\lambda_0+a\hat{\Lambda})^3-3a\lambda_0^2M(t))^{-\frac{1}{3}}, \tag{24}$$

and

$$\beta(t)=\frac{\mu_t\sigma_t^2\lambda_0}{\Sigma(t)(2\lambda_0+a\hat{\Lambda})^2}. \tag{25}$$

2.2 Equilibrium characteristics

2.2.1 Determinants of market liquidity. To study the determinants of market liquidity, we define a partial order among functions on [0, 1], which we use to identify increases or decreases in λ as parameters change in the function space on [0, 1].[6] For any f: $[0, 1]\to\mathbb{R}$, we say that *f increases on positive measure* if it is replaced by $\hat{f}$, where for a $Z\subseteq[0, 1]$ on positive Lebesgue measure, $\hat{f}(t)>f(t)$ for $t\in Z$ and $\hat{f}(t)=f(t)$ otherwise. The following proposition states the effects of parameters on market liquidity.

Proposition 3. *In equilibrium,*
(*i*) *If either one of r, ω, μ, or σ^2 increases on positive measure, $\lambda(t)$ decreases for all $t\in[0,1]$;*
(*ii*) *If a increases on positive measure, so does $\lambda(t)$ for all $t\in[0, 1]$;*
(*iii*) *If Σ_0 increases, so do $\lambda(t)$ and $\Sigma(t)$ for all $t\in[0, 1]$;*
(*iv*) *If Ω increases, so does $\lambda(t)$ for all $t\in[0, 1]$.*

[6] We focus on changes that preserve the equilibrium existence conditions.

The first result suggests a market-wide relationship between interest rates and illiquidity. Intuitively an increase in the discount rate decreases the importance of future risk for liquidity traders. As the discount rate increases, liquidity trading in the early periods intensifies, resulting in higher market depth. The empirical implication is that liquidity changes systematically with interest rates: the higher the interest rate, the higher the liquidity [measured, e.g., by the market impact cost; cf. Amihud, Mendelson, and Lauterbach (1997)]. One can also test the interaction of the interest rate and the longevity of private information suggested by our model. Other things being equal, lower interest rates are equivalent to a smaller opportunity window for trading on private information; differences in the due dates or frequencies of financial reports can be used as proxies. For instance, prior to 1970 Form 10-K had to be filed within 120 days after a company's fiscal year ended. This was shortened to 45 days. Foreign companies are required to file annual reports on Form 20-F within 6 months of their fiscal year end. Also prior to 1970, companies were not required to file full financial disclosures quarterly; disclosure requirements were made in 1970 and expanded in 1981. Differences in these requirements can be correlated with the longevity of private information and can be used to test how interest rates affect liquidity.

Consistent with Kyle (1985), as the potential liquidity trading interest increases, the market becomes more liquid, so increasing μ or σ^2 decreases λ, and as the information asymmetry increases, the market becomes less liquid. Further, if ω increases, more information about the security's value is autonomously revealed to the market, reducing the security's riskiness and increasing liquidity trading. Hence, if ω increases, λ decreases.

Part (*iv*) of Proposition 3 and Corollary 1 show that other things being equal, an increase in the total variance reduces liquidity and increases the adverse selection component of the spread. Thus the adverse-selection spread increases in both the informational asymmetry and the overall uncertainty about the security's value. Empirically, this implies that the adverse selection spread should be an increasing function of the security's total risk. This result is reminiscent of Ho and Stoll (1981), who found that the *inventory* component of the spread increases with total risk. An interesting empirical test would examine a cross section of securities, separate out the adverse selection component of the spread [cf. Huang and Stoll (1997), Easley, Hvidkjaer, and O'Hara (2002)] and test, using appropriate controls, the relationship between the adverse-selection spread and total risk.[7]

[7] Huang and Stoll (1997) separate out the adverse selection component of the bid-ask spread using a time-series model. Easley, Hvidkjaer, and O'Hara (2002) estimate the probability of information-based trading using maximum-likelihood estimation based on a theoretical microstructure model.

Our model implies that the adverse selection spread should increase with the uncertainty about events that are totally unpredictable and involve no inside information. Thus the time-series behavior of the adverse selection spread should be correlated with measures of macroeconomic and market-wide uncertainty. For example, the uncertainty about exchange rates that have an effect on the security's value should be positively correlated with the adverse-selection spread. Additional empirical implications follow from the cross-partial derivatives of the market impact coefficient (λ_0) given in Corollary 1. We have the following result.

Corollary 2. *With* $r_t = \omega_t = 0$, *and* $a_t = a$ *for all* $t \in [0, 1]$,
(*i*) $\frac{\partial^2 \lambda_0}{\partial M \partial \hat{\Lambda}} < 0$, (*ii*) $\frac{\partial^2 \lambda_0}{\partial \Sigma_0 \partial \hat{\Lambda}} > 0$, *and* (*iii*) $\frac{\partial^2 \lambda_0}{\partial M \partial \Sigma_0} < 0$.

While exogenous noise trading models imply that noise trading reduces the spread, part (*i*) of Corollary 2 shows that the slope of the spread on the liquidity trading interest M should be larger in absolute value for more risky securities. Amihud, Mendelson, and Uno (1999) measured M via the proportion of individual shareholders, as "informationless trades are typically associated with small, individual shareholders who buy or sell small quantities of the stock," and found that this measure was positively correlated with liquidity. Our result means that the effect of changing the investor base on liquidity (i.e., the slope coefficient) should be larger for riskier securities. Similar cross-effects can be tested using the size distribution of trades to quantify the amount of liquidity-motivated trading.

Part (*ii*) of the corollary says that larger total uncertainty about the security or the environment increases the negative effect of information asymmetry on market liquidity. This means, for example, that an upward shift in the uncertainty about macroeconomic variables or economic outlook will increase the coefficient of the adverse selection spread on measures of information asymmetry. Finally, part (*iii*) shows that the sign of the cross-partial of the market impact parameter with respect to information asymmetry and liquidity trading interest is negative, as in Kyle (1985).

2.2.2 Market efficiency. In Kyle (1985), changes in the liquidity trading interest have no effect on market efficiency, and in equilibrium, market uncertainty decreases linearly: $\Sigma(t) = \Sigma_0(1 - t)$. One expects an increase in liquidity trading volume to affect the diffusion of private information to the market. In fact, increased liquidity trading has two opposite effects. On the one hand, it induces more aggressive informed trading, resulting in faster information dissemination and a more efficient market. On the other hand, liquidity trading adds noise, which makes it more difficult to infer the security's value from the market price. In Kyle (1985), the two

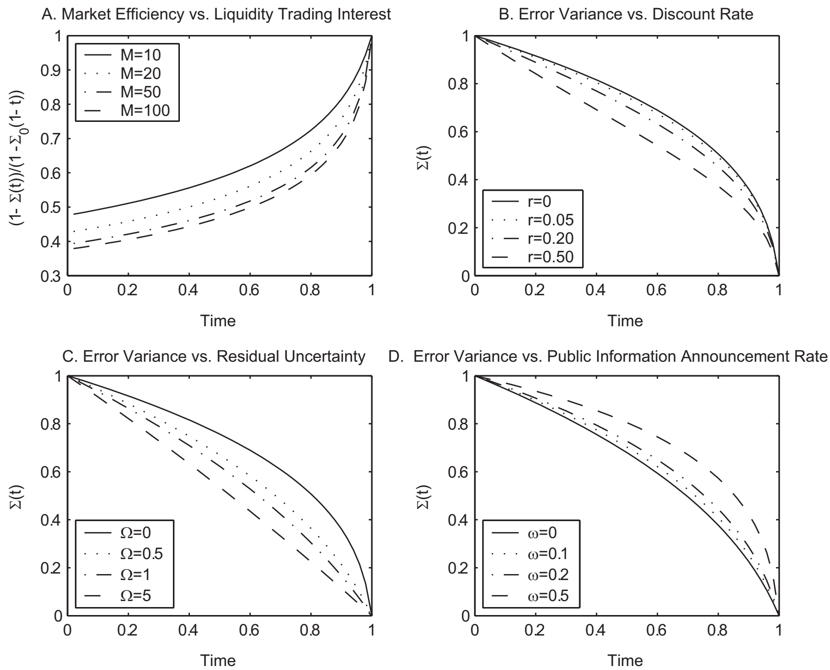

Figure 2
Panel A plots the ratio of (*i*) reduction in the conditional variance of the asset's value under our model to (*ii*) the corresponding variance under Kyle's model, for various constant liquidity trading interest levels ($\mu\sigma^2$). This ratio is a measure of relative market efficiency. $M = \mu\sigma^2$ and the parameter values are $r = \omega = \Omega = 0$. Panel B shows the conditional error variance of the informed trader's signal ($\Sigma(t)$) for various constant discount rate (r) levels. The parameter values are $\Omega = \omega = 0$, $\mu\sigma^2 = 50$. Panel C shows the error variance of the informed trader's signal for various levels of residual uncertainty (Ω). The parameter values are $r = \omega = 0$, $\mu\sigma^2 = 50$. Panel D shows the error variance of the informed trader's signal for various levels of public information announcement rate (ω). The parameter values are $r = 0$, $\Omega = 0.5$, $\mu\sigma^2 = 50$. For all panels, $\Sigma_0 = 1$ and $a = 1$.

effects are perfectly offsetting and efficiency is independent of market parameters.

Figure 2 presents the effects of several parameters on efficiency for our model. Figure 2A compares the percentage of private information incorporated in the market in our model, $(1 - \Sigma(t))$, to that of the Kyle model, $(1 - \Sigma(1 - t))$, for various levels of $\mu\sigma^2$. As $\mu\sigma^2$ increases, price efficiency decreases for all $t \in [0, 1]$. Thus increased liquidity trading interest reduces informational efficiency for all t—a major difference between our model and Kyle's.

The effect of increased r on market efficiency is illustrated in Figure 2B. As r increases, information dissemination is accelerated, the market becomes more efficient, and more liquidity trading takes place early on. Figure 2C demonstrates the effect on efficiency of changing the residual uncertainty (Ω). As Ω increases, so does the overall riskiness of the security, which reduces the informed trader's ability to hide his orders and results in faster information dissemination to the market. Increased residual uncertainty also makes liquidity trading less elastic. Hence, as Ω gets larger, the equilibrium approaches Kyle's (e.g., for $\Omega = 5$, $\Sigma(t)$ is almost linear).

Figure 2D shows the effect of the rate of public information release, ω, on the error variance of the signal. Faster public information release slows down information dissemination about the informed trader's signal. This is because as ω increases, the uncertainty about the liquidation value of the asset decreases through the end of the trading period, which increases liquidity trader aggressiveness at that time. This, in turn, induces the informed trader to trade less aggressively earlier and more aggressively later, holding back information from the market. Note that the first-order effect of an increase in ω is to increase the amount of liquidity trading at all times.

In the standard Kyle (1985) model, $\Sigma(t)$ declines linearly, that is, as $1 - t$. In our model, $\Sigma(t)$ declines concavely as $\sqrt[3]{(1-t)}$, resulting in slower information dissemination and a less efficient market. This difference invites an empirical test of the price adjustment process.

Under the conditions of Proposition 1, and with constant μ and σ^2, the residual market uncertainty for both the Kyle model and ours can be written as

$$\Sigma(t) = ((\psi_0)^{\zeta} - \psi_1 t)^{1/\zeta} - \psi_2, \tag{26}$$

where ψ_0, ψ_1, and ζ are constant parameters. Under the Kyle model, $\zeta = 1$, $\psi_0 = \psi_1 = \Sigma_0$, and $\psi_2 = 0$, whereas for our model, $\zeta = 3$ and ψ_0, ψ_1, and ψ_2 can be derived from Equations (22) and (23).

The speed of price adjustment has been tested for a number of different models in the literature [cf. Biais, Hillion, and Spatt (1999) and references therein]. Similar to Biais, Hillion, and Spatt, let I_t denote the information set available to the market at the end of period t. By Equation (26), $E[(p(t) - v)^2 - (((\psi_0)^{\zeta} - \psi_1 t)^{1/\zeta} - \psi_2) \mid I_{t-1}] = 0$. This means that for any instrumental variable y in I_{t-1}, we have the condition

$$E[((p(t) - v)^2 - (((\psi_0)^{\zeta} - \psi_1 t)^{1/\zeta} - \psi_2))y \mid I_{t-1}] = 0. \tag{27}$$

Thus, taking a fixed time horizon, using the closing price of the last day in the sample as a proxy for v [cf. Biais, Hillion, and Spatt (1999)] and the lagged prices as instruments, we can construct a GMM estimator for the parameters ζ, ψ_0, ψ_1, and ψ_2 and test the hypothesis $\zeta = 1$ versus the alternative $\zeta = 3$.

2.2.3 Informed trader profits.
Proposition 4.
(*i*) *The informed trader's expected ex ante profits are given by*

$$E[\pi_0^{inf}] = \frac{\Sigma_0}{\lambda_0 e^{2\int_0^1 r_t dt}}. \tag{28}$$

(*ii*) *When* $r = \omega = 0$ *and* μ, σ^2, *and a are constant, the informed trader's expected ex ante profit is given by*

$$E[\pi_0^{inf}] = \frac{3(\mu\sigma^2 - 4\Sigma_0)}{a\left(3(\Sigma_0 + 2\Omega) + \Sigma_0\sqrt{3\left(\frac{\mu\sigma^2}{\Sigma_0} - 1 + \frac{3\Omega(\Sigma_0+\Omega)\mu\sigma^2}{\Sigma_0^3}\right)}\right)}. \tag{29}$$

The informed trader's profits increase with μ and σ^2, which is intuitive. Moreover, they are decreasing in Ω, since higher residual uncertainty results in less liquidity trading and a thinner market. However, the effect of Σ_0 is nonmonotonic, which has implications on the informed trader's information acquisition decisions examined next.

3. Endogenous Information Acquisition and Market Efficiency

We now allow the informed trader to decide how much to invest in information acquisition, a decision that drives his informational advantage. We first assume that the informed trader's investment is observable by the market; in Section 3.4 we consider the case where it is unobservable.

Consider our model with no discounting and constant μ, σ^2 and a. The informed trader can choose to learn about v by privately observing an ex ante $N(0, \Sigma_0)$ distributed part of it, ξ. This observation leaves an independently distributed portion of v, say ϵ, unobserved. Let $\epsilon = \epsilon_0 + \epsilon_1$, where ϵ_0 denotes the component that he cannot acquire, and ϵ_1 is the component that he could have acquired but chose not to. That is, $v = \xi + \epsilon_1 + \epsilon_0$, where $\epsilon_1 + \xi$ represents the information the informed trader can acquire, which we call *tractable* information, and ϵ_0 is the information he cannot acquire, which we call *intractable* information. Let ξ, ϵ_1, and ϵ_0 be independently normally distributed with means p_0, 0, and 0 and variances Σ_0, $\Lambda - \Sigma_0$, and Ω_0, respectively. Thus the variance Σ_0 is endogenously determined by the informed trader and is observed by all market participants. We assume without loss of generality that at $t = 1$, the tractable information (i.e., $(\epsilon_1 + \xi) \sim N(p_0, \Lambda)$) is fully publicly revealed.[8] Observing a part of the full information with variance Σ_0 costs $c(\Sigma_0)$, where $c: \mathbb{R}^+ \to \mathbb{R}^+$ is increasing, differentiable, and convex with $c(0) = 0$.

We now have a two-stage game with the informed trader maximizing his expected net profit by first choosing how much to invest and then

[8] The intractable information ϵ_0 may be fully or partially revealed or not revealed at all.

exploiting his informational advantage through trading. Thus, the informed trader first determines Σ_0 and $\Omega = \Omega_0 + \Lambda - \Sigma_0$. Taking these as known parameter values, the market then operates as described in Section 2.

3.1 Market efficiency

To derive the equilibrium outcome of the endogenous information acquisition problem with an observed investment level, we need the following lemma:

Lemma 1. (*i*) *The expected trading profit of the informed trader for a chosen nonnegative* $\Sigma_0 \le \mu\sigma^2/4$ *is*

$$E[\pi_0^{inf}(\Sigma_0)] = \frac{3(\mu\sigma^2 - 4\Sigma)}{a\left(3(\Sigma_0 + 2(\Lambda + \Omega_0 - \Sigma_0)) + \Sigma_0\sqrt{3\left(\frac{\mu\sigma^2}{\Sigma_0} - 1 + \frac{3(\Lambda+\Omega_0)((\Lambda+\Omega_0)-\Sigma_0)\mu\sigma^2}{\Sigma_0^3}\right)}\right)}. \quad (30)$$

(*ii*) $E[\pi_0^{inf}(\Sigma_0)]$ *is a unimodal function of* Σ_0 *over the interval* $[0, \min\{\Lambda, \mu\sigma^2/4\}]$. *Let* x^* *be the unique root of the equation*

$$\tfrac{2}{3}(7x-1)^2 - x(1-8x)^2(1-x) = 0 \quad (31)$$

in the region $(\frac{1}{8}, \frac{1}{7})$ *and define* $\Lambda^* = \mu\sigma^2 x^*$.[9] *Then* $E[\pi_0^{inf}(\Sigma_0)]$ *is decreasing on a region* $\tilde{\Lambda} \le \Sigma_0 \le \min\{\Lambda, \mu\sigma^2/4\}$, *where* $\tilde{\Lambda} \ge \Lambda^*$.

Note that the net expected profit of the informed trader will be $E[\pi_0^{inf}(\Sigma_0)] - c(\Sigma_0)$. Next, we present the equilibrium outcome.

Proposition 5. *For the two-stage game described above, there exists a Nash equilibrium if and only if* $\Lambda \le \mu\sigma^2/4$. *If an equilibrium exists, it is subgame perfect. Let* Λ^* *be defined as in Lemma 1. The equilibrium* Σ_0, *denoted by* Σ_0^e, *is strictly positive, and there exist cutoff values* $\underline{\Lambda}$ *and* $\overline{\Lambda}$ *such that* $0 < \underline{\Lambda} \le \overline{\Lambda} \le \Lambda^*$ *and*

(*i*) *If* $\Lambda \le \underline{\Lambda}$, *then the informed trader acquires full information, that is* $\Sigma_0^e = \Lambda$; *and*
(*ii*) *If* $\Lambda > \overline{\Lambda}$, *then* $\Sigma_0^e < \Lambda$.

Proposition 5 says that when investment in information is observable and Λ is sufficiently large relative to $\mu\sigma^2$, then even for a low information acquisition cost, the informed trader chooses to limit the amount of

[9] The value of x^* up to first six significant digits is 0.137055.

information he acquires. This is driven by the endogeneity of liquidity traders' decisions, which reduces his profits when the informational asymmetry is large. Information acquisition costs aside, the informed trader is facing a trade-off between two effects: On the one hand, higher Σ_0 translates into superior information along with increased volatility, that translates into greater profit opportunities; but on the other hand, greater informational asymmetry reduces the volume of liquidity trading, resulting in a thinner market, which in turn reduces his expected profits. This effect is best illustrated by the following proposition, which shows that the informed trader limits the amount of information he acquires even when information acquisition is free.

Proposition 6. *Let* $c=0$. *If* $\Lambda \le \mu\sigma^2/4$, *there exists a unique subgame perfect Nash equilibrium. Let* Λ^* *be as defined in Lemma* 1. *Then*

(*i*) *If* $0 \le \Lambda + \Omega_0 \le \Lambda^*$, *then* Σ_0^e *is equal to* Λ.

(*ii*) *If* $\Lambda + \Omega_0 > \Lambda^*$, *then define* Σ_0^* *as the unique solution of the equation* $g(\Sigma_0^*(\Lambda)) - h(\Sigma_0^*(\Lambda)) = 0$ *on* $[0, min\{\Lambda + \Omega_0, \mu\sigma^2/4\}]$, *where*

$$g(\Sigma_0) = \mu\sigma^2(-3\sqrt{3}(\Lambda+\Omega_0)^2(\mu\sigma^2 - 12\Sigma_0) + \sqrt{3}\Sigma_0^2(\mu\sigma^2 + 2\Sigma_0) - 24(\Lambda+\Omega_0)\sqrt{3}\Sigma_0^2) \tag{32}$$

and

$$h(\Sigma_0) = 3(\mu\sigma^2 - 8(\Lambda+\Omega_0)) \times \Sigma_0\sqrt{3(\Lambda+\Omega_0)\mu\sigma^2((\Lambda+\Omega_0)-\Sigma_0)\Sigma_0 + \mu\sigma^2\Sigma_0^3 - \Sigma_0^4}. \tag{33}$$

There exists a Λ_c *such that* $\Sigma_0^*(\Lambda) < \Lambda$ *if and only if* $\Lambda > \Lambda_c$. *If* $\Lambda < \Lambda_c$, *then* $\Sigma_0^e = \Lambda$. *If* $\Lambda > \Lambda_c$, *then* $\Sigma_0^e = \min\{\Sigma_0^*, \Lambda\}$. Σ_0^* *is strictly increasing in* $\mu\sigma^2$ *and strictly decreasing in* Ω_0.

Proposition 6 shows that even when information acquisition is free, the informed trader may choose not to obtain all of it. This will never happen when liquidity trading is inelastic. Figure 3 presents the expected profits of the informed trader as a function of the initial error variance (determined by the amount of information he acquires). When $\Lambda/\mu\sigma^2$ is smaller than x^*, absent information acquisition costs, the informed trader wants to acquire full information. However, as $\Lambda/\mu\sigma^2$ becomes smaller, his profits are higher with less than full information.

With endogenous liquidity trading, efficiency is lower than is implied by models with exogenous noise trading. First, as shown in Section 2, information is disseminated more slowly, resulting in concave time patterns of

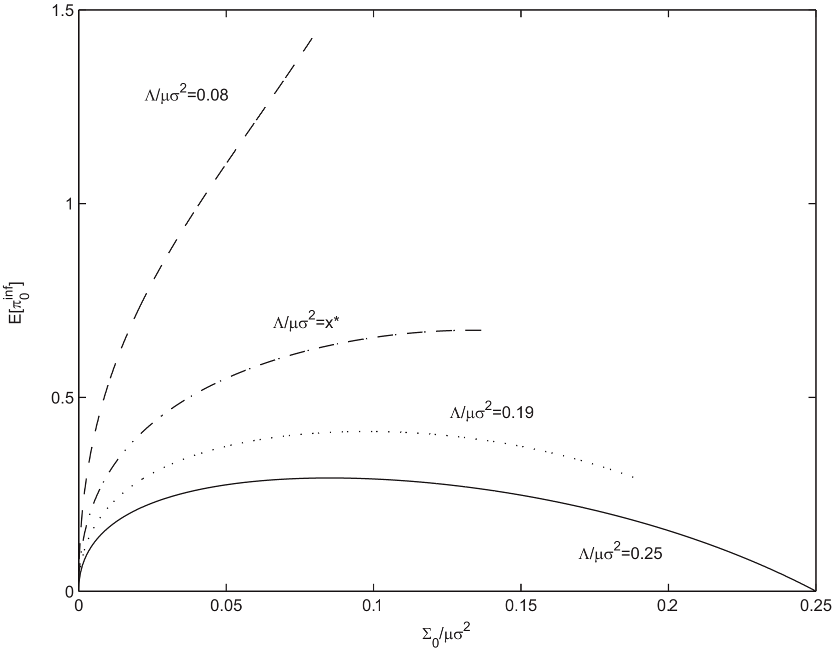

Figure 3
Informed trader's expected profit as a function of his chosen information acquisition (Σ_0) level (scaled by $\mu\sigma^2$) for various levels of (scaled) tractable information ($\Lambda/\mu\sigma^2$), $\mu\sigma^2 = a = 1$ and $\Omega_0 = 0$. x^* is as defined in Lemma 1.

the market's value uncertainty $(\Sigma(t) + (\Lambda - \Sigma_0))$. This is related empirically to the price adjustment process. Second, the informed trader may not acquire all tractable information, even at zero cost. The latter effect is captured by the *price jump* at $t = 1$, $p(1^+) - p(1^-)$, which can be interpreted as the surprise due to an announcement. The ratio of the price jump variance to the variance of the tractable information, $R_J = E[(p(1^+) - p(1^-))^2]/\Lambda$, is a measure of market inefficiency: the higher R_J, the less efficient the market price. The following corollary examines the effects of intractable information on R_J when $c = 0$ and the intractable information is not publicly released at $t = 1$.

Corollary 3.
(a) *There exists an* $m^* \geq 4\Lambda$ *such that*
 (i) *If* $\mu\sigma^2 \geq m^*$, *then* $\frac{\partial R_J}{\partial(\mu\sigma^2)} = 0$.
 (ii) *If* $4\Lambda < \mu\sigma^2 < m^*$, *then* $\frac{\partial R_J}{\partial(\mu\sigma)} < 0$.
(b) *Suppose no part of* ϵ_0 *is revealed to the market at* $t = 1$. *Then there exists an* $\Omega_0^* \geq 0$ *such that*
 (i) *If* $\Omega_0 \leq \Omega_0^*$, *then* $\frac{\partial R_J}{\partial \Omega_0} = 0$.
 (ii) *If* $\Omega_0 > \Omega_0^*$, *then* $\frac{\partial R_J}{\partial \Omega_0} > 0$.

Corollary 3 can be applied to the price reaction to earnings announcements. Part (*a*) suggests that at times when or for securities for which the uninformed investor base is larger, the price reaction following the announcement is smaller. As the investor base increases, the market becomes more liquid and the reaction to more asymmetric information declines, inducing the informed trader to acquire more information. Part (*b*) suggests that at times when or for securities for which the intractable information is larger, the price reaction following the announcement should be larger *even though* no part of the intractable information is released with the announcement. Across securities, these results can be related to the findings of Collins, Kothari, and Rayburn (1987), Ball and Kothari (1991), Zeghal (1984), Atiase (1985), Bamber (1987), Freeman (1987), and Ro (1988), that market reaction to earnings announcements are more pronounced for small firms than larger ones,[10] as firm size is likely to be positively related to the size of the uninformed investor base and negatively related to the amount of intractable information about the firm.

A more direct test of the uninformed investor base ($\mu\sigma^2$) effect can use the percentage of small investors [cf. Amihud, Mendelson, and Uno (1999)] or trade size-based measures as proxies. Then a cross-section study can relate the stock price jumps at earnings announcements to the uninformed investor base. A test of the relation between the price jump on the announcement and Ω_0 can examine its dependence on accounting regimes, using cross-country comparisons with appropriate controls. Ω_0 can also reflect macroeconomic or other market-wide uncertainties, suggesting a time-series test of the relation between earnings coefficients and market-wide uncertainty.

3.2 Information acquisition and liquidity trader surplus

The following proposition examines how the amount of information asymmetry, Σ_0^e, affects liquidity traders' welfare in equilibrium. Using this result, we derive the *optimum* amount of private information that maximizes liquidity traders' surplus.

Proposition 7.

(*i*) *The expected welfare of liquidity traders, defined as the sum of their expected utilities, is given by*

$$E[W^{liq}] = \frac{\Sigma_0(4\lambda_0 + a(2(\Lambda+\Omega_0) - \Sigma_0))}{4\lambda_0^2}. \tag{34}$$

(*ii*) *Denote the level of* Σ_0 *that maximizes Equation* (*34*) *over* $[0, \min\{\Lambda, \mu\sigma^2/4\}]$ *by* Σ_0^w. *Then* $0 < \Sigma_0^w \leq \Sigma_0^e$.

[10] For related accounting literature, see Lev and Ohlson (1982), Bernard (1989), Lev (1989), and Kothari (2001).

By Proposition 7, the amount of information asymmetry that maximizes liquidity traders' welfare, Σ_0^w, is strictly positive: *some* information asymmetry is always beneficial to them. However, to maximize his own profits, the informed trader acquires more information than is optimal for liquidity traders. Yet with low information acquisition costs, liquidity traders' welfare is still higher with the informed trader than without it. This is in contrast to pure noise trading models, where liquidity traders would always prefer zero informational asymmetry.

3.3 Voluntary information disclosure

Assume that after acquiring his private information, the informed trader can release some of it to the market before trading starts. Would he choose to do that? Because information release decreases the market's uncertainty about the security's value, it encourages liquidity trading and can increase the informed trader's profits. On the other hand, the more information the informed trader releases, the lower his informational advantage. The informed trader has to decide on the amounts of information to acquire and release so as to maximize his profits. The information release can be viewed as the release of a noisy signal about the informed trader's signal. Equivalently the informed trader's information can be interpreted as a collection of independent, verifiable bits, and he can decide how many bits to release publicly.[11]

This logic can apply, for example, to the case of a financial institution that spends resources to acquire information about a security. The institution bases its own trades on the information gathered by its research. The institution may have an incentive to release some of its information to reduce uncertainty and coax liquidity traders into trading higher quantities. Clearly it would not be in the institution's best interest to release all of the information at its disposal, since this would eliminate its entire informational advantage. Hence the institution may choose to add some form of noise or release only partial information.

We model voluntary information release using the following timeline. First, the informed trader chooses how much to spend on information acquisition and acquires private information accordingly. Second, he decides how much of the acquired information to release. Then, all other market participants update their beliefs, and finally the market operates as before. We denote the variance of released information by Σ_r ($\leq \Sigma_0$), that is, for the market, the conditional variance of ξ after the information release and before trading starts is $\Sigma_0 - \Sigma_r$. In addition, we assume $\Omega_0 > 0$.

[11] This can correspond to the informed trader performing a number of costly, noisy observations, and then deciding to release publicly the results of some of these observations.

The informed trader's problem is equivalent to choosing Σ_0 and Σ_r to maximize his expected trading profits less the cost of information acquisition:

$$\max_{0\leq\Sigma_r\leq\Sigma_0\leq\Lambda} E[\pi_0^{inf}(\Sigma_0,\,\Sigma_r)] - c(\Sigma_0), \tag{35}$$

where

$$\begin{aligned}
&E[\pi_0^{inf}(\Sigma_0,\Sigma_r)]\\
&=3(\mu\sigma^2-4(\Sigma_0-\Sigma_r))\cdot\Bigg\{a\Bigg(3((\Sigma_0-\Sigma_r)+2(\Omega_0+\Lambda-\Sigma_0))\\
&+(\Sigma_0-\Sigma_r)\sqrt{3\left(\frac{\mu\sigma^2}{\Sigma_0-\Sigma_r}-1+\frac{3(\Lambda+\Omega_0)(\Omega_0+\Lambda-\Sigma_0)\mu\sigma^2}{(\Sigma_0-\Sigma_r)^3}\right)}\Bigg)\Bigg\}^{-1}.
\end{aligned} \tag{36}$$

Denote the amount of information the informed trader acquires in equilibrium, a function of Λ, Ω_0, and c, by $\Sigma_0(\Lambda,\Omega_0,c)$, and the amount he releases by $\Sigma_r(\Lambda,\Omega_0,c)$. The equilibrium information acquisition and release strategies are given by the following proposition.

Proposition 8. *There is a unique subgame perfect equilibrium if and only if* $4\Lambda<\mu\sigma^2$.
(*i*) *If information acquisition is costless* $(c=0)$*, the informed trader always chooses to acquire all available information.*
(*ii*) *For each* Λ *and cost function c satisfying the above conditions, there exists an* $\overline{\Omega}_0(\Lambda,\,c)>0$ *such that if* $\Omega_0<\overline{\Omega}_0(\Lambda,\,c)$*, then* $\Sigma_0(\Lambda,\Omega_0,c)=\Lambda$ *and* $\Sigma_r(\Lambda,\Omega_0,c)=\Sigma_r(\Lambda,\Omega_0,0)$.

Proposition 8 states that if information acquisition is costless, with information release, the informed trader will acquire all available information. Further, even if information acquisition costs are high, with sufficiently low intractable information, he still acquires all tractable information, contrary to his behavior with no information release. This is because the public release of information increases liquidity trading, and with it, his profits. The amount of information released is independent of the information acquisition cost function.

Figure 4 illustrates the equilibrium outcome for a linear information acquisition cost function ($c(\Sigma_0)=\Sigma_0/2$). Figures 4A and B show the informed trader's expected net profits for the same level of Λ (0.2) and two different Ω_0 values. As Ω_0 increases, the profit-maximizing strategy shifts from acquiring all available information and releasing some of it to acquiring less information and releasing none. Figure 4C shows the percentage of available information that the informed trader acquires (Σ_0/Λ) and Figure 4D shows the percentage of information that he chooses to

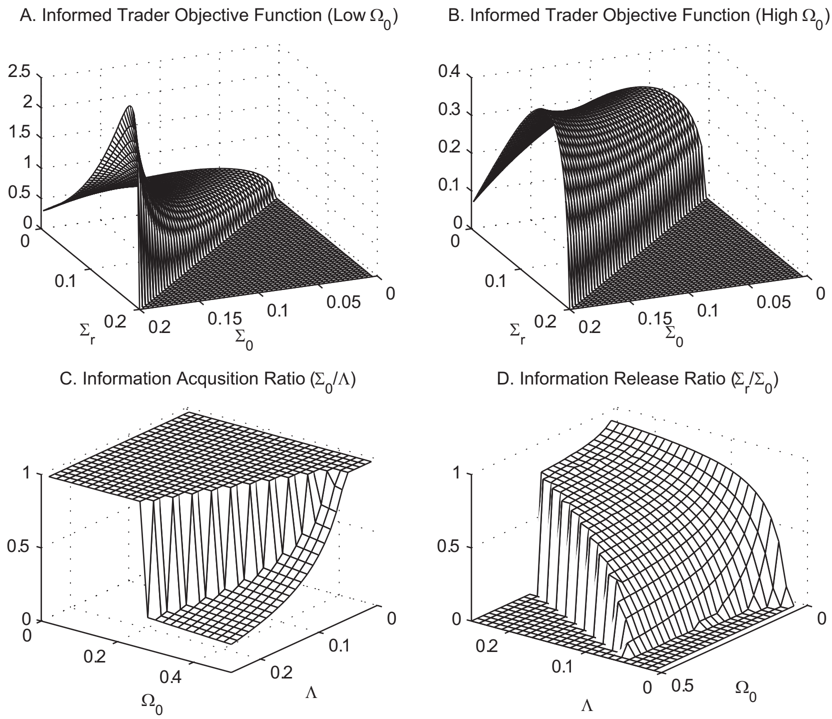

Figure 4
Panels A and B show the total expected profit of the informed trader with voluntary information disclosure for different levels of intractable information, $\Omega_0 = 0.02$ and $\Omega_0 = 0.19$, respectively. For both panels, $\Lambda = 0.2$. Panel C shows the fraction of acquired information (Σ_0/Λ) and panel B shows the fraction of released information (Σ_r/Σ_0) in equilibrium. Information acquisition costs are linear with $c = 0.5$, and for all panels $a = 1$.

release (Σ_r/Σ_0) for a range of values of Λ and Ω_0. For low Λ, it is more profitable to obtain all available information and release none. For high values of Λ, when Ω_0 is not too high, it is still optimal to obtain full information, but in this case he chooses to release most of it.

In summary, the informed trader chooses between two modes of operation: obtaining all available information and releasing some of it, or obtaining part of the available information and keeping it entirely to himself. The sharp transition between the two modes is a result of the bimodality of his objective function. Figure 4B shows how the global optimum shifts from one local optimum to another, resulting in a sharp change in strategy. Figure 4C shows that for lower Ω_0 and Λ values, the market is fully efficient in the sense that by the end of trading, *all* information available for acquisition is incorporated in the security's price, whereas for large intractable information (Ω_0), most of the information available for acquisition is not incorporated in the price. The reason for this is that when the amount of intractable information is large, it slows down liquidity trading and makes acquiring and relasing information less profitable for the informed trader. As a result, he chooses not to acquire much information and not to release any of it when Ω_0 is large. Thus intractable information has a significant impact on the amount of *tractable* information that gets incorporated in price by the end of trading. In general, the option to release information improves market performance. Not only does the market become more efficient, liquidity also increases. Information release alleviates the incentive to acquire less information (Section 3.1) and creates better alignment between the incentives of the informed and liquidity traders.

The foregoing results suggest that intractable information is an important driver of information acquisition and release strategies. Empirically, accounting regimes and firm disclosure policies affect the amount of intractable information. For example, cross-country differences in accounting practices and requirements affect information tractability. When a non-U.S. firm lists for trading in the United States, this typically makes some intractable information tractable. Our results in Section 2.2.1 imply that this should increase the stock's liquidity and result in a smaller bid-ask spread, even in its home market. The results in this section show that, in addition, this will increase the amount of information collected and analyzed about the firm, resulting in a more efficient market.

More broadly, accounting policies that provide better access to investors and analysts (thereby increasing tractability) should result in both lower bid-ask spreads and more information acquisition and assimilation about the firm. Within the United States, tractability can be proxied by the Financial Analysts Federation (FAF) scores developed by the Association for Investment Management Research to evaluate the disclosures provided by the firm's annual reports, other publications, and its investor relations department. For example, Lang and Lundholm (1996) found a strong correlation between FAF scores and the number of analysts following the firm, after controlling for other factors. Across countries, the Center for International Financial Analysis and Research [cf. Rajan and Zingales (1998)] developed the CIFAR index, which quantifies the quality of accounting regimes for 49 countries and can be used as a measure of information tractability in cross-country comparisons as well as in studying the effects of international listings.

3.4 Unobservable information acquisition investment

In this section we examine the rational expectations equilibrium when the information acquisition investment is not observable.[12] In equilibrium, the market conjectures the correct amount of information acquired by the

[12] We thank an anonymous referee for inspiring this analysis.

informed trader and the informed trader's best strategy is consistent with the market's conjecture.[13] Let Σ_0^{en} be the informed trader's acquired information in equilibrium when his decision is not observable. The equilibrium outcome for this case is given by the following proposition.

Proposition 9. *If the informed trader's information acquisition decision is not observable, then there is a unique equilibrium. Define*

$$\Psi(\Sigma_0) = \frac{3(\mu\sigma^2 - 4\Sigma_0)}{2a(3\Sigma_0(2(\Lambda+\Omega_0) - \Sigma_0) + \Sigma_0(3(\mu\sigma^2((\Sigma_0^2 + 3(\Lambda+\Omega_0) - \Sigma_0(\Lambda+\Omega_0))/\Sigma_0^3) - 1))^{\frac{1}{2}})}. \tag{37}$$

(*i*) *If* $\Psi(\Lambda) \geq c'(\Lambda)$, *then* $\Sigma_0^{en} = \Lambda$.
(*ii*) *If* $\Psi(\Lambda) < c'(\Lambda)$, *then* $\Sigma_0^{en} < \Lambda$ *and it solves* $\Psi(\Sigma_0^{en}) - c'(\Sigma_0^{en}) = 0$.

In equilibrium, unobservability can only reduce the informed trader's profits. Also, by Proposition 9, if the information acquisition cost is zero ($c = 0$), the informed trader will always acquire full information. Similarly, if the acquisition cost is low, the informed trader will acquire near-full information. Building on the discussion of Section 3.2, this reduces liquidity traders' welfare by pushing the information asymmetry further away from their welfare-maximizing level. This effect is especially severe when Λ is close to $\mu\sigma^2/4$. In this case, with low information acquisition costs and unobservability, the informed trader acquires near full information, λ_0 approaches infinity, trading volume approaches zero, and the equilibrium resembles a "no-trade" situation, hurting all market participants. Making his investment information acquisition observable can effectively remedy this undesirable outcome.

Because unobservability can only hurt the informed trader, he has an incentive to credibly inform the market of his information acquisition decision. In particular, he can hire an independent auditor to monitor his information acquisition investment and report it to the market. At zero cost, this is always (weakly) beneficial for the informed trader. Now suppose the auditor charges a fee, $\kappa \geq 0$. If hired, the auditor observes the informed trader's investment level and reports it to the market before trading starts; then the equilibrium unfolds as in Section 3.1. If the informed trader chooses not to hire the auditor, his investment level is unobservable and the equilibrium outcome is given by Proposition 9. Clearly the informed trader will chose to hire the auditor as long as κ is lower than the difference between the profit levels with and without observability.

Figure 5 compares the information acquired in the observable and unobservable cases for different levels of $c(\Sigma_0) = c \cdot \Sigma_0$, along with the

[13] The observable case (Section 3.1) trivially satisfies this condition.

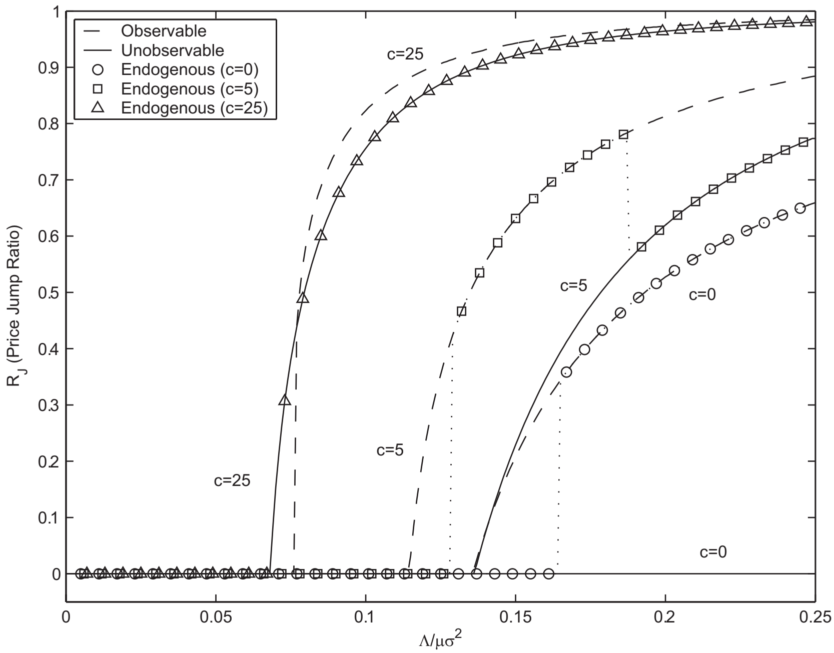

Figure 5
The price jump ratio R_J (i.e., $1 - \Sigma_0/\Lambda$) as the equilibrium outcome for a given level of $\Lambda/\mu\sigma^2$ and for different linear information acquisition costs (c). Shown are the cases of (*i*) observable information acquisition, (*ii*) unobservable information acquisition, and (*iii*) endogenous outcome of decision to hire an auditor with audit fee $\kappa = 0.1$. For all curves, $\mu\sigma^2 = a = 1$ and $\Omega_0 = 0$.

endogenous audit decision with auditor fee $\kappa = 0.1$. As before, we measure the price jump ratio $R_J = 1 - \Sigma_0/\Lambda$, that is, the fraction of tractable information not acquired by the informed. For $c = 0$, when the acquisition decision is unobservable, the informed trader always acquires full information, hence observability decreases the amount of information acquired for large values of Λ. The same holds for $c = 5$, except that the informed trader does not acquire all tractable information in the unobservable case. For large information acquisition costs ($c = 25$ in the figure), the curves for the observable and unobservable cases intersect. For low levels of Λ, the informed trader acquires more information when the acquisition decision is observable. The reason is that when his decision is observed by the market, the increased price reaction to the higher degree of information asymmetry increases the price volatility, which in turn increases his expected profits. For high values of Λ, the negative effect of information acquisition on liquidity prevails, inducing him to acquire less information when his decision is observable.

Which curve the informed trader will actually be on depends on the audit fee and information acquisition cost. Figure 5 shows the endogenous decision on hiring the auditor for $\kappa = 0.1$. When $c = 0$, for an extended range of Λ values he has no incentive to hire the auditor and acquires full information, which leads to zero price jump variance. Beyond a threshold value of Λ, however, the benefits of observability exceed the audit fee, inducing him to hire the auditor. When $c = 5$, the price jump curve shifts modes twice. For low Λ, the difference in profits due to observability is small and the informed trader chooses not to hire the auditor. In the middle range, observability substantially affects the amount of information acquired and consequently improves the informed trader's profits. Hence he chooses to hire the auditor and the price jump curve coincides with the observable case. However, as Λ increases, market uncertainty leads to illiquidity, which reduces the gains from observability to a level that does not justify the audit costs. Hence the informed trader again chooses not to hire the auditor and acquires more information than he would under observability, reducing the price jump. Finally, when information acquisition costs are very high (e.g., $c = 25$), the informed trader cannot acquire enough information to justify the audit fees. Therefore he chooses not to hire the auditor, and his information acquisition remains unobservable.

4. Conclusion

This article examined a dynamic financial market with endogenous liquidity trading under asymmetric information. Liquidity traders' decisions reflect their disparate valuations of the security, their expected losses to the informed trader, and the riskiness of the traded asset. We partition the information about the security's value into public information, tractable information that can be acquired by the informed trader, and intractable information that cannot be acquired. The informed trader acquires a portion of the tractable information set, generating informational asymmetry. The information acquired by the insider is gradually revealed to the market through the informed trader's trades. The intractable information may diffuse to the market over time. Our analysis shows how each of these types of information affect market performance.

We derive the determinants of the adverse selection component of the spread. As the interest rate, speed of public information release, or liquidity trading interest increase, so does the adverse selection spread. As the security's total risk or information asymmetry increases, so does the adverse selection spread. Our model identifies the importance of intractable information and its rate of diffusion to the market as determinants of liquidity and efficiency.

Our results show that compared to models with exogenous noise trading, less information is acquired by insiders and it is disseminated to the market more slowly when liquidity trading is endogenous. The acquisition of some private information by the informed trader increases liquidity traders' welfare, but the informed trader may acquire more information than needed to maximize liquidity traders' welfare. In essence, the informed trader plays the role of an insurer, with the liquidity traders paying him for risk reduction. This risk, however, is not exogenous, and the informed trader acquires more information than the liquidity traders would like.

The informed trader's information acquisition decision depends on whether the market can observe it. The informed trader often has an incentive to hire an auditor to monitor and report his information acquisition investment to the market. An interesting question is what happens if the information acquisition investment is *imperfectly* observed by the market. If the market observes the level of investment with noise, tractability is lost because the resulting distributions are no longer normal. It can be shown in a simpler setting with discrete asset values that the gist of our results remains intact when the market can observe the informed trader's information acquisition decision with noise.[14]

Our model of liquidity trading (Section 1) employs independent liquidity shocks that affect liquidity traders' immediacy preferences. An alternative approach [cf. Glosten (1989), Spiegel and Subrahmanyam (1992) (hereinafter "S&S")] assumes that liquidity trading is driven by hedging motives. The two approaches are substantively different due to the different effects of uncertainty on liquidity traders' behavior, and lead to different equilibrium sets, which we define as the set of parameters for which a linear equilibrium exists. Since the risk-averse hedgers in S&S increase hedging if the asset is more risky, an increase in asset riskiness induces them to trade more, which increases the equilibrium set and may make the market more liquid. With our modeling choice, liquidity traders trade in spite of the risk (rather than in order to offset it), so increased asset riskiness hurts market liquidity and reduces the equilibrium set.

An analysis that compares the results of a single-period version of our model with multiple informed traders (who obtain diverse signals) with those of S&S yields several differences.[15] First, in S&S, increasing the initial uncertainty about the asset's value expands the equilibrium set and sometimes increases market liquidity. With our modeling approach, such an increase always reduces the equilibrium set and decreases market liquidity. Second, in S&S, adding informed traders reduces the equilibrium set, decreases welfare per liquidity trader, and sometimes decreases market liquidity. In our model, such an increase always expands the equilibrium set and increases both the welfare per liquidity trader and

[14] This analysis is available from the authors upon request.

[15] Details are available from the authors.

overall market liquidity. Third, in S&S, adding liquidity traders sometimes decreases market liquidity, while in our model, it always increases liquidity. Fourth, contrary to our model, in S&S, increased liquidity trader risk aversion increases liquidity. Also, increased liquidity trader risk aversion expands the equilibrium set in S&S, whereas in our model it doesn't change the equilibrium set. In both models, liquidity trader risk aversion does not affect market efficiency.

Our results have a number of empirical implications:[16]

(i) The error variance of the informed trader's signal declines as the cubic root of $(1 - t)$, compared to a linear decline in Kyle (1985). We propose a GMM-based test of the price adjustment process in Section 2.2.2.

(ii) Liquidity is positively correlated with the interest rate (Section 2.2).

(iii) The adverse selection component of the spread is an increasing function of the security's total risk (Section 2.2).[17]

(iv) Uncertainty about macroeconomic events (e.g., exchange rates that affect the security's value) increases the adverse selection spread (Section 2.2).

(v) The effect of changing the investor base on liquidity is larger for riskier securities and for securities with a larger proportion of smaller trades (Section 2.2).

(vi) Both cross-sectionally and over time, the price reaction following an announcement is larger when the uninformed investor base in a security is smaller, where the latter can be proxied by the proportion of retail trades in a stock (Section 3).

(vii) Both cross-sectionally and over time, the price reaction following an announcement is larger when there is more intractable information, where the latter can be proxied by firm size or accounting regime (Section 3).

(viii) Listing a non-U.S. firm in the United States reduces the stock's bid-ask spread in its home market and increases information acquisition and research about the firm (Section 3).

(ix) Higher AIMR-FAF scores (for individual securities in the United States) and CIFAR index scores (across countries) are associated with more information acquisition and research (Section 3).

Empirical studies that pursue tests of these hypotheses can shed further light on the effects of endogenous liquidity trading on information acquisition, release, and market performance.

[16] "Other things being equal" is implied as appropriate.

[17] This is distinct from the result of Ho and Stoll (1981) that applies to the *inventory* component of the spread.

Appendix A: Technical Definitions for Continuous-Time Model

Define $\mathcal{F}_t = \sigma(\{\int_0^t dX(s) + dZ(s),\ \eta(t)\})$. We denote the informed trader's cumulative position in the security at time t as $X(t) \in \mathcal{H}^1$, where

$$\mathcal{H}^1 = \left\{ Y \mid dY \in \mathcal{L}^2,\ E\left[\left(\int_0^t (dY_s)^2 ds\right)^{1/2}\right] < \infty, \quad t \in [0,1] \right\} \tag{A1}$$

and $\mathcal{L}^2$ is the space of almost surely square integrable $\mathcal{F}_t \times \sigma(\xi)$-adapted processes. When X is differentiable, we define $dX(t)/dt = x(t)$. The cumulative net quantity of liquidity trading by time t is denoted by $Z(t)$.

Define

$$\pi_t^{inf} = \left(\int_t^1 (PV_{t,1}(v) - PV_{t,s}(p(s)))\, dX_i(s)\right). \tag{A2}$$

An *equilibrium* satisfies the following requirements:

(*a*) *At each time point t, each liquidity trader maximizes her utility given available information.* For a liquidity trader who arrives at time $t \in [0,1]$ and has immediacy value u, the equilibrium order quantity, z, satisfies

$$U(z(PV_{t,1}(v) + u - p(t)) \mid u, \mathcal{F}_t) \geq U(y(PV_{t,1}(v) + u - p(t)) \mid u, \mathcal{F}_t) \tag{A3}$$

for all $y \in \mathbb{R}$.

(*b*) *The informed trader maximizes his expected profits*:

The informed trader chooses the trading strategy $X(t)$ so that for any given trading strategy $Y(t) \in \mathcal{H}^1$ and for all $t \in [0,1]$:

$$E[\pi_t^{inf} \mid \xi, \mathcal{F}_t] = E\left[\int_t^1 (PV_{t,1}(v) - PV_{t,s}(p(s)))\, dX(s) \mid \xi, \mathcal{F}_t\right] \tag{A4}$$

$$\geq E\left[\int_t^1 (PV_{t,1}(v) - PV_{t,s}(p(s)))\, dY(s) \mid \xi, \mathcal{F}_t\right]. \tag{A5}$$

(*c*) *Market makers set prices efficiently*, that is, the market pricing function satisfies

$$p(t) = E[PV_{t,1}(v) \mid \mathcal{F}_t]. \tag{A6}$$

In addition to (a)–(c), we impose $\int_0^t \sigma_p^2(s)ds < \infty$ and $\int_0^t \sigma_z^2(s)ds < \infty$ for all t in [0,1], where σ_p and σ_z are the respective diffusion coefficients of the equilibrium price and liquidity trading processes. These conditions guarantee the stability of the price and cumulative liquidity trading in equilibrium.

We again concentrate on equilibria in linear strategies. Since the equilibrium market price reflects the present value of the security given all public information, the liquidity traders' strategies do not depend on the security's market price. Let the liquidity trader arriving at time t with valuation u play the linear order placement strategy $\gamma(t)u$.[18] The independence of the immediacy values implies that the cumulative net liquidity trading process, $Z(t)$, is an $\mathcal{F}_t$-adapted Brownian motion with zero drift and instantaneous variance $\mu_t \gamma^2(t)\sigma_t^2$.

Let $\hat{p}(t) = E[PV_{t,1}(\xi) \mid \mathcal{F}_t]$. In a linear equilibrium, the informed trader applies the following strategy:

$$dX(t) = \beta(t)\xi_t\, dt + \phi(t)E[\eta \mid \mathcal{F}_t] + \varphi(t)p(t), \tag{A7}$$

[18] In Section 1 (and in the associated proofs given in Appendix B), we in fact solve the liquidity traders' problem without assuming that the trading strategy is linear, and show that linear strategies are indeed optimal.

or alternatively

$$dX(t) = \beta(t)(\xi_t - \hat{p}(t)) + \phi(t)(E[\eta \mid \mathcal{F}_t] - p(t) + \hat{p}(t)), \tag{A8}$$

where $\xi_t = PV_{t,1}(\xi) = \xi \cdot e^{-\int_t^1 r_s ds}$.[19]

A linear pricing schedule for the market makers satisfies,[20] for λ, Λ_η real-valued functions defined on [0, 1],

$$\begin{aligned} dp(t) &= r_t p(t) + \lambda(t)(dX(t) + dZ(t)) + \lambda h(t) d\eta(t) \\ &= d\hat{p}(t) + r_t(p(t) - \hat{p}(t)) + \lambda_\eta(t) d\eta(t), \end{aligned} \tag{A9}$$

where

$$d\hat{p}(t) = r_t \hat{p}(t)\, dt + \lambda(t)\beta(t)(\xi_t - \hat{p}(t))\, dt + \lambda(t)\gamma(t)\sqrt{\mu_t}\bar{\sigma}_t\, dW_t \tag{A10}$$

and W is a standard Brownian motion on the filtration $\{\mathcal{F}_t \mid t \in [0, 1]\}$. We also define

$$\Omega_t = \left(\Omega - \int_0^t \omega_s ds\right) e^{-2\int_t^1 r_s ds}, \tag{A11}$$

$$\breve{\Sigma}(t) = E[(\xi_t - \hat{p}(t))^2 \mid \mathcal{F}_t], \tag{A12}$$

and

$$\Sigma(t) = E[(\xi - \hat{p}(t) e^{\int_t^1 r_s ds})^2 \mid \mathcal{F}_t]. \tag{A13}$$

Clearly, $\breve{\Sigma}(t) = \Sigma(t) e^{-2\int_t^1 r_s ds}$.

Appendix B: Proofs of Propositions

Proof of Proposition 1. The solution of the informed trader's problem is, a few details withstanding, similar to the Kyle case and skipped here. The normality of the end value and the cumulative noise trading yield the equations

$$\lambda_\tau = \frac{\beta_\tau \Sigma_{\tau-1}(1 + r_\tau \Delta t_\tau)}{\beta_\tau^2 \Sigma_{\tau-1} \Delta t_\tau + m_\tau \gamma_\tau^2 \sigma_\tau^2} \tag{B1}$$

$$\Sigma_\tau = \frac{m_\tau \gamma_\tau^2 \sigma_\tau^2 \Sigma_{\tau-1}(1 + r_\tau \Delta t_\tau)^2}{\beta_\tau^2 \Sigma_{\tau-1} \Delta t_\tau + m_\tau \gamma_\tau^2 \sigma_\tau^2}, \tag{B2}$$

from which the desired expressions for λ_τ and Σ_τ easily follow. Note that determination of $\lambda_{\eta\tau}$ and irrelevance of ϕ_τ follows from the market efficiency condition.

Liquidity trader j arriving at period τ solves

$$\max_{z_{j\tau}} U\left(z_{j\tau}\left(v_\tau + u_{j\tau} - p_0 - \lambda_\tau\left(\beta_\tau(\xi_{\tau-1} - p_{\tau-1}) - \sum_{k \neq j} \gamma_{k\tau} u_{ks} - z_{j\tau}\right)\right) \,\Big|\, u_{j\tau}, \mathcal{F}_\tau\right) \tag{B3}$$

from the first-order condition of which

$$\gamma_\tau = (2\lambda_\tau + a_\tau(\Omega_\tau + \Sigma_{\tau-1}((1 + r_\tau \Delta t_\tau) - \lambda_\tau \beta_\tau)^2 + \lambda_\tau^2 m_\tau \gamma_\tau^2 \sigma_\tau^2 \Delta t_\tau))^{-1} \tag{B4}$$

[19] In fact, we show that for equilibrium purposes, Equation (A8) and Equation (A9) are equivalent and ϕ is irrelevant. Considering this, the uniqueness of the equilibrium in the results presented in Section 1 should be considered as uniqueness up to a given irrelevant function ϕ.

[20] This pricing schedule also emerges from the necessary conditions for an equilibrium, and is consistent with the price process being a martingale with respect to the deflated measure (by r). See, e.g., Chapter 5 of O'Hara (1997).

and second-order condition

$$2\lambda_\tau + a_\tau(\Omega_\tau + \Sigma_{\tau-1}((1+r_\tau\Delta t_\tau) - \lambda_\tau\beta_\tau)^2 + \lambda_\tau^2 m_\tau \gamma_\tau^2 \sigma_\tau^2 \Delta t_\tau > 0 \tag{B5}$$

follow. Since the polynomial

$$\gamma_\tau(2\lambda_\tau + a_\tau(\Omega_\tau + \Sigma_{\tau-1}((1+r_\tau\Delta t_\tau) - \lambda_\tau\beta_\tau)^2 + \lambda_\tau^2 m_\tau \gamma_\tau^2 \sigma_\tau^2 \Delta t_\tau)) - 1 \tag{B6}$$

is monotonically increasing in γ_τ and negative at $\gamma_\tau = 0$, it follows that Equation (B4) has a unique root that is strictly positive.

Having obtained the necessary conditions for an equilibrium, we now show its existence. First note that by Equation (B1), λ_τ and β_τ must have the same sign. Moreover,

$$1 - \frac{\beta_\tau\lambda_\tau\Delta t_\tau}{1 + r_\tau\Delta t_\tau} = \frac{1}{2(1-\alpha_\tau\lambda_t)} = \frac{\gamma_\tau^2 m_\tau \sigma_\tau^2}{\beta_\tau^2\Sigma_{\tau-1}\Delta t_\tau + m_\tau\gamma_\tau^2\sigma_\tau^2} > 0. \tag{B7}$$

Combining these with Equation (11) we obtain

$$0 < \lambda_\tau < \frac{1}{2\alpha_\tau}, \tag{B8}$$

which verifies the second-order conditions for both the informed and liquidity trader's problems.

For any given nonnegative α_τ and Σ_τ, using Equations (11)–(14), we can deduce that λ_τ satisfies the fifth-order equation:

$$\begin{aligned}&\Sigma_\tau(1-2\alpha_\tau\lambda_\tau)\cdot\left(((1-2\alpha_\tau\lambda_\tau)+1)\left(2\lambda_\tau + a_\tau\Omega_\tau + a_\tau\Sigma_\tau\frac{m_\tau}{m_\tau+1}\right) + a_\tau\frac{\Sigma_\tau}{m_\tau+1}\right)^2\\ &\quad - ((1-2\alpha_\tau\lambda_\tau)+1)^3\lambda_\tau^2(m_\tau+1)\sigma_\tau^2\Delta t_\tau = 0,\end{aligned} \tag{B9}$$

which always has a unique root that satisfies Equation (B8) for $\alpha_\tau > 0$. For $\alpha_\tau = 0$ (i.e., $\tau = T$), it can be shown that a positive solution to Equation (B9) exists if an only if

$$\Sigma_T < \tfrac{1}{2}\bar{\sigma}_T^2 m_T \Delta t_T. \tag{B10}$$

and that this solution is unique. We can also easily verify that the resulting $\alpha_{\tau-1}$ and $\Sigma_{\tau-1}$ from this λ_τ for any $\alpha_\tau \geq 0$ are positive. Therefore we conclude that there exists a unique way to iterate the difference equation system backwards. Finally, we need to show that for each given Σ_0 that satisfies Equation (7), there is an Σ_T that satisfies the equation system. It can be shown that for each $0 \leq \tau \leq T-1$,

$$\tfrac{1}{2}\Sigma_\tau(1 + r_\tau\Delta t_\tau)^2 \leq \Sigma_{\tau-1} < \Sigma_\tau(1 + r_\tau\Delta t_\tau)^2, \tag{B11}$$

where the first inequality in Equation (B11) is satisfied as an equality if and only if $\tau = T-1$. Applying Equation (B11) iteratively, it follows that if Σ_0 satisfies Equation (7), then Σ_T satisfies Equation (B10). It is easy to see that if Σ_T is zero, then the Σ_0 resulting from the backwards iteration of the equation system is zero, and as $\Sigma_T \to \infty$ the corresponding Σ_0 also goes to infinity. Combining this with the continuity of Σ_0 as a function of Σ_T implies that there exists a $\Sigma_T \geq 0$ that generates a solution for each $\Sigma_0 \geq 0$ in the given range and this completes the proof. ■

Proof of Proposition 2. We start by introducing the following mathematical preliminaries. Let a process $P(V_t, t)$, where $\{V_t\}$ is the observed net trading volume process, be given. For any $y \in \mathbb{R}$, trading strategy $X \in \mathcal{H}^1$ and $0 \leq t_1 \leq t_2 \leq 1$ define

$$S_P(y, X, t_1, t_2) = E[(P(V_{t_2}(X), t_2) - y)^2 \mid \mathcal{F}_{t_1}]. \tag{B12}$$

Define *the set of exact control strategies of P for y over* $[t_1,t_2]$ as

$$\Theta_{P,y,t_1,\,t_2} = \{\theta \,|\, \theta \in \mathcal{H}^1,\, S_P(y,\, \theta,\, t_1,\, t_2) = 0\}. \tag{B13}$$

We call a process P *controllable* if Θ_{P,y,t_1,t_2} is nonempty for all $y \in \mathbb{R}$ and $0 \le t_1 \le t_2 \le 1$.

Now we show that the conditional expectation process defined by Equation (A10) is controllable under certain regularity conditions.

Lemma B.1. *The process generated by* Equation (A10), *where* $\lambda(t)$ *is nonzero a.e. and* $\lambda^2\mu\gamma^2\sigma^2$ *is integrable over* $[0, 1]$, *is controllable.*

Proof. We will show that there exists a linear exact control trading strategy $X \in \mathcal{H}^1$ where Θ_{P,y,t_1,t_2} is nonempty for any y, t_1, and t_2. To see this, first it can be shown that for any given X with $\frac{d}{dt}X(t) = x(t)$, $S_P(y, \theta, t_1, t)$ solves the differential equation

$$S_P'(y,\, X,\, t_1,\, t) = 2r_t S_P(y,\, X,\, t_1,\, t) - 2\lambda(t)E[x(t)(y - P(t))] + \lambda^2(t)\gamma^2(t)\mu_t\sigma_t^2, \tag{B14}$$

with boundary condition $S_P(y, X, t_1, t_1) = (P(t_1) - y)^2$. For a linear trading strategy with $x(t) = \beta(t)(y - P(t))$, Equation (B14) becomes

$$S_P'(y,\, X,\, t_1,\, t) = 2(r_t - \lambda(t)\beta(t))S_P(y,\, X,\, t_1,\, t) + \lambda^2(t)\gamma^2(t)\mu_t\sigma_t^2 \tag{B15}$$

and our problem becomes finding a $\beta : [0,1] \to \mathbb{R}$ that solves Equation (B15) with boundary conditions $S_P(y, X, t_1, t_1) = (P(t_1) - y)^2$ and $S_P(y, X, t_1, t_2) = 0$. For integrable $\lambda^2\mu\gamma^2\sigma^2$, the solution of this problem with arbitrary $S_P(y, X, t_1, t_2)$ (and hence free β) is

$$S_P(y,\, X,\, t_1,\, t) = \int_{t_1}^{t} e^{\int_s^t 2(r_u - \lambda(u)\beta(u))du}\lambda^2(s)\gamma^2(s)\mu_s\sigma_s^2 ds + (P(t_1) - y)^2 e^{\int_{t_1}^{t} 2(r_u - \lambda(u)\beta(u))du}. \tag{B16}$$

With almost everywhere nonzero λ it is easy to show that there can be found a β that satisfies $S_P(y, X, t_1, t_2) = 0$. Hence for any y, t_1, and t_2, Θ_{P,y,t_1,t_2} is nonempty and the proof of the lemma is complete. ■

Now consider the solution of the informed trader's problem fixing the strategies of market makers and the liquidity traders. Suppose $\lambda(t)$ and $\gamma(t)$ are given. Then, for any t,

$$\begin{aligned}
\pi^{inf}(t) &= \int_t^1 (ve^{-\int_t^1 r_u du} - p(s)e^{-\int_t^s r_u du})dX(s) \\
&= \int_t^1 e^{-\int_t^s r_u du}(ve^{-\int_s^1 r_u du} - p(s))dX(s) \\
&\quad + \int_t^1 ((\eta - (p(s) - \hat{p}(s))e^{\int_s^1 r_u du}) + \epsilon - \eta)e^{-\int_t^1 r_u du}dX(s)ds,
\end{aligned} \tag{B17}$$

where ξ_t, $\hat{p}(t)$, $p(t)$ satisfy the stochastic differential equation system

$$d\xi_s = r_s\xi_s ds \tag{B18}$$

$$d\hat{p}(s) = (r_s\hat{p}(s) + \lambda(s)x(s))ds + \lambda(s)\sqrt{\mu_s}\gamma(s)\bar{\sigma}_s dW_s \tag{B19}$$

$$dp(s) = d\hat{p}(s) + r_s(p(s) - \hat{p}(s))ds + \lambda_\eta(s)d\eta_s. \tag{B20}$$

First, clearly, unless almost everywhere on $[0, 1]$, $E[\eta \,|\, \mathcal{F}_s]e^{-\int_s^1 r_u du} - (p(s) - \hat{p}(s)) = \eta(s)e^{-\int_s^1 r_u du} - (p(s) - \hat{p}(s)) = 0$ is satisfied, $E[\pi^{inf}(t)]$ can be made arbitrarily large. This implies that $d((p(s) - \hat{p}(s))) = e^{-\int_s^1 r_u du}d\eta_t$ must hold, which requires $\lambda_\eta(t) = e^{-\int_s^1 r_u du}$. Moreover, since under this policy the coefficient of $\phi(t)$ is zero a.e., the choice of ϕ becomes irrelevant. Also, considering that $E[\epsilon - \eta \mid \xi, \mathcal{F}_t] = 0$ for all t, we conclude that

$$E[\pi^{inf}(t)] = E\left[\int_t^1 e^{-\int_t^s r_u du}(\xi_s - \hat{p}(s))x(s)\, ds\right]. \tag{B21}$$

Define

$$f(s, t, \xi_s, \hat{p}(s)) = \frac{e^{-\int_t^s r_u du}(\xi_s - \hat{p}(s))^2}{2\lambda(s)}. \tag{B22}$$

Then, by Itô's formula,

$$\begin{aligned} df(s, t, \xi_s, \hat{p}(s)) = e^{-\int_t^s r_u du}\Bigg\{ \Bigg(& \frac{(\xi_s - \hat{p}(s))^2 r_s}{\lambda(s)} - x(s)(\xi_s - \hat{p}(s)) \\ & + \frac{1}{2}(\xi_s - \hat{p}(s))^2 \left(\frac{d}{ds}\left(\frac{1}{\lambda(s)}\right) - \frac{r_s}{\lambda(s)} \right) + \frac{\sigma_p^2(s)}{\lambda(s)} \Bigg) ds \\ & + \frac{(\hat{p}(s) - \xi_s)\sigma_p(s)}{\lambda(s)} dW_s \Bigg\}, \end{aligned} \tag{B23}$$

where $\sigma_p^2(s) = \lambda^2(s)\mu_s\gamma^2(s)\sigma_s^2$. By Equation (B23), we obtain

$$\begin{aligned} \pi^{inf}(t) = \frac{1}{2}\Bigg\{ & \frac{(\xi_t - \hat{p}(t))^2}{\lambda(t)} - \frac{(\xi - \hat{p}(1))^2 e^{-\int_t^1 r_u du}}{\lambda(1)} \\ & + \int_t^1 e^{-\int_t^s r_u du}\left(\left(\frac{d}{ds}\left(\frac{1}{\lambda(s)}\right) + \frac{r_s}{\lambda(s)} \right)(\xi_s - \hat{p}(s))^2 + \frac{\sigma_p^2}{\lambda(s)} \right) ds \\ & + \int_t^1 e^{-\int_t^s r_u du}(\hat{p}(s) - \xi_s)\frac{\sigma_p(s)}{\lambda(s)} dW_s \Bigg\}. \end{aligned} \tag{B24}$$

Given that σ_p^2 and $\mu\gamma^2\sigma^2$ are integrable, by the above lemma, $\hat{p}$ is controllable and

$$M_t(\tau) = \int_t^\tau e^{-\int_t^s r_u du}(\hat{p}(s) - \xi_s)\frac{\sigma_p(s)}{\lambda(s)} dW_s \tag{B25}$$

is an $\mathcal{F}$-martingale. From this it follows that

$$\begin{aligned} E[\pi^{inf}(t) \mid \xi, \mathcal{F}_t] = \frac{1}{2}\Bigg\{ & \frac{(\xi_t - \hat{p}(t))^2}{\lambda(t)} - E\left[\frac{(\xi - \hat{p}(1))^2 e^{-\int_t^1 r_u du}}{\lambda(1)} \,\middle|\, \xi, \mathcal{F}_t \right] \\ & + E\left[\int_t^1 e^{-\int_t^s r_u du}\left(\frac{d}{ds}\left(\frac{1}{\lambda(s)}\right) + \frac{r_s}{\lambda(s)} \right)(\xi_s - \hat{p}(s))^2 ds \mid \xi, \mathcal{F}_t \right] \\ & + \int_t^1 e^{-\int_t^s r_u du}\lambda(s)\mu_s\gamma^2(s)\sigma_s^2 ds \Bigg\}. \end{aligned} \tag{B26}$$

From Equations (B24) and (B26), we can observe two things. First, in order to maximize $E[\pi^{inf}(t) \mid \xi, \mathcal{F}_t]$, $\lim_{t\to 1}\hat{p}(1) = \xi$ must hold. Since $\hat{p}$ is controllable, in equilibrium, the informed trader must ensure this equality. That is, in equilibrium

$$\Sigma(1) = 0 \tag{B27}$$

must hold. Second, we claim that in an equilibrium,

$$\frac{d}{ds}\left(\frac{1}{\lambda(s)}\right) + \frac{r_s}{\lambda(s)} = 0, \quad \text{a.e. on } \mathcal{S} \tag{B28}$$

must be satisfied for every $\mathcal{S} \in \mathcal{R}^1 \cap 2^{[0,1]}$, where $\mathcal{R}^1$ is the Borel sets of $\mathbb{R}$. To see this, first notice that given the information set, and the strategies of the market makers and liquidity traders, the first and the last terms of Equation (B26) are independent of the informed trader's strategy. Now suppose that $\frac{d}{ds}(\frac{1}{\lambda(s)}) + \frac{r_s}{\lambda(s)} > 0$ for such a set $\mathcal{S}_0 = (s_l, s_h)$. Then, by controllability of $\hat{p}$, for a given $\overline{p} > \xi(s_h) + \sqrt{\sup_{s\in\mathcal{S}}\{(\xi_s - \hat{p}(s))^2\}}$, the informed trader can bring $\hat{p}(s_m)$ up to $\overline{p}$ for a fixed $s_m \in \mathcal{S}_0$ and bring $\hat{p}$ back to $\hat{p}(s_h)$ at time s_h. Call the new path $\tilde{p}(s)$. Clearly $\tilde{p}$ can be chosen large enough to satisfy

$$E\left[\int_{\mathcal{S}_o} e^{-\int_t^s r_u du}\left(\frac{d}{ds}\left(\frac{1}{\lambda(s)}\right) + \frac{r_s}{\lambda(s)}\right)((\xi_s - \tilde{p}(s))^2 - (\xi_s - \hat{p}(s))^2)ds\right] > 0. \tag{B29}$$

Therefore, if $\frac{d}{ds}(\frac{1}{\lambda(s)}) + \frac{r_s}{\lambda(s)} > 0$, it is possible to improve $E[\pi^{inf}(t)]$ arbitrarily, hence this cannot hold in the equilibrium. Now suppose $\frac{d}{ds}(\frac{1}{\lambda(s)}) + \frac{r_s}{\lambda(s)} < 0$ on a given $\mathcal{S}_0 \in \mathcal{R}^1 \cap 2^{[0,1]}$. Then, unless $\xi_s = \hat{p}(s)$ holds for all $s \in \mathcal{S}_0$,

$$E\left[\int_{s\in\mathcal{S}_0} e^{-\int_t^s r_u du}\left(\frac{d}{ds}\left(\frac{1}{\lambda(s)}\right) + \frac{r_s}{\lambda(s)}\right)(\xi_s - \hat{p}(s))^2 ds \,|\, \xi, \mathcal{F}_t\right] \tag{B30}$$

cannot be maximized. However, we know that this cannot be satisfied since $\mu_t \sigma_t^2 > 0$ a.e., which implies $\sigma_p^2 > 0$ a.e. Therefore this case cannot hold in equilibrium either and we conclude that

$$\frac{d}{ds}\left(\frac{1}{\lambda(s)}\right) + \frac{r_s}{\lambda(s)} = 0, \quad \text{a.e. on } [0, 1]. \tag{B31}$$

This implies

$$\lambda(s) = \lambda_0 e^{\int_0^s r_u du} \tag{B32}$$

for some (positive) constant λ_0. The condition of Equation (B31) states that for the existence of equilibrium, the objective function has to be "flat" in the sense that as long as it ensures Equation (B27), any (smooth) trading strategy that the informed trader chooses maximizes his expected profits.

Next, consider the liquidity traders. The liquidity trader indexed by j and arriving at time t faces the following problem:

$$\max_{z_{jt}} E[W(z_{jt}, u_{jt}, p(t), v, t) \,|\, u_{jt}, \mathcal{F}_t] - a_t \text{var}[W(z_{jt}, u_{jt}, p(t), v, t) \,|\, u_{jt}, \mathcal{F}_t] \tag{B33}$$

where

$$\begin{aligned} W(z_{jt}, u_{jt}, p(t), v, t) = z_{jt}\Big(&v_t(1 + r_t dt) + u_{jt} - p(t)(1 + r_t dt) \\ &- \lambda_t(\beta(t)(\xi_t - p(t))dt) - \lambda(t)\sum_{k\neq j}\gamma_k(t)u_{kt} - \lambda(t)z_{jt}\Big) \end{aligned} \tag{B34}$$

and $v_t = v \cdot e^{-\int_t^1 r_s ds}$. Since $\{u_{jt}\}$ is independent and the market price is efficient, the optimization problem of Equation (B33) turns into

$$\max_{z_{jt}}\Big\{z_{jt}u_{jt} - z_{jt}^2(\lambda(t) + \frac{a_t}{2}\Big(\Omega_t + \breve{\Sigma}(t)(1 + (r_t - \lambda(t)\beta(t))dt)^2 + \lambda^2(t)\sigma_t^2\sum_{k\neq j}\gamma_k^2(t)dt)\Big)\Big\}. \tag{B35}$$

The first-order condition is

$$u_{jt} - \left(2\lambda(t) + \frac{a_t}{2}\left(\Omega_t + \breve{\Sigma}(t)(1 + (r_t - \lambda(t)\beta(t))dt)^2 + \lambda^2(t)\sigma_t^2\sum_{k\neq j}\gamma_k^2(t)dt\right)\right)z_{jt} = 0 \tag{B36}$$

and the second-order condition is

$$2\lambda(t) + \frac{a_t}{2}\left(\Omega_t + \breve{\Sigma}(t)(1 + (r_t - \lambda(t)\beta(t))dt)^2 + \lambda^2(t)\sigma_t^2\sum_{k\neq j}\gamma_k^2(t)dt\right) > 0. \tag{B37}$$

The solution of Equation (B36) is equivalent to $z_{jt} = u_{jt}(2\lambda(t) + a_t(\Omega_t + \breve{\Sigma}(t)))^{-1}$. The second-order condition is satisfied for all z, since $\lambda(t) > 0$ in the equilibrium, as we show below. Plugging in $\lambda(t) = \lambda_0 e^{\int_0^t r_s ds}$, we conclude that

$$\gamma(t) = (2\lambda_0 e^{\int_0^t r_s ds} + a_t(\Omega_t + \breve{\Sigma}(t)))^{-1}. \tag{B38}$$

Now consider the market efficiency condition. First,

$$p(t) = E[PV_{t,1}(v) \mid \mathcal{F}_t] = E[\xi_t \mid \mathcal{F}_t] + (E[\eta \mid \mathcal{F}_t] + E[\epsilon - \eta \mid \mathcal{F}_t])e^{-\int_t^1 r_s ds} \tag{B39}$$

implies that $p(t) - E[\xi_t \mid \mathcal{F}_t] = p(t) - \hat{p}(t) = \eta(t)e^{-\int_t^1 r_s ds}$, from which it follows that $\lambda_\eta(t) = e^{-\int_t^1 r_s ds}$. Second, the market-makers' observation system equations for ξ_t are

$$d\xi_t = r_t \xi_t dt \tag{B40}$$

$$dI(t) = \beta(t)\xi_t dt + dZ(t). \tag{B41}$$

By symmetry and independence of liquidity trading at every instant, $dZ(t) = (\mu_t \sigma_t^2)^{\frac{1}{2}} \gamma(t) dW(t)$ holds where $W(t)$ is a standard Brownian motion. The Kalman-Bucy filter equation for the system of Equations (B40) and (B41) is

$$dE[\xi_t \mid \mathcal{F}_t] = d\hat{p}(t) = \left(r_t - \frac{\beta(t)^2 \breve{\Sigma}(t)}{\mu_t \gamma^2(t) \sigma_t^2} \right) \hat{p}(t)\, dt + \frac{\beta(t)\breve{\Sigma}(t)}{\mu_t \gamma^2(t) \sigma_t^2} (\beta(t)\xi_t\, dt + dZ(t)). \tag{B42}$$

By Equation (A10),

$$d\hat{p}(t) = r_t \hat{p}(t)\, dt + \lambda(t)\beta(t)(\xi_t - \hat{p}(t))\, dt + dZ(t). \tag{B43}$$

From Equations (B42) and (B43) and the Ricatti equation for Equations (B40) and (B41) we obtain

$$\lambda(t) = \frac{\beta(t)\breve{\Sigma}(t)}{\gamma^2(t)\mu_t \sigma_t^2} \tag{B44}$$

and

$$\frac{d\breve{\Sigma}(t)}{dt} = 2r_t \breve{\Sigma}(t) - \lambda^2(t)\gamma^2(t)\mu_t \sigma_t^2. \tag{B45}$$

Plugging Equations (B32) and (B38) in Equation (B45), we obtain

$$\frac{d\breve{\Sigma}(t)}{dt} = 2r_t \breve{\Sigma}(t) - \frac{\lambda_0 e^{\int_0^t r_s ds} \mu_t \sigma_t^2}{(2\lambda_0 e^{\int_0^t r_s ds} + a_t(\breve{\Sigma}(t) + \Omega_t))^2} \tag{B46}$$

as desired. Now, substituting $\Sigma(t) = \breve{\Sigma}(t)e^{2\int_t^1 r_s ds}$ in Equation (B46) we obtain

$$\frac{d\Sigma(t)}{dt} = -\frac{\lambda_0^2 e^{2\int_1^0 r_s ds} \mu_t \sigma_t^2}{(2\lambda_0 e^{\int_0^t r_s ds} + a_t(\Sigma(t) + \Omega)e^{-2\int_t^1 r_s ds})^2}. \tag{B47}$$

This allows us to define the following setting. Define $F(\lambda_0, \Sigma) : \mathbb{R} \times \mathcal{C}^1_{[0,1]} \to \mathcal{C}^1_{[0,1]}$ as

$$F(\lambda_0, \Sigma, t) = \Sigma(t) - \Sigma_0 + \int_0^t \frac{\lambda_0^2 e^{2\int_0^1 r_u du} \mu_s \sigma_s^2}{(2\lambda_0 e^{\int_0^t r_u du} + a_s(\Sigma(s) + \Omega)e^{-2\int_s^1 r_u du})^2}\, ds. \tag{B48}$$

Note that $\mathcal{C}^1_{[0,1]}$ and $\mathbb{R} \times \mathcal{C}^1_{[0,1]}$ are Banach spaces with usual (and product) norms and F is continuously Fréchet differentiable. Also, by monotonicity, $F_\Sigma(\lambda_0, \Sigma)^{-1} : \mathcal{C}^1_{[0,1]} \to \mathcal{C}^1_{[0,1]}$ exists as a continuous linear operator. By Equation (B47) we have

$$F(\lambda_0, \Sigma) = 0, \tag{B49}$$

where $0 \in \mathcal{C}^1_{[0,1]}$. So by an application of the Hildebrandt-Graves implicit function theorem we obtain that there exists a continuously Fréchet differentiable function $y : \mathbb{R} \to \mathcal{C}^1_{[0,1]}$ defined on a neighborhood in $\mathbb{R} \times \mathcal{C}^1_{[0,1]}$ such that $y(\lambda_0) = \Sigma$ and that $y'(\lambda_0, t) < 0$ for all $t \in [0,1]$. However, for finite λ_0,

$$\int_0^1 \frac{\lambda_0^2 e^{2\int_1^0 r_u du} \mu_t \sigma_t^2}{(2\lambda_0 e^{\int_0^t r_u du} + a_t(\Sigma(t) + \Omega) e^{-2\int_t^1 r_u du})^2} dt < \frac{1}{4} \int_0^1 \mu_t \sigma_t^2 e^{2\int_t^1 r_u du} dt \tag{B50}$$

holds. Hence, in order for Equation (B49) to be satisfied for $t = 1$,

$$\int_0^1 \mu_t \sigma_t^2 e^{2\int_t^1 r_u du} dt > 4\Sigma_0 \tag{B51}$$

must hold. Conversely, given that Equation (B51) is satisfied, since the left-hand side of Equation (B50) converges to the right-hand side as $\lambda_0 \to \infty$ and $y'(\lambda_0, 1) < 0$ and by continuity of F, there exists a unique $0 < \lambda_0 < \infty$ such that Equation (B49) is satisfied.

In order to complete the proof we have to revisit two issues that we mentioned above. First, we have to show the positivity of λ_0 and integrability of $\lambda(t)$. From Equation (B26) we have

$$E[\pi^{inf}(t) \mid \xi, \mathcal{F}_t] = \frac{1}{2} \left\{ \frac{(\xi_t - p(t))^2}{\lambda(t)} - \frac{S_p(\xi, X, t, 1)}{\lambda(t)} + \lambda(t) \int_t^1 \mu_s \gamma^2(s) \sigma_s^2 \, ds \right\}. \tag{B52}$$

Suppose λ_0 is not positive. Then by Equation (B14), by making $E[x(t)(\xi - P(t))] > 0$, the informed trader can ensure an arbitrarily large positive $S(\xi, X, t, 1)$ leading to arbitrarily large informed trader profits. Therefore we conclude that $\lambda_0 > 0$ must hold. The integribility of $\lambda(t)$ immediately follows from Equation (B32).

Finally, we have to show that $M_t(\tau)$ is an $\mathcal{F}$-martingale in order to verify the solution of the informed trader's problem. To see this, notice that for a given $\lambda_0 > 0$ and t in [0,1],

$$\int_0^t \frac{\sigma_p^2(s)}{\lambda^2(s)} ds \leq \int_0^t \frac{\mu_s \sigma_s^2 e^{2\int_t^1 r_u du}}{4\lambda_0} ds < \infty. \tag{B53}$$

The integrability of σ_p^2 can be shown similarly and the proof is complete. ■

Proof of Corollary 1. For $r = 0$, $\breve{\Sigma}(t) = \Sigma$ and Equation (B46) becomes

$$\Sigma'(t) = -\frac{\lambda_0^2 \mu_t \sigma_t^2}{(2\lambda_0^2 + a(\Sigma(t) + \Omega))^2}. \tag{B54}$$

Solving Equation (B54), we obtain

$$\Sigma(t) = \frac{1}{a} \left((2\lambda_0 + a(\Sigma_0 + \Omega))^3 - 3a\lambda_0^2 \int_0^t \mu_s \sigma_s^2 ds)^{-\frac{1}{3}} - (2\lambda_0 + a\Omega) \right). \tag{B55}$$

Setting $\Sigma(1) = 0$, we obtain

$$(2\lambda_0 + a\Sigma_0 + a\Omega)^3 - 3a\lambda_0^2 \int_0^1 \mu_s \sigma_s^2 ds - (2\lambda_0 + a\Omega) = 0. \tag{B56}$$

Reorganizing Equation (B56), we obtain the quadratic equation

$$3\left(4\Sigma_0 - \int_0^1 \mu_s \sigma_s^2 ds\right)\lambda_0^2 + 6a\Sigma_0(\Sigma_0 + 2\Omega)\lambda_0 + a^2\Sigma_0(3\Omega^2 + 3\Sigma_0\Omega + \Sigma_0) = 0, \tag{B57}$$

the solution of which is

$$\lambda_0 = a\Sigma_0 \frac{3(\Sigma_0 + 2\Omega) \mp \Sigma_0(3(\int_0^1 \mu_t \sigma_t^2 dt \cdot ((\Sigma_0^2 + 3\Omega(\Sigma_0 + \Omega))/\Sigma_0^3) - 1))^{\frac{1}{2}}}{3(\int_0^1 \mu_t \sigma_t^2 dt - 4\Sigma_0)}. \tag{B58}$$

An examination of Equation (B58) gives us that $\lambda_0 > 0$ can be satisfied only when $\int_0^1 \mu_t \sigma_t^2 dt > 4\Sigma_0$ and if that is the case, only one of the roots is positive. Specifically, we obtain Equation (22). Equations (24) and (25) follow directly from their counterparts in Proposition 2. ■

Proof of Proposition 3. We will give the proof only for r. The proof for the remaining parameters will be similar. Let us start our analysis by a particular form of increase on positive measure. Specifically, suppose r is replaced by $\hat{r}$, where for a $Z \subseteq [0,1]$ with positive Lebesgue measure and scalar $k > 1$, $\hat{r}(t) = k \cdot r(t)$ for $t \in Z$ and $\hat{f}(t) = f(t)$ otherwise. Similar to before, let us define the function $F : \mathbb{R} \times (\mathbb{R} \times \mathcal{C}^1_{[0,1]}) \to \mathcal{C}^1_{[0,1]}$ as

$$F(k, \lambda_0, \Sigma, t) = \Sigma(t) - \Sigma_0 + \int_0^t f(k, \lambda_0, \Sigma(s), s) ds, \tag{B59}$$

where

$$f(k, \lambda_0, \Sigma(s), s) = \frac{\lambda_0^2 e^{2\int_1^0 k r_u du} \mu_s \sigma_s^2}{\left(2\lambda_0 e^{\int_0^t k r_u du} + a_s(\Sigma(s) + \Omega) e^{-2\int_s^1 k r_u du}\right)^2}. \tag{B60}$$

Note that $\frac{\partial f}{\partial k} > 0$, $\frac{\partial f}{\partial \lambda_0} > 0$, and $\frac{\partial f}{\partial \Sigma(s)} < 0$. By Equation (B49), we have $F(k, \lambda_0, \Sigma) = 0$, where $0 \in \mathcal{C}_{[0,1]}$. Again, similar to before, $F(k, \lambda_0, \Sigma)$ satisfies the conditions for the Hildebrandt-Graves implicit function theorem and there exists a continuosly Fréchet differentiable $y : \mathbb{R} \to \mathbb{R} \times \mathcal{C}^1_{[0,1]}$ such that $y(k) = (\lambda_0, \Sigma)$. Taking the total Fŕechet derivative of $F(k, \lambda_0, \Sigma)$ we obtain

$$\int_0^t \frac{\partial f(k, \lambda_0, \Sigma(s), s)}{\partial k} \frac{\partial \lambda_0}{\partial k} ds + \frac{\partial \Sigma(t)}{\partial k} + \int_0^t \frac{\partial f(k, \lambda_0, \Sigma(s), s)}{\partial \Sigma(s)} \frac{\partial \Sigma(s)}{\partial k} ds = 0 \tag{B61}$$

for all t in [0, 1]. Using the fact that in equilibrium $\frac{\partial \Sigma(t)}{\partial k} = 0$ at $t = 0$ and $t = 1$, and continuity of the parameter functions around $t = 0$, we conclude that if $\int_0^t \frac{\partial f(k, \lambda_0, \Sigma(s), s)}{\partial k} ds > 0$, as it would be when r increases on positive measure in this specific way, then $\frac{\partial \lambda_0}{\partial k} \geq 0$ cannot hold. Moreover, unless such an increase happens on at least a subset of [0,1], λ_0 cannot change. Repeating the same analysis for $\lambda_0 e^{\int_0^1 r_s ds}$ in place of λ_0 proves the proposition for r for the specific class of increases on positive measure described above. To generalize the proof to all kinds of increases with positive measure, suppose r_1 and r_2 are given, where r_1 is on positive measure greater than r_2. Notice that for any given ε, by a finite number of increases in the above specified class to r_2, we can obtain $r_2(\varepsilon)$ that satisfies $\|r_1 - r_2(\varepsilon)\| < \varepsilon$. Since each step of such an increase decreases λ, it follows that $\lambda_{r_2(\varepsilon)} < \lambda_{r_2}$. Combining this with the continuity of λ as a function of r around r_1 completes the proof. ■

Proof of Proposition 4. From Equations (B26), (B27), (B31), and (B32) it follows that the ex ante profit of the informed trader is given by

$$\begin{aligned} E[\pi^{inf}(0)] &= E[E[\pi^{inf}(0) \mid \xi, \mathcal{F}_0]] \\ &= E\left[\frac{(\xi(0) - p(0))^2}{2\lambda_0}\right] + \frac{1}{2}\int_0^1 e^{-\int_0^s r_u du} \lambda(s) \mu_s \gamma^2(s) \sigma_s^2 \, ds \\ &= e^{-2\int_0^1 r_s ds} \frac{\Sigma_0 + \int_0^1 \lambda_0^2 e^{2\int_0^1 r_s ds} \mu_s \gamma^2(s) \sigma_s^2 ds}{2\lambda_0}. \end{aligned} \tag{B62}$$

By Equation (B47), and since $\Sigma(1) = 0$, we have

$$\Sigma_0 = \int_0^1 \lambda_0^2 e^{2\int_0^1 r_u du} \mu_s \gamma^2(s) \sigma_s ds, \tag{B63}$$

which, when combined with (B62), leads to the desired result. Part (*ii*) is proved by plugging in the equilibrium values for λ_0, and $\breve{\Sigma}$ from Corollary 1 for $r=0$ and constant a, μ, and σ^2, evaluating the integral. ■

Proof of Lemma 1. Part (*i*) follows directly from part (*ii*) of Proposition 4. Proof of part (*ii*) is tedious and available from the authors upon request. ■

Proof of Proposition 5. In the two-stage game, the strategy space of the informed trader will be

$$\chi = \{(\Sigma_0, X) : 0 \leq \Sigma \leq \Lambda,\ X \in \mathcal{H}^1\}. \tag{B64}$$

Notice that if $\Lambda > \mu\sigma^2/4$, it is feasible for the informed trader to choose Σ_0 higher than $\mu\sigma/4$. However, as shown above, in this case the λ_0 that satisfies market efficiency cannot be positive, which allows the informed trader to make arbitrarily large profits and precludes the existence of an equilibrium. Thus we conclude that there is no equilibrium if $\Lambda > \mu\sigma^2/4$. Now suppose that $\Lambda < \mu\sigma^2/4$. Then, for any chosen non negative $\Sigma_0 \leq \mu\sigma^2/4$ the profit maximizing strategy in the second stage results in $E[\pi_0^{inf}]$ being as given in Equation (30) less $c(\Sigma_0)$. Taking the first derivative of $E[\pi_0^{inf}]$ with respect to Σ_0 shows that $\left.\frac{\partial E[\pi_0^{inf}]}{\partial \Sigma_0}\right|_{\Sigma_0=0} = \infty$. Since c is convex, we conclude that there exists $\underline{\Lambda}$ such that if $\Lambda \leq \underline{\Lambda}$ then for all $\Sigma_0 \in [0, \Lambda]$, $\frac{\partial(E[\pi_0^{inf}] - c(\Sigma_0))}{\partial \Sigma_0} > 0$ will hold, implying $\Sigma_0^e = \Lambda$. This proves (*i*). Moreover by Lemma 1, for $\Lambda \geq \Lambda^*$ and some $\Lambda^* < \tilde{\Lambda} < \Lambda$, $\frac{\partial E[\pi_0^{inf}]}{\partial \Sigma_0} < 0$ on $[\tilde{\Lambda}, \Lambda]$. Therefore there exists a $\overline{\Lambda} \leq \Lambda^*$ such that when $\Lambda \geq \overline{\Lambda}$, the curves $\frac{\partial E[\pi_0^{inf}]}{\partial \Sigma_0}$ and $\frac{dc(\Sigma_0)}{d\Sigma_0}$ intersect and, since c is convex, they intersect only once at some $\Sigma_0 < \Lambda$, which gives the profit-maximizing level of Σ_0 for the informed trader. This proves (*ii*). It is straightforward to verify the subgame perfectness of the equilibrium. ■

Proof of Proposition 6. Assume $c=0$. Given $\Lambda \leq \mu\sigma^2/4$ and $\Sigma_0 \leq \Lambda$, the total profit of the informed trader will be given by Equation (30). Taking the derivative of Equation (30) on the region $0 \leq \Sigma_0 \leq \mu\sigma^2/4$ shows that the sign of the derivative of $E[\pi_0^{inf}(\Sigma_0)]$ is equal to the sign of $g(\Sigma_0) - h(\Sigma_0)$. Moreover, if $\Lambda \leq x^*$, then $g(\Sigma_0) - h(\Sigma_0) > 0$ on $[0, \Lambda]$; hence on this region the informed trader's best strategy is choosing $\Sigma_0 = \Lambda$. When $\Lambda > \mu\sigma^2/4$, however, the value of Σ_0 that makes $g(\Sigma_0) - h(\Sigma_0) = 0$ is in $[0, \min\{\Lambda, \frac{\mu\sigma^2}{4}\}]$ and by Lemma 1, this Σ_0 is the maximizer of Equation (1) and hence it is the equilibrium Σ_0. The rest of the statement follows from the implicit function theorem. ■

Proof of Proposition 7. Substituting Equation (24) with constant μ and σ^2 in Equation (B34) and plugging in the utility function we obtain

$$W_{j,t}^{liq} = u_{jt}^2 (2((2\lambda_0 + a(\Sigma_0 + \Omega))^3 - 3a\lambda_0^2\mu\sigma^2 t)^{1/3})^{-1}. \tag{B65}$$

By taking the expectation over u_{jt}, we then obtain that the total expected welfare of the liquidity traders is

$$E[W^{liq}] = \int_0^1 \frac{\mu\sigma^2}{2\sqrt[3]{(2\lambda_0 + a(\Sigma_0 + \Omega))^3 - 3a\lambda_0^2\mu\sigma^2 t}}\, dt. \tag{B66}$$

Evaluating the integral gives Equation (34) and this proves part (*i*). To prove part (*ii*), notice that

$$E[W^{liq}] = E[\pi^{inf}(\Sigma_0)]\left(1 + \frac{a(2\Lambda - \Sigma_0)}{4\lambda_0}\right) \tag{B67}$$

and

$$\left.\frac{\partial W^{liq}(\Sigma_0)}{\partial \Sigma_0}\right|_{\Sigma_0=0} > 0. \tag{B68}$$

From Equation (B68), $\Sigma_0^w > 0$ follows. If $\Lambda \leq x^*$, where x^* is as defined earlier, then by Proposition (6), we have $\Sigma_0^i = \Lambda$ and hence $\Sigma_0^w \leq \Sigma_0^i$. Now suppose $\Lambda > x^*$. Taking the derivative of $W^{liq}(\Sigma_0)$ with respect to Σ_0 using Equation (B67) gives

$$\frac{\partial}{\partial \Sigma_0} E[W^{liq}] = \frac{\partial}{\partial \Sigma_0} E[\pi^{inf}(\Sigma_0)]\left(1 + \frac{a(2\Lambda - \Sigma_0)}{4\lambda_0}\right) + E[\pi^{inf}(\Sigma_0)]\frac{\partial}{\partial \Sigma_0}\left(\frac{a(2\Lambda - \Sigma_0)}{4\lambda_0}\right). \tag{B69}$$

Plugging $\Sigma_0 = \Sigma_0^i$ in Equation (B69) makes the first term zero. Since the second term is always negative,

$$\left.\frac{\partial W^{liq}(\Sigma_0)}{\partial \Sigma_0}\right|_{\Sigma_0=\Sigma_0^{inf}} < 0. \tag{B70}$$

Combining this with the unimodality of $E[\pi^{inf}(\Sigma_0)]$ in Σ_0 and the fact that $1 + \frac{a(2\Lambda - \Sigma_0)}{\lambda_0}$ is decreasing in Σ_0 implies that $\Sigma_0^w < \Sigma_0^i$ for $\Lambda > x^*$. This completes the proof. ■

Proof of Proposition 8. Extend the profit function given in Equation (36) to the region $\Lambda \geq \Sigma_r > \Sigma_0$ by defining $\pi^{inf}(\Sigma_0, \Sigma_r) = 0$ in this region. The strict positiveness of Ω_0 ensures the continuity of the objective function as $\Lambda \to 0$ and on the junction $\Sigma_0 = \Sigma_r$. In the rest of the domain, by its definition the objective is continuous. Combining this with the compactness of the domain $[0, \Lambda]^2 \subset \mathbb{R}^2$ ensures the existence of a solution to the informed trader's problem, and hence a subgame perfect equilibrium. The proof of part (*i*) follows immediately from costless expansion of the feasible domain by acquisition of more information. The first statement in part (*ii*) follows from the fact that $\frac{\partial}{\partial \Sigma_0} E[\pi_0^{inf}(\Sigma_0, \Sigma_r^1)] \to \infty$ as $\Sigma_0, \Omega_0 \to 0$ and the assumption of convexity of the cost function c. To show the second statement, we first claim that for given Λ and Ω_0, and two cost functions c_1 and c_2, if $\Sigma_0(\Lambda, \Omega_0, c_1) = \Sigma_0(\Lambda, \Omega_0, c_2)$, then $\Sigma_r(\Lambda, \Omega_0, c_1) = \Sigma_r(\Lambda, \Omega_0, c_2)$. Suppose the contrary. Then for two cost functions c_1 and c_2, optimal acquisition and release levels are given by (Σ_0, Σ_r^1) and (Σ_0, Σ_r^2) for some Σ_0, Σ_r^1 and Σ_r^2, and

$$E[\pi_0^{inf}(\Sigma_0, \Sigma_r^1)] - c_1(\Sigma_0) \;\geq\; E[\pi_0^{inf}(\Sigma_0, \Sigma_r^2)] - c_1(\Sigma_0) \tag{B71}$$

$$E[\pi_0^{inf}(\Sigma_0, \Sigma_r^2)] - c_2(\Sigma_0) \;>\; E[\pi_0^{inf}(\Sigma_0, \Sigma_r^1)] - c_2(\Sigma_0) \tag{B72}$$

both hold. As can be seen by canceling the cost components from both sides of Equations (B71) and (B72), this is a contradiction and we conclude that $\Sigma_r(\Lambda, \Omega_0, c_1) = \Sigma_r(\Lambda, \Omega_0, c_2)$ must hold. Given this, the second statement of this part follows immediately from (*i*) and the first statement of (*ii*). ■

Proof of Proposition 9. Denote the market's perceived Σ_0 by Σ_0^m. Plugging $r = a = 0$ in Equation (B62) with constant μ and σ and utilizing Equation (22), we obtain

$$E[\pi_0^{inf}] - c(\Sigma_0) = \Psi(\Sigma_0^m)(\Sigma_0 + \Sigma_0^m) - c(\Sigma_0). \tag{B73}$$

Therefore the informed trader's problem is to maximize Equation (B73) subject to $0 \leq \Sigma_0 \leq \Lambda$. Now,

$$\frac{d}{d\Sigma_0}(E[\pi_0^{inf}] - c(\Sigma_0)) = \Psi(\Sigma_0^m) - c'(\Sigma_0). \tag{B74}$$

The convexity of c implies that $E[\pi_0^{inf}] - c(\Sigma_0)$ is concave. Moreover, rational expectations imply that $\Sigma_0^m = \Sigma_0 = \Sigma_0^{en}$. By taking the first derivative and carrying out the algebra, we can show that $\Psi(\Sigma_0)$ is monotonically decreasing. Therefore, again by convexity of c, if $\Psi(\Lambda) \geq c'(\Lambda)$, then for any $0 \leq \Sigma_0^m \leq \Lambda$, we have $\Psi(\Sigma_0^m) \geq c'(\Sigma_0)$. Further, given $\Sigma_0^m = \Lambda$, $\Sigma_0 = \Lambda$ is the solution to the informed trader's problem. Thus we conclude that if $\Psi(\Lambda) \geq c'(\Lambda)$, then there exists a unique equilibrium in which $\Sigma_0^{en} = \Lambda$. This proves part (*i*).

To show part (*ii*), notice that $\Psi(0) = \infty$ and $\Psi(\mu\sigma^2/4) = 0$. Since $\Psi(\Sigma_0)$ is monotonically decreasing and c is convex, $\Psi(\Lambda) < c'(\Lambda)$ implies that there is a unique solution to the equation $\Psi(\Sigma_0) - c'(\Lambda) = 0$ in the region $0 < \Sigma_0 < \Lambda$. This solution gives us the unique Σ_0 that solves the informed trader's problem when $\Sigma_0^m = \Sigma_0$ also holds. It follows that this Σ_0 is the information acquisition level in the unique equilibrium in this case. ■

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