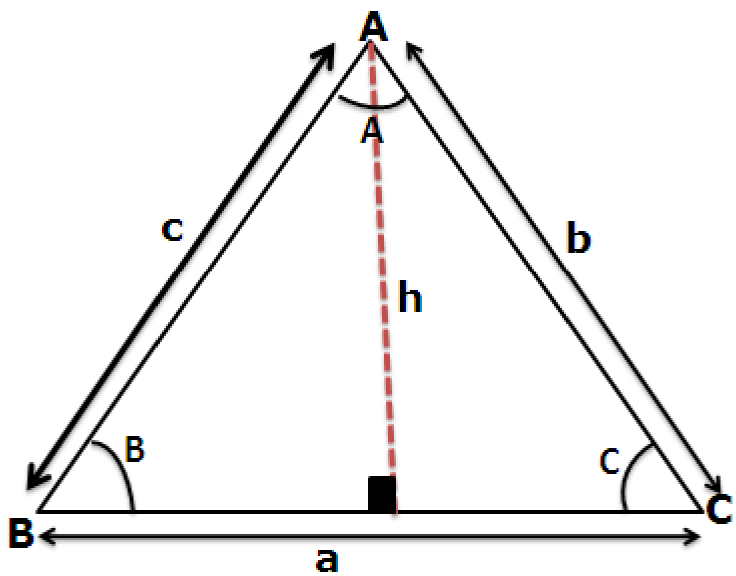
Area Of Triangles And Properties Related To Areas Of Triangles:

Area of a Triangle:



1. Area of ΔABC = ½ * Base * Height

   = ½ * a * h

2. Heron's Formula:

Area = $\sqrt{s(s-a)(s-b)(s-c)}$

Where, $s = \frac{a+b+c}{2}$ is the semi perimeter of ΔABC

3. Area = ½ * b* c* sinA
   = ½ * c * a * sinB
   = ½ * a* b * sinC

Q1. The sides of a triangle are 5,12 and 9 cms. Then, the altitude to the smallest side measures-

1. $\frac{2\sqrt{26}}{5}$ 2. $\frac{4\sqrt{26}}{5}$ 3. $\frac{8\sqrt{26}}{5}$ 4. $\frac{12\sqrt{26}}{5}$

Solution: The sides of the triangle are 5, 12 and 9 cms.

Semi perimeter = s = (5+12+9)/2 = 13

$Area = \sqrt{13 * (13-12)(13-5)(13-9)} = \sqrt{13*1*8*4} = 4\sqrt{26}$

Also we know that Area = ½ * base * height

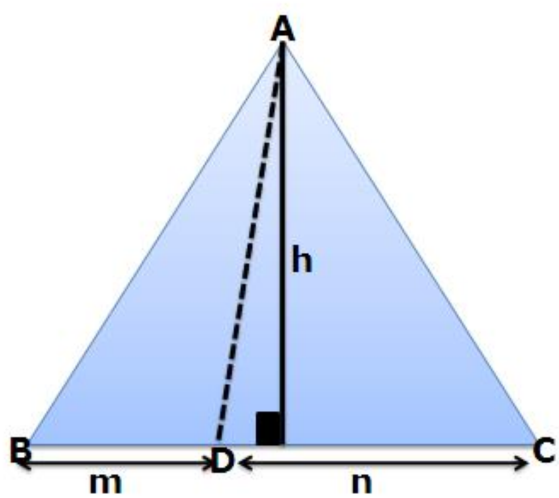
Here base = 5 cm {Smallest side}

$4\sqrt{26}$ = ½ * 5* h

h = $\frac{8\sqrt{26}}{5}$

Property:

In ΔABC, AD divides the base in two parts BD and CD. Let the length of BD and CD be m and n respectively.



Now, Area of ΔABD = ½ * BD * h (1)

Area of ΔADC = ½ * CD * h (2)

Divide (1) by (2)

$$\frac{Area(\Delta ABD)}{Area(\Delta ADC)} = \frac{BD}{CD} = \frac{m}{n}$$

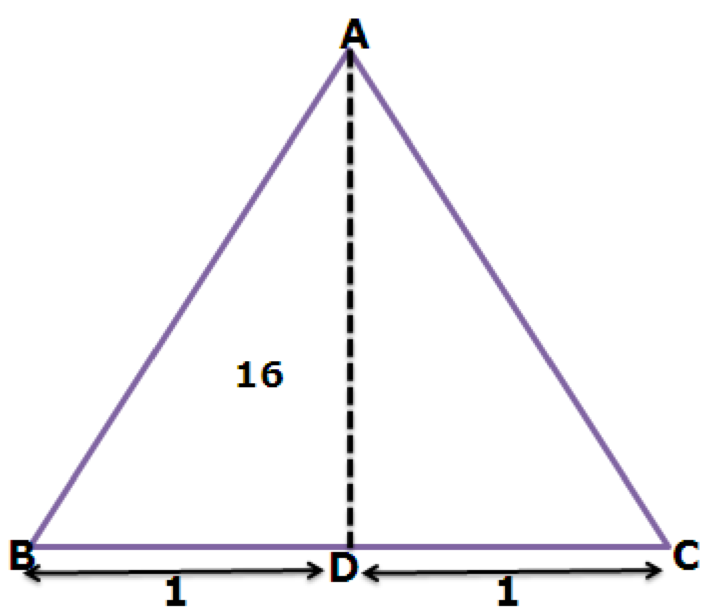
Thus, area of two triangles is in the ratio in which the base is divided given that the heights of two triangles is the same.

Note: Here we had considered 'm' and 'n' to be the lengths of BD and CD respectively. Even if the ratio in which the base is divided is given, then also the ratio of areas would be the same as the ratio in which the base is divided.

Q2. If D is the mid-point of side BC of ΔABC and the area of ΔABD is $16cm^2$, then the area of ΔABC is **(SSC- CGL 2012)**

1. $16cm^2$
2. $24cm^2$
3. $32\ cm^2$
4. $48\ cm^2$

Solution: As D is the mid-point of BC. Therefore BD = CD or BD:CD is 1:1.



So Area (ΔABD)/Area(ΔADC) = BD/CD = 1/1

Area(ΔADC) = Area(ΔABD) = 16

Hence, Area(ΔABC) = Area(ΔADC) + Area(ΔABD) = 16 + 16 = 32 units.

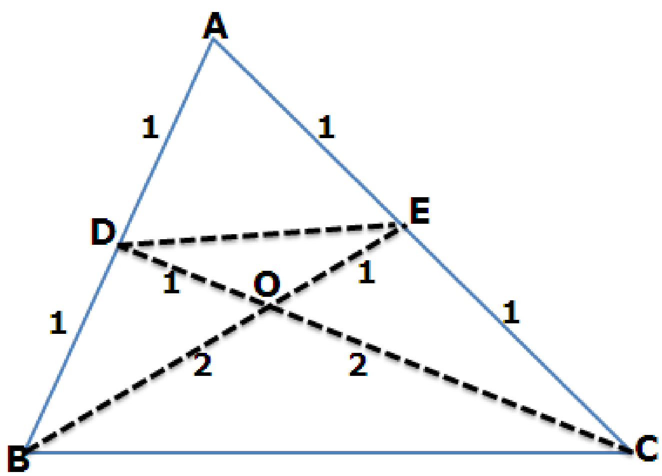
Q3. ABC is a triangle. The medians CD and BE intersect each other at O. Then ΔODE:ΔABC is **(SSC- CGL 2012)**

1. 1:3
2. 1:4
3. 1:6
4. 1: 12

Solution:

Note :-All the numbers given in the figure represent the ratio in which a particular side is divided.

As CD and BE are the medians, therefore they bisect AB and AC respectively. Also o is the centroid and we know that centroid divides the medians in the ratio of 2:1. Therefore OC : OD is 2:1 and also BO:OE is 2:1.

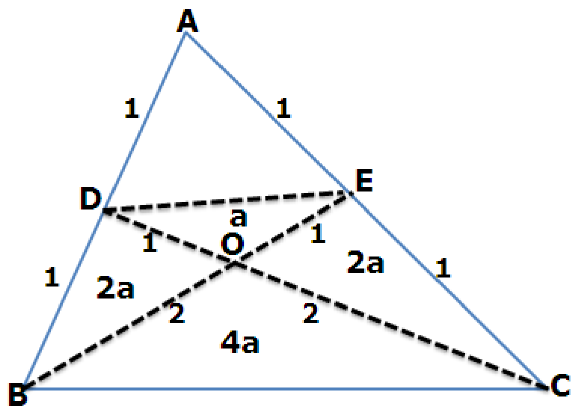


Now let the area of ΔODE be 'a' units.

Consider ΔBDE in which OB: OE is 2:1 and so the areas of ΔBDO and ΔDOE area in the ratio of 2:1. So, area(ΔBOD) = 2a

Now consider ΔDBC in which OC: OD is 2:1. So the area of ΔBOC and ΔBOD are in the ratio of 2:1. So, area ΔBOC = 2* Area(ΔBOD)= 4a

Thus, area of ΔDBC = Area(ΔBOD) + Area(ΔBOC) = 2a + 4a = 6a



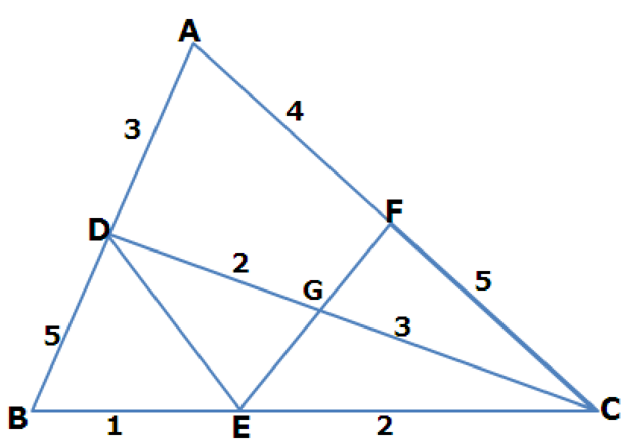
As CD is the median and we know that the median divides the area of triangle in two equal parts.

Thus, area (ΔABC) = 2 * area(ΔDBC) = 2*6a = 12a

Thus, area(ΔDOE)/Area(ΔABC) = a/12a = 1:12

Q4. In the given figure all the numbers represent the ratio in which a particular line segment is divided. For e.g. BD:AD = 5:3. If the area of ΔDEG is 20 units. What is the area of ΔGFC?

1. 12 2. 9 3. 15 4. 18



Solution: Consider ΔDEC, DG : GC = 2:3 , and so

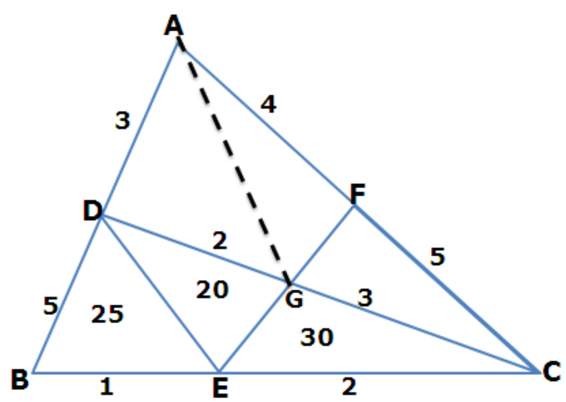
Area(ΔDEG)/Area(ΔGEC) = 2/3

Area (ΔGEC) = 30 units.

In ΔBDC, BE: EC = 1:2 , therefore Area(ΔBDE) = ½ Area(ΔDEC)

Area(ΔBDE) = ½ * 50 = 25 units.

Join A to G as shown in the figure



Area (ΔDBC) = 25 + 20 + 30 = 75 units

In ABC, CD divides AB in the ratio of 5:3

Area of (ΔADC) = (3/5)* 75 = 45 units

Consider (ΔADC)

DG: GC = 2:3

Area(ΔADG): Area(ΔAGC) = 2: 3

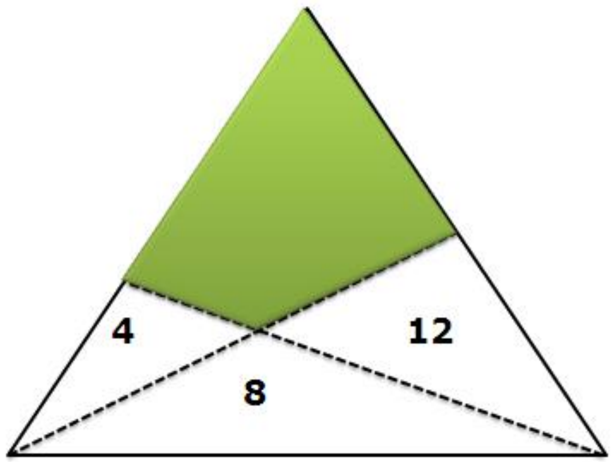
Area(ΔADG) = (2/5)* 45 = 18 and Area(ΔAGC) = 27 units

Now consider ΔAGC, AF : FC = 3:1

And so Area(ΔAGF) : Area(ΔGFC) = 3: 1

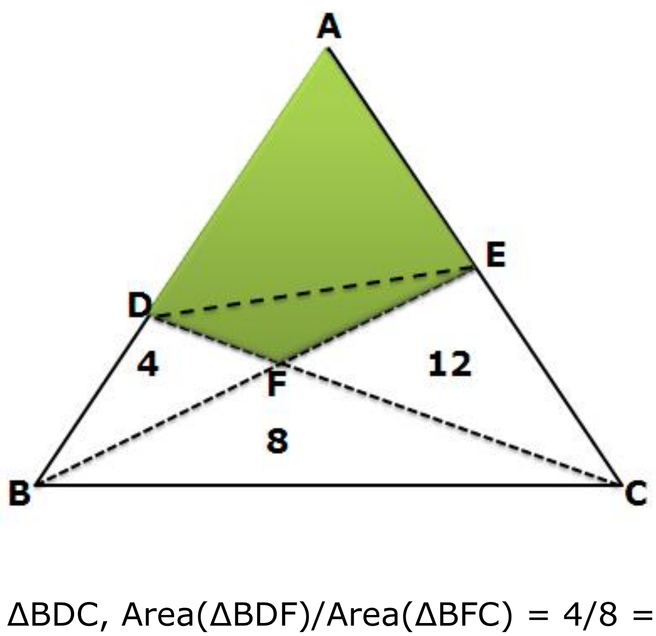
Hence Area(ΔGFC) = (5/9)* 27 = 15 units.

Q5. In given figure, areas of three regions are given. What is the area of the fourth region?



1. 2. 3. 4.

Solution: Naming the figure as shown below. Join D to E.

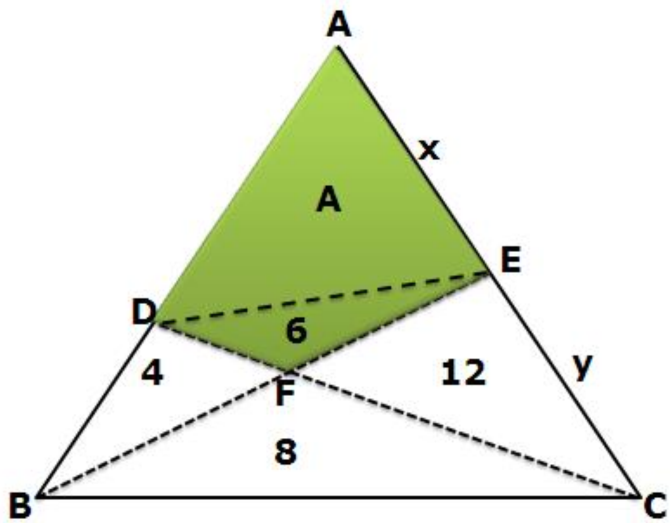


In ΔBDC, Area(ΔBDF)/Area(ΔBFC) = 4/8 = 1:2

So FD : FC = 1:2

In ΔDEC, DF : FC = 1:2 , So, Area(ΔDEF)/Area(ΔFEC) = 1:2

Hence, area(ΔDEF) = 6 units.



Now consider ΔADC, Let AE:EC = x:y

Area(ΔADE)/Area(ΔDEC) = A/(6+12) = x/y (1)

Now consider ΔABC, AE : EC = x: y

Area (ΔABE)/Area(ΔBEC) = (A+4+6)/(8+12) = x/y (2)

From (1) and (2) we get

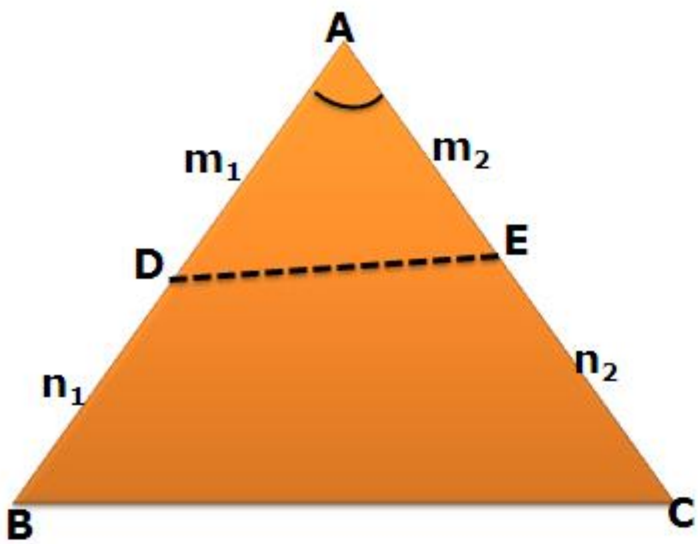
A/18 = (A+10)/20

Or A = 90

Thus, area of shaded region = 6 + 90 = 96 units.

Property:

Consider a triangle ABC in which DE divides AB such that AD = m_1 , BD = n_1, AE = m_2 , EC = n_2.



Area(ΔADE) = ½ *m_1 * m_2 * sinA (1)

Area (ABC) = ½ * (m_1+n_1)*(m_2+n_2) * sinA (2)

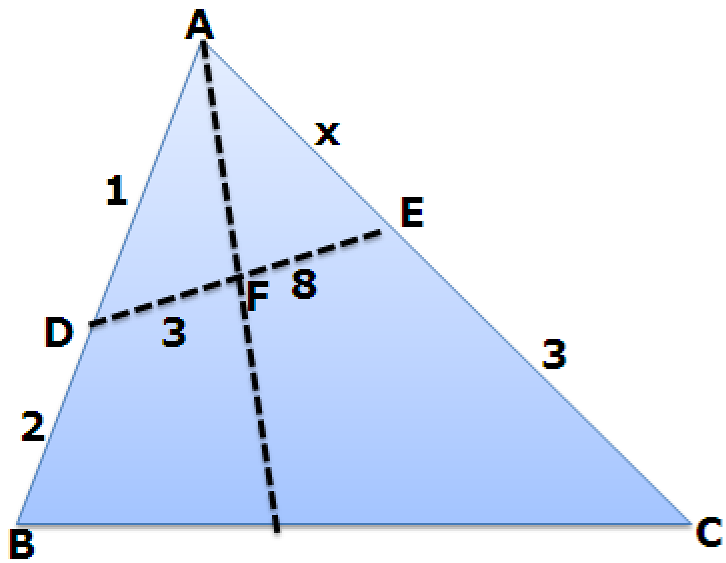
Divide (1) by (2) we get

$$\frac{Area(\Delta ADE)}{Area(\Delta ABC)} = \frac{m_1}{(m_1 + n_1)} * \frac{m_2}{(m_2 + n_2)}$$

<u>Note:</u> Here we have taken m_1, n_1 m_2 and n_2 as the lengths, even if we take them as the ratios in which sides are divided, still the ratio of areas would remain the same.

Q6. In the given figure, the numbers represent the ratios in which the sides are divided. If the area of ΔADF is 15 units and the area of ΔABC is 264 units, then the value of x is –

1. 2 2. 5 3. 7 4. 8



Solution: Area(ADF) = 15 units. Also DF : FE = 3: 8

So, area (AFE) = 40 units.

So, area of (ADE) = Area(ADF) + Area(AFE) = 15+40 = 55 units.

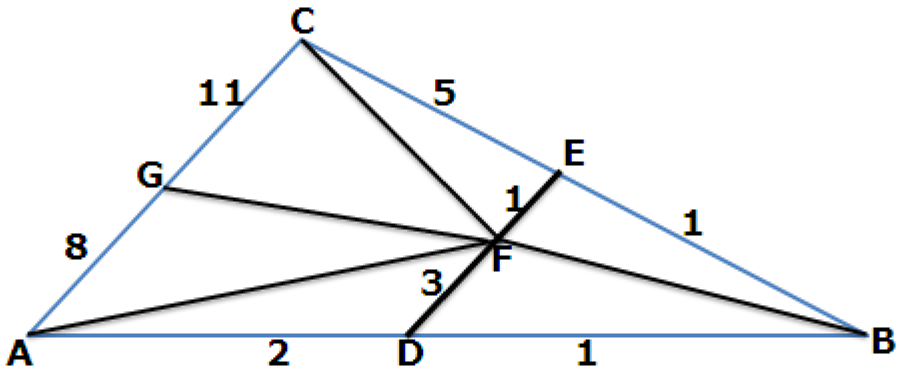
Also Area(ADE)/Area(ABC) = $\frac{1}{(1+2)} * \frac{x}{(x+3)} = \frac{x}{3(x+3)}$

$$\frac{55}{264} = \frac{x}{3(x+3)}$$

Solving we get x = 5

Q7. In the given figure all the numbers represent the ratio in which a particular side is divided. e.g. AD:DB is 2:1. What is the ratio of area of ΔAGF to area of ΔABC?

1. 1:3 2. 3: 8 3. 3:5 4. 4: 11



Solution: Let the area of ΔAFD = 6a

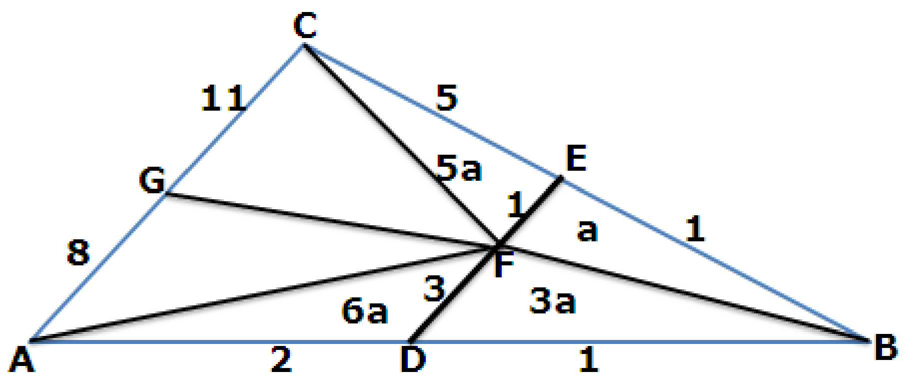
So, area of ΔDFB = 3a { As AD : DB is 2:1 }

Consider ΔBED, FD : FE is 3:1, so area (ΔFBD)/area (ΔBEF) = 3/1

Since area of ΔDFB = 3a, so area of ΔBEF =a

Consider Δ BFC, BE:EC is 1:5

So, area of ΔEFC = 5*Area of ΔBEF = 5a



$$\text{Now,}\frac{\text{Area}(\Delta\text{BED})}{\text{Area}(\Delta\text{ABC})} = \frac{1}{(1+2)} * \frac{1}{(1+5)} = \frac{1}{18}$$

$$\frac{4\text{a}}{\text{Area}(\Delta\text{ABC})} = \frac{1}{18}$$

Or Area(ΔABC) = 72 a

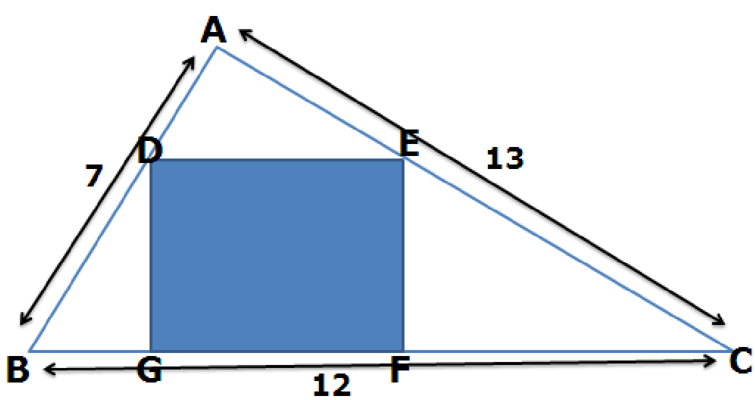
Thus, area of (ΔAFC) = 72a – 6a – 3a – a - 5a = 57a

Now In ΔAFC, AG : GC is 8:11, Hence

$$\text{Hence, Area}(\Delta \text{AFC}) = \frac{8}{(8+11)} * 57\text{a} = 24\text{a}$$

$$\text{Therefore } \frac{\text{Area}(\Delta\text{AFC})}{\text{Area}(\Delta\text{ABC})} = \frac{24\text{a}}{72\text{a}} = \frac{1}{3}$$

Q8. In ΔABC, AB = 7, BC = 12 and AC = 13 units. A square is inscribed in it and one side of the square lies along BC and the other two vertices lies on the other two sides as shown in the figure. What is the length of the side of the square?



Solution: First we find the area of ΔABC.

$$s = \frac{7 + 12 + 13}{2} = 16$$

$$A = \sqrt{16 * (16-7) * (16-12) * (16-13)} = 24\sqrt{3}$$

A = ½ * base * height

24√3 = ½ * 12 * h

Or h = 4√3

Now let the side of the square be x units.

Area(ΔABC) = Area(ΔADE) + Area(trapezium BDEC)

Height of ΔADE = ½ * x * (4√3 – x)

Area of trapezium BDEC = ½ * x * (x + 12)

{ Area of trapezium = ½ * height * Sum of parallel sides}

$$24\sqrt{3} = \frac{1}{2} * x * \left(4\sqrt{3} - x\right) + \frac{1}{2} * x * (x + 12)$$

$$48\sqrt{3} = 4\sqrt{3}x - x^2 + x^2 + 12x$$

$$x = \frac{12\sqrt{3}}{3 + \sqrt{3}}$$

$$x = 2\sqrt{3}(3 - \sqrt{3})$$

Or x = 6(√3 -1)