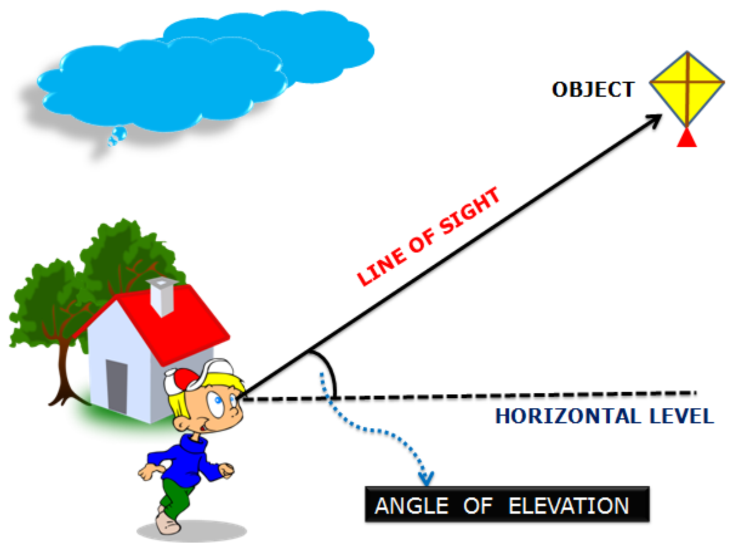
<u>**Heights And Distances**</u>

The chapter on Heights and Distances involves the application of Trigonometric Ratios in solving problems related to the calculation of heights and distances of various objects without actually measuring them. It finds its place in Astronomy and Geography where the distances between celestial bodies can be calculated and also heights of various mountain peaks can be determined by using simple trigonometric ratios. But, before we start solving problems, let's discuss some basic concepts and terms that we shall encounter while studying this chapter.

<u>**Angle of Elevation:**</u>

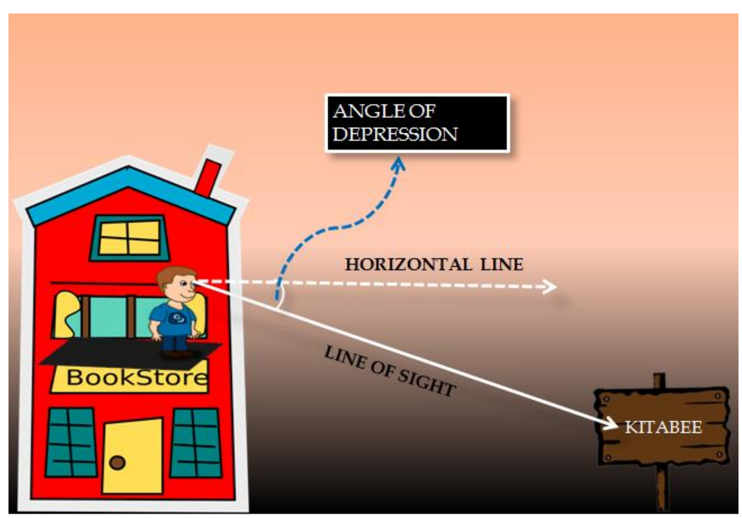
As shown in the figure, Line of sight is the line from the eye of the observer to the object being viewed.

Angle of elevation is the angle made by the line of sight with the horizontal. The object to be viewed must be above the horizontal.



<u>**Angle of Depression:**</u>

Angle of depression is the angle formed by the line of sight and the horizontal level when the object to be viewed is below the horizontal level.



With the help of a few solved examples you will understand how to make use of trigonometric ratios to find the height of objects or the distance between the objects involved in the questions.

Example 1: A small boy is standing 30 m away from a toy shop. The angle of elevation of the top of the shop from the eye of the boy is 60^0. What is the height of the shop?

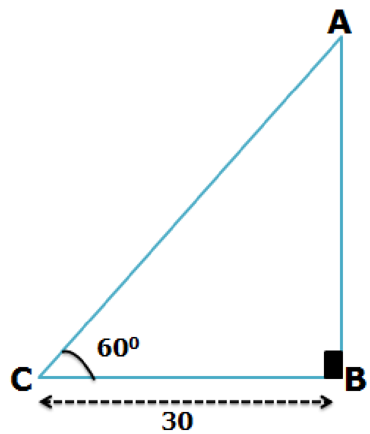
Solution: As shown in the figure, let AB be the shop and the boy is standing at a point C on the ground.

$\tan 60^0 = \frac{AB}{BC}$

Or $\sqrt{3} = \frac{AB}{30}$

Or AB = $30\sqrt{3}$

Hence, height of the shop is $30\sqrt{3}$ m



Example 2: The angles of elevation of a tower from the top and the bottom of a building are 30^0 and 45^0 respectively. If the height of the building is 10m, find the height of the tower.

Solution: Let AB be the building and CD be the tower as shown in the figure. Also AE ||BD.

Let the length of CD be x.

In ΔBDC, $\tan 45^0 = \frac{CD}{BD} = \frac{x}{BD}$

Or BD = x (As $\tan 45^0 = 1$)

Now consider ΔCAE.

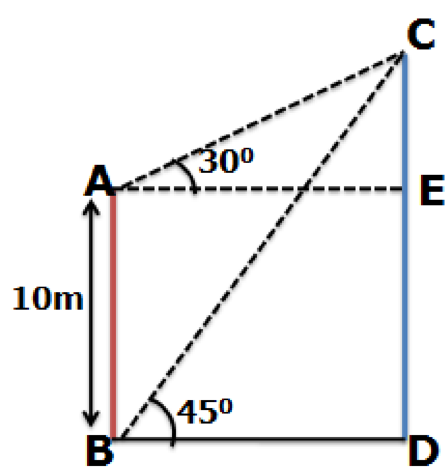
$\tan 30° = \frac{CE}{AE} = \frac{CE}{BD} = \frac{CE}{x}$

Or CE $= \frac{x}{\sqrt{3}}$ (As $\tan 30^0 = 1/\sqrt{3}$)

CD = ED + CE = 10 + $x/\sqrt{3}$

Or x = 10 + $x/\sqrt{3}$

Or x = $\frac{10\sqrt{3}}{(\sqrt{3}-1)} = 5\sqrt{3}(\sqrt{3}+1)$



Example 3: During heavy rain, a 15 m long tree broke down. The broken part of the tree formed a 60^0 angle of depression. Find the distance between the top and bottom of the tree.

Solution:

Let AB be the original tree and C be the point from which the tree broke. Let BC = x, so AC= CD = 15-x

In ΔCBD, $\sin 60^0$ = BC/CD

$$\frac{\sqrt{3}}{2} = \frac{x}{15 - x}$$

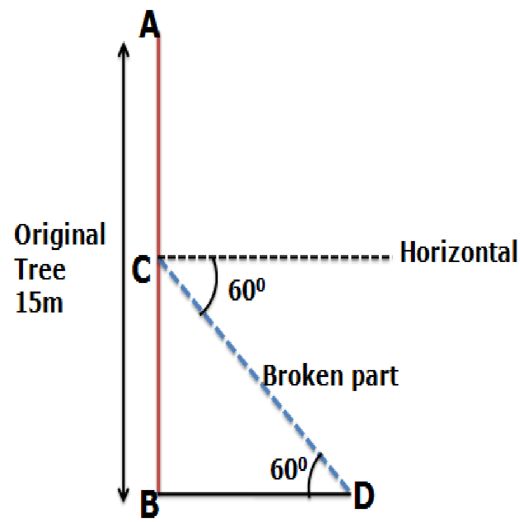
Or $x = \frac{15\sqrt{3}}{2+\sqrt{3}}$

Tan60 = BC/BD

Or BD = BC* $\cot 60^0$

BD $= \frac{15\sqrt{3}}{2+\sqrt{3}} * \frac{1}{\sqrt{3}} = \frac{15}{2+\sqrt{3}} = 15(2-\sqrt{3})$

Distance between the top and bottom $= 15(2-\sqrt{3})$



Example 4: From the top of a hill the angle of elevation to the top of a mountain is equal to the angle of depression to the foot of the mountain. Find a relation between the heights of the hill and the mountain.

Solution:

Let the height of the hill be 'h' and that of mountain be H. Let BD = x

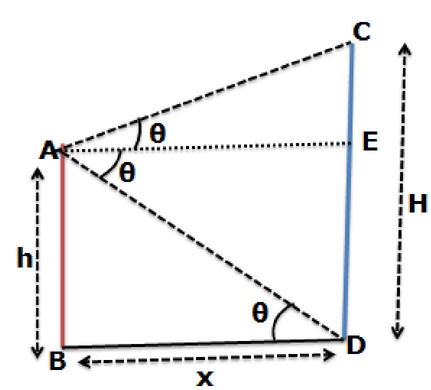
In ΔACE , $\tan\theta = \frac{CE}{AE} = \frac{H-h}{x}$ (1)

In ΔABD, $\tan\theta = \frac{AB}{BD} = \frac{h}{x}$ (2)

From (1) and (2)

$$\frac{H-h}{x} = \frac{h}{x}$$

Or h = H/2



Note: Whenever such a condition is achieved, it means the top and the bottom are both equidistant from the given point. This implies point A is exactly placed in the midpoint of BC. Thus, ED = CE. So h = H/2.

Example 5: An 8 m long rod is kept at an angle of 30^0 with a wall. The rod slides down after some time and the new angle made by the rod with the wall is 45^0. Find how much distance is travelled by the bottom of the rod?

Solution:

AB = A'B' = 8m (Length of the rod)

Here we have to find the length of B'B

In ΔABO, $cos60° = \frac{1}{2} = \frac{OB}{AB} = \frac{OB}{8}$

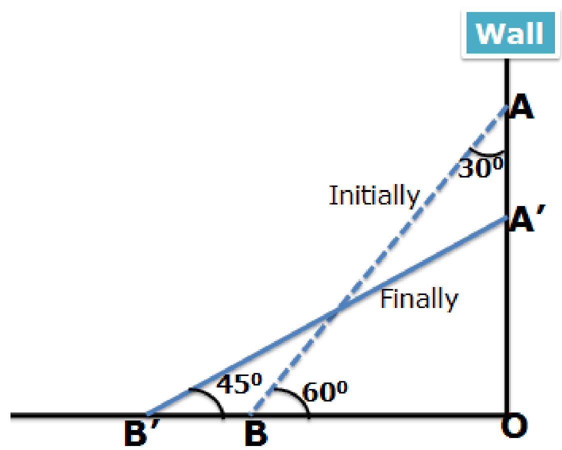
Thus OB = 4 units.

Consider ΔA'B'O

$cos45° = \frac{1}{\sqrt{2}} = \frac{OB'}{A'B'}$

O'B' = 4√2

Thus, B'B = 4√2 – 4 = 4(√2-1) m



Example 6: The angle of elevation of the top of a building is 60^0. I walked for some time and the angle of elevation again became 60^0. If the height of the building is 10m. What is the distance that I walked?

Solution:

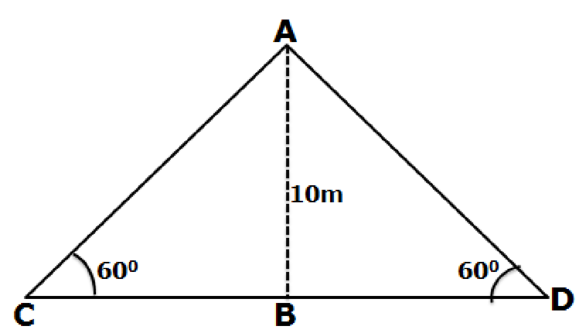
The person walks from C to D and AB is building. Now we can clearly see that ΔCBA is congruent to ΔDBA.

Thus, BC = BD

In ΔACB, tan600 = AB/BC = 10/BC

So BC = $\frac{10}{\sqrt{3}}$

CD = 2BC = $\frac{20}{\sqrt{3}}$



Example 7: An airplane is flying 10km above the ground level. From a point on the ground the angle of elevation of the plane is 45^0. After 5 seconds the angle of elevation changes to 30^0 from the same point. Find the speed of the airplane. (Take the value of √3 = 1.732)

Solution:

Consider ΔBCE,

$$tan45° = 1 = \frac{BC}{CE} = \frac{10}{CE}$$

Or CE = 10 km

Now consider ΔADE

$$tan30° = \frac{1}{\sqrt{3}} = \frac{AD}{DE} = \frac{10}{DC + 10}$$

DC + 10 = 10√3

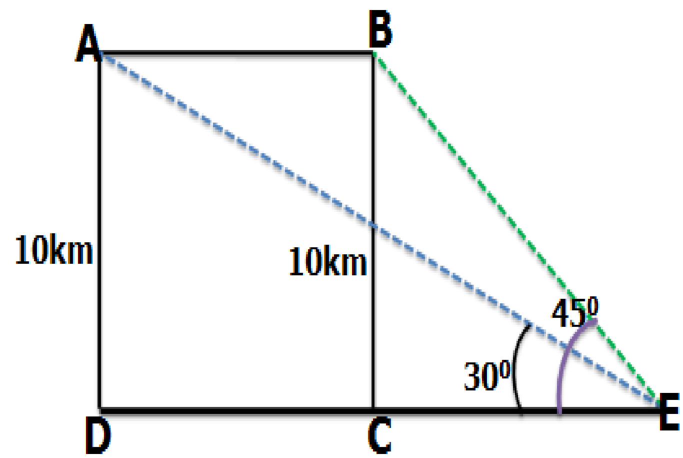
Or DC = AB = 10(√3-1)

= 10 (1.732-1)= 7.32 km

Speed = Distance/time

Speed of airplane in km/sec

= 7.32/5 = 1.464 km/sec



Example 8: A pole of height 50 ft has a shadow of length 50 ft at a particular instance of time. Find the angle of elevation of the sun at this point of that particular instant.

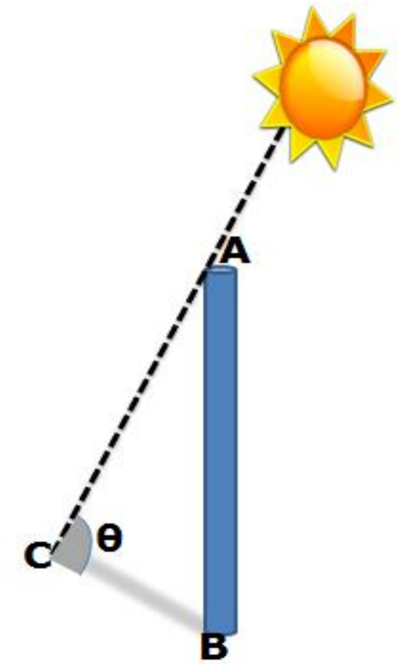
Soluion:

Here the endpoint of shadow, top of the object and the sun are in one straight line.

So, $\tan\theta = AB/BC = 50/50 = 1$

This implies $\theta = 45^0$

Thus, the angle of elevation of the sun is 45^0.



Example 9: Two men standing on the same side of the building have an angle of elevation of 60^0 and 45^0 to the top of the building. If the distance between them is 3m, find the length of the building. (Take $\sqrt{3} = 1.732$)

Solution:

Let AB be the building of height h and let BC be x units.

Consider Δ ABC,

$tan60° = \sqrt{3} = \frac{AB}{BC} = \frac{h}{x}$ (1)

Consider ΔABD,

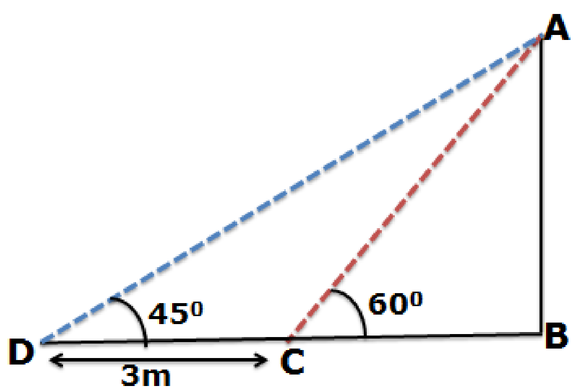
$$tan45° = 1 = \frac{AB}{BD} = \frac{h}{x+3}$$

Or h = x+3 (2)

From (1) and (2) we get

$$\sqrt{3} = \frac{h}{h-3}$$

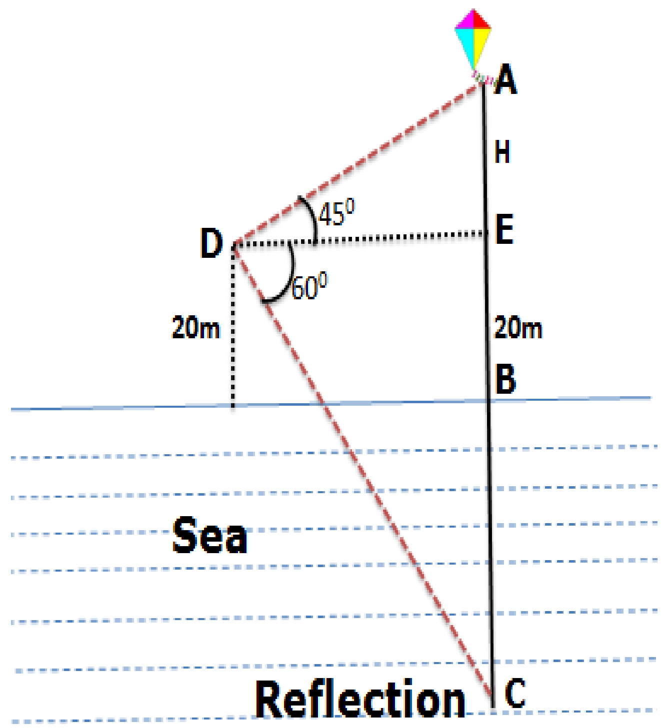
$Or\ h = \frac{3\sqrt{3}}{(\sqrt{3}-1)} = \frac{3\sqrt{3}(\sqrt{3}+1)}{2}$ = 7.1 m approx



Example 10: The angle of elevation of a kite from a point 20 m above the surface of a sea is 45^0. The angle of depression of the reflection of kite in the sea is 60^0 from the observation point. Find the distance between the kite and the observation point.

Solution:

Let D be the observation point and A be the kite. C is the reflection of the kite in the sea.



The distance of the kite from the surface of water is equal to the distance of its reflection from the surface, i.e. AB = BC

Consider ΔADE,

$$tan45° = 1 = \frac{AE}{DE} = \frac{H}{DE}$$

So, H = DE

Consider ΔDEC,

$$tan60° = \frac{EC}{DE} = \frac{BC + EB}{H} = \frac{AB + 20}{H}$$

$$\sqrt{3} = \frac{H + 20 + 20}{H} = \frac{H + 40}{H}$$

Or H = $20(\sqrt{3}+1)$

AD = $H*sec45^0 = 20\sqrt{2}(\sqrt{3}+1)$