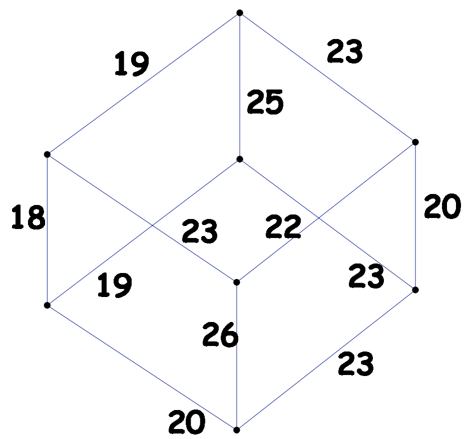
EXERCISES

1. Find a minimal spanning tree for the graph on the right.



2. The table to the right gives a describes a graph. An asterisk (*) indicates an edge of infinite weight. Use Kruskal's algorithm to find a minimal-weight spanning tree.

	A	B	C	D	E	F	G
A	*	5	8	7	*	*	*
B		*	5	*	4	5	*
C			*	2	2	*	3
D				*	*	*	2
E					*	3	1
F						*	3
G							*

3. (**Efficient upper and lower bounds for Hamiltonian cycles of minimal weight**) In this exercise we show how to obtain reasonable upper and lower bounds for the minimal weight of a Hamiltonian cycle in a weighted graph G.

   (a) **(Lower bound)** Notice that if we remove an edge from a Hamiltonian cycle we get a spanning tree. Therefore do this:

      i. Delete a vertex v and all the edges incident with v from the graph, call the resulting graph G_v.

      ii. Use Kruskal's algorithm to find a minimal spanning tree for G_v. Let the total weight of this tree be $\mathbf{W_v}$.

      iii. Replace the vertex v and two of the cheapest edges on v.

   Show that $\mathbf{W_v}+\mathbf{W} \leq$ total weight of a minimal-weight Hamiltonian cycle, where $\mathbf{W}$ denotes the sum of the weights of the two edges found in (iii), above. Thus we have efficiently ob-