BLOBs and SIFT features

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This lecture is being livestreamed and recorded

(hopefully)

Two feedback persons

Learning objectives

After this lecture you should be able to:

- implement and use BLOB detection using Difference-of-Gaussians
- analyse and use SIFT features and feature matching

Similarity

Basic idea

- Locally appearance between views is the same
- Variation can be handled via invariances

Local image features

SIFT – key elements

- Features localized at interest points
- Adapted to scale and invariant to appearance changes

SIFT – scale invariant feature transform (Lowe, 1999)

- Scale-space BLOB detection – difference of Gaussians
- Interest point localization
- Orientation assignment
- Interest point descriptor
- Note – SIFT is one example of interest point feature

Harris corners and BLOBs

Harris corners are features that have a large change of intensity in two orthogonal directions. They are:

- local,
- can be found at different scales by changing the Gaussian filters, and
- invariant to rotation.

Harris corners are found by first order derivatives whereas BLOBs are response to second order image derivatives.

BLOBs – Binary Large OBjects

Correspond to:

- a dark area surrounded by brighter intensities or,
- a bright area surrounded by darker intensities

Hessian

The Hessian matrix contains the second order derivatives

$$\mathbf{H}(x, y) = \begin{bmatrix} I_{xx}(x, y) & I_{xy}(x, y) \\ I_{xy}(x, y) & I_{yy}(x, y) \end{bmatrix},$$

where

$I_{xx}(x, y) = \frac{\partial^2 I(x,y)}{\partial x^2}$, $I_{yy}(x, y) = \frac{\partial^2 I(x,y)}{\partial y^2}$, and $I_{xy}(x, y) = \frac{\partial^2 I(x,y)}{\partial x \partial y}$.

Curvature

Second order derivatives measure curvature.

Eigenvalues of the Hessian (λ_1, λ_2) measure the principal curvature, i.e. the degree of change in derivative.

The eigenvectors measure the direction of that change

- the eigenvector corresponding to the largest eigenvalue (λ_1) is the direction of most change
- the second is orthogonal to that.

BLOB detection with Hessian

Similar to the Harris corner detector, we can use either of the measures

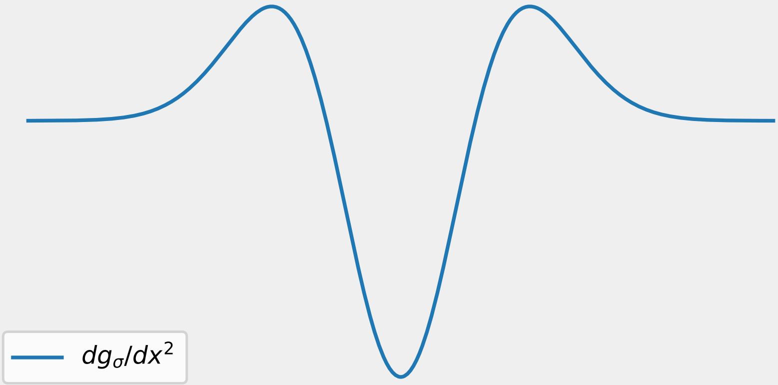
$$\det(\mathbf{H}) = \lambda_1 \lambda_2,$$
$$\operatorname{trace}(\mathbf{H}) = \lambda_1 + \lambda_2,$$

where λ_i are the eigenvalues of the Hessian.

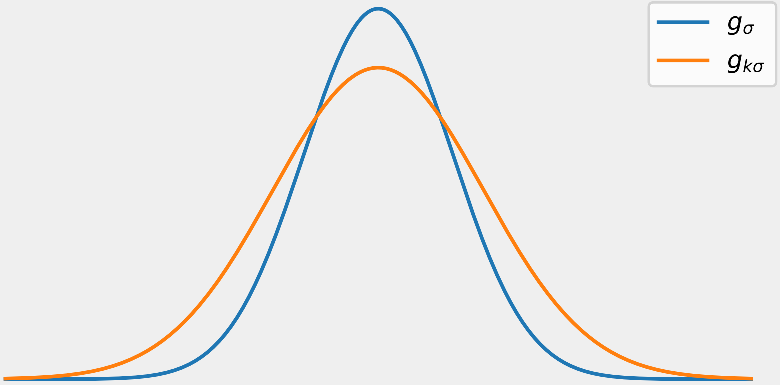
$\det(\mathbf{H})$ is the Gaussian curvature.

$\operatorname{trace}(\mathbf{H}) = \nabla^2 I$ is the Laplacian, which we use for BLOB detection.

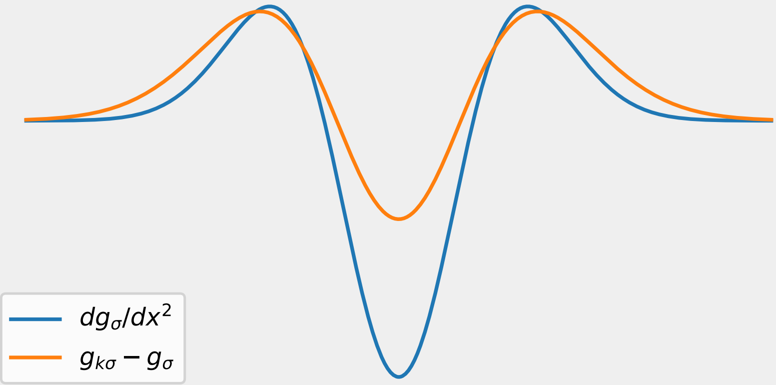
The Laplacian



Two Gaussians with different standard deviations



Difference of Gaussians vs Laplacian



BLOB detection with DoG

The Laplacian $\nabla^2 I$ can be approximated with the Difference-of-Gaussians (DoG).

Blurring the image with two different Gaussian kernels:

$$\nabla^2 I \approx D_\sigma = (G_{k\sigma} - G_\sigma) * I = G_{k\sigma} * I - G_\sigma * I,$$

where G_σ is a Gaussian with standard deviation σ and $k > 1$ is a scale factor.

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Why are we interested in this approximation?

BLOB detection with DoG

Figure 1: A dog detecting blobs

SIFT – Scale invariance

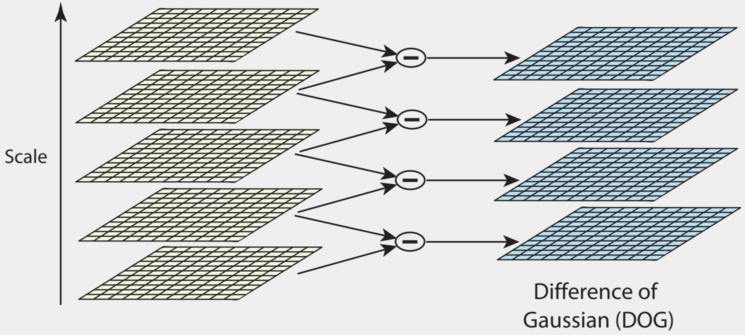
SIFT – Scale invariance

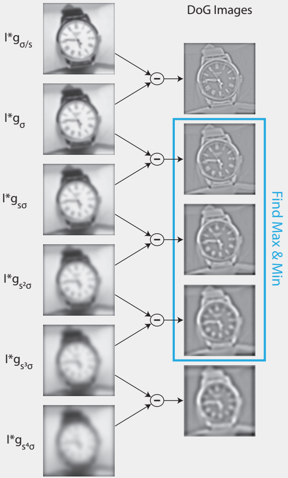
- Using difference of Gaussians for BLOB detection

$$\begin{aligned} D(x, y, \sigma) &= \big((G_{k\sigma} - G_\sigma) * I\big)(x, y) \\ &= (G_{k\sigma} * I)(x, y) - (G_\sigma * I)(x, y) \\ &= L(x, y, k\sigma) - L(x, y, \sigma) \end{aligned}$$

The DoG is computed by subtracting more and and more blurred images from each other.

SIFT - Difference of Gaussians





Gaussian scale space – Efficient

- Convolution of two Gaussians yield a new Gaussian
- Generate scale space by iteratively blurring already blurred images again
  - Otherwise we would need very large Gaussian kernels
- However the size of the kernel still grows
  - $(k\sigma)^2 = \sigma^2 + new^2$

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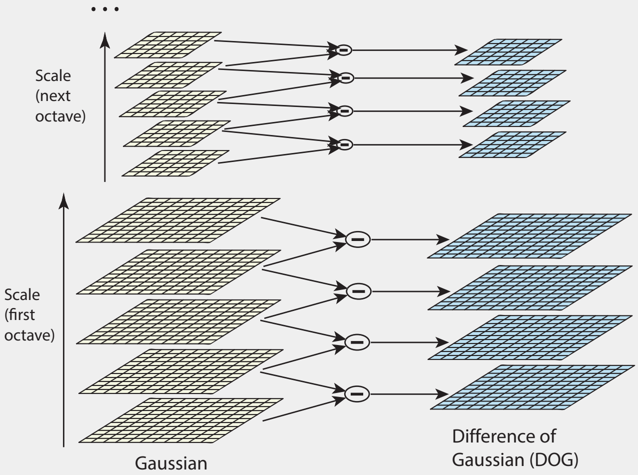
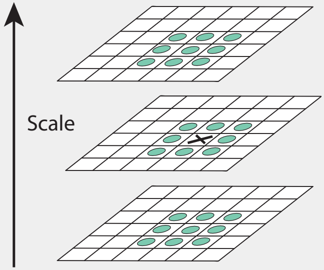
- However the size of the kernel still grows
  - $(k\sigma)^2 = \sigma^2 + new^2$
- We choose $k = 2^{\frac{1}{3}}$
- σ doubles after three images, the image is downsampled.
  - This is an octave.
  - We only need three Gaussians of constant size (precomputed)

SIFT – Estimation of DoG

Difference of Gaussians



Extrema localization



SIFT - Magnitude of the DoG response

Do we get smaller values in the difference of Gaussians for high values of σ?

Using the heat equation it can be shown that the response does not change as a function of sigma.

We can use the same threshold for the entire scale space.

SIFT – Subpixel localization

- Why is this necessary?

SIFT – Subpixel localization

- Why is this necessary?
- Second order Taylor approximation of DoG around local maximum:

$$D(\mathbf{x}) = D + \frac{\partial D^T}{\partial \mathbf{x}}\mathbf{x} + \frac{1}{2}\mathbf{x}^T \frac{\partial^2 D}{\partial \mathbf{x}^2}\mathbf{x}$$

- Setting the derivative of $D(\mathbf{x})$ to zero

$$\hat{\mathbf{x}} = -\frac{\partial^2 D}{\partial \mathbf{x}^2}^{-1} \frac{\partial D}{\partial \mathbf{x}}$$

SIFT – Subpixel localization

- We get

$$D(\hat{\mathbf{x}}) = D + \frac{1}{2}\frac{\partial D^T}{\partial \mathbf{x}}\hat{\mathbf{x}}$$

- If $|D(\hat{\mathbf{x}})| < 0.03$ the point is discarded
  - Removes points with low contrast.

SIFT – Interest point along edges discarded

- The eigenvalues of the Hessian are proportional to the principal curvatures

$$\mathbf{H} = \begin{bmatrix} I_{xx} & I_{xy} \\ I_{xy} & I_{yy} \end{bmatrix} = \begin{bmatrix} \frac{\partial^2 I}{\partial x^2} & \frac{\partial^2 I}{\partial x \partial y} \\ \frac{\partial^2 I}{\partial x \partial y} & \frac{\partial^2 I}{\partial y^2} \end{bmatrix}$$

Interest point along edges discarded

- λ_1 and λ_2 are the eigenvalues of the Hessian

$$\operatorname{trace}(\mathbf{H}) = I_{xx} + I_{yy} = \alpha + \beta$$
$$\det(\mathbf{H}) = I_{xx}I_{yy} - I_{xy}^2 = \alpha\beta$$

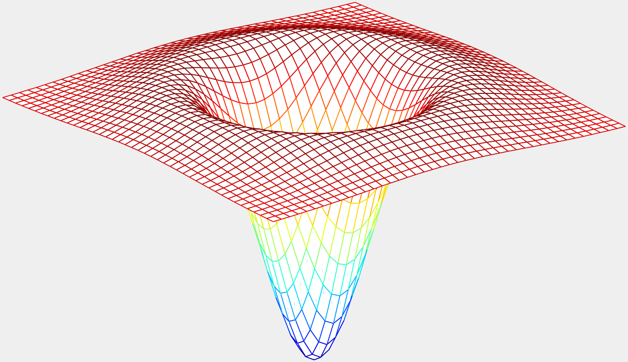
Points are kept if

$$\frac{\operatorname{trace}(\mathbf{H})^2}{\det(\mathbf{H})} < \frac{(r+1)^2}{r},$$

where $r = 10$ (found to be a good heuristic)

Short break

DoG measure

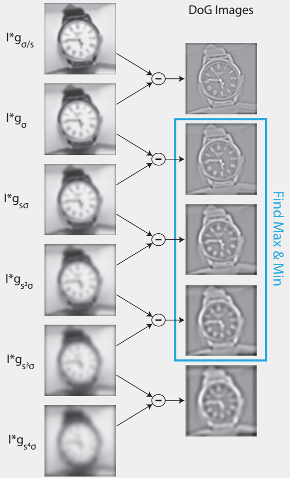


Features have $|D(\boldsymbol{x})| > \tau$, where τ is a threshold.

Dark BLOBs: $D(\boldsymbol{x}) > 0$, bright BLOBs: $D(\boldsymbol{x}) < 0$

DoG in scale pyramids

Scale pyramids are increasingly blurred of the same image.
Scale space DoG is subtraction between all layers adjacent scales.



Scale space BLOBs and DoG

DoGs at different scales makes for a scale invariant feature detector.
Small and large details are recoverable in different DoGs.

SIFT – Orientation assignment

Compute the orientation of gradients in a small region around the BLOB.

$$m(x, y) = \sqrt{L_x^2 + L_y^2}$$
$$\theta(x, y) = \arctan 2(L_y, L_x)$$

Where

$$L_x = L(x + 1, y) - L(x - 1, y)$$
$$L_y = L(x, y + 1) - L(x, y - 1)$$

SIFT – Orientation assignment

- Compute circular histogram of gradient orientations
    - Weighted by magnitude, smoothed, and has 36 bins

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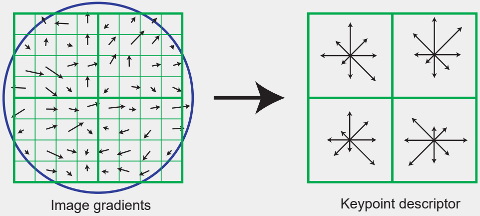
- This introduces rotation invariance.
- Can we have multiple peaks in histogram?

SIFT – Orientation assignment

- Compute circular histogram of gradient orientations
  - Weighted by magnitude, smoothed, and has 36 bins
- Use peak in histogram to assign orientation of point
- This introduces rotation invariance.
- Can we have multiple peaks in histogram?
  - Yes, this can happen at e.g. corners.
  - Create a new point at the same location if peak is over 80% of max.

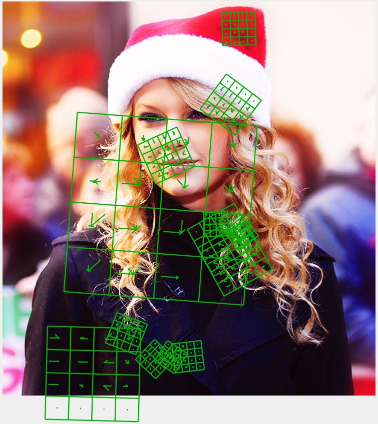
SIFT – Descriptor

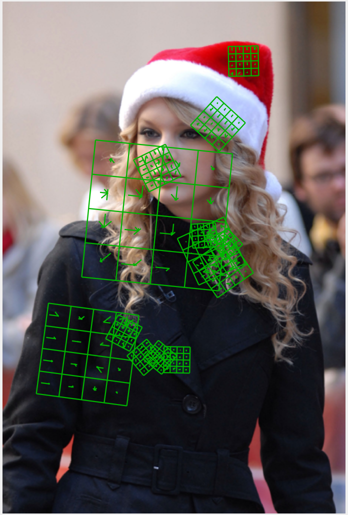
- Create local patch at scale and orientation of point
- Build a histogram of local gradient orientations



- Normalized using L_2 norm: $\mathbf{d}_n = \frac{1}{\sqrt{\sum_{i=1}^{128} \mathbf{d}(i)^2}}\mathbf{d}$

Taylor SIFT





Taylor SIFT

SIFT – Invariances

- Position
- Scale
- Rotation
- Linear intensity change
- Perspective changes?

SIFT – Matching of descriptors

- Use Euclidean distance between normalized vectors

$$\delta(\mathbf{d}_i, \mathbf{d}_j) = \sqrt{\sum_{n=1}^{128} (d_{i,n} - d_{j,n})^2}$$

- Note – for comparison the square root is not needed

RootSIFT

Simple trick to improve SIFT matching

- SIFT is a histogram
- Euclidean distance is dominated by large values
- RootSIFT is a transformation that measures distance using the Hellinger kernel.
  - L1 normalize
  - Take the square root of each element
  - L2 normalize the resulting vector
- Compare using Euclidean distance

SIFT – Matching of descriptors

- For each feature in image 1 ($\boldsymbol{d_{1,i}}$) find the closest feature in image 2 ($\boldsymbol{d_{2,j}}$)
    - This will give a lot of incorrect matches
- Cross checking
    - Only keep matches where $\boldsymbol{d_{2,j}}$ is also the closest to $\boldsymbol{d_{1,i}}$ of all features in image 1

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  - Only keep matches where $\boldsymbol{d_{2,j}}$ is also the closest to $\boldsymbol{d_{1,i}}$ of all features in image 1
- Ratio test
  - Compute the ratio between the closest and second closest match, and keep where this is below a threshold, e.g. 0.7.

SIFT – Summary

- SIFT is both a feature detector and descriptor
- Find local extrema of DoGs in scale space
- Place patch oriented along local gradients
- Compute histograms of gradients.
- Allows matching of images invariant to: scale, rotation, illumination and viewpoint
- Partly visible objects can be matched

Other descriptors

- SIFT is widely used. (74k+ citations)
  - Was patented until 2020.
- Meanwhile other similar methods were created
  - SURF, 2008 (14k+ citations)
  - ORB 2011, (12k+ citations)
  - BRIEF 2010, (5k+ citations)
  - BRISK 2011, (4k+ citations)

Learned descriptors

- Deep Learning has created improved feature detectors/descriptors.
- Mostly in improvement in invariance to changing lighting.
- Some examples:
  - R2D2: Repeatable and Reliable Detector and Descriptor [code]
  - Superpoint [code]

Learning objectives

After this lecture you should be able to:

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Exercise

Build a BLOB detector and match points with SIFT detector.

Python: Use OpenCV (4.2.0 or newer)

Matlab: Use VLFeat

https://www.vlfeat.org/overview/sift.html

Exercise time!