# Simple features

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**This lecture is being livestreamed and recorded**

(hopefully)

# Two feedback persons

# Learning objectives

After this lecture you should be able to:

- explain the image correspondence problem
- explain the use of image features (key points, interest points)
- understand and implement image filtering
- implement Harris corner detection
- run Canny edge detection

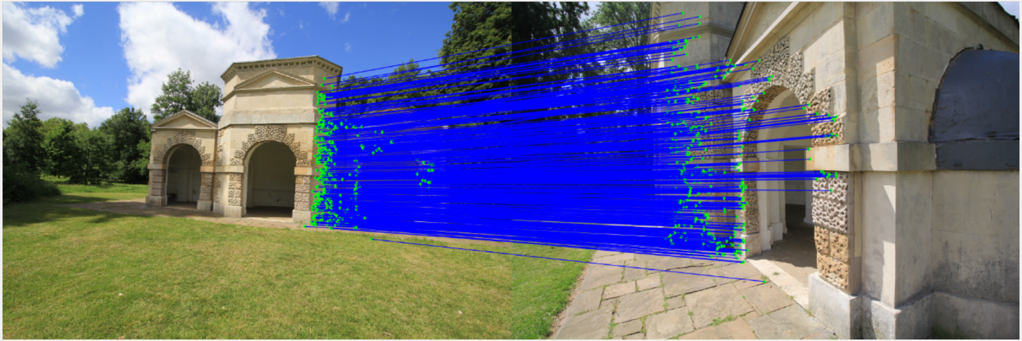
# Presentation topics

- Image correspondence problem
  - Features and feature descriptors
- Simple features
  - Filtering and convolution
  - Harris corners
  - Canny edges

# Image correspondence problem

# Image correspondence

The problem of matching two parts of different images:



# Image correspondence

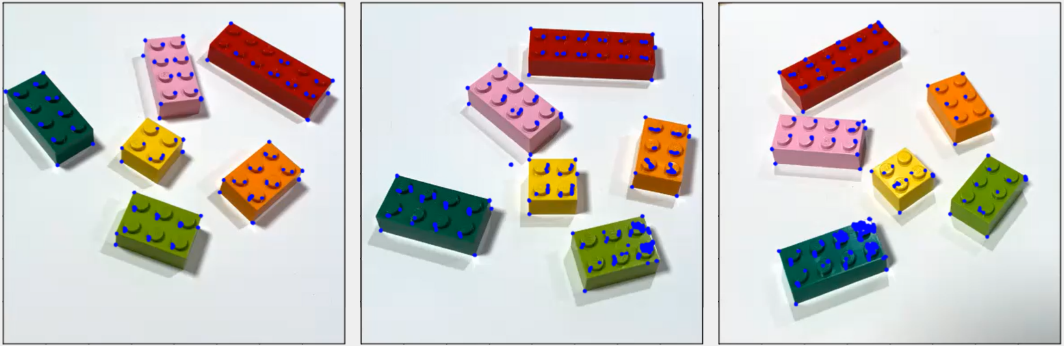
More images of the same scene

- Correspondence exists between parts that are visible in each image
- Not all corresponding points have a unique pattern
- Idea: Only choose points that can be identified uniquely

# What are good points?

# What are good points?

We are able to identify the same corners in multiple images



# Problems

Movement of camera

- Scale – distance of camera to object
- Rotation – objects and camera is rotated between frames
- Translation – movement of camera from one place to another

Result: Appearance change dependent on distance to camera

# Problems

Movement of camera

# Problems

Other issues

- Occlusion – objects can be in front of other objects
- Lighting change – darker/lighter, color of light (change in spectrum), direction of light
- Clutter – many objects in a scene

# Solution

Invariance to imaging problems

- Focus on points and a small area around each point
- Two aspects
  - identify key points (focus of today)
  - characterize pattern around point (week 8)

# Features and feature descriptors

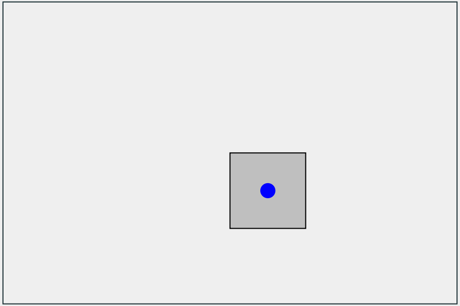
First some terminology:

Key points, interest points, and feature points are the same – namely points in an image with a coordinate position (also between pixels).

Descriptors, key point descriptors, interest point descriptors, and feature descriptors are the same and used for characterizing the pattern around a point. Usually a vector that can be matched between images.

# The image patch feature descriptor

One easy way of describing a feature is to use the local pixel values around the feature.



# The image patch matching

Stretch the image patch into a vector $\boldsymbol{f}$, and that is your descriptor.

A simple comparison operator $d(\boldsymbol{f}_1, \boldsymbol{f}_2)$ just uses the scalar product as the distance between features:

$$d(\boldsymbol{f}_1, \boldsymbol{f}_2) = \boldsymbol{f}_1^\top \boldsymbol{f}_2$$

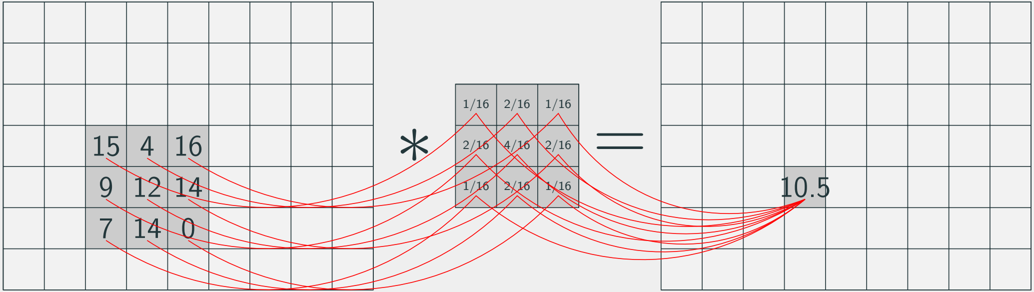
Good with only small transformations, low noise, and unchanged environment.

More robust (but more complicated) feature descriptors will follow in two weeks.

# Simple features

# Filtering and convolution

$$I(x;t) = \int_{\xi \in \mathbb{R}^2} I(x-\xi) g(\xi;t) d\xi \approx \sum_{i=-k}^{k} \sum_{j=-k}^{k} I(x-i, y-j) g(i,j)$$



# 2D convolution

We use filtering and convolution for the same operation and use the symbol $*$ for convolution. Convolution is commutative

$$I_g = g * I = I * g$$

2D Gaussian $g : \mathbb{R}^2 \times \mathbb{R}_+ \to \mathbb{R}$

$$g(x, y; \sigma^2) = \frac{1}{2\pi\sigma^2} \exp\left(\frac{-(x^2 + y^2)}{2\sigma^2}\right). \tag{1}$$

# Convolution – separability

Useful property: The (isotropic) Gaussian is separable

$$g(x, y; \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-x^2}{2\sigma^2}\right) \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-y^2}{2\sigma^2}\right)$$

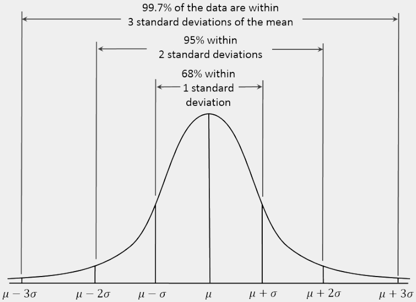
This separability means that

$$I_g = \left(\mathbf{g} * \mathbf{g}^\mathsf{T}\right) * I = \mathbf{g} * \left(\mathbf{g}^\mathsf{T} * I\right)$$

where g is a vector of the Gaussian.

# Size of Gaussian filter

- Ideally: Infinitely big – but this is impractical
- Empiric rule: $3\sigma$, $4\sigma$ or $5\sigma$ rule – example:
  - if $\sigma = 2$, filter size of Gaussian is $2 \cdot 5 \cdot 2 + 1 = 21$
  - if $\sigma = 20$, filter size of Gaussian is $2 \cdot 5 \cdot 20 + 1 = 201$



# Demo – Gaussian filtering

`cv2.sepFilter2D`

# Derivatives of an image

How can we find the derivative of an image in e.g. the $x$-direction?

What does it mean to take a derivative?

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How can we find the derivative of an image in e.g. the $x$-direction?

What does it mean to take a derivative?

An image is only discrete values.

If we can make the image continuous all our math works again

# Derivatives of an image using Gaussians

Let $\mathbf{g}$ be column vector of a Gaussian. Consider the blurred image

$$I_b = \mathbf{g} * \mathbf{g}^\top I.$$

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Let $\mathbf{g}$ be column vector of a Gaussian. Consider the blurred image

$$I_b = \mathbf{g} * \mathbf{g}^\mathsf{T} I.$$

Then the derivative of $I_b$ in the $x$-direction is

$$\begin{aligned}\frac{\partial}{\partial x} I_b &= \frac{\partial}{\partial x}\left(\mathbf{g} * \mathbf{g}^\mathsf{T} * I\right) \\ &= \mathbf{g} * \left(\frac{\partial}{\partial x}\mathbf{g}^\mathsf{T}\right) * I \\ &= \mathbf{g} * \mathbf{g}_d^\mathsf{T} * I,\end{aligned}$$

# Derivative of the Gaussian

Recall the one-dimensional Gaussian

$$g(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-x^2}{2\sigma^2}\right).$$

We can then compute the derivative

$$g_d(x) = \frac{d}{dx} g(x) = \frac{-x}{\sigma^2} g(x).$$

This is straightforward to implement on a computer.

# Short break

# Harris corners

Corners (and blobs) are great features because they are easy to describe and detect.

Let's see how much the intensity changes, for a small shift $\Delta_x, \Delta_y$

$$\Delta I(x, y, \Delta_x, \Delta_y) = I(x, y) - I(x + \Delta_x, y + \Delta_y)$$

Harris corners are detected as points with *locally maximum* change from a small shift.

A corner is then a local area where $\Delta I(x, y, \Delta_x, \Delta_y)^2$ is large no matter how we choose $\Delta_x, \Delta_y$.

# Harris corner measure

To improve robustness towards noise the “cornerness” should be true for a local area and not just a point. We apply a Gaussian blur to the above measure.

The measure to check for corners is then

$$
\begin{aligned}
c(x, y, \Delta_x, \Delta_y) &= g * \Delta I(x, y, \Delta_x, \Delta_y)^2 \\
&= g * \big(I(x, y) - I(x + \Delta_x, y + \Delta_y)\big)^2,
\end{aligned}
$$

where $g * \ldots$ is the convolution with the Gaussian.

# Replace with Taylor approximation

$$I(x+\Delta_x, y+\Delta_y) \approx I(x,y) - \frac{\partial I}{\partial x}(x,y)\,\Delta_x - \frac{\partial I}{\partial y}(x,y)\,\Delta_y$$

# Replace with Taylor approximation

$$I(x + \Delta_x, y + \Delta_y) \approx I(x, y) - \underbrace{\frac{\partial I}{\partial x}(x, y)}_{I_x} \Delta_x - \underbrace{\frac{\partial I}{\partial y}(x, y)}_{I_y} \Delta_y$$

# Replace with Taylor approximation

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&= I(x, y) - \begin{bmatrix} I_x & I_y \end{bmatrix} \begin{bmatrix} \Delta_x \\ \Delta_y \end{bmatrix}
\end{aligned}
$$

$I_x$ and $I_y$ are also values at $(x, y)$, but we omit it for readability.

# Harris corner measure derivation

The corner checking measure is then

$$\begin{aligned} c(x, y, \Delta_x, \Delta_y) &= g * \left(I(x, y) - I(x + \Delta_x, y + \Delta_y)\right)^2 \\ &\approx g * \left(\begin{bmatrix} I_x & I_y \end{bmatrix} \begin{bmatrix} \Delta_x \\ \Delta_y \end{bmatrix}\right)^2 \end{aligned}$$

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&\approx g * \left(\begin{bmatrix} I_x & I_y \end{bmatrix} \begin{bmatrix} \Delta_x \\ \Delta_y \end{bmatrix}\right)^2 \\
&= g * \left(\begin{bmatrix} \Delta_x & \Delta_y \end{bmatrix} \begin{bmatrix} I_x \\ I_y \end{bmatrix} \begin{bmatrix} I_x & I_y \end{bmatrix} \begin{bmatrix} \Delta_x \\ \Delta_y \end{bmatrix}\right) \\
&= g * \left(\begin{bmatrix} \Delta_x & \Delta_y \end{bmatrix} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} \begin{bmatrix} \Delta_x \\ \Delta_y \end{bmatrix}\right)
\end{aligned}
$$

# Harris corner measure derivation

Finally, we take the convolution with the Gaussian inside

$$
\begin{aligned}
c(x, y, \Delta_x, \Delta_y) &\approx g * \left( \begin{bmatrix} \Delta_x & \Delta_y \end{bmatrix} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} \begin{bmatrix} \Delta_x \\ \Delta_y \end{bmatrix} \right) \\
&= \begin{bmatrix} \Delta_x & \Delta_y \end{bmatrix} \underbrace{\begin{bmatrix} g * (I_x^2) & g * (I_x I_y) \\ g * (I_x I_y) & g * (I_y^2) \end{bmatrix}}_{\boldsymbol{C}(x,y)} \begin{bmatrix} \Delta_x \\ \Delta_y \end{bmatrix}.
\end{aligned}
$$

We end up with $\boldsymbol{C}(x, y)$, also known as the structure tensor.

# The structure tensor

Corners have a large $c(x, y, \Delta_x, \Delta_y)$ regardless of $\begin{bmatrix}\Delta_x & \Delta_y\end{bmatrix}$.

$$c(x, y, \Delta_x, \Delta_y) \approx \begin{bmatrix}\Delta_x & \Delta_y\end{bmatrix} \boldsymbol{C}(x, y) \begin{bmatrix}\Delta_x \\ \Delta_y\end{bmatrix}$$

How can we check if this the case from the $\boldsymbol{C}(x, y)$ matrix?

# The structure tensor

Corners have a large $c(x, y, \Delta_x, \Delta_y)$ regardless of $\begin{bmatrix}\Delta_x & \Delta_y\end{bmatrix}$.

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How can we check if this the case from the $\boldsymbol{C}(x, y)$ matrix?

When both eigenvalues of $\boldsymbol{C}(x, y)$ are large.

# The Harris corner metric continued

Let $\lambda_1$ and $\lambda_2$ be the eigenvalues of $\boldsymbol{C}(x, y) = \begin{bmatrix} a & c \\ c & b \end{bmatrix}$.

We then have the following

$$\det\left(\boldsymbol{C}(x, y)\right) = \lambda_1 \lambda_2 = ab - c^2,$$
$$\operatorname{trace}\left(\boldsymbol{C}(x, y)\right) = \lambda_1 + \lambda_2 = a + b, \text{ let us then define}$$
$$r(x, y) = \det\left(\boldsymbol{C}(x, y)\right) - k \ \operatorname{trace}\left(\boldsymbol{C}(x, y)\right)^2$$

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r(x, y) &= \det\left(\boldsymbol{C}(x, y)\right) - k \ \operatorname{trace}\left(\boldsymbol{C}(x, y)\right)^2 \\
&= \lambda_1 \lambda_2 - k(\lambda_1 + \lambda_2)^2 \\
&= ab - c^2 - k(a + b)^2
\end{aligned}$$

# The Harris corner metric

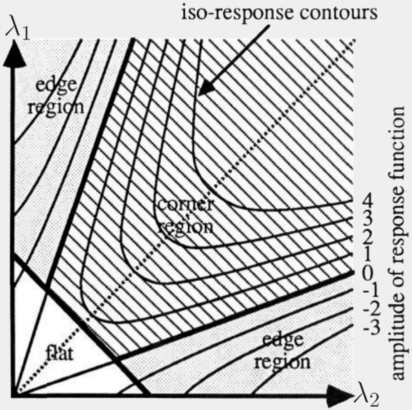
Now we have the Harris corner metric

$$r(x, y) = \lambda_1 \lambda_2 - k(\lambda_1 + \lambda_2)^2.$$

$k$ is a free parameter, typically $k = 0.06$.

Notice that $r(x, y)$ is:

- negative for one eigenvalue much greater than the other
- large and positive for large eigenvalues
- small and positive for small eigenvalues



# The Harris corner detector

We can now detect corners by finding points where $r(x, y)$ is greater than some threshold $\tau$.

Typically, you can choose $\tau$ to be

$$0.1 \cdot \max\big(r(x, y)\big) < \tau < 0.8 \cdot \max\big(r(x, y)\big).$$

# Non-maximum suppression

Find the local maximum of one pixel compared to neighbours

$$\big(I(x,y) - I(x',y')\big) > 0 \;\; \forall \;\; x' \in \mathcal{N}(x,y)$$

where $\mathcal{N}(x,y)$ is a neighbourhood around the point $(x,y)$

Suggested procedure

- Initialize a new array **M** with $I(x,y) > \tau$
- Compare original image with one neighbour (e.g to the right)
- Set pixels in **M** to 0 where original not larger than neighbour
    - Repeat for all neighbours
- Find coordinates of all pixels that are 1

# Canny edges

Often, we might also consider detecting lines.

To detect (non-straight) lines, we can use the Canny edge detector.

# Canny edges

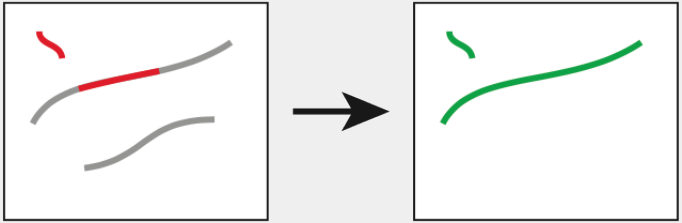
The metric in the canny edge detector is simply the gradient magnitude

$$m(x, y) = \sqrt{I_x^2(x, y) + I_y^2(x, y)}.$$

And the edges are detected by thresholding the magnitude $m(x, y)$, however, in two stages: seed and grow.

# Canny edges

- Seed: label edges by a threshold where $m(x, y) > \tau_1$
- Grow: with a second threshold where $m(x, y) > \tau_2$, label iif the new points are next to previously labelled edges.
- The threshold values are chosen such that $\tau_1 > \tau_2$



# Exercise

During the exercise, you will

- implement the Harris corner detector
- try the Canny edge detector

Suggestion: After finishing your implementation – try on your own images and perhaps draw some features and see how it works

# Learning objectives

After this lecture you should be able to:

- explain the image correspondence problem
- explain the use of image features (key points, interest points)
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# Exercise time!