Robust Model Fitting

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02504 Computer vision course lectures,
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This lecture is being livestreamed and recorded

(hopefully)

Two feedback persons

Learning objectives

After this lecture you should be able to:

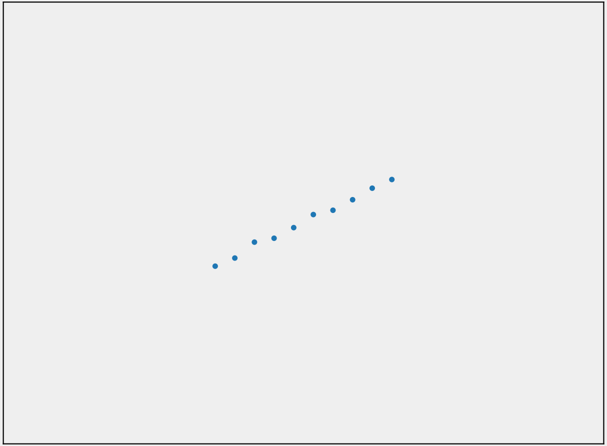
- explain how the Hough transform works
- understand and implement RANSAC

Presentation topics

- Hough Transform
- RANSAC
  - Recap of lines in homogeneous coordinates

Fitting models

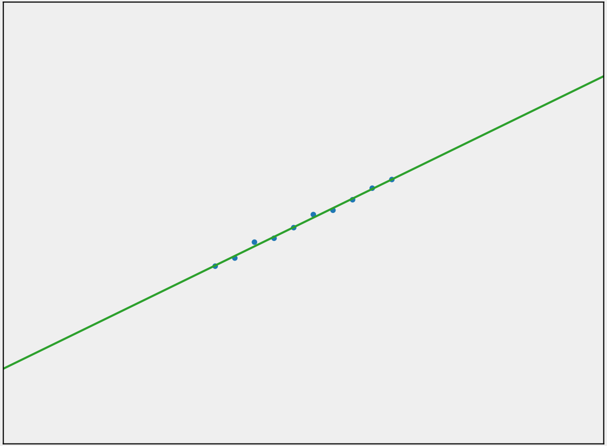
Can we fit a straight line?



Fitting models

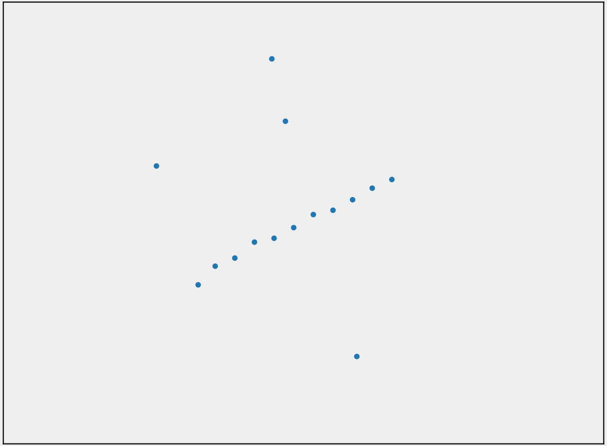
Can we fit a straight line?

Yes we can!



Fitting models

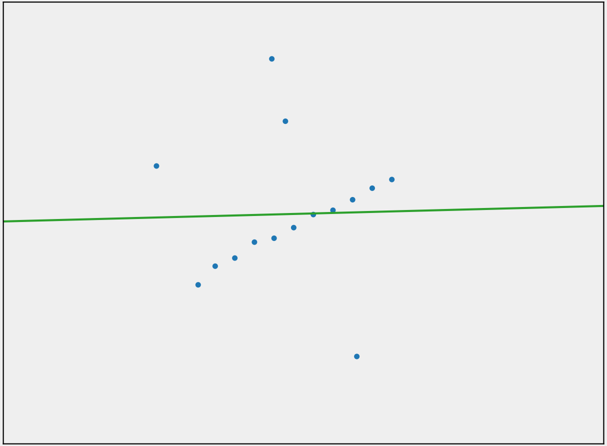
Can we fit a straight line when there are a few outliers?



Fitting models

Can we fit a straight line when there are a few outliers?

Not really...
We get a very bad fit.
We need robust ways to fit models!



About lines

- This presentation uses fitting straight lines to 2D points for all examples.
- Do we really care that much about fitting straight lines?

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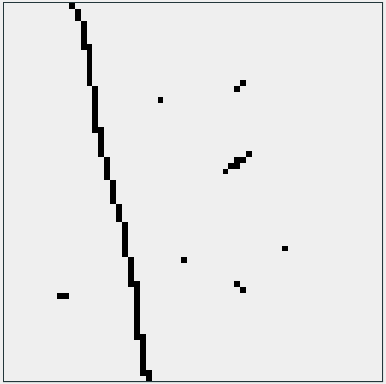
- This presentation uses fitting straight lines to 2D points for all examples.
- Do we really care that much about fitting straight lines?
- The principles *generalize* to other models!

Hough Transform

Hough Transform

Is a transformation of an edge image where lines can be extracted.

Example image



Hough transform of image

Hough Transform

How to represent a line?

- $y = ax + b$?
  - Has singularities for vertical lines
- Homogeneous coordinates?
  - Is over-parametrized

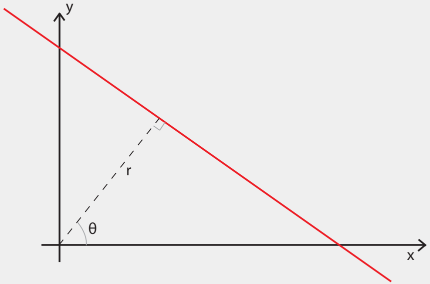
Hough Transform - r, θ representation

Represent using:

- θ angle of the line
- r closest distance from origin to line

Closely related to the homogeneous line representation

$$\boldsymbol{l} = \begin{bmatrix} \cos(\theta) & \sin(\theta) & -r \end{bmatrix}$$



Hough Transform

We can represent any line with these two parameters.

- $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}[$
- $r \in [-d, d]$, where d is the diagonal length of the image.

We discretize these values and represent them on a 2D grid

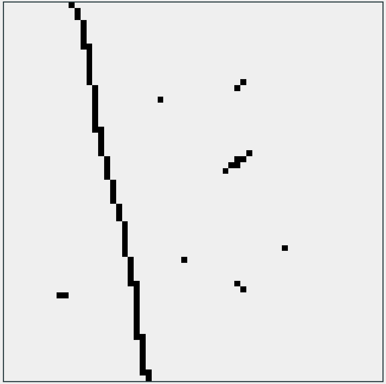
Hough Transform

Idea: Let points vote on which line is the best!

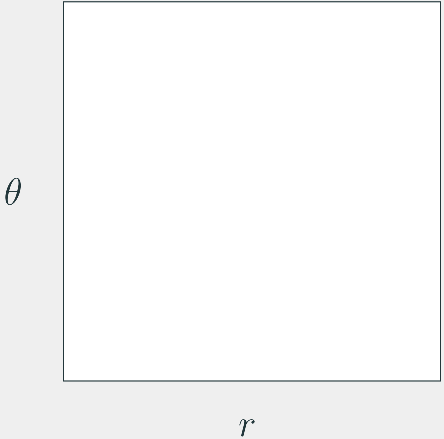
- Each point can be part of (infinitely) many lines.
- All potential lines going through this point are of interest
- Each point votes on all lines that go through its
  - This corresponds to a line in Hough space
- Repeat for all points

Hough Transform – Example

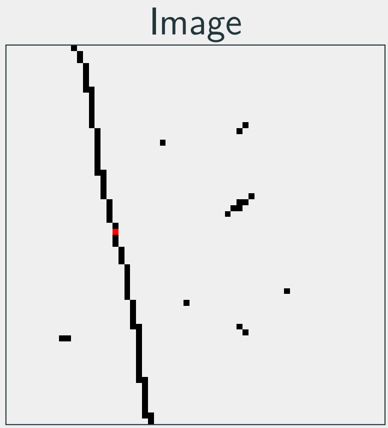
Image

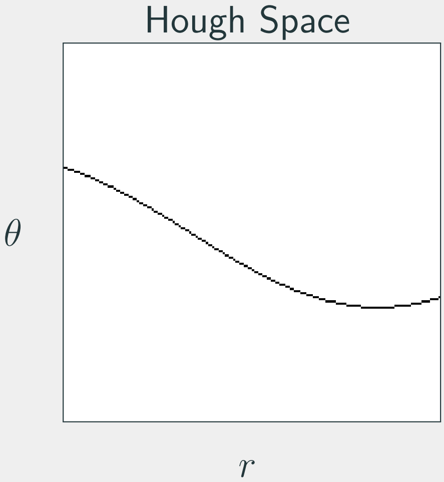


Hough Space

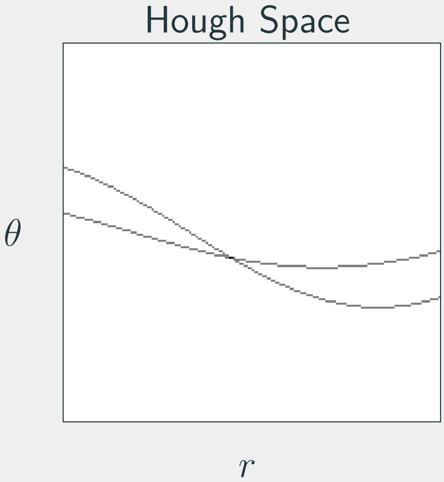


Hough Transform – Example

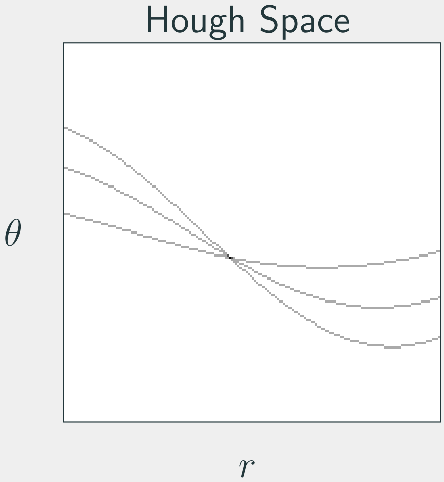




Hough Transform – Example



Hough Transform – Example



Hough Transform – Example

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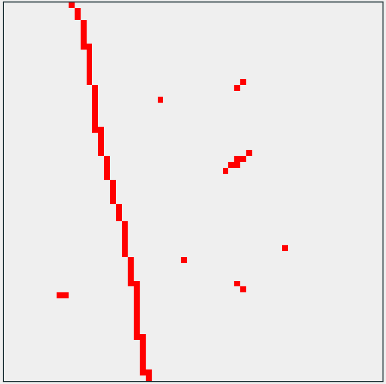
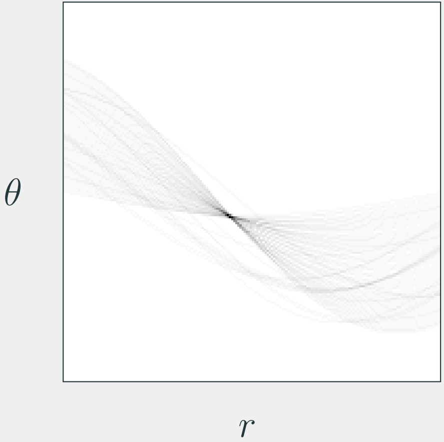


Hough Space



Hough Transform

- A peak in Hough space corresponds to a line in the image.
- Can be found using non-maximum suppression.

Generalized Hough Transform

- We can generalize the Hough transform for more complex models.
  - e.g. for circles, the Hough space is now in 3D and each point becomes a conic.
- Hough space has same number of dimensions as the model we fit has degrees of freedom.
- Impractical for more than three degrees of freedom

RANSAC

Random sample consensus (RANSAC)

Idea!

Instead of computing the Hough space, what if we could sample points directly in Hough space?

Random sample consensus (RANSAC)

Idea!

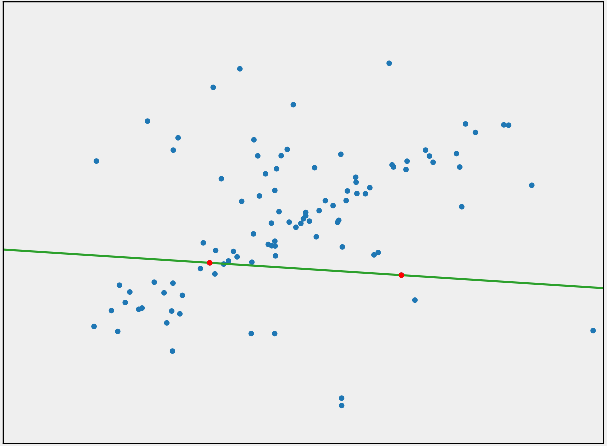
Instead of computing the Hough space, what if we could sample points directly in Hough space?

What if we could sample with the value in hough space being proportional to the probability of sampling the point?

RANSAC

- Randomly sample the minimum number of points we need to fit our model
- Fit the model to these samples

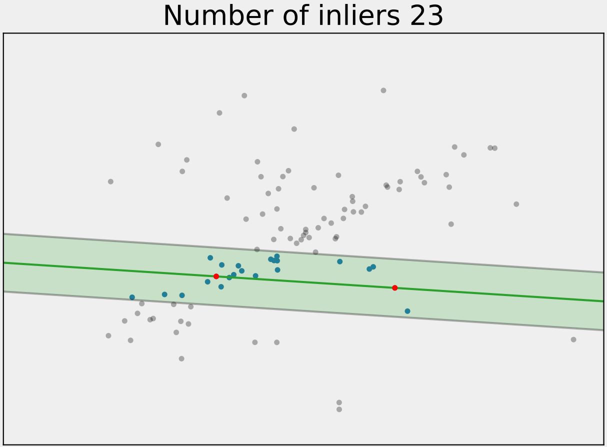
Does this line fit the data well?



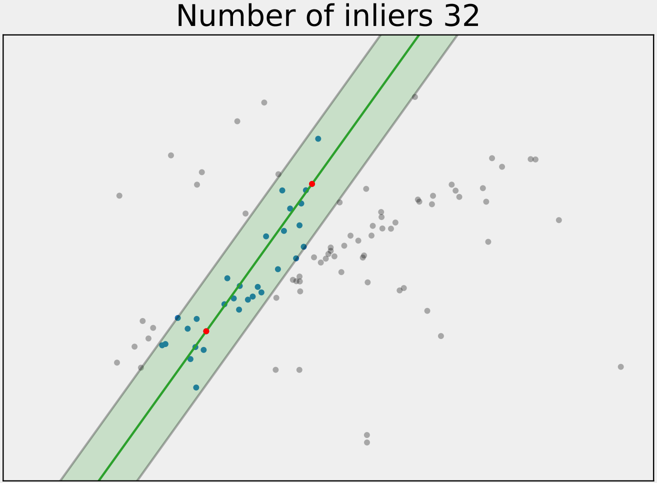
Measure inliers

Points closer than a certain threshold to the line are inliers!

Number of inliers is indication of how well the line fits.



Example line



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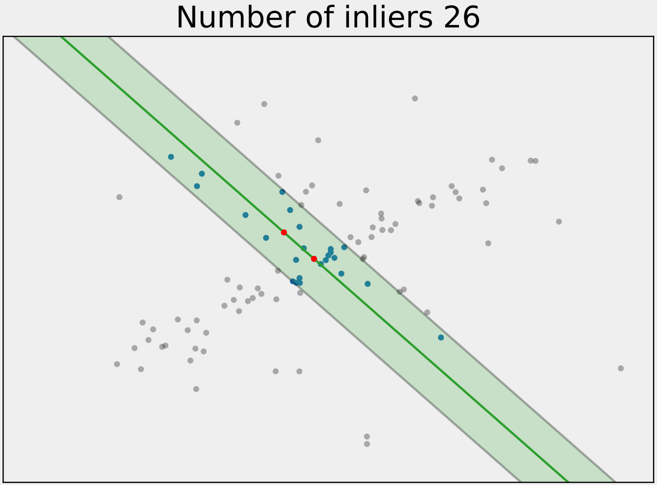

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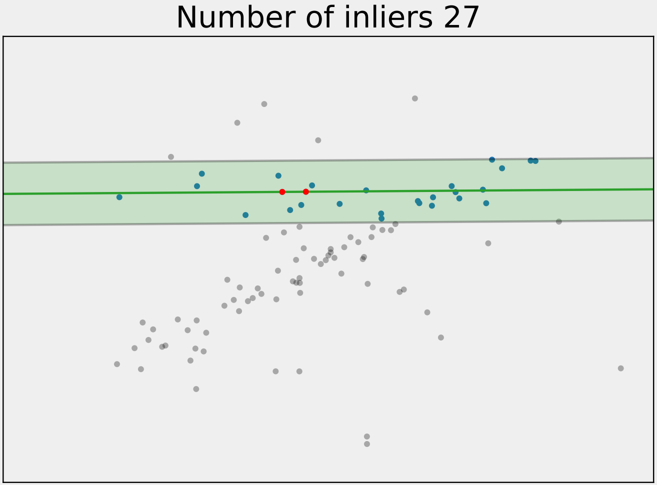

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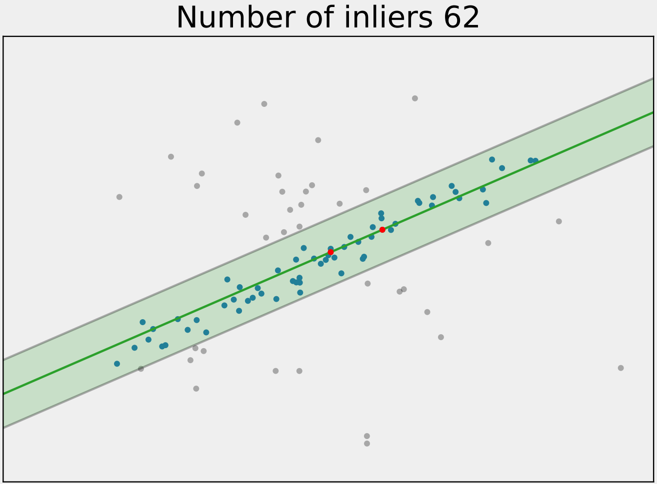

The RANSAC algorithm

Keep track of which fit had the most inliers so far, and update this if a fit with more inliers is found

The RANSAC algorithm

- Sample minimum number of points required to fit model
  - Fit model to these
- Data points with an error less than τ are inliers with respect to fitted model
  - If number of inliers is higher than the highest number of inliers seen so far, update best model.

The RANSAC algorithm

- Sample minimum number of points required to fit model
  - Fit model to these
- Data points with an error less than τ are inliers with respect to fitted model
  - If number of inliers is higher than the highest number of inliers seen so far, update best model.
- Repeat for N iterations.
- Final step:
  - Re-fit model to all inliers of the best model

Implementation details

- Represent the lines using homogeneous coordinates $\begin{bmatrix} a & b & c \end{bmatrix}$
  - Makes it easy to compute distance to line
  - Scale the line such that $a^2 + b^2 = 1$
- Recall that distance to line is given by: $|\boldsymbol{l} \cdot \Pi^{-1}(\boldsymbol{p})|$
  - When $a^2 + b^2 = 1$

RANSAC

- Sample in Hough space without computing it.
- Useful for fitting models when outliers are present.
- We must select the threshold for inliers and the number of iterations carefully.
- Being able to fit a model to the least amount of data points is of interest

How many iterations?

- We can come with some idea of how many iterations we need
- Assume fraction of outliers is ϵ
- i.e. $\epsilon = 0.1$ means 10% of data are outliers.

How many iterations?

- We need n data points to fit a single model

$$P(\text{one sample has only inliers}) = (1-\epsilon)^n$$

- Set p to a high value such as $p = 0.99$. Useful if we know ϵ.

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P(\text{at least one of } N \text{ samples has only inliers}) &= 1-(1-(1-\epsilon)^n)^N = p \\
\Leftrightarrow N &= \frac{\log(1-p)}{\log\left(1-(1-\epsilon)^n\right)}
\end{aligned}
$$

- Set p to a high value such as $p = 0.99$. Useful if we know ϵ.

Determining number of iterations adaptively

- We can estimate an upper bound of ϵ while running RANSAC.
- Let be s the number of inliers in the best model found so far
- Let be m the total number of data points
- $\hat{\epsilon} = 1 - \frac{s}{m} > \epsilon$

Determining number of iterations adaptively

- We can estimate an upper bound of ϵ while running RANSAC.
- Let be s the number of inliers in the best model found so far
- Let be m the total number of data points
- $\hat{\epsilon} = 1 - \frac{s}{m} > \epsilon$
- We can now estimate an upper bound on the number of iterations required
- $\hat{N} = \frac{\log(1-p)}{\log(1-(1-\hat{\epsilon})^n)} > N$
- Terminate once we have done more than $\hat{N}$ iterations.

Recap of lines in homogeneous coordinates

- How do we fit a line to two points ($\mathbf{p}_1$ and $\mathbf{p}_2$) in homogeneous coordinates?
- What do we know about the line $\mathbf{l}$?
- $\mathbf{l} \cdot \mathbf{p}_1 = 0$ and $\mathbf{l} \cdot \mathbf{p}_2 = 0$.

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- The dot product is zero only when two vectors are perpendicular.
- How can we find $\mathbf{l}$ such that it is perpendicular to $\mathbf{p}_1$ and $\mathbf{p}_2$?

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- The dot product is zero only when two vectors are perpendicular.
- How can we find $\mathbf{l}$ such that it is perpendicular to $\mathbf{p}_1$ and $\mathbf{p}_2$?
- Use the cross product! $\mathbf{l} = \mathbf{p}_1 \times \mathbf{p}_2$.

Learning objectives

After this lecture you should be able to:

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Midterm Evaluation

Exercise time!