

Geometry Constrained Feature Matching

Morten R. Hannemose, mohan@dtu.dk

April 4, 2025

02504 Computer vision course lectures,
DTU Compute, Kgs. Lyngby 2800, Denmark



**This lecture is being
livestreamed and recorded
(hopefully)**

Two feedback persons

Learning objectives

After this lecture you should be able to:

- explain and implement the eight point algorithm for estimating the fundamental matrix
- explain and implement estimation of the fundamental matrix with RANSAC
- choose the threshold for RANSAC using χ^2

Presentation topics

Estimating the fundamental matrix

- Linear algorithm

- Incorporating RANSAC

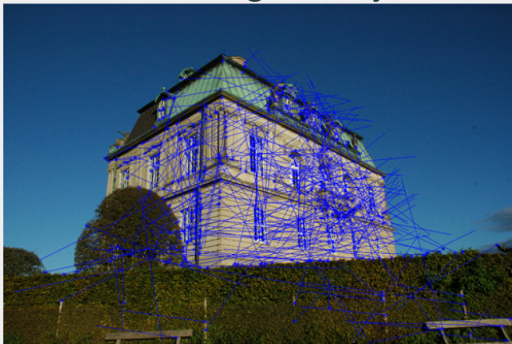
Thresholding for RANSAC

Setting the scene

- Stereo geometry
- (SIFT) Features
- RANSAC

Geometry constrained feature matching

Use multi view geometry to filter matches



Fundamental matrix – Recap

\mathbf{R} and \mathbf{t} describe the relative pose between the cameras.

$$\mathbf{E} = [\mathbf{t}]_{\times} \mathbf{R}$$

$$\mathbf{F} = \mathbf{K}_2^{-\top} \mathbf{E} \mathbf{K}_1^{-1}$$

$$0 = \mathbf{q}_2^{\top} \mathbf{F} \mathbf{q}_1$$

The fundamental matrix expresses that two corresponding points lie on their corresponding epipolar lines.

Estimating the fundamental matrix

Fundamental matrix problem

The fundamental matrix is defined with the relation $\mathbf{q}_2^\top \mathbf{F} \mathbf{q}_1 = 0$.

Consider the corresponding points \mathbf{q}_{1i} and \mathbf{q}_{2i} projected into cameras one and two, respectively. The relation is then

$$\begin{aligned} 0 &= \mathbf{q}_{2i}^\top \mathbf{F} \mathbf{q}_{1i} \\ &= \begin{bmatrix} x_{2i} \\ y_{2i} \\ 1 \end{bmatrix}^\top \begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix} \begin{bmatrix} x_{1i} \\ y_{1i} \\ 1 \end{bmatrix} \end{aligned}$$

Fundamental matrix problem

Rearrange the terms:

$$0 = \mathbf{q}_{2i}^\top \mathbf{F} \mathbf{q}_{1i} ,$$

$$0 = \mathbf{B}^{(i)} \text{flatten}(\mathbf{F}) ,$$

where

$$\begin{aligned} \mathbf{B}^{(i)} &= \begin{bmatrix} x_{1i}x_{2i} & y_{1i}x_{2i} & x_{2i} & x_{1i}y_{2i} & y_{1i}y_{2i} & y_{2i} & x_{1i} & y_{1i} & 1 \end{bmatrix} \\ &= \text{flatten}(\mathbf{q}_{2i} \mathbf{q}_{1i}^\top) , \end{aligned}$$

$$\text{flatten}(\mathbf{F}) = \begin{bmatrix} F_{11} & F_{12} & F_{13} & F_{21} & F_{22} & F_{23} & F_{31} & F_{32} & F_{33} \end{bmatrix}^\top$$

Fundamental matrix solution

Define B

$$B = \begin{bmatrix} B^{(1)} \\ B^{(2)} \\ \vdots \\ B^{(n)} \end{bmatrix}.$$

Subject to $\|\text{flatten}(\mathbf{F})\|_2 = 1$ the solution is the singular vector with the smallest singular value.

Degrees of freedom – Eight point algorithm

F has 9 numbers, and is scale invariant.

Each pair of corresponding points fixes a degree of freedom. Eight points is enough to estimate the fundamental matrix.

This is the **eight point algorithm**.

Degrees of freedom – Seven point algorithm

$[t]_{\times}$ has rank 2, and thus F is also rank deficient, i.e. $\det(F) = 0$.

Thus F has 7 degrees of freedom, and can be found from 7 matches.

B will have two singular vectors with singular value 0.

Denote these F' and F^{\dagger} .

$F = \alpha F' + (1 - \alpha)F^{\dagger}$, where α is chosen such that $\det(F) = 0$.

This is the **seven point algorithm**.

Degrees of freedom – Seven point algorithm

$[t]_{\times}$ has rank 2, and thus F is also rank deficient, i.e. $\det(F) = 0$.

Thus F has 7 degrees of freedom, and can be found from 7 matches.

B will have two singular vectors with singular value 0.

Denote these F' and F^{\dagger} .

$F = \alpha F' + (1 - \alpha)F^{\dagger}$, where α is chosen such that $\det(F) = 0$.

This is the **seven point algorithm**. Discuss with your neighbour why it is preferable to estimate from 7 instead of 8.

F can be estimated from eight point correspondences easily.

Possible to estimate from just seven.

Estimating the fundamental matrix with RANSAC

To use RANSAC, we need a way to measure how well a datapoint fits a specific model. For lines, we measured the distance from a point to the line.

What is a datapoint when talking about the fundamental matrix?

Estimating the fundamental matrix with RANSAC

What does $\mathbf{q}_{2i}^T \mathbf{F} \mathbf{q}_{1i}$ equal for non corresponding points?

Estimating the fundamental matrix with RANSAC

What does $\mathbf{q}_{2i}^\top \mathbf{F} \mathbf{q}_{1i}$ equal for non corresponding points?

$\mathbf{q}_{2i}^\top \mathbf{F}$ and $\mathbf{F} \mathbf{q}_{1i}$ are epipolar lines.

Distance from point to line

$$d = \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} x & y & 1 \end{bmatrix}^\top$$

should have $a^2 + b^2 = 1$, this is not guaranteed for the epipolar lines.

Symmetric epipolar distance

We want to measure the distance

- from \mathbf{q}_{2i} to $\mathbf{F}\mathbf{q}_{1i}$ and
- from \mathbf{q}_{1i} to $\mathbf{F}^\top\mathbf{q}_{2i}$

We can compute these as

$$\frac{\mathbf{q}_{2i}^\top \mathbf{F} \mathbf{q}_{1i}}{\sqrt{(\mathbf{F} \mathbf{q}_{1i})_1^2 + (\mathbf{F} \mathbf{q}_{1i})_2^2}} \quad \text{and} \quad \frac{\mathbf{q}_{2i}^\top \mathbf{F} \mathbf{q}_{1i}}{\sqrt{(\mathbf{q}_{2i}^\top \mathbf{F})_1^2 + (\mathbf{q}_{2i}^\top \mathbf{F})_2^2}}$$

where x_i^2 refers to the square of i^{th} element of \mathbf{x} .

Symmetric epipolar distance

We can normalize the distance to both epipolar lines, using their first two coordinates.

The squared symmetric epipolar distance is then given by

$$\left(\mathbf{q}_{2i}^\top \mathbf{F} \mathbf{q}_{1i}\right)^2 \left(\frac{1}{(\mathbf{q}_{2i}^\top \mathbf{F})_1^2 + (\mathbf{q}_{2i}^\top \mathbf{F})_2^2} + \frac{1}{(\mathbf{F} \mathbf{q}_{1i})_1^2 + (\mathbf{F} \mathbf{q}_{1i})_2^2} \right),$$

Sampson's Distance

A similar distance is Sampson's distance.

$$d_{\text{Samp}}(\mathbf{F}, \mathbf{q}_{1i}, \mathbf{q}_{2i}) = \frac{(\mathbf{q}_{2i}^{\top} \mathbf{F} \mathbf{q}_{1i})^2}{(\mathbf{q}_{2i}^{\top} \mathbf{F})_1^2 + (\mathbf{q}_{2i}^{\top} \mathbf{F})_2^2 + (\mathbf{F} \mathbf{q}_{1i})_1^2 + (\mathbf{F} \mathbf{q}_{1i})_2^2}$$

Performs slightly better than the geometric distance in practice.

Is a squared distance.

Thresholding for RANSAC

Thresholding distances

- How to choose the threshold for RANSAC?

Thresholding distances

- How to choose the threshold for RANSAC?
- Introduce assumptions! 😊
- Assume that the errors of inliers follow a normal distribution.

Dimensionality of the error

- Fundamental/essential matrix
 - The error is the distance to the epipolar line
 - This is an error in one dimension
- Homography
 - The error is the distance from mapped point to true point
 - This is an error in two dimensions
- Pose estimation/camera calibration
 - Again it is a distance between points
 - This is an error in two dimensions

This is called the **codimension** of the problem.

We denote this m .

Choosing the threshold

- Assume that the error of each sample follows an m -dimensional normal distribution with standard deviation σ
- The summed and squared error is χ_m^2 distributed (by definition)
- Always work with squared distances
 - Not necessary to take square root when comparing distances

Choosing the threshold

- Assume that the error of each sample follows an m -dimensional normal distribution with standard deviation σ
- The summed and squared error is χ_m^2 distributed (by definition)
- Always work with squared distances
 - Not necessary to take square root when comparing distances
- Choose a confidence level e.g. 95%
 - i.e. we want our threshold to correctly identify 95% of true inliers.
- Look up the CDF for our χ_m^2 distribution.
- E.g. for a fundamental matrix and 95%, $\tau^2 = 3.84 \cdot \sigma^2$

Some values from the CDF of χ_m^2

$m \backslash 1 - p$	90%	95%	99%
1	2.71	3.84	6.63
2	4.61	5.99	9.21

Learning objectives

After this lecture you should be able to:

- explain and implement the eight point algorithm for estimating the fundamental matrix
- explain and implement estimation of the fundamental matrix with RANSAC
- choose the threshold for RANSAC using χ^2

Exercise time!