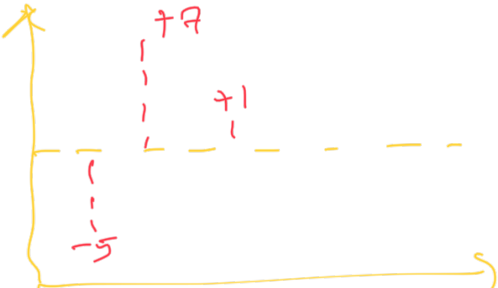
Loss/ Cost and optimisation

Recap:- ① Univariate → where the future values based mean based Best fit li(ne)



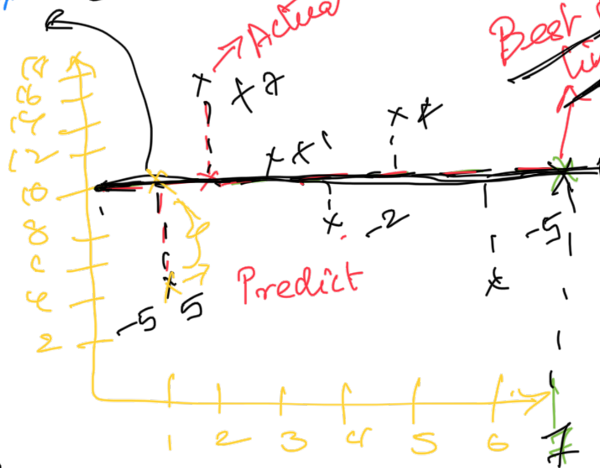
Drawbacks:- ① Residual values are high. → mere Estimate

② → Bill amount / Tip amount

what will be the ne(xt) tip?

$y = 0(x) + 10$

Meal#	Total Tip amount
1	5
2	17
3	11
4	8
5	14
6	5
→ 7 Mean	10



Sum of Residuals = Total (…)

$\Rightarrow -5 + 7 + 1 + -2 + 4 + -5 = 0$

Residuals ⇒ Errors

SSE → Sum of Squared Erro(r)

$\boxed{SSE = 120}$

Meal#	Total Bill (x)	Tip (y)	$(x_i-\bar{x})$	$(y_i-\bar{y})$	×	$(x_i-\bar{x})^2$
	34	5	40	-5	200	160

	x	y (actual)	$x_i-\bar{x}$	$y_i-\bar{y}$	$(x_i-\bar{x})(y_i-\bar{y})$	
1						
2	108	17	34	7	238	115
3	64	11	-10	1	-10	100
4	88	8	14	-2	-28	19
5	99	14	25	4	100	62
6	51	5	-23	-5	115	52
	$\bar{x} = 74$	$\bar{y} = 10$			615	42

60\$

Line Equation!!

$$m = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$$

$$m = \frac{615}{4206} = 0.146$$

$mx + c$

$$c = \bar{y} - m\bar{x} \Rightarrow 10 - 0.146(74)$$
$$\Rightarrow 10 - 10.81$$
$$= 0.8188$$

$$\hat{y} = 0.1462x - 0.818$$

Centroid $\Rightarrow (\bar{x}, \bar{y})$

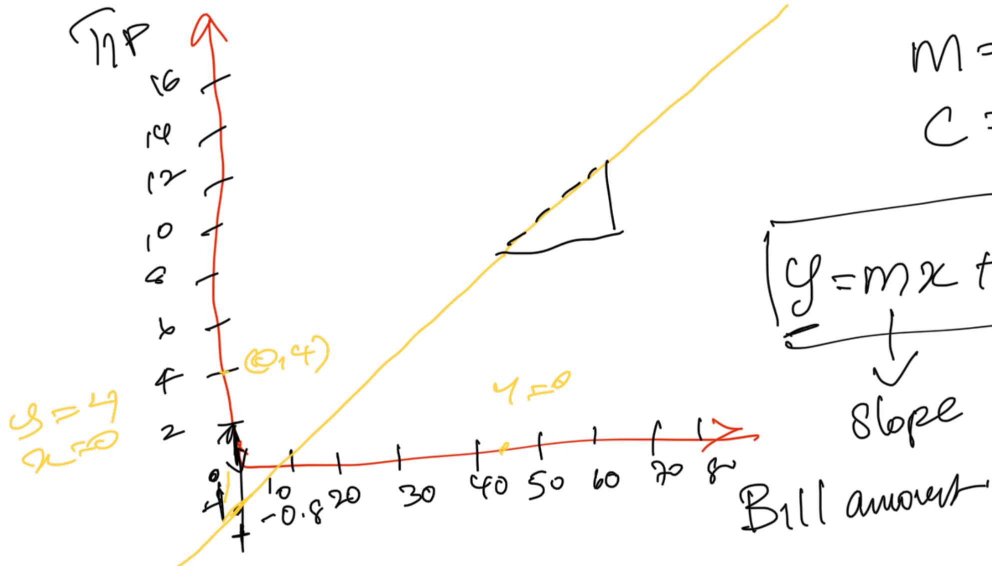
$c = -0.81$

m = Slope

c = y-Intercept

$$\boxed{y = mx + c}$$

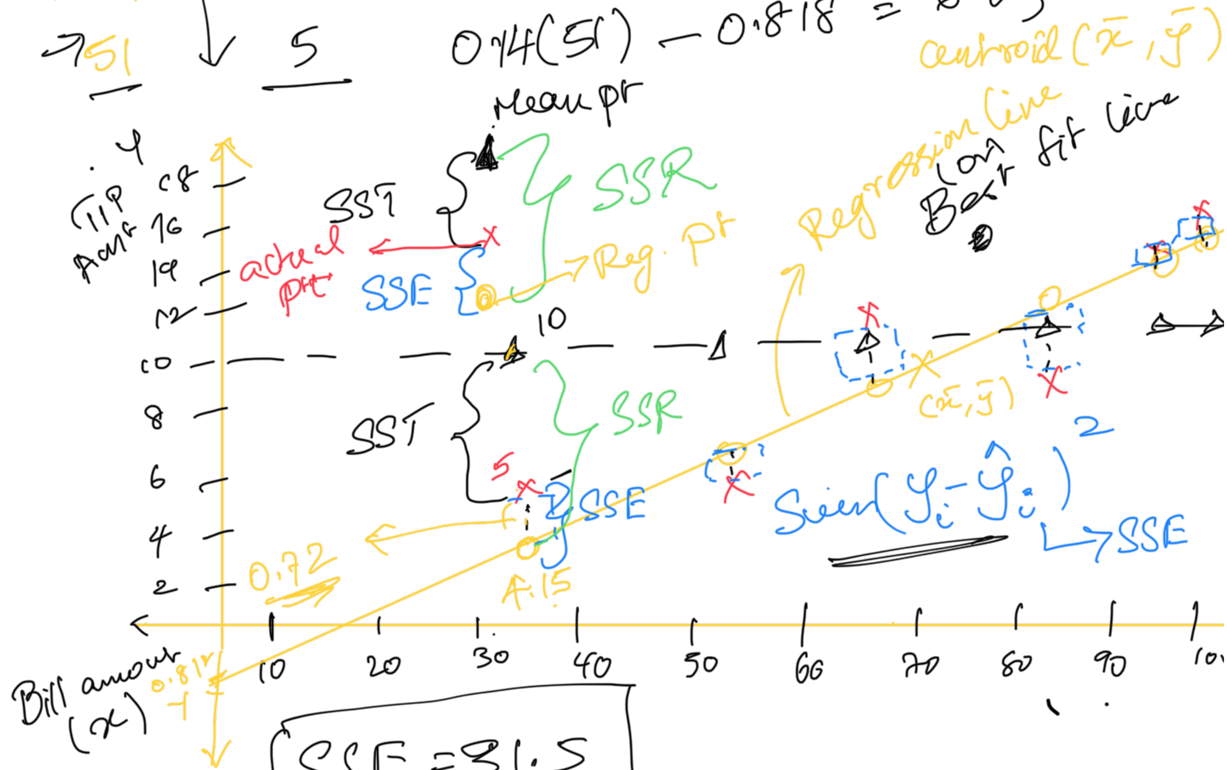
Slope



$\rightarrow y = 0.14x - 0.818$ $\rightarrow$ Goodness of fit line

X	Tip y	Prediction $\hat{y}$
34	5	$0.14(34) - 0.818 = 3.94$
108	17	$0.14(108) - 0.818 = 14.97$
	11	$0.14(64) - 0.818 = 8.53$

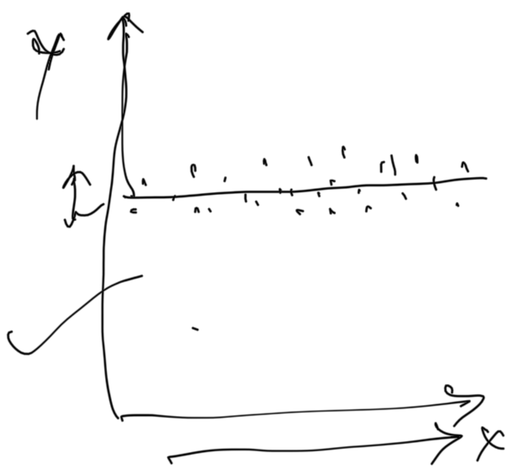
→ 64	11	0.14(64)
→ 88	8	0.14(88) − 0.818 = 12.04 ✓
→ 99	14	0.14(99) − 0.818 = 13.65 ✓
→ 51	5	0.14(51) − 0.818 = 6.63 ✓

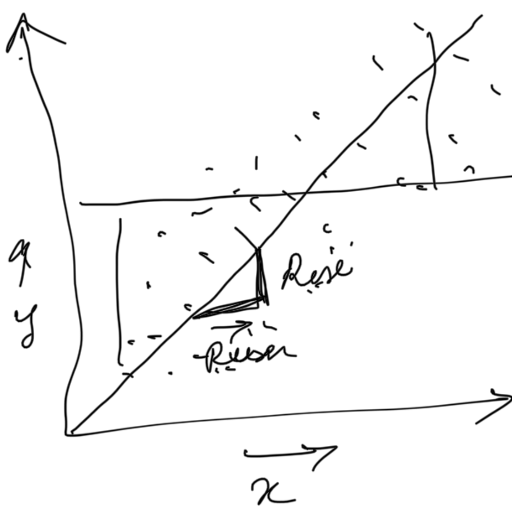


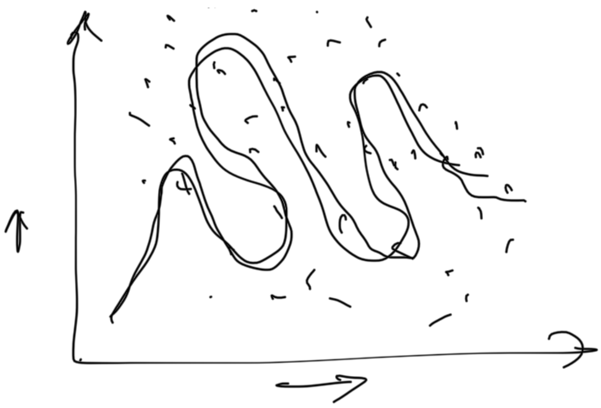
$\boxed{SSE = 31.5}$ Linear Regn.

$\boxed{SSE = 120}$ mean line

Goodness of fit → Regression line







Linear → line

Total Bill	Obsn	$\hat{y}$	$y-\hat{y}_i$	SE
39	5	4.15	0.849	0.72
108	17	14.96	2.03	4.12
69	11	8.53	2.46	6.0
88	8	12.04	−4.04	16.3
99	14	13.65	0.34	0.12
51	5	6.63	−1.63	2.6
				$\Sigma = 30.0$

SSE → 120 → SST

Regression

↳ SSR ⇒ 30.07

SST → Sum of Square diff - bet. actual & mean

SSE → Sum of Square Errors = differnce bet actual & Predicted.

→ Error

SSR → Sum of Squares due to the Regression line

Regression line

SSE = 31.5

SST = 120

SSR ⇒ sum of Squ Res.

→ SST

SST → SSR + SSE

Total

explains how much

Error & can reduce.

120 → 89.925 + 30.075 left

Total

reduced from Total SST by regression line

Goodness of fit. $r^2 = \frac{SSR}{SST}$ → Regression line reduced / Total Error.

Coefficient of Determin.

SST → SSR, SSE

$r^2 = \frac{89.925}{120} = 0.75$ → 75%

75% of the total Sum of Squ can be explained by regression eqn, remainder is error.

Mean Square

SSE =

$MSE = \frac{1}{n} SSE$

reduce

$r^2 = 1$ → Perfect regression line.

We will have to Reduce the

SSE → ... SSE → 120 → MSE

SSE

Mean Squared Error $= \frac{1}{n} \sum_{i \to n} (y_i - \hat{y}_i)^2$

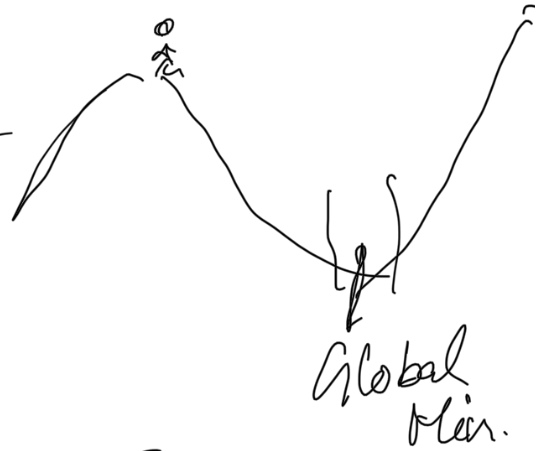
sum

Cost function

S - S - E

sum

$\sum (y - \hat{y})^2$



① cost fn = MSE

Mean Squared Error.

② alter or change the values of m & c → Intercept & Slope.

→ MSE

reduction or Alteration we try to perform should also include MSE

differential Calculus:

new Slope = Older slope − (± delta)

new intercept = older Intercept − (± delta)

$y = m_0 x + c_0$

$y = m_{new} x + c_{new}$

$\hookrightarrow f(2) = (2)^2 + 2 = 4 + 2 = 6$

① one i/p var.

$\frac{d\,f(x)}{dx} = 2x + 0$

$\rightarrow \frac{d(x)^n}{dx} = n x^{n-1}$

dx

only 1 ind. 'x'

$\frac{d(\text{constant})}{dx} = 0$

② 2 variable

Partial differentiation

$f(x, y)$ → Partial differentiation

new slope → Old Slope − P.D (MSE w.r.t Slope)

new intercept ⇒ old intercept − P.D (MSE w.r.t Intercept)

$\frac{d(MSE)}{dm}$ → Slope delta × L.R

$\frac{d(MSE)}{dc}$ × L.R = intercept delta

$$\text{Cost fn} = \frac{\sum_{i=1}^{n} (y_i - (mx + c)_i)^2}{n}$$

$$\hat{y} = mx + c$$

$\frac{d(\text{cost fn})}{dm}$, $\frac{f(\text{cost})}{fc}$

$$(x+y)^2 = x^2 + y^2 + 2xy$$

$$\frac{d(C.F)}{dm} \Rightarrow \frac{\sum (y^2 + m^2x^2 + c^2 + 2mxc - 2ymx - 2yc)}{n}$$

$$\Rightarrow \frac{\sum (0 + 2mx^2 + 0 + 2xc - 2yx - 0)}{n}$$

$$\frac{2\sum (mx^2 + xc - yx)}{n}$$

$\Rightarrow n$

$$\Rightarrow \frac{2x}{n} \sum (mx + c - y)$$

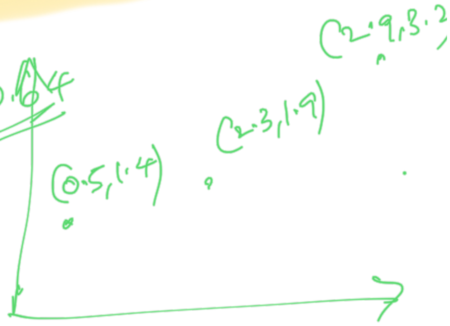
Step × slope $= -\frac{2}{n}\sum_{i=0}^{n} x_i [y_i - \hat{y}_i]$

Step × Intercept $= -\frac{2}{n}\sum_{i=0}^{n} (y_i - \hat{y}_i)$

x	y
0.5	1.4
2.3	1.9
2.9	3.2

$m = 0$
$c = 0$

$m = 0.64$



$m = 0.64$ $c = 0$

$\hat{y}_1 = mx_1 + c$
$\Rightarrow 0.64(0.5) + 0$
$\Rightarrow 0.32$
0.02

$\hat{y}_2 = mx_2 + c$
$= 0.64(2.3) + 0$
$= 1.47$
0.02

$\hat{y}_3 = mx_3 +$
$= 0.64(2.9)$
$= 1.856$

0.05

$SSE = (1.4 - 0.32)^2 + (1.9 - 1.47)^2 + (3.2 - 1.85$
$= 1.16 + 0.18 + 1.82$
$= 3.17$
SSE

$(1.4 - 0.32)$
$= 1.08$

$(1.9 - 1.47)$
$= 0.43$

$(3.2 - 1.8$
$= 1.35$

$-2 [1.08 + 0.42 + 1.35] = -1.91$

3 L

Learning rate = 0.001

→ 0.01

new Intercept = C − (0.01)(−1.91)

Learning Rate

Global mi

new Intercept = 0 + 0.02

(new eqn) ⇒ 0.64x + 0.02

Model will decide

Lowest MSE

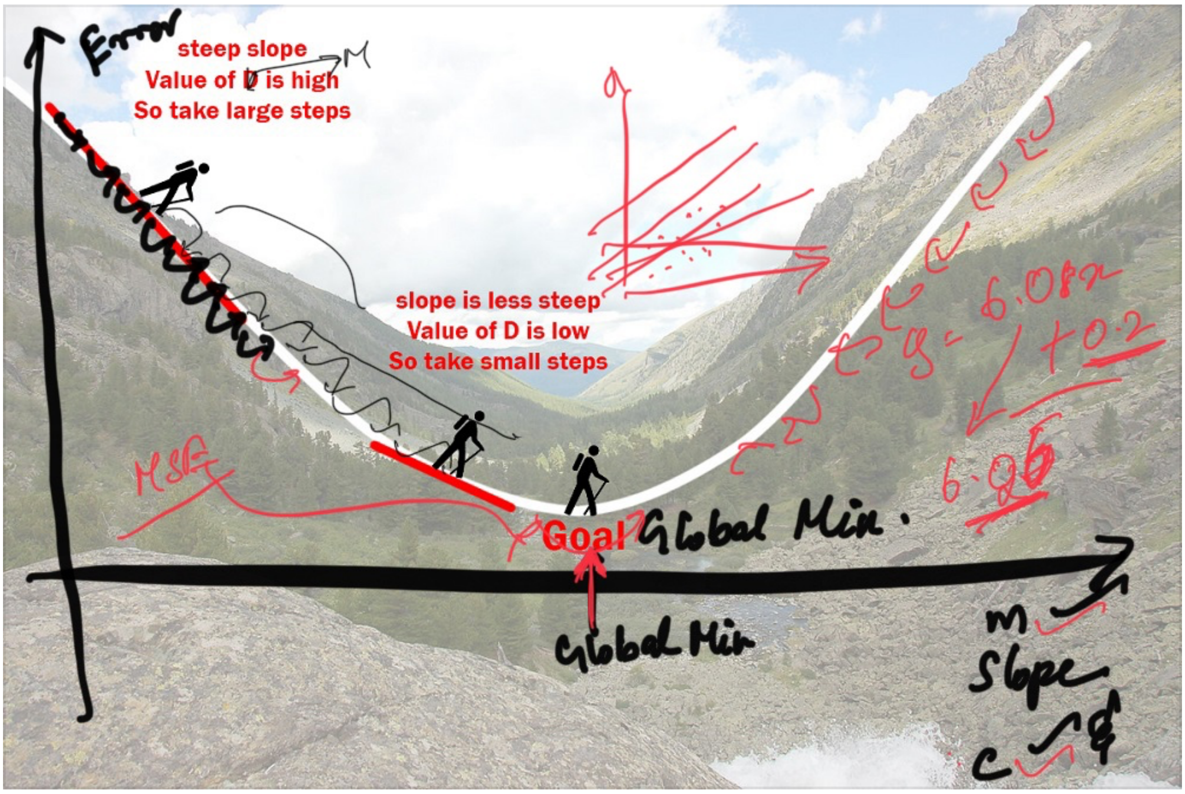
0.64 0.8 0.9 0.955 1.0

0.05 0.07 0.09 0.15

Write this using for code.

Pseudo code

1. Pre define → m & c = 0
2. delta → formula
3. MSE → print
4. Learning rate = 0.01
5. n iteration = 1000


Error
steep slope
Value of D is high
So take large steps
slope is less steep
Value of D is low
So take small steps
Goal
Global Min.
Global Min
MSE
m
slope
c