SOROBAN

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THE LEAGUE OF JAPAN ABACUS ASSOCIATIONS

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Preface

The development of internationalization has tremendously increased the number of foreign visitors to Japan. During their stay in this country, they may have occasions to have their accounts settled at stores and business establishments, where tradesman and clerks compute correct sums and balances in no time, deftly clicking beads on their abacus or soroban, the Japanese adding machine. In many instances they give the total by mental calculation. To foreign eyes, this must appear as nothing but short of a miracle, for foreign tourists observe from time to time that Japanese clerks and accountants seem to have a calculating machine in their head. Today large Japanese corporations and business firms are all equiped with a whole array of up-to-date electronic and electric calculating appliances. However, most calculations at these huge establishments, to say nothing of private stores and households, are done by the handy and simple abacus, which is still unrivaled as the most convenient and efficient instrument of everyday business calculation.

The most convicing evidence attesting to the extraordinary popularity of the soroban, which has remained a favorite companion of the Japanese for 500 years, is the fact that an amazingly large number of young people take examinations for the first, second and third grade abacus operator's licenses which are held three times a year under the auspices of the Japan Chamber of Commerce and Industry. In recente years the annual number has risen to 1,000,000 representing as large as 1.1 per cent of the total Japanese population. There is hardly any town or village where no abacus school is stablished. Each year every Japanese community holds on inter-grade and inter-high school abacus contest participated in by hundreds of eager contestants. Furthermore, popular lessons in abacus calculation broadcast over the national television and radio network serve to enhance young people's skill to operate the soroban, one of the requisies for their employment in business firms and corporations.

Japan yearly exports more than 330,000 sorobans. Now that the utility

of the soroban is recognized all over the world, we sincerely hope that the correct use of the soroban will be learned abroad. This handbook written after careful study is intended to provide a home-study course for correct and speedy abacus calculation, a suitable introduction for foreign beginners. Dealing mainly with addition and subtraction, it also touches upon multiplication and division. One word of caution - the student should use a standard abacus designed for special international use. He will then be able to acquire the secret of wonderfully speedy calculation.

The Japan Chamber of Commerce and Industry.

The League of Japan Abacus Associations.

Contents

History of the Abacus

1 Dust Abacus

The original meaning of the word, abacus, is presumed to have been a board covered with dust or fine sand. The surface covered with pretty dust or powder was divided with lines, each of which represented a different numerical place. Numbers and quantities were calculated by means of various signs drawn along lines. It is easy to imagine that such a primitive abacus was devised in the early primitive age of mankind. Probably the early civilization of Mesopotamia may have been the cradle of such a crude calculator. This type of abacus is called a dust abacus.

2 Line Abacus

In time the dust abacus developed into a ruled board, upon which pebbles or counters for calculation were placed on lines somewhat like checkers on a backgammon board. Its wide use in Egypt, Rome, Greece, India and other ancient civilized countries is well attested. Herodotus (484-425 B.C.) in his record probably refers to a line abacus: The Egyptians move their hand from right to left in calculation, while the Greeks from left to right. In the Athens Museum is preserved the Saramis Abacus, which is a white marble abacus, 149cm wide and 75cm height, with lines drawn on the board.

3 Grooved Abacus

In addition to the line abacus, the Romans made use of a more advanced abacus. On its board were carved several grooves, along which counters were moved up and down in calculation. One counter was laid in each of the upper grooves, while four in each of the lower grooves, with some additional counters laid at the right to facilitate the calculation of fractions.

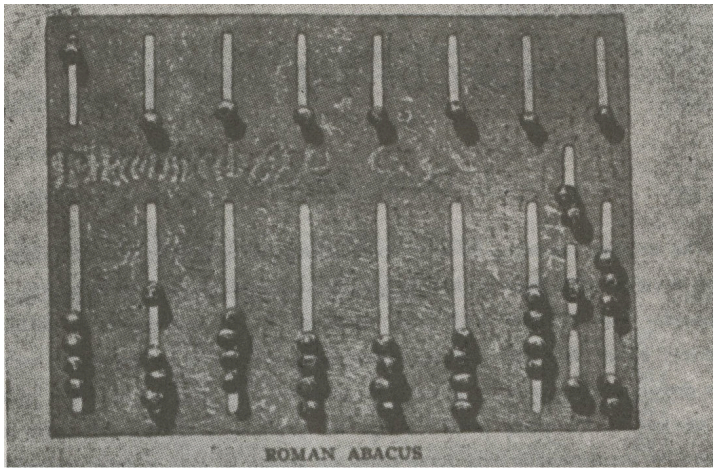


Figure 1: Roman Grooved Abacus.

4 Ancient Chinese Abacus

The earliest chinese abacus resembles the ancient Roman grooved abacus. The picture below represents the ancient Chinese abacus imagined from its desciption given in a book entitled Mathematical Treatises by Ancients written by Hsu Yo toward the close of the Later Han Dynasty about 1,700 years ago and annotated by Chen Luan 1,400 years ago.

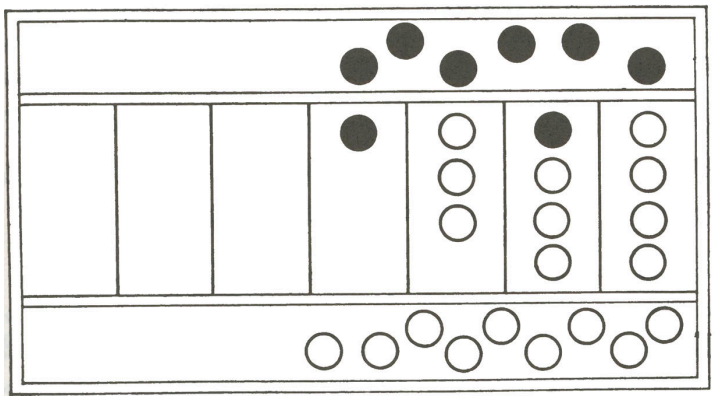


Figure 2: The abacus described in Mathematical Treatises by Ancients.

The Abacus is closely similar to the Roman grooved abacus both in construction and in the method of calculation. From these and other evidences, it may be well assumed that this was an improvement upon the ancient Roman grooved abacus which had been imported into China in earlier days.

5 Chinese Abacus

In China the abacus came into common use in the Ming Dinasty. A book entitled Ch'o Ching Lu gives a proverbial expression: "A servant, some time after he is hired, comes to do nothing more than he is ordered to. Therefore, he is like an abacus counter". A book written by Wu Ching-hsin-min in 1450 gives descriptions of the abacus. A large number of books published toward the close of the Ming Dinasty attests to the fact that the abacus had come into popular use as the people's favorite tool. Many of the books published in those days have been preserved until this day. The abacus in those days had two counters above the bar and five below. In China this type of abacus has continued in use down to this day.

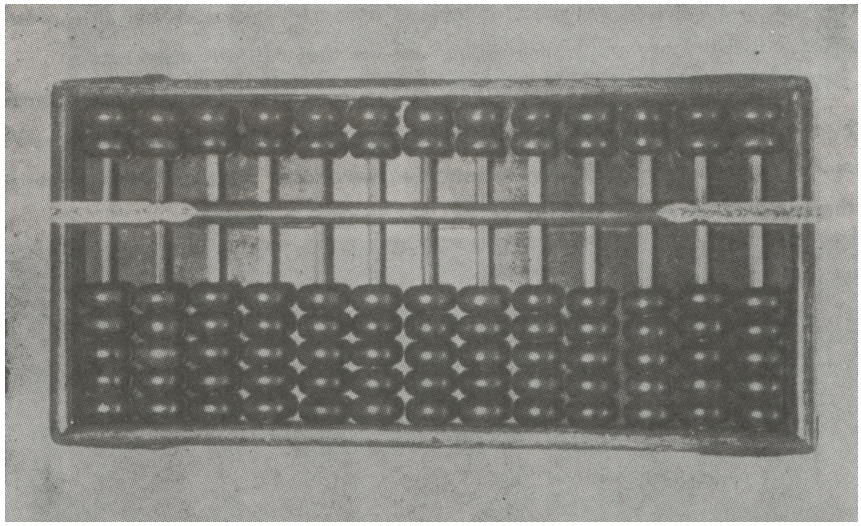


Figure 3: Ancient Chinese Abacus.

6 Japanese Abacus

A little past the middle of the fifteenth century the Chinese abacus and its operational techniques were introduced into Japan. Shortly afterward Japan entered into a long period of peace, which fostered the development of her cities and commerce. Mathematician's constant, diligent study developed the distinctive Japanese of abacus operation superior to the original Chinese method. The large-sized Chinese abacus was improved into a handier smaller-sized one, and toward the close of the ninteenth century the modern Japanese abacus with one five-unit counter and four-unit counters on each rod came into usage along with the older-typed one with one-unit counters on each rod and five one-unit counters on each rod. In 1938 the technique of abacus operation was included in the national grade-school textbooks on arithmetic complied by the Education Ministry, and today abacus technique is a required study in the third and upper grades. Now the abacus with one-five unit counter and four one-unit counters on each rod is now a standard one in universal use. It should be also noted that the older Chinese division method was formerly replaced by the present Japanese division method which makes use of the multiplication table.

The inclusion of abacus technique in the Curriulum of Japanese compulsory education and the enforcement of the abacus Efficiency Tests system since its inception in 1928 have been the two major factors which have led to the present universal popularity of the abacus in Japan.

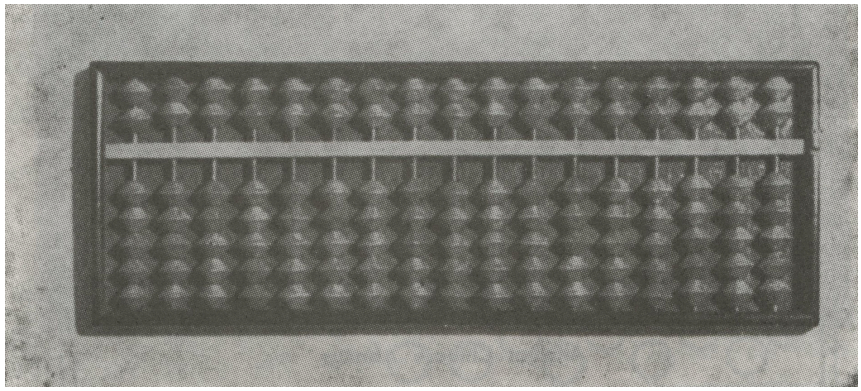


Figure 4: Japanese abacus (used till the 19th century).

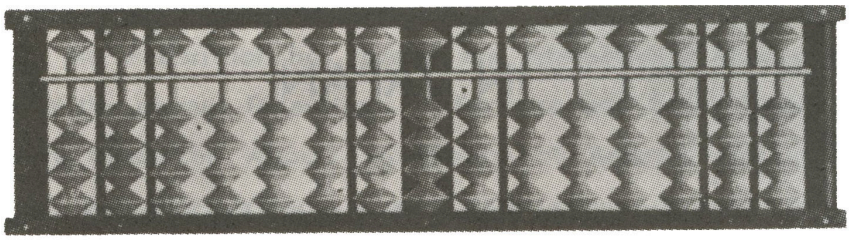


Figure 5: Abacus (Soroban) for foreigner's use.

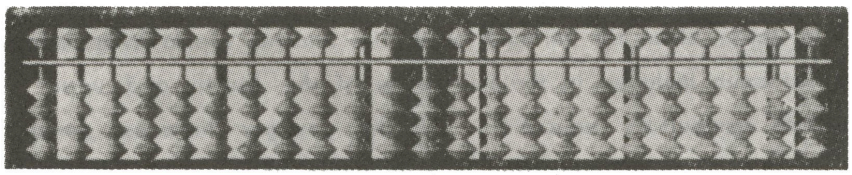


Figure 6: Current Abacus (Soroban) in Japan.

Present activities of abacus circles in Japan

Since its introduction in Japan, the abacus has attained a high development. The large-sized Chinese abacus was improved into the Japanese soroban which is handier to carry and the two five-unit counters and five one-unit counters on each rod of the former were reduced to one five-unit counter and four one-unit counters on that of the latter. The remarkable economic development from the nineteenth to the twentieth century saw the improvement of the abacus into a more scientific one easier to understand and learn. The methods of addition, subtraction, multiplication and division introduced in this book are those used by the majority of Japanese people and taught by the majority of abacus instructors as well.

In Japan since the termination of the Second World War, abacus operation has been included in the curriculum of fourth and upper grades of Japanese compulsory education. So the younger Japanese generation who has received postwar education is well aware of the great utility of abacus calculation, and the increasing number of younger parents send their grade-school children to extra-curriculum abacus schools. Applicants for the first, second and third-grade abacus operators' licenses have multiplied in recent years so much, so that they yearly number one-million or about 10 per cent of the total population of this country. The third-grade license is the lowest qualification for the competency of professional calculators in business firms. The TV and radio lessons in abacus calculation aim at helping listeners acquire the third-grade license. The immense number of abacus license holders is an eloquent testimony to the tremendous popularity of abacus calculation and to the high prestige of the abacus licensing system.

In addition, once a year we hold a national abacus contest in Tokyo, in which the abacists who pass local preliminary contests participate for the all-Japan abacus championship. We dub this an abacus festival.

A total of 6,500 abacus instructors who hope for the healthy growth of abacus calculation has formed the All-Japan Federation of Abacus Operators to make joint efforts for the effective teaching of abacus techniques, the establishment of the science of abacus calculation and the adavancement of the culture and welfare of the members.

Recently the Federation has formed the International Abacus Association in league with the abacus federations in Formosa and South Korea.

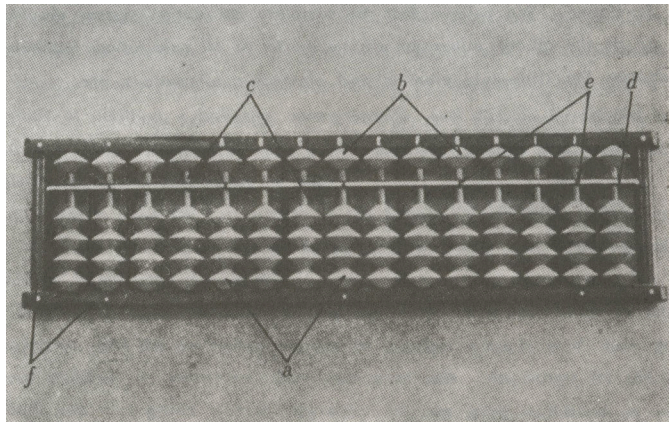
The newly organized Abacus Association of America has asked for its membership in our International Abacus Association. So we are enlivened with the reassuring hopes that our International Abacus Association will grow up in time into a world-wide association in name and deed.

The abacus is by no means a relic of the past, it awaits yet to be more fully developed. We believe that the complete calculation system should be built up on the all-out utilization of the merits of all calculation facilities - the abacus, the slide-rule electric and electronic calculators, etc.

Preliminary knowledge

A. Construction of the abacus

Caclulation by means of the abacus is briefly called abacus calculation. First of all, you have to learn the terms and basic principles of abacus calculation. Compare your abacus with the picture of the abacus below and learn the terms for various parts of the abacus.



a - 1-unit counter... a counter that represents 1, 10, 100, 1000, etc.

b - 5-unit counter... a counter that represents 5, 50, 500, 5000, etc.

c - rod

d - bar

e - unit point... a marker dot placed on every third rod serving as marker for setting a number.

f - frame

B. Proper posture for operating the abacus

Now look at the pictures below. Fig. A shows a proper posture as seen from the front. Hold the left end of the abacus from above. See that the wrist or the elbow of your right arm does not touch the abacus or the desk.

Fig. B is a proper posture as seen sideways. Sit straight with your burst slightly bending forward.

Next, you use the thumb and the forefinger of the right hand in operating the abacus, closing your other three fingers lightly.

Fig. A

Fig. B

C. Getting ready for calculation

Now get ready for calculation. Before beginning calculation, you must see that all counters of the abacus represent zero. You can do this by slanting your abacus toward you and moving down all the counters. Next level the abacus on the desk, and as shown in Figure D, move up all the 5-unit counters by running the forefinger of your right hand between the 5-unit counters and the bar from left to right. Now the abacus is ready for calculation.

Fig. C

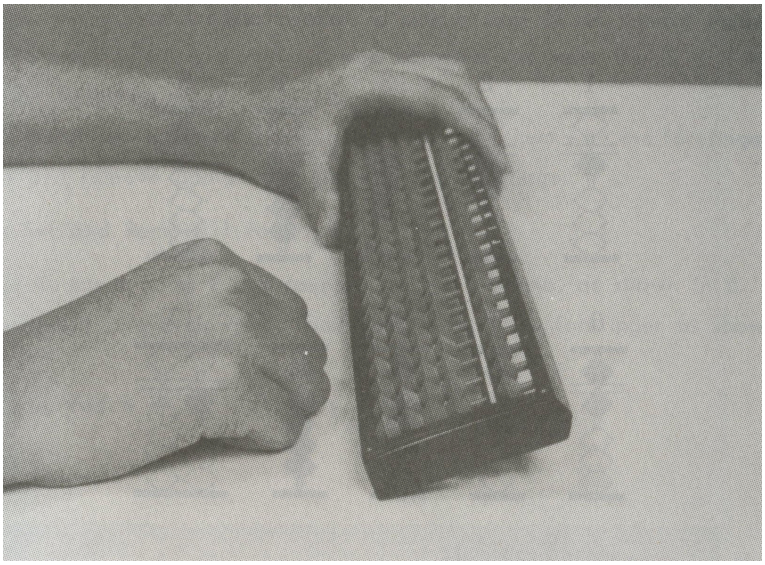
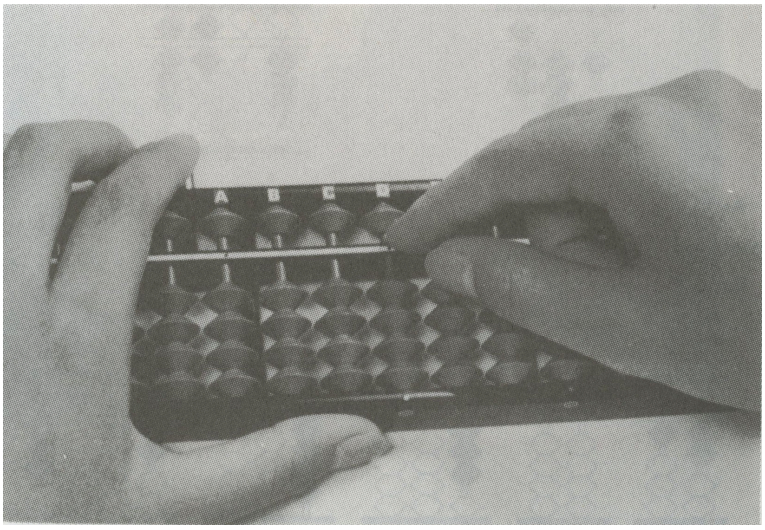
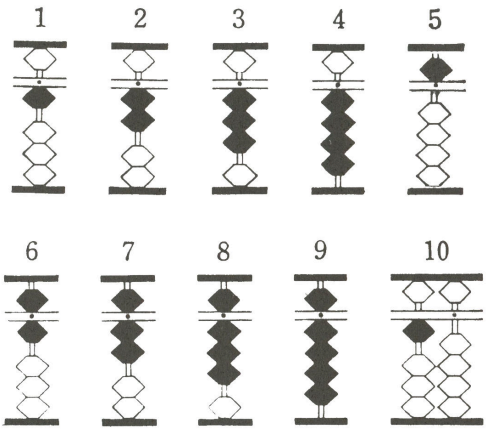


Fig. D

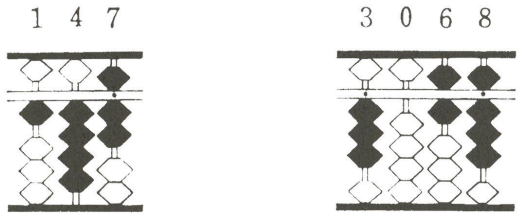


D. How to form numbers

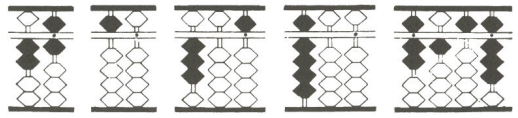
On the abacus numbers are formed by moving counters close to the bar. Numbers from 1 to 10 are formed as follows respectively.



147 and 3,068 are set as follows.



As the above figure shows, you get zero on a rod when both the 1-unit counter and the 5-unit counter are away from the bar. Read the numbers represented in the figures below.



If you can read the numbers correctly, let us go on to the next page.

E. How to set and remove numbers

In putting a number on the abacus, you set or enter it, and in taking a number away, you remove or clear it.

In setting and removing a number, you correctly use your thumb and forefinger in the following ways:

a. Set 1-unit counters with the thumb and remove them with the forefinger.

b. Set and remove 5-unit counters with the forefinger.

1. How to set and remove 1 to 9:

a. To set1, move up a 1-unit counter, with the thumb, as shown in E.

b. To remove 1, move down a 1-unit counter with the forefinger as shown in F.

c. Set 2, 3 and 4 in the same way.

Fig. E

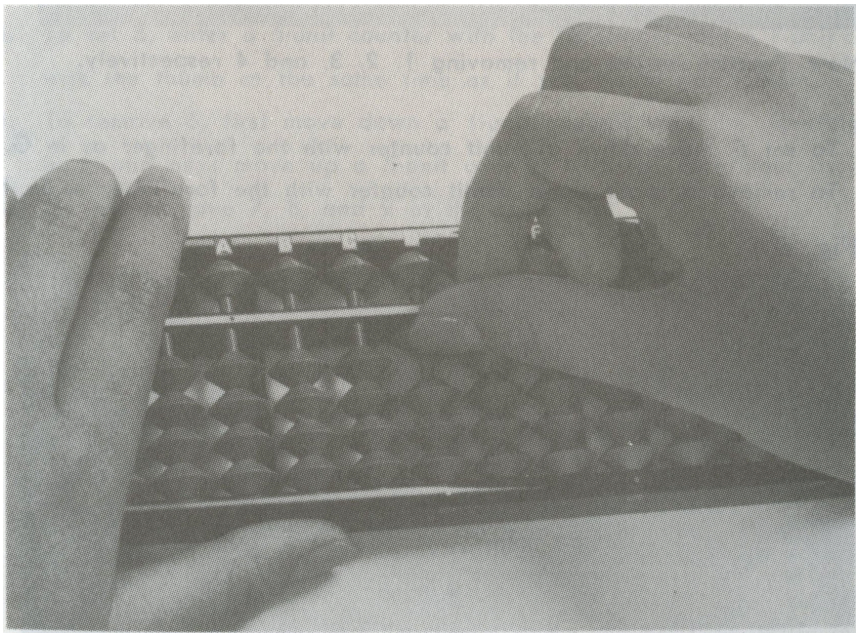
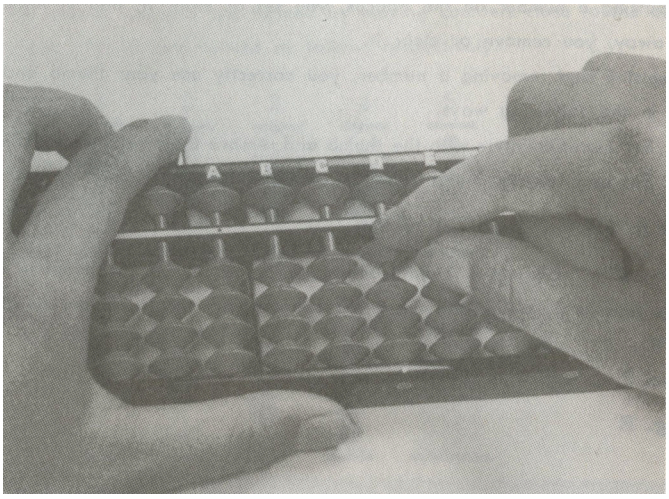


Fig. F



Now practice setting and removing 1, 2, 3 and 4 respectively.
a. To set 5, move down a 5-unit counter with the forefinger as in G.
b. To remove 5, move up a 5-unit counter with the forefinger as in H.

Fig. G

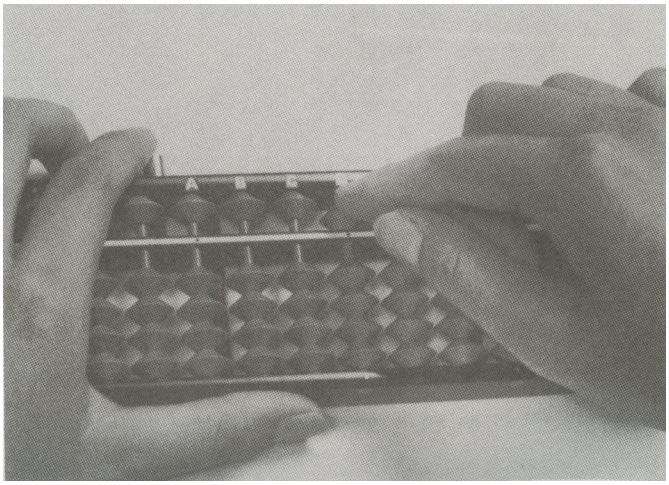
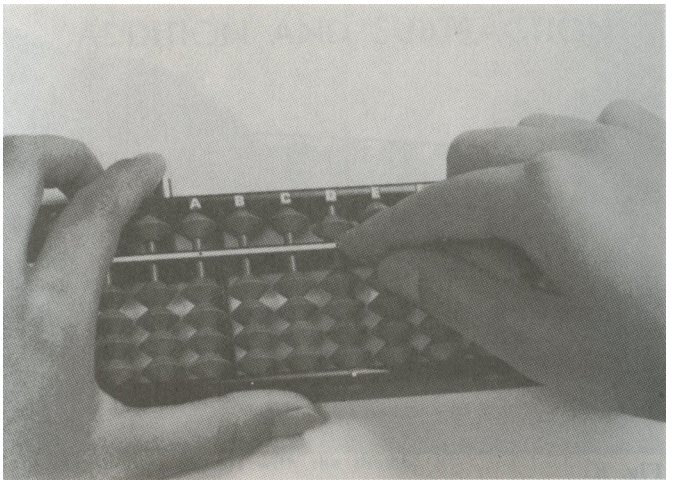


Fig. H



a. To set 6, enter a 5-unit counter with the forefinger and a 1-unit counter with the thumb at the same time as if you would pinch them.

b. To remove 6, first move down a 1-unit counter with the forefinger as in J, and next move up a 5-unit counter with the forefinger as in K.

c. Set and remove 7, 8 and 9 in the same way.

Fig. I

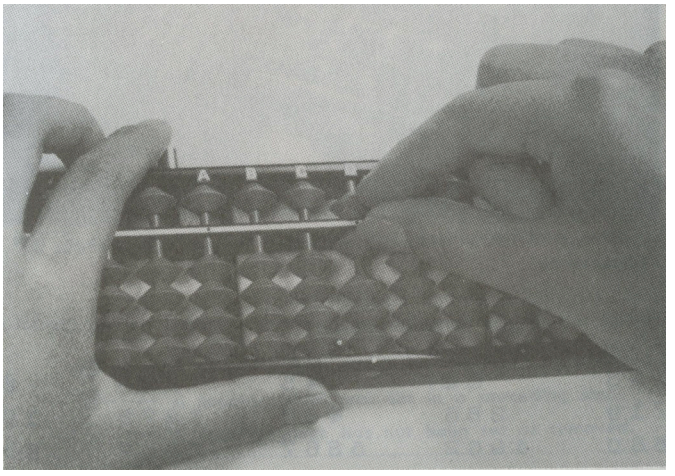


Fig. J

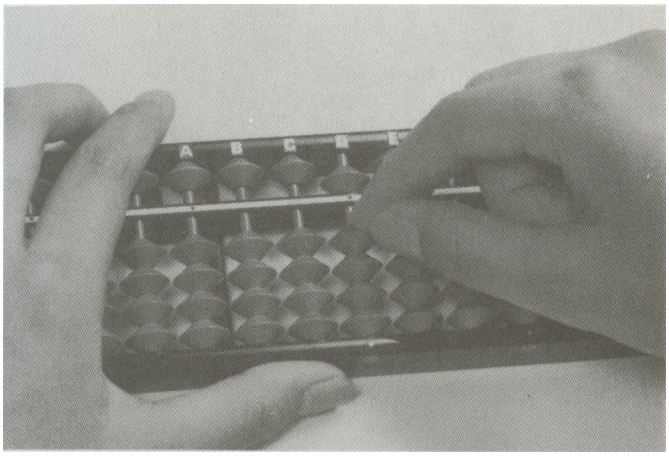
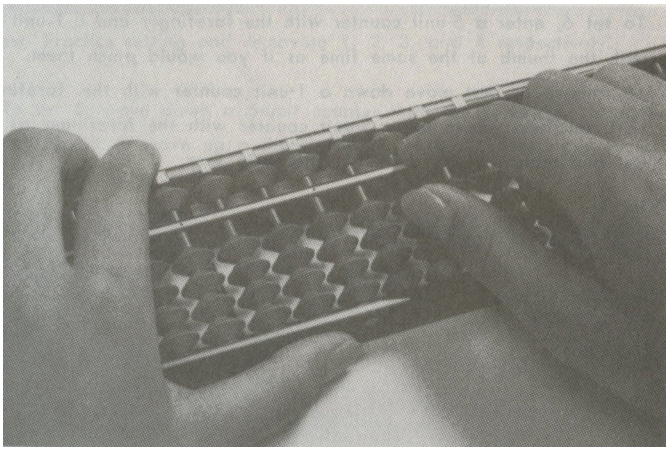


Fig. K



Now practice setting and removing 6, 7, 8 and 9 five times respectively.

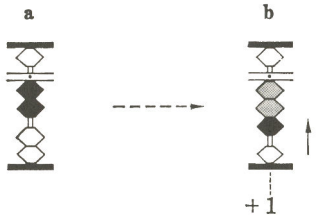
Exercises

Practice setting and removing the following numbers, paying special attention to numerical places, such as 1, 10, 100, 1000, etc.

23	34	45	125	601
719	286	400	1278	1050
3560	4902	5867	2359	9300

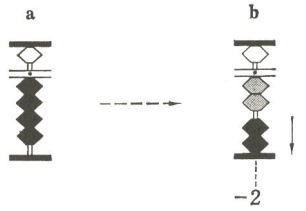
Addition and Subtraction

Example 1. 2 + 1 = 3

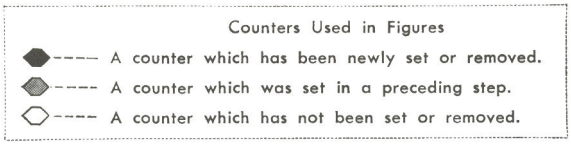


a. Set two 1-unit counters with the thumb.
b. To add 1, enter one 1-unit counter with the thumb.

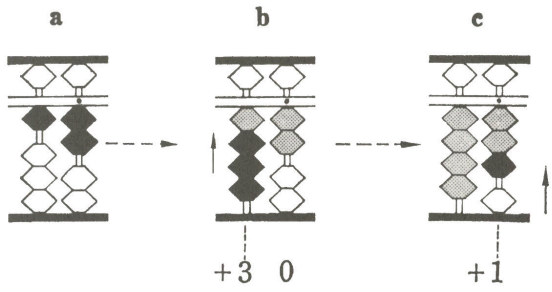
Example 2. 4 - 2 = 2



a. Set four 1-unit counters with the thumb.
b. To subtract 2, remove two 1-unit counters with the forefinger.

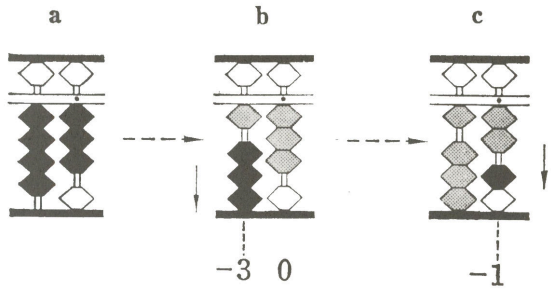


Example 3. 12 + 31 = 43



Begin calculation with tens in the same way as you calculate ones.
a. Set 12 with the thumb.
b. To add 31, first set the 30 of 31 with the thumb.
c. Next set the 1 of 31 with the thumb.

Example 4. 43 - 31 = 12



a. Set 43 with the thumb.
b. To subtract 31, first remove the 30 of 31 with the forefinger.
c. Next remove the 1 of 31 with the forefinger.

Exercises 1

(1)
$$\begin{array}{r} 21 \\ +\ 13 \\ \hline \end{array}$$

(2)
$$\begin{array}{r} 20 \\ +\ 24 \\ \hline \end{array}$$

(3)
$$\begin{array}{r} ¥\ 31 \\ +\ 11 \\ \hline \end{array}$$

(4)
$$\begin{array}{r} \$\ 10 \\ +\ 23 \\ \hline \end{array}$$

(5)
$$\begin{array}{r} 43 \\ -\ 32 \\ \hline \end{array}$$

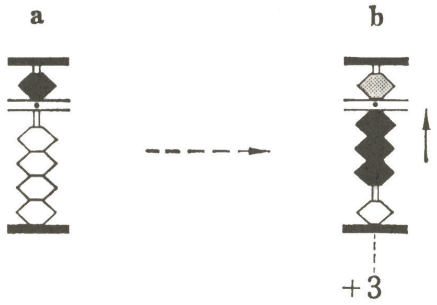
(6)
$$\begin{array}{r} 32 \\ -\ 11 \\ \hline \end{array}$$

(7)
$$\begin{array}{r} ¥\ 42 \\ -\ 22 \\ \hline \end{array}$$

(8)
$$\begin{array}{r} \$\ 34 \\ -\ 31 \\ \hline \end{array}$$

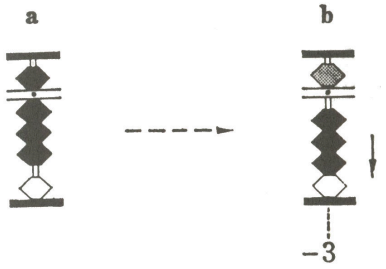
Answer
(1) 34 (2) 44 (3) ¥42 (4) $33 (5) 11 (6) 21 (7) ¥20 (8) $3

Example 5. $5 + 3 = 8$



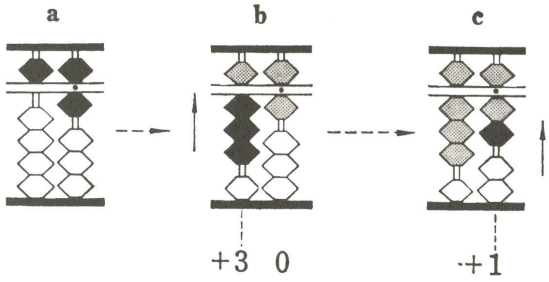
a. Set a 5-unit counter with the forefinger.
b. To add 3, enter three 1-unit counters with the thumb.

Example 6. 8 - 3 = 5



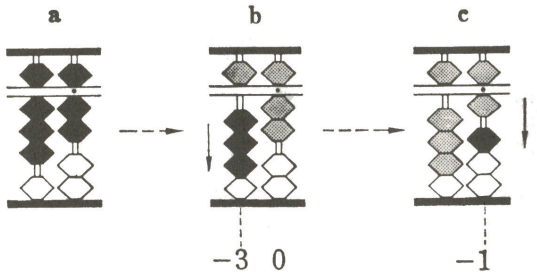
a. Set 8 with the thumb and the forefinger at the same time.
b. To subtract 3, remove three 1-unit counters with the forefinger.

Example 7. 56 + 31 = 87



a. Set the 5 in the tens place with the forefinger and set the 6 in the ones place with the thumb and the forefinger as if you would pinch it together.
b. To add 31, first enter the 3 in the tens place with the thumb.
c. Next enter the 1 in the ones place with the thumb.

Example 8. 87 - 31 = 56



a. Set 87 with the thumb and the forefinger as if you would pinch it together.
b. To subtract 31, first remove the 3 in the tens place with the forefinger.
c. Next remove the 1 in the ones place with the forefinger.

Exercises 2

(1)
$$\begin{array}{r} 50 \\ +\ 39 \\ \hline \end{array}$$

(2)
$$\begin{array}{r} 78 \\ +\ 21 \\ \hline \end{array}$$

(3)
$$\begin{array}{r} ¥\ 65 \\ +\ 32 \\ \hline \end{array}$$

(4)
$$\begin{array}{r} \$\ 76 \\ +\ 12 \\ \hline \end{array}$$

(5)
$$\begin{array}{r} 96 \\ -\ 21 \\ \hline \end{array}$$

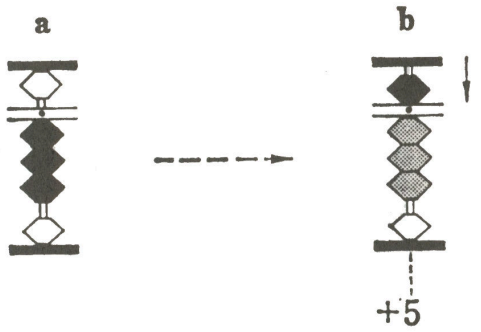
(6)
$$\begin{array}{r} 87 \\ -\ 32 \\ \hline \end{array}$$

(7)
$$\begin{array}{r} ¥\ 98 \\ -\ 48 \\ \hline \end{array}$$

(8)
$$\begin{array}{r} \$\ 79 \\ -\ 13 \\ \hline \end{array}$$

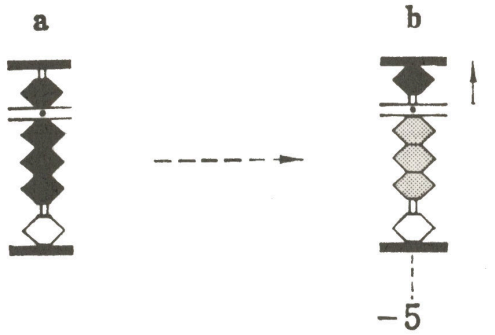
Answer
(1) 89 (2) 99 (3) ¥97 (4) $88 (5) 75 (6) 55 (7) ¥50 (8) $66

Example 9. $3 + 5 = 8$



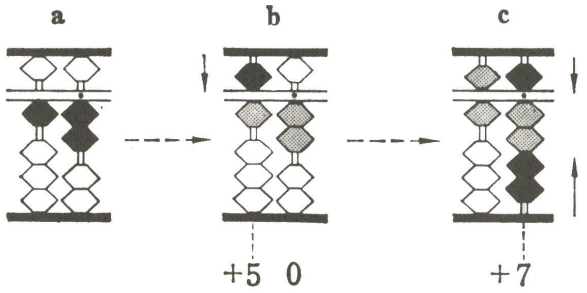
a. Set three 1-unit counters with the thumb.
b. To add 5, enter the 5-unit counter with the forefinger.

Example 10. $8 - 5 = 3$



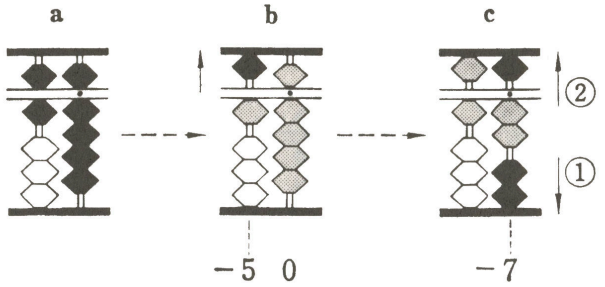
a. Set 8 with the thumb and the forefinger at the same time.
b. To subtract 5, remove the 5-unit counter with the forefinger.

Example 11. 12 + 57 = 69



a. Set 12 with the thumb.
b. To add 57, first enther the 5 in the tens place with the forefinger.
c. Next enter the 7 in the ones place with the thumb and the forefinger as if you would pinch it together.

Example 12. 69 - 57 = 12



a. Set 69 with the thumb and the forefinger as if you would pinch it together.
b. To subtract 57, first remove the 5 in the tens place with the forefinger.
c. Next subtract the 7 in the ones place. To subtract 7, first remove 2 and next 5 with the forefinger.

Exercises 3

(1)
$$\begin{array}{r} 12 \\ +\ 67 \\ \hline \end{array}$$

(2)
$$\begin{array}{r} ¥\ 42 \\ +\ 56 \\ \hline \end{array}$$

(3)
$$\begin{array}{r} \$\ 31 \\ +\ 68 \\ \hline \end{array}$$

(4)
$$\begin{array}{r} ¢\ 12 \\ +\ 75 \\ \hline \end{array}$$

(5)
$$\begin{array}{r} 89 \\ -\ 76 \\ \hline \end{array}$$

(6)
$$\begin{array}{r} ¥\ 67 \\ -\ 57 \\ \hline \end{array}$$

(7)
$$\begin{array}{r} \$\ 79 \\ -\ 68 \\ \hline \end{array}$$

(8)
$$\begin{array}{r} ¢\ 99 \\ -\ 75 \\ \hline \end{array}$$

Answer
(1) 79 (2) ¥98 (3) $99 (4) ¢87 (5) 13 (6) ¥10 (7) $11 (8) ¢24

Practice 1

(1) 11+21+56=

(2) 21+13+65=

(3) 51+15+32=

(4) 30+58−21=

(5) 76+21−52=

(6) 89−57−11=

(7) 99−41−56=

(8)
$$\begin{array}{r} 56 \\ 41 \\ -\ 15 \\ \hline \end{array}$$

(9)
$$\begin{array}{r} 82 \\ 12 \\ -\ 63 \\ \hline \end{array}$$

(10)
$$\begin{array}{r} ¥\ 21 \\ 57 \\ -\ 13 \\ \hline \end{array}$$

(11)
$$\begin{array}{r} ¥\ 55 \\ 41 \\ -\ 25 \\ \hline \end{array}$$

(12)
$$\begin{array}{r} \$\ 73 \\ 11 \\ -\ 62 \\ \hline \end{array}$$

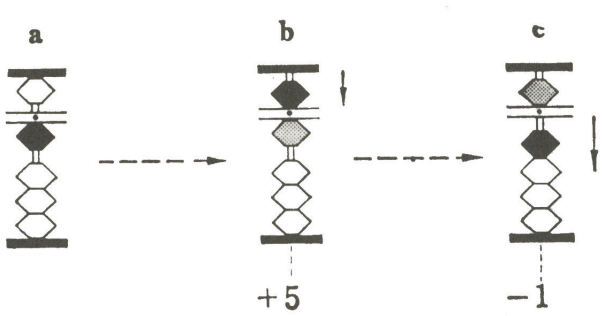
(13)
$$\begin{array}{r} \$\ 11 \\ 68 \\ -\ 53 \\ \hline \end{array}$$

(14)
$$\begin{array}{r} \$\ 50 \\ 25 \\ -\ 65 \\ \hline \end{array}$$

(15)
$$\begin{array}{r} ¢\ 15 \\ 53 \\ -\ 12 \\ \hline \end{array}$$

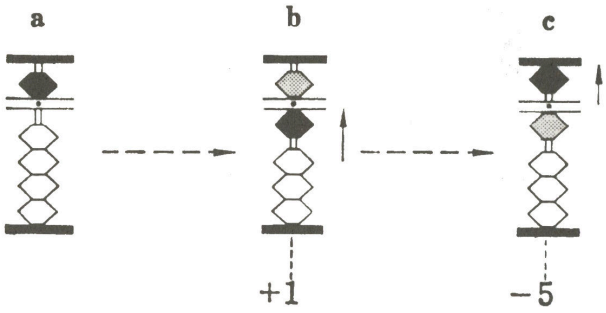
Answer
(1) 88 (2) 99 (3) 98 (4) 67 (5) 45 (6) 21 (7) 2 (8) 82 (9) 31
(10) ¥65 (11)¥71 (12) $22 (13) $26 (14) $10 (15) ¢56

Example 13. 1 + 4 = 5



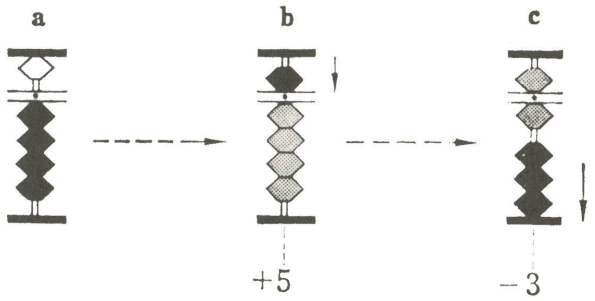
a. Set 1 with the thumb.
b. You cannot add 4, as there are only 1-unit counter that can be added. So first add 5 with the forefinger.
c. In adding 4, you have added 5. This means that you have added 1 too many. So with the forefinger remove the 1 that you have added in excess.

Example 14. 5 - 4 = 1



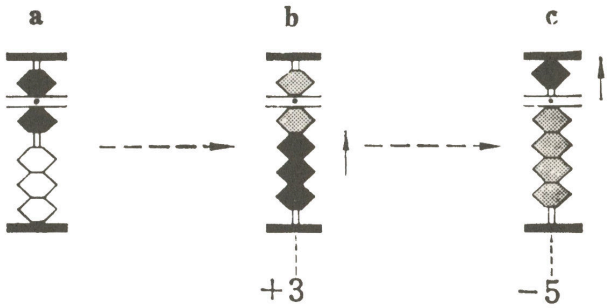
a. Set 5 with the forefinger.
b. You cannot subtract 4, as there are no 1-unit counters to be subtracted on the rod. So thinking of subtracting 4 from 5, first add 1 with the thumb.
c. Next remove 5 with the forefinger.

Example 15. 4 + 2 = 6



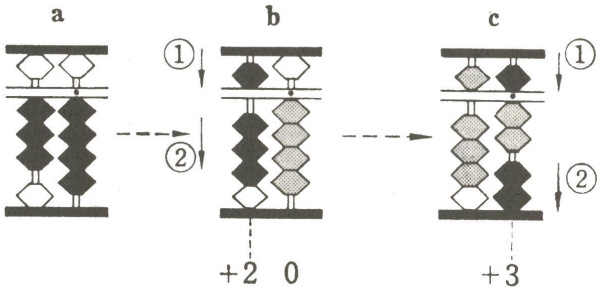
a. Set 4 with the thumb.
b. You cannot add 2, as there are no 1-unit counters that can be added. So first add 5 with the forefinger.
c. In adding 2, you have added 5. This means that you have added 3 too many. So remove 3 which you have added in excess.

Example 16. 6 - 2 = 4



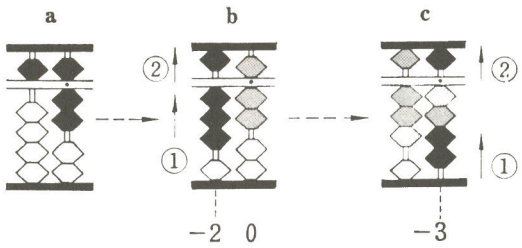
a. Set 6 as if you would pinch it with the thumb and the forefinger.
b. You cannot subtract 2, as there is only one 1-unit counter that can be subtracted. So think of subtracting 2 from 5 counter. If you subtract 2 from 5, you have 3 left, so first add 3.
c. Next remove 5 with the forefinger.

Example 17. 34 + 23 = 57



a. Set 34 with the thumb.
b. To add 23, first calculate tens. You cannot add the 20 of 23 by using 1-unit counters. So with the forefinger first enter 5 and next remove the 3 which you have added in excess.
c. Next calculate ones. You cannot add the 3 of 23 by using 1-unit counters, so enter 5 and remove the 2 which you have added in excess.

Example 18. 57 - 23 = 34



a. First set 5 in the tens place and next set 7 in the ones place as if you would pinch it.
b. First calculate tens. You cannot subtract the 20 of 23 by using 1-unit counters. So thinking of subtracting 2 from 5, add the 3 that are left with the thumb and remove 5 with the forefinger.
c. Next you come to the calculation of ones. You cannot subtract the 3 of 23 by using 1-unit counters. So thinking of subtracting 3 from 5, add the 2 that are left with the thumb and remove 5 with the forefinger.

Exercises 4

(1)
$$\begin{array}{r} 14 \\ +\ 43 \\ \hline \end{array}$$

(2)
$$\begin{array}{r} ¥\ 33 \\ +\ 32 \\ \hline \end{array}$$

(3)
$$\begin{array}{r} \$\ 44 \\ +\ 24 \\ \hline \end{array}$$

(4)
$$\begin{array}{r} ¢\ 22 \\ +\ 34 \\ \hline \end{array}$$

(5)
$$\begin{array}{r} 56 \\ -\ 23 \\ \hline \end{array}$$

(6)
$$\begin{array}{r} ¥\ 77 \\ -\ 34 \\ \hline \end{array}$$

(7)
$$\begin{array}{r} \$\ 85 \\ -\ 41 \\ \hline \end{array}$$

(8)
$$\begin{array}{r} ¢\ 65 \\ -\ 24 \\ \hline \end{array}$$

Answer

(1) 57 (2) ¥65 (3) $68 (4) ¢56 (5) 33 (6) ¥43 (7) $44 (8) ¢41

Practice 2

(1) 13+11+33−12=

(2) 21+22−11+46=

(3) 44−12+57−70=

(4) 12+31+24−51=

(5) 11+47−34−13=

(6) 43+14+22−68=

(7) 24+31+40−13=

(8)
$$\begin{array}{r} 55 \\ -\ 23 \\ 34 \\ -\ 22 \\ \hline \end{array}$$

(9)
$$\begin{array}{r} ¥\ 76 \\ -\ 34 \\ 47 \\ -\ 58 \\ \hline \end{array}$$

(10)
$$\begin{array}{r} 44 \\ 23 \\ 31 \\ -\ 77 \\ \hline \end{array}$$

(11)
$$\begin{array}{r} ¥\ 13 \\ 42 \\ 31 \\ -\ 54 \\ \hline \end{array}$$

(12)
$$\begin{array}{r} \$\ 87 \\ -\ 14 \\ -\ 32 \\ 26 \\ \hline \end{array}$$

(13)
$$\begin{array}{r} \$\ 68 \\ 21 \\ -\ 45 \\ 14 \\ \hline \end{array}$$

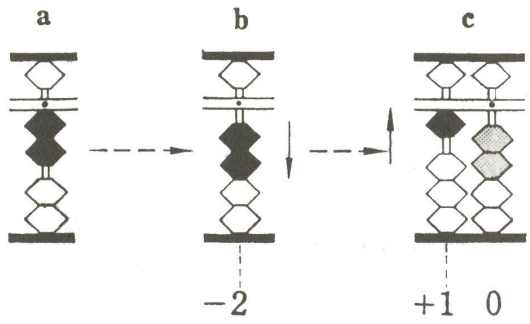
(14)
$$\begin{array}{r} \$\ 63 \\ 14 \\ -\ 43 \\ 35 \\ \hline \end{array}$$

(15)
$$\begin{array}{r} ¢\ 89 \\ -\ 66 \\ 42 \\ -\ 34 \\ \hline \end{array}$$

Answer

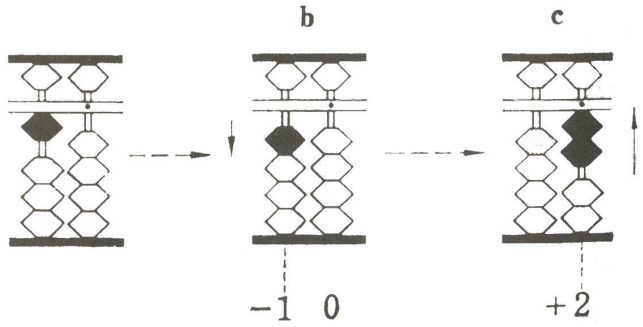
(1) 45 (2) 78 (3) 19 (4) 16 (5) 11 (6) 11 (7) 82 (8) 44 (9) ¥ 31
(10) 21 (11) ¥32 (12) $67 (13) $58 (14) $69 (15) ¢31

Example 19. 2 + 8 = 10



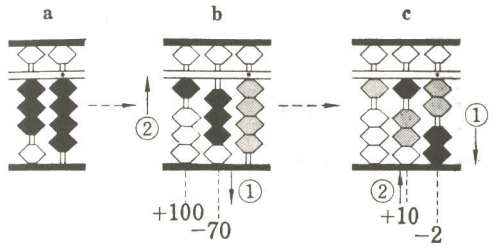
a. Set 2 with the thumb.
b. You cannot add 8 on the rod which has 2. So think, "8 and what equals 10". That is 2. So remove 2 with the forefinger.
c. 8 plus 2 equals 10. So form 1 on the first rod to the left.

Example 20. 10 - 8 = 2



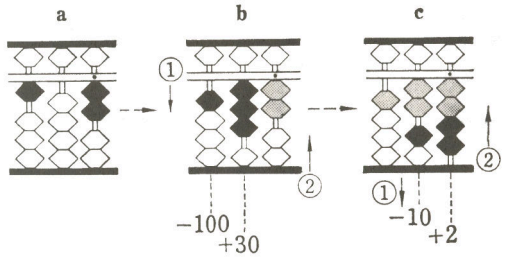
a. Set 10 with the thumb.
b. You cannot subtract 8 from ones. So think of subtracting 8 from 10, and subtract 10 first.
c. After subtracting 8 from 10, you have 2 left, so enter 2 on the ones rod.

Example 21. 34 + 78 = 112



a. Set 34 with the thumb.
b. First calculate tens. You cannot add the 70 of 78 on the tens rod. So think, "7 and what equals 10". That is 3. So remove 3, and enter 1 on the hundreds rod.
c. Next calculate ones. You cannot add the 8 of 78 on the rod that has 4. So think what is needed to make 8 into 10. That is 2. So remove 2 and shift up 1 to the tens rod.

Example 22. 102 - 78 = 24



a. Set 102 with the thumb.
b. Calculate tens first. You cannot subtract 70 of 78 from the zero on the tens place. So think of the 1 on the hundreds rod as 10, and subtract 7 from 10. Then you have 3 left. With this idea, remove 1 from the hundreds rod and add 3 on the tens rod.
c. Next you cannot subtract the 8 of 78 from the 2 on the ones rod. So thinking of subtracting 8 from 10, subtract 10 and add the remainder 2 to the 2 on the ones rod.

Exercises 5

(1)

$$\begin{array}{r} 14 \\ +\ 97 \\ \hline \end{array}$$

(2)

$$\begin{array}{r} 34 \\ +\ 86 \\ \hline \end{array}$$

(3)

$$\begin{array}{r} ¥\ 42 \\ +\ 89 \\ \hline \end{array}$$

(4)

$$\begin{array}{r} \$\ 33 \\ +\ 79 \\ \hline \end{array}$$

(5)

$$\begin{array}{r} \$\ 42 \\ +\ 98 \\ \hline \end{array}$$

(6)

$$\begin{array}{r} 100 \\ -\ 87 \\ \hline \end{array}$$

(7)

$$\begin{array}{r} ¥100 \\ -\ 79 \\ \hline \end{array}$$

(8)

$$\begin{array}{r} \$\ 113 \\ -\ 89 \\ \hline \end{array}$$

(9)

$$\begin{array}{r} \$\ 102 \\ -\ 69 \\ \hline \end{array}$$

(10)

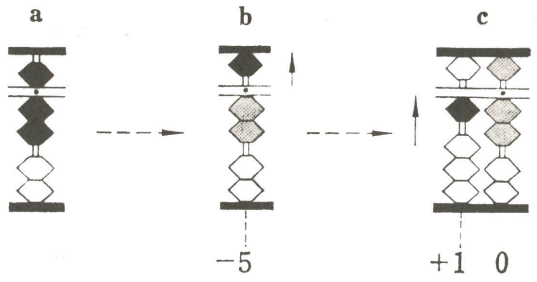
$$\begin{array}{r} \$\ 112 \\ -\ 98 \\ \hline \end{array}$$

Answer

(1) 111 (2) 120 (3) ¥131 (4) $112 (5) $140 (6) 13 (7) ¥21

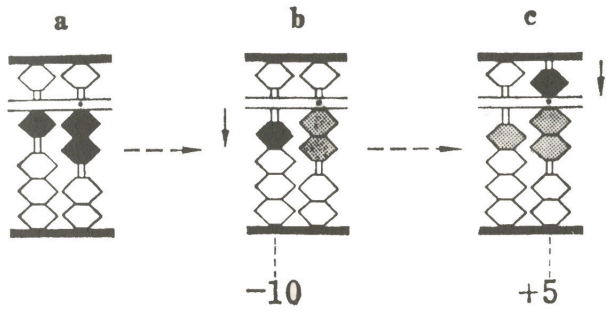
(8) $24 (9) $33 (10) $14

Example 23. $7 + 5 = 12$



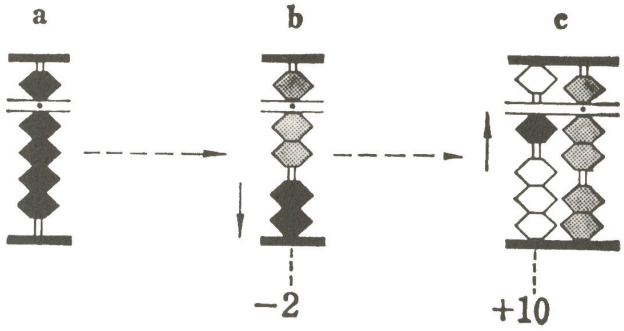
a. Set 7 with the thumb and the forefinger.
b. You cannot add 5 on the ones rod. So think, "5 and what equals 10". 5 plus 5 equals 10. So remove 5 with the forefinger.
c. Shift up 1 to the tens rod.

Example 24. 12 - 5 = 7



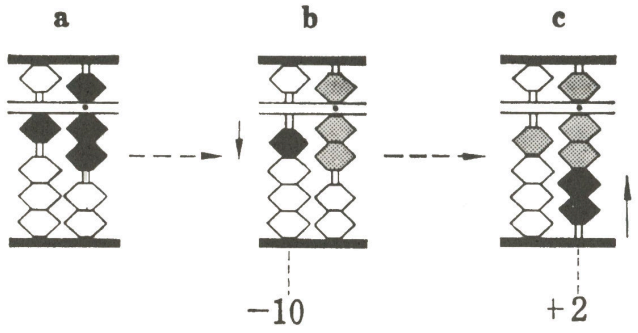
a. Set 12 with the thumb.
b. You cannot subtract 5 on the ones rod which has only 2. So think of subtracting 5 from 10 and subtract 10.
c. 10 minus 5 equals 5. So add 5 on the ones rod.

Example 25. 9 + 8 = 17



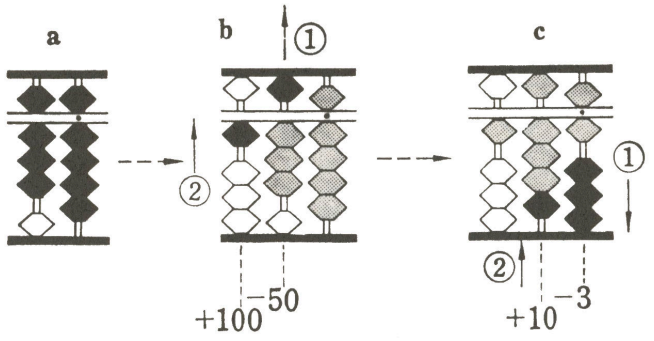
a. Set 9 with the thumb and the forefinger.
b. You cannot add 8 on the ones rod which has 9. So think, "8 and what equals 10". 8 plus 2 equals 10. So remove 2 with the forefinger.
c. Shift up 1 to the tens place.

Example 26. 17 - 8 = 9



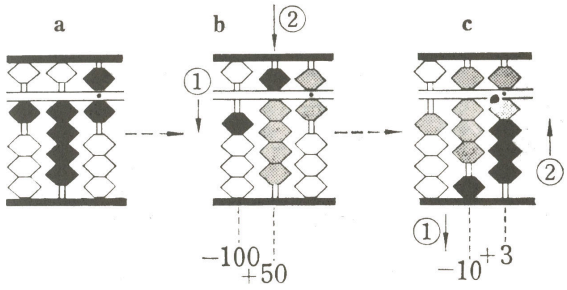
a. First set 17.
b. You cannot subtract 8 on the ones rod that has only 7. So think of subtracting 8 from 10 and subtract 10.
c. 10 minus 8 equals 2, so add 2 on the ones place.

Example 27. 89 + 57 = 146



a. Set 89.
b. You cannot add the 50 of 57 on the tens rod that has 8. So thinking, "5 plus 5 equals 10", remove 5 on the tens rod and shift up 1 to the hundreds rod.
c. You cannot add the 7 of 57 on the rod that has 9. So thinking, "7 plus 3 equals 10", remove 3 on the ones rod and shift 1 to the tens rod.

Example 28. 146 - 57 = 89



a. Set 146.
b. You cannot subtract the 50 of 57 from the tens rod that has 4. So think of subtracting 5 from 10, and take 1 from the hundreds rod and add the remainder 5 to the tens rod.
c. You cannot subract 7 from the ones rod that has 6. So taking away 10, add the remainder 3 to the ones rod.

Exercises 6

(1)
```
  59
+ 58
```

(2)
```
  97
+ 75
```

(3)
```
¥ 88
+ 87
```

(4)
```
¥ 79
+ 96
```

(5)
```
$ .89
+ .59
```

(6)
```
 117
− 59
```

(7)
```
 165
− 98
```

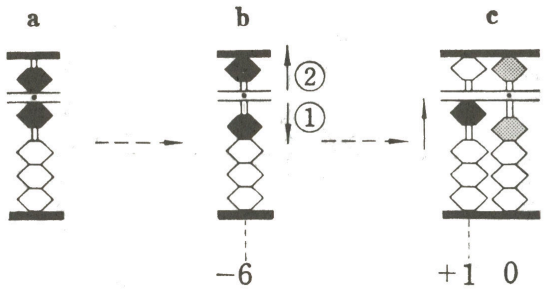
(8)
```
¥ 174
−  85
```

(9)
```
$ 165
−  89
```

(10)
```
$ 186
−  97
```

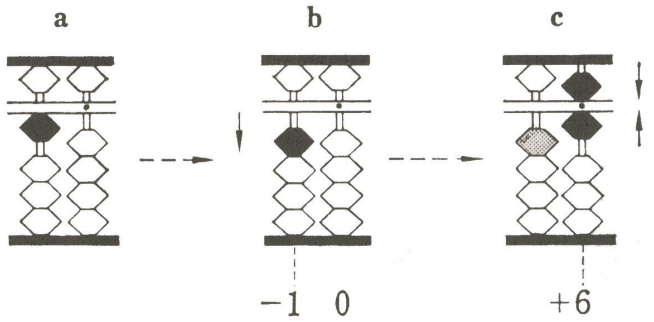
Answer
(1) 117 (2) 172 (3) ¥175 (4) ¥175 (5) $1.48 (6) 58 (7) 67
(8) ¥89 (9) $ 76 (10) $ 89

Example 29. 6 + 4 = 10



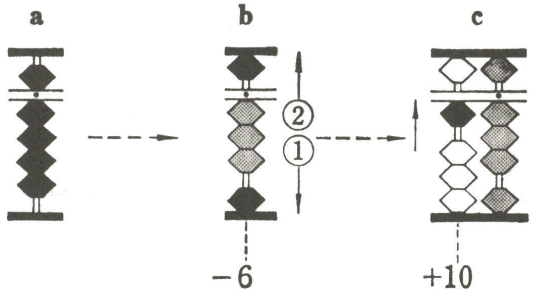
a. Set 6.
b. You cannot add 4 on the rod that has 6, so think "4 plus what equals 10". 4 plus 6 equals 10. So remove 6.
c. Shift up 1 to the tens rod.

Example 30. 10 - 4 = 6



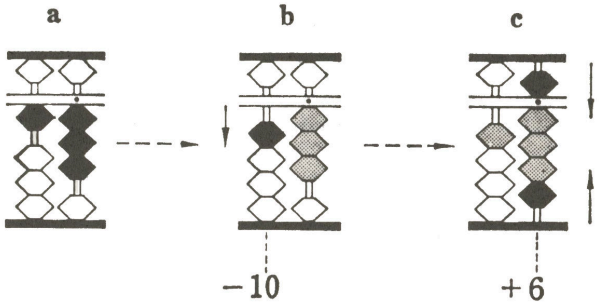
a. Set 10.
b. You cannot subtract 4 on the ones rod. So think of subtracting it feom 10 and then take away 10.
c. 10 minus 4 equals 6, so enter 6.

Example 31. 9 + 4 = 13



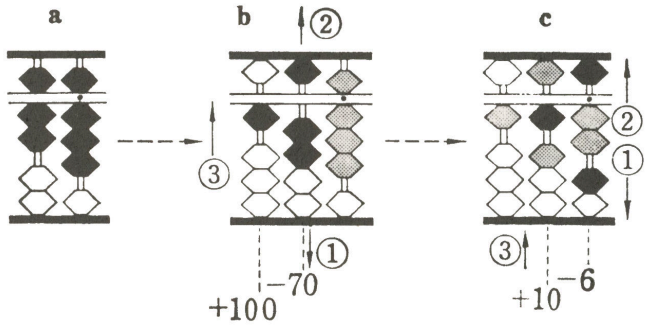
a. Set 9.
b. You cannot add 4 on the rod that has 9. So think, "4 plus what equals 10". 4 plus 6 equals 10. So remove 6.
c. Shift up 1 to the tens rod.

Example 32. 13 - 4 = 9



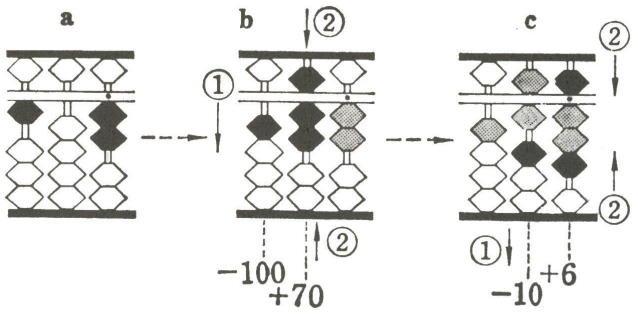
a. Set 13.
b. You cannot subtract 4 from 3 on the ones rod. So thinking of subtracting it from 10, take 10.
c. If you subtract 4 from 10, you get 6 left. So add 6 to the ones rod.

Example 33. 78 + 34 = 112



a. Set 78.
b. You cannot add the 30 of 34 on the tens rod that has 7. So think "3 plus 7 equals 10", remove 7 and shift up 1 to the hundreds.
c. You cannot add the 4 of 34 on the ones rod that has 8. So think "4 plus 6 equals 10", remove 6 and shift up 1 to the tens rod.

Example 34. 102 - 34 = 68



a. Set 102.
b. You cannot subtract the 30 of 34 from the zero on the tens rod. So take the 1 on the hundreds rod, and thinking that you have got 10, add the remainder 7 on the tens rod.
c. Next you cannot subtract the 4 of 34 from 2 on the ones rod. So taking 10, add the remainder 6 to the 2 on the ones rod.

Exercises 7

(1) 98 + 13

(2) 79 + 43

(3) ¥ 98 + 42

(4) $ 97 + 23

(5) ¢ 68 + 44

(6) 100 − 23

(7) 112 − 43

(8) ¥ 101 − 13

(9) $ 1.02 − .44

(10) $ 1.31 − .42

Answer

(1) 111 (2) 122 (3) ¥140 (4) $120 (5) $1.12

(6) 77 (7) 69 (8) ¥ 88 (9) $ .58 (10) $ .89

Practice 3

(1) 21 + 18 + 35 + 81

(2) 42 + 32 + 94 + 59

(3) 21 + 55 + 34 + 97

(4) ¥ 62 + 93 + 41 + 84

(5) ¥ 32 + 24 + 39 + 42

(6) $ 13 + 48 + 27 + 34

(7) $ 43 + 24 + 31 + 79

(8) $ 34 + 98 + 24 + 51

(9) $ .44 + .34 + .48 + .39

(10) $ .48 + .94 + .76 + .49

(11) 155 − 76 − 32 − 28

(12) 266 − 87 − 43 − 92

(13) 208 − 49 − 24 − 51

(14) ¥352 − 69 − 44 − 27

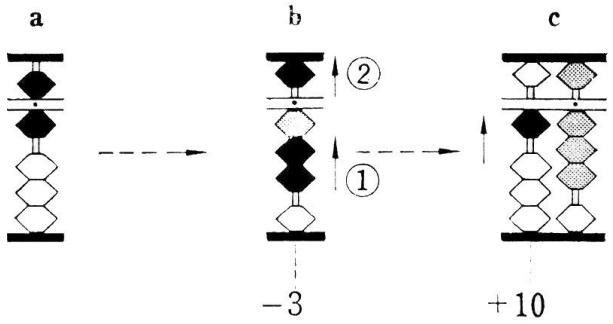
(15) ¥350 − 84 − 34 − 53

(16)	(17)	(18)	(19)	(20)
¥ 301	$ 455	$ 260	$ 3.65	$ 4.00
− 38	− 27	− 34	− .99	− .21
− 49	− 56	− 42	− .34	− .48
− 21	− 38	− 43	− .15	− .93

Answer

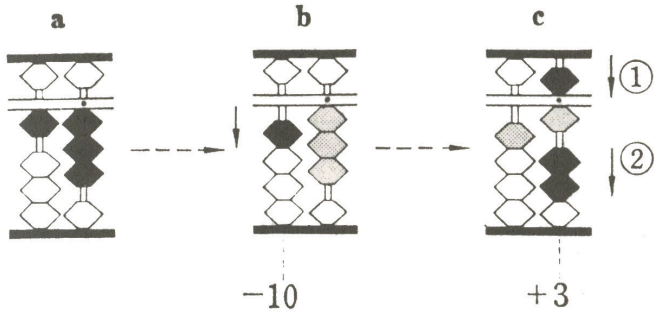
(1) 155	(2) 227	(3) 207	(4) ¥ 280	(5) ¥ 137
(6) $122	(7) $177	(8) $207	(9) $1.65	(10) $2.67
(11) 19	(12) 44	(13) 84	(14) ¥ 212	(15) ¥ 179
(16) ¥193	(17) $334	(18) $141	(19) $2.17	(20) $2.38

Example 35. $6 + 7 = 13$



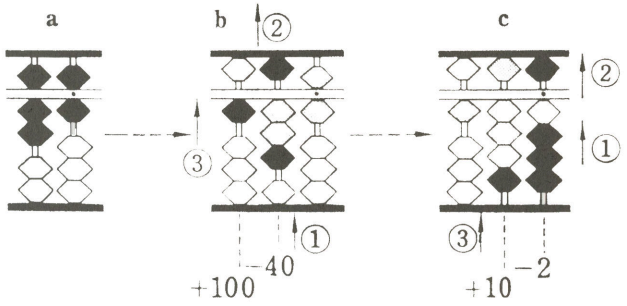
a. Set 6.
b. You cannot add 7 on the rod that has 6. So thinking 7 plus 3 equals 10, subtract 3 from 6. In subtracting 3 from 6, you subtract 3 from the 5 of 6. In this subtraction, first enter the remainder 2 and then remove 5.
c. Next shift up 1 to the ten rod, as 7 plus the 3 you have removed equals 10.

Example 36. 13 - 7= 6



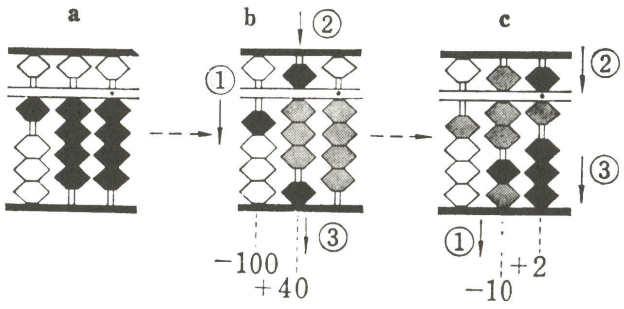
a. Set 13.
b. You cannot subtract 7 from the 3 on the ones rod. So take 1 from the tens rod with the idea of subtracting 7 from 10.
c. Subtract 7 from the 10 that you have got and add the remainder 3 to the 3 on the ones rod. There is only one counter left to be added. So add 5 with the forefinger and subtract the 2 which you have added in excess.

Example 37. 76 + 68 = 144



a. Set 76.
b. First add the 60 of 68 to the 70 of 76. Thinking "6 plus 4 equals 10", subtract 4 from the 5 of 7 and shift up 1 to the hundresds rod.
c. You cannot add the 8 of 68 on the ones rod. So thinking "8 plus 2 equals 10", subtract 2 from the 5 of 6 and shift up 1 to the tens place.

Example 38. 144 - 68 = 76



a. Set 144.
b. You cannot subtract the 6 of 68 from the 4 on the tens place. So taking 1 from the hundreds place, think of it as a 10 and add on the tens place the remainder 4 you have got after subtracting 6 from 10. In this step, first enter 5 and next remove 1.
c. You cannot subtract the 8 of 68 from the 4 on the ones place. So taking 10, subtract 8 from it and add the remainder 2 to the 4 one the ones place. In this step, firts enter 5 and next remove 3.

Exercises 8

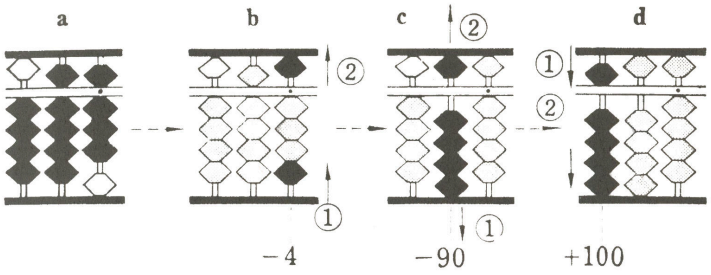
(1)	(2)	(3)	(4)	(5)
58 + 76	75 + 69	¥ 57 + 87	¥ 65 + 76	$ .66 + .68

(6)	(7)	(8)	(9)	(10)
134 − 68	342 − 67	¥ 134 − 79	¥ 223 − 68	$ 3.41 − .76

Answer

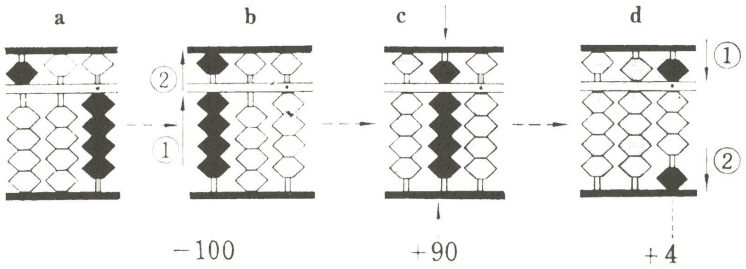
(1) 134	(2) 144	(3) ¥ 144	(4) ¥ 141	(5) $1.34
(6) 66	(7) 275	(8) ¥ 55	(9) ¥ 155	(10) $2.65

Example 39. 498 + 6 = 504



a. Set 498.
b. You cannot add 6 to the 8 on the ones rod. So thinking, "6 plus 4 equals 10", subtract 4 from the 8 on the ones rod and shift up 1. In this operation, move up 1 first and then move up 5.
c. The 10 that has been shifted up plus 90 equals 100, so remove 90 and shift up 1 to the hundreds rod.
d. In adding 1, enter 5 first and then remove 4.

Example 40. 504 -6 = 498



a. Set 504.
b. You cannot subtract 6 from the 4 on the ones rod. So take 10, but as you cannot take it from the tens place, take 100.
c. Take 10 from 100, and set the remainder 90 on the tens rod.
d. Subtract 6 from the 10 that you have taken and add the remainder 4 to 4 on the ones rod. In this operation, enter 5 and remove 1.

Practice 4

(1)

$$\begin{array}{r} 11 \\ 14 \\ 98 \\ 43 \\ +\ 76 \\ \hline \end{array}$$

(2)

$$\begin{array}{r} 88 \\ 79 \\ 47 \\ 24 \\ +\ 96 \\ \hline \end{array}$$

(3)

$$\begin{array}{r} ¥\ 12 \\ 35 \\ 26 \\ 48 \\ 84 \\ \hline \end{array}$$

(4)

$$\begin{array}{r} \$\ .32 \\ .43 \\ .69 \\ .18 \\ .86 \\ \hline \end{array}$$

(5)

$$\begin{array}{r} \$\ .57 \\ .67 \\ .43 \\ .76 \\ .52 \\ \hline \end{array}$$

(6)

$$\begin{array}{r} 323 \\ -\ 37 \\ -\ 34 \\ -\ 86 \\ -\ 62 \\ \hline \end{array}$$

(7)

$$\begin{array}{r} 244 \\ -\ 67 \\ -\ 43 \\ -\ 76 \\ -\ 34 \\ \hline \end{array}$$

(8)

$$\begin{array}{r} ¥\ 414 \\ -\ 78 \\ -\ 63 \\ -\ 58 \\ -\ 41 \\ \hline \end{array}$$

(9)

$$\begin{array}{r} \$\ 3.42 \\ -\ .87 \\ -\ .14 \\ -\ .79 \\ -\ .97 \\ \hline \end{array}$$

(10)

$$\begin{array}{r} \$\ 4.00 \\ -\ .78 \\ -\ .69 \\ -\ .47 \\ -\ .98 \\ \hline \end{array}$$

(11)

$$\begin{array}{r} 85 \\ 97 \\ 61 \\ -\ 76 \\ 67 \\ \hline \end{array}$$

(12)

$$\begin{array}{r} 58 \\ 86 \\ -\ 68 \\ 36 \\ -\ 94 \\ \hline \end{array}$$

(13)

$$\begin{array}{r} ¥\ 43 \\ 67 \\ 55 \\ -\ 22 \\ -\ 76 \\ \hline \end{array}$$

(14)

$$\begin{array}{r} \$\ 1.55 \\ -\ .21 \\ -\ .78 \\ .86 \\ -\ .67 \\ \hline \end{array}$$

(15)

$$\begin{array}{r} \$\ .55 \\ .76 \\ .44 \\ .67 \\ -\ .76 \\ \hline \end{array}$$

Answer

(1) 242 (2) 334 (3) ¥205 (4) $2.48 (5) $2.95

(6) 74 (7) 24 (8) $174 (9) $ .65 (10) $1.08

(11) 234 (12) 18 (13) ¥ 67 (14) $ .75 (15) $1.66

Practice 5

(1)

$$\begin{array}{r} 76 \\ 55 \\ 26 \\ 98 \\ 45 \\ 89 \\ +\ 21 \\ \hline \end{array}$$

(2)

$$\begin{array}{r} 16 \\ 95 \\ 89 \\ 33 \\ 97 \\ 15 \\ +\ 56 \\ \hline \end{array}$$

(3)

$$\begin{array}{r} ¥\ 22 \\ 79 \\ 45 \\ 58 \\ 97 \\ 69 \\ +\ 71 \\ \hline \end{array}$$

(4)

$$\begin{array}{r} \$\ 67 \\ 99 \\ 85 \\ 96 \\ 68 \\ 37 \\ +\ 88 \\ \hline \end{array}$$

(5)

$$\begin{array}{r} £\ 89 \\ 34 \\ 43 \\ 37 \\ 98 \\ 25 \\ +\ 76 \\ \hline \end{array}$$

(6)

$$\begin{array}{r} 99 \\ 23 \\ 15 \\ 67 \\ -\ 46 \\ -\ 35 \\ -\ 76 \\ \hline \end{array}$$

(7)

$$\begin{array}{r} 66 \\ 87 \\ 93 \\ -\ 48 \\ 36 \\ -\ 87 \\ 26 \\ \hline \end{array}$$

(8)

$$\begin{array}{r} ¥\ 76 \\ 78 \\ -\ 59 \\ 27 \\ 82 \\ 99 \\ -\ 58 \\ \hline \end{array}$$

(9)

$$\begin{array}{r} \$\ 31 \\ 12 \\ 65 \\ 96 \\ -\ 69 \\ -\ 84 \\ 93 \\ \hline \end{array}$$

(10)

$$\begin{array}{r} £\ 24 \\ 97 \\ -\ 76 \\ 58 \\ -\ 27 \\ 89 \\ 36 \\ \hline \end{array}$$

(11)

$$\begin{array}{r} 627 \\ 89 \\ 72 \\ 18 \\ 195 \\ 581 \\ +369 \\ \hline \end{array}$$

(12)

$$\begin{array}{r} 789 \\ 876 \\ 85 \\ 64 \\ 188 \\ 36 \\ +\ 67 \\ \hline \end{array}$$

(13)

$$\begin{array}{r} \$\ 987 \\ 559 \\ 81 \\ 79 \\ 294 \\ 93 \\ +\ 417 \\ \hline \end{array}$$

(14)

$$\begin{array}{r} \$\ 5.08 \\ 5.97 \\ .26 \\ .68 \\ 7.03 \\ .48 \\ +\ 4.79 \\ \hline \end{array}$$

(15)

$$\begin{array}{r} \$\ .93 \\ .57 \\ .86 \\ 8.94 \\ .48 \\ 3.26 \\ +\ 4.98 \\ \hline \end{array}$$

(16)

$$\begin{array}{r} 999 \\ -\ 65 \\ -\ 86 \\ -\ 49 \\ -378 \\ -166 \\ -\ 58 \\ \hline \end{array}$$

(17)

$$\begin{array}{r} 888 \\ -\ 53 \\ -\ 36 \\ -452 \\ -\ 48 \\ -\ 13 \\ -\ 87 \\ \hline \end{array}$$

(18)

$$\begin{array}{r} \$\ 777 \\ -\ 34 \\ -\ 87 \\ -\ 59 \\ -\ 49 \\ -\ 224 \\ -\ 126 \\ \hline \end{array}$$

(19)

$$\begin{array}{r} \$\ 6.66 \\ -\ .54 \\ -\ .16 \\ -\ .53 \\ -\ 4.38 \\ -\ .31 \\ -\ .47 \\ \hline \end{array}$$

(20)

$$\begin{array}{r} \$\ 5.55 \\ -\ .49 \\ -\ .27 \\ -\ .31 \\ -\ .36 \\ -\ .17 \\ -\ 1.96 \\ \hline \end{array}$$

Answer

(1) 410	(2) 401	(3) ¥ 441	(4) $ 540	(5) £ 402
(6) 47	(7) 173	(8) ¥ 245	(9) $ 144	(10) £ 201
(11) 1,951	(12) 2,105	(13) $ 2,510	(14) $ 24.29	(15) $ 20.02
(16) 197	(17) 199	(18) $ 198	(19) $ .27	(20) $ 1.99

Practice 6

(1)	(2)	(3)	(4)	(5)
¥ 673	$ 2.18	$ 3.21	£ 4.03	£ 5.17
105	4.93	7.60	9.58	8.05
842	9.70	− 6.49	4.65	− 2.63
239	1.04	8.35	6.32	− 9.72
410	5.62	− 1.58	2.71	5.41
951	7.25	− 9.34	7.84	− 4.86
786	8.36	7.20	1.09	9.30

(6)	(7)	(8)	(9)	(10)
¥ 562	$ 6.74	$ 8.76	£ 9.81	£ 5.69
703	9.58	6.92	2.07	7.40
− 128	8.01	− 1.09	1.26	− 3.18
697	1.93	− 3.18	5.34	− 2.90
804	5.27	− 7.54	6.95	6.03
− 315	2.36	8.25	3.70	8.41
− 468	7.04	4.30	8.49	− 5.72

Answer

(1) ¥4,006 (2) $39.08 (3) $ 8.95 (4) £36.22 (5) £10.72

(6) ¥1,855 (7) $40.93 (8) $16.42 (9) £37.62 (10) £15.73

How to calculate a Column of Numbers

A column of numbers like the following example is calculated in the following way.

Example

$$\begin{array}{r} 16 \\ 73 \\ -52 \\ 95 \\ -28 \\ \hline 104 \end{array}$$

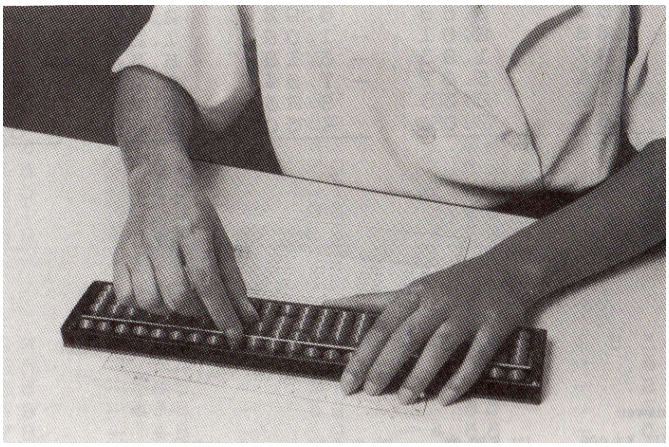


Fig. A

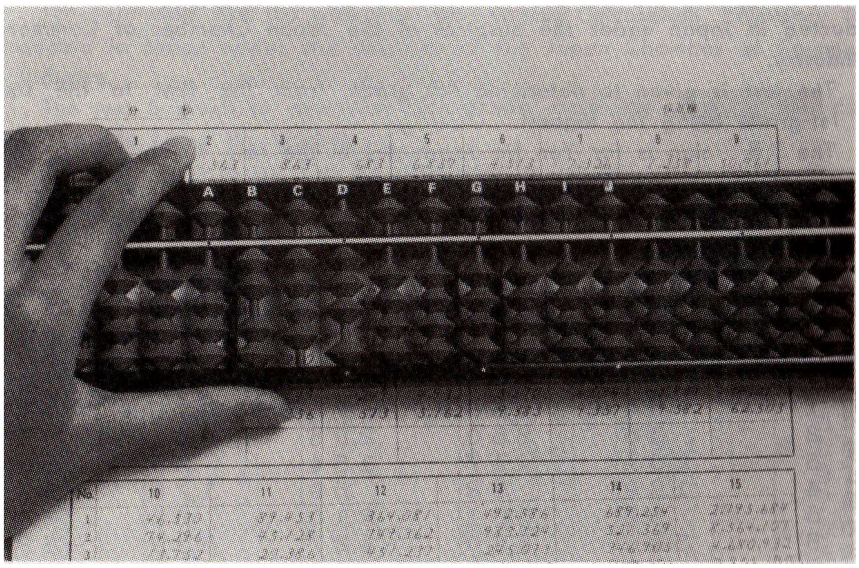


Fig. B

Place a column of numbers right in front of you and put the upper edge of the abacus right below the first number and form it on the board. Then move the abacus down till the next number appears right over the abacus and calculate it on the board, and continue in this fashion to the end of the problem.

ABACUS EFFICIENCY TEST

The following are the problems for Abacus Calculation Efficiency Test conducted in Japan under the auspices of the Japan Chamber of Commerce and Industry.

The test is given to determine 6th grade from the first to the sixth, the 1st-grade being the highest.

The "$" sign is for Yen (in Japanese) in the Test.

The Abacus Efficiency Tests
under the Auspices
of
The Japan Chamber of Commerce and Industry
6 th grade Problems of Addition and Subtraction

(Time limit 10 minutes)

No.	1	2	3	4	5
1	764	¥ 638	¥ 78	$ 948	$ 418
2	35	250	203	350	37
3	201	417	164	26	206
4	189	92	59	170	590
5	52	301	610	21	−21
6	340	540	−427	539	−375
7	83	69	−350	467	640
8	672	781	−91	82	89
9	901	824	823	103	136
10	47	35	64	741	524
11	596	967	571	86	−71
12	180	23	980	950	−890
13	329	180	−29	289	−362
14	58	749	−438	53	45
15	476	65	567	647	789
total					

No.	6	7	8	9	10
1	846	¥ 635	¥ 75	$ 863	$ 57
2	730	48	340	702	34
3	25	720	826	419	620
4	192	192	91	51	189
5	301	−31	102	−230	42
6	58	−504	563	−647	301
7	467	869	487	85	598
8	209	72	14	904	760
9	16	310	209	123	14
10	534	487	38	−59	253
11	970	56	657	−768	160
12	89	−291	901	−14	897
13	123	−803	37	302	29
14	647	−74	925	596	748
15	58	659	864	87	365
total					

ABACUS EFFICIENCY TEST

The following are the problems for Abacus Calculation Efficiency Test conducted in Japan under the auspices of the Japan Chamber of Commerce and Industry.

The test is given to determine 5th grade from the first to the sixth, the 1st-grade being the highest.

The "$" sign is for Yen (in Japanese) in the Test.

The Abacus Efficiency Tests
under the Auspices
of
The Japan Chamber of Commerce and Industry
5 th grade Problems of Addition and Subtraction
(Time limit 10 minutes)

No.	1	2	3	4	5
1	715	¥ 8,403	¥ 516	$ 8,503	$ 352
2	8,036	725	7,048	642	6,047
3	294	961	390	917	918
4	328	240	132	2,054	−724
5	1,407	1,536	6,824	316	−1,503
6	659	179	−957	780	−860
7	2,510	4,802	−2,403	1,479	639
8	936	635	−715	723	2,415
9	874	480	569	856	370
10	103	3,279	108	9,102	189
11	4,527	716	9,623	534	4,628
12	768	9,058	847	269	957
13	690	193	309	3,801	−709
14	512	624	−427	498	−5,614
15	3,849	578	−1,658	657	238
total					

No.	6	7	8	9	10
1	514	¥ 7,631	¥ 163	$ 8,304	$ 142
2	736	284	7,502	526	635
3	9,208	4,509	849	9,107	9,708
4	841	920	2,390	230	256
5	3,620	−813	476	−451	413
6	157	−356	758	−579	860
7	2,095	−1,078	5,029	−6,082	7,091
8	543	625	316	613	524
9	968	3,749	184	1,749	439
10	270	201	902	208	1,287
11	1,436	856	3,675	358	6,302
12	370	−9,487	940	−4,967	589
13	819	−702	4,183	−709	173
14	4,628	139	621	613	8,094
15	759	654	578	254	756
total					

Multiplication and Division

Multiplication Table

Do you know the table given below? It is called a multiplication table. It is so arranged that you can find at a glance the product of the multiplication of any two digits. For instance, if you want to find the product of 5x7, look down File 5 to where it crosses Rank 7, and you can find the product 35. Now do you understand how to look at the table?

	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54
7	7	14	21	28	35	42	49	56	63
8	8	16	24	32	40	48	56	64	72
9	9	18	27	36	45	54	63	72	81

Multiplication and division are done by making use of multiplications of the digits given in this table. So you had better memorize the whole table so that you do not have to take the trouble of looking for the product of any two digits, as 5 x 7 = 35, 6 x 4 = 24, etc.

What is the number that fits into each given below?

(1) $6 \times 8 = \square$

(2) $3 \times 4 = \square$

(3) $2 \times 6 = \square$

(4) $7 \times 5 = \square$

(5) $9 \times 2 = \square$

(6) $1 \times 7 = \square$

(7) $4 \times 9 = \square$

(8) $5 \times 8 = \square$

(9) $8 \times 3 = \square$

(10) $6 \times 4 = \square$

(11) $3 \times \square = 18$

(12) $6 \times \square = 42$

(13) $4 \times \square = 32$

(14) $5 \times \square = 45$

(15) $2 \times \square = 12$

(16) $7 \times \square = 21$

(17) $8 \times \square = 72$

(18) $2 \times \square = 8$

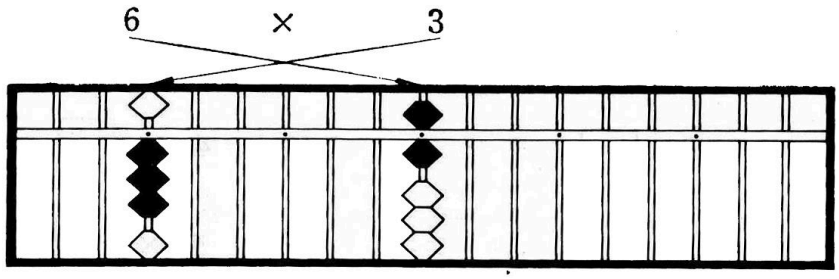
(19) $9 \times \square = 27$

(20) $4 \times \square = 28$

Multipliers

How to set Multiplicands and Multipliers

Example 6 x 3 = 18

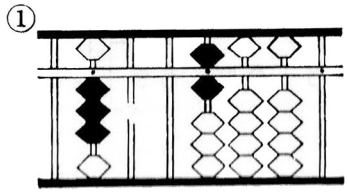


As in the above figure, place the multiplicand about the middle of the board and the multiplier to its left.

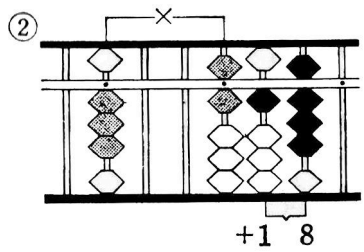
How to multiply One-Digit Numbers

Example 1. 6 x 3 = 18

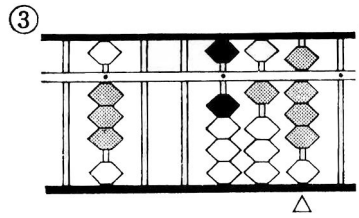
1. Set the problem as in the figure given at the right.



2. Using the multiplication formula, 6x3=18, set the product 18 to the right of the multiplicand, with the first rod to the right of the multiplicand as the tens rod of the product.

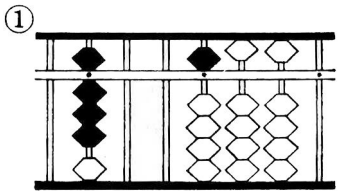


3. Clear the multiplicand 6. When the multiplier is a one-digit number, the ones place of the product is formed on the second rod to the right of that of the multiplicand. The answer is 18.

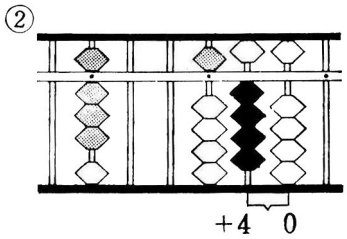


Example 2. 5 x 8 = 40

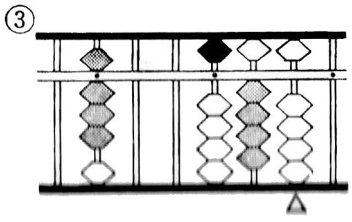
1. Set the problem as in the figure given at the right.



2. 5x8 =40. So enter 40, with the first rod to the right of the multiplicand, as the tens rod.

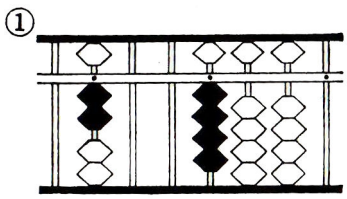


3. Clear the multiplicand 5. The ones place of the product is formed on the second rod to the right of that of the multiplicand. So the answer is 40.

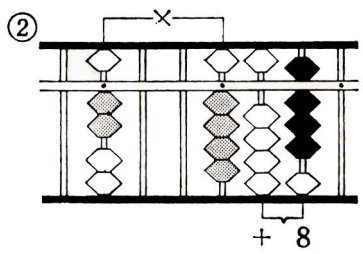


Example 3. 4 x 2 = 8

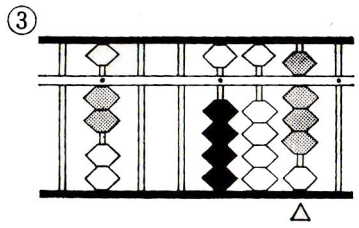
1. Set the problem as in the figure given at the right.



2. Using the formula 4x2=8, set the product on the second rod to the right of the multiplicand. In this operation, the first rod to the right of the multiplicand always becomes the tens place of the product.



3. Clear the multiplicand 4. The ones place of the product is formed on the second rod to the right of that of the multiplicand. So the answer is 8.



Exercises 1

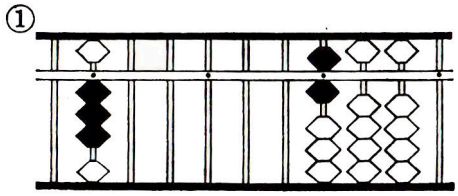
(1) 2 × 8 =
(2) 7 × 3 =
(3) 6 × 4 =
(4) 9 × 5 =
(5) 4 × 7 =
(6) 5 × 4 =
(7) 6 × 5 =
(8) 3 × 2 =
(9) 2 × 4 =
(10) 2 × 2 =

Answer

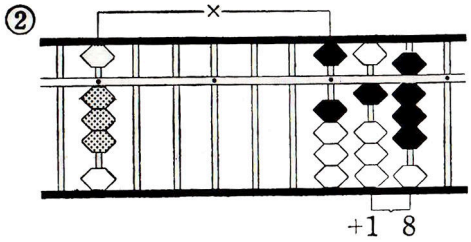
(1) 16 (2) 21 (3) 24 (4) 45 (5) 28 (6) 20 (7) 30 (8) 6 (9) 8 (10) 4

Example 4. 76 x 3 = 228

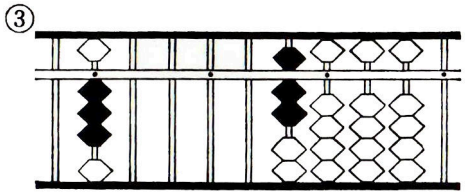
1. First let us try 6x3. Set the two digits as in the figure at the right.



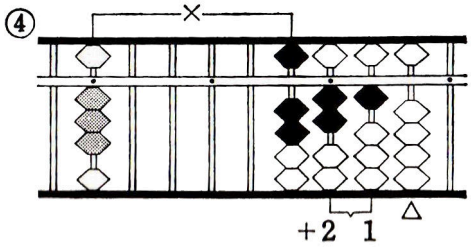
2. Using 6x3=18, set the product 18 with the first rod to the right of the multiplicand 6 as its tens place and clear the multiplicand 6. The answer is 18.



3. Next let us try 70x3. Set the two digits as in the figure at the right.



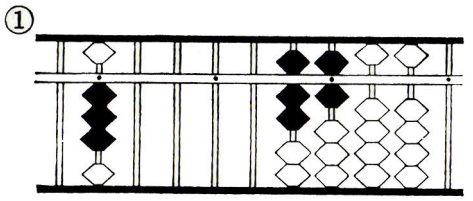
4. Using 7x3=21, set the product 21, with the first rod to the right of the multiplicand 7 as its tens place and clear the multiplicand. The ones place of the product is formed on the second rod to the right of that of the multiplicand, 70. The answer is 210.



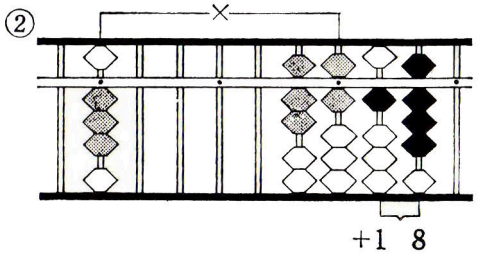
$$\left.\begin{array}{r} 6\times 3= \ 18 \\ 70\times 3=210 \end{array}\right\} \ 18+210=228$$

On the abacus, the above two calculations are not made separately but are made jointly as follows.

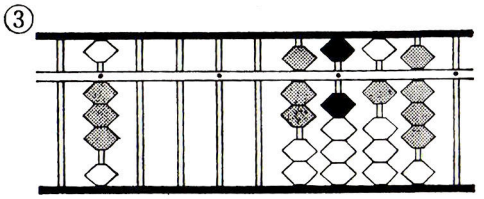
1. Set the problem as in the figure at the right.



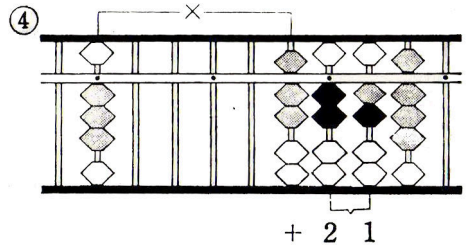
2. First calculate 6x3. 6x3=18. So set the product to 18, with the first rod to the right of that of the multiplicand digit as its tens place.



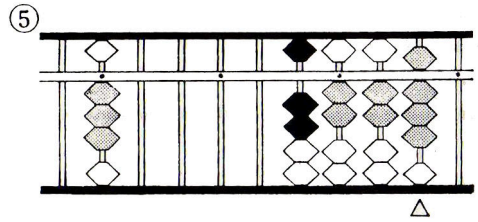
3. Clear the 6 of the multiplicand 76.



4. Next calculate the multiplication of 70 by 3. 7x3=21. Add the product 21, with the first rod to the right of that of the multiplicand digit as its tens place.

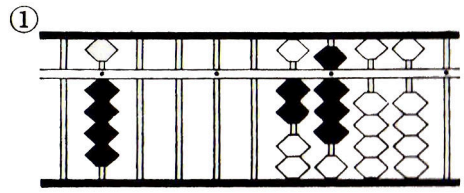


5. Clear 7. As the ones place of the product is formed on the second rod to the right of that of the multiplicand of this problem, the answer is 228.

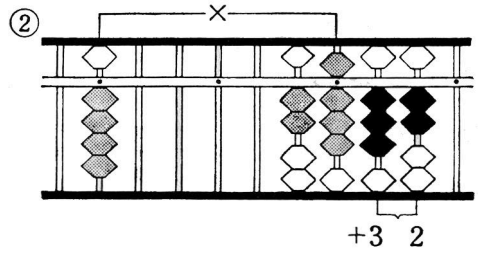


Example 5. 28 x 4 = 112

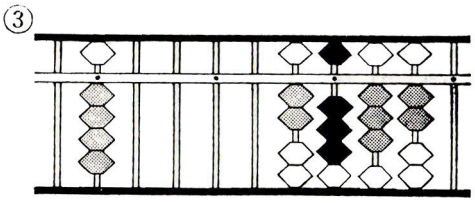
1. Set the problem as in the figure at the right.



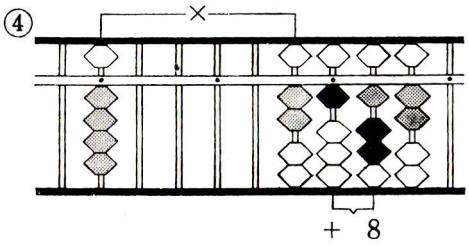
2. First calculate 8x4. 8x4=32. So set the product 32, with the first rod to the right of that of the multiplicand digit 8 as its tens place.



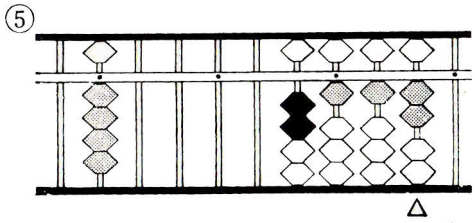
3. Clear the multiplicand digit 8.



4. Next calculate the multiplication of the 20 of 28 by 4. 2x4=8. So set the product 8, with the first rod to the multiplicand digit 2 as its tens rod.



5. Clear the multiplicand digit 2, as the ones place of the product of the problem is formed on the second rod to the right of the problem, the answer is 112.



Exercises 2

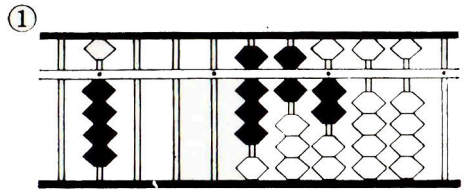
(1) 24 × 6 =
(2) 37 × 4 =
(3) 49 × 3 =
(4) 72 × 8 =
(5) 21 × 4 =
(6) 32 × 3 =
(7) 92 × 4 =
(8) 46 × 2 =
(9) 51 × 6 =
(10) 29 × 3 =
(11) 17 × 6 =
(12) 83 × 3 =

Answer

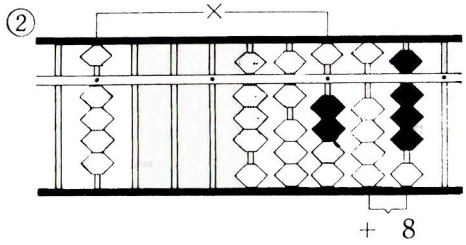
(1) 144 (2) 148 (3) 147 (4) 576 (5) 84 (6) 96
(7) 368 (8) 92 (9) 306 (10) 87 (11) 102 (12) 249

Example 6. 862 x 4 = 3,448

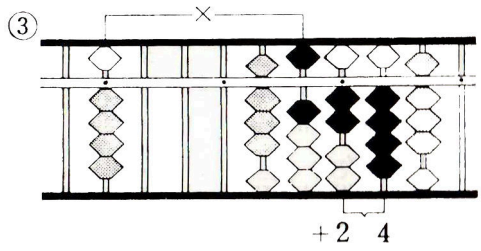
1. Set the problem as in the figure at the right.



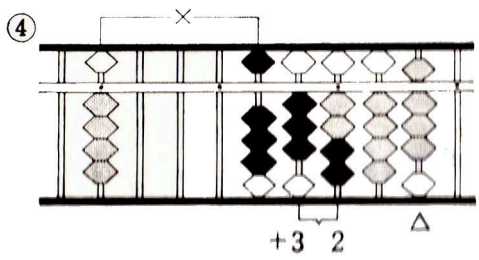
2. First multiply 2 by 4. 2x4=8. So set the product 8 with the first rod to the right of the multiplicand digit 2 as its tens place and clear 2.



3. Next multiply 6 in the tens place by 4. 6x4=24. Add 24, with the first rod to the right of the multiplicand digit 6 as its tens place and clear the multiplicand 6.

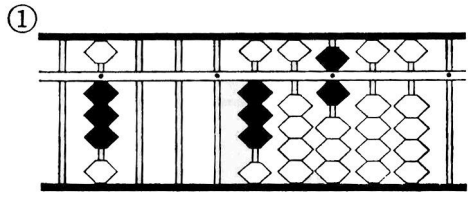


4. Finally multiply 8 in the hundreds place by 4. 8x4=32. Add 32, with the first rod to the right of the multiplicand digit 8 as the tens place, and clear the multiplicand digit 8. The ones place of the product of the problem is formed on the second rod to the right of that of the multiplicand of the problem. The answer is 3,448.

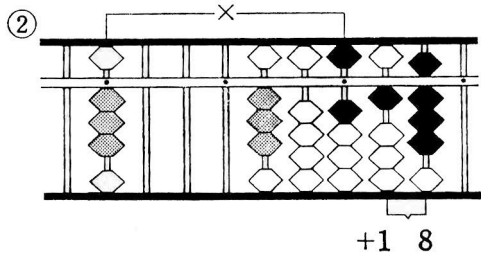


Example 7. 306 x 3 = 918

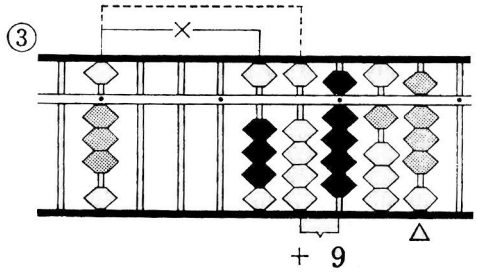
1. Set the problem as in the figure at the right.



2. First multiply 6 by 3. 6x3=18. So set 18, with the first rod to the right of the multiplicand digit 6 as its tens place and clear 6.



3. Next multiply the zero in the tens place by 3. 0x3=0. So you do not have to move counters. So multiply 3 in the hundreds place by 3. 3x3=9. Add 9, with the first rod to the right of the multiplicand digit 3 as its tens place and clear 3. The ones place of the product is formed on the second rod to the right of the multiplicand, so the answer is 918.



Exercises 3

(1) 768 × 3 =
(2) 973 × 2 =
(3) 826 × 4 =
(4) 157 × 6 =
(5) 631 × 3 =

(6) 129 × 4 =
(7) 382 × 2 =
(8) 213 × 3 =
(9) 208 × 4 =
(10) 401 × 5 =

Answer

(1) 2,304	(2) 1,946	(3) 3,304	(4) 942	(5) 1,893
(6) 516	(7) 764	(8) 639	(9) 832	(10) 2,005

Practice 1

(1) 746 × 8 =	(16) 873 × 3 =
(2) 358 × 4 =	(17) 145 × 6 =
(3) 614 × 7 =	(18) 302 × 5 =
(4) 435 × 2 =	(19) 697 × 8 =
(5) 269 × 3 =	(20) 534 × 2 =
(6) 527 × 6 =	(21) 218 × 4 =
(7) 173 × 9 =	(22) 706 × 9 =
(8) 982 × 4 =	(23) 859 × 3 =
(9) 841 × 5 =	(24) 421 × 7 =
(10) 397 × 8 =	(25) 960 × 5 =
(11) 402 × 2 =	(26) 308 × 2 =
(12) 716 × 3 =	(27) 791 × 6 =
(13) 659 × 6 =	(28) 542 × 8 =
(14) 108 × 7 =	(29) 197 × 4 =
(15) 523 × 4 =	(30) 623 × 6 =

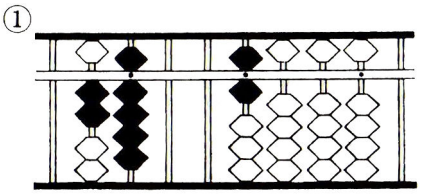
Answer

(1) 5,968	(2) 1,432	(3) 4,298	(4) 870	(5) 807
(6) 3,162	(7) 1,557	(8) 3,928	(9) 4,205	(10) 3,176
(11) 804	(12) 2,148	(13) 3,954	(14) 756	(15) 2,092
(16) 2,619	(17) 870	(18) 1,510	(19) 5,576	(20) 1,068
(21) 872	(22) 6,354	(23) 2,577	(24) 2,947	(25) 4,800
(26) 616	(27) 4,746	(28) 4,336	(29) 788	(30) 3,738

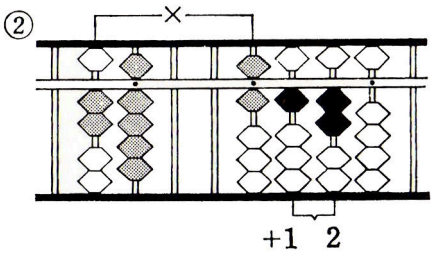
Multiplication by Two-Digit Numbers

Example 8. 6 x29 = 174

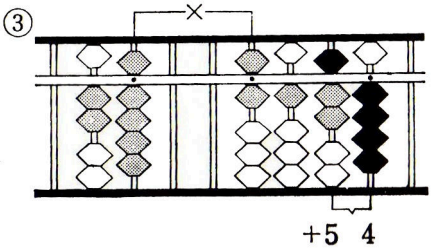
1. Set the problem as in the figure at the right.



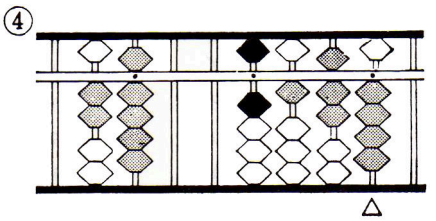
2. First multiply the 6 by the 2 in the tens place. 6x2=12. Set 12, with the first rod to the right of the multiplicand 6 as its tens place.



3. Next multiply the 6 by the 9 in the ones place. 6x9=54. Add the product 54, with the first rod to the right of the tens rod of the preceding product 12 as its tens place.

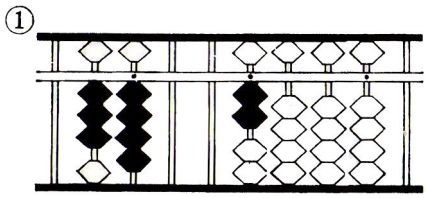


4. Clear the multiplicand 6. When the multiplier is a two-digit number, the ones place of the product moves to the third rod to the right of that of the multiplicand. The answer is 174.

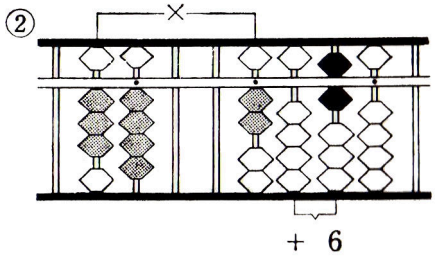


Example 9. 2 x 34 = 68

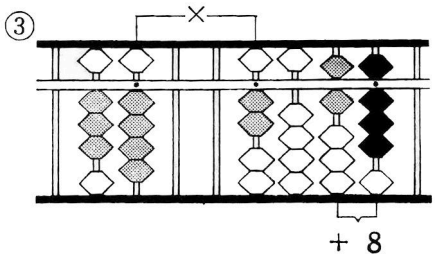
1. Set the problem as in the figure at the right.



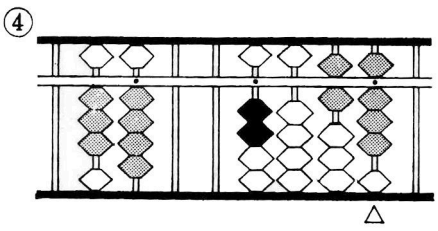
2. First multiply the 2 by the 3 in the tens place of the multiplier. 2x3=6. So set 6, with the first rod to the right of the multiplicand 2 as its tens place.



3. Next multiply the 4 in the ones place of the multiplier. 2x4=8. Now add 8, with the first rod to the right of the tens place of the preceding product as its tens place.

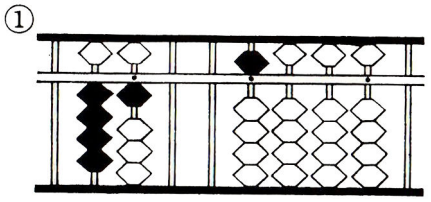


4. Clear the multiplicand 2. The ones place of the product is formed on the third rod to the right of that of the multiplicand. So the answer is 68.

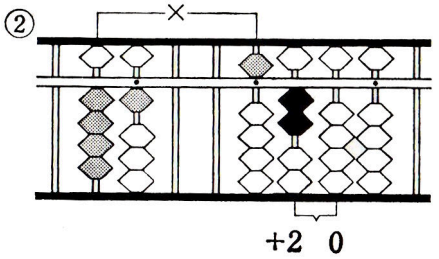


Example 10. 5 x 41 = 205

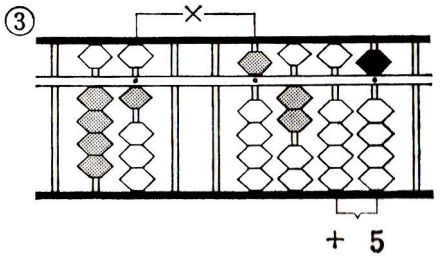
1. Set the problem as in the figure at the right.



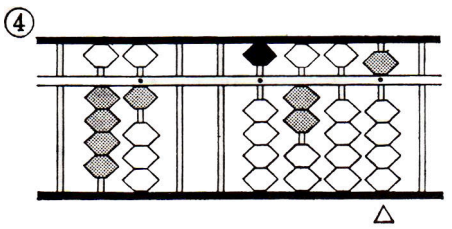
2. First multiply the 5 by the 4 in the tens place of the multiplier. 5x4=20. So set 20, with the first rod to the right of the multiplier as its tens place.



3. Next multiply the 5 by the 1 in the ones place of the multiplier. 5x1=5. Add 5, with the first rod to the right of the tens place of the preceding product 20 as its tens place.



4. Clear the multiplicand 5. The ones place of the product is formed on the third rod to the right of that of the multiplicand. So the answer is 205.



Exercises 4

(1) $8 \times 49 =$
(2) $7 \times 36 =$
(3) $6 \times 58 =$
(4) $2 \times 42 =$
(5) $3 \times 12 =$
(6) $4 \times 72 =$
(7) $9 \times 13 =$
(8) $3 \times 28 =$
(9) $5 \times 14 =$
(10) $6 \times 51 =$

Answer

(1) 392 (2) 252 (3) 348 (4) 84 (5) 36
(6) 288 (7) 117 (8) 84 (9) 70 (10) 306

Practice 2

(1) $9 \times 45 =$
(2) $8 \times 76 =$
(3) $4 \times 23 =$
(4) $2 \times 51 =$
(5) $6 \times 37 =$
(6) $3 \times 92 =$
(7) $5 \times 14 =$
(8) $7 \times 89 =$
(9) $6 \times 56 =$
(10) $2 \times 38 =$
(11) $4 \times 92 =$
(12) $5 \times 21 =$
(13) $8 \times 45 =$
(14) $4 \times 13 =$
(15) $7 \times 64 =$
(16) $6 \times 97 =$
(17) $3 \times 18 =$
(18) $9 \times 74 =$
(19) $7 \times 69 =$
(20) $2 \times 83 =$
(21) $4 \times 52 =$
(22) $5 \times 31 =$
(23) $8 \times 26 =$
(24) $2 \times 79 =$
(25) $9 \times 45 =$
(26) $4 \times 17 =$
(27) $3 \times 32 =$
(28) $6 \times 91 =$
(29) $5 \times 28 =$
(30) $7 \times 43 =$

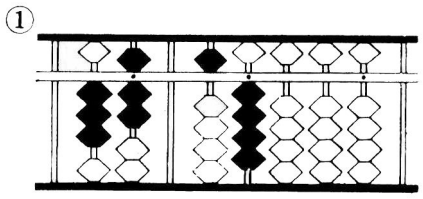
Answer

(1) 405 (2) 608 (3) 92 (4) 102 (5) 222 (6) 276 (7) 70 (8) 623 (9) 336 (10) 76
(11) 368 (12) 105 (13) 360 (14) 52 (15) 448 (16) 582 (17) 54 (18) 666 (19) 483 (20) 166
(21) 208 (22) 155 (23) 208 (24) 158 (25) 405 (26) 68 (27) 96 (28) 546 (29) 140 (30) 301

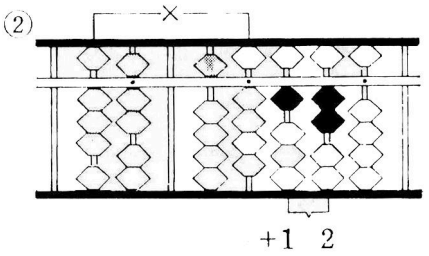
Example 11. 54 x 37 = 1,998

54 x 37 = 4 x 37 + 50 x 37. These two sets of calculation are made on the same rods jointly.

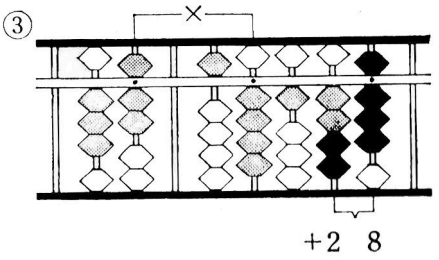
1. Set the problem as in the figure at the right.



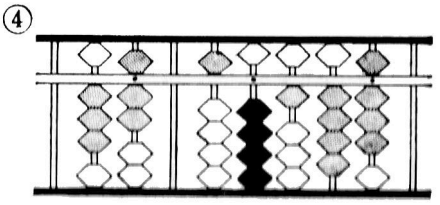
2. First multiply the 4 of 54 by the 3 in the tens place. 4x3=12. Set 12, with the right of the first rod to the right of the multiplicand figure 4 as its tens place of the multipliers.



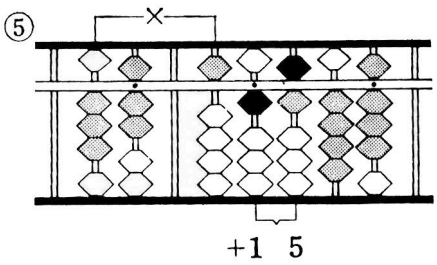
3. Next multiply the 4 of 54 by the 7 in the ones place of the multiplier. 4x7=28. Add 28, with the tens place of the preceding product 12 as its tens place.



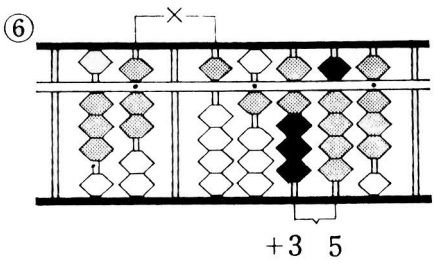
4. Clear the multiplicand digit 4. This has finished the calculation of 4x37.



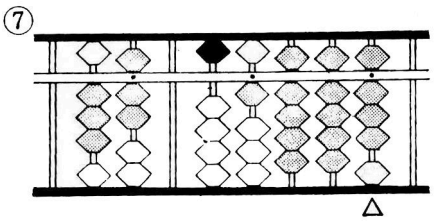
5. Next multiply the 5 of 54 and the 3 in the tens place of the multiplier. 5x3=15. Add 15, with the first rod to the right of the multiplicand figure 5 as its tens place.



6. In succession, multiply the 5 of 54 and the 7 in the ones place of the multiplier. 5x7=35. Add 35, with the first rod to the right of the tens rod of the preceding product 15 as its tens place.



7. Clear the multiplicand digit 5. The ones place of the product moves to the third rod to the right of that of the multiplicand. The answer is 1,998.



Exercises 5

(1)	67×48=	(6)	26×32=
(2)	59×23=	(7)	61×17=
(3)	34×89=	(8)	85×41=
(4)	43×12=	(9)	42×73=
(5)	21×34=	(10)	54×28=

Answer

(1) 3,216	(2) 1,357	(3) 3,026	(4) 516	(5) 714
(6) 832	(7) 1,037	(8) 3,485	(9) 3,066	(10) 1,512

Practice 3

(1) 72×31=
(2) 68×54=
(3) 23×15=
(4) 45×62=
(5) 16×83=
(6) 94×26=
(7) 37×94=
(8) 51×47=
(9) 82×61=
(10) 19×78=
(11) 56×23=
(12) 24×95=
(13) 71×32=
(14) 43×81=
(15) 68×49=
(16) 35×76=
(17) 89×14=
(18) 27×57=
(19) 73×82=
(20) 91×69=
(21) 13×42=
(22) 96×73=
(23) 41×59=
(24) 25×16=
(25) 58×67=
(26) 37×24=
(27) 62×85=
(28) 84×31=
(29) 79×58=
(30) 53×37=
(31) 29×43=
(32) 35×76=
(33) 42×94=
(34) 87×15=
(35) 63×29=
(36) 11×84=
(37) 24×67=
(38) 78×58=
(39) 96×31=
(40) 52×42=

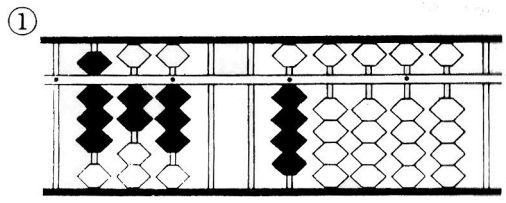
Answer

(1) 2,232	(2) 3,672	(3) 345	(4) 2,790	(5) 1,328	(6) 2,444	(7) 3,478
(8) 2,397	(9) 5,002	(10) 1,482	(11) 1,288	(12) 2,280	(13) 2,272	(14) 3,483
(15) 3,332	(16) 2,660	(17) 1,246	(18) 1,539	(19) 5,986	(20) 6,279	(21) 546
(22) 7,008	(23) 2,419	(24) 400	(25) 3,886	(26) 888	(27) 5,270	(28) 2,604
(29) 4,582	(30) 1,961	(31) 1,247	(32) 2,660	(33) 3,948	(34) 1,305	(35) 1,827
(36) 924	(37) 1,608	(38) 4,524	(39) 2,976	(40) 2,184		

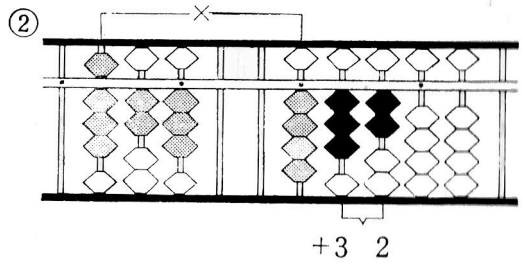
Multiplication by Three-Digit Numbers

Example 12. 4 x 823 = 3,292

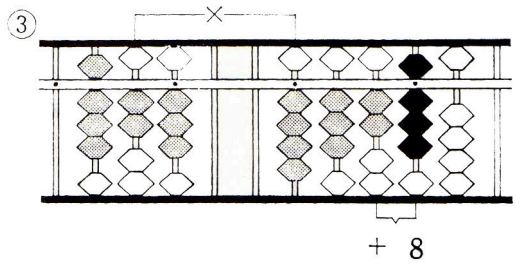
1. Set the problem as in the figure at the right.



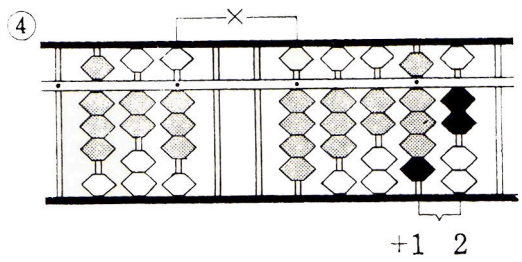
2. First multiply the 4 by the 8 in the hundreds place of the multiplier. 4x8=32. Set 32, with the first rod to the right of the multiplicand 4 as its tens place.



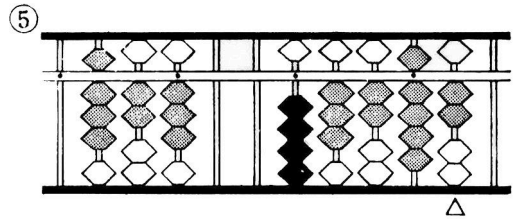
3. Next multiply the 4 by the 2 in the ones place of the multiplier. 4x2=8. Add 8, with the first rod to the right of the tens place of the preceding product 32 as its tens place.



4. Finally multiply the 4 by the 3 in the ones place of the multiplier. 4x3=12. Add 12, with the first rod to the right of the tens rod of the preceding product as its tens place.

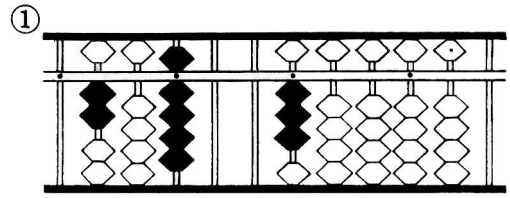


5. Clear the multiplicand 4. When the multiplier is a three-digit number, the ones place of the prodcut moves to the fourth rod to the right of that of the multiplicand. The answer is 3,292.

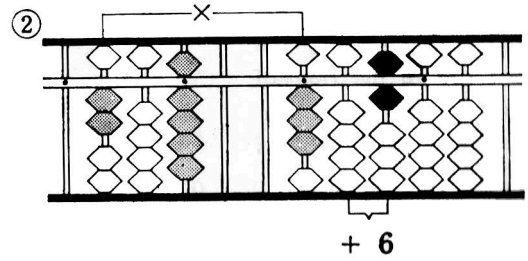


Example 13. 3 x 209 = 627

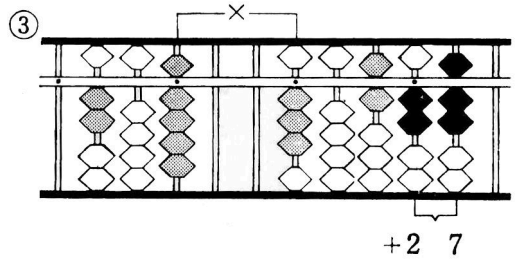
1. Set the problem as in the figure at the right.



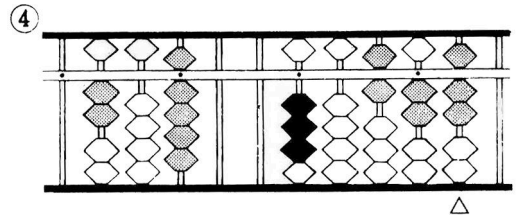
2. First multiply the 3 by the 2 in the hundreds place of the multiplier. 3x2=6. Set 6, with the first rod to the right of the multiplicand as its tens place.



3. Next multiply 3 by the digit in the tens place. But as the digit in the tens place is zero, multiply 3 by 9 in the ones place. 3x9=27. Add 27, with the second rod to the right of the tens place of the preceding product 6 as its tens place. In this operation, be sure to skip over one rod in adding the product 27.



4. Clear the multiplicand 3. The ones place of the product is formed on the fourth rod to the right of that of the multiplicand. So the answer is 627.



Exercises 6

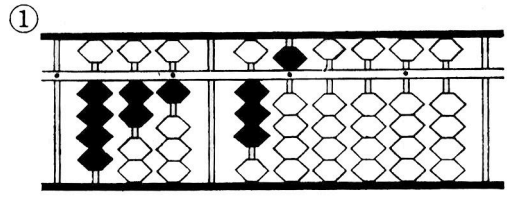
(1) 6 × 764 =
(2) 4 × 893 =
(3) 8 × 276 =
(4) 5 × 749 =
(5) 2 × 423 =
(6) 3 × 312 =
(7) 4 × 617 =
(8) 3 × 283 =
(9) 2 × 251 =
(10) 7 × 175 =
(11) 9 × 806 =
(12) 4 × 502 =

Answer

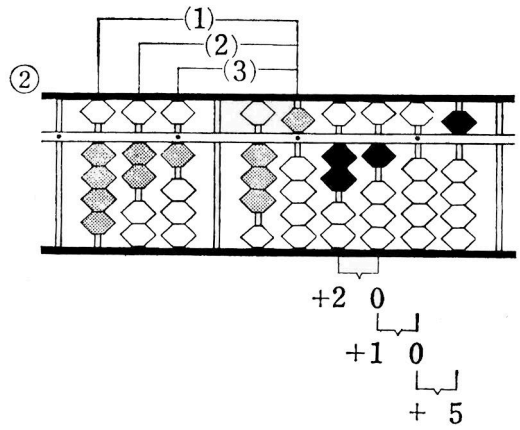
(1) 4,584 (2) 3,572 (3) 2,208 (4) 3,745 (5) 846 (6) 936
(7) 2,468 (8) 849 (9) 502 (10) 1,225 (11) 7,254 (12) 2,008

Example 14. 35 x 421 = 14,735

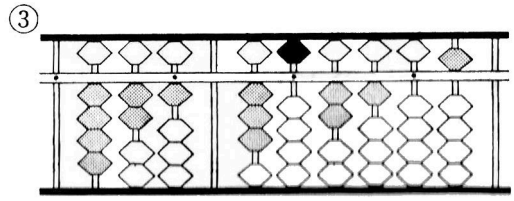
1. Set the problem as in the figure at the right.



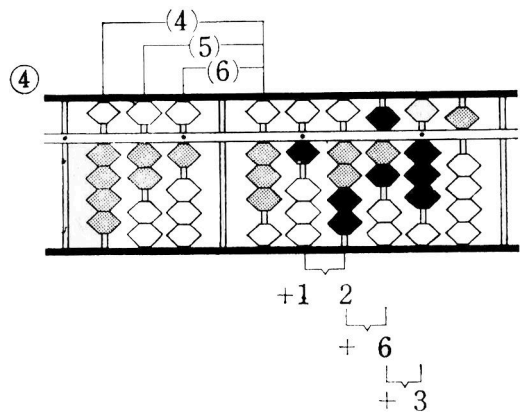
2. First calculate 5x421. To begin with, calculate 5 x 4 = 20. Set 20, with the first rod to the right of the multiplicand digit 5 as its tens place. Next 5x2=10. Add 10, with the first rod to the right of tens place of the preceding product 20 as its tens place. Subsequently, calculate 5x1=5. Add 5, with the first rod to the right of the tens place of the preceding product 10 as its tens place.



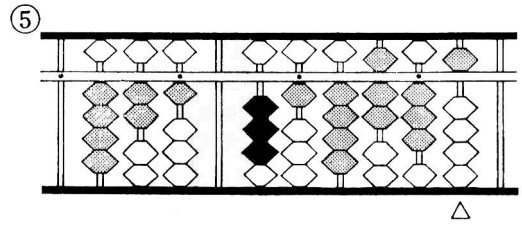
3. Clear the multiplicand digit 5. This has finished the calculation of 5 x 421.



4. Next calculate 3x421. Next calculate 3x4=12. Add 12, with the first rod to the right of the multiplicand digit 3 as its tens place. Next 3x2=6. Add 6, with the tens place of the first rod to the right of the tens place of the preceding product 12 as its tens place. Subsequently, 3x1=3. Add 3, with the first rod to teh right of the tens place of the preceding product 6 as its tens place.



5. Clear 3. The ones place of the product is formed on the fourth rod to the right of that of the multiplicand. The answer is 14,735.



Exercises7

(1)	74×648=	(7)	62×342=
(2)	68×392=	(8)	38×613=
(3)	93×549=	(9)	27×284=
(4)	52×657=	(10)	85×416=
(5)	21×231=	(11)	76×704=
(6)	32×123=	(12)	43×209=

Answer

(1) 47,952 (2) 26,656 (3) 51,057 (4) 34,164 (5) 4,851 (6) 3,936
(7) 21,204 (8) 23,294 (9) 7,668 (10) 35,360 (11) 53,504 (12) 8,987

Practice 4

(1)	35×267=	(21)	86×349=
(2)	72×846=	(22)	59×604=
(3)	48×523=	(23)	13×582=
(4)	97×195=	(24)	28×713=
(5)	61×354=	(25)	45×937=
(6)	84×619=	(26)	67×201=
(7)	23×781=	(27)	71×849=
(8)	56×438=	(28)	34×526=
(9)	39×862=	(29)	92×178=
(10)	14×273=	(30)	58×405=
(11)	72×152=	(31)	49×261=
(12)	61×938=	(32)	26×534=
(13)	38×491=	(33)	62×302=
(14)	54×719=	(34)	18×793=
(15)	97×386=	(35)	84×650=
(16)	23×245=	(36)	37×487=
(17)	45×678=	(37)	75×108=
(18)	16×924=	(38)	21×355=
(19)	89×532=	(39)	96×820=
(20)	32×417=	(40)	43×916=

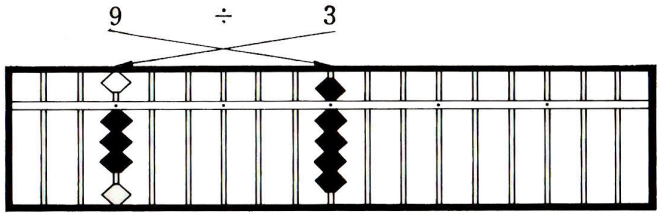
Answer

(1)	9,345	(2)	60,912	(3)	25,104	(4)	18,915	(5)	21,594	(6)	51,996
(7)	17,963	(8)	24,528	(9)	33,618	(10)	3,822	(11)	10,944	(12)	57,218
(13)	18,658	(14)	38,826	(15)	37,442	(16)	5,635	(17)	30,510	(18)	14,784
(19)	47,348	(20)	13,344	(21)	30,014	(22)	35,636	(23)	7,566	(24)	19,964
(25)	42,165	(26)	13,467	(27)	60,279	(28)	17,884	(29)	16,376	(30)	23,490
(31)	12,789	(32)	13,884	(33)	18,724	(34)	14,274	(35)	54,600	(36)	18,019
(37)	8,100	(38)	7,455	(39)	78,720	(40)	39,388				

Division

How to set Dividend and Divisor

Example $9 \div 3 = 3$



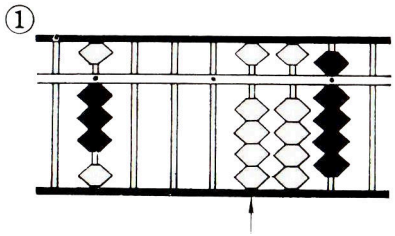
As in the above figure, the dividend is set about the middle of the abacus and the divisor to its left.

↑ ·····Sign to indicate the quotient

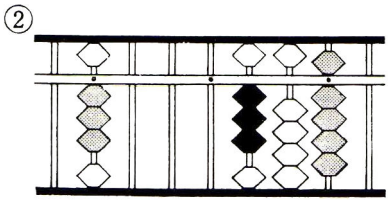
Division by One-Digit Numbers

Example 1. $9 \div 3 = 3$

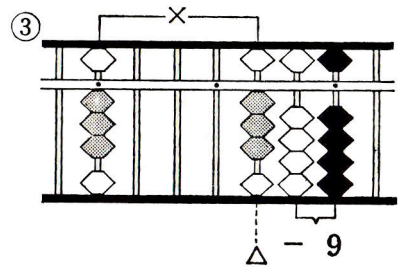
1. Set the problem as in the figure at the right. Compare the dividend 9 and the divisor 3 and you will see that the former is larger. In such a case, set the quotient on the second rod to the left of the dividend.



2. To find the quotient, use the formula 3x□=9 and set the quotient 3.

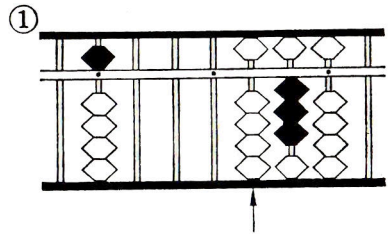


3. Using the formula 3 x 3 = 9, subtract 9, with the first rod to the right of the quotient as the tens place of the product. The ones place of the quotient is formed on the second rod to the left of that of dividend. The answer is 3.

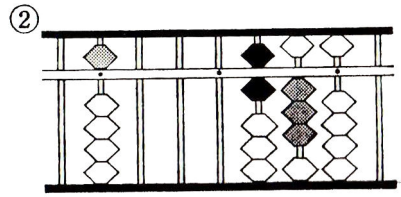


Example 2. $30 \div 5 = 6$

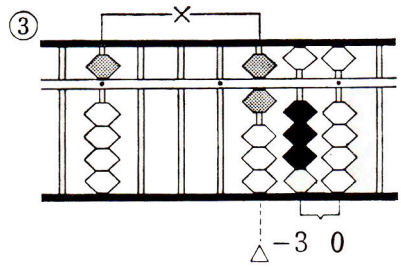
1. Set the problem as in the figure at the right. Compare the first digit 3 of the dividend with the divisor 5, and you will see that the former is smaller. In such a case, set the quotient on the first rod to the left of the dividend.



2. To find the quotient, think 5 x □ = 30 and set 6 as a quotient.

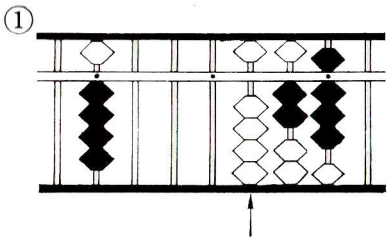


3. Using the formula 6x5=30, subtract 30, with the first rod to the right of the quotient 6 as the tens place of the product. The ones place of the quotient is formed on the second rod to the left of that of the dividend. The answer is 6.

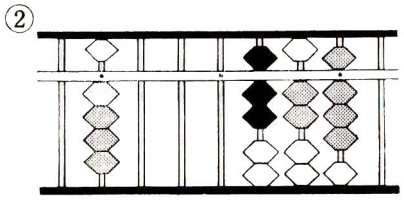


Example 3. $28 \div 4 = 7$

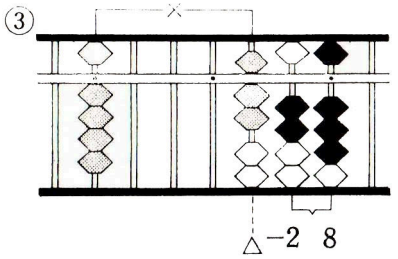
1. Set the problem as in the figure at the right. Compare the first digit of the dividend 2 with the divisor 4, and you will see that the former is smaller. So set the quotient on the first rod to the left of the dividend.



2. To find the quotient, think 4 x □ = 28 and set the quotient 7.



3. Using the formula 7x4=28, subtract 28, with the first rod to the right of the quotient 7 as the tens place of the dividend. The ones place of the quotient is formed on the second rod to the left of that of the dividend. The answer is 7.



Exercises 1

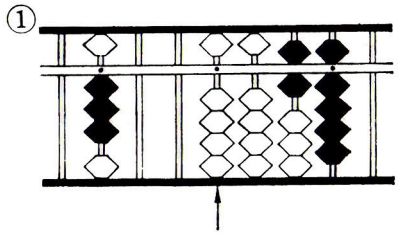
(1) 6÷2=
(2) 8÷4=
(3) 6÷3=
(4) 40÷5=
(5) 20÷4=
(6) 24÷6=
(7) 18÷3=
(8) 42÷7=
(9) 56÷8=
(10) 32÷4=

Answer

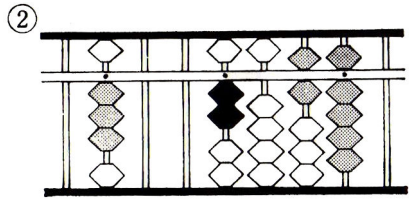
(1) 3 (2) 2 (3) 2 (4) 8 (5) 5 (6) 4
(7) 6 (8) 6 (9) 7 (10) 8

Example 4. $69 \div 3 = 23$

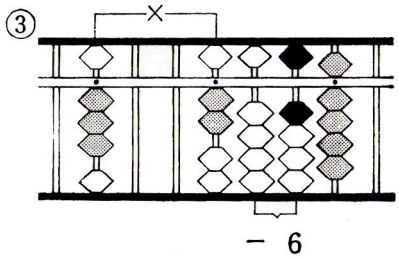
1. Set the problem as in the figure at the right. Compare the first digit 6 of the dividend with the divisor 3, and you will find that the former is larger. So set the quotient on the second rod to the left of the dividend figure 6.



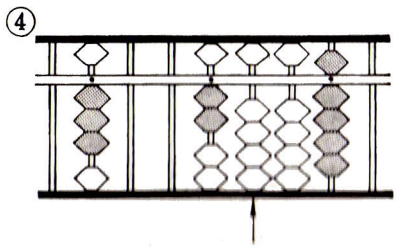
2. To find the quotient, think 3 x □ = 6 and set the quotient 2.



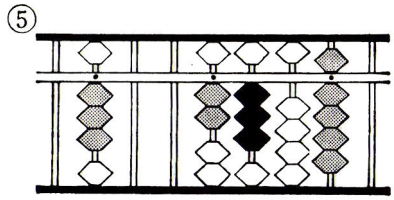
3. Using the formula 2x3=6, subtract 6, with the first rod to the right of the quotient 2 as the tens place of the product. Still you have 9 left. So calculate $9 \div 3$ in close succession.



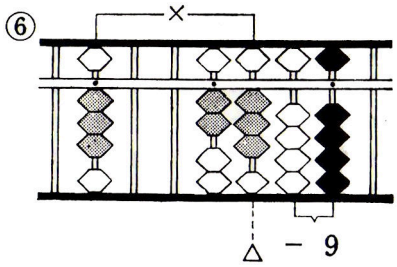
4. Compare the second dividend figure 9 with the division 3 and you will find that the former is larger. So set the quotient figure on the second rod to the left of the dividend figure 9.



5. To find the quotient, think 3 x □ = 9 and set the quotient 3.

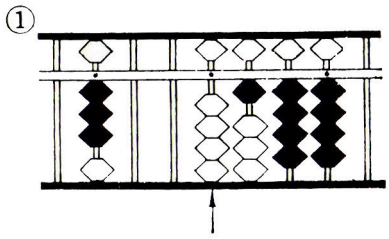


6. Using the formula 3x3=9, subtract 9. As the ones place of the quotient is formed on the second rod to the left of that of the dividend. The answer is 23.

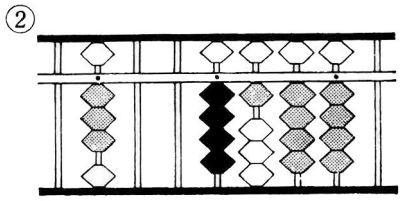


Example 5. $144 \div 3 = 48$

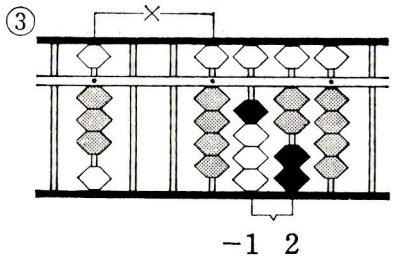
1. Set the problem as in the figure at the right. Compare the first digit 1 of the dividend with the divisor 3, and you will find that the former is smaller. So set the quotient on the first rod to the left of the dividend.



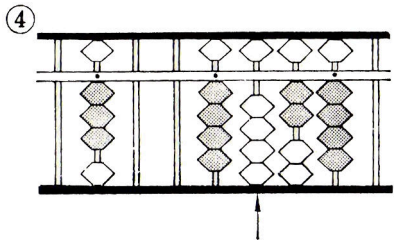
2. Mark off the first two digits of the dividend and think of the nearest digit that divides into 3 x □ = 14. 3x4=12. So set 4 as a quotient digit.



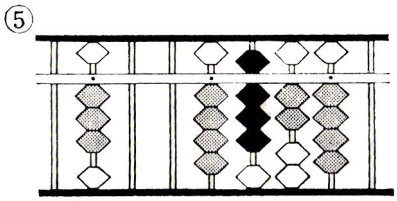
3. Using the formula 4x3=12, subtract 12, with the first rod to the right of the first quotient digit 4 as the tens place of the product. You still have 24 left. So calculate 24 ÷ 3 in succession.



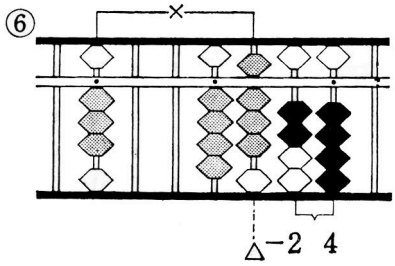
4. Compare 2, the first digit of the remaining dividend, with the divisor 3. The former is smaller. So set the second quotient figure on the first rod to the left of the remaining dividend.



5. In finding the second quotient digit, think 3 x □ = 24, and set 8 as a quotient digit.

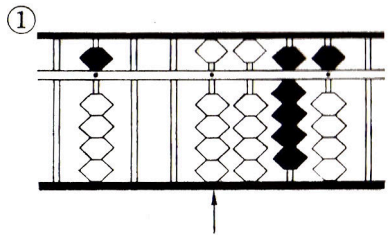


6. Using the formula 8x3=24, subtract 24, with the first rod to the right of the second quotient digit 8 as the tens place of the product. The ones place of the quotient is formed on the second rod to the left of that of the dividend. The answer is 48.

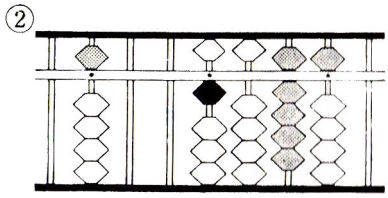


Example 6. $95 \div 5 = 19$

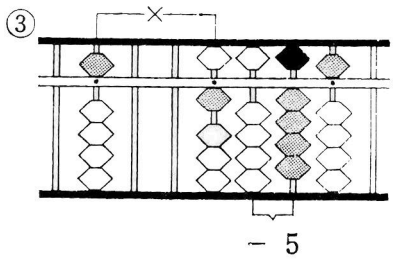
1. Set the problem as in the figure at the right. Compare 9, the first digit of the dividend with the divisor 5. The foremer is larger. So set the first quotient digit on the second rod to the left of the dividend.



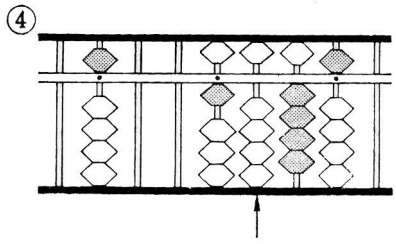
2. To find the quotient figure, think of the nearest digit that divides into the formula 5 x □ = 9. 5x1 equals 5. So set 1 as a quotient digit.



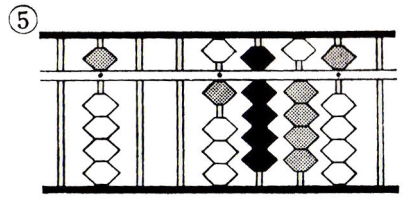
3. Using the formula 1x5=5, subtract 5, with the first rod to the right of the first quotient digit 1 as the tens place of the product. You still have 45 left. So calculate $45 \div 5$ in succession.



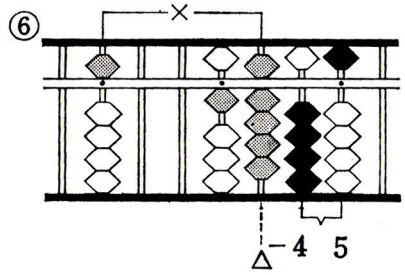
4. Compare 4, the first digit of the remaining dividend, with the divisor 5. The former is smaller. So set the second quotient figure on the first rod to the left of the remaining dividend.



5. To find the quotient figure, think 5 x □ = 45, and set 9 as a quotient figure.



6. Using the formula 9x5=45, subtract 45, with the first rod to the right of the second quotient digit 9 as the tens rod of the product. The ones place of the quotient is formed on the second rod to the left of that of the dividend. So the answer is 19.



Exercises 2

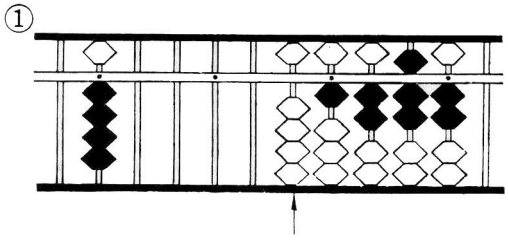
(1)	93 ÷ 3 =	(11)	290 ÷ 5 =
(2)	48 ÷ 4 =	(12)	201 ÷ 3 =
(3)	86 ÷ 2 =	(13)	234 ÷ 6 =
(4)	96 ÷ 3 =	(14)	243 ÷ 9 =
(5)	64 ÷ 2 =	(15)	623 ÷ 7 =
(6)	224 ÷ 4 =	(16)	96 ÷ 4 =
(7)	138 ÷ 6 =	(17)	728 ÷ 8 =
(8)	152 ÷ 2 =	(18)	111 ÷ 3 =
(9)	294 ÷ 7 =	(19)	414 ÷ 9 =
(10)	592 ÷ 8 =	(20)	72 ÷ 4 =

Answer

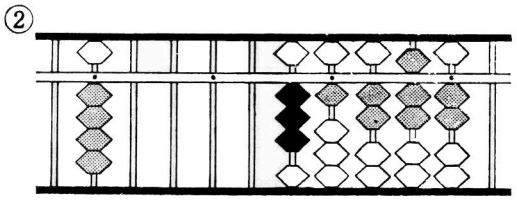
(1) 31	(2) 12	(3) 43	(4) 32	(5) 32	(6) 56	(7) 23
(8) 76	(9) 42	(10) 74	(11) 58	(12) 67	(13) 39	(14) 27
(15) 89	(16) 24	(17) 91	(18) 37	(19) 46	(20) 18	

Example 7. $1,272 \div 4 = 318$

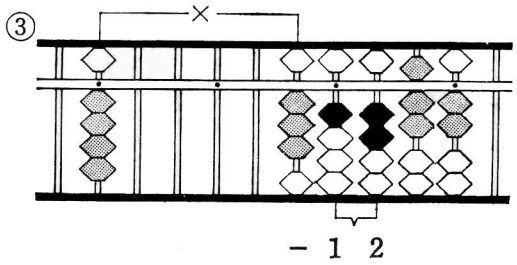
1. Set the problem as in the figure at the right. Compare 1, the first digit of the dividend with 4, the divisor. The foremer is smaller. So set the quotient digit on the first rod to the left of the dividend.



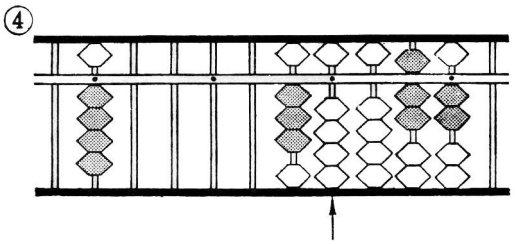
2. Mark off the first two digits of the dividend and calculate $12 \div 4$. Think 4 x $\square = 12$ and set 3 as a quotient figure.



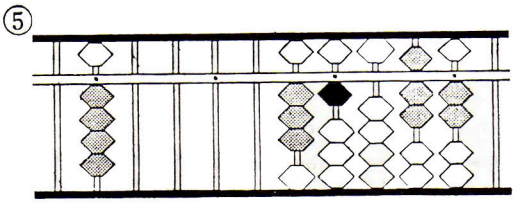
3. Thinking 3x4=12, subtract 12, with the first rod to the right of the quotient digit as the tens palce of the product.



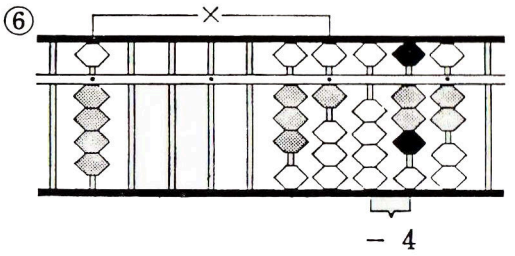
4. Compare 7, the first digit of the remaining dividend, with the divisor 4. The former is larger. So set the second quotient figure on the second rod to the left of the remaining dividend.



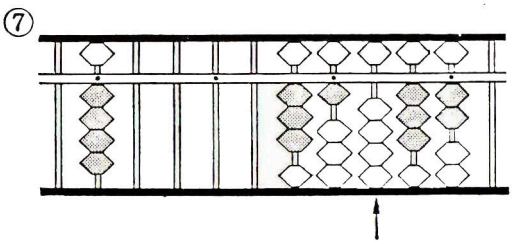
5. Think of the nearest digit that divides into 4 x □ = 7. 4x1=4. So set 1 as the second quotient digit.



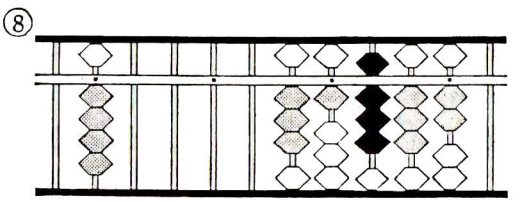
6. Thinking 1x4=4, subtract 4, with the first rod to the right of 1, the second quotient digit, as the tens place of the product.



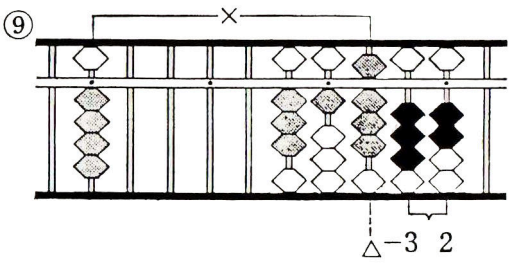
7. Compare 3, the first digit of the remaining dividend, with the divisor 4. The former is smaller. So set the third quotient figure on the first rod to the left of the remaining dividend.



8. To find the third quotient figure, think 4 x □ = 32, and set 8 as a quotient digit.

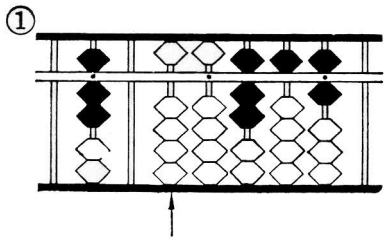


9. Thinking 8x4=32, subtract 32, with the first rod to the right of 8, the third quotient digit, as the tens place of the product. The ones place of the quotient is formed on the second rod to the left of that of the dividend. The answer is 318.

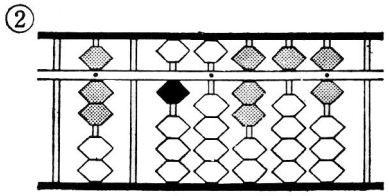


Example 8. $756 \div 7 = 108$

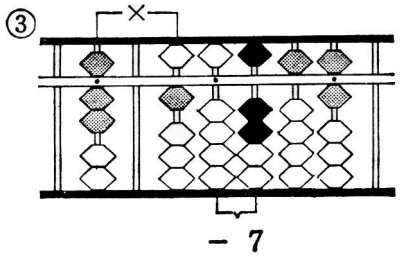
1. Set the problem as in the figure at the right. The first digit of the dividend and the divisor are the same. In cases when the divisor is a one-digit number if such a situation occurs, set the quotient on the second rod to the left of the dividend.



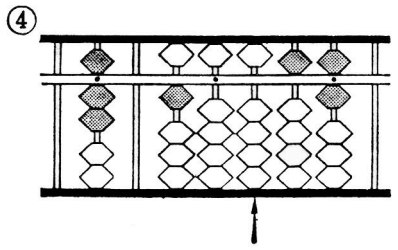
2. To find the quotient, think 7 x □ = 7 and set 1 as a quotient.



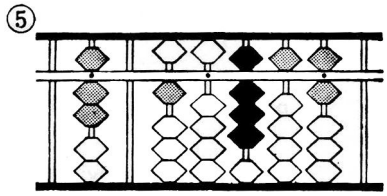
3. Thinking 1x7=7, and subtract 7, with the first rod to the right of the quotient digit as the tens place of the product. You still have 56 left. So calculate $56 \div 7$ in succession.



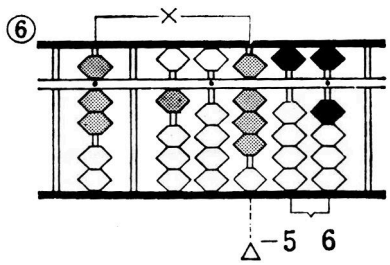
4. Compare 5, the first digit of the remaining dividend, with the divisor 7. The former is smaller. So set the second quotient digit on the first rod to the left of the remaining dividend.



5. To find the second quotient digit, think $7 \times \square = 56$ and set 8 as a quotient digit.



6. Thinking 8x7=56, subtract 56, with the first rod to the right of the second quotient digit 8 as the tens place of the product. The ones place of the quotient is formed on the second rod to the left of the dividend. The answer is 108.



Exercises 3

(1)	624 ÷ 2 =	(11)	1,584 ÷ 3 =
(2)	693 ÷ 3 =	(12)	964 ÷ 4 =
(3)	484 ÷ 4 =	(13)	852 ÷ 2 =
(4)	2,992 ÷ 8 =	(14)	822 ÷ 6 =
(5)	3,792 ÷ 6 =	(15)	849 ÷ 3 =
(6)	2,480 ÷ 5 =	(16)	4,284 ÷ 7 =
(7)	3,416 ÷ 4 =	(17)	1,424 ÷ 8 =
(8)	3,312 ÷ 9 =	(18)	1,296 ÷ 4 =
(9)	5,432 ÷ 8 =	(19)	926 ÷ 2 =
(10)	5,215 ÷ 7 =	(20)	7,101 ÷ 9 =

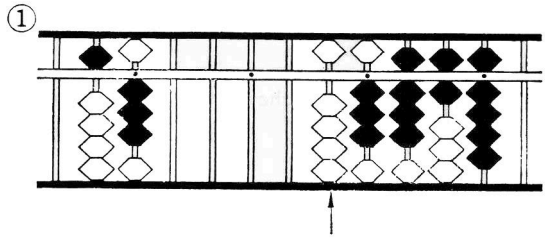
Answer

(1) 312	(2) 231	(3) 121	(4) 374	(5) 632	(6) 496
(7) 854	(8) 368	(9) 679	(10) 745	(11) 528	(12) 241
(13) 426	(14) 137	(15) 283	(16) 612	(17) 178	(18) 324
(19) 463	(20) 789				

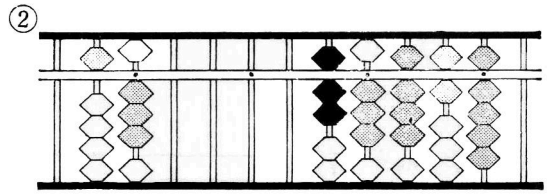
Division by Two-Digit Numbers

Example 9. $3,869 \div 53 = 73$

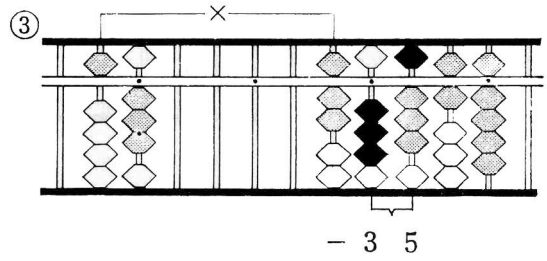
1. Set the problem as in the figure at the right. In division by a two-digit number, the quotient is set in the same way as in division by a one-digit number. Compare 3, the first digit of the dividend with 5, the first digit of the divisor. The former is smaller. So set the first quotient digit on the first rod to the left of the dividend.



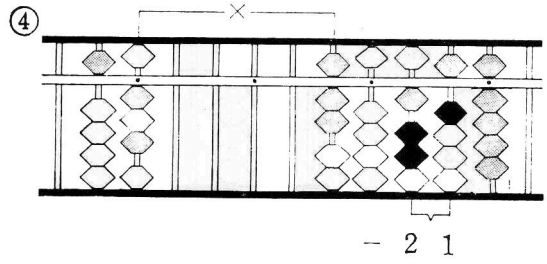
2. Mark off the first two digits of the dividend and think 5 x $\square = 38$. Thinking 5x7=35, set 7 as a quotient digit.



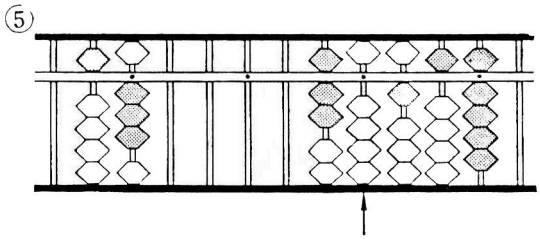
3. Multiply 7, the quotient digit, and 5, the first digit of the divisor. Then thinking 7x5=35, subtract 35, with the first rod to the right of the 7 as the tens place of the product.



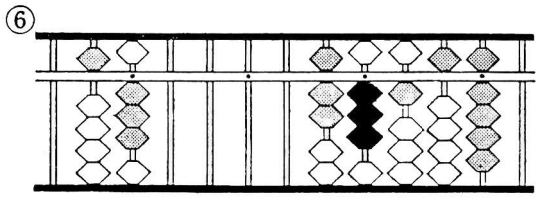
4. In succession, multiply the 7 and the 3, the second digit of the divisor. Then thinking 7x3=21, subtract 21, with the first rod to the right of the tens place of the preceding product as its tens place. You still have 159 left. So next calculate $159 \div 53$.



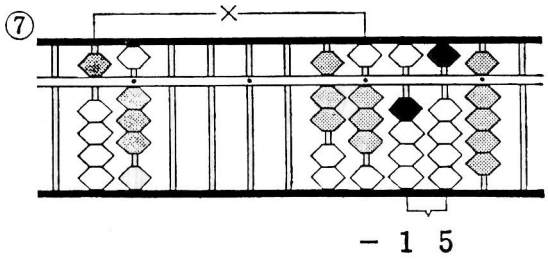
5. Compare 1, the first digit of the remaining dividend, with 5, the first digit of the divisor. The formar is smaller. So set the second quotient digit on the first rod to the left of the remaining dividend.



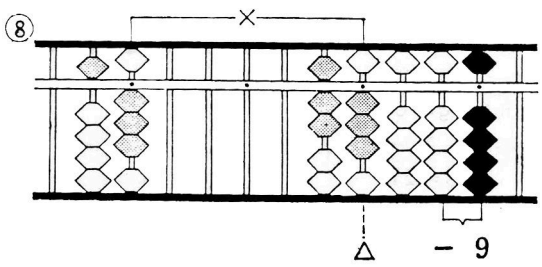
6. Thinking 6 x □ = 15, set 3 as the second quotient digit.



7. Multiply 3, the second quotient digit, and 5 the first digit of the divisor. Then thinking 3 x 5 = 15, subtract 15.



8. Multiply 3, the second quotient digit, and 3 the second digit of the divisor. Then thinking 3x3=9, subtract 9, with the first rod to the right of the preceding product as its tens place. The ones place of the quotient is formed on the third rod to the left of the dividend. The answer is 73.

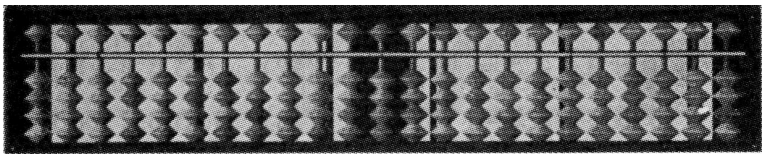


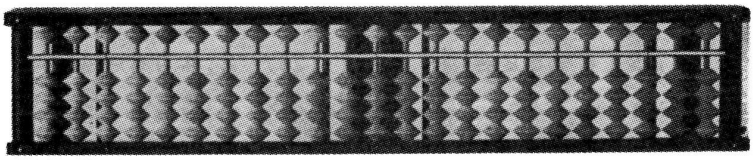
Exercises 4

(1)	384÷12=	(21)	2,397÷51=
(2)	989÷23=	(22)	338÷26=
(3)	736÷32=	(23)	5,382÷69=
(4)	3,328÷64=	(24)	6,417÷93=
(5)	4,875÷75=	(25)	540÷15=
(6)	4,316÷52=	(26)	2,496÷48=
(7)	984÷41=	(27)	6,888÷82=
(8)	2,336÷32=	(28)	850÷34=
(9)	1,764÷21=	(29)	6,552÷72=
(10)	3,526÷86=	(30)	800÷25=
(11)	4,588÷74=	(31)	8,148÷97=
(12)	2,132÷82=	(32)	368÷16=
(13)	3,441÷93=	(33)	4,788÷84=
(14)	540÷45=	(34)	465÷31=
(15)	5,063÷61=	(35)	828÷23=
(16)	1,353÷33=	(36)	4,189÷59=
(17)	728÷52=	(37)	4,864÷76=
(18)	4,368÷84=	(38)	1,344÷48=
(19)	8,835÷95=	(39)	3,038÷62=
(20)	4,745÷73=	(40)	3,515÷37=

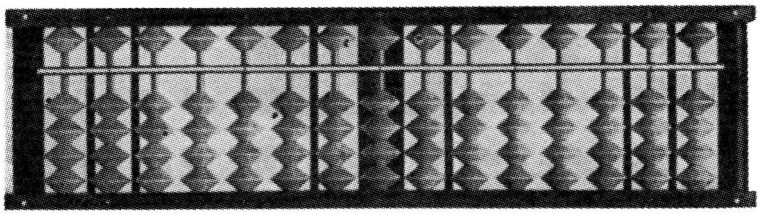
Answer

(1) 32	(2) 43	(3) 23	(4) 52	(5) 65	(6) 83
(7) 24	(8) 73	(9) 84	(10) 41	(11) 62	(12) 26
(13) 37	(14) 12	(15) 83	(16) 41	(17) 14	(18) 52
(19) 93	(20) 65	(21) 47	(22) 13	(23) 78	(24) 69
(25) 36	(26) 52	(27) 84	(28) 25	(29) 91	(30) 32
(31) 84	(32) 23	(33) 57	(34) 15	(35) 36	(36) 71
(37) 64	(38) 28	(39) 49	(40) 95		





Model B 3 and G soroban are most suitable for students, officials, office clerks, bankers and financiers. After tens of years study the size of counter-beads on these model are made most ideal for the touch of the fingers. These standard model hold 85% of the entire output of soroban in Japan.



GO 150 is made specially for beginners with larger beads for easy fingering and this soroban is recommended both by the Japan Chamber of Commerce and Industry and the League of Japan Abacus Association.

Photo by courtesy of Tomoe Soroban Co., Ltd., 14-3, Uchikanda 2-chome, Chiyoda-ku, Tokyo.