Babak's Problem

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1 Description of the Problem

We are given an isosceles triangle ABC the angle $\angle BAC$ equal to $20°$. Let point D be a point placed on the line segment $\overline{AB}$, where the distance between the two points A and D is the same as the distance between B and C. Find the angle θ that is defined as the angle $\angle BDC$.

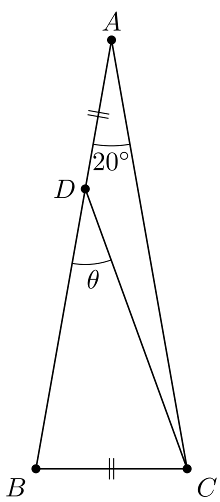

Figure 1: Babak's initial problem

1.1 The Solution

Let's begin by imagining making a copy of the triangle ABC, placing the transformed points B' and C' on the original points A and D respectively, giving us the transformed triangle ADE.

The triangle ACE is an isosceles triangle since the length of the line segments $\overline{AC}$ and $\overline{AE}$ have equal length. Thusly, by calculating the angle of $\angle CAE$ we can calculate the other two angles of the triangle. We know that the angle $\angle CAE$ is equal to $60°$, since the angle $\angle BAE$ is equal to $80°$ and the angle $\angle BAC$ is equal to $20°$. This means that the other two angles of the triangle ACE are also equal to $60°$, showing us that the triangle ACE is in fact an equilateral triangle.

Let's shift out focus to the triangle CED. We can show that this triangle is an isosceles triangle, in the same way we did for the triangle ACE, since the two line segments $\overline{AE}$ and $\overline{CE}$ have equal length. Since the two angles $\angle ADE$ and $\angle ACE$ are known, we can calculate the angle of $\angle DEC$ to be equal to $40°$. Considering the aforementioned fact, we know that the two remaining angles of our triangle CED are both equal to $70°$, including the angle $\angle CDE$.

The three angles θ, $\angle CDE$, and $\angle EDA$ added together is equal to 180°. As the angle $\angle EDA$ is the same as the angle $\angle ACB$, which is equal to 80° and

We first need to recognize that the angle $\angle CAE$ is equal to 60°, since the angle $\angle BAE$ is equal to 80° and the angle $\angle BAC$ is equal to 20°.

We can conclude that the triangle ACE is an isosceles.