Algebraic Identities Part 4

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1 Problems

A Let $a, b, c \in \mathbb{R}$ where $a \neq b$. If $c^3 = b^3 + b^3 + 3abc$, prove that $c = a + b$.

Proof.

Theorem 1. *Let $a, b, c \in \mathbb{R}$ If $a^3 + b^3 + c^3 = 3abc \Leftrightarrow a = b = c$ or $a + b + c = 0$*

$$
\begin{aligned}
b^3 + b^3 + 3abc &= c^3 \\
b^3 + b^3 - c^3 &= -3abc \\
-a^3 - b^3 + c^3 &= 3abc
\end{aligned}
$$

Using theorem 1 we can conclude

$$
\begin{array}{rcr}
-a = -b = c & \quad or \quad & -a - b + c = 0 \\
\text{which is false since} & & c = a + b \\
a \neq b & &
\end{array}
$$

□