INTERNATIONAL
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Problems of the Week, Week 12, 2020/03/18

Algebraic Identities, Part 04

A *simple* calculator is the only digital aid allowed.

Problems

A Let a, b, and c be real numbers and let $a \neq b$. If $c^3 = a^3 + b^3 + 3abc$, prove that $c = a + b$.

B Let x, y, z, and a be non-zero real numbers such that

$$x + y + z = a, \quad \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{1}{a} \cdot$$

Show that at least one of the numbers x, y, or z is equal to a.

Hint: Show that $(a - x)(a - y)(a - z) = 0$.

C Challenging! Solve the following system:

$$\begin{cases} x + y + z &= 10 \\ x^2 + y^2 + z^2 &= 100 \\ x^3 + y^3 + z^3 &= 1000 \end{cases}$$

D Challenging! Let a, b, c be real numbers such that $a+b+c = 0$. Show that $2(a^5+b^5+c^5) = 5abc(a^2+b^2+c^2)$.

Hint: Considering $abc = \frac{1}{3}(a^3 + b^3 + c^3)$, start from the right hand side.

E Challenging! Let a, b, c be non-zero numbers such that $a + b + c = 0$ and $a^3 + b^3 + c^3 = a^5 + b^5 + c^5$. Determine the exact value of $a^2 + b^2 + c^2$.

Hint: Expand $(a^2 + b^2 + c^2)(a^3 + b^3 + c^3)$.

F Challenging! Solve the following equaiton:

$$(x+1)^{63} + (x+1)^{62}(x-1) + (x+1)^{61}(x-1)^2 + \cdots + (x+1)^2(x-1)^{61} + (x+1)(x-1)^{62} + (x-1)^{63} = 0.$$

Hint: Multiply both sides of the equation by $2 = (x + 1) - (x - 1)$.

Answers

B $x = 10, y = 0, z = 0$ or $x = 0, y = 10, z = 0$ or $x = 0, y = 0, z = 10$

E $\frac{6}{5}$

F $x = 0$