Forward kinematic

المعطيات :

$\theta_1 = 20°$

$\theta_2 = 20°$

$L_1 = 3.8m$

$L_2 = 3.5m$

المجاهيل

X

Y

θ

“القياسات دقيقة, بحيث المنقلة والمسطرة”

ذراع روبوتية :

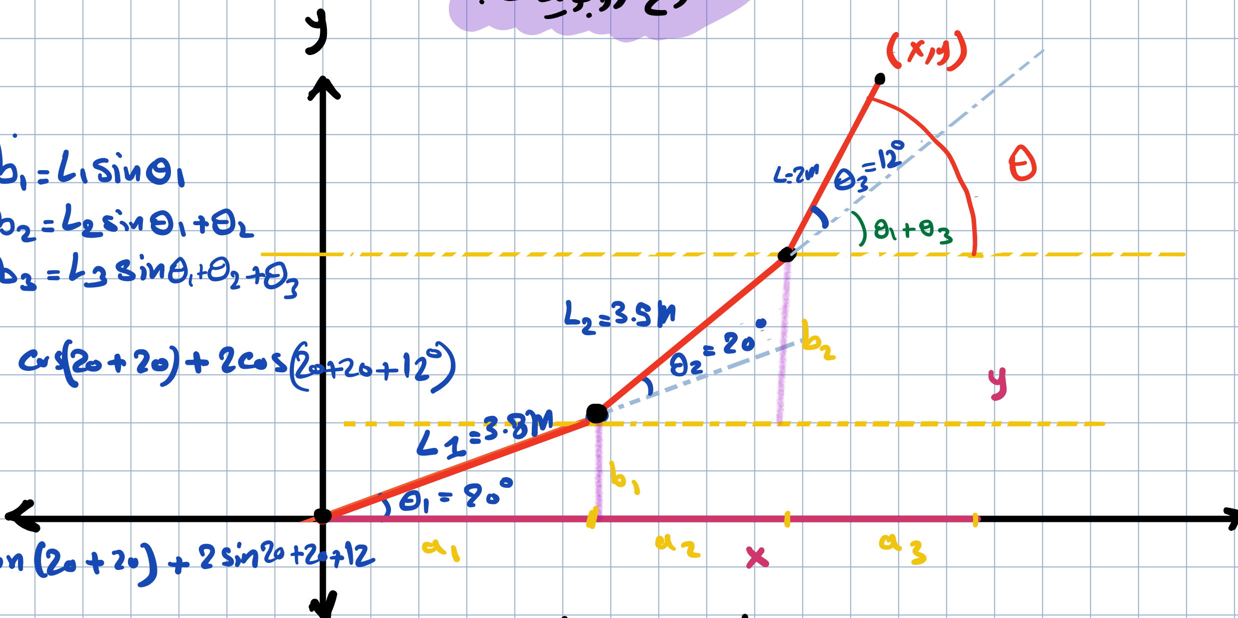
$X = a_1 + a_2 + a_3$

$y = b_1 + b_2 + b_3$

$\theta = \theta_1 + \theta_2 + \theta_3$

$a_1 = L_1 \cos \theta_1$	$b_1 = L_1 \sin \theta_1$
$a_2 = L_2 \cos \theta_1 + \theta_2$	$b_2 = L_2 \sin \theta_1 + \theta_2$
$a_3 = L_3 \cos \theta_1 + \theta_2 + \theta_3$	$b_3 = L_3 \sin \theta_1 + \theta_2 + \theta_3$

$X = 3.8 \cos 20 + 3.5 \cos(20+20) + 2\cos(20+20+12°)$
$= 7.48m$



$y = 3.8 \sin 20 + 3.5 \sin(20+20) + 2 \sin 20 + 20 + 12$
$= 5.13m$

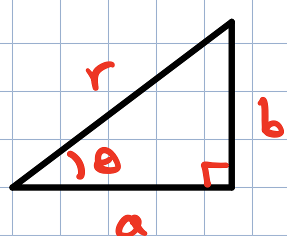
I need to find each of x, y, θ

$\theta = 20 + 20 + 12 = 52°$

$(x, y) \rightarrow (7.48, 5.13)$ ✓

$\theta \rightarrow 52°$ ✓

فيثاغورس



$a^2 + b^2 = r^2$

$\sin \theta = \frac{b}{r}$

$\cos \theta = \frac{a}{r}$

we need $a_1 + a_2$ for X_1 and $b_1 + b_2$ for Y } To solve inverse kinematic

$X_1 = 3.8 \cos 20 + 3.5 \cos 40 = 6.25m$

$Y_1 = 3.8 \sin 20 + 3.5 \sin(40) = 3.55m$

Inverse kinematic

نستخدم النواتج التي حصلنا عليها في المثال السابق لافتراض كل من المعطيات التالية
x، y، θ.

المعطيات	المجاهيل
$x = 7.48$	θ_1
$y = 5.13$	θ_2
$\theta = 52^\circ$	θ_3
$L_1 = 3.5\ m$	
$L_2 = 3.8\ m$	
$L_3 = 2\ m$	
$X_1 = 6.25\ m$	
$Y_1 = 3.55\ m$	

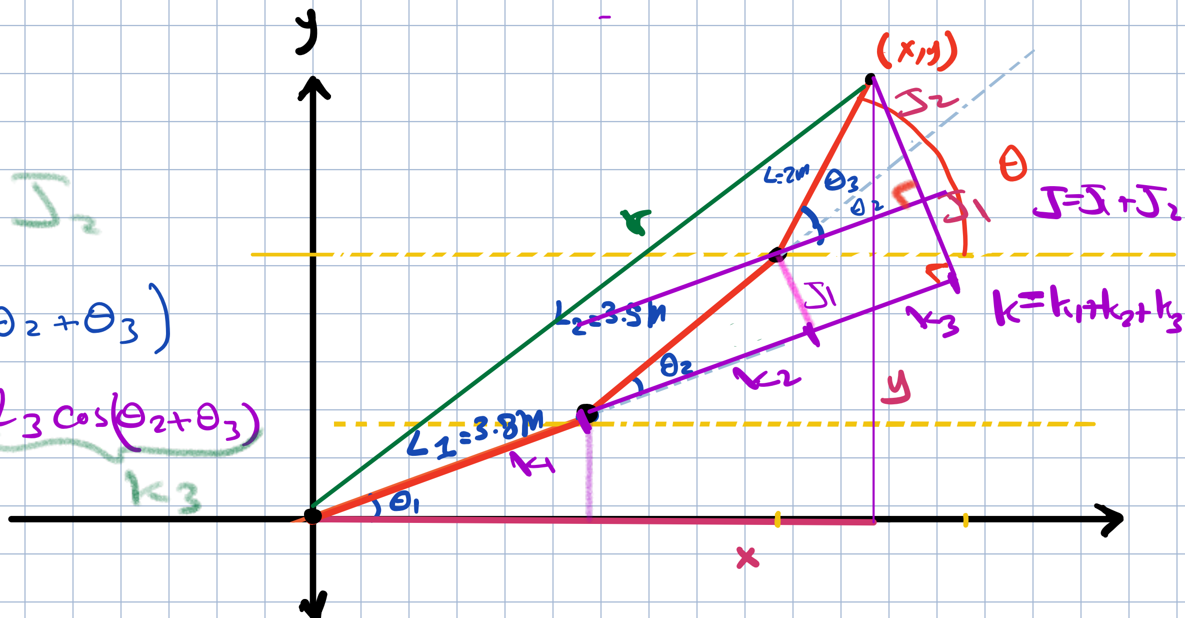
$$x^2 + y^2 = r^2$$

$$k^2 + J^2 = r^2$$

$$J = J_1 + J_2$$

$$J = \overbrace{L_2 \sin\theta_2}^{J_1} + \overbrace{L_3 \sin(\theta_2 + \theta_3)}^{J_2}$$

$$k = \underbrace{L_1}_{k_1} + \underbrace{L_2 \cos\theta_2}_{k_2} + \underbrace{L_3 \cos(\theta_2 + \theta_3)}_{k_3}$$



$$x^2+y^2 = K^2+J^2$$

$$x^2+y^2 = \left[L_1 + L_2\cos\theta_2 + L_3\cos(\theta_2+\theta_3)\right]^2 + \left[L_2\sin\theta_2 + L_3\sin(\theta_2+\theta_3)\right]^2$$

$$\Rightarrow (L_1+L_2\cos\theta_2)^2 + 2L_3(L_1+L_2\cos\theta_2)\cos(\theta_2+\theta_3) + L_3^2\cos^2(\theta_2+\theta_3) +$$

$$L_2^2\sin^2\theta_2 + 2L_2L_3\sin\theta_2\sin(\theta_2+\theta_3) + L_3^2\sin^2(\theta_2+\theta_3) = x^2+y^2$$

$$\Leftrightarrow L_1^2 + 2L_1L_2\cos\theta_2 + L_2^2\cos^2\theta_2 + 2L_1L_3\cos(\theta_2+\theta_3) +$$

$$2L_2L_3\cos\theta_2\cos(\theta_2+\theta_3) + L_3^2\cos^2(\theta_2+\theta_3)$$

$$L_2^2\sin^2\theta_2 + 2L_2L_3\sin\theta_2\sin(\theta_2+\theta_3) + L_3^2\sin^2(\theta_2+\theta_3) = x^2+y^2$$

$[\sin^2\theta + \cos^2\theta = 1]\, L_2^2$

L.H.S = R.H.S

L.H.S أصل

R.H.S ل

$$L_2^2\cos^2\theta_2 + L_2^2\sin^2\theta_2 = L_2^2$$

$[\sin^2\theta + \cos^2\theta = 1]\, L_3^2$

L.H.S = R.H.S

L.H.S أصل

R.H.S ل

$$L_3^2\cos^2(\theta_2+\theta_3) + L_3^2\sin^2(\theta_2+\theta_3) = L_3^2$$

$$\cos(\cancel{\theta_2} - (\cancel{\theta_2}+\theta_3))$$

$$+ 2L_2L_3\left[\cos\theta_2\cos(\theta_2+\theta_3) + \sin\theta_2\sin(\theta_2+\theta_3)\right]$$

$$= x^2+y^2 - L_1^2$$

$$\boxed{\begin{gathered}\cos(a\pm b) = \cos a\cos b \mp \sin a\sin b \\ a=\theta_2,\quad b=\theta_2+\theta_3\end{gathered}}$$

$$+2L_2L_3\cos(\cancel{\theta_2} - (\cancel{\theta_2}+\theta_3)) \Longleftrightarrow \cos(-\theta_3) = \cos(\theta_3)$$

$$= 2L_2L_3\cos(\theta_3)$$

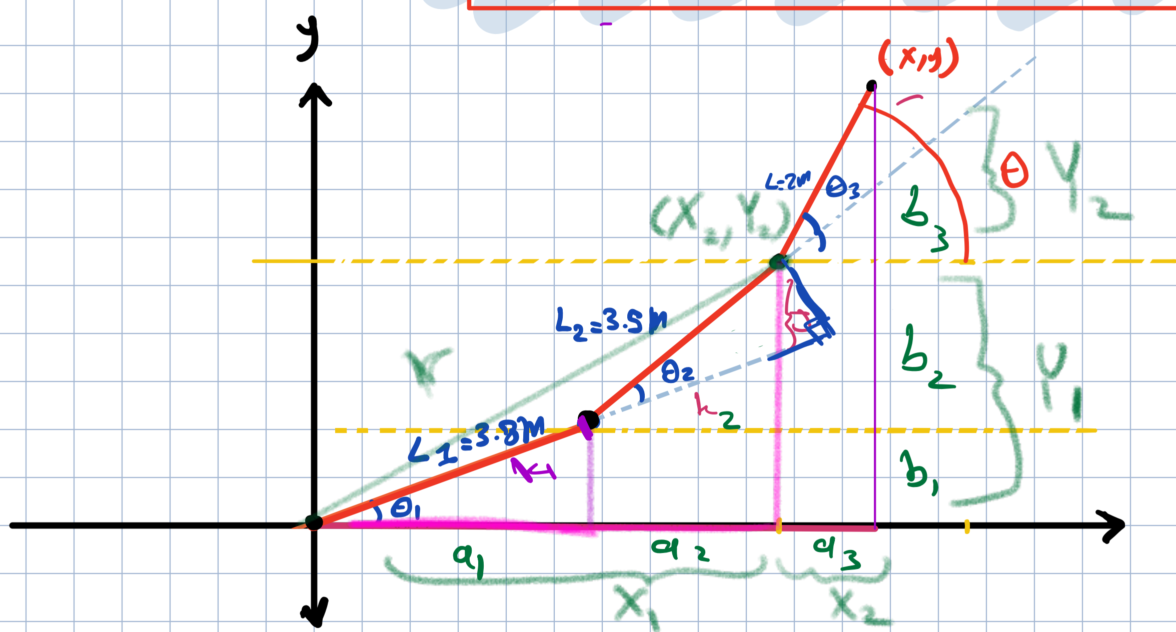
$$\Rightarrow L_1^2 + L_2^2 + L_3^2$$
$$+ 2L_1L_2\cos\theta_2$$
$$+ 2L_1L_3\cos(\theta_2+\theta_3)$$
$$+ 2L_2L_3\cos(\theta_3)$$
$$= x^2 + y^2$$

$$\Leftrightarrow L_1L_2\cos(\theta_2) + L_2L_3\cos(\theta_3) + L_1L_3\cos(\theta_2+\theta_3)$$
$$= \frac{1}{2}\left[x^2+y^2-L_1^2-L_2^2-L_3^2\right] \longrightarrow (1)$$

we can notice here we have 2 unknown, So, we can't solve the eq

But, Let solve this problem:

$\boxed{X_1, Y_1 \text{ Should be given}}$



$X = x_1 + x_2 + x_3 \qquad Y = y_1 + y_2 + y_3$

$$X = X_1 + X_2$$
$$Y = Y_1 + Y_2$$

where $X_1 = a_1 + a_2$ $\qquad X_2 = a_3$

$= L_1 \cos\theta_1 + L_2 \cos(\theta_1 + \theta_2)$

$Y_1 = b_1 + b_2$ $\qquad Y_2 = b_3$

$= L_1 \sin\theta_1 + L_2 \sin(\theta_1 + \theta_2)$

$$X_1^2 + Y_1^2 = r^2$$

To find θ_2:

$k = [L_1 + L_2 \cos\theta_2]$

$J = L_2 \sin\theta_2$

$r^2 = J^2 + k^2$, $r^2 = X^2 + Y^2$

$$X^2 + Y^2 = J^2 + k^2$$

$$X_1^2 + Y_1^2 = [L_2 \sin\theta_2]^2 + [L_1 + L_2 \cos\theta_2]^2$$

$$X_1^2 + Y_1^2 = \underbrace{L_2^2 \sin^2\theta_2} + L_1^2 + 2L_1L_2 \cos\theta_2 + \underbrace{L_2^2 \cos^2\theta_2}$$

$\sin^2\theta + \cos^2\theta = 1 \Leftarrow$

$L_2^2 \sin^2\theta_2 + L_2^2 \cos^2\theta_2 = L_2^2$

$$X_1^2 + Y_1^2 = L_2^2 + L_1^2 + 2L_1L_2 \cos\theta_2 +$$

$$\cos\theta_2 = \frac{1}{2L_1L_2} [X_1^2 + Y_1^2 - L_2^2 - L_1^2]$$

$$\therefore \theta_2 = \cos^{-1}\left[\frac{X_1^2 + Y_1^2 - L_2^2 - L_1^2}{2L_1L_2}\right]$$

$$\theta_2 = \cos^{-1}\left[\frac{6.25^2 + 3.55^2 - 3.8^2 - 3.5^2}{2(3.8)(3.5)}\right]$$

$$\theta_2 = 20.13$$

we need θ_1 & θ_3 :

now we can use eq (1)

$$\Longleftrightarrow L_1L_2\cos(\theta_2) + L_2L_3\cos(\theta_3) + L_1L_3\cos(\theta_2+\theta_3)$$

$$= \frac{1}{2}\left[x^2+y^2-L_1^2-L_2^2-L_3^2\right] \longrightarrow (1)$$

$$L_2L_3\cos(\theta_3) + L_1L_3\cos(\theta_2+\theta_3) = \frac{1}{2}\left[x^2+y^2-L_1^2-L_2^2-L_3^2\right] - L_1L_2\cos\theta_2$$

now sub:

$$(3.5)(3.8)\cos(\theta_3) + (3.8)(2)\cos(20+\theta_3) = \frac{1}{2}\left[(7.48)^2+(5.13)^2 - (3.8)^2 - (3.5)^2 - (2)^2\right] - (3.8)(3.5)\cos 20$$

$$\Longleftrightarrow 13.3\cos\theta_3 + 7.6\cos(20+\theta_3) = 13.3$$

θ_3 has so many solutions

Therefore using Matlab here is a wise

المعطيات

x = 7.48

Y = 5.13

θ = 52°

L_1 = 3.8

L_2 = 3.5 m

L_3 = 2m

x_1 = 6.25 m

Y_1 = 3.55 m

desecion to find determinant solution!

$$\tan\theta_1 = \frac{\sin\theta_1}{\cos\theta_1}$$

$$\theta_1 = \tan^{-1}\left[\frac{\sin\theta_1}{\cos\theta_1}\right]$$

$$\theta = \theta_1 + \theta_2 + \theta_3$$

$$\theta_3 = \theta - (\theta_1 + \theta_2)$$