# Robo-advisor


## Abstract

This project built a robo-advisor website for users to better invest money into stocks and cryptocurrencies which can provide investment strategies and current information related to the financial market. This paper states the factors which lead to creating this innovative idea and how the idea is well implemented by applying deep learning and CAPM model to build up the core algorithm of the website.

# Contents

# 1 Introduction

After the 2008 financial crisis, investment banks, asset management companies and startups have started to notice the development of automated investment and financial advisory platforms. In addition to building their own robo-advisors, some of them have invested in this area, such as Blackrock acquires FutureAdvisor, Aberdeen acquires Parmenion, and Charles Schwab launches Intelligent Portfolios, etc.

Several factors have resulted in the vigorous development of automated investment and wealth management platforms. First of all, the development of machine learning and artificial intelligence technology. To improve the robo-advisor, good algorithms are needed. In addition to tracking the change of indexes, it is also critical for robo-advisors to improve the portfolio according to the market news, black swan and so on. Secondly, the changes of investment behavior. The young investors are highly dependent on Internet technology. Robotic investment advisors provide a convenient user interface, a low and transparent fee structure, and a low threshold for opening an account. Therefore, they are attractive for young investors to use such services. During this pandemic caused by COVID-19, global economies, stock markets, small and large businesses were all affected negatively. Surprisingly, robo-advisors have seen an unexpected increase in users. The signups for robo-advisory services have surged like never before. In the first quarter of 2020, an average rise of 3.1% across all providers and platforms was reported. Individually, Vanguard reported a 14% growth in assets and 35% surge in customer numbers and PensionBee's number of users increased by 14%. AJ Bell Youinvest and Hargreaves Lansdown were not far behind with growth in assets of 13% and 7.4%, respectively.(1)(2)

Vanguard, which manages more than $6 trillion, debuted a digital-only financial advice service this year geared toward a younger clientele called Vanguard Digital Advisor. Two-thirds of investors who sign up for the service are either millennials or Gen Z. According to this firm's survey, most millennials have never received professional financial advice, but nearly half say their interest has increased due to Covid-19. Additionally, they are twice as likely as some older investors to consider using a robo-advisor.(3)

Analysts speculate that the sudden rise can be due to the large number of millennials taking advantage of the buying opportunity in the bear market.(4) Such investors have a longer time horizon and have a higher tolerance for economic damage. Amidst the crashing equity markets, the do-it-yourself investors are relying more on the advice of the automated robots than their shaken knowledge and experience.

In conclusion, we want to exploit the robo-advisor market, focusing mainly on young investors who are new to the investment world. If our users are either scared to invest due to limited knowledge or they think their budget is too small to invest, we can help. Robo-advisors are best for risk-averse, low-budget customers because of their cheaper fees and lower saving entry points compared to conventional alternatives. In June 2019, the six largest robo-advisors managed over ten billion U.S. dollars each. This fact alone testifies the potential of such technology as it is expected that by 2023, assets managed by robo-advisors will jump to 2.5 trillion U.S. dollars.

# 2 Motivation and Significance

There are currently already well-established robo-advisors on the market. What motivates us to build another product?

The fact that all the robo-advisors excluded investment in cryptocurrencies. A robo-advisor encouraging its users to invest in a market considered too volatile will be an innovation.

At the time of writing of our proposal, on the 30th of September, the market capitalization of cryptocurrencies was approximately 342 billion U.S. dollars, as obtained from https://coinmarketcap.com. At the time of writing of this report, on the 23rd November, the market capitalization of cryptocurrencies is approximately 534 billion U.S. dollars. In the time of one month and a half, the cryptocurrencies' market increased by nearly 200 billion U.S. dollars.

The straightforward reason for such exponential increase in the market size of cryptocurrencies is the presence of the pandemic caused by COVID-19. However, the pandemic had a negative impact on stock markets worldwide, when they reported, on 28 February 2020, their largest single-week declines since the 2008 financial crisis. Just like Satoshi Nakamoto, the creator of Bitcoin, envisioned during the financial crisis of 2008-09, the creation of decentralized digital currencies did help during the current financial crisis. Despite the potential of high returns, investors are still skeptical about investing in the cryptocurrency market due to high volatility. Limited access to expertise in that sector discouraged common people who are excited about this technology to invest. Therefore, we came up with Tomorrow's Robo-Advisor, an advisor that can help you not only in investing stock but also in cryptocurrencies.

# 3 Methodology

## 3.1 Portfolio Models

In this project, there are two main portfolio models which are capital asset price model (CAPM) and 1/N model being used, both of which are able to generate the optimal portfolio. First, in an attempt to address and quantify portfolio effects in the cryptocurrency investment universe, CAPM model combined with the traditional mean-variance portfolio selection framework as proposed by Markowitz can reveal preliminary evidence on portfolio effects, i.e. properties of multiple cryptocurrency investments. Second, the reason why 1/N is also adopted in this project is that "1/N-portfolio outperformed single cryptocurrencies and more than 75% of mean-variance optimal portfolios".(5)

#### 3.1.1 CAPM Model

In CAPM model, given a specific level of rate of return, there will be various portfolios with different variance, and only the portfolio with minimal variance is the optimal one. Then the model will adjust the rate of return into different values ranging from the minimal return of those selected cryptocurrencies to the maximal one of them. Consequently, an efficient frontier is constructed based on set of optimal portfolios that offer the lowest risk for a given level of expected return. Thus, the optimal portfolio of the CAPM model consists of a risk-free asset and an optimal risky asset portfolio, and the optimal risky asset portfolio is at the point where the capital allocation line (CAL) is tangent to the efficient frontier. This portfolio is optimal because the slope of CAL is the highest, which means the model can achieve the highest returns per additional unit of risk. Therefore, the optimal weights of cryptocurrencies can be obtained from the optimal portfolio, which are used to construct future investment strategies.

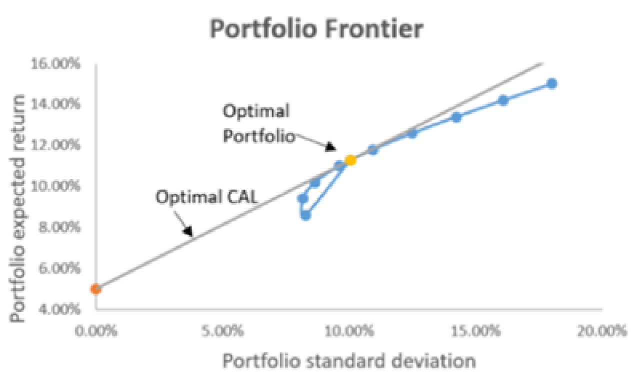


Figure 1: Optimal Portfolio on Efficient Frontier

#### 3.1.2 1/N Model

1/N model is used to assign each cryptocurrency with equal weight, which may be seen as a benchmark for data-driven portfolios. The reason why the naively diversified 1/N equal-weighted portfolio is adopted is because price changes in each cryptocurrency are unpredictable , which results in the inaccuracy of calculated portfolios. Therefore, 1/N model can also be used to compare with other portfolio models.

## 3.2 Optimization Algorithms

This project mainly focuses on two optimization methods to solve the optimization problems in the CAPM model. In specific, the portfolio risk of return is quantified by $\sigma_p^2$ and in mean-variance analysis, only the first two moments ($\sigma_p^2$ and $\mu_p$) are considered in the portfolio model. In order to minimize the portfolio risk no matter whether the portfolio model allows short-selling or not, the first method is to apply closed form combined with quadratic programming; the second one is to use a function named "minimize" from the package "scipy.optimize" in Python.

### 3.2.1 Closed Form and Quadratic Programming

The first approach is to apply closed form to get the optimal portfolio with short-selling and quadratic programming to get the optimal one without short-selling since there is no closed-form solution in inequality constrained optimization problem.

Firstly, for closed from, the solution can be calculated in the equality constrained optimization problem. To solve the problem, Markowitz's mean-variance analysis should be used and its mathematical formulation of given by:

$$\min \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}(w_i w_j \sigma_{ij})$$

subject to $\sum_{i=1}^{N} w_i R_i = \mu_p$ and $\sum_{i=1}^{N} w_i = 1$. Given the target expected rate of return of portfolio $\mu_p$, find the portfolio strategy that minimizes $\sigma_p^2$. To form the Lagrangian:

$$L = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}(w_i w_j \sigma_{ij}) - \lambda_1(\sum_{i=1}^{N} w_i - 1) - \lambda_2(\sum_{i=1}^{N} w_i R_i - \mu_p)$$

where $\lambda_1$ and $\lambda_2$ are Lagrangian multipliers. Then differentiate $L$ with respect to $w_i$ and Lagrangian multipliers, and set the derivative to zero.

$$\frac{\partial L}{\partial w_i} = \sum_{j=1}^{N} \sigma_{ij} w_j - \lambda_1 - \lambda_2 R_i = 0, i = 1, 2, ..., N$$

$$\frac{\partial L}{\partial \lambda_1} = \sum_{i=1}^{N} w_i - 1 = 0$$

$$\frac{\partial L}{\partial \lambda_2} = \sum_{i=1}^{N} w_i R_i - \mu_p = 0$$

Let $\Omega$ denote the covariance matrix of cryptocurrencies of the portfolio so that $a = 1^T\Omega^{-1}1$, $b = 1^T\Omega^{-1}\mu$, $c = \mu^T\Omega^{-1}\mu$. Solving for $\lambda_1$ and $\lambda_2$: $\lambda_1 = \frac{c-b\mu_p}{\Delta}$ and $\lambda_2 = \frac{a\mu_p-b}{\Delta}$, where $\Delta = ac - b^2$. Thus, the optimal weight is given by:

$$w^* = \Omega^{-1}(\lambda_1 1 + \lambda_2\mu)$$

Secondly, for quadratic programming, Markowitz's algorithm with no short selling requires the weights of all cryptocurrencies to be positive or zero. With inequality constraints, the Lagrange multiplier method no longer works because it imposes an equality in the constraint. This optimization problem must be solved numerically, e.g. using the cvxopt.solver.qp in Python or the function solve.QP() in the R package quadprog. Quadratic programming problems are of the form:

$$\min \frac{1}{2}x^T Dx - d^T x \ s.t.$$

$$A_{neq}^T x \geq b_{neq}$$

$$A_{eq}^T x = b_{eq}$$

where $D$ is an $n \times n$ matrix, $x$ and $d$ are $n \times 1$ vectors, $A_{neq}^T$ is an $m \times n$ matrix, $b_{neq}$ is an $m \times 1$ vector, $A_{eq}^T$ is an $l \times n$ matrix, $b_{eq}$ is an $l \times 1$ vector. Now, consider the portfolio optimization problem:

$$\min_w \sigma^2 = w^T\Omega w \ s.t.$$

$$w^T R = \mu_p$$

$$w^T 1 = 1$$

$$w_i \geq 0 \ i = 1, 2, ..., N$$

Then combine the inequality constraints and equality constraints into matrices separately and use Python or R to solve the inequality constrained optimization problem. Thus, the optimal weight will be computed.

#### 3.2.2 "scipy.optimize.minimize"

The second approach is to use Python package "scipy.optimize" to solve the minimization problem regardless of whether the optimal portfolio enables short-selling or not. The default method of "scipy.optimize.minimize" is BFGS to solve both unconstrained minimization problems and constrained minimization problems.The package provides L-BFGS-B algorithm (L-BFGS-B), a modification of Powell's method (Powell), and a truncated Newton algorithm (TNC) for bound-constrained minimization, and constrained optimization BY linear approximation method (COBYLA), sequential least squares programming (SLSQP), and trust-region algorithm (trust-constr) for constrained minimization.

### 3.3 WaveNet

WaveNet (6) was first developed for generating raw audio. The architecture of WaveNet allows it to exploit the advantages of convolution layers while being able to learn long-term dependencies of input data. Because of such characters, it is naturally suitable for time series forecasting. For the sake of applying WaveNet for stock price prediction, we replace the original softmax output layers with a feed-forward network. In addition, we omit the conditional architecture of Wavenet as there is no extra form of input data in the stock dataset. In figure 2 we show the complete architecture of the model, which can receive a multivariate time series and output the next y days prediction.

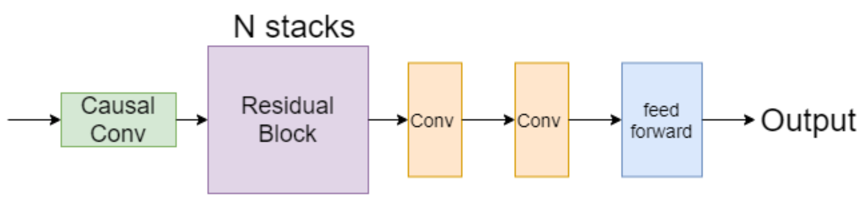


Figure 2: Complete architecture of WaveNet

#### 3.3.1 Causal Convolution Layer

One of the important components of WaveNet is the causal convolution layer, which enables the model to have the autoregressive property. The output of the units of the next layer only depends on past and current inputs. Therefore, the causal convolution layer can be represented by the following equation:

$$p(x) = \prod_{t=1}^{T} p(x_t|x_1, ..., x_{t-1}) \tag{1}$$

Here, $p(x)$ is a joint probability of sequence $x = (x_1, ..., x_T)$ and each input $x_t$ is conditioned on the samples at all previous timesteps.

#### 3.3.2 Dilated Causal Convolution Layer

One problem with causal convolution is that they require many layers, or large filters to increase the receptive field. The complexity of the model will grow exponentially as the length of the input increases. To properly handle the input with large timesteps, WaveNet introduces a mechanism called **dilated**. Figure 3 shows the exact architecture of the dilated causal convolution layer. We can see that the filters of the next layer skip constant inputs in between each of the inputs from the previous layers, which enables the model to have very large receptive fields with just a few layers. It allows WaveNet to handle long-term dependencies of lengthy input data while maintaining computational efficiency.

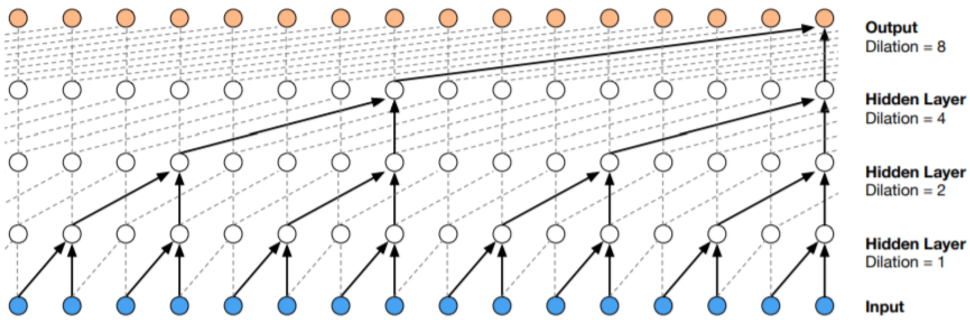


Figure 3: Dilated Causal Convolution Layer

#### 3.3.3 Residual Block and Skip Connections

Both residual (7) and parameterized skip connections are used throughout the network, to speed up convergence and enable training of much deeper models. The output of the residual block is the combination of the input and output of the dilated causal convolution layer, and the stack is built by multiple layers of the residual block, where outputs of some layers are skipped to speed up convergence.

#### 3.3.4 Training

We trained our models on a stock dataset consist of 10 chosen stocks with Adam optimizer with $\beta_1 = 0.9$, $\beta_2 = 0.999$. We split the the dataset into training set and test set in 8:2 ratio respectively, and evaluate their performance on the test sets by mean square error. The models are trained with a learning rate of 0.0005 in the first 20 epochs and a learning rate of 0.0001 after. The training will stop once the training loss no longer decreases.

# 4 Innovative Element Design

Firstly, the system ensures ease of interaction with the advisor by changing the menu button for better navigation and orientation, implementing one long scrollable website for easier navigation, and stricter coupling of elements' functionality. Users are able to input their desired stocks and investment objectives (eg. risk tolerance and expected return) in their personal pages with convenient and intelligible user interfaces. We make the process of constructing the portfolio as simple as possible because the most likely users of robo-advisor are those who have a small amount of budget and little knowledge of finance. Compared with the existing products, Our robo-advisor also addresses the needs of small investors by providing an accessible and user-friendly service.

Secondly, the system is designed to achieve work efficiency. It will store users' personal data and investment styles as well as investment preferences. Furthermore, the system will analyze legal and regulatory constraints such as government restrictions on portfolio contents or laws against insider trading. The system will construct the users' portfolios by determining suitable and optimal allocations to various asset classes based on their personal data and expectations about macroeconomic variables such as inflation, interest rates, and GDP growth. And the system will automatically monitor and rebalance the portfolios to adjust asset class allocations and securities holdings in response to market performance.

# 5 Experimentation and Application

In this section, we give an overview of the architecture and workflow of the robo-advisor. We also briefly discuss the application testing and design of the survey.

## 5.1 Architecture

The system architecture shown below is relatively simple. First, the user can browse our website and send data to the user interface, which requests data from the database. The database then retrieves the relevant information and responds to the user interface. Finally, the user interface displays results to the user.

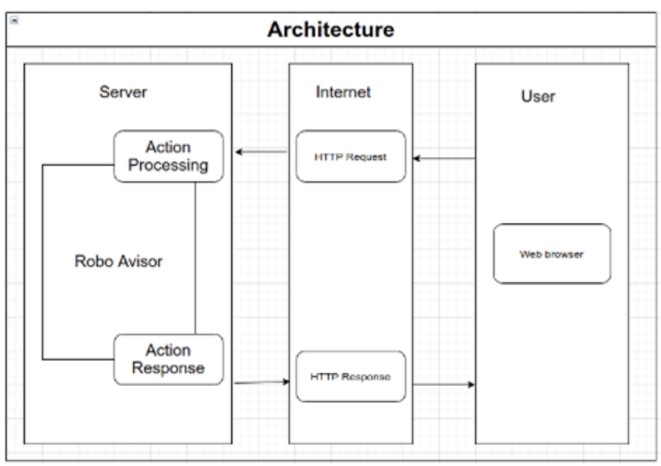


Figure 4: Architecture

The robo-advisor will be deployed on the AWS server. Any valid HTTP request will be processed and corresponding actions will be performed.

## 5.2 Application Design

### 5.2.1 A Brief Introduction

The application is designed for users to have their own customized portfolio according to their risk tolerance, and users can know the composition and the prediction stock price of that portfolio from the graph. Our application split into three parts, questionnaire, portfolio generation, and stock prediction.

### 5.2.2 Workflow

The image below shows the workflow of our website:

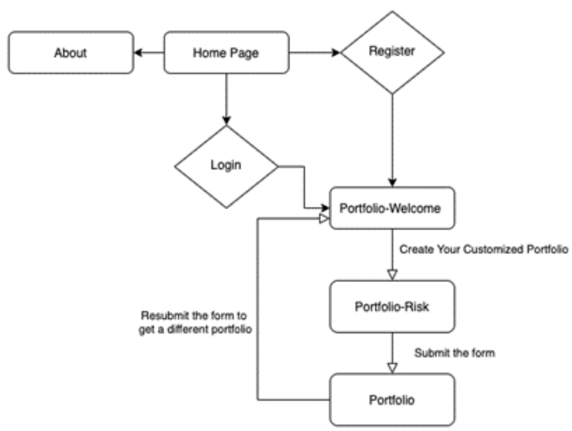


Figure 5: Workflow

- Home page: The basic information of the robo-advisor
- About page: Brief introduction about concept
- Portfolio risk: The questionnaires to measure the risk tolerance
- Portfolio page: The information of your customized portfolio and stock prediction
- Storage: All the data stored in the database

### 5.2.3 Functions

As mentioned, we mainly have three features for our application: questionnaire, customized portfolio, and stock prediction.

First for the questionnaire, our questionnaire will evaluate your risk tolerance according to these 6 questions. Each option accounts for different weights. An account of the weight of 5, B is 4, and so on. E is 1. The questions include :

1. What percentage of your savings are you investing?

   A Less than 20%

   B Between 20% and 30%

   C Between 30% and 40%

   D Between 40% and 50%

   E More than 50%

2. How long is your expected investment period?

   A Above 5 years

   B 4 - 5 years

   C 2 - 4 years

   D 1- 2 years

   E Below 1 year

3. What percentage of your monthly income do you usually spend, excluding investments in stocks, bonds, and cryptocurrencies?

   A Less than 50%

   B Between 50% and 60%

   C Between 60% and 70%

   D Between 70% and 80%

   E More than 80%

4. What is your net worth?

   A Above 500,000 HKD

   B Between 400,000 HKD and 500,000 HKD

   C Between 300,000 HKD and 400,000 HKD

   D Between 200,000 HKD and 300,000 HKD

   E Below 200,000 HKD

5. What is your age?

   A Above 60 years old

   B 50 - 60 years old

   C 40 - 50 years old

   D 30 - 40 years old

E 20 - 30 years old

6. What is your status about investment?

   A I want to invest in a simple way

   B I am eager to learn

   C I am an experienced investor

According to the choice of these 6 questions, if the total score is 22-30, the user is a more risk-averse person; if the total score is 15-22, the user is a risk-neutral person; if 8 - 14, the user is a risk-loving person. Our customized portfolio will adjust according to different risk types.

Second one is the portfolio part. We assume the user is more risk-averse since our targeted customers are millennials. So the below portfolio is the one with a total score between 22 and 30. For the first graph, we can clearly see the maximized return and the optimal portfolio and we use a table to show the contained stocks and cryptocurrencies. The second graph uses the bar chart to present for better understanding. The negative weights represent a short sell. We can clearly consider this portfolio a conservative portfolio since the portfolio contains more apple stock and only contains few cryptocurrencies.

## Optimal Portfolio

Maximal Return: 0.2333

Sharpe Ratio: 0.0135

### Stocks

| Symbol | Name | Buying Price | Price | Weight |
|---|---|---|---|---|
| AAPL | Apple Inc. | 117.34 | 113.85 | 2.3855 |
| IBM | International Business Machines | 116.94 | 120.09 | -0.6063 |
| GS | Goldman Sachs Group, Inc. (The) | 223.35 | 228.83 | -1.2228 |

### Cryptocurrency

| Symbol | Name | Buying Price | Price | Weight |
|---|---|---|---|---|
| BTC-USD | Bitcoin USD | 18569.46 | 18437.80 | 0.5077 |
| ETH-USD | Ethereum USD | 584.74 | 611.17 | -0.0641 |

Figure 6: Optimal Portfolio

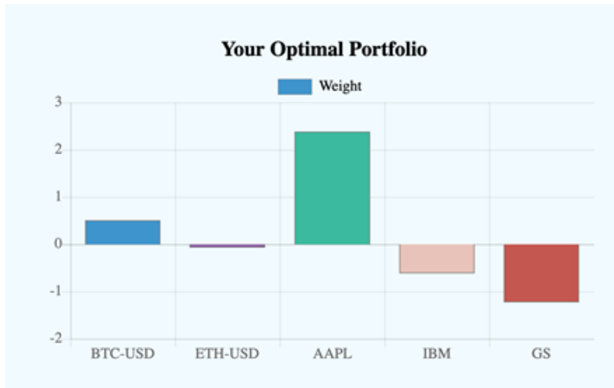


Figure 7: Optimal Weights

Last but not least, the stock prediction part. We use WaveNet to predict the stock prices for the following days. The price is not perfect since the model is not fully trained.

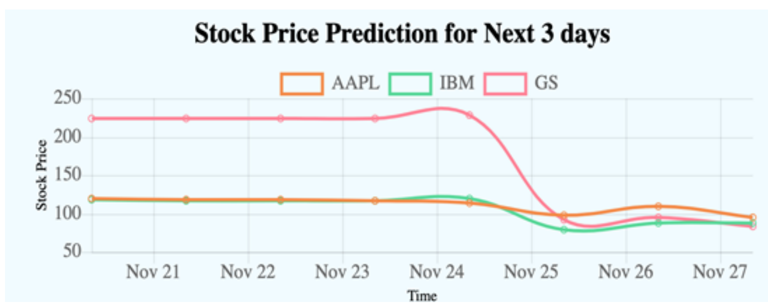


Figure 8: Stock Price Prediction

To sum up, these three features are the main characteristics of our application design. By integrating the portfolio and the web app, we show the optimal portfolio and the stock prediction in an easily understood manner.

### 5.3 Application Testing

The application testing consists of multiple steps that ensure that the robo-advisor is fully functional and runs smoothly and securely. We create several test conditions to see if the functions of the robot advisor return the expected outputs. We then test the application under different environments to ensure it is compatible with all browsers and devices. Finally, we make sure it is protected against unauthorized access and harmful actions through viruses or other malicious software. For example, we check whether unauthorized users can access restricted files.

# 6 Conclusion

In this project, we have applied the CAPM model and WaveNet on the web application to provide investment advice about stocks and cryptocurrency. Our robo-advisor addresses the need for small investors by providing an accessible and user-friendly service. Although in terms of production there is still room for improvement, especially in the training of WaveNet, our robo-advisor still provides a comprehensive advisory service. In conclusion, we have successfully implemented a robo-advisor that is fully functioning and extensible.

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