

Exercise sheet 3

Deadline: Tuesday, 14.11.2017 11:00 AM

Exercise 1 (6 points)

Show that $\log_2 n = \mathcal{O}(n)$ holds. Directly use the definition of \mathcal{O} by determining n_0 and C such that $\log_2 n \leq C \cdot n$ holds for all $n \geq n_0$.

Show that $\log_2 n = \Omega(n)$ does not hold. Directly use the definition of Ω by showing that for each given $C > 0$ and n_0 there exists a $n \geq n_0$ which violates the definition of Ω (i.e. that $\log_2 n \leq C \cdot n$). Consider also that C can be smaller than 1.

Exercise 2 (7 points)

Argue that the propositions in Exercise 1 do not only hold for \log_2 , but in the general case \log_b for any single given $b > 1$ that does not depend on n . Remark: That is why in runtime analyses you often find \log written without the base. Why is it important that $b > 1$? Describe what happens if $b = 1$ or $b < 1$. Why is it important that b does not depend on n ? Give an example where b does depend on n and one of the propositions above does not hold anymore (you can choose which one you want to use).

Exercise 3 (7 points)

Order the following functions f_1, f_2, f_3, f_4, f_5 according to their runtime complexity such that $f_i = \mathcal{O}(f_{i+1})$ holds for $i = 1, 2, 3, 4$. Also determine for which i $f_i = \Theta(f_{i+1})$ holds and for which not. Justify your decisions, particularly for the i cases where $f_i = \Theta(f_{i+1})$ does not hold. You can use the limit definition of \mathcal{O} and Θ for all your justifications.

$$\begin{aligned} & n^2 \\ & n \log_{10} n \\ & n^2 \log_2(n^2) \\ & \sqrt{n} \\ & n \log_2(n^2) \end{aligned}$$

Commit

Commit your solutions in PDF format within a new subdirectory **uebungsblatt_03** into your SVN repository. Also commit a text file *erfahrungen.txt*. Therein describe your experience with the exercise sheet in a few sentences: was it manageable for you? how much time did it take you? Did you have problems with specific parts?