APS1022: Final Project – Computational Project

University of Toronto

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Contents

Initial Problem Description:

We will be building 4 portfolios using the Risk Adjusted Return Mean-Variance Optimization Model, the Robust Risk Adjusted Return Mean-Variance Optimization Model, Risk Parity Optimization Model, and the Market weights for the following Assets:

F (Ford Motor Co.); CAT (Catepillar Inc.); DIS; MCD; KO; PEP; WMT;
C; WFC; JPM; AAPL; IBM; PFE; JNJ; XOM; MRO; ED; T; VZ; NEM

In order to do so, historical Adjusted Closing Prices were collected using: ca.finance.yahoo.com from January-2008 until June-2011. The collected data was from **December 2007 until June 2011** to cover this period.

The major portfolios' quantities (Portfolio Return, Portfolio Variance and Standard Deviation, and The Sharp Ratio) will be compared with different methods.

Table 1 shows the equations used to estimate our portfolios' parameters:

Table 1: Equations

$r_{it} = \dfrac{Last\ Day\ of\ Month_t}{Last\ Day\ of\ Month_{t-1}} - 1$	r_{it} = Return of asset i for month t
$\mu_i = \dfrac{(\sum_{t=1}^{T} r_{it})}{T}$	μ_i = Arithmetic average of asset i, T = Total number of months
$\sigma_{ij} = \dfrac{\left(\sum_{t=1}^{T}(r_{it} - \bar{r}_i)(r_{jt} - \bar{r}_j)\right)}{T}$	σ_{ij} = Covariance between assets i and j

The Portfolio Variance = $\sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j \sigma_{ij}$, where x_i is the weight of asset i from the investment capital, x_j is the weight of asset j from the investment capital, and σ_{ij} = Covariance between assets i and j. It can be written in the form of matrices as x^TQx, where Q is a symmetric matrix representing the covariances.

We will also be using Matlab to code an algorithm which will be producing out portfolios:

- Quadprog is a matlab function which would solve quadratic programs in the following form: $\min \frac{1}{2} w^T Q w + c^T w$, Subjected to: $Aw \leq b$, $A_{eq}\, w = b_{eq}$, $lb \leq w \leq ub$
- Fmincon is a matlab function that would minimize functions of various forms and degrees. We will be specifying the objective function and the constraints and feeding them to the fmincon to obtain results.

To plot the graphs, the resulting data will be taken into a python code to make use of the different options provided there.

1. Risk Adjusted Return Mean-Variance Optimization Model:

Table 2 shows the optimization problem at hand and how it will be input in matlab's quadprog:

Table 2: Risk Adjusted MVO Model

Max $\mu^T x - \lambda x^T Q x$ Subjected to $e^T x = 1$	OR	Min $\frac{1}{2}\lambda x^T Q x - \frac{1}{2}\mu^T x$ Subjected to $e^T x = 1$

Where $\mu \in R^n$ is the vector of expected returns , $Q \in R^{n*n}$ is the covariance matrix, and $e \in R^n$ is a vector of n ones. Short selling is allowed. $\lambda > 0$ is a risk aversion parameter.

λ would be the quantity that Idzorek suggests = $(E(r_{mkt}) - r_f)/ \sigma^2_{mkt}$
To get the market expected return and covariance matrix, we assume the market is the 20 asset stocks:

$$E(r_{mkt}) = \sum_{i=1}^{n} w_i \mu_i$$

$$\sigma^2_{mkt} = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij} = w^T Q w$$

Where w_i is the market weights = Asset i Market Cap / Total Market Cap

Since obtaining the Outstanding shares for each asset was unavailable/difficult, we obtained the end of year Market Capitalization for 2010 and 2011 from https://companiesmarketcap.com/ , Asset i Market Cap is assumed to be the average of these two years.

The remaining parameter to calculate the risk aversion parameter λ is the risk-free rate. From the Federal Reserve Bank Website a short-term treasury bond (3-Month Treasury Bill) in June 2011 is considered to get r_f

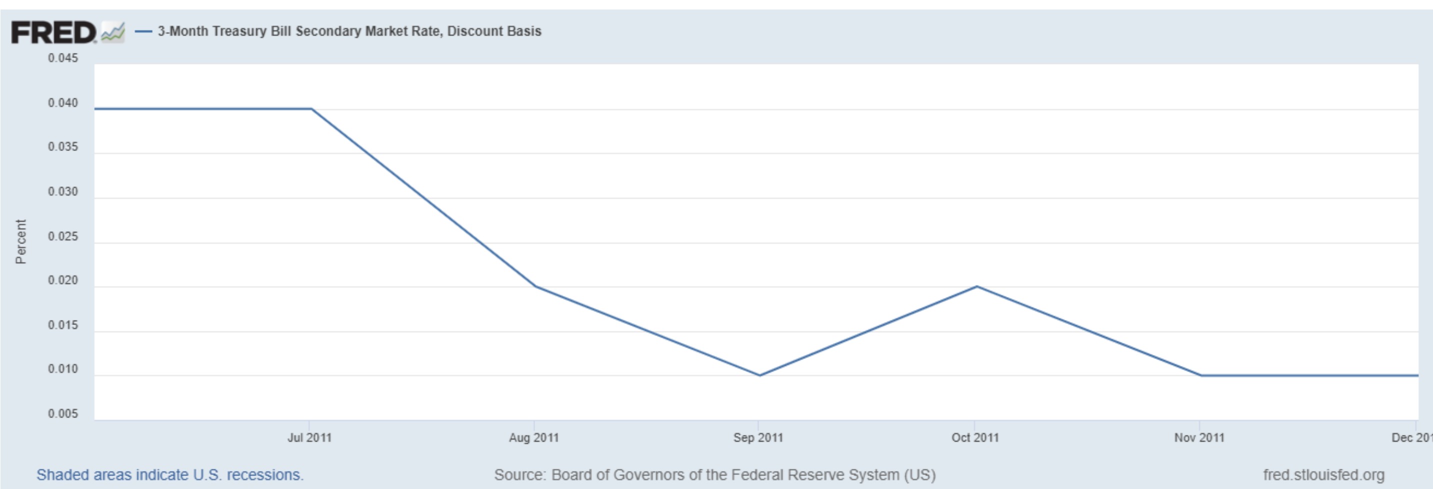

Figure 1: Risk Free Asset – June-November 2011

End of Month:	Risk Free Return
June - 2011	4% (0.33% monthly)
July - 2011	2% (0.16% monthly)
August - 2011	1% (0.083% monthly)
September - 2011	2% (0.16% monthly)
October - 2011	1% (0.083% monthly)
November - 2011	1% (0.083% monthly)

2. Robust Risk Adjusted Return Mean-Variance Optimization Model:

Table 3 shows the optimization problem at hand and how it will be input in matlab's quadprog:

Table 3: Robust Risk Adjusted MVO Model

Min $\frac{1}{2}\lambda x^T Qx - \frac{1}{2}(\mu^T x - \delta^T y)$ Subjected to $e^T x = 1$ $y - x \geq 0$ $y + x \geq 0$	OR	Min $\frac{1}{2}\lambda x^T Qx - \frac{1}{2}(\mu^T x - \delta^T x)$ Subjected to $e^T x = 1$ ${A_1}^T x \leq 0$ ${A_2}^T x \leq 0$

Where: $Q \in \mathrm{R}^{2n*2n}$ is the Covariance matrix surrounded by zeros: e.g. $\begin{bmatrix} \sigma & \sigma & 0 & 0 \\ \sigma & \sigma & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$x \in \mathrm{R}^{2n}$ is a vector consisting of both x and y: e.g. $\begin{bmatrix} x \\ x \\ y \\ y \end{bmatrix}$

$e \in \mathrm{R}^n$ is a vector of n ones

$A_1 \in \mathrm{R}^{n*2n}$ is a non-equality matrix consisting of zeros, ones and negative ones: e.g. $\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}$

$A_2 \in \mathrm{R}^{n*2n}$ is a non-equality matrix consisting of zeros and negative ones: e.g. $\begin{bmatrix} -1 & 0 & -1 & 0 \\ 0 & -1 & 0 & -1 \end{bmatrix}$

(These changes are to make it compatible with Matlab Quadprog Function)

We will be using the box uncertainty, subtracting it from the expected return to penalize it into considering possible returns. The following parameters will be calculated:

$\delta = \varepsilon * \Theta^{1/2}$ | $\Theta^{1/2} = \sigma / T^{1/2}$ | ε 90% = 1.645 | ε 95% = 1.96

3. Risk Parity Optimization Model:

We will be using (fmincon) function in matlab, and we will be processing the following alternative from the Risk Parity Optimization (*Table 4*) since it is computationally easier:

Table 4: Risk Parity Optimization Model

Min $\sum_{i=1}^{n}(x^T R_i x - \theta)^2$ Subjected to $e^T x = 1$ $x \geq 0$

Where $R_i \in \mathrm{R}^n$ is a sparse symmetric matrix of Q for x_i

To check the risk parity results, $x^T R_i x - \theta$ was calculated for each R_i, the values were close to zero and almost identical.

4. Market Weights

Based on each asset's market capitalization, weights are calculated (same method mentioned in calculating the risk aversion parameter) and the expected return, variance and sharpe ratio are generated.

Part A: Comparing Portfolios Using Historical Data Since Dec-2007

- Numerical data for the quantities here are provided in the appendix.

Resulting Expected Returns (Arithmetic Average of 42 months):

Table 5: 42 Months Arithmetic Average Returns

F	CAT	DIS	MCD	KO	PEP	WMT	C	WFC	JPM
0.04534	0.02065	0.00878	0.01228	0.00636	0.00211	0.00568	-0.0173	0.01093	0.00611
AAPL	**IBM**	**PFE**	**JNJ**	**XOM**	**MRO**	**ED**	**T**	**VZ**	**NEM**
0.01957	0.01451	0.00411	0.00386	7.88E-06	0.00392	0.00762	0.00159	0.00431	0.01009

Table 6: 42 Months Geometric Average Returns

F	CAT	DIS	MCD	KO	PEP	WMT	C	WFC	JPM
0.01723	0.01166	0.00543	0.01124	0.004781	0.00066	0.00446	-0.0442	0.00016	-7.15E-06
AAPL	**IBM**	**PFE**	**JNJ**	**XOM**	**MRO**	**ED**	**T**	**VZ**	**NEM**
0.01264	0.01261	0.00188	0.00265	-1.45E-03	-0.0011	0.0066	-0.0003	0.00255	0.00319

We can notice from the Arithmetic Average Returns, most of the assets' returns were positive and higher than the risk-free return (Except for Citigroup Inc., which almost collapsed and was declared insolvent during the 2008 financial crisis), while in the geometric returns many were negative, and a significant number is lower than the risk-free return.

Even though the geometric average is usually a more suitable method to get the expected returns, taking into consideration the chosen period had the year 2008 included when the global financial crisis occurred, the arithmetic average will be used.

$$\lambda = 1.383$$

Table 7: Risk Parity Check Values

F	CAT	DIS	MCD	KO	PEP	WMT	C	WFC	JPM
0.00005	0.00002	0.00003	-0.00003	0.00000	-0.00001	-0.00005	0.00001	0.00002	0.00003
AAPL	IBM	PFE	JNJ	XOM	MRO	ED	T	VZ	NEM
0.00004	-0.00001	0.00000	-0.00001	-0.00003	0.00004	-0.00006	-0.00001	0.00000	-0.00001

The remaining difference of each asset's variance contribution is almost 0, assuring the risk parity function effectiveness.

Main Check Code:

```
RP_CHECK = zeros(1, n);
for i = 1:n
    RP_CHECK(i) = Risk_Parity_X(1:n)'* R{i} * Risk_Parity_X(1:n) - Risk_Parity_X(n+1);
end
```

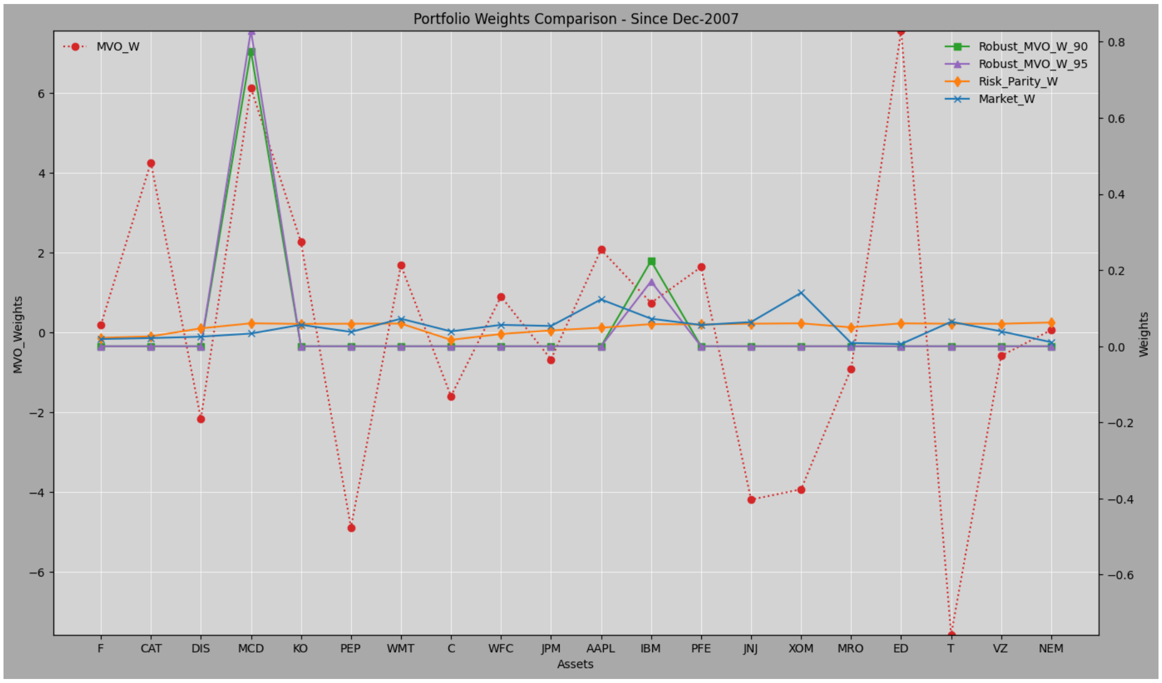

Figure 2: Portfolios Weights Comparison - Since Dec-2007

From (*figure 2*), we notice the weights resulting from the risk adjusted returns MVO had a larger scale than the rest (left vertical axes to represent them); since it was not panelized with robustness, and short selling was allowed, these values went as high or as low as they could to maximize the return.

Additionally, with both robust MVO optimization, most of the assets' weight was driven to zero from the initial investment; since we took high protection values, the results were more conservative to reduce variance. We also notice how they both behave the same in different magnitudes.

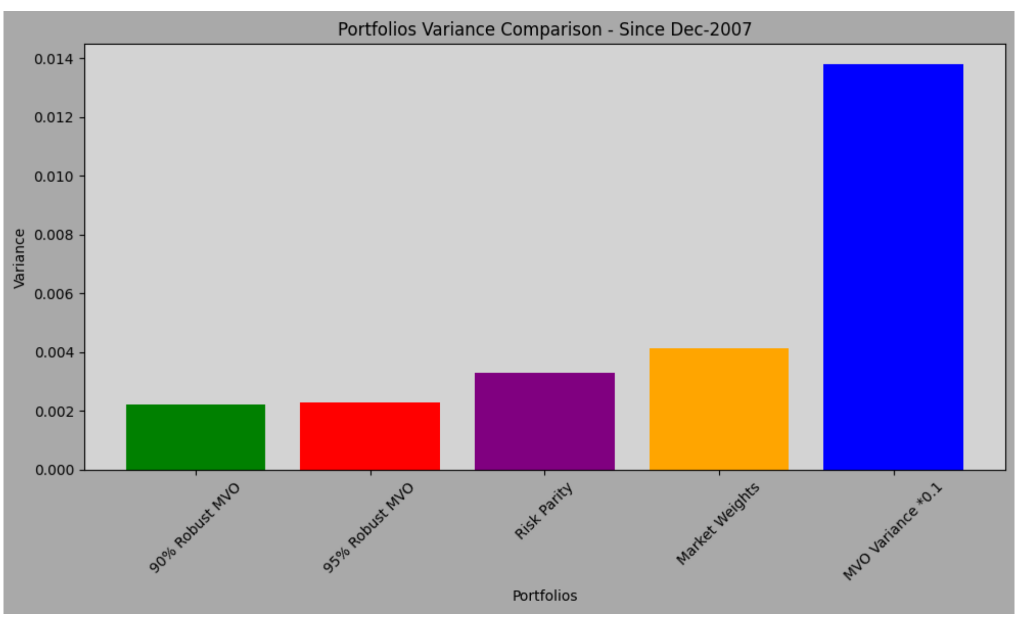

Figure 3: Portfolios Variance Comparison - Since Dec-2007

After seeing the high weights values for the MVO portfolio, it was expected to have a high variance (around 0.14, standard deviation 0.37). The rest of the portfolios had low and close variances; The market has an acceptable variance of 0.004, and the least of which is for the Robust MVO with 90% confidence level.

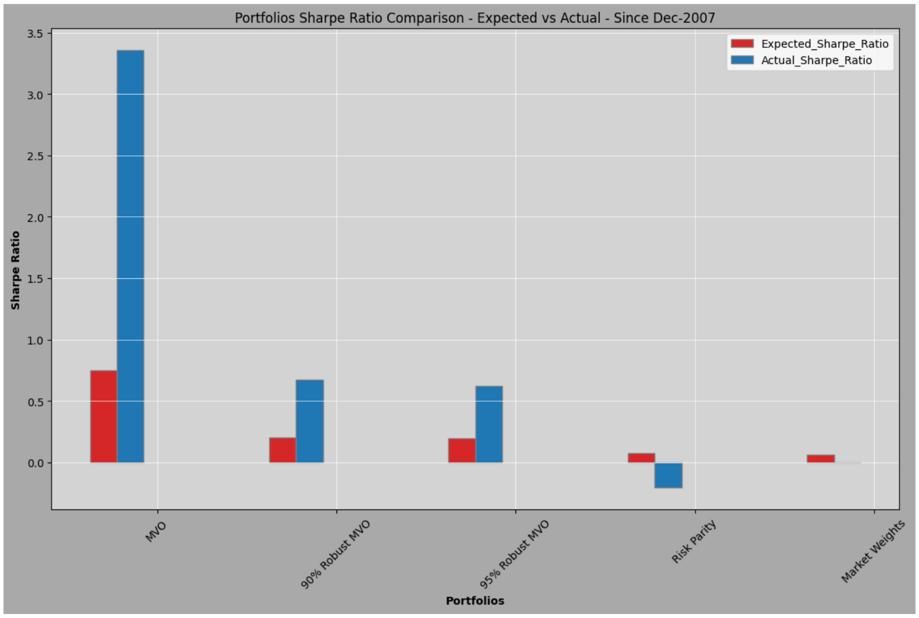

Figure 4: Portfolios Sharpe Ratio - Since Dec-2007

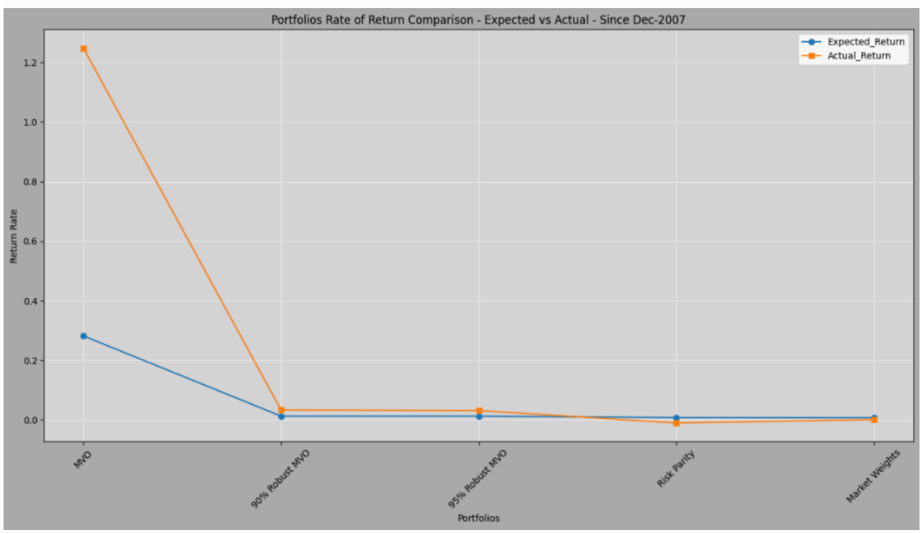

Figure 5: Portfolios Return Rate - Since Dec-2007

Figures 4 and 5 demonstrate the difference between the portfolios expected return and Sharpe Ratio. Both MVO and Robust MVO performed better than the expectation, while Risk Parity and Market Weights performed better similar/slightly worse.

In regards of the MVO portfolio, it was more daring with high risk, which resulted in favor of the portfolio, and it performed better than expected. However, the risk parity allowed each asset variance contribution to be equal, since this market only has 20 asset (not very diversified) and the 2008 data was involved, this was not the best approach and it resulted in a negative Sharpe ratio, indicating the portfolio performed worse than the risk-free rate!

The robust MVO portfolios performed the closest in ratio to their expectations (for the 90% and 95% 3.3% / 1.27% and 3.1% / 1.26% respectively), which was expected since they had a low variance. The market weights portfolio had a Sharpe ratio of almost zero, indicating it's performing as good as the risk-free rate.

In general, in this part the best performing portfolio in regards of returns was the MVO, while the most stable were the robust MVO.

Part B: Rolling 6 Months Horizon Test (21 Months)

- Numerical data for the quantities here are provided in the appendix.

Estimating the expected returns using 6 months' worth of historical data resulted in a non-PSD covariance matrix; thus, our objective function is not convex and reaching an optimal value would be hard.
The following code was used to check if the covariance matrix is PSD or not:

```
% Compute the eigenvalues of the matrix
eigenvalues = eig(Assets_covariance);

% Check if all eigenvalues are non-negative
if all(eigenvalues >= 0)
    disp('The matrix is PSD.');
else
    disp('The matrix is not PSD.');
end
```

After gradually testing the number of months by increasing it each time by one, the smallest number of months that would result in a PSD covariance matrix was 21, and this is what we will use in this part:

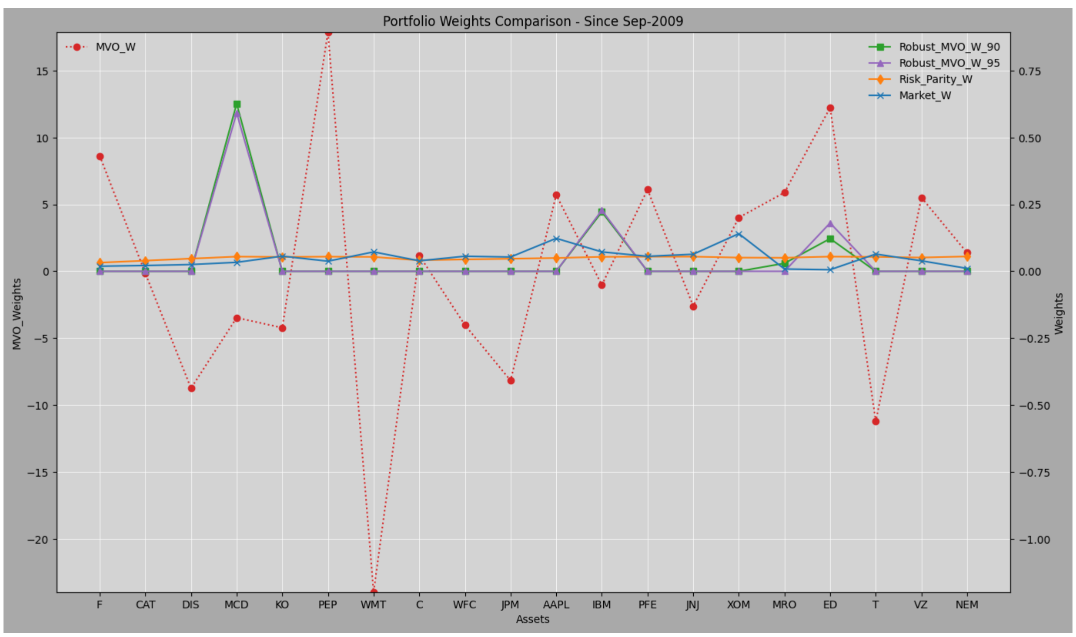

Figure 6: Portfolios Weights Comparison - Since Sep-2009

Similar to the first part, *Figure 6* shows MVO weights were largely scaled (read from left vertical axis), and the Robust MVO dragged most of the asset's weights to zero. Market weights do not change since they are based on June-2011 market capitalizations only.

Expectations are not uniform across the 20 stocks between the portfolios. There was no agreement whether a certain stock would rise or fall (except maybe for the two robust MVO which only differ with the delta value).

This shows the high volatility between the stocks throughout the past months and instability of the economy.

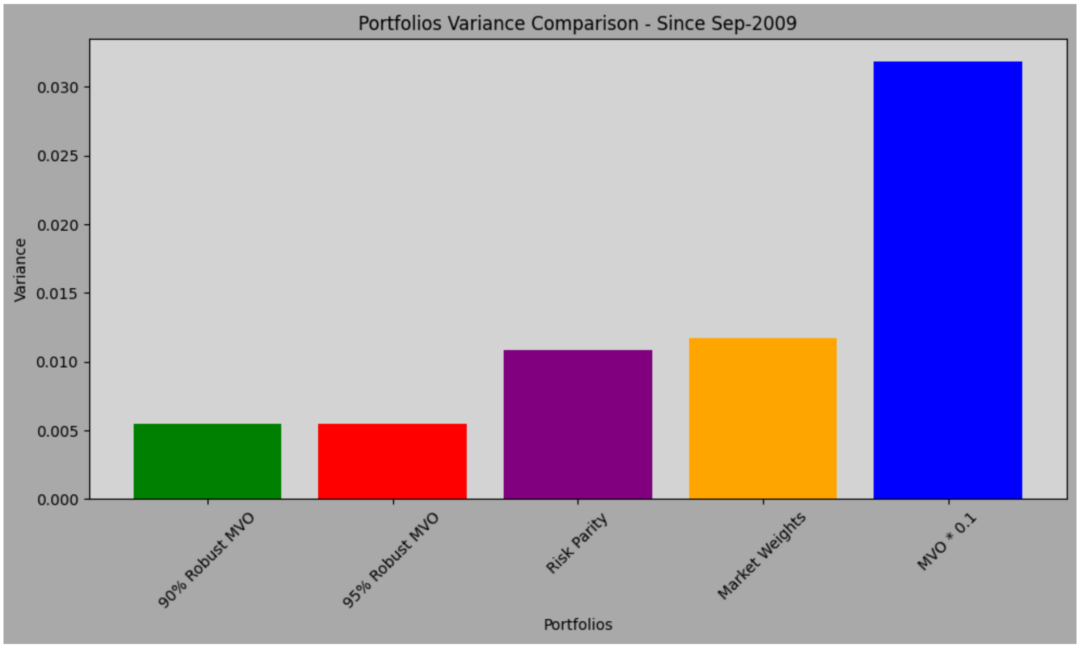

Figure 7: Portfolios Variance Comparison - Since Sep-2009

In *figure 7*, we can clearly see an increase in variance. MVO has a variance of 0.3 (std = 0.56) which is very volatile and will be risky. But the portfolios behaved in the same manner and have the same relation between each other.

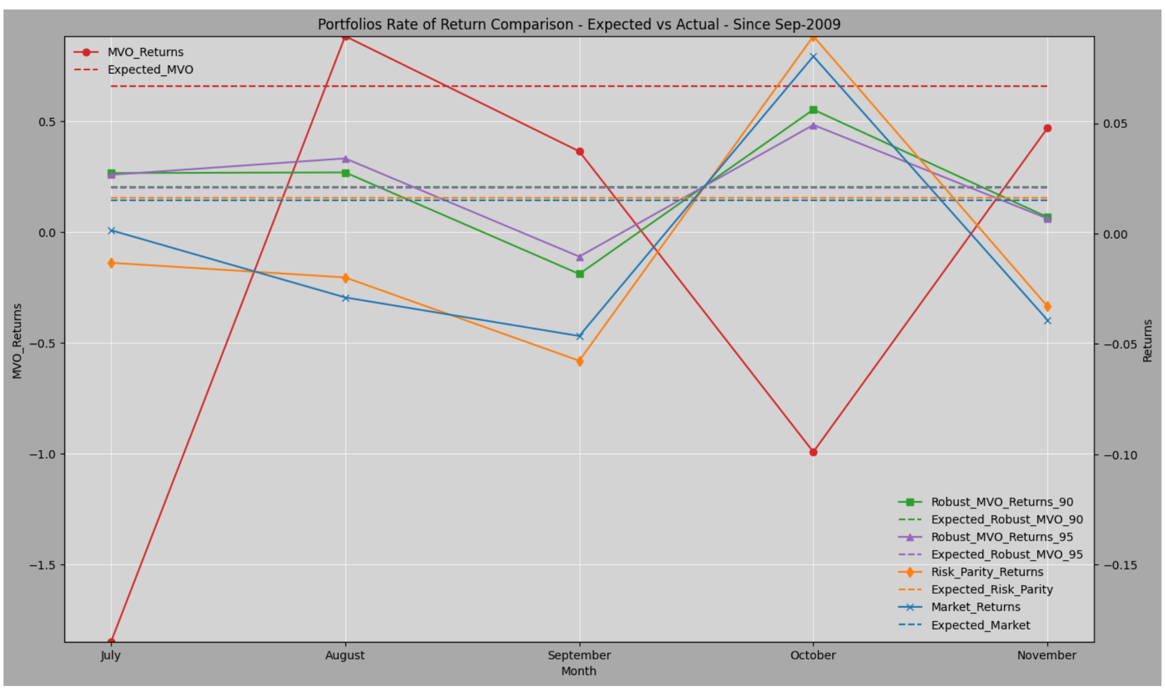

Figure 8: Portfolios Return Rate - Since Sep-2009

The dashed lines in *Figure 8* are the expected values of each portfolio. With an expectation of 65%, the MVO behaved in an extremely volatile and unforeseen manner. While both Robust MVOs stayed around their expected return of 2%. The Risk Parity and Market Weights were somewhere in between, showing moderation.

Starting with a major loss, the volatility of the MVO portfolio took the best of its performance and caused a -185% loss, making the investment worth -85% of its original value. Since short selling was allowed the investor ended up owing money.

The rest of the portfolios had the same pattern across the 5 months. It seems like the economy was bad in September, but it got back up again in October.

Robust MVO had the most linear/stable returns, but that also meant lower return. With this method, robust MVO were the best performing and the most stable as well.

Part C: Portfolios Cumulative wealth

- Numerical data for the quantities here are provided in the appendix.

To calculate the cumulative wealth across the months from June till November, the following equation is used:

$$Month's\ CW = \left(\prod_{t=1}^{T} (1 + \ Portfolio\ Retutn_t)\right)$$

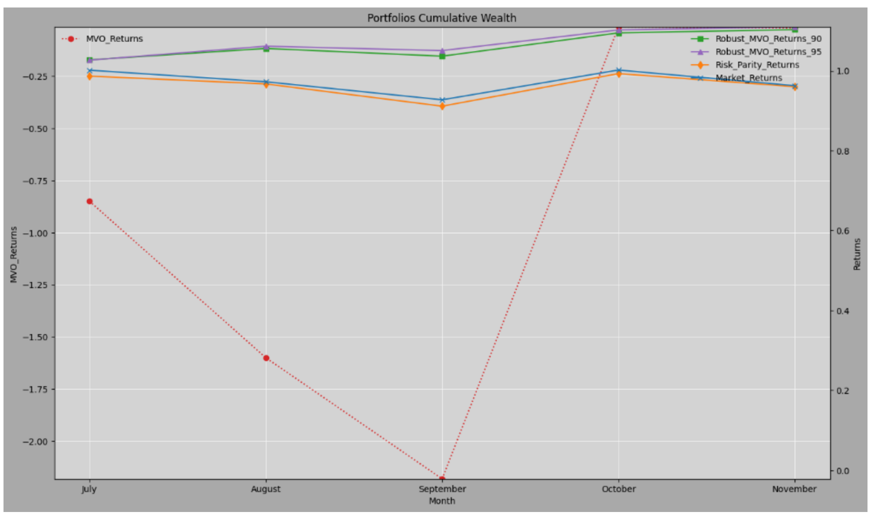

Figure 9: Portfolios Cumulative Wealth

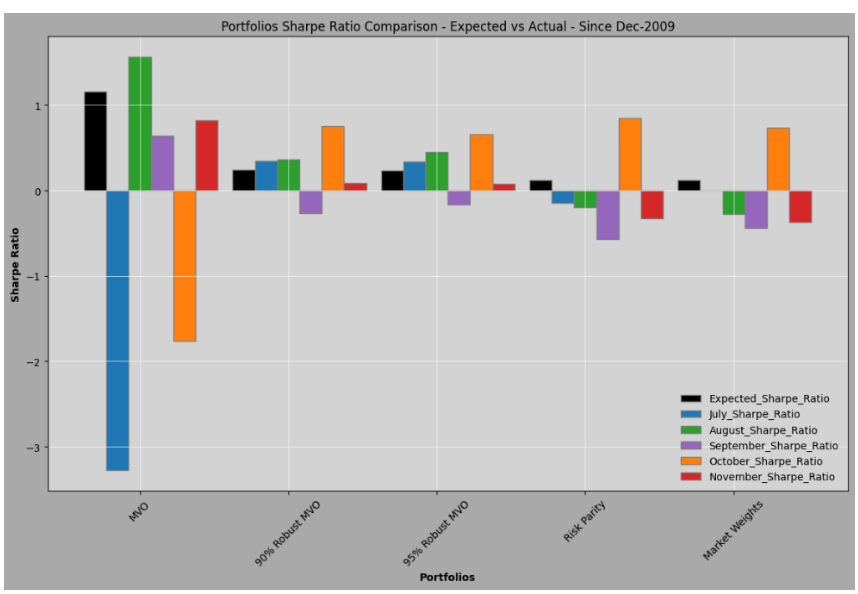

Figure 10: Portfolios Sharpe Ratio - Since Sep-2009

Part D: Discussion

According to the scaling rule, the covariance can be well estimated with larger number of historical datapoints, reducing the error. This is shown in how close the values were to their expected returns in *figure 5* – part 1, while they differed more in part 2. It's also shown with the lower variance for the 42 months based estimations than the 21 months.

The high volatility of the Risk Adjusted Return MVO optimization compared with other values is worrying. The fact it performed greatly when 42 months were involved can be considered a shot in the dark, especially with a high dependency over the assets which performed well under the previous economic recession. Once the data was taken from September 2009, it resulted in a big loss to the investor.

It was interesting to see how MVO expected Pepsico's stocks to skyrocket (suggesting buying 17.8 times the initial investment value worth of stock) and that the Walmart stocks would go down (suggests to short sell 23.9 times the initial investment value worth of stock) when in fact Pepsico's stocks held a 9.07% loss in July alone.

While plotting the cumulative wealth, since the MVO portfolio-based investment was in debt since July, it is assumed the investment kept on going, until the losses went down to -2% instead of the initial -85%

The way the portfolios behaved in *figure 8*, suggesting the economy went down in September and back up in October checks out with the actual return values for the assets (*Tables 10 & 11*), in September most of the assets had a negative return, while in October they all had positive returns.

In *Figure 9* we see both Risk Parity and Market Portfolios had almost the same cumulative wealth, around 0.96 times the initial investment, resulting in a slight loss. They were also behaving the same throughout the studied months, which suggests the market is behaving in a risk parity sort of way: *Tables 22 and 23*.

And *Figure 10* demonstrates the Sharpe Ratio of each portfolio across each month, all portfolios had positive and negative values across the studied months, which shows no model would always outperform the risk-free rate. MVO usually gave the highest, either in the positive or negative direction, between -3.3 and 1.6, this is because of it's more daring strategies without penalties or robustness.
The Risk Parity portfolio underperformed most of the months. The assets we have on hand had a generally high volatility; when each one is given the same opportunity to contribute to the portfolio variance, the model had a high chance to underperform.
Both Robust MVO were stable, scoring a maximum of 0.75 Sharpe Ratio and a minimum -0.27. their stability and protectiveness of the return resulted in a good outcome. It is also worth mentioning they performed much better than expected; it is safe to assume they were overly protective, and the confidence level can be taken down a notch and be riskier.

In conclusion, Robust Risk Adjusted Return Mean Variance Portfolio had the best outcome, it had a 1.1 cumulative return by the end of November, a 10% increase which is impressive. While MVO had a -102% lost driving the whole portfolio value under zero.
Risk Parity and Market weights had similar

Instructions To Run The Code:

Open each folder location in Matlab, Keeping the documents in the folder so they can be read, and simply hit run.

Part A and Part B are in 2 separate folders and code files for easier variable access.

Graphs are made using python for better visualization. Variable values were taken into a Jupyter Notebook file and plotted there. The Jupyter Notebook is provided within the submitted file.

Code variables name:

Risk Adjusted Return

Weights:	`MVO_X`
Expected Return:	`MVO_r`
Variance:	`MVO_VAR`
Sharpe Ratio:	`MVO_SR`
Actual July Return	`MVO_July_r`

Robust Risk Adjusted Return with 90% Confidence

Weights:	`MVO_R_90_X`
Expected Return:	`MVO_R_90_r`
Variance:	`MVO_R_90_VAR`
Sharpe Ratio:	`MVO_R_90_SR`
Actual July Return	`MVO_R_90_July_r`

Robust Risk Adjusted Return with 95% Confidence

Weights:	`MVO_R_95_X`
Expected Return:	`MVO_R_95_r`
Variance:	`MVO_R_95_VAR`
Sharpe Ratio:	`MVO_R_95_SR`
Actual July Return	`MVO_R_95_July_r`

Risk Parity:

Weights:	`Risk_Parity_X`
Expected Return:	`Risk_Parity_r`
Variance:	`Risk_Parity_VAR`
Sharpe Ratio:	`Risk_Parity_SR`
Actual July Return	`Risk_Parity_July_r`

Market Weights:

Weights:	`Market_Weights`
Expected Return:	`Market_Weight_r`
Variance:	`Market_Weight_VAR`
Sharpe Ratio:	`Market_Weight_SR`
Actual July Return	`Market_Weight_July_r`

Actual Months Return (Represented with July):

Assets Returns in July:	`July_Returns`
MVO July Return:	`MVO_July_r`
MVO July Sharpe Ratio:	`MVO_July_SR`
90% Robust MVO July Return:	`MVO_R_90_July_r`
90% Robust MVO July Sharpe Ratio:	`MVO_R_90_July_SR`
95% Robust MVO July Return:	`MVO_R_95_July_r`
95% Robust MVO July Sharpe Ratio:	`MVO_R_95_July_SR`
Risk Parity July Return:	`Risk_Parity_July_r`
Risk Parity July Sharpe Ratio:	`Risk_Parity_July_SR`
Market Weights July Return:	`Market_Weight_July_r`
Market Weights July Sharpe Ratio:	`Market_Weight_July_SR`

Appendix

The Four Models Used:

1. Risk Adjusted Return Mean Variance Optimization Model
2. Robust Risk Adjusted Return Mean Variance Optimization Model with 90% Confidence
3. Robust Risk Adjusted Return Mean Variance Optimization Model with 95% Confidence
4. Risk Parity Optimization Model
5. Market Weights Portfolio

Actual Months Returns:

Table 8: July Actual Returns

F	CAT	DIS	MCD	KO	PEP	WMT	C	WFC	JPM
-0.11458	-0.06813	-0.01076	0.02562	0.01070	-0.09073	-0.00809	-0.07901	-0.00428	-0.00590
AAPL	**IBM**	**PFE**	**JNJ**	**XOM**	**MRO**	**ED**	**T**	**VZ**	**NEM**
0.16329	0.06004	-0.06553	-0.02601	-0.01954	-0.01444	-0.01202	-0.05137	-0.03972	0.03039

Table 9: August Actual Returns

F	CAT	DIS	MCD	KO	PEP	WMT	C	WFC	JPM
-0.08927	-0.07885	-0.11807	0.05252	0.03588	0.01419	0.01641	-0.19014	-0.06171	-0.07145
AAPL	**IBM**	**PFE**	**JNJ**	**XOM**	**MRO**	**ED**	**T**	**VZ**	**NEM**
-0.01447	-0.05054	-0.00303	0.02464	-0.06619	-0.12585	0.08070	-0.02666	0.02494	0.12606

Table 10: September Actual Returns

F	CAT	DIS	MCD	KO	PEP	WMT	C	WFC	JPM
-0.13040	-0.18857	-0.11450	-0.02865	-0.03448	-0.03927	-0.02425	-0.17488	-0.07586	-0.19808
AAPL	**IBM**	**PFE**	**JNJ**	**XOM**	**MRO**	**ED**	**T**	**VZ**	**NEM**
-0.00912	0.01722	-0.06849	-0.03207	-0.01878	-0.19837	0.01441	0.00140	0.01742	0.00997

Table 11: October Actual Returns

F	CAT	DIS	MCD	KO	PEP	WMT	C	WFC	JPM
0.20786	0.28633	0.15650	0.05728	0.01125	0.01696	0.09287	0.23302	0.07421	0.16421
AAPL	**IBM**	**PFE**	**JNJ**	**XOM**	**MRO**	**ED**	**T**	**VZ**	**NEM**
0.06152	0.05581	0.08937	0.01099	0.07518	0.20621	0.01491	0.04879	0.01889	0.06164

Table 12: November Actual Returns

F	CAT	DIS	MCD	KO	PEP	WMT	C	WFC	JPM
-0.14469	-0.04150	-0.02523	0.01414	-0.02414	0.01128	0.02556	-0.20075	-0.06610	-0.17837
AAPL	**IBM**	**PFE**	**JNJ**	**XOM**	**MRO**	**ED**	**T**	**VZ**	**NEM**
-0.07802	-0.01605	0.01741	-0.01596	-0.00899	0.02425	0.01191	-0.04265	-0.00946	-0.02304

Part A Optimized Weights:

Table 13: A - MVO Weights

F	CAT	DIS	MCD	KO	PEP	WMT	C	WFC	JPM
0.18561	4.26206	-2.1647	6.13176	2.26644	-4.8888	1.68438	-1.598	0.90219	-0.6754
AAPL	**IBM**	**PFE**	**JNJ**	**XOM**	**MRO**	**ED**	**T**	**VZ**	**NEM**
2.07709	0.7419	1.63966	-4.1904	-3.9318	-0.9102	7.56832	-7.5776	-0.585	0.0623

Table 14: A - 90% Robust MVO Weights

F	CAT	DIS	MCD	KO	PEP	WMT	C	WFC	JPM
1.01E-09	3.93E-10	1.60E-10	0.77563	4.10E-10	-3.30E-13	1.88E-09	2.83E-12	1.17E-10	9.10E-11
AAPL	**IBM**	**PFE**	**JNJ**	**XOM**	**MRO**	**ED**	**T**	**VZ**	**NEM**
5.23E-10	0.22437	7.26E-11	1.09E-10	-1.62E-11	-7.84E-11	1.10E-07	2.51E-10	6.27E-10	-2.67E-11

Table 15: A - 95% Robust MVO Weights

F	CAT	DIS	MCD	KO	PEP	WMT	C	WFC	JPM
1.50E-09	1.89E-10	1.97E-11	0.82937	6.86E-10	7.91E-11	1.03E-08	-2.46E-11	4.87E-11	1.37E-10
AAPL	**IBM**	**PFE**	**JNJ**	**XOM**	**MRO**	**ED**	**T**	**VZ**	**NEM**
1.30E-09	0.17062	8.58E-11	3.61E-10	1.25E-10	-1.64E-10	1.24E-06	7.97E-10	2.86E-09	-1.39E-10

Table 16: A - Risk Parity Weights

F	CAT	DIS	MCD	KO	PEP	WMT	C	WFC	JPM
0.02228	0.02614	0.04719	0.0604	0.0588141	0.059363	0.05968	0.016895	0.03232	0.041739
AAPL	**IBM**	**PFE**	**JNJ**	**XOM**	**MRO**	**ED**	**T**	**VZ**	**NEM**
0.04874	0.05824	0.05747	0.05932	0.0603088	0.049774	0.06028	0.059265	0.05918	0.062606

Table 17: A - Market Weights

F	CAT	DIS	MCD	KO	PEP	WMT	C	WFC	JPM
0.01905	0.02162	0.02527	0.03351	0.0565397	0.0378	0.07244	0.03914	0.05631	0.053355
AAPL	**IBM**	**PFE**	**JNJ**	**XOM**	**MRO**	**ED**	**T**	**VZ**	**NEM**
0.12318	0.07248	0.05599	0.06371	0.1406564	0.00856	0.00595	0.06443	0.03921	0.010806

Part A Major Portfolios Quantities:

Table 18: A - Major Portfolios Quantities

	MVO	90% Robust MVO	95% Robust MVO	Risk Parity	Market Weights
Variance	0.13798	0.00221	0.00231	0.00329	0.00412
STD	0.37146	0.04700	0.04805	0.05740	0.06420
Expected Sharpe Ratio	0.75011	0.20105	0.19414	0.07710	0.06420
Actual Sharpe Ratio	3.35503	0.67398	0.62064	-0.20712	-0.00214
Expected Return	0.28197	0.01278	0.01266	0.00776	0.00745
Actual Return	1.24793	0.03334	0.03149	-0.01022	0.00153

Part B Optimized Weights:

Table 19: B - MVO Weights

F	CAT	DIS	MCD	KO	PEP	WMT	C	WFC	JPM
8.63243	-0.15289	-8.73963	-3.46649	-4.22265	17.87469	-23.96728	1.19221	-4.01618	-8.16391
AAPL	IBM	PFE	JNJ	XOM	MRO	ED	T	VZ	NEM
5.71102	-1.03073	6.11939	-2.59815	4.01328	5.88895	12.23636	-11.20958	5.51732	1.38185

Table 20: B - 90% Robust MVO Weights

F	CAT	DIS	MCD	KO	PEP	WMT	C	WFC	JPM
-1.08E-12	3.65E-11	-4.76E-13	0.62614	9.71E-13	6.12E-13	-5.13E-13	-3.64E-13	2.54E-12	2.02E-14
AAPL	IBM	PFE	JNJ	XOM	MRO	ED	T	VZ	NEM
2.52E-10	2.22E-01	-2.24E-12	2.12E-13	2.35E-14	0.02990	0.12207	1.64E-12	2.97E-12	1.44E-12

Table 21: B - 95% Robust MVO Weights

F	CAT	DIS	MCD	KO	PEP	WMT	C	WFC	JPM
3.91E-12	8.73E-11	-5.40E-12	0.59140	-8.33E-13	3.13E-12	1.61E-13	1.58E-11	-6.87E-12	7.74E-12
AAPL	IBM	PFE	JNJ	XOM	MRO	ED	T	VZ	NEM
1.42E-10	0.22803	4.70E-12	-9.09E-13	7.90E-12	2.62E-08	0.18057	1.50E-12	7.10E-12	1.51E-12

Table 22: B - Risk Parity Weights

F	CAT	DIS	MCD	KO	PEP	WMT	C	WFC	JPM
0.03240	0.03981	0.04741	0.05485	0.05367	0.05464	0.05375	0.04125	0.04457	0.04688
AAPL	IBM	PFE	JNJ	XOM	MRO	ED	T	VZ	NEM
0.04981	0.05401	0.05473	0.05473	0.05122	0.05064	0.05487	0.05387	0.05137	0.05552

Table 23: B - Market Weights

F	CAT	DIS	MCD	KO	PEP	WMT	C	WFC	JPM
0.019046	0.02162	0.025267	0.033509	0.05654	0.0378	0.072442	0.039141	0.056311	0.053355
AAPL	IBM	PFE	JNJ	XOM	MRO	ED	T	VZ	NEM
0.123177	0.07248	0.055988	0.063712	0.140656	0.00856	0.005954	0.06443	0.039208	0.010806

Part B Major Portfolios Quantities:

	MVO	90% Robust MVO	95% Robust MVO	Risk Parity	Market Weights
Variance	0.31887	0.00547	0.00548	0.01084	0.01171
STD	0.56468	0.07396	0.07400	0.10412	0.10820
Expected Sharpe Ratio	1.15741	0.24090	0.23405	0.12385	0.12385
July Sharpe Ratio	-3.27892	0.34881	0.33788	-0.14297	-0.00127
August Sharpe Ratio	1.56182	0.36402	0.44960	-0.19882	-0.27511
September Sharpe Ratio	0.63953	-0.26983	-0.16324	-0.56952	-0.44437
October Sharpe Ratio	-1.76117	0.74909	0.65484	0.85194	0.73528
November Sharpe Ratio	0.82611	0.08977	0.08136	-0.32464	-0.36968
Expected Return	0.65690	0.02115	0.02065	0.01623	0.01504
July Return	-1.84988	0.02746	0.02667	-0.01322	0.00153
August Return	0.88277	0.02775	0.03410	-0.01987	-0.02894
September Return	0.36280	-0.01829	-0.01041	-0.05763	-0.04642
October Return	-0.99367	0.05623	0.04929	0.08953	0.08039
November Return	0.46732	0.00747	0.00685	-0.03297	-0.03917

Part C Cumulative Wealth:

Table 24: C - Cumulative Wealth

	MVO	90% Robust MVO	95% Robust MVO	Risk Parity	Market Weights
July Cumulative Wealth	-0.84988	1.02746	1.02667	0.98678	1.00153
August Cumulative Wealth	-1.60013	1.05598	1.06168	0.96718	0.97255
September Cumulative Wealth	-2.18065	1.03667	1.05063	0.91144	0.92741
October Cumulative Wealth	-0.01381	1.09496	1.10242	0.99304	1.00197
November Cumulative Wealth	-0.02027	1.10314	1.10997	0.96031	0.96272

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