hw5-6

October 31, 2023

1 Lab: SVD and PCA Analysis – Layth Aljorani

```
[2]: import numpy as np
```

1.1 1. SVD and PCA basics

1.1.1 setup

SVD factorization leads to $A = U * sigma * V^T$.

For $i = 1, \ldots, N$, the i-th column of V is a unit vector that defines the i-th principle component.

```
[5]: array([[17, 24, 1, 8, 15], [23, 5, 7, 14, 16], [4, 6, 13, 20, 22], [10, 12, 19, 21, 3], [11, 18, 25, 2, 9]])
```

1.1.2 Exercise 1:

Use numpy to perform singular value decomposition (SVD) and principal component analysis (PCA). (matrix is centered around zero)

1. Use X, s, Y = np.linalg.svd(A) to perform SVD analysis.

```
[8]: X, s, Y = np.linalg.svd(A)
```

2. Calculate and print the variance explained by the i-th component, i.e.,

```
[10]: var = (s ** 2) / np.sum(s ** 2) var
```

```
[10]: array([0.76470588, 0.09201289, 0.08513021, 0.03251685, 0.02563417])
[11]: # same things as above
      summ = np.sum(s ** 2)
      variance = []
      for i in s:
          tmp = (i ** 2) / summ
          variance.append(tmp)
      variance
[11]: [0.764705882352941,
       0.09201288836360606,
       0.0851302114999976,
       0.03251684732353188,
       0.025634170459923528]
       3. To verify the factorization is correct, use X, s, Y to recover matrix A (let's denote the recovered
          matrix as B). Is Y = V or Y = V T?
     Answer: Y is equal to V.T
[13]: s
[13]: array([65.
                         , 22.54708869, 21.68742536, 13.403566 , 11.90078954])
[14]: np.diag(s)
                          , 0.
                                                     , 0.
[14]: array([[65.
                                                                  , 0.
                                                                                ],
             [ 0.
                          , 22.54708869, 0.
                                                        0.
                                                                  , 0.
                                                                                ],
             [ 0.
                          , 0.
                                       , 21.68742536,
                                                                  , 0.
                                                       0.
                                                                                ],
             [ 0.
                                                 , 13.403566 , 0.
                                                                                ],
                             0.
                                       , 0.
             Γ0.
                             0.
                                          0.
                                                        0.
                                                                   , 11.90078954]])
[15]: B = X @ np.diag(s) @ Y
[15]: array([[17., 24., 1., 8., 15.],
             [23., 5., 7., 14., 16.],
             [4., 6., 13., 20., 22.],
             [10., 12., 19., 21., 3.],
             [11., 18., 25., 2., 9.]])
        4. Use np.linalg.norm to calculate the difference of matrices A and B.
[17]: A - B
[17]: array([[-3.55271368e-15, 1.06581410e-14, 1.93178806e-14,
              -2.48689958e-14, -1.59872116e-14],
```

```
[ 0.00000000e+00, 1.77635684e-14, 1.33226763e-14,
-2.66453526e-14, -1.42108547e-14,
[ 5.77315973e-15, -5.32907052e-15, -1.77635684e-14,
-1.06581410e-14, -7.10542736e-15],
[ 3.55271368e-15, -3.19744231e-14, -1.42108547e-14,
 1.77635684e-14, 6.66133815e-15],
[-5.32907052e-15, -3.55271368e-14, -1.42108547e-14,
 2.17603713e-14, 1.95399252e-14]])
```

```
[18]: np.linalg.norm(A - B)
```

[18]: 8.523187293941492e-14

5. Define W as the matrix containing the first 2 columns of V, which defines a plane formed by the first 2 principal components.

Answer: Y = V.T ==> V = Y.T

```
[20]: V = Y.T
      ٧
```

```
[20]: array([[-4.47213595e-01, -4.04516436e-01, 2.46564896e-01,
             -6.62726001e-01, 3.69278287e-01],
             [-4.47213595e-01, -5.56615971e-03, 6.62726001e-01,
              2.46564896e-01, -5.47694274e-01],
             [-4.47213595e-01, 8.20165192e-01, -3.22455040e-14,
             -3.69575263e-17, 3.56831975e-01],
             [-4.47213595e-01, -5.56615971e-03, -6.62726001e-01,
             -2.46564896e-01, -5.47694274e-01],
             [-4.47213595e-01, -4.04516436e-01, -2.46564896e-01,
              6.62726001e-01, 3.69278287e-01]])
```

```
[21]: W = V[:,:2]
      W
```

```
[21]: array([[-0.4472136 , -0.40451644],
             [-0.4472136, -0.00556616],
             [-0.4472136, 0.82016519],
             [-0.4472136, -0.00556616],
             [-0.4472136 , -0.40451644]])
```

6. Project the matrix A to the new plane by perform the dot product of A and W, denoted the reduced matrix as Ar

```
[23]: Ar = np.dot(A, W)
      Ar
```

```
[23]: array([[-2.90688837e+01, -1.23024779e+01],
             [-2.90688837e+01, -1.01407417e+01],
             [-2.90688837e+01, -5.29465360e-13],
             [-2.90688837e+01, 1.01407417e+01],
             [-2.90688837e+01, 1.23024779e+01]])
     1.1.3 Exercise 2: Use Scikit-Learn to perform PCA analysis in Exercise 1.
     (this matrix will be centered around the mean)
        1. Load library: from sklearn.decomposition import PCA.
[26]: from sklearn.decomposition import PCA
        2. Initialize a PCA model with 2 components.
[28]: model = PCA(n_components=2)
        3. Use Model.fit(A) to fit the model.
[30]: model.fit(A)
[30]: PCA(n_components=2)
[31]: print(f"components: {model.components_}")
      print(f"explained variance: {model.explained_variance_}")
      print(f"explained variance ratio: {model.explained_variance_ratio_}")
      print(f"n components: {model.n_components_}")
      print(f"n samples: {model.n_samples_}")
      print(f"n features in: {model.n_features_in_}")
     components: [[ 4.04516436e-01 5.56615971e-03 -8.20165192e-01 5.56615971e-03
        4.04516436e-01]
      [-2.46564896e-01 -6.62726001e-01 -3.54216581e-15 6.62726001e-01
        2.46564896e-01]]
     explained variance: [127.09280205 117.58610463]
     explained variance ratio: [0.39105478 0.3618034 ]
     n components: 2
     n samples: 5
     n features in: 5
        4. Use Model.transform(A) to transform reduce the dimension of matrix A.
[33]: model.transform(A)
[33]: array([[ 1.23024779e+01, -1.10967458e+01],
             [ 1.01407417e+01, 4.23857973e+00],
             [-6.25055563e-14, 1.37163321e+01],
```

[-1.01407417e+01, 4.23857973e+00],

```
[-1.23024779e+01, -1.10967458e+01]])
```

5. Print the percentage of variance explained by each of the selected components.

explained variance ratio: [0.39105478 0.3618034] per component total variance explained by all components 0.7528581744203147

- 6. Compare the reduced matrix here with that obtained in Exercise 1. What is your observation? Answer: they are not the same
 - 7. If 80% of information, how many components should we use?

explained variance ratio: [0.39105478 0.3618034 0.1381966] per component total variance explained by all components 0.8910547755453252

1.1.4 Exercise 3: Revisit Exercise 1

1. Center the data by performing A_centered = A_i - mean(A)

```
[41]: A_centered_around_mean = A - np.mean(A)
A_centered_around_mean
```

2. Repeat the tasks in Exercise 1 for A center , and compare the reduced matrix here with that obtained in Exercise 2

```
[43]: Xc, sc, Yc = np.linalg.svd(A_centered_around_mean) # c = centered var = (sc ** 2) / np.sum(sc ** 2) var
```

```
[43]: array([3.91054776e-01, 3.61803399e-01, 1.38196601e-01, 1.08945224e-01, 4.85090051e-34])
```

```
[44]: Bc = Xc @ np.diag(sc) @ Yc
      Вс
[44]: array([[ 4.00000000e+00, 1.10000000e+01, -1.20000000e+01,
             -5.00000000e+00, 2.00000000e+00],
             [ 1.00000000e+01, -8.00000000e+00, -6.00000000e+00,
              1.00000000e+00, 3.0000000e+00],
             [-9.00000000e+00, -7.00000000e+00, 4.46161833e-15,
              7.0000000e+00, 9.0000000e+00],
             [-3.00000000e+00, -1.00000000e+00, 6.00000000e+00,
              8.00000000e+00, -1.00000000e+01],
             [-2.00000000e+00, 5.00000000e+00, 1.20000000e+01,
              -1.10000000e+01, -4.0000000e+00]])
[45]: Vc = Yc.T
      Wc = Vc[:,:2]
      A_centered_around_mean_reduced = np.dot(A_centered_around_mean, Wc)
      A_centered_around_mean_reduced
[45]: array([[-1.23024779e+01, 1.10967458e+01],
             [-1.01407417e+01, -4.23857973e+00],
             [ 6.25055563e-14, -1.37163321e+01],
             [1.01407417e+01, -4.23857973e+00],
             [ 1.23024779e+01, 1.10967458e+01]])
```

The matrix is the same as the one obtained in exercise 2 value-wise but complementary sign-wise which is fine. it is due to picking different pos and negative axes

1.2 2. SVD and PCA for image compression

1.2.1 setup

(1200, 1600, 3)

Load the picture "swim.jpg" from Homework 4, and define the data as "image". It is a RGB color image. Split the three colors, e.g.

```
red = image[:, :, 0] green = image[:, :, 1] blue = image[:, :, 2]
```

```
[50]: import imageio
  import matplotlib.pyplot as plt

[51]: image = imageio.imread("./swim.jpg") # W x H x D (where D is 3)
  print(f"{np.shape(image)}")
  plt.imshow(image)
  plt.show()
```

```
200 - 400 - 600 800 1000 1200 1400
```

```
[52]: red = image[:,:,0]
      green = image[:,:,1]
      blue = image[:,:,2]
      red
[52]: Array([[ 90, 106, 116, ..., 89,
                                     94, 99],
             [ 93, 107, 117, ..., 84,
                                     86,
                                          86],
             [ 95, 107, 117, ...,
                                77,
                                     76, 76],
             [ 57, 61, 63, ...,
                                88,
                                     94,
                                          87],
                                72, 101,
             [ 60, 61, 62, ...,
                                          87],
             [ 58, 57, 58, ..., 58, 83, 94]], dtype=uint8)
```

1.2.2 Exercise 4: Use SVD to perform image compression.

- 1. For each color,
- (a) Perform SVD, e.g., Ur, sr, VrT = np.linalg.svd(red).
- (b) Compute the k-rank approximation, e.g.,

```
[55]: # Compressed color matrices
k = 20

Ur, sr, VTr = np.linalg.svd(red) # r = red, T = Transpose
Ug, sg, VTg = np.linalg.svd(green) # r = green, T = Transpose
```

```
Ub, sb, VTb = np.linalg.svd(blue) # b = blue, T = Transpose

comp_red = 0
comp_green = 0
comp_blue = 0
for j in range(k): # k-rank approximation
        comp_red += sr[j] * np.outer(Ur[:,j], VTr.T[:,j])
        comp_green += sg[j] * np.outer(Ug[:,j], VTg.T[:,j])
        comp_blue += sb[j] * np.outer(Ub[:,j], VTb.T[:,j])

print(f"shape: {np.shape(comp_red)}")
print(f"shape: {np.shape(comp_green)}")
print(f"shape: {np.shape(comp_blue)}")
```

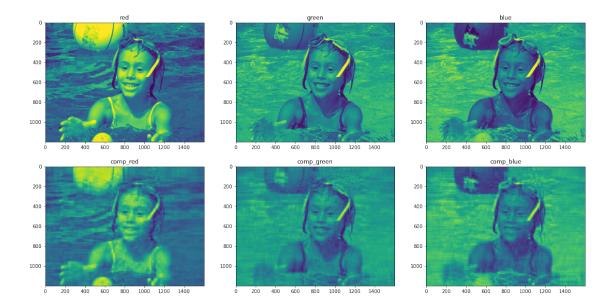
shape: (1200, 1600)
shape: (1200, 1600)
shape: (1200, 1600)

2. Make a plot with 2 $\,$ 3 subplots. The first row shows the original image of each color, while the second row shows the corresponding compressed image with k=20.

```
[57]: fix, axes = plt.subplots(2, 3, figsize=(20,10))

    axes[0,0].set_title("red")
    axes[0,0].imshow(red)
    axes[0,1].set_title("green")
    axes[0,1].imshow(green)
    axes[0,2].set_title("blue")
    axes[0,2].imshow(blue)

    axes[1,0].set_title("comp_red")
    axes[1,0].imshow(comp_red)
    axes[1,1].set_title("comp_green")
    axes[1,1].imshow(comp_green)
    axes[1,2].set_title("comp_blue")
    axes[1,2].imshow(comp_blue)
```



3. Combine three compressed image into a RGB image. Make a plot with 2—1 subplots to compare the compressed image with the original one.

```
[59]: # keeping both versions as a constructive example

new_comp_image1 = np.zeros((1200,1600,3), np.uint8) # uses 0..255

new_comp_image2 = np.zeros((1200,1600,3)) # gets divided bt 255, thus uses 0...

1 float values (similar to using np.float64)

new_comp_image1[:,:,0] = comp_red

new_comp_image1[:,:,1] = comp_green

new_comp_image2[:,:,2] = comp_blue

new_comp_image2[:,:,0] = comp_red

new_comp_image2[:,:,1] = comp_green

new_comp_image2[:,:,2] = comp_blue

# np.stack((comp_red, comp_green, comp_blue), axis=-1) # axis = -1 because we_uere inputting back into the 3rd dim
```

```
[60]: fix, axes = plt.subplots(1, 3, figsize=(20,10))

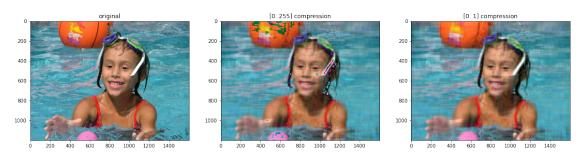
axes[0].set_title("original")
axes[0].imshow(image)

axes[1].set_title("[0..255] compression")
axes[1].imshow(new_comp_image1)

axes[2].set_title("[0..1] compression")
```

```
axes[2].imshow(new_comp_image2/255)
plt.show()
```

Clipping input data to the valid range for imshow with RGB data ([0..1] for floats or [0..255] for integers).



[0..255] creats more jumps in RGB data while [0..1] is more continuous

```
[62]: new_comp_image1[:,:,0]
[62]: array([[90, 90, 89, ..., 76, 76, 76],
             [91, 91, 90, ..., 76, 77, 77],
             [91, 91, 91, ..., 77, 78, 78],
             [63, 65, 65, ..., 62, 61, 59],
             [64, 65, 65, ..., 59, 59, 57],
             [64, 65, 65, ..., 57, 57, 55]], dtype=uint8)
[63]: new_comp_image2[:,:,0]/255
[63]: array([[0.35465665, 0.35431185, 0.35246822, ..., 0.29921647, 0.3005075,
              0.30114719],
             [0.35736373, 0.35716577, 0.35548999, ..., 0.30110658, 0.30255269,
              0.30311513],
             [0.35934212, 0.35934207, 0.35779764, ..., 0.30583563, 0.30729739,
              0.30760134],
             [0.25056304, 0.25558621, 0.25638659, ..., 0.24450831, 0.24251457,
              0.23424918],
             [0.25132471, 0.25639013, 0.25750143, ..., 0.23527269, 0.23352885,
              0.22547061],
             [0.25250537, 0.25767154, 0.25879431, ..., 0.22708176, 0.22543199,
              0.21730521]])
```

1.2.3 Exercise 5: Use PCA to perform image compression.

- 1. For each color,
- (a) Perform PCA, e.g.,
 - pcar = PCA(n components = 10)
 - reduced red = pcar.fit transform(red)

It reduces the original dimension to, e.g. 10, dimensions.

(b) Use pcar.inverse transform(reduced red) to decompress it back to original dimensions.

```
[66]: from sklearn.decomposition import PCA

[67]: k = 10

# Red
PCAr = PCA(n_components = k)
reduced_red = PCAr.fit_transform(red)
reduced_red = PCAr.inverse_transform(reduced_red)

# Green
PCAg = PCA(n_components = k)
reduced_green = PCAg.fit_transform(green)
reduced_green = PCAg.inverse_transform(reduced_green)

# Blue
PCAb = PCA(n_components = k)
reduced_blue = PCAb.fit_transform(blue)
reduced_blue = PCAb.inverse_transform(reduced_blue)
```

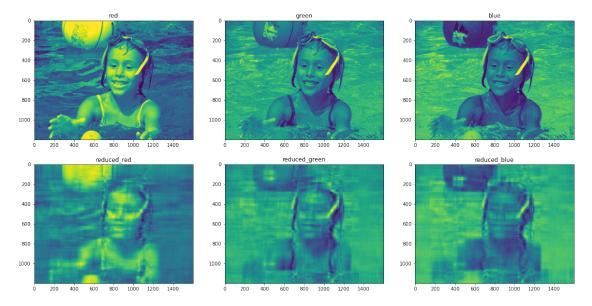
2. Make a plot with 2—3 subplots. The first row shows the original image of each color, while the second row shows the corresponding reduced (i.e., compressed) image with 20 components.

```
[69]: fix, axes = plt.subplots(2, 3, figsize=(20,10))

    axes[0,0].set_title("red")
    axes[0,1].set_title("green")
    axes[0,1].imshow(green)
    axes[0,2].set_title("blue")
    axes[0,2].imshow(blue)

axes[1,0].set_title("reduced_red")
    axes[1,0].imshow(reduced_red)
    axes[1,1].set_title("reduced_green")
    axes[1,1].imshow(reduced_green)
    axes[1,2].set_title("reduced_blue")
    axes[1,2].imshow(reduced_blue)
```

plt.show()



3. Combine three compressed image into a RGB image. Make a plot with 2—1 subplots to compare the compressed image with the original one.

```
fix, axes = plt.subplots(1, 2, figsize=(20,10))

axes[0].set_title("original")
axes[0].imshow(image)

axes[1].set_title("reduced")

#normalize_reduced_image = new_reduced_image / np.max(new_reduced_image)
#clipped_reduced_image = np.clip(new_reduced_image/255, 0., 1.)
axes[1].imshow(new_reduced_image/255)

plt.show()
```

Clipping input data to the valid range for imshow with RGB data ([0..1] for floats or [0..255] for integers).





1.3 3. PCA for dimensionality reduction

1.3.1 Exercise 6: Consider the data diagnosis.csv for cancer diagnosis.

1. Use Scikit-Learn to perform PCA analysis for the features of data, and reduce the data to two dimensions.

```
[76]: import pandas as pd from sklearn.preprocessing import MinMaxScaler, LabelEncoder
```

```
[77]: cancer_data = pd.read_csv("diagnosis.csv", delimiter=",", header=0).

drop('Unnamed: 32',axis=1)
cancer_data
```

	cancer_data							
[77]:		id	diagnosis	radius_mean	texture_mean	perimeter_mean	area_mean	\
	0	842302	М	17.99	10.38	122.80	1001.0	
	1	842517	M	20.57	17.77	132.90	1326.0	
	2	84300903	M	19.69	21.25	130.00	1203.0	
	3	84348301	M	11.42	20.38	77.58	386.1	
	4	84358402	M	20.29	14.34	135.10	1297.0	
		•••	•••	•••	***	•••		
	564	926424	M	21.56	22.39	142.00	1479.0	
	565	926682	M	20.13	28.25	131.20	1261.0	
	566	926954	M	16.60	28.08	108.30	858.1	
	567	927241	M	20.60	29.33	140.10	1265.0	
	568	92751	В	7.76	24.54	47.92	181.0	
		smoothness_mean 0.11840 0.08474		mpactness_mea	n concavity_me	.cavity_mean concave points_m		
	0			0.2776		10	0.14710 0.07017	
	1			0.0786		90		
	2	C	0.10960	0.1599	0 0.197	40	0.12790	

```
3
              0.14250
                                  0.28390
                                                    0.24140
                                                                           0.10520
4
              0.10030
                                  0.13280
                                                    0.19800
                                                                           0.10430
. .
                  •••
                                                    0.24390
                                                                           0.13890
564
              0.11100
                                  0.11590
565
              0.09780
                                  0.10340
                                                    0.14400
                                                                           0.09791
566
                                                                           0.05302
              0.08455
                                  0.10230
                                                    0.09251
567
              0.11780
                                  0.27700
                                                    0.35140
                                                                           0.15200
568
                                                    0.00000
              0.05263
                                  0.04362
                                                                           0.00000
        radius_worst
                        texture_worst perimeter_worst
                                                           area_worst
                                                                2019.0
0
               25.380
                                 17.33
                                                  184.60
1
               24.990
                                 23.41
                                                  158.80
                                                                1956.0
2
               23.570
                                 25.53
                                                  152.50
                                                                1709.0
3
               14.910
                                 26.50
                                                   98.87
                                                                567.7
4
               22.540
                                 16.67
                                                  152.20
                                                                1575.0
. .
564
               25.450
                                 26.40
                                                                2027.0
                                                  166.10
565
               23.690
                                 38.25
                                                  155.00
                                                                1731.0
566
                                 34.12
               18.980
                                                  126.70
                                                                1124.0
567
               25.740
                                 39.42
                                                  184.60
                                                                1821.0
568
                9.456
                                 30.37
                                                   59.16
                                                                 268.6
     smoothness_worst
                         compactness_worst
                                              concavity_worst
               0.16220
                                    0.66560
                                                        0.7119
0
1
               0.12380
                                    0.18660
                                                        0.2416
2
               0.14440
                                    0.42450
                                                        0.4504
3
               0.20980
                                    0.86630
                                                        0.6869
4
               0.13740
                                    0.20500
                                                        0.4000
. .
564
               0.14100
                                    0.21130
                                                        0.4107
565
               0.11660
                                    0.19220
                                                        0.3215
566
               0.11390
                                    0.30940
                                                        0.3403
567
               0.16500
                                    0.86810
                                                        0.9387
568
                                    0.06444
                                                        0.0000
               0.08996
     concave points_worst
                             symmetry_worst
                                               fractal_dimension_worst
0
                    0.2654
                                      0.4601
                                                                 0.11890
1
                    0.1860
                                      0.2750
                                                                 0.08902
2
                     0.2430
                                      0.3613
                                                                 0.08758
3
                     0.2575
                                      0.6638
                                                                 0.17300
4
                     0.1625
                                      0.2364
                                                                 0.07678
                        •••
                                       •••
564
                     0.2216
                                      0.2060
                                                                 0.07115
565
                     0.1628
                                      0.2572
                                                                 0.06637
566
                    0.1418
                                      0.2218
                                                                 0.07820
567
                    0.2650
                                      0.4087
                                                                 0.12400
568
                    0.0000
                                      0.2871
                                                                 0.07039
```

[569 rows x 32 columns]

```
[78]: # Encoding categorical column
      label_encoder = LabelEncoder()
      label_encoder.fit(cancer_data["diagnosis"])
      cancer_data['diagnosis'] = label_encoder.transform(cancer_data['diagnosis'])
      X = cancer_data.iloc[:,2:] # features, avoiding the id and diagnosis columns
      y = cancer_data["diagnosis"] # labels
[79]: k = 2
      PCA_cancer = PCA(n_components = k)
      reduced_cancer_data = PCA_cancer.fit_transform(X)
      reduced_cancer_data
[79]: array([[1160.1425737, -293.91754364],
             [1269.12244319, 15.63018184],
             [ 995.79388896, 39.15674324],
             [ 314.50175618, 47.55352518],
             [1124.85811531, 34.12922497],
             [-771.52762188, -88.64310636]])
[80]: print("shape of reduced data: ", np.shape(reduced_cancer_data))
     shape of reduced data: (569, 2)
       2. What is the total percentage of variance explained by this reduced data?
[82]: print(f"explained variance ratio: {PCA_cancer.explained_variance_ratio_} per_
       print(f"total variance explained by the components above: {sum(PCA cancer.
       ⇔explained_variance_ratio_)}")
     explained variance ratio: [0.98204467 0.01617649] per component
     total variance explained by the components above: 0.9982211613741724
       3. Use the reduced data to perform the logistic regression (split 25% data as test data with
          random state=20).
[84]: from sklearn.linear_model import LogisticRegression
      from sklearn.model_selection import train_test_split
[85]: x_train, x_test, y_train, y_test = train_test_split(reduced_cancer_data, y,_

stest_size=0.25, shuffle=True, random_state=20)

      model = LogisticRegression()
```

```
model.fit(x_train, y_train)
      print("trained model parameters are:")
      print(model.coef_, model.intercept_)
     trained model parameters are:
     [[ 0.01286424 -0.03663826]] [1.22540627]
       4. Print out the in-sample and out-sample confusing matrix and accuracy.
[87]: from sklearn.metrics import confusion matrix, accuracy_score
[88]: ypred_train
                        = model.predict(x_train)
      accuracy_train
                         = accuracy_score(y_train, ypred_train)
      conf_matrix_train = confusion_matrix(y_train, ypred_train)
      print('Accuracy for training data (R^2): ', accuracy_train, '\n')
      print('Confusion matrix for training data:\n', conf matrix train, '\n')
      ypred_test
                         = model.predict(x_test)
      accuracy_test
                         = accuracy_score(y_test,ypred_test)
      conf_matrix_test = confusion_matrix(y_test, ypred_test)
      print('Accuracy for test data (R^2): ', accuracy_test, '\n')
      print('Confusion matrix for test data:\n', conf_matrix_test, '\n')
     Accuracy for training data (R^2): 0.9295774647887324
     Confusion matrix for training data:
      [[261 10]
      [ 20 135]]
     Accuracy for test data (R^2): 0.9370629370629371
     Confusion matrix for test data:
      [[84 2]
      [ 7 50]]
     1.3.2 Exercise 7: Revisit Exercise 6. Normalize the reduced data before performing
            the logistic regression.
[90]: from sklearn.preprocessing import MinMaxScaler
[91]: scaler = MinMaxScaler().fit(reduced_cancer_data)
      reduced_normalized_data = scaler.transform(reduced_cancer_data)
      print(np.shape(reduced_normalized_data))
      reduced_normalized_data
```

(569, 2)

```
[91]: array([[0.42772682, 0.267593],
             [0.4507654, 0.48695099],
             [0.39298315, 0.50362286],
             [0.24895651, 0.50957316],
             [0.42026761, 0.50006016],
             [0.01936754, 0.41305874]])
[92]: x_train, x_test, y_train, y_test = train_test_split(reduced_normalized_data, y,__
      stest_size=0.25, shuffle=True, random_state=20)
      norm_model = LogisticRegression()
      norm_model.fit(x_train, y_train)
      print("trained model parameters are:")
      print(norm_model.coef_, norm_model.intercept_)
     trained model parameters are:
     [[ 8.1668785 -0.526361 ]] [-1.79846019]
[93]: ypred_train
                      = norm_model.predict(x_train)
                       = accuracy_score(y_train, ypred_train)
      accuracy train
      conf_matrix_train = confusion_matrix(y_train, ypred_train)
      print('Accuracy for training data (R^2): ', accuracy_train, '\n')
      print('Confusion matrix for training data:\n', conf_matrix_train, '\n')
      ypred_test
                        = norm_model.predict(x_test)
                       = accuracy_score(y_test,ypred_test)
      accuracy_test
      conf_matrix_test = confusion_matrix(y_test, ypred_test)
      print('Accuracy for test data (R^2): ', accuracy_test, '\n')
      print('Confusion matrix for test data:\n', conf_matrix_test, '\n')
     Accuracy for training data (R^2): 0.8568075117370892
     Confusion matrix for training data:
      [[270
              1]
      [ 60 95]]
     Accuracy for test data (R^2): 0.8601398601398601
     Confusion matrix for test data:
      [[86 0]
      [20 37]]
 []: We can see that the results are around
[94]: !pip install nbconvert
```

Defaulting to user installation because normal site-packages is not writeable Looking in links: /usr/share/pip-wheels Requirement already satisfied: nbconvert in /opt/conda/envs/anaconda-2022.05-py39/lib/python3.9/site-packages (6.4.4) Requirement already satisfied: nbclient<0.6.0,>=0.5.0 in /opt/conda/envs/anaconda-2022.05-py39/lib/python3.9/site-packages (from nbconvert) (0.5.13) Requirement already satisfied: jupyter-core in /opt/conda/envs/anaconda-2022.05-py39/lib/python3.9/site-packages (from nbconvert) (4.9.2) Requirement already satisfied: pandocfilters>=1.4.1 in /opt/conda/envs/anaconda-2022.05-py39/lib/python3.9/site-packages (from nbconvert) (1.5.0) Requirement already satisfied: jupyterlab-pygments in /opt/conda/envs/anaconda-2022.05-py39/lib/python3.9/site-packages (from nbconvert) (0.1.2) Requirement already satisfied: nbformat>=4.4 in /opt/conda/envs/anaconda-2022.05-py39/lib/python3.9/site-packages (from nbconvert) (5.3.0) Requirement already satisfied: traitlets>=5.0 in /opt/conda/envs/anaconda-2022.05-py39/lib/python3.9/site-packages (from nbconvert) (5.1.1) Requirement already satisfied: defusedxml in /opt/conda/envs/anaconda-2022.05-py39/lib/python3.9/site-packages (from nbconvert) (0.7.1) Requirement already satisfied: mistune<2,>=0.8.1 in /opt/conda/envs/anaconda-2022.05-py39/lib/python3.9/site-packages (from nbconvert) (0.8.4) Requirement already satisfied: testpath in /opt/conda/envs/anaconda-2022.05-py39/lib/python3.9/site-packages (from nbconvert) (0.5.0) Requirement already satisfied: beautifulsoup4 in /opt/conda/envs/anaconda-2022.05-py39/lib/python3.9/site-packages (from nbconvert) (4.11.1) Requirement already satisfied: bleach in /opt/conda/envs/anaconda-2022.05-py39/lib/python3.9/site-packages (from nbconvert) (4.1.0) Requirement already satisfied: pygments>=2.4.1 in /opt/conda/envs/anaconda-2022.05-py39/lib/python3.9/site-packages (from nbconvert) (2.11.2) Requirement already satisfied: jinja2>=2.4 in /opt/conda/envs/anaconda-2022.05-py39/lib/python3.9/site-packages (from nbconvert) (2.11.3) Requirement already satisfied: entrypoints>=0.2.2 in /opt/conda/envs/anaconda-2022.05-py39/lib/python3.9/site-packages (from nbconvert) (0.4)

/opt/conda/envs/anaconda-2022.05-py39/lib/python3.9/site-packages (from

Requirement already satisfied: MarkupSafe>=0.23 in

```
jinja2 >= 2.4 - nbconvert) (2.0.1)
Requirement already satisfied: nest-asyncio in
/opt/conda/envs/anaconda-2022.05-py39/lib/python3.9/site-packages (from
nbclient<0.6.0,>=0.5.0->nbconvert) (1.5.5)
Requirement already satisfied: jupyter-client>=6.1.5 in
/opt/conda/envs/anaconda-2022.05-py39/lib/python3.9/site-packages (from
nbclient<0.6.0,>=0.5.0->nbconvert) (6.1.12)
Requirement already satisfied: tornado>=4.1 in
/opt/conda/envs/anaconda-2022.05-py39/lib/python3.9/site-packages (from jupyter-
client>=6.1.5->nbclient<0.6.0,>=0.5.0->nbconvert) (6.1)
Requirement already satisfied: pyzmq>=13 in
/opt/conda/envs/anaconda-2022.05-py39/lib/python3.9/site-packages (from jupyter-
client>=6.1.5->nbclient<0.6.0,>=0.5.0->nbconvert) (22.3.0)
Requirement already satisfied: python-dateutil>=2.1 in
/opt/conda/envs/anaconda-2022.05-py39/lib/python3.9/site-packages (from jupyter-
client>=6.1.5->nbclient<0.6.0,>=0.5.0->nbconvert) (2.8.2)
Requirement already satisfied: fastjsonschema in
/opt/conda/envs/anaconda-2022.05-py39/lib/python3.9/site-packages (from
nbformat>=4.4->nbconvert) (2.15.1)
Requirement already satisfied: jsonschema>=2.6 in
/opt/conda/envs/anaconda-2022.05-py39/lib/python3.9/site-packages (from
nbformat>=4.4->nbconvert) (4.4.0)
Requirement already satisfied: attrs>=17.4.0 in
/opt/conda/envs/anaconda-2022.05-py39/lib/python3.9/site-packages (from
jsonschema>=2.6->nbformat>=4.4->nbconvert) (21.4.0)
Requirement already satisfied: pyrsistent!=0.17.0,!=0.17.1,!=0.17.2,>=0.14.0 in
/opt/conda/envs/anaconda-2022.05-py39/lib/python3.9/site-packages (from
jsonschema>=2.6->nbformat>=4.4->nbconvert) (0.18.0)
Requirement already satisfied: six>=1.5 in
/opt/conda/envs/anaconda-2022.05-py39/lib/python3.9/site-packages (from python-
dateutil>=2.1->jupyter-client>=6.1.5->nbclient<0.6.0,>=0.5.0->nbconvert)
Requirement already satisfied: soupsieve>1.2 in
/opt/conda/envs/anaconda-2022.05-py39/lib/python3.9/site-packages (from
beautifulsoup4->nbconvert) (2.3.1)
Requirement already satisfied: webencodings in
/opt/conda/envs/anaconda-2022.05-py39/lib/python3.9/site-packages (from
bleach->nbconvert) (0.5.1)
Requirement already satisfied: packaging in
/opt/conda/envs/anaconda-2022.05-py39/lib/python3.9/site-packages (from
bleach->nbconvert) (21.3)
Requirement already satisfied: pyparsing!=3.0.5,>=2.0.2 in
/opt/conda/envs/anaconda-2022.05-py39/lib/python3.9/site-packages (from
packaging->bleach->nbconvert) (3.0.4)
```

[4]: !jupyter nbconvert --to pdf "hw5-6.ipynb"

[NbConvertApp] Converting notebook hw5-6.ipynb to pdf

```
[NbConvertApp] Support files will be in hw5-6_files/
[NbConvertApp] Making directory ./hw5-6_files
[NbConvertApp] Writing 94324 bytes to notebook.tex
[NbConvertApp] Building PDF
[NbConvertApp] Running xelatex 3 times: ['xelatex', 'notebook.tex', '-quiet']
[NbConvertApp] Running bibtex 1 time: ['bibtex', 'notebook']
[NbConvertApp] WARNING | bibtex had problems, most likely because there were no citations
[NbConvertApp] PDF successfully created
[NbConvertApp] Writing 2654145 bytes to hw5-6.pdf
```

[]: