hw4

October 19, 2023

1 HW 4 – KMeans – Layth Aljorani

1.1 Exercise 1

the file hw data.txt contains 25,000 records, each involving three quantities, an index, a height in inches, and a weight in pounds. We want to select the first 350 records, and plot height versus weight, with the mean surrounded by three red rings of radius 1, 2 and 3 standard deviations. Write a program exercise1.py and:

- use np.loadtxt() to read data from hw data.txt;
- use np.shape() to get and print the number of rows and columns;
- print the first five rows of data;
- now reduce data to the first 350 rows, and last two columns of information.
- compute and print min, mean, max, variance, std of data;
- create data2, a standardized copy of data;
- use plt.scatter ( ) to plot values of height (column 0) versus weight (column 1) of your standardized data;
- add 3 red rings to your plot, of radius 1, 2 and 3 standard deviations:

[5]:
```
import numpy as np
from sklearn.preprocessing import StandardScaler
import matplotlib.pyplot as plt
```

[6]:
```
data = np.loadtxt("./hw_data.txt") # import data
print(f"shape of data: {np.shape(data)}") # print shape
print(f"first five rows:\n{data[:5]}") # print first 5 rows
```

```
shape of data: (25000, 3)
first five rows:
[[  1.        65.78331 112.9925 ]
 [  2.        71.51521 136.4873 ]
 [  3.        69.39874 153.0269 ]
 [  4.        68.2166  142.3354 ]
 [  5.        67.78781 144.2971 ]]
```

[7]:
```
data = data[:350,-2:] # reducing data
print(f"shape of data: {np.shape(data)}")
```

```
shape of data: (350, 2)
```

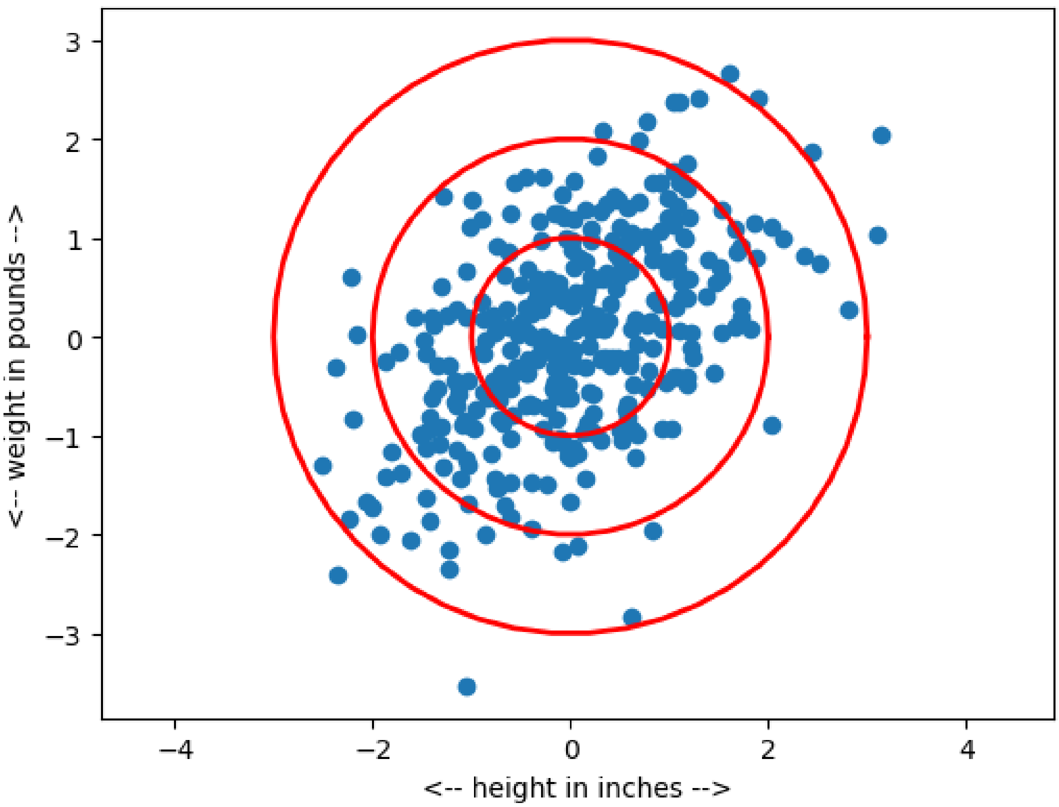
[8]:
```
print(f"Min of data: {np.min(data):.2f}")
print(f"Mean of data: {np.mean(data):.2f}")
print(f"Max of data: {np.max(data):.2f}")
print(f"Variance of data: {np.var(data):.2f}")
print(f"Standarad Deviation of data: {np.std(data):.2f}")
```

```
Min of data: 63.19
Mean of data: 97.40
Max of data: 158.96
Variance of data: 941.52
Standarad Deviation of data: 30.68
```

[9]:
```
xs = ((data[:,0] - np.mean(data[:,0])) / np.std(data[:,0])) # standaradize the x
ys = ((data[:,1] - np.mean(data[:,1])) / np.std(data[:,1])) # standaradize the y

# or
std_scaler = StandardScaler().fit(data)
data2 = std_scaler.transform(data)
data2[:5]
```

[9]:
```
array([[-1.14219365, -1.15067636],
       [ 1.87735868,  0.800458  ],
       [ 0.76240693,  2.17399521],
       [ 0.13965818,  1.28611563],
       [-0.08622745,  1.44902574]])
```

[10]:
```
plt.scatter(xs, ys)
t = np.linspace(0, 2 * np.pi, 51) # generate 51 points from 0 to 2pi
x = np.cos(t)
y = np.sin(t)
plt.plot(x, y, 'r-', linewidth = 2)
plt.plot(2.0*x, 2.0*y, 'r-', linewidth = 2)
plt.plot(3.0*x, 3.0*y, 'r-', linewidth = 2)

plt.xlabel("<-- height in inches -->")
plt.ylabel("<-- weight in pounds -->")
plt.axis("equal") # to remove directional bias
plt.savefig('exercise1.jpg')
plt.show()
```



1.2 Prepare for Exercise 2

We will now read a set of two-dimensional data. We will consider regarding this data as either a single cluster, or a pair of clusters. We want to compare the energy of the one and two cluster cases. To split our data into k clusters, we use the implementation of the K Means algorithm available in scikit-learn. To use the algorithm, we need the statement

To use the algorithm, we need the statement - `from sklearn.cluster import KMeans`

if we wish to create k clusters, we must define the word kmeans as follows: - `kmeans = KMeans(n_clusters=k, n_init='auto')`

to cluster data, we then write - `kmeans.dfit(data)`:

Following this command, we can collect some information as follows: - Z = `kmeans.cluster_centers` - C = `kmeans.labels_` - E = `kmeans.inertia_`

Here, each of the n values C[i] satisfies 0   C[i] < k, and indicates that data item i belongs to cluster C[i]. If we wish to plot all the points belonging to cluster 0, we might use a scatter() command like this:

- `plt.scatter(data[C==0, 0], data[C==0, 1], c='red'`

with similar commands to plot points from other clusters, perhaps in blue, green, and so on. Each cluster has a center. The coordinates of these centers are in the k × d array Z. It's important to know how the energy changes as we increase the number of clusters. Therefore, if we want to compare the energy for the one-cluster and two-cluster cases, then for k = 1 and then k = 2, we have to: - set k; - define kmeans; - apply kmeans() to our data; - print E=kmeans.inertia ;

This was a lot of preparation, but now we are ready to deal with our tricky two-cluster data from the Old Faithful geyser.

1.3 Exercise 2

Write a program exercise2.py and: - use np.loadtxt() to read data from faithful data.txt; - use np.shape() to get and print the number of rows and columns; - print the first five rows of data; - compute and print min, max, mean, variance, std of data; - create data2, a standardized copy of data; - use plt.scatter (x values, y values) to plot eruption time (column 0) versus wait (column 1); - apply the KMeans algorithm to data2, requesting k=1 clusters, and print the energy - apply the KMeans algorithm to data2, requesting k=2 clusters, and print the energy; - in a scatter plot show cluster C=0, using c = 'red' and C=1 using c = 'cyan'; - in the same plot, add the two cluster centers Z, using c = 'black' and marker = '*' for both

You should notice that the energy decreased a lot when we went from k=1 to k=2. In your scatterplot, you should expect the red and cyan dots to be correctly clustered around a cluster center. Turn in: a copy of your final plot, exercise2.jpg.

[16]:
```
from sklearn.cluster import KMeans
```

[17]:
```
data = np.loadtxt("./faithful_data.txt")
print(f"shape of data: {np.shape(data)}")
print(f"First five rows:\n{data[:5]}")
```

```
shape of data: (272, 2)
First five rows:
[[ 3.6   79.   ]
 [ 1.8   54.   ]
 [ 3.333 74.   ]
 [ 2.283 62.   ]
 [ 4.533 85.   ]]
```

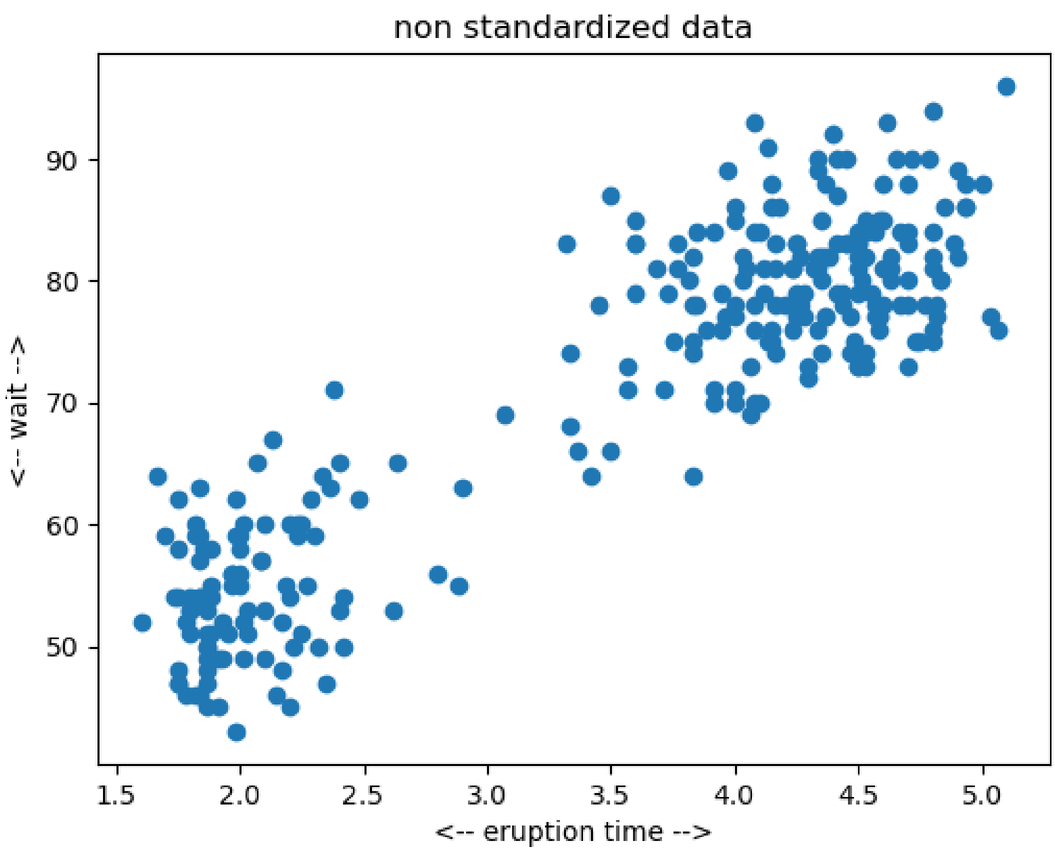
[18]:
```
print(f"Min of data: {np.min(data):.2f}")
print(f"Mean of data: {np.mean(data):.2f}")
print(f"Max of data: {np.max(data):.2f}")
print(f"Variance of data: {np.var(data):.2f}")
print(f"Standarad Deviation of data: {np.std(data):.2f}")
```

```
Min of data: 1.60
Mean of data: 37.19
Max of data: 96.00
Variance of data: 1228.72
Standarad Deviation of data: 35.05
```

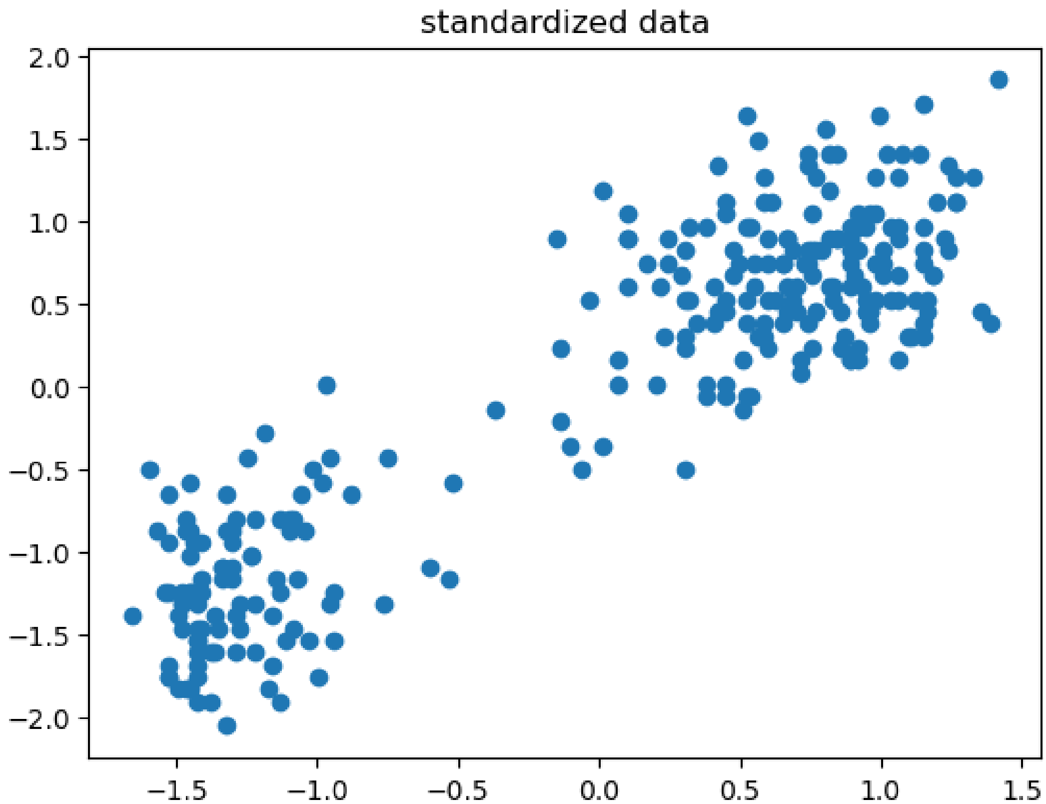
[19]:
```
# standardize the data
std_scaler = StandardScaler().fit(data)
data2 = std_scaler.transform(data)
data2[:5]
```

[19]:
```
array([[ 0.09849886,  0.59712344],
       [-1.48145856, -1.24518118],
       [-0.13586149,  0.22866251],
       [-1.05750332, -0.6556437 ],
       [ 0.91744345,  1.03927655]])
```

[20]:
```
plt.scatter(data[:,0], data[:,1])
plt.title("non standardized data")
plt.xlabel("<-- eruption time -->")
plt.ylabel("<-- wait -->")
plt.show()
```



[21]:
```
plt.scatter(data2[:,0], data2[:,1])
plt.title("standardized data")
plt.show()
```

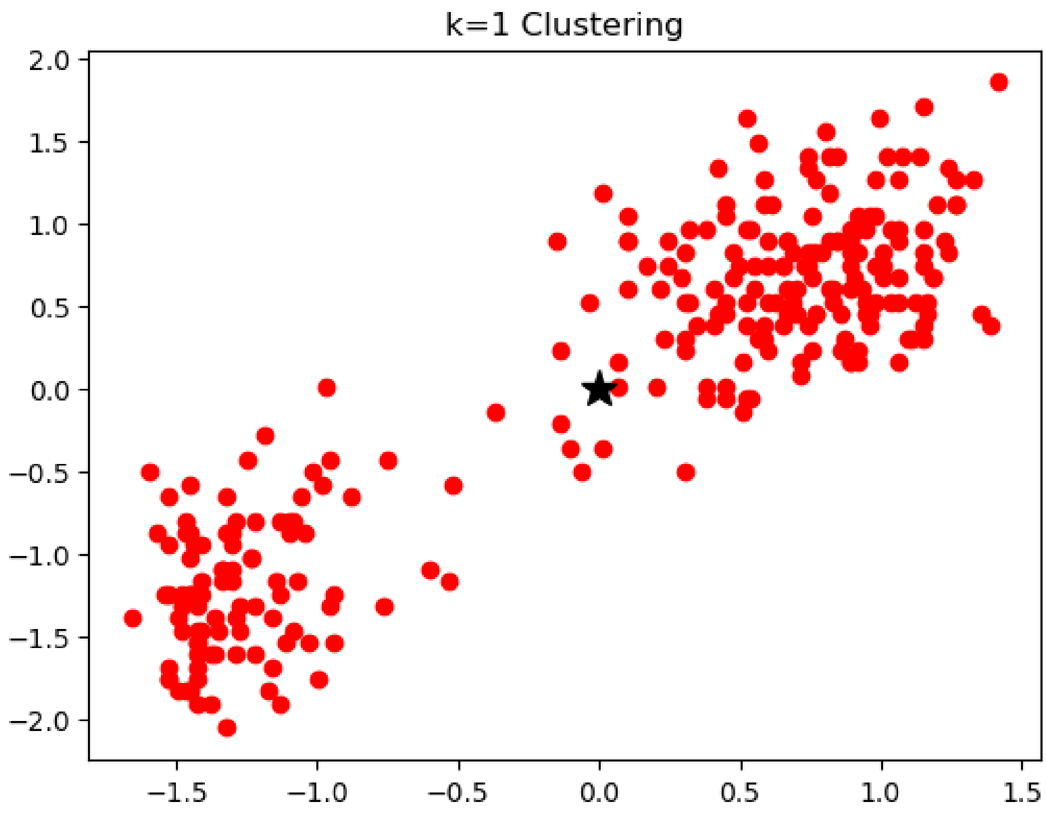


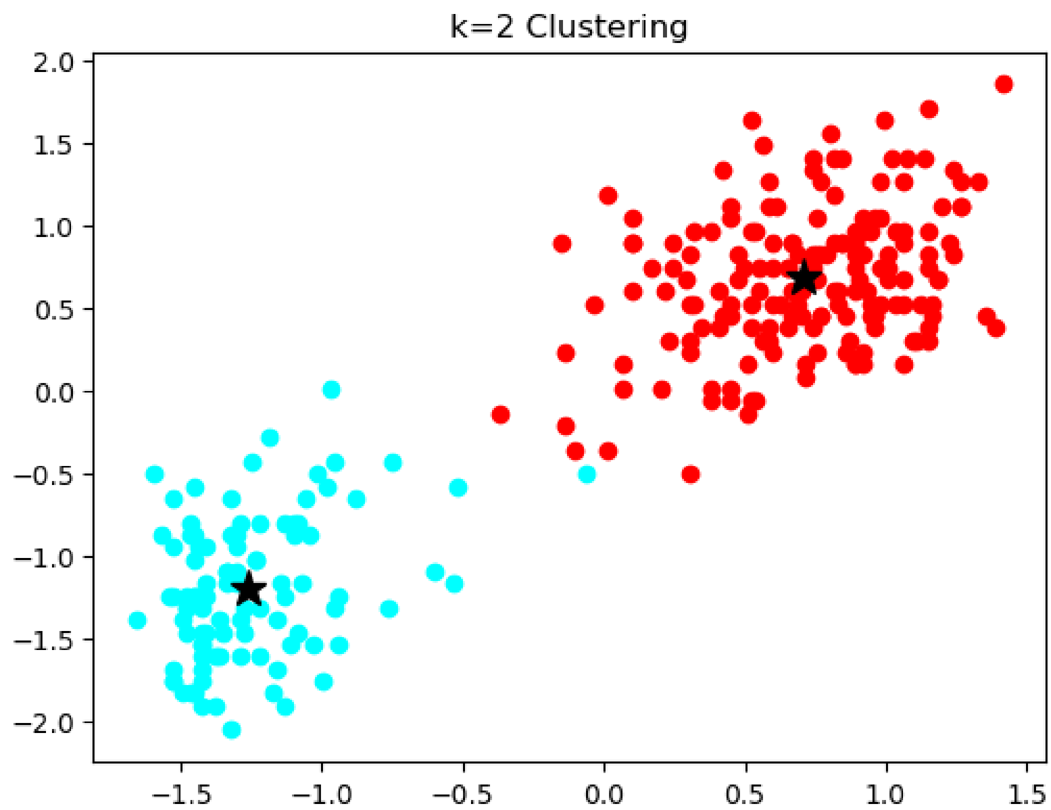
[22]:
```
ks = [1, 2] # easily scale your k
k_dict = {} # {k: (Z,C)}
for k in ks:
    kmeans = KMeans(n_clusters=k, n_init='auto') # create a kmeans
    kmeans.fit(data2) # fit it on standardized data

    # fetch and save properties
    Z = kmeans.cluster_centers_
    C = kmeans.labels_
    E = kmeans.inertia_
    k_dict[k] = (Z,C)

    print(f"@ K= {k}; Energy= {E:.2f}")
```

```
@ K= 1; Energy= 544.00
@ K= 2; Energy= 79.58
```

[23]:
```
for key, value in k_dict.items():
    Z = value[0] # k=1 ; index 0 (Z,C) ; center coordinates
    C = value[1] # k=1 ; index 1 (Z,C) ; labels
    colors = ['red', 'cyan', 'magenta', 'yellow'] # NOTE: len(colors) >= k
```

```python
for i in range(key):
    plt.scatter(data2[C==i, 0], data2[C==i, 1], c=colors[i % len(colors)])

for capital in Z: # print capital of cluster in a black star (k*)
    plt.plot(capital[0], capital[1], 'k*', markersize=14)

# titles
plt.title(f"k={key} Clustering")
if key == 2: plt.savefig('exercise2.jpg')
plt.show()
```





1.4 Exercise 3

The Ruspini data naturally breaks into a number of clusters. We will use KMeans to cluster the data into 1 k 10 clusters, compute the energy each time, and plot the sequence of energy values, looking for a sort of "elbow" or "hockey stick" in the plot, which suggests a natural value of k to choose.

Write a program explore.py which searches for a good value of k: - use np.loadtxt() to read data from ruspini data.txt; - use np.shape() to get and print the number of rows and columns; - print the first five rows of data; - create data2, a standardized copy of data; - use plt.scatter (x values, y values) to plot x (column 0) versus y (column 1); - for 1 k 10, set up kmeans, cluster the data, store E[k-1]=kmeans.inertia ; - print the values k,E[k-1]; - plot the values k,E[k-1];

Write a second program exercise3.py which uses your chosen value of k: - based on your energy plot, choose a value of k; - using your chosen value of k, define kmeans, use kmeans() to cluster data2; - in a scatter plot, display each set of clustered data; - in the same scatter plot, add the cluster centers, using c = 'black' and marker = '*'

If you have chosen k well, your plot should show nicely clustered data.

Turn in: a copy of your final plot, exercise3.jpg.

1.4.1 Exploring

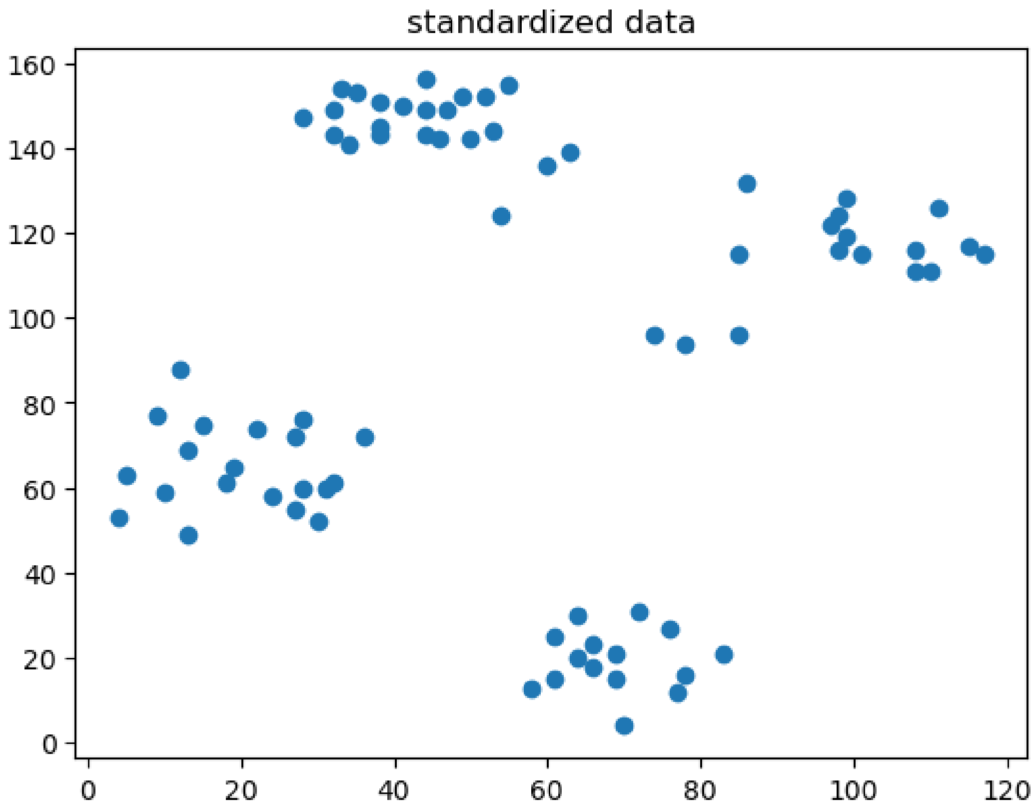
[27]:
```
data = np.loadtxt("./ruspini_data.txt")
print(f"shape of data: {np.shape(data)}")
print(f"First five rows:\n{data[:5]}")
print(f"Min of data: {np.min(data):.2f}")
print(f"Mean of data: {np.mean(data):.2f}")
print(f"Max of data: {np.max(data):.2f}")
print(f"Variance of data: {np.var(data):.2f}")
print(f"Standarad Deviation of data: {np.std(data):.2f}")
```

```
shape of data: (75, 2)
First five rows:
[[ 4. 53.]
 [ 5. 63.]
 [10. 59.]
 [ 9. 77.]
 [13. 49.]]
Min of data: 4.00
Mean of data: 73.45
Max of data: 156.00
Variance of data: 1974.13
Standarad Deviation of data: 44.43
```

[28]:
```
# standardize the data
std_scaler = StandardScaler().fit(data)
data2 = std_scaler.transform(data)
data2[:5]
```

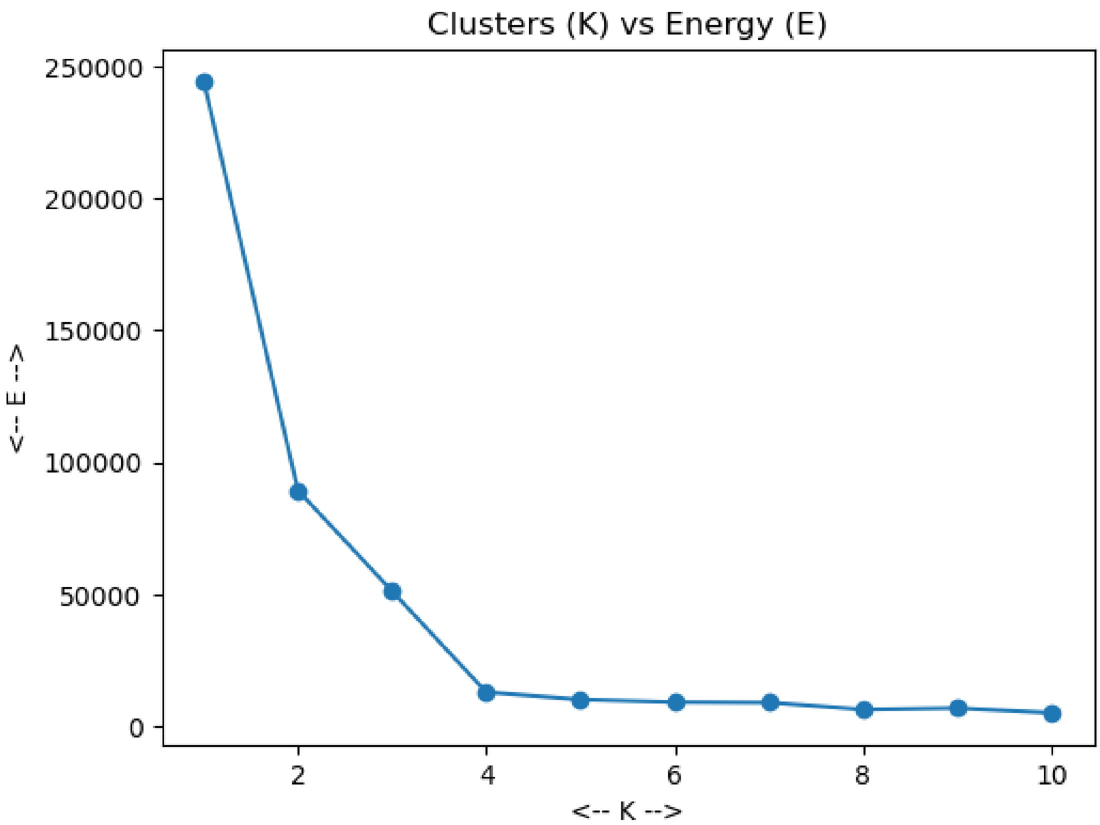
[28]:
```
array([[-1.67929121, -0.806722  ],
       [-1.64628627, -0.60001155],
       [-1.48126158, -0.68269573],
       [-1.51426652, -0.31061691],
       [-1.38224677, -0.88940619]])
```

[29]:
```
plt.scatter(data[:,0], data[:,1])
plt.title("standardized data")
plt.show()
```



[30]:
```
ks = range(1,11) # easily scale your k
k_dict = {} # {k: (Z,C}}
Energy = np.zeros(len(ks)) # init to zeros
for k in ks:
    kmeans = KMeans(n_clusters=k, n_init='auto') # create a kmeans
    kmeans.fit(data) # fit it on raw data

    # fetch and save properties
    Z = kmeans.cluster_centers_
    C = kmeans.labels_
    E = kmeans.inertia_
    k_dict[k] = (Z,C,E)
    Energy[k - 1] = E

    print(f"@ K= {k}; Energy= {E:.2f}")

plt.plot(ks, Energy, marker='o')
plt.title("Clusters (K) vs Energy (E)")
plt.xlabel("<-- K -->")
plt.ylabel("<-- E -->")
plt.show()
```

```
@ K= 1; Energy= 244373.87
@ K= 2; Energy= 89337.83
@ K= 3; Energy= 51155.41
@ K= 4; Energy= 12881.05
@ K= 5; Energy= 10126.72
@ K= 6; Energy= 9158.73
@ K= 7; Energy= 8988.05
@ K= 8; Energy= 6351.22
@ K= 9; Energy= 6875.27
@ K= 10; Energy= 5096.64
```

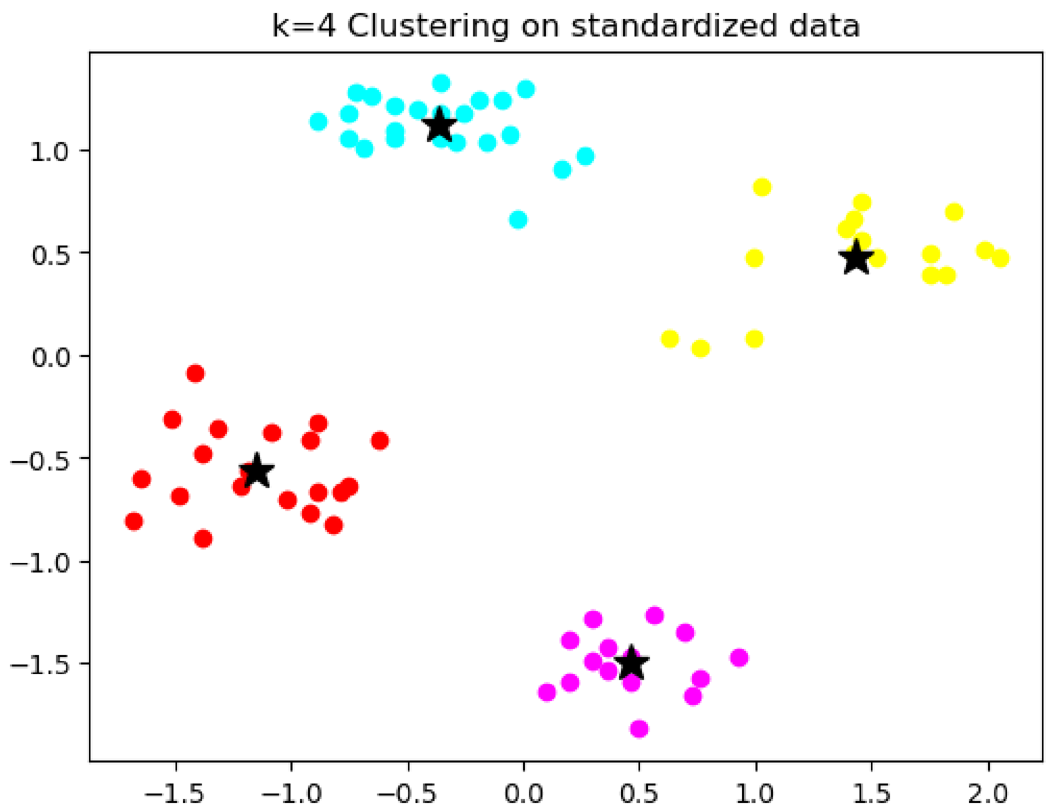


Not sure if the section above wanted raw data or standardized data, but I decided to use raw since the next section uses standardized

1.4.2 Main

[33]:
```python
k = 4
kmeans = KMeans(n_clusters=k, n_init='auto') # create a kmeans
kmeans.fit(data2) # fit it on standardized data

Z = kmeans.cluster_centers_
```

```python
C = kmeans.labels_
E = kmeans.inertia_

colors = ['red', 'cyan', 'magenta', 'yellow'] # NOTE: len(colors) >= k
for i in range(k):
    plt.scatter(data2[C==i, 0], data2[C==i, 1], c=colors[i % len(colors)])

for capital in Z: # print capital of cluster in a black star (k*)
    plt.plot(capital[0], capital[1], 'k*', markersize=14)

# titles
plt.title(f"k={k} Clustering on standardized data")
plt.savefig('exercise3.jpg')
plt.show()
```



1.5 Prepare for Exercise 4

in this exercise, we will explore how the K-Means algorithm can be used to manipulate an image. We will look at a simple kind of compression operation in which we reduce the number of colors used. You should be familiar with the idea that in computer graphics, most images are described as an array whose entries are essentially triples of (R,G,B) (red, green, blue) values. Typically, these values are unsigned integers between 0 and 255. The arithmetic type is called uint8. This means that Python sees an image as a 3-dimensional array of shape w × h × d of uint8 values, where d is 3.We would like to regard the colors as data to be clustered. In particular, we want to create K color clusters, such that each color used in the image is assigned to one of these clusters. Each cluster can be represented by its center value Z. We reason that if each color is close to its cluster center Z, we might be able to simplify the picture so that only K colors are used, namely, the K cluster centers Z. We can try to create a new image, using just these K colors. This is a standard process in image compression and image analysis. Because the image is stored as a triple index array of unsigned integers, we need to do some mysterious manipulations to convert the image to a numpy array, process it with kmeans(), and then convert the result back into something that looks like an image that we can display!

1.6 Exercise 4

[36]:
```
import imageio.v2 as imageio
```

[37]:
```
image = imageio.imread("./swim.jpg") # W x H x D (where D is 3)
plt.imshow(image)
plt.show()
```

The image currently is represented by unsigned 8 bit integers. We want to convert this to floating point numbers between 0 and 1:

[39]:
```
image = np.array(image, dtype=np.float64) / 255 # where 255 is the max color␣
    ↪value (0, 255)
print(f"shape: {np.shape(image)}")
print(f"first 3 elements: \n {image[:3]}")
```

```
shape: (1200, 1600, 3)
first 3 elements:
 [[[0.35294118 0.39607843 0.36470588]
  [0.41568627 0.43921569 0.42352941]
  [0.45490196 0.45490196 0.44705882]
  …
  [0.34901961 0.37647059 0.41568627]
  [0.36862745 0.39607843 0.43529412]
  [0.38823529 0.41176471 0.45882353]]

 [[0.36470588 0.41568627 0.38039216]
  [0.41960784 0.45882353 0.42745098]
  [0.45882353 0.45882353 0.45098039]
  …
  [0.32941176 0.36862745 0.40784314]
  [0.3372549  0.37647059 0.41568627]
  [0.3372549  0.38823529 0.42352941]]

 [[0.37254902 0.42352941 0.38823529]
  [0.41960784 0.45882353 0.42745098]
  [0.45882353 0.45882353 0.45098039]
  …
  [0.30196078 0.37254902 0.41176471]
  [0.29803922 0.37647059 0.41176471]
  [0.29803922 0.37647059 0.41176471]]]
```

The kmeans() code expects to work on an array with two dimensions, but our image data is in 3D. We need to reshape the array so that all the pixel data is in the first dimension. we must reshape the array so it looks like a 2-dimensional numpy array that kmeans() can handle:

[41]:
```
w = np.shape(image)[0]
h = np.shape(image)[1]
d = np.shape(image)[2]
image2 = np.reshape(image, (w*h, d))
print(f"new shape: {np.shape(image2)}")
```

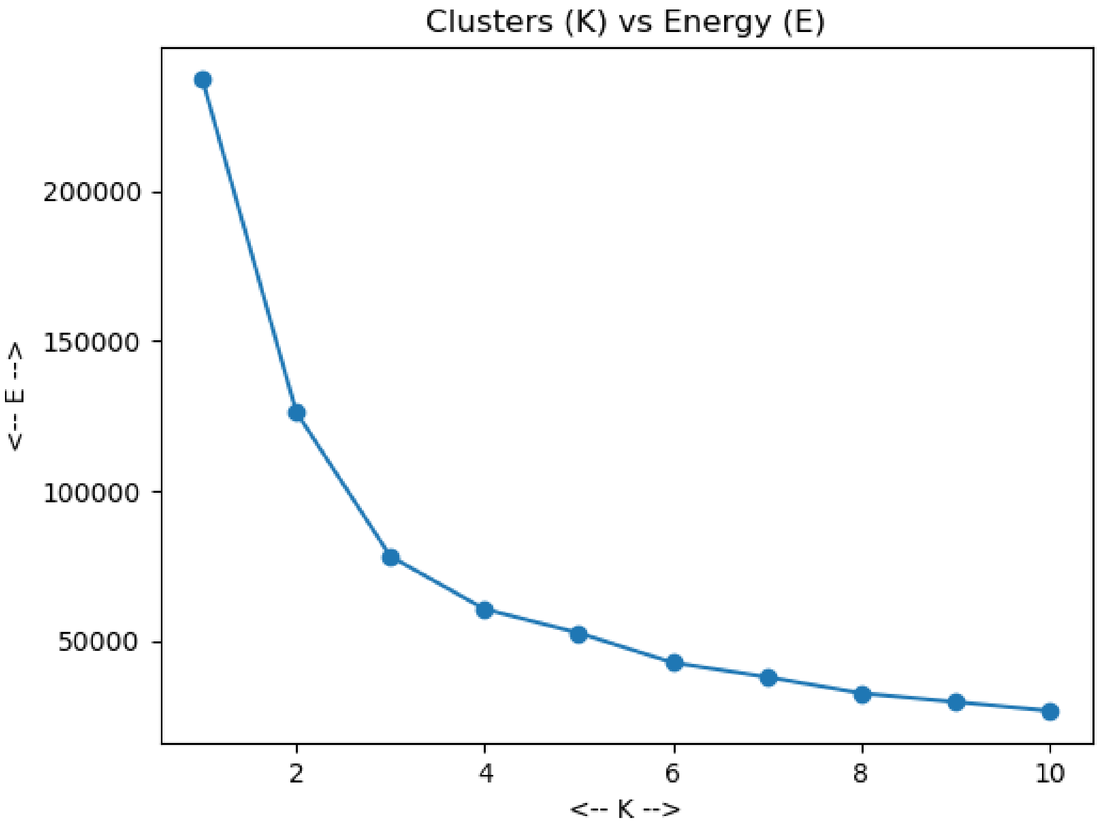
```
new shape: (1920000, 3)
```

I will, first, test which K value to choose.

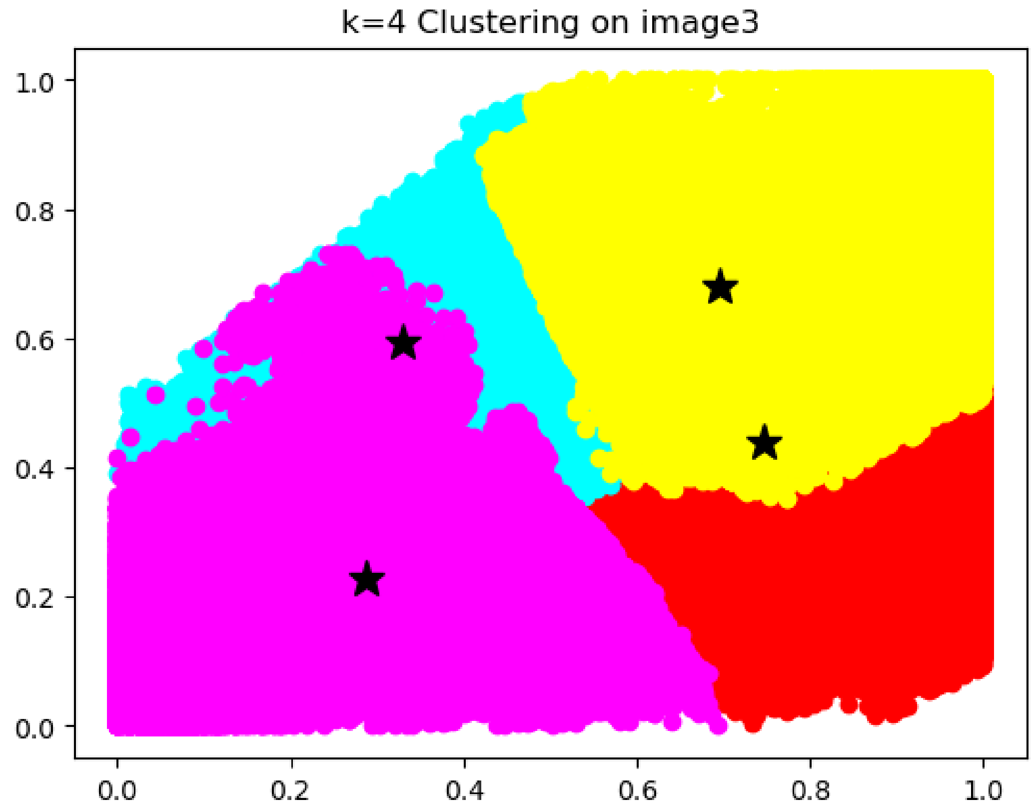
[43]:
```
ks = range(1,11) # easily scale your k
k_dict = {} # {k: (Z,C)}
Energy = np.zeros(len(ks)) # init to zeros
```

```python
for k in ks:
    kmeans = KMeans(n_clusters=k, n_init='auto') # create a kmeans
    kmeans.fit(image2) # fit it on reshaped image

    # fetch and save properties
    Z = kmeans.cluster_centers_
    C = kmeans.labels_
    E = kmeans.inertia_
    k_dict[k] = (Z,C,E)
    Energy[k - 1] = E

    print(f"@ K= {k}; Energy= {E:.2f}")

plt.plot(ks, Energy, marker='o')
plt.title("Clusters (K) vs Energy (E)")
plt.xlabel("<-- K -->")
plt.ylabel("<-- E -->")
plt.show()
```

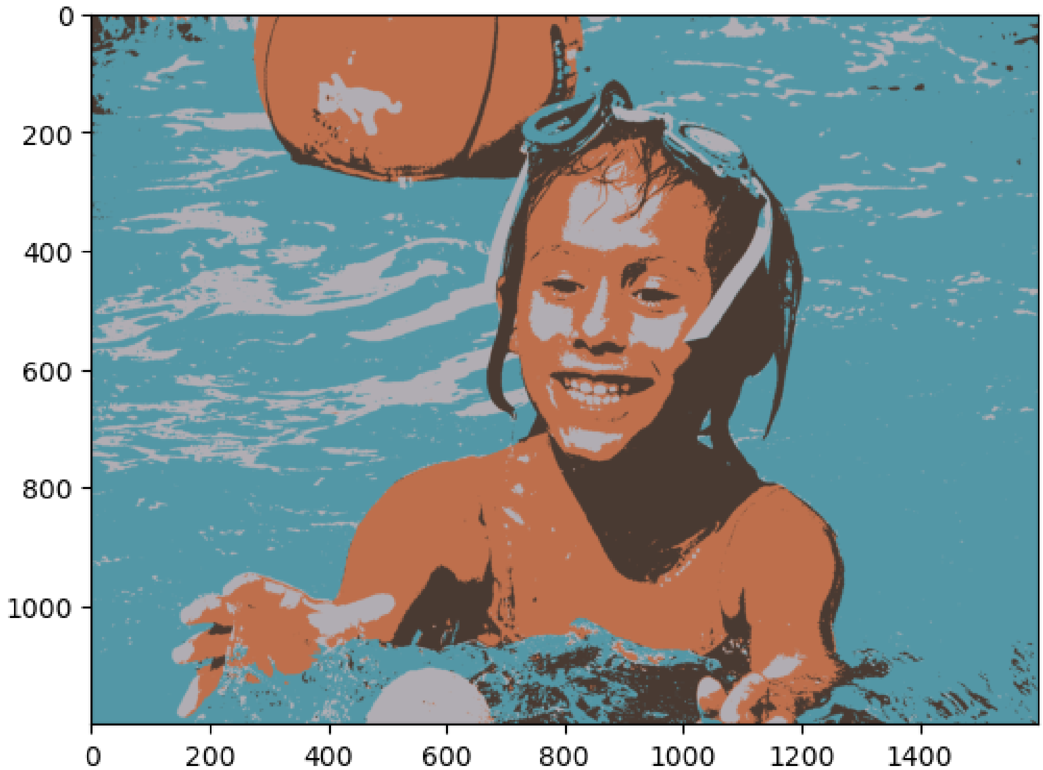
```
@ K= 1; Energy= 237514.96
@ K= 2; Energy= 126206.05
@ K= 3; Energy= 78039.21
@ K= 4; Energy= 60442.94
@ K= 5; Energy= 52472.21
@ K= 6; Energy= 42572.81
@ K= 7; Energy= 37772.11
@ K= 8; Energy= 32366.18
@ K= 9; Energy= 29478.94
@ K= 10; Energy= 26520.38
```



[44]:
```
k = 4
kmeans = KMeans(n_clusters=k, n_init='auto') # create a kmeans
kmeans.fit(image2) # fit it on reshaped image
Z = kmeans.cluster_centers_
C = kmeans.labels_

colors = ['red', 'cyan', 'magenta', 'yellow'] # NOTE: len(colors) >= k
for i in range(k):
    plt.scatter(image2[C==i, 0], image2[C==i, 1], c=colors[i % len(colors)])

for capital in Z: # print capital of cluster in a black star (k*)
    plt.plot(capital[0], capital[1], 'k*', markersize=14)

# titles
plt.title(f"k={k} Clustering on image3")
plt.show()

# A hot mess
```



[45]:
```
image3 = Z[C]   # Replace every color by its 'center'
image3 = image3.reshape(w,h,d) # restore original image shape

plt.imshow(image3)
plt.show()
```



You can peek at the result using the imshow() and show() commands.

[47]:
```
image3 = np.array(255 * image3, dtype=np.uint8)
imageio.imwrite('exercise4.jpg', image3)
```