Week #4

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https://github.com/LazarGlumac/MATB24-TUT3-Notes

start @ $12^{\underline{10}}$

HW1 marks released
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1. Let X, Y be subspaces of a vector space V. Prove $X \cap Y$ is a subspace of V. try it!

wts

① non empty
② closed under "+"
③ closed under scalar mult.

① since X, Y are S.S. $\vec{0} \in X$, $\vec{0} \in Y$
thus, $\vec{0} \in (X \cap Y)$ ✓

② Let $\vec{u}, \vec{v} \in (X \cap Y)$ wts $(\vec{u} + \vec{v}) \in (X \cap Y)$

$\vec{u}, \vec{v} \in (X \cap Y)$

$\Rightarrow \vec{u}, \vec{v} \in X \wedge \vec{u}, \vec{v} \in Y$

since X, Y are S.S.

$(\vec{u} + \vec{v}) \in X \wedge (\vec{u} + \vec{v}) \in Y$

since X, Y are s.s.

$(\vec{u}+\vec{v}) \in X \wedge (\vec{u}+\vec{v}) \in Y$ ✓

$\Rightarrow (\vec{u}+\vec{v}) \in (X \wedge Y)$

③ Let $\vec{u} \in (X \wedge Y)$, Let $c \in F$

$\vec{u} \in (X \wedge Y)$

$\Rightarrow \vec{u} \in X \wedge \vec{u} \in Y$

since X, Y is a s.s

$c\vec{u} \in X \quad c\vec{u} \in Y$

$\Rightarrow c\vec{u} \in (X \wedge Y)$ ✓

Therefore, by ①, ②, ③ $(X \wedge Y) \subseteq_{s.s} V$

2. Prove or disprove: $X := \{ A \in M_{n \times n}(\mathbb{R}) \mid \underline{A \text{ is invertible}} \} \subseteq_{s.s} M_{n \times n}(\mathbb{R})$

$O_n \notin$

<u>disprove</u> - counter-example

$A = I \quad \Rightarrow A \in X$

$B = -I \quad \Rightarrow B \in X$

$A + B = I + (-I)$

$= O \notin X$

3. Find all solution of the vector eqn.

$x_1\vec{v}_1 + x_2\vec{v}_2 + x_3\vec{v}_3 = \vec{0}$, (what does this tell us?)

where $v_1 = \begin{pmatrix}1\\0\\0\end{pmatrix}$, $v_2 = \begin{pmatrix}1\\2\\0\end{pmatrix}$, $v_3 = \begin{pmatrix}0\\1\\1\end{pmatrix}$

$$\Rightarrow x_1\begin{pmatrix}1\\0\\0\end{pmatrix} + x_2\begin{pmatrix}1\\2\\0\end{pmatrix} + x_3\begin{pmatrix}0\\1\\1\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$$

$$\Rightarrow \begin{cases} x_1 + x_2 + 0x_3 = 0 \\ 0x_1 + 2x_2 + x_3 = 0 \\ 0x_1 + 0x_2 + x_3 = 0 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \overset{R_2 \to \frac{1}{2}(R_2 - R_1)}{\sim} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$R_1 \to R_1 - R_2$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \Rightarrow \begin{array}{l} x_1 = 0 \\ x_2 = 0 \\ x_3 = 0 \end{array}$$

only the trivial soln exists.

Therefore, the vectors are l.i

4. For what values of v does the system $\begin{pmatrix} 1 & 0 & 0 \\ 2 & 2 & 6 \\ 1 & 1 & 3 \end{pmatrix} \vec{x} = \begin{pmatrix} 1 \\ 2 \\ v \end{pmatrix}$ have a soln.

Find the general soln.

try it

$$\Rightarrow \begin{cases} x_1 + 0x_2 + 0x_3 = 1 \\ 2x_1 + 2x_2 + 6x_3 = 2 & (2) \\ x_1 + x_2 + 3x_3 = v & (3) \end{cases}$$

eqn (2) = 2 · eqn (3)

$\Leftrightarrow v = \frac{2}{2} = 1$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 2 & 2 & 6 & 2 \\ 1 & 1 & 3 & 1 \end{array}\right) \underset{R_3 \to R_3 - \frac{1}{2}R_3}{\sim} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 2 & 2 & 6 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\underset{R_2 \to R_2 - 2R_1}{\sim} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 2 & 6 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

Let $x_3 = s \in \mathbb{R}$, $2x_2 + 6x_3 = 0$ $\qquad x_1 = 1$

$2x_2 = -6x_3$

$x_2 = -3s$

$$\vec{x} = \begin{pmatrix} 1 \\ -3s \\ s \end{pmatrix}$$

$= \begin{pmatrix} 1 \end{pmatrix} + \begin{pmatrix} 0 \\ -3s \end{pmatrix}$

$$= \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -3s \\ s \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + s \begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix}$$

$$\vec{x} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \mathrm{sp}\left\{ \begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix} \right\}$$