Solving second order analogue filter

Second order polynomial:

$P(s) = s^2 + 1.414s + 1$

Coefficients:

$a_1 = 1.414, a_0 = 1$

Substitutions:

$$s = j2\pi\omega, \qquad \omega c = 2\pi f c, \qquad a_1 = \frac{1}{Q}, \qquad \sqrt{a_0} = FSF$$

Hence,

$Q = 0.707, \qquad FSF = 1$

For filter with gain K=1, equations from Texas Instruments:

$FSF \times f_c = \frac{1}{2\pi RC\sqrt{mn}}, \; Q = \frac{\sqrt{mn}}{m+1}$ where $R_1 = mR, R_2 = R, C_1 = C, C_2 = nC$

Solving for ratio of m and n

$$0.707 = \frac{\sqrt{mn}}{m+1}$$

$$0.707^2 = \frac{mn}{m^2 + 2m + 1}$$

$$0.707^2 m^2 + (2 \times 0.707^2 - n)m + 0.707^2 = 0$$

$$m = \frac{n - 2 \times 0.707^2 + \sqrt{(2 \times 0.707^2 - n)^2 - 4 \times 0.707^4}}{2 \times 0.707^2}$$

The above gives us a graph for n to m ratio. Here we are free to pick any convenient ratio for the filter.

Choose $\qquad n = 2, m = 1$

Desired $\qquad f_c = 25Hz$

Substituting in the equations from Texas Instruments to obtain C and R near standard values:

$$25 = \frac{1}{\sqrt{2} \times 2\pi RC}$$

Choose $\qquad C = 0.015\ \mu F$ (standard value)

$R = \frac{1}{\sqrt{2} \times 25 \times 2\pi \times 0.015 \times 10^{-6}} = 300105 \approx 300K$ (standard value as well)

Check the actual cutoff frequency f_c

$f_c = \frac{1}{\sqrt{2} \times 2\pi \times 300 \times 10^3 \times 0.015 \times 10^{-6}} = 25\ Hz$ matches expected f_c

<u>Results: $R_1 = 300K, R_2 = 300K, C_1 = 0.015\mu F, C_2 = 0.030\mu F$</u>

Appendix:

Graph for n and m