Solving second order analogue filter

Second order polynomial:

$P(s) = s^2 + 1.414s + 1$

Coefficients:

$a_1 = 1.414, a_0 = 1$

Substitutions:

$$s = j2\pi\omega, \qquad \omega c = 2\pi f c, \qquad a_1 = \frac{1}{Q}, \qquad \sqrt{a_0} = FSF$$

Hence,

$Q = 0.707, \qquad FSF = 1$

For filter with gain K=1, equations from Texas Instruments:

$FSF \times f_c = \frac{1}{2\pi RC\sqrt{mn}},\ Q = \frac{\sqrt{mn}}{m+1}$ where $R_1 = mR, R_2 = R, C_1 = C, C_2 = nC$

Solving for ratio of m and n

$$0.707 = \frac{\sqrt{mn}}{m+1}$$

$$0.707^2 = \frac{mn}{m^2 + 2m + 1}$$

$$0.707^2 m^2 + (2 \times 0.707^2 - n)m + 0.707^2 = 0$$

$$m = \frac{n - 2 \times 0.707^2 + \sqrt{(2 \times 0.707^2 - n)^2 - 4 \times 0.707^4}}{2 \times 0.707^2}$$

The above gives us a graph for n to m ratio. Here we are free to pick any convenient ratio for the filter.

Choose $n = 2, m = 1$

Desired $f_c = 25Hz$

Substituting in the equations from Texas Instruments to obtain C and R near standard values:

$$25 = \frac{1}{\sqrt{2} \times 2\pi RC}$$

As R is predetermined by internal resistance on each output of ADXL:

Choose $R = 32K$ (standard value)

$C = \frac{1}{\sqrt{2} \times 25 \times 2\pi \times 32 \times 10^3} = 0.1407 \times 10^{-6} \approx 0.15\mu F$ (standard value as well)

Results: $R_1 = 32K, R_2 = 32K, C_1 = 0.15\mu F, C_2 = 0.33\mu F$

Check the actual cutoff frequency f_c

$f_c = \frac{1}{2\pi\times32\times10^3\times\sqrt{0.15\times0.33}\times10^{-6}} = 22.35\ Hz$ - a bit lower than expected f_c

Let's adjust value for R_2 as we can choose it freely as long as Q stays around 0.707

Try: $R_2 = 27K$

Check for Q

$Q = \frac{\sqrt{32\times10^3\times27\times10^3\times0.15\times10^{-6}\times0.33\times10^{-6}}}{(32+27)\times10^3\times0.15\times10^{-6}} = 0.739$ (expected 0.707)

As Q is so far off, behaviour of filter might be altered. That calls for reworking coefficients m and n

Choose $m = 3.95, n = 3.1$, since it is easy to find capacitors for such ratio n

As it is predetermined by ADXL that $R_1 = 32K$

$R_2 = \frac{R_1}{m} = \frac{32\times10^3}{3.95} = 8.10 \times 10^3 \approx 8.2K$ (table value)

Compute actual m

$m = \frac{32}{8.2} = 3.90$

Now, calculate C for chosen f_c

$C = \frac{1}{\sqrt{3.1\times3.90}\times25\times2\pi\times8.2\times10^3} = 2.23 \times 10^{-7} \approx 0.22\mu F$ (table value)

$C_2 = nC = 0.22 \times 10^{-6} \times 3.1 = 0.682 \times 10^{-6} \approx 0.68\mu F$ (table value)

<u>Results: $R_1 = 32K, R_2 = 8.2K, C_1 = 0.22\mu F, C_2 = 0.68\mu F$</u>

Check for actual f_c

$$f_c = \frac{1}{2\pi \times \sqrt{32 \times 8.2 \times 0.22 \times 0.68} \times 10^3 \times 10^{-6}} = 25.4Hz$$

Taking into account tolerance of 15% on R_1, f_c is expected to be within 23.7 – 27.6Hz

Finally, check for actual Q

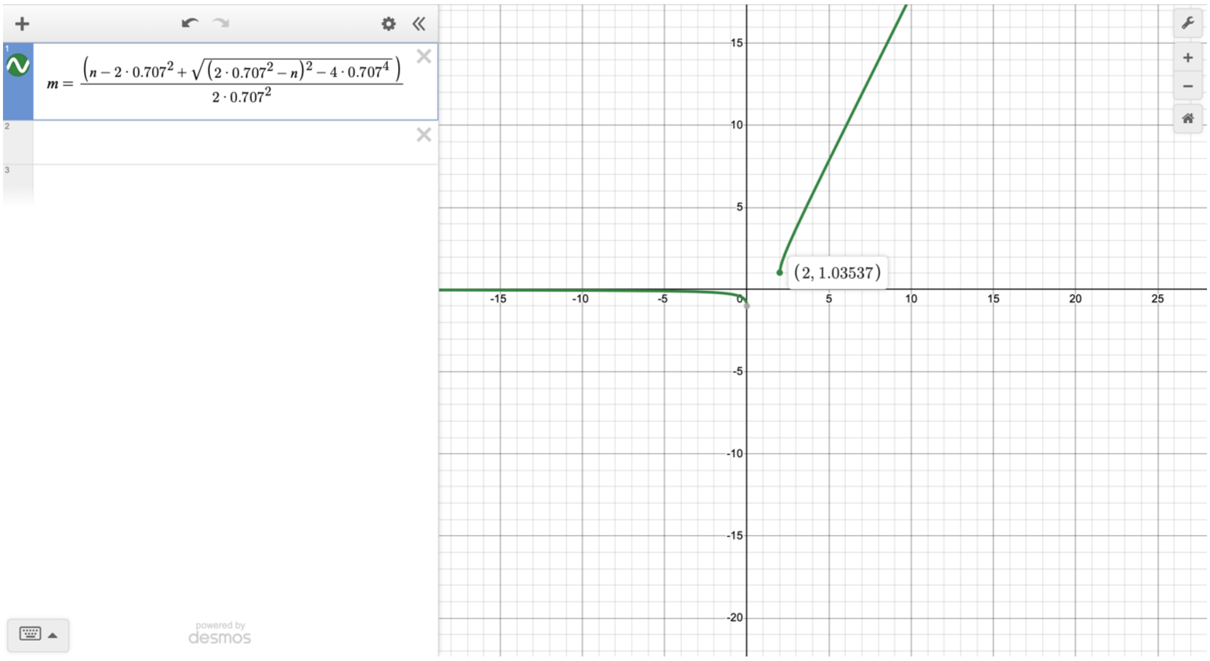
$$Q = \frac{\sqrt{32\times10^{3}\times8.2\times10^{3}\times0.22\times10^{-6}\times0.68\times10^{-6}}}{(32+8.2)\times10^{3}\times0.22\times10^{-6}} = 0.708$$ - very close to expected 0.707

Taking into account tolerance of 15% on R_1, Q is expected to be within 0.679 – 0.742

To negate very low tolerance on R_1, it was decided to not use the resistor from ADXL and rather place another 32K resistor with higher tolerance, separated from ADXL using buffer amplifier.

Appendix:

A. Graph for n and m



B. Another trial for m and n:

Choose n=2.2, m = 1.87

Hence R1 = 32K (from datasheet)
R2 = 17.65 ~ 18K

Calculate C for cutoff frequency 25Hz
C = 0.17 microF – not table value. Try again
Can be used if fc is changed to 29Hz or 18-19Hz

C. Calculations brief (for slides)

Second order polynomial:

$$\underline{P(s) = s^2 + 1.414s + 1}$$

From the coefficients of the polynomial:

$$\boldsymbol{Q = 0.707}, \qquad \boldsymbol{FSF = 1}$$

For filter with gain K=1, equations from Texas Instruments:

$FSF \times f_c = \frac{1}{2\pi RC\sqrt{mn}}$, $Q = \frac{\sqrt{mn}}{m+1}$ where $R_1 = mR, R_2 = R, C_1 = C, C_2 = nC$

Solving for ratio of m and n we get the graph for m

$$m = \frac{n - 2 \times 0.707^2 + \sqrt{(2 \times 0.707^2 - n)^2 - 4 \times 0.707^4}}{2 \times 0.707^2}$$

Choose $\boldsymbol{m = 3.95}, \boldsymbol{n = 3.1}$, since it is easy to find capacitors for such ratio n

After choosing $R_1 = 32K$ and calculating the rest of components:

$$\boldsymbol{R_1 = 32K, R_2 = 8.2K, C_1 = 0.22\mu F, C_2 = 0.68\mu F}$$

Check for actual f_c

$$f_c = \frac{1}{2\pi \times \sqrt{32 \times 8.2 \times 0.22 \times 0.68} \times 10^3 \times 10^{-6}} = \boldsymbol{25.4Hz}$$

Finally, check for actual Q

$$Q = \frac{\sqrt{32 \times 10^3 \times 8.2 \times 10^3 \times 0.22 \times 10^{-6} \times 0.68 \times 10^{-6}}}{(32 + 8.2) \times 10^3 \times 0.22 \times 10^{-6}} = \boldsymbol{0.708}$$