Solving second order analogue filter

Second order polynomial:

$$P(s) = s^2 + 1.414s + 1$$

Coefficients:

$$a_1 = 1.414, a_0 = 1$$

Substitutions:

$$s = j2\pi\omega, \qquad \omega c = 2\pi fc, \qquad a_1 = \frac{1}{Q}, \qquad \sqrt{a_0} = FSF$$

Hence,

$$Q = 0.707, \qquad FSF = 1$$

For filter with gain K=1, equations from Texas Instruments:

$FSF \times f_c = \frac{1}{2\pi RC\sqrt{mn}}$, $Q = \frac{\sqrt{mn}}{m+1}$ where $R_1 = mR, R_2 = R, C_1 = C, C_2 = nC$

Solving for ratio of m and n

$$0.707 = \frac{\sqrt{mn}}{m+1}$$

$$0.707^2 = \frac{mn}{m^2 + 2m + 1}$$

$$0.707^2 m^2 + (2 \times 0.707^2 - n)m + 0.707^2 = 0$$

$$m = \frac{n - 2 \times 0.707^2 + \sqrt{(2 \times 0.707^2 - n)^2 - 4 \times 0.707^4}}{2 \times 0.707^2}$$

The above gives us a graph for n to m ratio. Here we are free to pick any convenient ratio for the filter.

Choose $n = 2, m = 1$

Desired $f_c = 25Hz$

Substituting in the equations from Texas Instruments to obtain C and R near standard values:

$$25 = \frac{1}{\sqrt{2} \times 2\pi RC}$$

Choose $C = 2.2\ \mu F$ (standard value)

$R = \frac{1}{\sqrt{2} \times 25 \times 2\pi \times 2.2 \times 10^{-6}} = 2046 \approx 2K$ (standard value as well)

<u>Results: $R_1 = 2K, R_2 = 2K, C_1 = 2.2\mu F, C_2 = 4.7\mu F$</u>

Check the actual cutoff frequency f_c

$$f_c = \frac{1}{2\pi \times 2\times 10^3 \times \sqrt{2.2\times 4.7}\times 10^{-6}} = 24.7\ Hz$$ near expected f_c

Appendix:

Graph for n and m