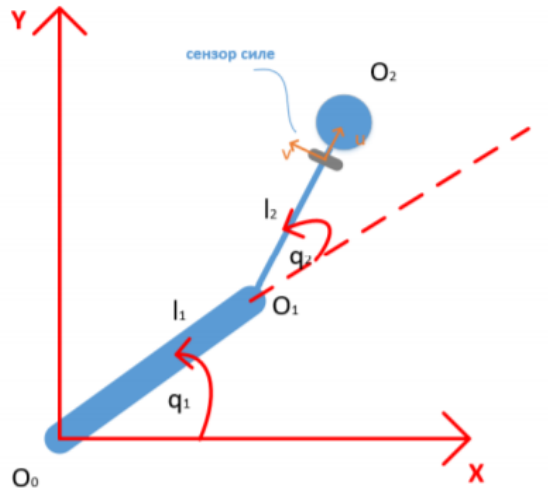
Izvođenje modela robota



Слика 1 Скица робота

```
clear all
sympref('AbbreviateOutput',false);
syms l_1 l_2 q_1 q_2 m_1 m_2 g
x = l_1*cos(q_1)+l_2*cos(q_2+q_1)
```

x = $l_2\cos(q_1+q_2)+l_1\cos(q_1)$

```
y = l_1*sin(q_1)+l_2*sin(q_2+q_1)
```

y = $l_2\sin(q_1+q_2)+l_1\sin(q_1)$

```
J(1,1) = diff(x,q_1);
J(2,1) = diff(y,q_1);
J(1,2) = diff(x,q_2);
J(2,2) = diff(y,q_2);
J
```

J =

$$\begin{pmatrix} -l_2\sin(q_1+q_2)-l_1\sin(q_1) & -l_2\sin(q_1+q_2) \\ l_2\cos(q_1+q_2)+l_1\cos(q_1) & l_2\cos(q_1+q_2) \end{pmatrix}$$

```
syms q_dot_1 q_dot_2 q_ddot_1 q_ddot_2
vx = diff(x,q_1)*q_dot_1+diff(x,q_2)*q_dot_2;
vy = diff(y,q_1)*q_dot_1+diff(y,q_2)*q_dot_2;
v2=simplify(sqrt(vx^2+vy^2));
Ek = simplify(1/6*m_1*(q_dot_1*l_1)^2+1/2*m_2*v2^2)
```

Ek =

$$\frac{m_2\left(l_1^2\,\dot{q}_1^{\,2}+2\cos(q_2)\,l_1\,l_2\,\dot{q}_1^{\,2}+2\cos(q_2)\,l_1\,l_2\,\dot{q}_1\,\dot{q}_2+l_2^2\,\dot{q}_1^{\,2}+2\,l_2^2\,\dot{q}_1\,\dot{q}_2+l_2^2\,\dot{q}_2^{\,2}\right)}{2}+\frac{l_1^2\,m_1\,\dot{q}_1^{\,2}}{6}$$

```
Ep = l_1/2*m_1*sin(q_1)*g + y*m_2*g
```

Ep =

$$g\,m_2\,\left(l_2\sin(q_1+q_2)+l_1\sin(q_1)\right)+\frac{g\,l_1\,m_1\sin(q_1)}{2}$$

```
L=Ek-Ep;

dL_ddq1 = diff(L,q_dot_1);
dL_ddq2 = diff(L,q_dot_2);

syms q_1(t) q_2(t)
q_1 = q_1(t);
q_2 = q_2(t);
q_dot_1 = diff(q_1,t);
q_dot_2 = diff(q_2,t);

eq1=simplify(diff(eval(dL_ddq1),t)-functionalDerivative(eval(L),q_1));
eq2=simplify(diff(eval(dL_ddq2),t)-functionalDerivative(eval(L),q_2));

collect(eq1,[diff(q_1,t,2) diff(q_2,t,2)] )
```

ans =

$$\left(\frac{2\,l_1^2\,m_1}{3}+2\,l_1^2\,m_2+2\,l_2^2\,m_2+4\,l_1\,l_2\,m_2\cos(q_2(t))\right)\frac{\partial^2}{\partial t^2}\,q_1(t)+\left(2\,m_2\,l_2^2+2\,l_1\,m_2\cos(q_2(t))\,l_2\right)\frac{\partial^2}{\partial t^2}\,q_2(t)\,\cdot$$

```
collect(eq2,[diff(q_1,t,2) diff(q_2,t,2)] )
```

ans =

$$\left(2\,m_2\,l_2^2+2\,l_1\,m_2\cos(q_2(t))\,l_2\right)\frac{\partial^2}{\partial t^2}\,q_1(t)+\left(2\,l_2^2\,m_2\right)\frac{\partial^2}{\partial t^2}\,q_2(t)+l_1\,l_2\,m_2\sin(q_2(t))\,\left(\frac{\partial}{\partial t}\,q_1(t)\right)^2-l_1\,l_2\,m_2\,\mathrm{si}$$

```
H(1,1)=subs(eq1,[diff(q_1,t,2),diff(q_1,t) diff(q_2,t,2),diff(q_2,t)],[1 0 0 0]);
H(1,2)=subs(eq1,[diff(q_1,t,2),diff(q_1,t) diff(q_2,t,2),diff(q_2,t)],[0 0 1 0]);
H(2,1)=subs(eq2,[diff(q_1,t,2),diff(q_1,t) diff(q_2,t,2),diff(q_2,t)],[1 0 0 0]);
H(2,2)=subs(eq2,[diff(q_1,t,2),diff(q_1,t) diff(q_2,t,2),diff(q_2,t)],[0 0 1 0]);
H=subs(H,g,0)
```

H =

$$\begin{pmatrix}\frac{2\,l_1^2\,m_1}{3}+2\,l_1^2\,m_2+2\,l_2^2\,m_2+4\,l_1\,l_2\,m_2\cos(q_2(t)) & 2\,m_2\,l_2^2+2\,l_1\,m_2\cos(q_2(t))\,l_2\\ 2\,m_2\,l_2^2+2\,l_1\,m_2\cos(q_2(t))\,l_2 & 2\,l_2^2\,m_2\end{pmatrix}$$