Solutions Sheet

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Exercise 1

(a) We have the random variable $X \sim \mathcal{U}(c,d)$. Therefore we have the pdf :

$$p_x(X) = \begin{cases} \frac{1}{c-d} & c \leq x \leq d \\ 0 & \text{Otherwise} \end{cases}$$

We now create the new random variable $Y(X) = aX + b$. Therefore we can deduct the inverse function:

$$X(Y) = \frac{y-b}{a}$$

Furthermore we can deduct:

$$\frac{\delta X(Y)}{\delta Y} = \frac{1}{a}$$

Since Y(X) is an increasing monotonic function, because $Y'(X) = a > 0$ according to therorem 7.1 :

$$p_y(Y) = p_x(X(Y))\frac{\delta X(Y)}{\delta Y} = \begin{cases} \frac{1}{(c-d)a} & c \leq \frac{y-b}{a} \leq d \\ 0 & \text{Otherwise} \end{cases}$$

(b) Wit the same argumentation as in a we can deduct

$$p_y(Y) = \frac{1}{a(\sqrt{2\pi\sigma^2})} exp(-\frac{(\frac{Y-b}{a} - \mu)^2}{2\sigma^2})$$

Exercise 2

Exercise 2