Solutions Sheet

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Exercise 1

Exercise 2

Exercise 3

(a) *Proof.* Let $k(x_1, x_2) = C$ with $C \in \mathbb{R}_{>0}$. then for $x \in \mathbb{R}^n$ we have:

$$x^T k_{\mathbf{xx}} x = C(\sum_{i=1}^{n} x_i)(\sum_{j=1}^{n} x_j)$$

We will show that this sum is greater or equal to 0. To show that let I be the set of indices from 1 to n. Let further be :

$$P \subseteq I := \{i \in I : x_i \geq 0\}$$

$$N \subseteq I := \{i \in I : x_i < 0\}$$

Then we can write :

$$C(\sum_{i=1} x_i)(\sum_{j=1} x_j) = C(\sum_{i\in P} x_i + \sum_{j\in N} x_j)(\sum_{l\in P} x_l + \sum_{k\in N} x_k)$$

We can now distinguish two cases:

Case 2: $\sum_{i\in P} x_i \geq \sum_{j\in N} |x_j|$ Then we have

$$C(\underbrace{\sum_{i\in P} x_i + \sum_{j\in N} x_j}_{\geq 0})(\underbrace{\sum_{l\in P} x_l + \sum_{k\in N} x_k}_{\geq 0}) \geq 0$$

Case 1: $\sum_{i\in P} x_i < \sum_{j\in N} |x_j|$ Then we have

$$C\,\underbrace{(\underbrace{\sum_{i\in P} x_i + \sum_{j\in N} x_j}_{<0})(\underbrace{\sum_{l\in P} x_l + \sum_{k\in N} x_k}_{<0})}_{>0} > 0$$

So we have

$$x^T k_{\mathbf{xx}} x = C(\sum_{i=1}^{n} x_i)(\sum_{j=1}^{n} x_j) \geq 0$$

And k is thus positive semidefinite $\square$