Solutions Sheet

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Exercise 1

Given: Let $m \leq n \leq k, y \in \mathbb{R}^m, b \in \mathbb{R}^k$ and $A \in \mathbb{R}^{mxn}, B \in \mathbb{R}^{kxn}$ We are considering the follwoing optimization optimization problem:

$$\min_{x \in \mathbb{R}^n} ||Ax - y||_2^2$$
$$\text{s.t.} Bx = b$$

Task: Find a matrix $P \in \mathbb{R}^{(n+k)x(n+k)}$ and a vector $p \in \mathbb{R}^{n+k}$ such that solving :

$$P \begin{bmatrix} x \\ \lambda \end{bmatrix} = p$$

gives a critical point for the optimization problem.

Solution: We will start by defining the Langragian function associated to this problem:

$$L(\lambda) = ||Ax - y||_2^2 + \lambda \cdot (Bx - b)$$

Exercise 2

Exercise 3

Exercise 4