Solutions Sheet

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Exercise 1

(a)

Let $x, b \in \mathbb{R}^m, \alpha \in \mathbb{R}$ and $A \in R^{mxm}$. We want to find the gradient of :

$$\psi = x^T Ax + b^T x + \alpha$$

We have:

$$\begin{aligned} d\psi &= d(x^T Ax + b^T x + \alpha) \\ &= dx^T Ax + db^T x + d\alpha \\ &= dx^T Ax + db^T x \\ &= x^T (A + A^T)dx + db^T x \\ &= x^T (A + A^T)dx + d(b)^T x + b^T dx \\ &= (x^T (A + A^T) + b^T)dx \end{aligned}$$

So:

$$D\psi = x^T (A + A^T) + b^T$$

(b)

Let $x_i \in \mathbb{R}^n$ and $y_i \in \{0, 1\}$ for $1 \leq i \leq n$ We are looking for the dervative of

$$\tau = \sum_{i=1}^{n} (y_i - \sigma(x_i^T \omega + b))^2$$

,where:

$$\sigma(\alpha) = \frac{1}{1 + exp(\alpha)}$$

We have:

$$
\begin{aligned}
d\tau &= d(\sum_{i=1}^{n}(y_i - \sigma(x_i^T\omega + b))^2) \\
&= \sum_{i=1}^{n} d((y_i - \sigma(x_i^T\omega + b))^2) \\
&= \sum_{i=1}^{n} 2(y_i - \sigma(x_i^T\omega + b))^2 + d(y_i - \sigma(x_i^T\omega + b))^2
\end{aligned}
$$

Exercise 2

Exercise 3

Exercise 4