ES 9

Exo 3

We have

$$\mathcal{L}(\theta, \lambda) = \ell(\theta | D) + \lambda \left(\sum_k \pi_k - 1 \right)$$

We will now calculate the derivatives

$$\frac{\partial \mathcal{L}}{\partial \pi_i} = \frac{\partial}{\partial \pi_i} \ell(\theta | D) + \frac{\partial}{\partial \pi_i} \lambda \left(\sum_k \pi_k - 1 \right)$$

$$= \sum_n \frac{\partial}{\partial \pi_i} \log \sum_j \pi_j f_j(x_n | \theta_j) + \lambda$$

$$= \sum_n \frac{1}{\sum_j \pi_j f_j(x_n | \theta_j)} \frac{\partial}{\partial \pi_i} \sum_j \pi_j f_j(x_n | \theta_j) + \lambda$$

$$= \sum_n \frac{1}{\sum_j \pi_j f_j(x_n | \theta_j)} f_i(x_n | \theta_i) + \lambda$$

$$= \sum_n \frac{f_i(x_n | \theta_i)}{\sum_j \pi_j f_j(x_n | \theta_j)} + \lambda$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \frac{\partial}{\partial \lambda} \ell(\theta | D) + \frac{\partial}{\partial \lambda} \lambda \left(\sum_k \pi_k - 1 \right)$$

$$= \sum_k \pi_k - 1$$

b) By setting $\frac{\partial \mathcal{L}}{\partial \lambda} = 0$ we have:

$$\sum_k \pi_k - 1 = 0 \iff \sum_k \pi_k = 1$$

c) By setting $\frac{\partial \mathcal{L}}{\partial \pi_i} = 0$, we obtain: