Name: Section: 202L- Date:

Experiment 6: The Lorentz Force

Objectives

- To learn the basics of good lab practice
- To learn how to estimate uncertainties
- To learn how to propagate uncertainties
- To study the magnetic field produced by a pair of Helmholtz coils
- To study the Lorentz force on a charged particle in a magnetic field

Equipment

- One (1) Cathode Ray Tube (CRT) fixture with two Helmholtz coils (n=320, r=6.8 cm)
- One (1) Cathode Ray Tube (CRT) containing an electron gun, fluorescent grid, and deflection plates.
- One (1) Package of Cables containing:
  - Two (2) long (86 cm) black cables
  - One (1) long red cable
  - Two (2) medium (61 cm) blue cables
  - Four (4) short (36 cm) black cables
  - Two (2) short (36 cm) red cables
  - Three (3) red male-to-female cables
  - One barrel connector
- One (1) Frederiksen 3630 24V Power Supply
- Two (2) Frederiksen 3670 6 kV Power Supplies
- One (1) PASCO 850 interface
- One (1) PASCO Magnetic Field Sensor

Safety---Electrical Shock Hazards

You will be working with voltages significantly above 50 V and currents above 50 mA. You must be ***extra cautious*** when working with the energized equipment in this experiment. Following these instructions will help keep you and the equipment safe:

- ***No food or drink is allowed.*** Water and electrical equipment do not play well together.
- ***Never "get between" a voltage source and ground.*** Do not touch metal parts unless you are **certain** that they do not have voltage on them. Currents as low as 10 mA are painful and currents of 20 mA and higher can do harm if they are allowed to travel through your body---especially across your chest.
- ***Never touch the positive and negative leads together to form a "short" circuit.*** Doing so may cause a spark which can destroy sensitive equipment or cause an electrical fire.
- ***Always turn off the power supply before connecting or disconnecting new circuit elements.*** Doing so will minimize the chance of causing sparks.

Introduction

The interaction of electric *currents* with electric and magnetic fields was first proposed by James Clerk Maxwell when he published his famous equations in *A Dynamical Theory of the Electromagnetic Field* in 1864. A decade later, Henry Rowland discovered that moving electric charges would create a magnetic field just like we find in a current-carrying wire.

In 1897, J.J. Thompson discovered the electron by studying the "cathode rays" that appeared in a Crooke's tube (also known as a "Cathode Ray Tube"). One of the experiments that he performed to show that these "cathode rays" were actually a beam of charged particles was to push them around using both electric and magnetic fields. He was able to easily move them around using magnetic fields, but had a hard time affecting the electrons using electric fields until he was able to produce a much better vacuum within the tube.

J.J. Thompson was able to formulate a basic force law that was obeyed by electrons in a magnetic field, but it took Hendrik Antoon Lorentz to put it in the form we know today:

$$\vec{F} = q\vec{v} \times \vec{B} \quad (1)$$

Using somewhat more modern equipment that is designed to measure these forces, you will be able to reproduce J.J. Thompson's experiment and, using the charge that you determined for the electron in Experiment 3, you will be able to verify Lorentz' equation as well as show that electrons traveling through crossed electric and magnetic fields will remain undeflected if their velocity is given by $v=E/B$.

Theory

Determining the Speed of the Electron

From conservation of energy, we know that the electron will be ejected from an "electron gun" with a kinetic energy that is related to the accelerating voltage, V_a :

$$\frac{1}{2} mv^2 = qV_a \quad (2)$$

We can re-arrange this equation for the speed of the electron:

$$v = \sqrt{\frac{2qV_a}{m}} \quad (3)$$

where m is the electron's mass.

Computing the Applied Fields

The electric and magnetic fields themselves are not known, but the voltages and currents that will be used to make them will be measurable.

The Magnetic Field:

The law of Biot and Savart tells us that the magnetic field produced on the axis of one Helmholtz coil of N turns of wire with a radius of R is given by

$$B = \frac{\mu_0 N I}{2} \frac{R^2}{(R^2 + y^2)^{3/2}} \quad (4)$$

where y is the distance away from the plane of the coil along the axis. In the experiment, a pair of Helmholtz coils will be held apart a distance equal to the radius of one of the coils. At the center of this configuration, the magnetic field will be

$$\begin{aligned} B &= 2\left(\frac{\mu_0 N I}{2}\right)\frac{R^2}{\left(R^2 + (R/2)^2\right)^{3/2}} \\ &= \mu_0 N I \frac{1}{R} \frac{1}{\left(1 + \frac{1}{4}\right)^{3/2}} \\ &= \left(\frac{4}{5}\right)^{3/2} \frac{\mu_0 N}{R} I \end{aligned} \quad (5)$$

The Electric Field:

We could compute the electric field from the plate voltage, V, from

$$E = \frac{V_p}{d_{eff}} \quad (6)$$

where d is the *effective* distance between the plates. It is not the *actual* distance because the parallel-plate capacitor approximation isn't quite good enough for these long, thin plates. What we *do* know (or, better yet, can measure) is the deflection of the electron beam after it travels the 10 cm across the measurement grid.

The time it takes the electrons to traverse the grid is: $t = \Delta x/v$. The vertical acceleration of the electrons is given by

$$a=\frac{F}{m}=\frac{qE}{m} \tag{7}$$

The vertical distance that the electron then travels is given by kinematics:

$$\Delta y=\frac{1}{2}at^2=\frac{qE}{2m}t^2=\frac{qE}{2m}\left(\frac{\Delta x}{v}\right)^2 \tag{8}$$

If we have measured the deflection, we can find the electric field by rearranging Equation (8):

$$E=\frac{2m(\Delta y)v^2}{q(\Delta x)^2}=4V_a\frac{\Delta y}{(\Delta x)^2} \tag{9}$$

Where we have substituted in Equation (3) for the velocity of the electron. Setting E=V/d we find that

$$d_{eff}=\frac{V_p}{\Delta y}\frac{(\Delta x)^2}{4V_a} \tag{10}$$

Determining the Lorentz Force

By Radius of Curvature:

If we apply a magnetic field at right angles to the electron's motion, we find that the electron moves in circular motion. Equating the Lorentz Force with the centripital force required to keep the electron moving in a circle we find that:

$$qvB=m\frac{v^2}{R} \tag{11}$$

If we now rearrange this equation and substitute in equation (3) for the speed of the electron, we find that:

$$BR=\frac{B}{1/R}=\sqrt{\frac{2mV_a}{q}} \tag{12}$$

The odd-looking fraction gives us a way to graphically determine the validity of the Lorentz force law (Equation 1): If we create a graph of B vs. $1/R$, the slope should be the constant value given in Equation (12).

By Balancing with an Electric Force:

If, instead of measuring the radius of curvature of the electron's path, we apply an *electric* field of a known strength, the electric and magnetic forces are balanced when:

$$qE=qvB \quad \Rightarrow \quad v=\frac{E}{B} \tag{13}$$

Uncertainty Analysis

We have many cases where we will want the uncertainty in a quantity: the velocity of the electron, the applied magnetic field, and the effective plate distance. We have three equations (3, 5, and 10) with which we could use propagation of errors, but as you already know, computing these values is very tedious.

Equation (5), for example, tells us what the magnetic field will be given the radius, number of turns, and applied current in a pair of perfectly-made coils exactly a distance R apart. Experimentally, however, the magnetic field must be slightly different from the theoretical predictions because the coils have a finite thickness and the separation between the coils will not be exactly R. Instead of using the product rule to estimate uncertainties, it is better (and easier!) to actually measure the magnetic field as a function of current. Then, by fitting the measured B vs. I curve with a linear fit, the slope tells us the constant of proportionality between the applied current and generated magnetic field:

$$B = \gamma I \tag{14}$$

It is much easier to perform propagation of errors on this equation knowing the measured uncertainty in γ and the uncertainty in the applied current, *I*.

Similarly, equation (10) gives us the effective plate spacing but it is much easier to plot plate voltage, *V*, versus deflection distance, Δy. Once again, this slope (multiplied by a constant) gives us our effective plate gap and, using the reported uncertainty in that slope, we can use propagation of errors on equation (6) instead.

Experimental Procedure

Measuring the Magnetic Field Produced by the Helmholtz Coils

To begin, we will measure the magnetic field produced by the Helmholtz coils and compare that field with equation (5).

1. Plug in the Magnetic Field Sensor into Analog channel "A" and turn on the PASCO 850 interface.
2. Make sure the switches on the sensor are set to "Radial" and "1x."
3. Start the Capstone program and drag one Graph, one Digits, and one Table onto the main window.
4. Click on the "Hardware Setup" tab in the in the left pane.
5. Click on the Analog channel "A" port in the diagram and select the "Magnetic Field Sensor."
6. Close the tab by clicking "Hardware Setup" again.
7. For the "<Select Measurement>" button on the Digits display, select "Magnetic Field Strength (1x)."
8. In the title bar for the Digits display, click on the left-most (greenish) button which increases the number of decimal digits shown. You should click this button until you see five decimal places displayed.

The graph can be used to verify that you are getting good values for the magnetic field. Otherwise, your data can simply be read off the Digits display.

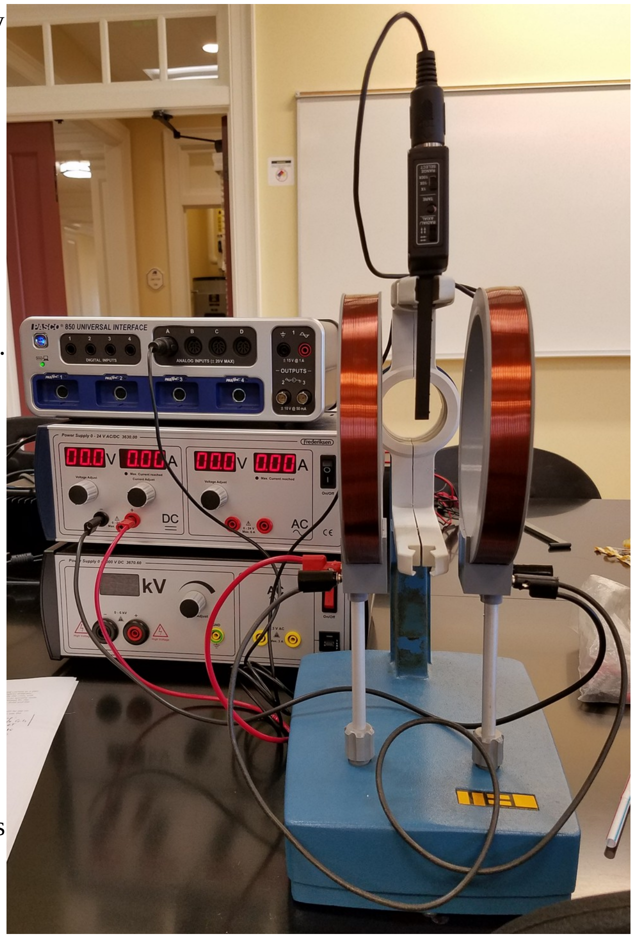

Figure 1: Helmholtz Coil Configuration

The Helmholtz coils have a maximum current rating of 2A. We will now configure the Helmholtz coils and set the current limiter on the 24 V power supply:

1. Ensure that both Helmholtz coils are securely installed into the fixture with the "A" and "Z" connection labels facing away from each other.
2. With the power supply **turned off** and both (DC) knobs rotated **fully CCW**, plug the positive side of the power supply into the "A" port of the left coil and the negative side into the "A" port of the right coil.
3. Connect the two "Z" ports together with a short cable.
4. Turn on the power supply and rotate the **voltage** knob fully clockwise.
5. ***Slowly*** rotate the **current** knob until the current reading is 2.0 A. The current limit is now set. ***Do not touch this knob for the remainder of the experiment.*** If you do, you will have to start again with step 1.

6. Turn the **voltage** knob fully CCW.

You are now ready to take data:

1. Carefully set the Hall Probe in the upper arm of the fixture (see Figure 1.) You might use a piece of tape to help secure it in position with the controls facing towards you.
2. Carefully press the “TARE” button.
3. Take magnetic field data every 0.2 A by **rotating the voltage knob only**. Enter this data into your table and compare the slope of your graph to equation (5).
4. Rotate the voltage knob fully CCW and turn off the power supply when you are done.

N.B. The Hall probe reading may drift at a rate of up to 0.5 μT/s, so your precision will be limited and you will want to try and capture your data within 5–10 minutes of pressing the “TARE” button.

The slope of your graph (and its uncertainty) are the values you will need to utilize equation (14).

Measuring the Electric Field Produced by the Deflection Plates

We now need to insert the Cathode Ray Tube into the holder and connect it to power.

I. Mechanical Assembly:

A. Remove the Hall probe and set it aside.

B. Remove both coils from the assembly. You can leave the banana cables plugged into them if possible.

C. Call your instructor over and have them carefully insert the cathode ray tube into position.

D. Replace the two coils in the same orientation and with the same wiring as in the previous section.

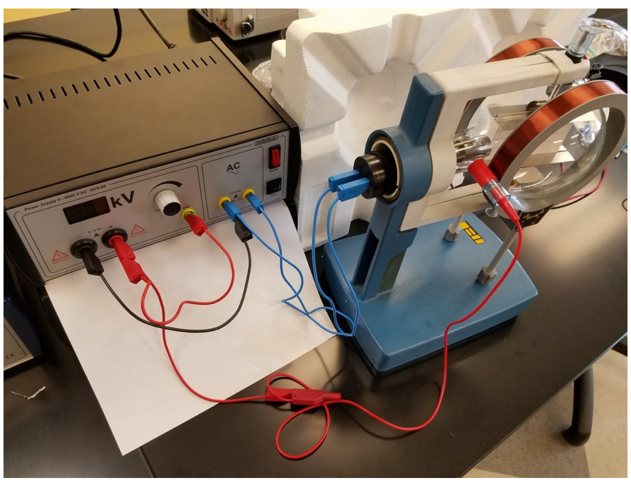

Figure 2: Electron Gun Configuration

II. Electron Gun Connections:

A. Pick one of the two HV power supplies to run the electron gun. **Ensure that it is switched OFF and the voltage knob is rotated fully CCW**. It will have two high-voltage terminals, a ground terminal, and two AC voltage terminals. You will need:

1. Two (2) **blue** banana plug cables
2. One (1) short **red** banana plug cable
3. Two (2) short **black** banana plug cables
4. One (1) long **red** female-to-female banana cable.

B. **Power the heater:** Connect the AC voltage sockets (yellow) to the heater inputs on the back of the black cap of the Cathode Ray Tube (CRT) with the two **blue** banana plug cables.

C. **Power the accelerating grid:**

1. Connect the **negative (black)** high-voltage terminal to one of the blue heater cables (it doesn’t matter which one) with a short **black** cable.
2. Plug the short **red** cable into the **positive (red)** high-voltage terminal.
3. Plug the other end of the **red** cable into the “plastic brick” end of the long **red** female-to-female cable.
4. Connect the other end of the long **red** female-to-female cable to the anode pin on the neck of the CRT. *Be gentle:* you’re working with a thin glass tube!
5. Connect the **positive (red)** high-voltage terminal to the ground (GND) terminal with a short **black** cable.

At this point, the connections should look like those in Figure 2.

III. Electric Field Plate Connections:

A. Your other HV power supply will power the electric field plates. You will only need to use the black and red HV connections and the ground terminal. You will not need the AC connections. **Again, make sure the power supply is switched off and the voltage control knob is fully CCW.** You will need:

1. One (1) barrel connector
2. One (1) long **black** cable
3. One (1) short **red** cable
4. One (1) **red** female-to-female cable
5. One (1) short **black** cable

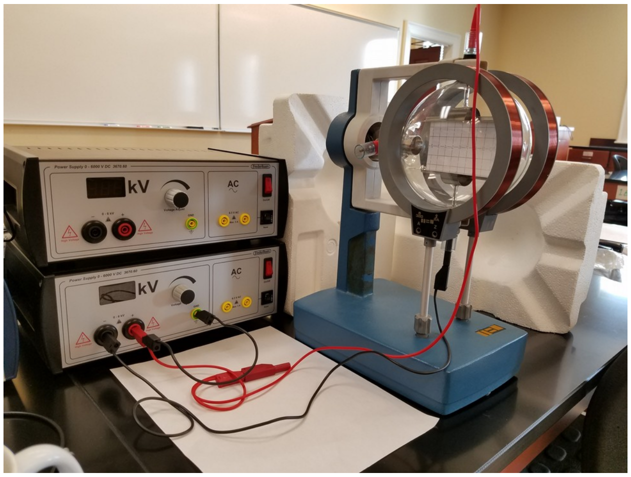

Figure 3: Electric Field Plate Configuration

B. Attach a barrel connector to the lower pin on the CRT. <u>Be Gentle.</u> It should not need too much force.

C. Connect the **negative (black)** terminal of the HV power supply to the barrel connector with a long **black** cable.

D. Plug the short **red** cable into the **positive (red)** terminal of the HV power supply.

E. Plug the other end of the **red** cable into the "plastic brick" end of the long **red** female-to-female cable.

F. Connect the other end of the long **red** female-to-female cable to the upper pin on the CRT. <u>Be gentle</u>. It should not need much force.

G. Connect the **positive (red)** high-voltage terminal to the ground (GND) terminal with a short **black** cable.

These connections should look like those in Figure 3

Have your instructor check all three sets of connections before continuing!

IV. Data Acquisition

You are now ready to compute the slope of a graph of equation (10).

A. Turn on the HV power supply connected to the electron gun and set the accelerating voltage to 2.50 kV.

B. Wait until you see the glowing blue trail of electrons on the mica screen.

C. Note carefully the height of the beam at the x=10 cm mark.

D. Turn on the HV power supply connected to the electrode plates and slowly increase the voltage until the beam is 0.5 cm above its original height. Record the voltage. Do this for heights of 0.5, 1.0, 1.5, and 2.0 cm.

E. Reduce the plate voltage to zero volts.

F. Plot the data and use equation (10) to compute the effective plate gap.

G. You can now use equation (6) to compute the applied electric field from the plate voltage.

Measuring the Lorentz Force on the Electron

You are now ready to verify the Lorentz Force law. Make sure that the electron beam is visible in the tube and is perfectly horizontal.

1. Increase the current (by increasing the voltage) on the Helmholtz coils by 0.05 A.
2. Determine the height of the beam at every "x" value from 2 cm to 10 cm.
3. Use the Mathematica notebook "circlefit.nb" to find the best-fit radius of curvature and its uncertainty.

4. Increase the plate voltage until the beam is once again horizontal.
5. Record your results in the Raw Data Table using equations (12) and (14).
6. Return the plate voltage to zero.
7. Repeat steps 1–6 until the current is at 0.25 A (or you've taken at least five data points)

Clean Up

Once you've completed the experiment, return your apparatus to a safe state by following these steps *in this order*:

1. Ensure you have all the data you need to write your report (see the next page). You might consider uploading your data to your Google Drive.
2. Rotate all knobs on all Power Supplies fully CCW and turn them off.
3. *Gently* unplug all cables from your assembly and return them to the bag.
4. **<u>Carefully</u>** remove the Helmholtz coils from the assembly. **Do not remove the CRT.**
5. Shut down the Capstone program and reboot the computer.

Troubleshooting Capstone's Curve Fitting Algorithm

To determine the slopes of your plotted data, you will use Capstone's curve fitting algorithm (most notably, the "linear fit"). Sometimes this algorithm needs help in finding the correct fit. You can tell that this is true when the drawn curve poorly matches the data points. The slope might be exactly zero or one and the Mean Squared Error (MSE) will also be over 0.1.

To help Capstone attempt a better fit, first, click on the box where the incorrect values are reported. On the left pane will be a "Curve Fit Editor" tab that you should click. Within that tab are the starting values Capstone is assuming. Some of these values will be "locked" (not be allowed to vary) when they should be and vice versa. You will need to make some educated guesses as to the correct fit parameters, enter them into the boxes, and click "Update Fit." You will know if the fit has updated properly, but if you're not sure, ask your instructor because incorrect data will lead to incorrect results!

Raw Data

(1) Printout of Helmholtz Coil “Magnetic Field vs. Applied Current” Table and Graph.

Slope of graph (with uncertainty):

(2) Printout of “Electrode Plate Voltage vs. Deflection” Table and Graph.

Slope of graph (with uncertainty):

(3) Fill out this table:

Accelerating Voltage:

Helmholtz Current (A)	**Computed Magnetic Field (T)**	**Radius of Curvature (m)**	**Applied Plate Voltage (V)**	**Computed Electric Field (V/m)**	**Ratio of Electric Field to Magnetic Field (E/B)**

(4) Printout of “Magnetic Field vs. Reciprocal of Path Radius of Curvature” Table and Graph.

Slope of graph (with uncertainty):

(5) Printout of “Electric Field vs. Magnetic Field” Table and Graph.

Slope of graph (with uncertainty):

Your Lab Report

Introduction

Write a few sentences about what you set out to measure and how you will compare the measured values with theory. Do *not* include details here. That is the job for the rest of your report. (Hint: write this section *last.* This way you'll have the whole experiment in your head when you write it and can properly foreshadow the results.)

Theory

Your theory section should describe the mathematical model that you expect the experiment to match. It should also detail the mathematical method by which you will compute your uncertainties. Make a prediction of your results in this section. Read your Lab Manual for instructions on how to format equations for this section.

Procedure

In one or two paragraphs, describe the methods you used to take your data, any problems you encountered and how you solved them. **You will be graded on your ability to clearly describe what <u>you</u> did.** The description should be clear enough that someone else could reproduce your results by reading this section. **Never use the second person "you" in your lab report.** ***Be Careful:*** There is a fine line between giving enough information so that a competent student could reproduce your results and writing *way too much detail*. The idea is to be <u>concise.</u> If this section is longer than two pages, it is too long.

Data

You must provide the following plots and report their slopes and uncertainties.
(1) The speed of the electrons ejected from the electron gun.
(2) Magnetic Field vs. Current for the Helmholtz coils.
(3) Applied Voltage vs. Vertical Displacement for the Electrode plates. Report the effective plate gap.
(4) Applied Magnetic Field vs. Reciprocal Radius of Curvature for the electron path.
(5) Applied Electric Field vs. Applied Magnetic Field required to straighten out the electron path.

These plots must appear as separate figures in your report. If you need help, ask your instructor how to create a new graph of each from your data using Mathematica, Matlab, Python/MatPlotLib, Excel, or Capstone to create an image you can import into your report.

Consult your Lab Manual for instructions on proper formatting of graphs.

Results and Conclusion

This is the most important section of your report as this section must compare your results with your theoretical predictions. It must be in paragraph form. Make sure you address the following points:

- How does the slope of your Magnetic Field vs. Applied Current graph for the Helmholtz Coils compare to your theoretical value from Equation (5)?
- How does the effective plate separation compare to the physical distance between the plates? You can use the grid on the fluorescent screen to estimate the physical distance.
- How does the slope of your Magnetic Field vs. Reciprocal Radius of Curvature graph compare to your theoretical value from Equation (12)? **What does this say about your measurement of the Lorentz Force Law?**
- How does the slope of your Applied Electric Field vs. Applied Magnetic field compare to your theoretical value from Equation (13)?
- Does the speed of the electrons that you calculated from Equation (3) make sense with the data you collected?
- What sources of uncertainty did you have? How could you have removed or reduced them?

Every comparison you make must be numerical!! Use percent errors. You will lose points if you use subjective comparisons such as (but not limited to): "about," "almost," "close to," "kind of," "roughly," or "sort of." You must quote uncertainties for every value you present!

Appendix

Include your signed and completed raw data sheet(s) and Capstone graphs.
Include a sample calculation for every computation you made.