

## Experiment 3: Electric Field Mapping

### Objectives

- To obtain equipotential lines from measurements
- To construct electric field lines from equipotential lines
- To calculate the magnitude of the electric field from the spacing of the equipotential lines.

### Equipment

- Field mapping board with “Parallel Plate,” “Two Points,” and “Unknown” field plates and U-shaped probe
- Plastic template of probe patterns at each station.
- BK Precision 1651A Power Supply (set to output 12 VDC),
- Protek B940 Digital Multimeter,
- Two (2) pair of banana-plug to alligator-clip hook-up wires
- 5x5 mm quadrule ruled paper.
- Colored pencils,
- French Curve, 30 cm Ruler.

## Introduction

In physics, a “field” is the assignment of a value to every point in space. A scalar field assigns a number to every point in space. Typical examples of scalar fields would be a temperature field which would describe the temperature at every point within of a region of space or a pressure field which would describe the pressure of a fluid at every point within the fluid. When we talk about forces, we create a *vector* field: we assign a vector to every point in space which represents the strength and direction of a force.

When a force is “conservative,” we can store energy by doing work against the force. The energy we store is called “potential energy.” Gravity and electricity are forces which are conservative. If you lift an object, the work you do against gravity is returned when the object is dropped. If you push an electron towards a negatively charged plate, this electron will return the energy you stored when it is released. Friction, however, is not conservative---any work done against friction is lost.

If you lift the same object to the same height above the ground, you will store the same amount of gravitational potential energy. By connecting all the points on Earth that would store the same amount of energy for that object, you have drawn a “surface of equal potential” which is usually called an **equipotential surface**. If we compare the equipotential surfaces to the vector fields that represent the force, we find that every vector is normal (perpendicular) to the equipotential surface at that location. See **Figure 1**.

In the figure, the dashed lines are **equipotential lines**. Each line represents an equal step in potential energy. Two 1~kg objects will have the same amount of potential energy if they live on the same equipotential line. Notice that the equipotential lines near the Earth (**Figure 1a**) are horizontal, and the ones for distances large enough that we must take into account the inverse-squared law (**Figure 1b**) are circular and centered on the Earth. The blue arrows represent the vector force of gravity---the gravitational *field*,  $\vec{g}$ . Notice that the gravitational field lines are always perpendicular to the equipotential surface at that location.

Because the force law for electrical forces is of precisely the same form as for the gravitational force, we can draw many parallels between the two forces. The object of this lab is to determine the electric field that surrounds charged objects based on the equipotentials that can be measured.

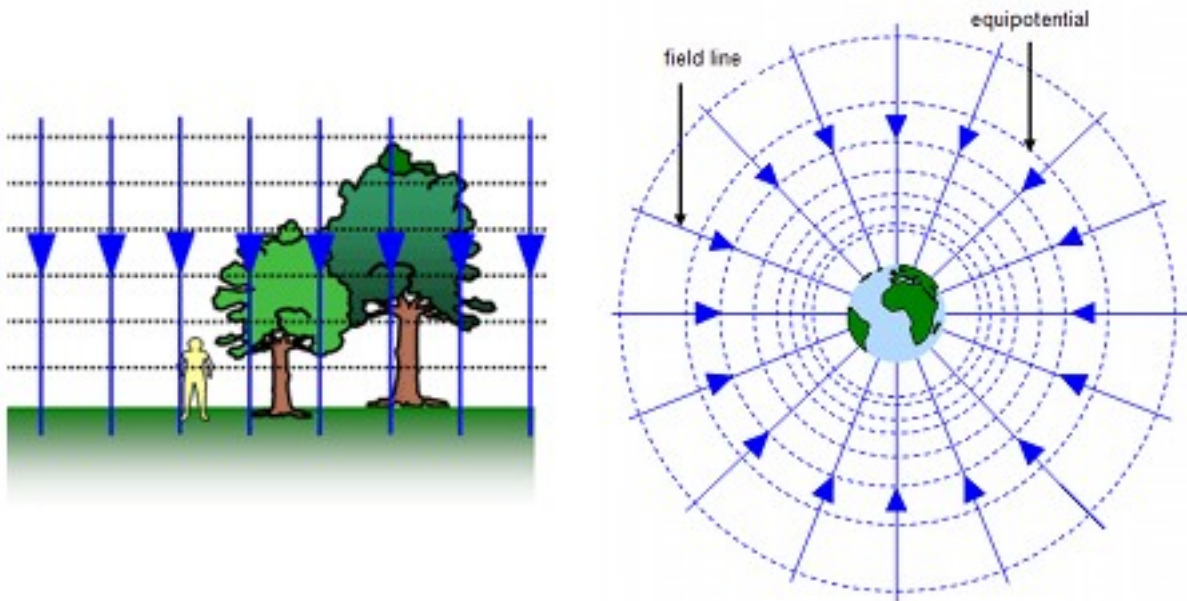


Figure 1: Gravitational Equipotentials and Field Lines

## Theory

### The Electric Field

Equations (1a) and (1b) show the strength of the gravitational and electrical forces:

$$\vec{F}_{gravity} = \frac{G m_1 m_2}{r^2} \quad \vec{F}_{electric} = \frac{k q_1 q_2}{r^2} \quad (1)$$

where  $G$  and  $k$  are constants to make sure the force comes out in Newtons,  $m$  is the mass of each object in kilograms,  $q$  is the charge of each object in Coulombs, and  $r$  is the distance between the two objects in meters. The direction of each force points along the line connecting the two objects.

The vector force field answers the question “How does the second object ‘know’ that the first object is there?” The first object creates a field in all of space and the second object reacts to that field. We define the field created by the first object by either

$$\vec{g} = \frac{G m_1}{r^2} \quad \vec{E} = \frac{k q_1}{r^2} \quad (2)$$

We call  $\vec{g}$  the “gravitational field” produced by  $m_1$  and  $\vec{E}$  the “electric field” produced by  $q_1$ . The direction of either field at a point, P, points along the line connecting the object creating the field to the point, P.

The gravitational field can only point *towards* a massive object because masses can only be positive. Electrical charges, however, can be either positive or negative. A positive charge has an electric field which points *away* from it in all directions and a negative charge has an electric field which points *towards* it from all directions. See **Figure~2**.

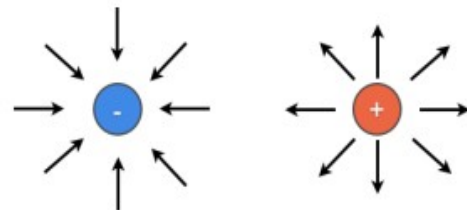


Figure 2: The Electric Field Near Negative and Positive Charges

Equations (1a) and (1b) tell us that the force felt by the second object due to each field is given by:

$$\vec{F}_{gravity} = m_2 \vec{g} \quad \vec{F}_{electric} = q_2 \vec{E} \quad (3)$$

## The Electrical Potential Energy (Voltage)

From equation (3b), it would seem that all we need to do to measure an electric field would be to measure the force on a known charge,  $q$ , and then divide the force by the charge. While it isn't too hard to measure forces (with springs or scales or pendulums), it turns out to be very difficult to measure charge accurately. If it is at all humid, water molecules are attracted to any excess charge and provide a conductive coating which drains the charge away to ground. You may have had trouble keeping a charge on your equipment in the electrostatics experiment because of this. So we need another way to measure electric fields.

Just like gravity, the electric force is conservative. This means we can “store” energy by working against the force. The amount of potential energy we store in an electric field by pushing a charge,  $q$ , “uphill,” is given by the amount of work the field *loses*:

$$\Delta U = -W = -\int \vec{F} \cdot d\vec{r} = -\int q_2 \vec{E} \cdot d\vec{r} \quad (4)$$

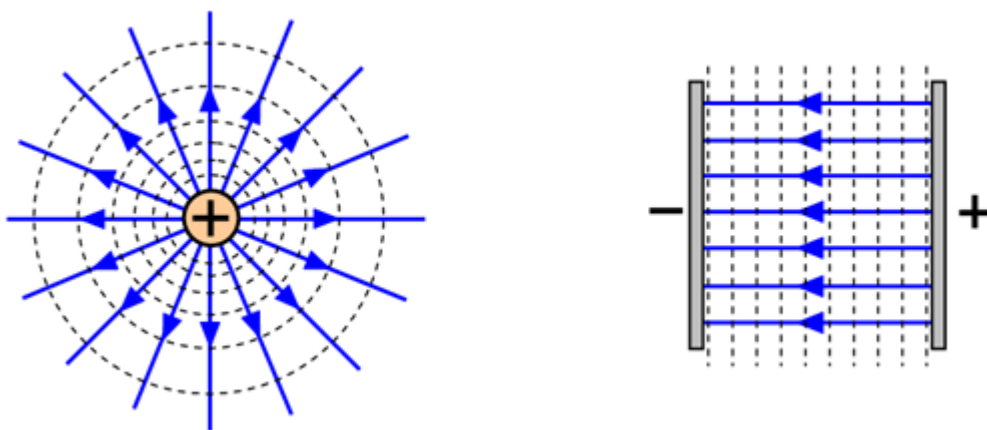
Because it is difficult to measure charge, we divide the far right and far left of equation (4) by the charge,  $q$ :

$$\frac{\Delta U}{q_2} \equiv \Delta V = -\int \vec{E} \cdot d\vec{r} \quad (5)$$

We *define* the **voltage** (electrical potential),  $V$ , as the potential energy of a charge,  $q$ , divided by the value of that charge. For a small enough change in distance, the electric field can be considered constant and the integral in equation (5) becomes

$$\Delta V = -E \Delta x \quad \text{or} \quad E = -\frac{\Delta V}{\Delta x} \quad (6)$$

So if we can measure the change in voltage over a change in distance, we can determine the electric field at that location. The equipotential (equal voltage) lines (dashed) and electric field (blue arrows) are depicted both for a point charge and for a parallel-plate capacitor in **Figure 3**. Once again, the electric field lines are always normal to the equipotential lines at that location.



**Figure 3: Electrical Equipotentials (dashed) and Field Lines (solid) for:  
(a) a Point Charge and (b) Parallel Plates**

## Uncertainty Analysis

Your Lab Manual and the previous labs can guide you in estimating the uncertainties in your measurements and propagating those errors into your computed electric field. The uncertainty in your electrical potential will be determined from either the implied precision of the DMM or the standard deviation of your data, whichever is higher. The uncertainty in your distance will be determined by your measurement error.

## Experimental Procedure

You will use field-mapping boards to locate lines of equipotential by measuring the voltage at various points on the board and connecting points with the same voltage. These data will allow you to construct electric field lines. The field-mapping boards will have one of three different electrode configurations on their underside. You will need to use each one to complete this lab.

## Set Up

1. Set the “A” Channel of the Power Supply to 12 V.
2. Connect a black wire from the black (-) terminal on the power supply to the binding post marked “Bat. Or Osc” closest to the “E1” terminal.
3. Connect a red wire from the red (+) terminal on the power supply to the binding post marked “Bat. Or Osc” closest to the “E7” terminal.
4. Connect a black wire from the binding post marked “Bat. Or Osc” closest to the “E1” terminal to the “COM” port of the DVM.
5. Connect a red wire from the binding post on the probe to the “V, $\Omega$ ,C,Hz” port on the DVM. See **Figure 4** below.
6. Set the DVM knob to “V===”

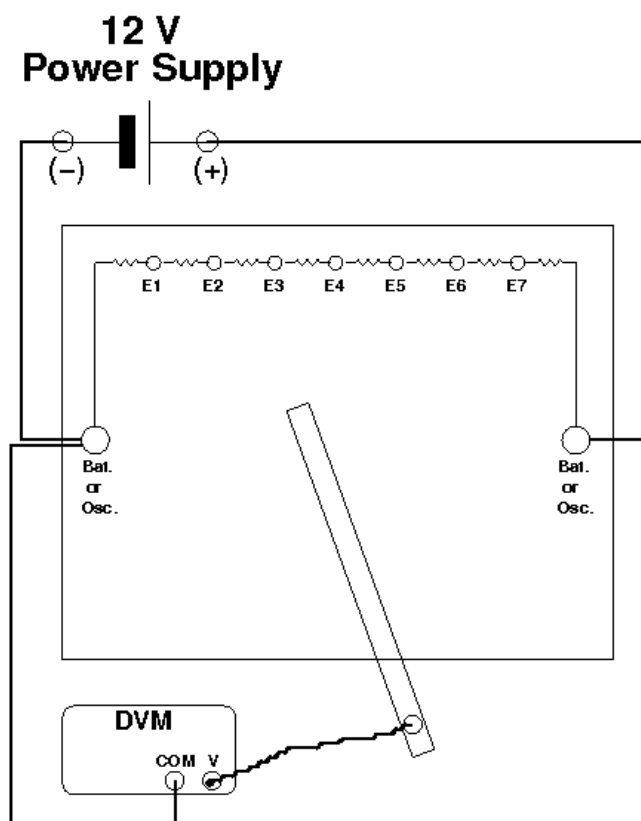


Figure 4: Schematic of Experimental Setup

## Data Acquisition

1. Fasten a sheet of 11 x 8.5 inch graph paper to the top surface of the field-mapping board. Secure the paper under the four rubber bumpers on the board (if you press down on the board, the bumpers will “lift up”). **A small amount of tape should also be used keep the paper from slipping.**
2. If you're using the “Parallel Plate” or “Two Points” board, select the clear plastic template that carries the field plate design underneath the board. Each template has several designs on it. Place the template on the pins just above the upper edge of the paper and trace the design onto the paper to correspond with that of the electrodes on the underside. **Make sure to orient the template properly.**
3. To start plotting equipotential points, start by locating a point which reads 2.0 V on the DVM. It will be difficult to hold the probe steady enough to get exactly 2.000 V, but try to get as close as you can. This location may be marked on the paper by making a pencil mark through the circular hole in the probe. **Do not squeeze the probe---this will give you inaccurate readings.**

4. Move the probe to another point where the voltage is 2.0 V and record it by both placing a mark on the paper and by recording the voltage you measured in the appropriate data table. Continue to find at least seven to ten of these points spread across the board.
5. Label this collection of circles by writing “2.0 V” with a pencil whose color is the same as that used for the circles themselves. Compute the standard deviation of the voltages you recorded. This is the uncertainty in your voltage reading,  $\delta V$ .
6. Repeat steps 3—5 for 4.0 V. Plot the equipotential points for this new voltage with a pencil of a different color (so you will not confuse these points with your 2.0 V data) and use this same color of pencil to label these circles as you did before. Be sure you compute your new standard deviation.
7. Repeat this procedure until the equipotential points are plotted for voltages from 2.0 V through 10.0 V in steps of 2 V.
8. You will be asked to compute the magnitude and direction of the electric field at the center of each pattern. Be sure you have enough data to do this. You may need to measure voltages in steps smaller than 2.0 V to get a reasonable result.

Once you’ve completed this process with one board, select a different field-mapping station and repeat the previous steps. **Do not remove the probe pattern from your board**; simply move to a station with a new board. You will map the equipotential points for three different probe patterns: parallel planes, two points, and an unknown board.

**Before starting the next section, make a Xerox copy of the data for each member of your lab team.**

## Procedure---Equipotentials and Field Lines

1. For every set of equipotential points (*e.g.* 6.0 V), draw a smooth equipotential curve (freehand or with a French curve, **not with a ruler**) which passes through all of the points smoothly. Use a pencil whose color is the same as that of the points themselves. Do you expect some symmetry in the equipotentials? You should account for these symmetries when drawing the equipotential curves.
2. Draw at least seven electric field lines using your equipotentials. Include arrowheads to indicate direction. Use a regular pencil (instead of a colored one).
3. Calculate the magnitude and direction of the electric field at the center of each board. Use Equation (6) to determine  $E$ . Note that  $\Delta x$  is the shortest distance between the equipotential lines, not the horizontal distance. By estimating the uncertainty in  $\Delta x$  and  $\Delta V$  and using uncertainty propagation (Rules R1 and R2), it is possible to estimate how well the electric field has been determined.

## Notes on Drawing Electric Field Lines

To draw a field line:

- Choose a point on the highest equipotential
- Lightly draw the shortest possible line between this point and the next lowest equipotential, with which it will intersect at a point.
- From this new point, lightly draw the shortest possible line between it and the next lowest equipotential. Remember that field lines must be **perpendicular** to every equipotential line they cross and electric field lines *never cross each other*.
- When you reach the lowest equipotential, go back and make the field line smoother and correct errors.
- Along the field line, draw arrows directed from high-potential to low potential.
- Choose another point on the highest equipotential, and repeat.
- Attempt to space your field lines evenly and symmetrically.

## Raw Data

### **Parallel Plate Measured Voltage**

Equipotential	2 V	4 V	6 V	8 V	10 V
Point 1					
Point 2					
Point 3					
Point 4					
Point 5					
Point 6					
Point 7					
Point 8					
Point 9					
Point 10					
Mean Voltage					
Std. Dev.					

### **Two Points Measured Voltage**

Equipotential	2 V	4 V	6 V	8 V	10 V
Point 1					
Point 2					
Point 3					
Point 4					
Point 5					
Point 6					
Point 7					
Point 8					
Point 9					
Point 10					
Mean Voltage					
Std. Dev.					

### **Unknown Configuration Voltage Data**

Equipotential							
Point 1							
Point 2							
Point 3							
Point 4							
Point 5							
Point 6							
Point 7							
Point 8							
Point 9							
Point 10							
Mean Voltage							
Std. Dev.							

# Lab Report

## Introduction

Write a few sentences about what you set out to measure and how you will compare the measured values with the theory. Do *not* include details here. That is the job for the rest of your report. (Hint: write this section *last*. This way you'll have the whole experiment in your head when you write it and can properly foreshadow the results.)

## Theory

Your theory section should describe the mathematical model that you expect the experiment to match. This section should describe the predicted electric field patterns for the two known field plates (the “Parallel Plate” and “Two Points” plates). It should also detail the mathematical method by which you will compute your uncertainties. Make a prediction of your results in this section. Read your Lab Manual for instructions on how to format equations for this section.

## Methods

In one or two paragraphs, describe the methods you used to take your data, any problems you encountered and how you solved them. **You will be graded on your ability to clearly describe what you did.** The description should be clear enough that someone else could reproduce your results by reading this section. **Never use the second person in your lab report.** ***Be Careful:*** There is a fine line between giving enough information so that a competent student could reproduce your results and writing *way too much detail*. The idea is to be concise. If this section is longer than two pages, it is too long.

## Data

Include the following items in this section:

- Your three plots.
- The value of the electric field (along with your uncertainties!) at the center of each plot.
- For each plot, a table of your voltages and their standard deviations for each equipotential.

## Results and Conclusion

Compare your expected results from your Theory section to the results you obtained in your Data section. **Where appropriate, use proper numerical comparisons such as percent differences or percent errors.** Discuss your uncertainties and how you might be able to reduce them. Include the answers to the following questions:

- *Qualitatively* describe the field maps for each of the three different electrode combinations, including discussion of both the equipotential lines and the field lines. Carefully discuss the reason for your electric field measurement on the unknown plate.
- Do your results for the parallel-plate configuration show a constant electric-field between the lines? Make sure you consider the uncertainties in your measurements!
- Compare the shape of the electric field for the parallel-plate electrode board and that of the theoretical gravitational field near the surface of the Earth.

## Appendix

- Your Signed Raw Data sheet(s)
- One sample calculation for each equation you used.