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Implementing Kernel Regression for OFDM System Channel Estimation

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ABSTRACT This paper presents two kernel regression algorithms for channel estimation in OFDM system design, comparing their performance with the Linear Interpolation algorithm. The findings indicate that both the Radial Basis Function kernel regression and Gaussian kernel regression significantly outperform Linear Interpolation in terms of estimation accuracy. Moreover, the channel estimations derived from the kernel algorithms closely match the true channel conditions for each OFDM symbol.

INDEX TERMS channel estimation, Radial Basis Function (RBF) Kernel Regression, Gaussian Kernel Regression, linear interpolation, MATLAB, states tracking, bit error rate.

I. INTRODUCTION

Kernel regression is a non-parametric technique, which is used to estimate the conditional expectation of a dependent variable[1]. Kernel regression is comprehensively used for curve smoothing, and non-linear estimation, offering robust solutions to both regression and classification problems. It can be integrated with machine learning and neural networks for sample training, which extends beyond ordinary applications. The adaptability of kernel regression ensures it processes large amounts of data accurately. It is particularly useful for real-time non-linear data processing and classification.

Gaussian kernel regression and RBF kernel regression are the extensions of the kernel regression algorithm, which adjusts the weights assigned to kernels around the raw estimates, allowing for the adaptive scaling of weights. This approach ensures that the weights of the kernel function are proportional to the system input, enhancing the stability of the kernel regression and the reliability of the estimates. [2].

Linear interpolation is a technique in mathematics, that uses linear polynomials to estimate the unknown values. It is widely used in data science for predicting and visualizing the unknown pattern in a set of data.

OFDM is a critical technology in wireless communications, which marks the crucial advancement from 3G to 4G communications. Nowadays, it is extensively used in wireless communication systems, including 4G, and 5G. Considering, its robustness against multipath fading and spectrum efficiency.

Channel estimation is the process of determining the characteristics of a channel through which a signal has passed, by analyzing the received signal. This step is essential because the signal that travels from the transmitter to the receiver undergoes various changes—often referred to as fading. Channel estimation plays a significant role in the subsequent process of channel equalization, where the target is to recover the originally transmitted symbols from those that have faded during transmission, utilizing the knowledge of the estimated channel h[n,k][3].

Communications channels are subjected to a range of impairments. These include interference from other signals, noise that can be random distortion that alters the signal shape, and fading. Each of these factors can significantly degrade the overall quality of the received signal, making it challenging to accurately interpret the original transmission.

To counteract these impairments, an accurately estimated channel is indispensable.

The evolution of wireless communication technologies and the increasing demand for high-speed and high-performance real-time channel estimation necessitates an easily implementable and power consumption-optimized channel estimation algorithm. Such an algorithm enables more effective channel equalization while maintaining low power consumption. In response to this demand, this paper presents algorithms that utilize RBF kernel regression and Gaussian kernel regression for real-world scenario channel estimation.

II. SYSTEM ANALYSIS

A. System analysis: problem modeling and solutions

The OFDM system is a key modulation and multiplexing technique used in 5G New Radio (NR) to efficiently manage the high data rates and bandwidth demands of modern wireless communication. OFDM works by splitting the currently available spectrum into multiple orthogonal sub-carriers, which significantly reduces interference and maximizes spectral efficiency since the orthogonality of each sub-carrier, makes it ideal for the dense, high-speed requirements of 5G cellular networks. An OFDM system typically comprises several modules, including the transmitter, which involves data serialization, pilot insertion (reference signals) module, Inverse Fourier Transform (IFFT) operations, and adding a cyclic prefix (CP) insertion to manage inter-symbol interference (ISI). On the receiving end, processes such as Fast Fourier Transform (FFT), CP removal, data parallelization, channel estimation, and equalization[4]. The structure of the OFDM is shown in Fig. 1.

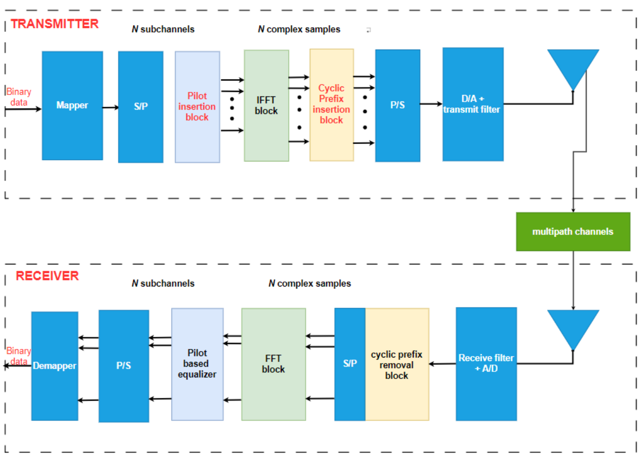

FIGURE 1. The structure of the OFDM system.

In the 5G new radio standard, we can model the channel estimation and equalization as the following process [5]:

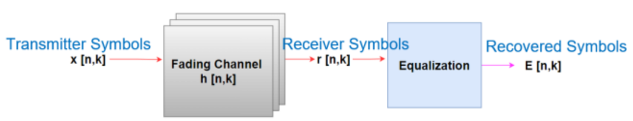

FIGURE 2. The process of channel estimation and equalization.

Where x[n,k] is the transmitted symbol through the fading channel h[n,k]. r[n,k] is the received signal. After equalization, we obtain the raw estimated symbols from the faded symbols E[n,k]. Therefore, we can express the process as:

$$\boldsymbol{r[n,k] = h[n,k]x[n,k] + n[n,k]} \tag{1}$$

$$\boldsymbol{x[n,k] \approx E[n,k] = \frac{r[n,k]}{h[n,k]}} \tag{2}$$

or

$$\boldsymbol{x[n,k] \approx E[n,k] = \frac{h[n,k]r[n,k]}{|h[n,k]|^2 + n[n,k]}} \tag{3}$$

However, the raw estimated symbols from the equalization are too noisy and inaccurate to determine the true symbols. Thus, a lightweight and easy-to-implement algorithm for smoothing the raw symbols is in demand for high-speed and high-performance real-time channel estimation applications.

To solve the issues of inaccurate estimates, this paper introduces two effective methodologies: RBF kernel regression and Gaussian kernel regression. These methods are evaluated against the traditional linear interpolation method to highlight their relative effectiveness and performance.

Kernel regression's core principle is to smoothly fit a curve through data points with minimal deviation from the raw estimates. This fitting is achieved by assigning weights to the kernels. Kernels closer to the raw estimates have more influence than those further away. The "kernel" in kernel regression acts as a weighting function, determining the weight for raw estimates contributes to the regression.

RBF Kernel Regression uses a radial basis function for calculating the weights. Then it assigns the magnitude to each kernel around the raw estimates.

The raw estimates can be obtained by the following equation:

$$\boldsymbol{\hat{h}(n,k)_{raw} \approx \frac{r(n,k)}{x(n,k)_{pilot}}} \tag{4}$$

This methodology obtains the final estimates by convoluting these values and dividing the convoluted estimates by the convoluted indicators. This method does not normalize the impact of each pilot signal based on each kernel weight around the raw estimates[6]. The RBF kernel regression algorithm can be expressed as:

$$\boldsymbol{\hat{h}[n] = \frac{(h_0 * w)(n)}{(z * w)(n)}} \tag{5}$$

Where $\boldsymbol{\hat{h}[n]}$ is the RBF kernel regression estimated channel at n-th subcarrier; $\boldsymbol{\vec{w}}$ is the vector of Gaussian kernel weights. It can be calculated by $\boldsymbol{w = e^{\left(-\frac{1}{2}\left(-\frac{kernel\ length}{\sigma}\right)^2\right)}}$; $\boldsymbol{\overrightarrow{h_0}}$ is the vector of received pilot values placed at their respective subcarrier indices with 0 elsewhere; $\boldsymbol{\vec{z}}$ is an indicator vector that marks the presence of pilot signals 1 at pilot positions and 0 elsewhere.

Gaussian Kernel Regression calculates the normalized weights for each kernel using a Gaussian function through iterations to determine the number of kernels around the raw estimate.

The raw estimates can also be obtained by the following equation:

$$\boldsymbol{\hat{h}(n,k)_{raw} \approx \frac{r(n,k)}{x(n,k)_{pilot}}} \tag{6}$$

This method emphasizes stability and reliability in the estimation process by ensuring a balanced weight distribution, addressing the challenges of channel estimation in wireless communications [7]. Gaussian kernel regression algorithm can be represented as:

$$\widehat{\boldsymbol{h}}[\boldsymbol{n}] = \frac{\sum_{i=1}^{n} w_i h_0[k]}{\sum_{i=1}^{n} w_i} \tag{7}$$

Where $\widehat{\boldsymbol{h}}[\boldsymbol{n}]$ is the Gaussian kernel regression estimated channel n-th; $\boldsymbol{h_0}[\boldsymbol{k}]$ is the received pilot values; $\sum_{\substack{i=1\\j=1}}^{n} \boldsymbol{w_j} = e^{(-\frac{(i-p_j)^2}{2\sigma^2})}$ are the weights calculated using the Gaussian kernel for the distance between the $\boldsymbol{i}$-th subcarrier and the $\boldsymbol{j}$-th pilot subcarrier index $\boldsymbol{p_j}$;

B. System analysis: systems design considerations and tradeoffs

Energy efficiency is a critical consideration in the design of wireless communications systems. Achieving a balance between energy efficiency and high accuracy is essential. Utilizing RBF and Gaussian kernel regression algorithms, systems can more accurately estimate optimal communication channels in real-time. This adaptability is crucial for improving system performance without compromising on energy efficiency. These algorithms significantly enhance transmission accuracy, leading to substantial energy savings by minimizing the need for retransmissions and error correction[8].

The exceptional accuracy and resilience of RBF and Gaussian kernel regression algorithms, especially in non-linear and rapidly evolving environments where the channel has a lot of impairments, highlight the necessity of selecting the appropriate algorithm for channel estimation. System designers must evaluate the capabilities of these algorithms to boost communication reliability under various conditions. This involves understanding the trade-offs associated with deploying more computationally demanding algorithms to achieve the requisite level of system robustness and accuracy[9].

III. SIMULATION DESIGN & IMPLEMENTATION

The simulation begins with the setup of necessary parameters. This includes the definition of constants, simulation options, and initial conditions that will be used throughout the simulation. The parameters set at this stage will define the operating environment for the generation of QPSK symbols and the simulation of channels[10].

Once the simulation parameters are established, the next step involves generating QPSK symbols and channels. These symbols are randomly generated and serve as the input for the communication channel[11].

The optimum kernel window is determined to improve the estimation of the transmitted signal after passing through the channel. Finding the optimal kernel window involves calculating which window width provides the MMSE (minimum mean square error) between the true channel and the estimated channel.

The simulation moves to the transmission stage, where the generated QPSK symbols are modulated onto subcarriers using OFDM. The transmission through a simulated channel introduces impairments such as noise and multipath effects, fading, followed by the reception of the symbols.

For implementing the Gaussian kernel regression algorithm, This method uses the optimum kernel window to perform channel estimation. In parallel to the Gaussian kernel approach, an RBF kernel regression algorithm also uses the optimum kernel window width to perform the channel estimation.

The simulation includes an analysis phase where the Bit Error Rate (BER) is calculated against the Signal-to-Noise Ratio (SNR) to evaluate the performance of the signal transmission and the effectiveness of the kernel regression algorithms for both normalized, unnormalized, and linear interpolation approaches. The flowchart of the simulation design and implementation is shown in Fig. 3.

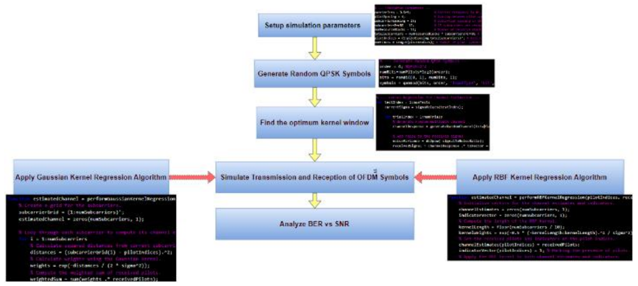

FIGURE 3. **The flowchart of the simulation design and implementation.**

IV. SIMULATION RESULTS FOR VERIFICATION

A. Linear Interpolation

The 3D surface plot in Fig. 4 shows the true channel conditions derived from the linear interpolation channel estimation simulation.

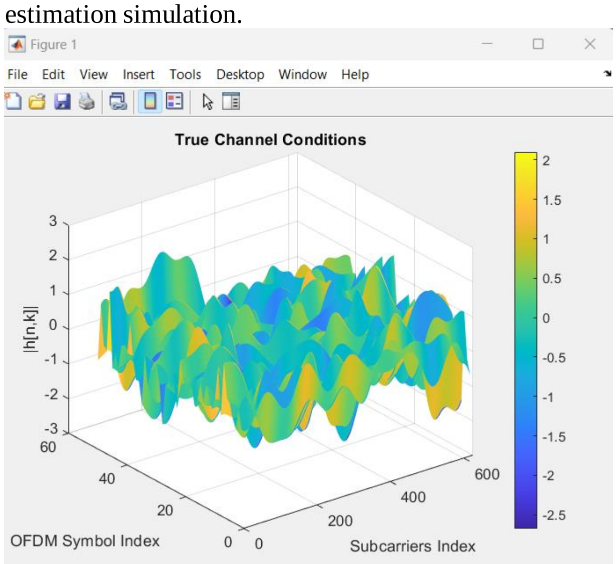

FIGURE 4. **The true channel conditions from the linear interpolation simulation.**

In Figure 5, the 3D surface plot shows the channel estimates obtained via linear interpolation across the OFDM symbol and subcarrier indices at SNR of 10 dB. This visualization clearly shows a noisy estimation result, which can severely degrade the reliability of the subsequent equalization process and the accuracy of data recovery at the receiver side.

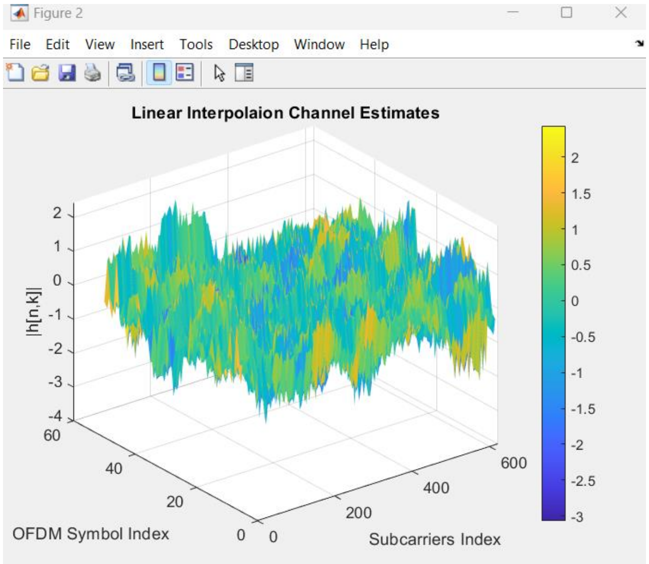

FIGURE 5. The estimated channels using the linear interpolation.

Consequently, the reliability and accuracy of this approach may be further compromised in real-world applications due to non-linear and rapidly evolving environments. Figures 6 and 7 illustrate the results of the single-symbol channel estimation.

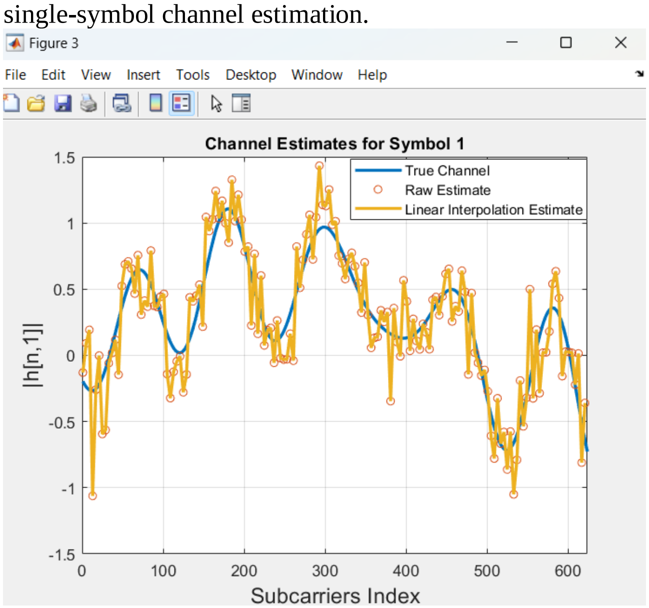

FIGURE 6. The estimated channel of symbol 1 using the linear interpolation.

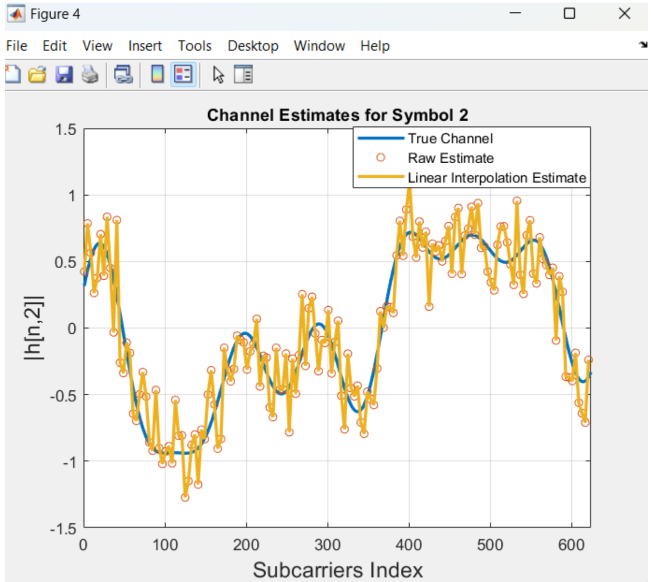

FIGURE 7. The estimated channel of symbol 2 using the linear interpolation.

B. Gaussian Kernel Regression

The 3D surface plot of the true channel conditions, derived from the Gaussian kernel regression simulation, is presented in Figure 8. The 3D surface plot for the Gaussian kernel regression channel estimates at SNR of 10 dB is depicted in Figure 9. By comparing Figures 8 and 9 with Figure 5, this can demonstrate the effectiveness of the Gaussian kernel regression algorithm. It not only closely replicates the true channel conditions but also effectively mitigates the noisy components during the estimation process.

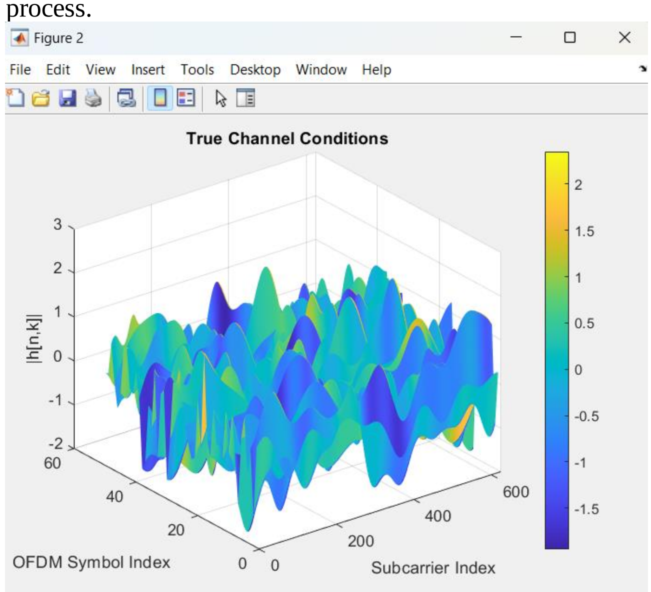

FIGURE 8. The true channel conditions from the Gaussian kernel regression simulation.

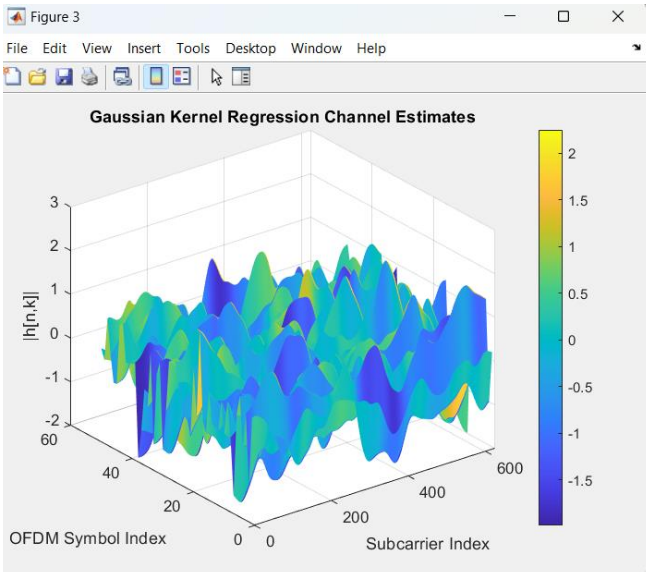

FIGURE 9. The estimated channels using the Gaussian kernel regression

Figures 10 and 11 show the performance of the Gaussian kernel regression algorithm for single-symbol channel estimation. The graphs present a comparative analysis between the true channel conditions and the estimated channels using the Gaussian kernel regression algorithm. The close alignment of the two lines across subcarriers demonstrates the precision of the Gaussian kernel regression in capturing the true state of the channel. This precision is maintained despite inherent noise and estimation errors, proving the algorithm's robustness. The consistent replication of the true channels is shown in both figures.

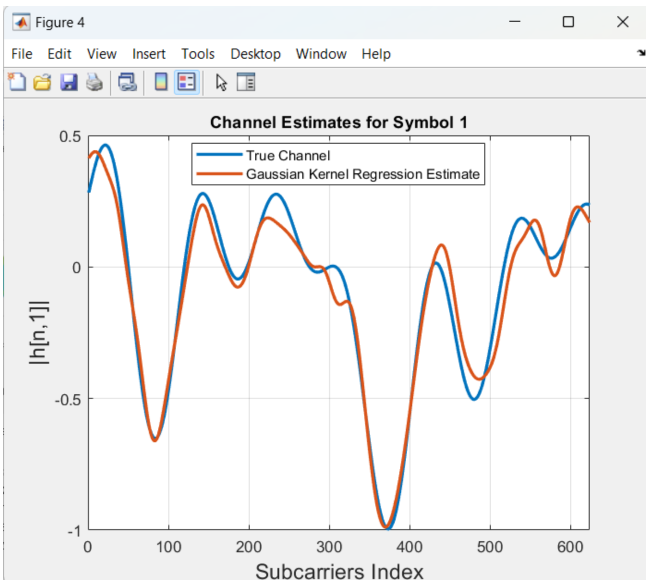

FIGURE 10. The estimated channel of symbol 1 using the Gaussian kernel regression.

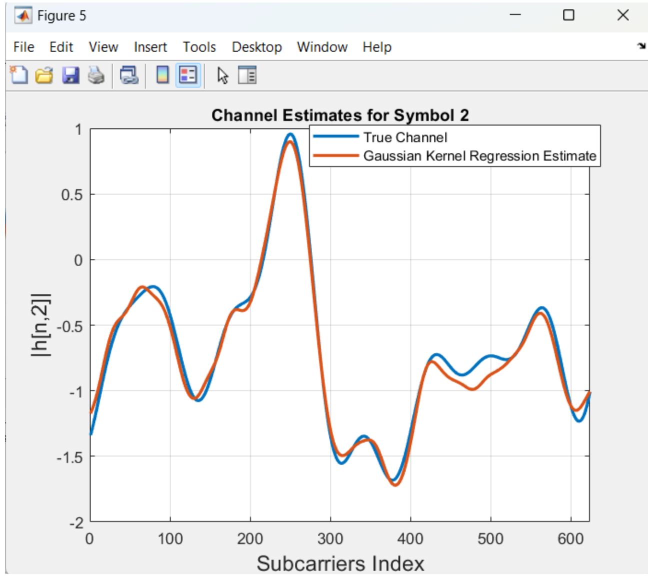

FIGURE 11. The estimated channel of symbol 2 using the Gaussian kernel regression.

C. RBF Kernel Regression

Figure 12 presents a 3D surface plot of the true channel conditions, derived from a simulation using RBF kernel regression. Figure 13 shows a 3D surface plot of the channel estimates obtained through RBF kernel regression at SNR of 10 dB. Comparing Figures 12 and 13 shows the estimation performance of the RBF kernel regression is more reliable and superior to the linear interpolation approach.

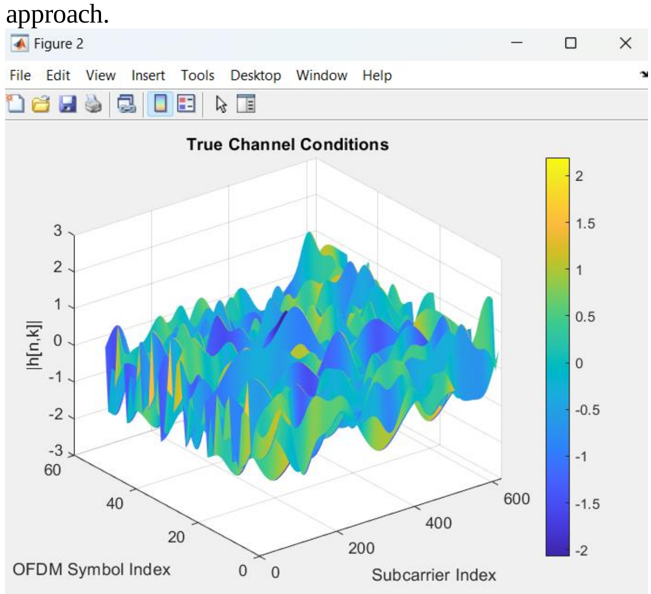

FIGURE 12. The true channel conditions from the RBF kernel regression simulation

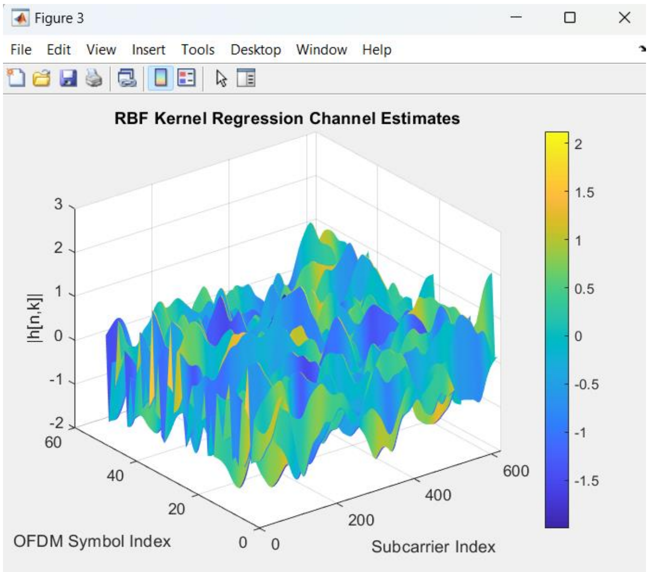

FIGURE 13. The estimated channels using the RBF kernel regression

Figures 14 and 15 display the channel estimation performance of the RBF kernel regression algorithm of single-symbol. These figures can evidently prove the effectiveness of the RBF kernel regression for channel estimation.

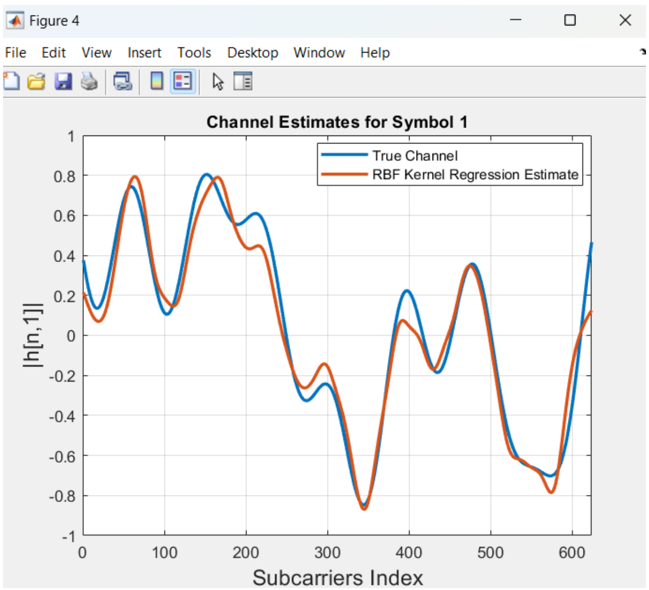

FIGURE 14. The estimated channel of symbol 1 using the RBF kernel regression.

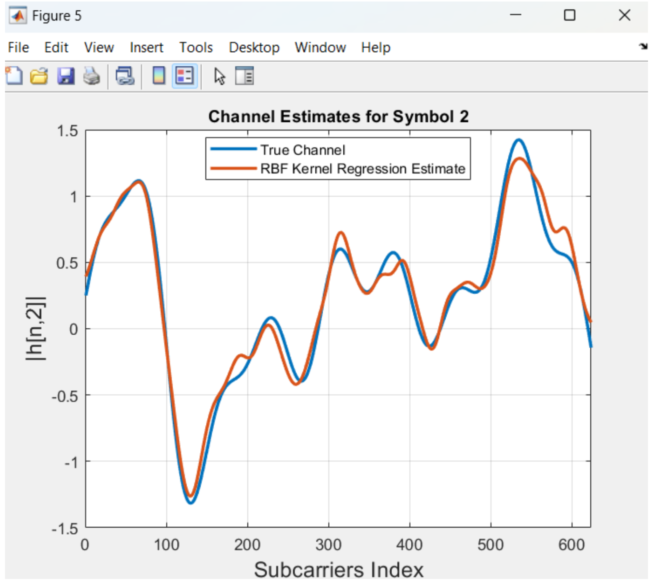

FIGURE 15. The estimated channel of symbol 2 using the RBF kernel regression.

D. BER vs SNR of three approaches

In Fig. 16, we can observe the BER vs SNR for kernel algorithms. The graph plots theoretical values alongside the estimated values from the RBF kernel regression and Gaussian kernel regression algorithms. We can observe both the Gaussian and RBF kernel regression algorithms exhibit remarkably excellent performance across various SNR levels. This underscores the efficiency of both methods in maintaining reliability in channel estimation tasks in noisy conditions.

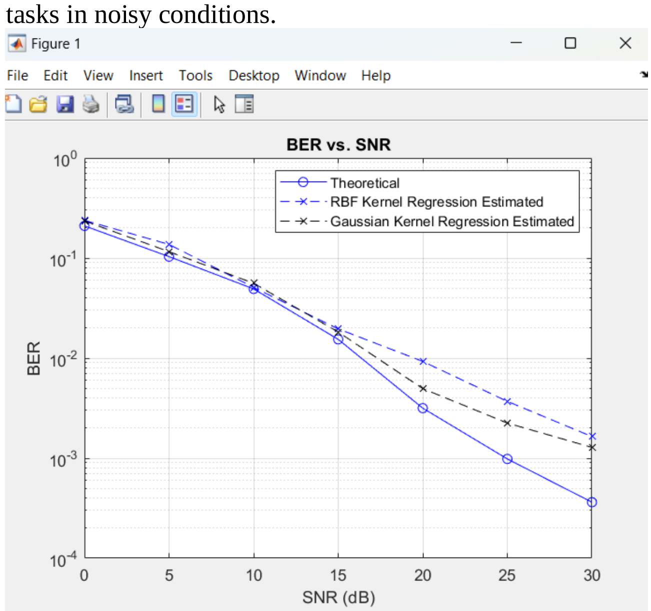

FIGURE 16. The BER vs SNR plot of kernel algorithms

V. RESEARCH FINDINGS & APPLICATION POTENTIAL & FUTURE WORK

The different kernel widths have different MSE between the actual and the estimated channels. This applies to both Gaussian and RBF kernel algorithms. The relationship between MSE and kernel width for both Gaussian kernel regression and RBF kernel regression are shown in Fig. 17 and 18. Thus, in practice, it's essential to identify the minimal kernel window width before proceeding with channel estimation using these algorithms[11] [12].

Given the superior channel estimation performance of both Gaussian kernel regression and RBF kernel regression, these algorithms can be implemented in real-world scenarios of OFDM systems, such that they adapt to non-linear and rapidly evolving environments where the channel has a lot of impairments[13]. This approach provides robust estimation outcomes and optimizes power consumption by reducing the demand for retransmission in real-world applications. Therefore, the potential for practical application of these two algorithms is significant.

Future work will focus on optimizing the computational complexity of these algorithms to prevent computation overhead in channel estimation tasks, despite their excellent performance. Another area of future research aims to achieve further improvements in MSE for enhanced accuracy of these algorithms, considering the restricted requirements of real-world practices in terms of estimation accuracy[14].

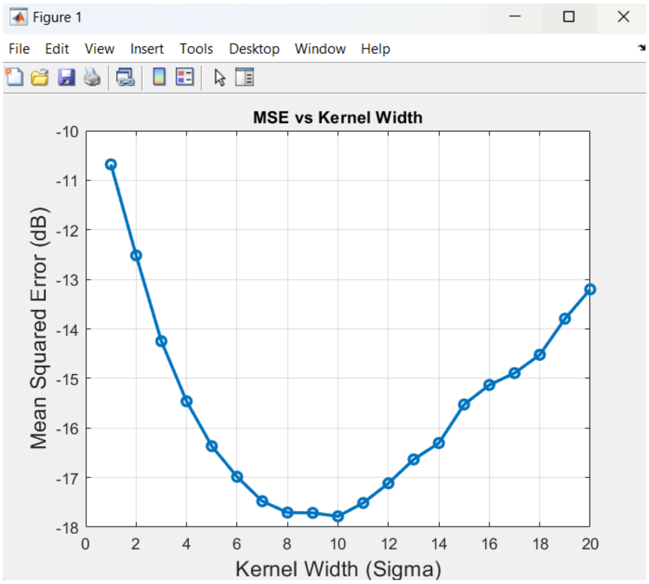

FIGURE 16. **The MSE vs Kernel Width of the Gaussian kernel regression algorithm.**

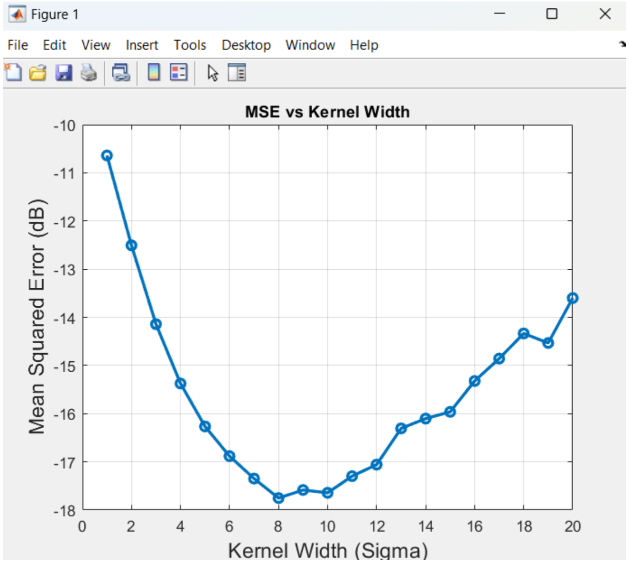

FIGURE 16. **The MSE vs the window width of the RBF kernel regression algorithm.**

VI. CONCLUSION

In conclusion, the application of Gaussian kernel regression and RBF kernel regression for real-time channel estimation has been proven viable. Simulations and experiments affirm the effectiveness and efficiency of these algorithms, demonstrating their excellent accuracy in channel estimation compared to actual channel conditions and in BER vs. SNR performance. These results validate the algorithms' capability to handle channel estimation tasks in real-world scenarios while maintaining low power consumption, making them suitable for various OFDM communication system implementations.

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