

PERFORMATIVE POLICY GRADIENT: OPTIMALITY IN PERFORMATIVE REINFORCEMENT LEARNING

Debabrota Basu, Udvash Das, Brahim Driss

Univ. Lille, Inria, CNRS,
Centrale Lille, UMR 9189 – CRISyAL
F-59000 Lille, France

Uddalak Mukherjee

ACMU, Indian Statistical Institute, Kolkata,
West Bengal 700108, India

ABSTRACT

Post-deployment machine learning algorithms often influence the environments they act in, and thus *shift* the underlying dynamics that the standard reinforcement learning (RL) methods ignore. While designing optimal algorithms in this *performative* setting has recently been studied in supervised learning, the RL counterpart remains under-explored. In this paper, we prove the performative counterparts of the performance difference lemma and the policy gradient theorem in RL, and further introduce the **Performative Policy Gradient** algorithm (**PePG**). **PePG** is the first policy gradient algorithm designed to account for performativity in RL. Under softmax parametrisation, and also with and without entropy regularisation, we prove that **PePG** converges to *performatively optimal policies*, i.e. policies that remain optimal under the distribution shifts induced by themselves. Thus, **PePG** significantly extends the prior works in Performative RL that achieves *performative stability* but not optimality. Furthermore, our empirical analysis on standard performative RL environments validate that **PePG** outperforms standard policy gradient algorithms and the existing performative RL algorithms aiming for stability.

1 INTRODUCTION

Reinforcement Learning (RL) studies the dynamic decision making problems under incomplete information (Sutton & Barto, 1998). Since an RL algorithm tries and optimises an utility function over a sequence of interactions with an unknown environment, RL has emerged as a powerful tool for algorithmic decision making. Specially, in the last decade, RL has underpinned some of the celebrated successes of AI, such as championing Go with AlphaGo (Silver et al., 2014), controlling particle accelerators (St. John et al., 2021), aligning Large Language Models (LLMs) (Bai et al., 2022), reasoning (Havrilla et al.), to name a few. But the existing paradigm of RL assumes that the underlying environment with which the algorithm interacts stays static over time and the goal of the algorithm is to find the utility-maximising, aka optimal policy for choosing actions over time for this specific environment. But *this assumption does not hold universally*.

In this digital age, algorithms are not passive. Their decisions also shape the environment they interact with, inducing distribution shifts. This phenomenon that predictive AI models often trigger actions that influences their own outcomes is termed as *performativity*. In the supervised learning setting, the study of *performative prediction* is pioneered by Perdomo et al. (2020), and then followed by an extensive literature encompassing stochastic optimisation, control, multi-agent RL, games (Izzo et al., 2021; 2022; Miller et al., 2021; Li & Wai, 2022; Eriksson et al., 2022; Narang et al., 2023; Pilipouras & Yu, 2023; Góis et al., 2024; Barakat et al., 2025) etc. There has been several attempts to achieve performative optimality or stability for real-life

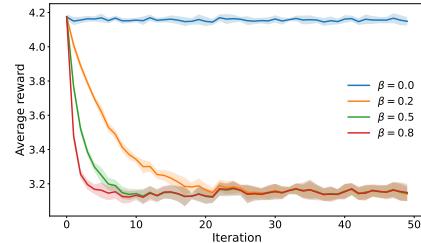


Figure 1: Average reward (over 10 runs) obtained by ERM and Performative Optimal policies across performative strength β .

tasks—recommendation systems (Eilat & Rosenfeld, 2023), measuring the power of firms (Hardt et al., 2022; Mofakhami et al., 2023), healthcare (Zhang et al., 2022) etc. Performativity of algorithms is also omnipresent in practically deployed RL systems. For example, an RL algorithm deployed in a recommender system does not only aim to maximise the user satisfaction but also shifts the preferences of the users in the long-term (Chaney et al., 2018; Mansouri et al., 2020). To clarify the impact of performativity, let us consider an example.

Example 1 (Performative RL in loan approval). *Let us consider a loan approval problem, where an applicant obtains a loan (or get rejected) according to their credit score x , and x depends on the capital of the applicant and that of the population. At each time t , a loan applicant arrives with a credit score x_t sampled from $\mathcal{N}(\mu_t, \sigma^2)$. The bank chooses whom to give a loan by applying a softmax binary classifier $\pi_\theta : \mathbb{R} \rightarrow \{0, 1\}$ on x with threshold parameter θ . This decision has two effects. (a) The bank receives a positive payoff R , if the loan applicant who was granted a loan repays, or else, loses by L . Thus, the bank's expected utility for policy π_θ is $U(\theta, \mu) = \mathbb{E}_{x \sim \mathcal{N}(\mu, \sigma^2)} [\pi_\theta(x)(\mathbb{P}(\text{repayment}|x)R - (1 - \mathbb{P}(\text{repayment}|x))L)]$. (b) Since the amount of capital both the applicant and the population influence the credit score, we model that the change in the population mean μ_{t+1} depends on the bank's policy, via a grant rate $\mathbb{E}_{x \sim \mathcal{N}(\mu_t, \sigma_t^2)} [\pi_\theta(x)]$. Specifically, $\mu_{t+1} = (1 - \beta)\mu_t + \beta f(\mathbb{E}_{x \sim \mathcal{N}(\mu_t, \sigma_t^2)} [\pi_\theta(x)])$, where $\beta \in [0, 1]$ is the performative strength and $f : \mathbb{R} \rightarrow [-M, M]$. Now, if one ignores the performative nature of this decision making problem, and try to find out the optimal with respect to a static credit distribution, it obtains $\theta^{\text{ERM}} \triangleq \arg \max_\theta U(\theta, \mu_0)$. In contrast, if it considers performativity, it obtains $\theta^{\text{Perf}} \triangleq \arg \max_\theta U(\theta, \mu^*(\theta))$. In Figure 1, we show that the average reward obtained by θ^{ERM} and θ^{Perf} are significantly different. This demonstrates why performativity is a common phenomenon across algorithmic decision making problems, and how it changes the resulting optimal solution. Further details are in Appendix B.*

These problem scenarios have motivated the study of performative RL. Though Bell et al. (2021) were the first to propose a setting where the transition and reward of an underlying MDP depend non-deterministically on the deployed policy, Mandal et al. (2023) formally introduced *Performative RL*, and its solution concepts, i.e., performatively stable and optimal policies. Performative stable policies do not get affected or changed due to distribution shifts after deployment. Performatively optimal policies yield the highest expected return once deployed in the performative RL environment. Mandal et al. (2023) proposed direct optimization and ascent based techniques that attains performative stability upon repeated retraining. Extending this work, Rank et al. (2024) and Mandal & Radanovic (2024) managed to solve the same problem with delayed retraining for gradually shifting and linear MDPs. However, *there exists no algorithm yet in performative RL that provably converges to the performative optimal policy*.

As we know from the RL literature, the Policy Gradient (PG) type of algorithms that treats policy as a parametric function and updates the parameters through gradient ascent algorithms are efficient and scalable (Williams, 1992; Sutton et al., 1999; Kakade, 2001). Some examples of successful and popular policy gradient methods include TRPO (Schulman et al., 2015), PPO (Schulman et al., 2017), NPG (Kakade, 2001), which are widely used in modern RL applications. Recent theoretical advances also establish finite-sample convergence guarantees and complexity analyses (Agarwal et al., 2021; Yuan et al., 2022) of PG algorithms. Motivated by the simplicity and universality of the PG algorithms, we ask these two questions in the context of performative RL:

1. How to design PG-type algorithms for performative RL environments to achieve optimality?
2. What are the minimal conditions under which PG-type algorithms converge to the performatively optimal policy?

Our contributions address these questions affirmatively, and showcases the difference of optimality-seeking and stability-seeking algorithms in performative RL.

I. Algorithm Design: We propose the first Performative Policy Gradient algorithm, **PePG**, for performative RL environments. Specifically, we extend the classical vanilla PG and entropy-regularised PG algorithms to Performative RL settings. Though the general algorithm design stays same, we derive a performative policy gradient theorem that shows, evaluation of the gradient involves two novel gradient terms in performative RL – (a) the expected gradient of reward, and (b) the expected gradient of log-transition probabilities times its impact on the expected cumulative return. We lever-

age this theorem to propose an estimator of the performative policy gradient under any differentiable parametrisation.

II. Convergence to Performative Optimality. We further analyse **PePG** (with and without entropy regularisation) for softmax policies, and softmax Performative Markov Decision Processes (PeMDPs), i.e. the MDPs with softmax transition probabilities and linear rewards with respect to the parameters of the softmax policy. We provide a minimal recipe to prove convergence of **PePG** using (a) smoothness of the performative value function, and (b) approximate gradient domination lemma for performative policy gradients. This allows us to show that **PePG** converges to an $(\epsilon + \frac{1}{1-\gamma})$ -ball around performative optimal policy in $\Omega\left(\frac{|\mathcal{S}||\mathcal{A}|^2}{\epsilon^2(1-\gamma)}\right)$ iterations, where $|\mathcal{S}|$ and $|\mathcal{A}|$ are the number of states and actions, respectively.

Specifically, [Mandal et al. \(2023\)](#) frames the question of using policy gradient to find stable policies as an open problem. The authors further contemplate, as PG functions in the policy space, whether it is possible to converge towards a stable policy. In this paper, we affirmatively solve an extension to this open problem for tabular softmax PeMDPs with softmax policies.

III. Stability- vs. Optimality-seeking Algorithms in Performative RL. We further theoretically and numerically contrast the performances of stability-seeking and optimality-seeking algorithms. Theoretically, we derive the performative performance difference lemma that distinguished the effect of policy update in these two types of algorithms. Numerically, we compare the performances of **PePG** with the state-of-the-art MDRR (Mixed Delayed Repeated Retraining ([Rank et al., 2024](#))) algorithm for finding performatively stable policies in the multi-agent environment proposed by ([Mandal et al., 2023](#)). We show that **PePG** yields significantly higher values functions than MDRR, while MDRR achieves either similar or lower distance from stable state-action distribution than **PePG**.

2 PRELIMINARIES: FROM RL TO PERFORMATIVE RL

Now, we formalise the RL and performative RL problems, and provide the basics of policy gradient algorithms in RL.

2.1 RL: INFINITE-HORIZON DISCOUNTED MDPs

In RL, we mostly study Markov Decision Processes (MDPs) defined via the tuple $(\mathcal{S}, \mathcal{A}, \mathbf{P}, r, \gamma)$, where $\mathcal{S} \subseteq \mathbb{R}^d$ is the state space and $\mathcal{A} \subseteq \mathbb{R}^d$ is the action space. Both the spaces are assumed to be compact. At any time step $t \in \mathbb{N}$, an agent plays an action $a_t \in \mathcal{A}$ at a state $s_t \in \mathcal{S}$. It transits the MDP environment to a state s_{t+1} according to a transition kernel $\mathbf{P}(\cdot | s_t, a_t) \in \Delta(\mathcal{S})$. The agent further receives a reward $r(s_t, a_t) \in \mathbb{R}$ quantifying the goodness of taking action a_t at s_t . The strategy to take an action is represented by a stochastic map, called *policy*, i.e. $\pi : \mathcal{S} \rightarrow \Delta(\mathcal{A})$. Given an initial state distribution $\rho \in \Delta(\mathcal{S})$, the goal is to find the optimal policy π^* that maximises the expected discounted sum of rewards, i.e., the *value function*: $V_\pi(\rho) \triangleq \mathbb{E}_{s_0 \sim \rho, s_{t+1} \sim \mathbb{P}(\cdot | s_t, \pi(s_t))} [\sum_{t=0}^{\infty} \gamma^t r(s_t, \pi(s_t))]$, where $\gamma \in (0, 1)$ is called the *discount factor*. γ indicates how much a previous reward matters in the next step, and bounds the effective horizon of a policy to $\frac{1}{1-\gamma}$.

Algorithm 1 Vanilla Policy Gradient

- 1: **Input:** Learning rate $\eta > 0$.
 - 2: **Initialize:** Policy parameter $\theta_0(s, a) \forall s \in \mathcal{S}, a \in \mathcal{A}$.
 - 3: **for** $t = 1$ to T **do**
 - 4: Estimate the gradient $\nabla_\theta V^\pi(\rho) |_{\theta=\theta_t}$
 - 5: **Gradient ascent step:** $\theta_{t+1} \leftarrow \theta_t + \eta \nabla_\theta V^\pi(\rho) |_{\theta=\theta_t}$
 - 6: **end for**
-

gradient ascent on the policy parameter at each step $t \in \mathbb{N}$. As the goal is to maximise $V^\pi(\rho)$, we update θ towards $\nabla_\theta V^\pi(\rho)$, which is the direction improving the value $V^\pi(\rho)$ with a fixed learning

rate $\eta > 0$. For vanilla PG, the policy gradient takes the convenient form leading to estimators computable only with policy rollouts.

Theorem 1 (Policy Gradient Theorem (Sutton et al., 1999)). *Fix a differentiable parameterisation $\theta \mapsto \pi_\theta(a | s)$ and an initial distribution ρ . Let us define the Q-value function $Q^{\pi_\theta}(s, a) \triangleq \mathbb{E}_{s_{t+1} \sim \mathbf{P}_\pi(\cdot | s_t, \pi(s_t))} [\sum_{t=0}^{\infty} \gamma^t r(s_t, \pi(s_t)) | s_0 = s, a_0 = a]$, and advantage function $A^{\pi_\theta}(s, a) \triangleq Q^{\pi_\theta}(s, a) - V^{\pi_\theta}(s)$. Then,*

$$\begin{aligned}\nabla_\theta V^{\pi_\theta}(\rho) &= \frac{1}{1-\gamma} \mathbb{E}_{\tau \sim \mathbb{P}_{\pi_\theta}^{\pi_\theta}} \left[\sum_{t=0}^{\infty} \gamma^t Q^{\pi_\theta}(s, a) \nabla_\theta \log \pi_\theta(a | s) \right] \\ &= \frac{1}{1-\gamma} \mathbb{E}_{\tau \sim \mathbb{P}_{\pi_\theta}^{\pi_\theta}} \left[\sum_{t=0}^{\infty} \gamma^t A^{\pi_\theta}(s, a) \nabla_\theta \log \pi_\theta(a | s) \right].\end{aligned}$$

Since the value function is not concave in the policy parameters, achieving optimality with PG has been a challenge. But practical scalability and efficiency of these algorithms has motivated a long-line of work to understand the minimum conditions and parametric forms of policies leading to convergence to the optimal policy (Agarwal et al., 2021; Mei et al., 2020; Wang & Zou, 2022; Yuan et al., 2022). Our work extends these algorithmic techniques and theoretical insights to performative RL.

2.2 PERFORMATIVE RL: INFINITE-HORIZON DISCOUNTED PEMDPs

Given a policy set $\Pi \in \Pi$, we denote the Performative Markov Decision Process (PeMDP) is defined as the set of MDPs $\{\mathcal{M}(\pi) | \pi \in \Pi\}$, where each MDP is a tuple $\mathcal{M}(\pi) \triangleq (\mathcal{S}, \mathcal{A}, \mathbf{P}_\pi, \mathbf{r}_\pi, \gamma)$. Note, that the transition kernel and rewards distribution are no more invariant with respect to the policy. They shift with the deployed policy $\pi \in \Delta(\mathcal{A})$ (Mandal et al., 2023; Mandal & Radanovic, 2024). In this setting, the probability of generating a trajectory $\tau_\pi \triangleq (s_t, a_t)_{t=0}^\infty$ under policy π with underlying MDP $\mathcal{M}(\pi')$ is given by¹ $\mathbb{P}_{\pi'}^\pi(\tau | \rho) \triangleq \rho(s_0) \prod_{t=0}^{\infty} \pi(a_t | s_t) \mathbf{P}_{\pi'}(s_{t+1} | s_t, a_t)$, where $\rho \in \Delta(\mathcal{S})$ is the initial state distribution. Furthermore, the state-action occupancy measure for deployed policy π and environment-inducing policy π' is defined as $\mathbf{d}_{\pi', \rho}^\pi \triangleq (1-\gamma) \mathbb{E}_{\tau \sim \mathbb{P}_{\pi'}^\pi} [\sum_{t=0}^{\infty} \gamma^t \mathbf{1}(s_t = s, a_t = a) | s_0 \sim \rho]$. Now, we are ready to define the performative expected return, referred as the performative value function that we aim to maximise while solving PeMDP.

Definition 1 (Performative Value Function). *Given a policy $\pi \in \Pi$ and an initial state distribution $\rho \in \Delta(\mathcal{S})$, the performative value function $V_\pi^\pi(\rho)$ is*

$$V_\pi^\pi(\rho) \triangleq \mathbb{E}_{\tau \sim \mathbb{P}_\pi^\pi} \left[\sum_{t=0}^{\infty} \gamma^t \mathbf{r}_\pi(s_t, \pi(s_t)) | s_0 \sim \rho \right]. \quad (1)$$

Equation (2) gives the total expected return that captures the performativity aspect in PeMDPs as the underlying dynamics changes with a deployed policy $\pi(\cdot | s)$.

On a similar note, we define the performative Q-value function (or action-value function) of a policy π as follows.

Definition 2 (Performative Q-value). *Given a policy $\pi \in \Pi$ and a state-action pair $(s, a) \in (\mathcal{S}, \mathcal{A})$, the performative Q-value function $Q_\pi^\pi(s, a)$ is*

$$Q_\pi^\pi(s, a) \triangleq \mathbb{E}_{\tau \sim \mathbb{P}_\pi^\pi} \left[\sum_{t=0}^{\infty} \gamma^t r_\pi(s_t, a_t) | s_0 = s, a_0 = a \right] \quad (2)$$

The Q-value satisfies $Q_\pi^\pi(s, a) = r_\pi(s, a) + \gamma \mathbb{E}_{s' \sim \mathbf{P}_\pi(\cdot | s, a)} [V_\pi^\pi(s')]$. Note that, we can maximise performative value function in two ways: (i) considering π as both the environment-inducing policy and the policy the RL agent deploys, or (ii) deploying π to fix it as the environment-inducing policy and agent plays another policy π' . At this vantage point, let us introduce the notion of optimality and stability of policies in PeMDPs (Mandal et al., 2023).

¹Hereafter, for relevant quantities, π in superscript denotes the deployed policy, and π' in the subscript denotes the environment-inducing, i.e. the policy inducing the transition kernel and reward function that the algorithm interacts with.

Definition 3 (Performative Optimality). *A policy π_o^* is performatively optimal if it maximizes the performative value function.*

$$\pi_o^* \in \arg \max_{\pi \in \Delta(\mathcal{A})} V_\pi^\pi(\rho). \quad (3)$$

Thus, if we play the policy π in the environment induced by policy π to maximise the expected return, we land on the performatively optimal policy.

Definition 4 (Performative Stability). *A policy π_s^* is performatively stable if there is no gain in performative value function due to deploying any other policy than π_s^* in the environment induced by π_s^* .*

$$\pi_s^* \in \arg \max_{\pi \in \Delta(\mathcal{A})} V_{\pi_s^*}^\pi(\rho). \quad (4)$$

As noted by [Mandal et al. \(2023\)](#), a performatively optimal policy may not be performatively stable, i.e., π_o^* may not be optimal for a changed underlying environment $\mathcal{M}(\pi_o^*)$, when it is deployed. Also, in general, the performative value function of π_o^* might be equal to or higher than that of π_s^* . In this paper, we design PG algorithms computing the performative optimal policy for a given set of MDPs, and reinstate their differences with performatively stable policies.

The existing literature on PeMDPs ([Mandal et al., 2023; Mandal & Radanovic, 2024; Rank et al., 2024; Pollatos et al., 2025; Chen et al., 2024](#)) focused primarily on finding a performatively stable policy, i.e. a π_s^* according to Definition 4. In practice, while the notion of stable policies matters for very specific applications, a stable policy may not always suffice. But they might show large sub-optimality gaps, which are often not desired for real-life tasks. *We fill up this gap in literature and propose the first provably converging and computationally efficient PG algorithm for PeMDPs.* Later on, we also empirically show the deficiency of the existing stability finding algorithms if we aim for optimality (Section 5).

Entropy Regularised PeMDPs. Entropy regularisation has emerged as a simple but powerful technique in classical RL to design smooth and efficient algorithms with sufficient exploration. Thus, we study another variant of the performative value function that is regularised using discounted entropy ([Mei et al., 2020; Neu et al., 2017; Liu et al., 2019; Zhao et al., 2019; Flet-Berliac & Basu, 2022](#)). In this setting, the original value function in Definition 1 is regularised using the discounted entropy $H_\pi(\rho) \triangleq \mathbb{E}_{\tau \sim \mathbb{P}_\pi^\pi} [-\sum_{t=0}^{\infty} \gamma^t \log \pi(a_t | s_t)]$. This is equivalent to maximising the expected reward with a shifted reward function $\tilde{r}_\pi(\pi(s_t), s_t) \triangleq r_\pi(\pi(s_t), s_t) - \lambda \log(\pi(a_t | s_t))$ for some $\lambda \geq 0$. \tilde{r}_π is referred as the “soft-reward” in MDP literature ([Wang & Uchibe, 2024; Herman et al., 2016; Shi et al., 2019](#)). This allows us to define the soft performative value function.

Definition 5 (Entropy Regularised (or Soft) Performative Value Function). *Given a policy $\pi \in \Pi$, a starting state distribution $\rho \in \Delta(S)$, and a regularisation parameter $\lambda \geq 0$, the soft performative value function $V_\pi^\pi(\rho)$ is*

$$\begin{aligned} \tilde{V}_\pi^\pi(\rho) &\triangleq \mathbb{E}_{\tau \sim \mathbb{P}_\pi^\pi} \left[\sum_{t=0}^{\infty} \gamma^t (r_\pi(s_t, \pi(s_t)) - \lambda \log \pi(a_t | s_t)) \mid s_0 \sim \rho \right] \\ &= \mathbb{E}_{\tau \sim \mathbb{P}_\pi^\pi} \left[\sum_{t=0}^{\infty} \gamma^t \tilde{r}_\pi(s_t, \pi(s_t)) \mid s_0 \sim \rho \right]. \end{aligned} \quad (5)$$

Since policies belong to the probability simplex, the entropy regularisation naturally lends to smoother and stable PG algorithms. Later, we show that the discounted entropy is a smooth function of the policy parameters for PeMDPs extending the optimization-wise benefits of entropy regularisation to PeMDPs. Additionally, using the notion of soft rewards, we can further define soft performatively optimal and stable policies for entropy regularised PeMDPs. Leveraging it, we *unifiedly design PG algorithms for both the unregularised and the entropy regularised PeMDPs*.

3 POLICY GRADIENT ALGORITHMS IN PERFORMATIVE RL

In this section, we first study the impact of policy updates in PeMDPs. Then, we leverage it to derive the performative policy gradient theorem and design Performative Policy Gradient ([PePG](#)) algorithm for any differentiable parametric policy class.

3.1 IMPACT OF POLICY UPDATES ON PEMDPs

Performance difference lemma has been central in RL to understand the impact of changing policies in terms of value functions (Kakade & Langford, 2002a). It has been also central to analysing and developing PG-type methods (Agarwal et al., 2021; Silver et al., 2014; Kallel et al., 2024). But the existing versions of performance difference cannot handle performativity. Here, we derive the performative version of the performance difference lemma that quantifies the shift in the performative value function due to change the deployed and environment-inducing policies.

Lemma 1 (Performative Performance Difference Lemma). *The difference in performative value functions induced by π and $\pi' \in \Pi$ while starting from the initial state distribution ρ is*

$$\begin{aligned} V_\pi^\pi(\rho) - V_{\pi'}^{\pi'}(\rho) &= \frac{1}{1-\gamma} \mathbb{E}_{(s,a) \sim d_{\pi',\rho}} [A_{\pi'}^{\pi'}(s, a)] \\ &+ \frac{1}{1-\gamma} \mathbb{E}_{(s,a) \sim d_{\pi',\rho}} \left[(r_\pi(s, a) - r_{\pi'}(s, a)) + \gamma (\mathbf{P}_\pi(\cdot|s, a) - \mathbf{P}_{\pi'}(\cdot|s, a))^T V_\pi^\pi(\cdot) \right]. \end{aligned} \quad (6)$$

where $A_{\pi'}^{\pi'}(s, a) \triangleq Q_{\pi'}^{\pi'}(s, a) - V_{\pi'}^{\pi'}(s)$ is the performative advantage function for any state $s \in \mathcal{S}$ and action $a \in \mathcal{A}$.

The crux of the proof is decomposing the performative value through environment-inducing and deployed policies

$$V_\pi^\pi(s_0) - V_{\pi'}^{\pi'}(s_0) = \underbrace{V_\pi^\pi(s_0) - V_{\pi'}^{\pi'}(s_0)}_{\text{performative shift term}} + \underbrace{V_{\pi'}^{\pi'}(s_0) - V_{\pi'}^{\pi'}(s_0)}_{\text{performance difference term}}.$$

- (1) *Connection to Classical RL.* In classical RL, the performance difference lemma yields $V^\pi(\rho) - V^{\pi'}(\rho) = \frac{1}{1-\gamma} \mathbb{E}_{(s,a) \sim d_\rho} [A^{\pi'}(s, a)]$. The first term in Lemma 1 is equivalent to the classical result in the environment induced by π' . But due to environment shift, two more terms appear in the performative performance difference incorporating the impacts of reward shifts and transition shifts.
- (2) *Connection to Performative Stability.* If we ignore the reward and transition shift terms, the performance difference term $V_{\pi'}^{\pi'}(s_0) - V_{\pi'}^{\pi'}(s_0)$ quantifies the impact of changing the deployed policy from π' to π in an environment induced by π' . Thus, a stability seeking algorithm would like to minimise this term, while an optimality seeking algorithm has to incorporate all of the terms.

Now, we ask: *how much do the new environment shift terms change the performative performance difference?*

For simplicity, we focus on the commonly studied PeMDPs with bounded rewards and gradually shifting environments, i.e. the ones with Lipschitz transitions and rewards with respect to the deployed policies (Rank et al., 2024).

Assumption 1 (Bounded reward). *We assume that the rewards are bounded in $[-R_{\max}, R_{\max}]$.*

This is the only assumption needed through the paper and is standard in MDP literature (Mei et al., 2020; Li & Yang, 2023).

Lemma 2 (Bounding Performative Performance Difference for Gradually Shifting Environments). *Let us assume that both rewards and transitions are Lipschitz functions of policy, i.e. $\|r_\pi - r_{\pi'}\|_1 \leq L_r \|\pi - \pi'\|_1$ and $\|\mathbf{P}_\pi - \mathbf{P}_{\pi'}\|_1 \leq L_P \|\pi - \pi'\|_1$, for some $L_r, L_P \geq 0$. Then, under Assumption 1, the performative shift in the sub-optimality gap of a policy π_θ satisfies*

$$\begin{aligned} &\left| V_{\pi_o^*}^{\pi_o^*}(\rho) - V_{\pi_\theta}^{\pi_\theta}(\rho) - \frac{1}{1-\gamma} \mathbb{E}_{(s,a) \sim d_{\pi_\theta,\rho}} [A_{\pi_\theta}^{\pi_\theta}(s, a)] \right| \\ &\leq \frac{2\sqrt{2}}{1-\gamma} (L_r + \frac{\gamma}{1-\gamma} L_P R_{\max}) \mathbb{E}_{s_0 \sim \rho} D_H(\pi_o^*(\cdot|s_0) \| \pi_\theta(\cdot|s_0)). \end{aligned} \quad (7)$$

where $D_H(\mathbf{x} \| \mathbf{y})$ denotes the Hellinger distance between \mathbf{x} and \mathbf{y} .

Implication. Lemma 2 shows novel characterisation of the extra cost we pay to adapt to performativity of the environment in terms of Hellinger distance between the true performatively optimal policy π_o^* and any other parametrised policy π_θ . This implies that the order of difference between

Algorithm 2 PePG: Performative Policy Gradient

-
- 1: **Input:** Transition Feature Map $\psi(s) \forall s \in \mathcal{S}, \xi \in [-R_{\max}, R_{\max}]$ and discount factor γ .
2: **Initialize:** Initial policy parameters θ_0 , initial value function parameters ϕ_0
3: **for** $k = 1, 2, \dots$ **do**
4: **Collect trajectories:** $\mathcal{D}_k = \{\tau_i\}_{i=1}^I$, where each $\tau_i \triangleq \{(s_{i,t}, a_{i,t}, s_{i,t+1}, r_{i,t})\}_{t=0}^{T-1}$ by playing $\pi_{\theta_k} = \pi(\theta_k)$
5: Compute returns $R_k \triangleq \{R_{k,i}\}_{i=1}^I$, where $R_{k,i} = \{R_{k,i,t}\}_{t=0}^{T-1}$
6: Compute advantage estimates $\hat{A}_k(\tau_i)$ using value function $\hat{V}_{\phi_k}(\tau_i)$ for each $\tau_i \in \mathcal{D}_k$ (estimate of $V_{\pi_{\theta_k}}(\tau_i)$ obtained from fitted value network with parameters ϕ_k)
7: **Gradient estimation:** Estimate policy gradient using (11)
8: **Gradient ascent step:** Update policy parameters using (8)
9: Fit value function $V_{\phi_{k+1}}$:
- $$\phi_{k+1} \leftarrow \arg \min_{\phi} \frac{1}{IT} \sum_{i=1}^I \sum_{t=0}^{T-1} (\hat{V}_{\phi_k}(s_t \in \tau_i) - R_{k,i,t})^2$$
- 10: **end for**
-

the optimal performative value function and that of any stability-seeking algorithm is $\Theta(\frac{1}{1-\gamma})$. This significantly improves the known order of sub-optimality achieved by existing algorithms. Specifically, [Mandal et al. \(2023\)](#) show that using repeated policy optimisation algorithms converges to a suboptimality gap $\mathcal{O}\left(\max\left\{\frac{S^{5/3}A^{1/3}\epsilon^{2/3}}{(1-\gamma)^{14/3}}, \frac{\epsilon S}{(1-\gamma)^4}\right\}\right)$. Thus, we see an opportunity to improve on the existing works and design algorithms that can achieve suboptimality gap of order $\Theta(\frac{1}{1-\gamma})$.

Additionally, we note that an optimality-seeking algorithm tries to minimise both the advantage function and the effect of the shifts in the environment quantified by the Hellinger distance, i.e., $D_H(\pi_o^*(\cdot|s_0)\|\pi_\theta(\cdot|s_0))$. While it suffices for a stability-seeking algorithm to minimise the advantage function, and thus, we cannot minimise the RHS of Equation (7) lower than $D_H(\pi_o^*(\cdot|s_0)\|\pi_\theta(\cdot|s_0))$. Thus, optimality-seeking algorithms can achieve a lower performative performance difference than the stability-seeking algorithms if they also learn and incorporate the performative shifts in the environment.

3.2 ALGORITHM DESIGN: PERFORMATIVE POLICY GRADIENT (PePG)

To achieve performative optimality, the goal is to maximise value function at the end of learning process. Gradient ascent is a standard first-order optimisation method to find maxima of a function. Similar to Algorithm 1, the crux of performative policy gradient method lies in the ascent step:

$$\theta_{t+1} \leftarrow \begin{cases} \theta_t + \eta_t \nabla_{\theta} V_{\pi_{\theta}}^{\pi_{\theta}}(\tau) |_{\theta=\theta_t}, & \text{for unregularised objective} \\ \theta_t + \eta_t \nabla_{\theta} \hat{V}_{\pi_{\theta}}^{\pi_{\theta}}(\tau) |_{\theta=\theta_t}, & \text{for Entropy-regularised objective.} \end{cases} \quad (8)$$

Given this ascent step, we have to evaluate the gradient at each time step from the roll-outs of the present policy. In classical PG, the policy gradient theorem serves this purpose ([Williams, 1992](#); [Sutton et al., 1999](#); [Silver et al., 2014](#)). Thus, we derive the performative counterpart of the classic policy gradient theorem.

Theorem 2 (Performative Policy Gradient Theorem). *The gradient of the performative value function w.r.t θ is as follows:*

(a) *For the unregularised objective,*

$$\begin{aligned} & \nabla_{\theta} V_{\pi_{\theta}}^{\pi_{\theta}}(\tau) \\ &= \mathbb{E}_{\tau \sim \mathbb{P}_{\pi_{\theta}}^{\pi_{\theta}}} \left[\sum_{t=0}^{\infty} \gamma^t (A_{\pi_{\theta}}^{\pi_{\theta}}(s_t, a_t) (\nabla_{\theta} \log \pi_{\theta}(a_t | s_t) + \nabla_{\theta} \log P_{\pi_{\theta}}(s_{t+1} | s_t, a_t)) + \nabla_{\theta} r_{\pi_{\theta}}(s_t, a_t)) \right], \end{aligned} \quad (9)$$

(b) *For the entropy-regularised objective, we define the soft advantage, soft Q, and soft value functions with respect to the soft rewards $\tilde{r}_{\pi_{\theta}}$ satisfying $\tilde{A}_{\pi_{\theta}}^{\pi_{\theta}}(s, a) = \tilde{Q}_{\pi_{\theta}}^{\pi_{\theta}}(s, a) - \tilde{V}_{\pi_{\theta}}^{\pi_{\theta}}(s)$ that further*

yields

$$\begin{aligned} \nabla_{\boldsymbol{\theta}} \tilde{V}_{\pi_{\boldsymbol{\theta}}}^{\pi_{\boldsymbol{\theta}}}(\tau) = \\ \mathbb{E}_{\tau \sim \mathbb{P}_{\pi_{\boldsymbol{\theta}}}^{\pi_{\boldsymbol{\theta}}}} \left[\sum_{t=0}^{\infty} \gamma^t \left(\tilde{A}_{\pi_{\boldsymbol{\theta}}}^{\pi_{\boldsymbol{\theta}}}(s_t, a_t) (\nabla_{\boldsymbol{\theta}} \log \pi_{\boldsymbol{\theta}}(a_t | s_t) + \nabla_{\boldsymbol{\theta}} \log P_{\pi_{\boldsymbol{\theta}}}(s_{t+1} | s_t, a_t)) + \nabla_{\boldsymbol{\theta}} \tilde{r}_{\pi_{\boldsymbol{\theta}}}(s_t, a_t | \boldsymbol{\theta}) \right) \right]. \end{aligned} \quad (10)$$

PePG: To elaborate on the design of PePG (Algorithm 2), we focus on the REINFORCE update and softmax policy parametrisation. With the appropriate parameter choices, and initialisation of the policy parameter $\boldsymbol{\theta}$ and value function parameter ϕ , for each episode $k = 1, 2, \dots$, PePG collects I trajectories to calculate return R^i and estimates advantage function \hat{A}_k (Line 4-6). For a particular trajectory τ_i , the estimated advantage for a given state-action is $\hat{A}_{\pi_{\boldsymbol{\theta}_k}}^{\pi_{\boldsymbol{\theta}_k}}(s_t^i, a_t^i) = R_{t,k}^i - V_{\phi_k}(s_t^i)$, where $R^i = \sum_{t=0}^{T-1} \gamma^t r_{\pi_{\boldsymbol{\theta}_k}}(s_t^i, a_t^i)$.

Gradient Estimation (Line 7). With the necessary estimates in hand for all the collected I trajectories, PePG computes average gradient estimate over all the trajectories using

$$\begin{aligned} \widehat{\nabla_{\boldsymbol{\theta}_k} V_{\pi_{\boldsymbol{\theta}_k}}^{\pi_{\boldsymbol{\theta}_k}}} = \frac{1}{I} \sum_{i=1}^I \sum_{t=0}^T \gamma^t & \left(\hat{A}_{\pi_{\boldsymbol{\theta}_k}}^{\pi_{\boldsymbol{\theta}_k}}(s_t^i, a_t^i) \left(\nabla_{\boldsymbol{\theta}_k} \log \pi_{\boldsymbol{\theta}_k}(a_t^i | s_t^i) + \nabla_{\boldsymbol{\theta}_k} \log P_{\pi_{\boldsymbol{\theta}_k}}(s_{t+1}^i | s_t^i, a_t^i) \right) \right. \\ & \left. + \nabla_{\boldsymbol{\theta}_k} r_{\pi_{\boldsymbol{\theta}_k}}(s_t^i, a_t^i | \boldsymbol{\theta}_k) \right) \end{aligned} \quad (11)$$

where all the individual gradients $\nabla_{\boldsymbol{\theta}_k} \log P_{\pi_{\boldsymbol{\theta}_k}}$, $\nabla_{\boldsymbol{\theta}_k} r_{\pi_{\boldsymbol{\theta}_k}}$ and $\nabla_{\boldsymbol{\theta}_k} \log \pi_{\boldsymbol{\theta}_k}$ have the closed form expressions for softmax parametrisation according to Equation (33). Further, in Line 8, PePG updates the policy parameter for the next episode using a gradient ascent step leveraging the estimated average gradient over all I trajectories. Specifically, we plug in $\widehat{\nabla_{\boldsymbol{\theta}_k} V_{\pi_{\boldsymbol{\theta}_k}}^{\pi_{\boldsymbol{\theta}_k}}}$ to both the unregularised and entropy-regularised update rules are given in Equation (8). For the next episode, we again run a regression to update the value network plugging in the current estimates and resume the learning process further.

4 CONVERGENCE ANALYSIS OF PePG: SOFTMAX POLICIES AND SOFTMAX PeMDPs

For rigorous theoretical analysis of PePG, we restrict ourselves to *softmax policy class*, and *softmax PeMDPs*. We define the softmax PeMDPs as the ones having softmax transition kernels with feature map $\psi(\cdot) : \mathcal{S} \rightarrow \mathbb{R}$, and linear reward functions with respect to the policy parameters, for all state $s \in \mathcal{S}$ and action $a \in \mathcal{A}$. Specifically, the class of softmax PeMDPs is $\{\mathcal{M}(\boldsymbol{\theta}) = \mathcal{M}(\pi_{\boldsymbol{\theta}}) \mid \boldsymbol{\theta} \in \mathbb{R}^{|\mathcal{S}| \times |\mathcal{A}|}\}$ such that

$$\pi_{\boldsymbol{\theta}}(a|s) = \frac{e^{\boldsymbol{\theta}_{s,a}}}{\sum_{a'} e^{\boldsymbol{\theta}_{s,a'}}}, \quad \mathbf{P}_{\pi_{\boldsymbol{\theta}}}(s'|s, a) = \frac{e^{\boldsymbol{\theta}_{s,a}\psi(s')}}{\sum_{s''} e^{\boldsymbol{\theta}_{s,a}\psi(s'')}}, \quad r_{\pi_{\boldsymbol{\theta}}}(s, a) = \mathcal{P}_{[-R_{\max}, R_{\max}]}[\xi \boldsymbol{\theta}_{s,a}] \quad (12)$$

where ψ is non-negative and upper bounded by ψ_{\max} , and $\xi \in [0, R_{\max}]$ to align with Assumption 1.

Remark 1. *To prove convergence of policy gradients, one needs to assume a smooth parametrization of the policy and the PeMDPs. Here, we choose softmax PeMDPs as softmax transitions belong to the exponential family - a canonical and representative class for modeling discrete conditional distributions (Xu et al., 2021), which is a standard assumption in RL theory. We would like to highlight that, for our proposed convergence analysis, it is enough to assume smooth paramterisation of transitions and rewards without fixing a specific parametric family. However, we consciously choose to work on softmax PeMDPs to (a) show explicit gradient computations, and (b) establish an explicit relation between parametrisation of PeMDP and smoothness of performatve value functions.*

Thus, we derive the derivatives of policy, transitions, and rewards as

$$\begin{aligned} \frac{\partial}{\partial \theta_{s',a'}} \log \pi_\theta(a|s) &= \mathbb{1}[s = s', a = a'] - \pi_\theta(a'|s) \mathbb{1}[s = s'], \\ \frac{\partial}{\partial \theta_{s',a'}} \log \mathbf{P}_{\pi_\theta}(s''|s, a) &= \psi(s'') \mathbb{1}[s = s', a = a'] (1 - \mathbf{P}_{\pi_\theta}(s''|s, a)) , \\ \frac{\partial}{\partial \theta_{s',a'}} r_{\pi_\theta}(s, a) &= \xi \mathbb{1}[s = s', a = a']. \end{aligned} \quad (13)$$

Given the derivatives, we can now readily estimate the policy gradient and deploy [PePG](#) for softmax PeMDPs.

Convergence Analysis: Challenges and Three Step Analysis. The main challenge to prove convergence of [PePG](#) is that the performative value function is not concave in the parameterisation θ , in general, and also in softmax PeMDPs. The similar issue occurs while proving convergence of PG-type algorithms in classical RL, which has been overcome by leveraging smoothness properties of the value functions and by deriving the local Polyak-Lojasiewicz (PL)-type conditions, known as *gradient domination*, with respect to the policy parameterisation. Leveraging these insights, we devise a three step convergence analysis for [PePG](#).

Step 1: Smoothness of Performative Value Functions. First, we prove that the unregularised performative value function is $\mathcal{O}(\frac{|\mathcal{A}|}{(1-\gamma)^2})$ smooth. As we show that the entropy is also a smooth function for softmax PeMDPs, then under proper choice of the regularisation parameter, i.e., $\lambda = \frac{1-\gamma}{1+2\log|\mathcal{A}|}$, entropy regularised performative value function is also $\mathcal{O}(\frac{|\mathcal{A}|}{(1-\gamma)^2})$ smooth. Since gradient ascent/descent methods can work well in smooth functions, we proceed thoroughly.

Step 2: Gradient Domination for Softmax PeMDPs. Now, the next step is to relate the performative performance difference with the performative policy gradient. This allows us to connect the per iteration improvement in the performative value function with the performative gradient descent at that step. These are known as PL-type inequalities. For non-concave objectives, PL inequalities guarantee convergence to global maxima by showing that the gradient of the objective at any parameter dominates the sub-optimality w.r.t. that parameter.

Lemma 3 (Performative Gradient Domination for Softmax PeMDPs). *Let us consider PeMDPs defined in (12).*

(a) *For unregularised value function,*

$$V_{\pi_o^*}^*(\rho) - V_{\pi_\theta}^*(\rho) \leq \sqrt{|\mathcal{S}||\mathcal{A}|} \left\| \frac{d_{\pi_\theta,\rho}^*}{d_{\pi_\theta,\nu}} \right\|_\infty \|\nabla_\theta V_{\pi_\theta}^*(\nu)\|_2 + \frac{R_{\max}}{1-\gamma} \left(1 + \frac{2\gamma}{1-\gamma} \psi_{\max} \right). \quad (14)$$

(b) *For entropy-regularised value function,*

$$\begin{aligned} \tilde{V}_{\pi_o^*}^*(\rho) - \tilde{V}_{\pi_\theta}^*(\rho) &\leq \sqrt{|\mathcal{S}||\mathcal{A}|} \left\| \frac{d_{\pi_\theta,\rho}^*}{d_{\pi_\theta,\nu}} \right\|_\infty \|\nabla_\theta V_{\pi_\theta}^*(\nu)\|_2 \\ &\quad + \frac{R_{\max}}{1-\gamma} \left(1 + \frac{2\gamma}{1-\gamma} \psi_{\max} \left(1 + \frac{\lambda}{R_{\max}} \log |\mathcal{A}| \right) \right) + \frac{\lambda}{1-\gamma} (1 + 2 \log |\mathcal{A}|). \end{aligned} \quad (15)$$

Remark 2 (Minimal Assumption on PeMDPs for Convergence of [PePG](#)). *Following the proof framework of Lemma 3, a performative gradient domination lemma can be established for a more general class of rewards and transitions with Lipschitz continuous gradients. In this setting, the terms involving the Lipschitz constants of the reward and transition functions, and their gradients reappear in the final gradient domination inequality as additive constants alongside the gradient-norm term. This is also known to be bounded sensitivity condition on the underlying environmental dynamics in control theory ([He et al., 2025](#)) and smoothness in optimisation literature ([Nesterov, 2018](#); [Mahdavi et al., 2013](#)). We derive a new performative gradient domination lemma and corresponding convergence analysis of [PePG](#) under these minimal structural assumption in Appendix J.*

Step 3: Iterative Application of Gradient Domination for Smooth Functions. Now, we can apply gradient domination along with the classic iterative convergence proof of gradient ascent for smooth

Algorithms	Regulariser λ	Min. #samples	Environment
RPO FS (Mandal et al., 2023)	$\mathcal{O}\left(\frac{ \mathcal{S} + \gamma \mathcal{S} ^{3/2}}{(1-\omega)(1-\gamma)^4}\right)$	$\frac{ \mathcal{A} ^2 \mathcal{S} ^3}{\epsilon^4(1-\gamma)^6\lambda^2} \ln (\#\text{iter})$	Direct PeMDPs + quadratic-regul. on occupancy ω -dependence between two envs.
MDRR (Rank et al., 2024)	$\mathcal{O}\left(\frac{ \mathcal{S} + \gamma \mathcal{S} ^{3/2}}{(1-\omega)(1-\gamma)^4}\right)$	$\frac{ \mathcal{A} ^2 \mathcal{S} ^3}{\epsilon^4(1-\gamma)^6\lambda^2} \ln (\#\text{iter})$	Direct PeMDPs + quadratic-regul. on occupancy ω -dependence between two envs.
PePG (This paper)	$\frac{R_{\max}(1-\gamma)}{1+\log(\mathcal{A})}$	$\frac{ \mathcal{S} \mathcal{A} ^2}{\epsilon^2(1-\gamma)^3}$	softmax PeMDPs + entropy regul. on policy
PePG (This paper)	0	$\frac{ \mathcal{S} \mathcal{A} }{\epsilon^2} \max\left\{\frac{\gamma R_{\max} \mathcal{A} }{(1-\gamma)^3}, \frac{\gamma^2}{(1-\gamma)^4}\right\}$	unregularised softmax PeMDPs

Table 1: Comparison of theoretical performance of SOTA stability-seeking algorithms and PePG.

functions. The intuition is that since the per-step sub-optimality is dominated by the gradient and the smooth functions are bounded by quadratic envelopes of parameters, applying gradient ascent iteratively would bring the sub-optimality down to small error level after enough iterations. We formalise this in Theorem 3.

Theorem 3 (Convergence of PePG in softmax PeMDPs). *Let $\text{Cov} \triangleq \max_{\theta,\nu} \left\| \frac{d\pi_{\theta,\nu}^*}{d\pi_{\theta,\nu}} \right\|_\infty$. The gradient ascent algorithm on $V_{\pi_\theta}^*(\rho)$ (Equation (8)) satisfies, for all distributions $\rho \in \Delta(\mathcal{S})$.*

(a) *in the unregularised case with $\eta = \Omega(\min\{\frac{(1-\gamma)^2}{\gamma|\mathcal{A}|}, \frac{(1-\gamma)^3}{\gamma^2}\})$, $\min_{t < T} \left\{ V_{\pi_\theta^*}^*(\rho) - V_{\pi_{\theta_t}}^*(\rho) \right\} \leq \epsilon + \mathcal{O}\left(\frac{1}{1-\gamma}\right)$ when $T = \Omega\left(\frac{|\mathcal{S}||\mathcal{A}|\text{Cov}^2}{\epsilon^2} \max\left\{\frac{\gamma R_{\max}|\mathcal{A}|}{(1-\gamma)^3}, \frac{\gamma^2}{(1-\gamma)^4}\right\}\right)$.*

(b) *in the entropy regularisation scenario with $\lambda = \frac{(1-\gamma)R_{\max}}{1+2\log|\mathcal{A}|}$ and $\eta = \Omega\left(\frac{(1-\gamma)^2}{\gamma|\mathcal{A}|}\right)$, $\min_{t < T} \left\{ \tilde{V}_{\pi_\theta^*}^*(\rho) - \tilde{V}_{\pi_{\theta_t}}^*(\rho) \right\} \leq \epsilon + \mathcal{O}\left(\frac{1}{1-\gamma}\right)$ when $T = \Omega\left(\frac{|\mathcal{S}||\mathcal{A}|^2\text{Cov}^2}{\epsilon^2(1-\gamma)^3}\right)$.*

Implications. (1) We observe that PePG converges to an ϵ -optimal policy in $\frac{|\mathcal{S}||\mathcal{A}|^2}{\epsilon^2(1-\gamma)^3}$ iterations. This reduces the sample complexity required for the existing stability-seeking algorithms by at least an order $\frac{|\mathcal{S}|^2}{\epsilon^2(1-\gamma)^3}$, and shows efficiency of using PePG than the algorithms directly optimising the occupancy measures. (2) Additionally, the regularisation parameters needed for the existing algorithms are pretty big and bigger than $\frac{|\mathcal{S}|}{(1-\gamma)^4}$. This is counter-intuitive and does not match the experimental observations. Here, we prove that setting the regularisation parameter to $\frac{(1-\gamma)R_{\max}}{1+2\log|\mathcal{A}|}$ suffices for proving convergence to optimality. (3) The minimum number of samples required to achieve convergence is proportional to the square of coverage for the softmax PeMDP. This is a ubiquitous quantity dictating convergence of PG-methods in classical RL (Agarwal et al., 2021; Mei et al., 2020), and retraining methods in performative RL (Mandal et al., 2023; Rank et al., 2024). (4) The $\mathcal{O}\left(\frac{1}{1-\gamma}\right)$ suboptimality gap appearing in Theorem 3 is analogous to the effect of using relaxed weak gradient domination result (Yuan et al., 2022, Corollary 3.7). It argues that if the policy gradient in classical MDPs satisfies the relaxed weak gradient domination, i.e., $\epsilon' + \|\nabla_\theta V(\theta)\| \geq 2\sqrt{\mu}(V^* - V(\theta))$ for some $\mu > 0$ and $\epsilon' > 0$, then the corresponding policy gradient method guarantees $\min_{t \in \{0, \dots, T\}} (V^* - V(\theta_t)) \leq \mathcal{O}(\epsilon) + \mathcal{O}(\epsilon')$ for big enough T . Lemma 3 constructs the performative counterpart of this relaxed weak gradient domination property with $\epsilon' = \mathcal{O}\left(\frac{1}{1-\gamma}\right)$.

Remark 3 (Convergence for Generic PeMDPs with Softmax Policy). *We can generalise the convergence analysis of PePG to PeMDPs with Lipschitz gradients (Appendix J). Though the range of the step-size η , the regularisation parameter λ , and the minimum convergence time T remain the same, we pay an extra cost in terms of the unavoidable bias. In softmax PeMDPs, the final bias is $\mathcal{O}(\frac{1}{1-\gamma})$. For general PeMDPs, the final bias increases to $\mathcal{O}(\frac{\text{Cov}}{(1-\gamma)^2})$. This term shows two things. First, PePG can reach closer to optimality whereas a stability seeking algorithm can yield policies which are $\mathcal{O}(\frac{\text{Cov}}{(1-\gamma)^4})$ far away from the optimal value function (Mandal & Radanovic, 2024, Theorem 7). In the literature, we find existing algorithms achieve $\mathcal{O}(\frac{1}{(1-\gamma)^6})$ (Mandal et al., 2023) and $\mathcal{O}(\frac{1}{(1-\gamma)^8})$ (Sahitaj et al., 2025) suboptimality in different settings of performative RL.² This*

²We omit the coverage term as it's expression varies under each setting.

at the same time shows the lacunae of stability seeking algorithms and effectiveness of PePG to achieve optimality. Second, this existence of an irreducible bias at convergence for both stability and optimality seeking algorithms indicate that this might be inherent to performative RL. We further observe that this effect vanishes for PePG if the Lipschitz constants for reward and transition shifts decay to 0 over time (i.e. L_r and L_P in Lemma 2). In future, it would be interesting to prove a lower bound supporting this phenomenon.

5 EXPERIMENTAL ANALYSIS

In this section, we empirically compare the performance of PePG in the performative reinforcement learning setting and analyse its behaviour against the state-of-the-art stability-finding methods.³

Performative RL Environment. We evaluate PePG in the Gridworld test-bed (Mandal et al., 2023), which has become a standard benchmark in performative RL. This environment consists of a grid where two agents A_1 (the principal) and A_2 (the follower), jointly control an actor navigating from start positions (S) to the goal (G) while avoiding hazards. The environment dynamics are as follows: Agent A_1 proposes a control policy for the actor by selecting one of four directional actions. Agent A_2 can either accept this action (not intervene) or override it with its own directional choice. *This creates a performative environment for A_1 , as its effective policy outcomes depend on A_2 's responses to its deployed strategy.*

The cost structure follows: visiting blank cells (S) incurs penalty of -0.01 , goal cells (F) cost -0.02 , hazard cells (H) impose a severe penalty of -0.5 , and any intervention by A_2 results in an additional cost of -0.05 for the intervening agent. The response model also follows that of Mandal et al. (2023), i.e., the agent A_2 responds to A_1 's policy using a Boltzmann softmax operator. Given A_1 's current policy π_1 , we compute the optimal Q-function $Q^{*|\pi_1}$ for each follower agent A_j relative to a perturbed version of the grid world, where each cell types matches A_1 's environment with probability 0.7. We then define an average Q-function over the follower agents and determine the collective response policy via Boltzmann softmax $Q^{*|\pi_1}(s, a) = \frac{1}{n} \sum_{j=2}^{n+1} Q_j^{*|\pi_1}(s, a), \pi_2(a|s) = \frac{\exp(\beta \cdot Q^{*|\pi_1}(s, a))}{\sum_{a'} \exp(\beta \cdot Q^{*|\pi_1}(s, a'))}.$

It is important to note that our experimental setup deliberately uses the immediate response model from the original performative RL framework, rather than the gradually shifting environment introduced by Rank et al. (2024) that assumes slow shifts in the environment. Our choice to use the immediate response model presents a more challenging performative setting where the environment responds instantaneously to policy changes. This allows us to demonstrate that unlike MDRR (Rank et al., 2024), PePG can handle the fundamental performative challenge without requiring environmental assumptions that artificially slows down the feedback loop, thereby highlighting the robustness of the proposed PePG approach.

Experimental Setup. We evaluate PePG (with and without entropy regularisation) alongside Mixed Delayed Repeated Retraining (MDRR), which represents the current state-of-the-art in performative reinforcement learning under gradually shifting environments (Rank et al., 2024), and Repeated Policy Optimization with Finite Samples (RPO FS). MDRR has demonstrated significant improvements over traditional repeated retraining methods, by leveraging historical data from multiple deployments, while RPO FS is included as the baseline method from (Mandal et al., 2023) for direct comparison with the original performative RL approach.

All experiments use a 8×8 grid with $\gamma = 0.9$, exploration parameter $\epsilon = 0.5$ for initial policy construction, one follower agent A_2 , and 100 trajectory samples per iteration. The algorithms share common parameters of $T = 100$ iterations. For regularization, RPO FS and MDRR use $\lambda = 0.1$ from their original experiments, while entropy-regularized PePG uses $\lambda = 2.0$ (ablation studies for this choice are provided in the appendix). PePG uses learning rate $\eta = 0.1$, MDRR employs memory weight $v = 1.1$ for historical data utilization, delayed round parameter $k = 3$, and FTRL parameters $N = B = 10$, while RPO FS follows the finite-sample optimization from Mandal et al.

³Code of PePG is available in [Link](#). Further ablation studies with respect to hyperparameters are in Appendix H.

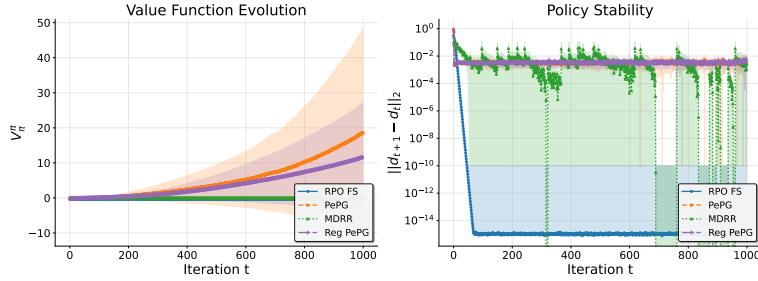


Figure 2: Comparison of evolution in expected average return (both regularised and unregularised) and stability of **PePG** with SOTA stability-achieving methods. Each algorithm is run for 20 random seeds and 1000 iterations.

Results and Observations. Our experimental evaluation across 1000 iterations reveals fundamental differences between **PePG** and MDRR and RPO in the immediate response performative setting.

I. Results: Optimality: The left panel reveals a clear performance hierarchy among the four methods. **PePG** achieves the highest value function performance, reaching approximately 18 for standard **PePG** and 11 for regularized **PePG** (Reg **PePG**) by iteration 1000, both showing consistent improvement from initial values around 0. This sustained upward progression over 1000 iterations highlights **PePG**’s effectiveness in discovering better performative equilibria rather than settling for the first stable solution encountered. MDRR shows modest improvement, reaching approximately 5, while RPO FS exhibits minimal improvement, remaining near 0 throughout the entire training period.

II. Results: Comparison of Optimality- and Stability-seeking Algorithms. The results expose a critical limitation of algorithms designed primarily for stability rather than optimality. RPO FS maintains the highest stability with successive occupancy differences below 10^{-14} throughout training, achieving its design goal of rapid convergence. However, this stability comes at the cost of solution quality, as RPO FS remains at the lowest performance level near 0. MDRR shows intermediate behavior with stability fluctuations around 10^{-2} to 10^{-4} , punctuated by occasional sharp drops to near 10^{-14} (approximately every 200-300 iterations). Despite these periodic stabilization events, MDRR achieves only modest performance improvement, demonstrating that stability-focused methods can become trapped at suboptimal equilibria. In contrast, both **PePG** variants maintain consistent moderate variability around 10^{-2} throughout the 1000 iterations, indicating persistent exploration without convergence. The results demonstrate that optimality-seeking algorithms benefit from maintaining policy variability rather than prematurely stabilizing, even though this comes with higher occupancy measure distances between iterations. Also, **PePG** exhibits marginally better performance compared to its regularised variant albeit with larger variance across its runs.

6 DISCUSSIONS, LIMITATIONS, AND FUTURE WORKS

We study the problem of Performative Reinforcement learning in tabular MDPs (PeMDPs) using softmax parametrised policies with entropy-regularised objective function, where any action taken by the agent cause potential shift in the MDP’s underlying reward and transition dynamics. We are the first to develop PG-type algorithm, **PePG**, that attains performatively optimality against the existing performative stability-seeking algorithms, affirmatively solving an extended open problem in (Mandal et al., 2023). We also derive the novel performative counterpart of classic Performance Difference Lemma and Policy Gradient Theorem that affirmatively captures this performative nature of the environment we act. We provide a sufficient conditions to prove that **PePG** converges to an $(\epsilon + \frac{1}{1-\gamma})$ -ball around performative optimal policy in $\Omega(\frac{|S||A|^2}{\epsilon^2(1-\gamma)^3})$ iterations.

As we develop a PG-type algorithm, it will be interesting to see how much can we reduce the variance (Wu et al., 2018; Papini et al., 2018) in estimation while achieving optimality. We are still in the tabular setting with finite set of state-actions. A potential future direction would be to scale **PePG** to continuous state-space with large number of state-actions.

REFERENCES

- Alekh Agarwal, Sham M Kakade, Jason D Lee, and Gaurav Mahajan. On the theory of policy gradient methods: Optimality, approximation, and distribution shift. *Journal of Machine Learning Research*, 22(98):1–76, 2021. [2](#), [3](#), [4](#), [6](#), [10](#), [20](#)
- Yuntao Bai, Andy Jones, Kamal Ndousse, Amanda Askell, Anna Chen, Nova DasSarma, Dawn Drain, Stanislav Fort, Deep Ganguli, Tom Henighan, et al. Training a helpful and harmless assistant with reinforcement learning from human feedback. *arXiv preprint arXiv:2204.05862*, 2022. [1](#)
- Anas Barakat, John Lazarsfeld, Georgios Piliouras, and Antonios Varvitsiotis. Multi-agent online control with adversarial disturbances. *arXiv preprint arXiv:2506.18814*, 2025. [1](#), [20](#)
- James Bell, Linda Linsefors, Caspar Oesterheld, and Joar Skalse. Reinforcement learning in new-comblike environments. *Advances in Neural Information Processing Systems*, 34:22146–22157, 2021. [2](#), [20](#)
- Gavin Brown, Shlomi Hod, and Iden Kalemaj. Performative prediction in a stateful world. In *International conference on artificial intelligence and statistics*, pp. 6045–6061. PMLR, 2022. [20](#)
- Songfu Cai, Fei Han, and Xuanyu Cao. Performative control for linear dynamical systems. *Advances in Neural Information Processing Systems*, 37:70617–70658, 2024. [20](#)
- Allison JB Chaney, Brandon M Stewart, and Barbara E Engelhardt. How algorithmic confounding in recommendation systems increases homogeneity and decreases utility. In *Proceedings of the 12th ACM conference on recommender systems*, pp. 224–232, 2018. [2](#)
- Qianyi Chen, Ying Chen, and Bo Li. Practical performative policy learning with strategic agents. *arXiv preprint arXiv:2412.01344*, 2024. [5](#)
- Kamil Ciosek and Shimon Whiteson. Expected policy gradients for reinforcement learning. *Journal of Machine Learning Research*, 21(52):1–51, 2020. [3](#)
- Thomas M. Cover and Joy A. Thomas. *Elements of Information Theory*. Wiley-Interscience [John Wiley & Sons], Hoboken, NJ, 2nd edition, 2006. ISBN 978-0-471-24195-9. [39](#)
- Itay Eilat and Nir Rosenfeld. Performative recommendation: diversifying content via strategic incentives. In *International Conference on Machine Learning*, pp. 9082–9103. PMLR, 2023. [2](#), [20](#)
- Hannes Eriksson, Debabrota Basu, Mina Alibeigi, and Christos Dimitrakakis. Risk-sensitive bayesian games for multi-agent reinforcement learning under policy uncertainty. *arXiv preprint arXiv:2203.10045*, 2022. [1](#)
- Yannis Flet-Berliac and Debabrota Basu. Saac: Safe reinforcement learning as an adversarial game of actor-critics. In *RLDM 2022-The Multi-disciplinary Conference on Reinforcement Learning and Decision Making*, 2022. [5](#)
- Mohammad Ghavamzadeh and Yaakov Engel. Bayesian policy gradient algorithms. *Advances in neural information processing systems*, 19, 2006. [3](#)
- António Góis, Mehrnaz Mofakhami, Fernando P Santos, Gauthier Gidel, and Simon Lacoste-Julien. Performative prediction on games and mechanism design. *arXiv preprint arXiv:2408.05146*, 2024. [1](#), [20](#)
- Moritz Hardt, Meena Jagadeesan, and Celestine Mendler-Dünner. Performative power. *Advances in Neural Information Processing Systems*, 35:22969–22981, 2022. [2](#), [20](#)
- Alexander Havrilla, Yuqing Du, Sharath Chandra Raparthi, Christoforos Nalmpantis, Jane Dwivedi-Yu, Eric Hambro, Sainbayar Sukhaaatar, and Roberta Raileanu. Teaching large language models to reason with reinforcement learning. In *AI for Math Workshop@ ICML 2024*. [1](#)

- Zhiyu He, Saverio Bolognani, Florian Dörfler, and Michael Muehlebach. Decision-dependent stochastic optimization: The role of distribution dynamics. *arXiv preprint arXiv:2503.07324*, 2025. 9
- Michael Herman, Tobias Gindele, Jörg Wagner, Felix Schmitt, and Wolfram Burgard. Inverse reinforcement learning with simultaneous estimation of rewards and dynamics. In *Artificial intelligence and statistics*, pp. 102–110. PMLR, 2016. 5
- Zachary Izzo, Lexing Ying, and James Zou. How to learn when data reacts to your model: performative gradient descent. In *International Conference on Machine Learning*, pp. 4641–4650. PMLR, 2021. 1, 20
- Zachary Izzo, James Zou, and Lexing Ying. How to learn when data gradually reacts to your model. In *International Conference on Artificial Intelligence and Statistics*, pp. 3998–4035. PMLR, 2022. 1, 20
- Sham Kakade and John Langford. Approximately optimal approximate reinforcement learning. In *Proceedings of the nineteenth international conference on machine learning*, pp. 267–274, 2002a. 6
- Sham M Kakade. A natural policy gradient. *Advances in neural information processing systems*, 14, 2001. 2
- Sham M. Kakade. A natural policy gradient. In *Advances in Neural Information Processing Systems 14 (NIPS 2001)*, pp. 1531–1538, 2002. 20
- Sham M. Kakade and John Langford. Approximately optimal approximate reinforcement learning. In *International Conference on Machine Learning*, 2002b. URL <https://api.semanticscholar.org/CorpusID:31442909>. 21
- Mahdi Kallel, Debabrota Basu, Riad Akroud, and Carlo D’Eramo. Augmented bayesian policy search. In *The Twelfth International Conference on Learning Representations (ICLR)*, 2024. URL <https://openreview.net/forum?id=OvlcyABNQT>. 6
- Vijay R. Konda and John N. Tsitsiklis. Actor-critic algorithms. In *Advances in Neural Information Processing Systems 13 (NIPS 2000)*, pp. 1008–1014, 2000. 20
- Qiang Li and Hoi-To Wai. State dependent performative prediction with stochastic approximation. In *International Conference on Artificial Intelligence and Statistics*, pp. 3164–3186. PMLR, 2022. 1, 20
- Qiang Li, Chung-Yiu Yau, and Hoi-To Wai. Multi-agent performative prediction with greedy deployment and consensus seeking agents. *Advances in Neural Information Processing Systems*, 35:38449–38460, 2022. 20
- Shengshi Li and Lin Yang. Horizon-free learning for markov decision processes and games: stochastically bounded rewards and improved bounds. In *International Conference on Machine Learning*, pp. 20221–20252. PMLR, 2023. 6
- Jingbin Liu, Xinyang Gu, and Shuai Liu. Policy optimization reinforcement learning with entropy regularization. *arXiv preprint arXiv:1912.01557*, 2019. 5
- Mehrdad Mahdavi, Lijun Zhang, and Rong Jin. Mixed optimization for smooth functions. *Advances in neural information processing systems*, 26, 2013. 9
- Debmalya Mandal and Goran Radanovic. Performative reinforcement learning with linear markov decision process. *arXiv preprint arXiv:2411.05234*, 2024. 2, 4, 5, 10, 20
- Debmalya Mandal, Stelios Triantafyllou, and Goran Radanovic. Performative reinforcement learning, 2023. URL <https://arxiv.org/abs/2207.00046>. 2, 3, 4, 5, 7, 10, 11, 12, 20
- Masoud Mansouri, Himan Abdollahpour, Mykola Pechenizkiy, Bamshad Mobasher, and Robin Burke. Feedback loop and bias amplification in recommender systems. In *Proceedings of the 29th ACM international conference on information & knowledge management*, pp. 2145–2148, 2020. 2

- Jincheng Mei, Chenjun Xiao, Csaba Szepesvari, and Dale Schuurmans. On the global convergence rates of softmax policy gradient methods. In *International conference on machine learning*, pp. 6820–6829. PMLR, 2020. [3](#), [4](#), [5](#), [6](#), [10](#), [28](#), [37](#)
- Celestine Mendler-Dünner, Juan Perdomo, Tijana Zrnic, and Moritz Hardt. Stochastic optimization for performative prediction. *Advances in Neural Information Processing Systems*, 33:4929–4939, 2020. [20](#)
- John P Miller, Juan C Perdomo, and Tijana Zrnic. Outside the echo chamber: Optimizing the performative risk. In *International Conference on Machine Learning*, pp. 7710–7720. PMLR, 2021. [1](#), [20](#)
- Volodymyr Mnih, Adrià Puigdomènech Badia, Mehdi Mirza, Alex Graves, Timothy Lillicrap, Tim Harley, David Silver, and Koray Kavukcuoglu. Asynchronous methods for deep reinforcement learning. In *Proceedings of the 33rd International Conference on Machine Learning (ICML)*, pp. 1928–1937, 2016. [20](#)
- Mehrnaz Mofakhami, Ioannis Mitliagkas, and Gauthier Gidel. Performative prediction with neural networks. In *International Conference on Artificial Intelligence and Statistics*, pp. 11079–11093. PMLR, 2023. [2](#), [20](#)
- Adhyyan Narang, Evan Faulkner, Dmitriy Drusvyatskiy, Maryam Fazel, and Lillian J Ratliff. Multiplayer performative prediction: Learning in decision-dependent games. *Journal of Machine Learning Research*, 24(202):1–56, 2023. [1](#), [20](#)
- Yurii Nesterov. Smooth convex optimization. In *Lectures on convex optimization*, pp. 59–137. Springer, 2018. [9](#)
- Gergely Neu, Anders Jonsson, and Vicenç Gómez. A unified view of entropy-regularized markov decision processes. *arXiv preprint arXiv:1705.07798*, 2017. [5](#)
- Matteo Papini, Damiano Binaghi, Giuseppe Canonaco, Matteo Pirotta, and Marcello Restelli. Stochastic variance-reduced policy gradient. In *International conference on machine learning*, pp. 4026–4035. PMLR, 2018. [12](#)
- Juan Perdomo, Tijana Zrnic, Celestine Mendler-Dünner, and Moritz Hardt. Performative prediction. In *International Conference on Machine Learning*, pp. 7599–7609. PMLR, 2020. [1](#), [20](#)
- Georgios Piliouras and Fang-Yi Yu. Multi-agent performative prediction: From global stability and optimality to chaos. In *Proceedings of the 24th ACM Conference on Economics and Computation*, pp. 1047–1074, 2023. [1](#), [20](#)
- Vasilis Pollatos, Debmalya Mandal, and Goran Radanovic. On corruption-robustness in performative reinforcement learning. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 39, pp. 19939–19947, 2025. [5](#)
- Ben Rank, Stelios Triantafyllou, Debmalya Mandal, and Goran Radanovic. Performative reinforcement learning in gradually shifting environments. *arXiv preprint arXiv:2402.09838*, 2024. [2](#), [3](#), [5](#), [6](#), [10](#), [11](#), [20](#)
- Mitas Ray, Lillian J Ratliff, Dmitriy Drusvyatskiy, and Maryam Fazel. Decision-dependent risk minimization in geometrically decaying dynamic environments. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 36, pp. 8081–8088, 2022. [20](#)
- Rilind Sahitaj, Paulius Sasnauskas, Yiğit Yalın, Debmalya Mandal, and Goran Radanović. Independent learning in performative markov potential games. *arXiv preprint arXiv:2504.20593*, 2025. [10](#)
- John Schulman, Sergey Levine, Philipp Moritz, Michael I. Jordan, and Pieter Abbeel. Trust region policy optimization. In *Proceedings of the 32nd International Conference on Machine Learning (ICML)*, pp. 1889–1897, 2015. [2](#), [20](#)
- John Schulman, Filip Wolski, Prafulla Dhariwal, Alec Radford, and Oleg Klimov. Proximal policy optimization algorithms. In *arXiv preprint arXiv:1707.06347*, 2017. [2](#), [20](#)

- Wenjie Shi, Shiji Song, and Cheng Wu. Soft policy gradient method for maximum entropy deep reinforcement learning. *arXiv preprint arXiv:1909.03198*, 2019. [5](#)
- David Silver, Guy Lever, Nicolas Heess, Thomas Degrif, Daan Wierstra, and Martin Riedmiller. Deterministic policy gradient algorithms. In *International conference on machine learning*, pp. 387–395. Pmlr, 2014. [1](#), [6](#), [7](#)
- Jason St. John, Christian Herwig, Diana Kafkes, Jovan Mitrevski, William A Pellico, Gabriel N Perdue, Andres Quintero-Parra, Brian A Schupbach, Kiyomi Seiya, Nhan Tran, et al. Real-time artificial intelligence for accelerator control: A study at the fermilab booster. *Physical Review Accelerators and Beams*, 24(10):104601, 2021. [1](#)
- Richard S Sutton and Andrew G Barto. *Reinforcement learning: An introduction*, volume 1. MIT press, 1998. [1](#)
- Richard S. Sutton, David A. McAllester, Satinder Singh, and Yishay Mansour. Policy gradient methods for reinforcement learning with function approximation. In *Advances in Neural Information Processing Systems 12 (NIPS 1999)*, pp. 1057–1063. The MIT Press, 1999. [2](#), [4](#), [7](#), [20](#)
- Jiexin Wang and Eiji Uchibe. Reward-punishment reinforcement learning with maximum entropy. In *2024 International Joint Conference on Neural Networks (IJCNN)*, pp. 1–7. IEEE, 2024. [5](#)
- Xiaolu Wang, Chung-Yiu Yau, and Hoi To Wai. Network effects in performative prediction games. In *International Conference on Machine Learning*, pp. 36514–36540. PMLR, 2023. [20](#)
- Yue Wang and Shaofeng Zou. Policy gradient method for robust reinforcement learning. In *International conference on machine learning*, pp. 23484–23526. PMLR, 2022. [3](#), [4](#)
- Ronald J. Williams. Simple statistical gradient-following algorithms for connectionist reinforcement learning. *Machine Learning*, 8:229–256, 1992. [2](#), [3](#), [7](#), [20](#)
- Cathy Wu, Aravind Rajeswaran, Yan Duan, Vikash Kumar, Alexandre M Bayen, Sham Kakade, Igor Mordatch, and Pieter Abbeel. Variance reduction for policy gradient with action-dependent factorized baselines. *arXiv preprint arXiv:1803.07246*, 2018. [12](#)
- Tengyu Xu, Haoyang Wang, and Yingbin Liang. Improved sample complexity for policy gradient via Poincaré inequality. In *Advances in Neural Information Processing Systems (NeurIPS)*, volume 33, pp. 15728–15739, 2020. [20](#)
- Tingting Xu, Henghui Zhu, and Ioannis Ch Paschalidis. Learning parametric policies and transition probability models of markov decision processes from data. *European journal of control*, 57: 68–75, 2021. [8](#)
- Rui Yuan, Robert M Gower, and Alessandro Lazaric. A general sample complexity analysis of vanilla policy gradient. In *International Conference on Artificial Intelligence and Statistics*, pp. 3332–3380. PMLR, 2022. [2](#), [4](#), [10](#), [20](#)
- Angela Zhang, Lei Xing, James Zou, and Joseph C Wu. Shifting machine learning for healthcare from development to deployment and from models to data. *Nature biomedical engineering*, 6(12):1330–1345, 2022. [2](#), [20](#)
- Kaiqing Zhang, Zhuoran Yang, and Tamer Basar. On the global convergence of policy gradient methods with variance reduction. In *Proceedings of the 37th International Conference on Machine Learning (ICML)*, pp. 11106–11115. PMLR, 2020. [20](#)
- Rui Zhao, Xudong Sun, and Volker Tresp. Maximum entropy-regularized multi-goal reinforcement learning. In *International Conference on Machine Learning*, pp. 7553–7562. PMLR, 2019. [5](#)

Appendix

Table of Contents

A Notations	18
B Details of the Toy Example: Loan Approval Problem	19
C Extended Related Works	20
D Impact of Policy Updates on PeMDPs (Section 3.1)	21
E Smoothness of Performative Value Function and Entropy Regulariser	24
F Derivation of Performative Policy Gradients	30
G Convergence of PePG : Proofs of Section 4	32
G.1 Proofs for Unregularised Value Function	32
G.2 Proofs for Entropy-regularised or Soft Value Function	36
H Ablation Studies	42
H.1 Entropy regularisation	42
H.2 Learning rate	42
I Technical Lemmas	43
J Convergence results under minimal assumptions	43

A NOTATIONS

Notation	Description
\mathcal{S}	state space
\mathcal{A}	action space
γ	discount factor
π_θ	policy parametrized by θ
$\Pi(\Theta)$	policy space
\mathbf{P}_π	transition under the environment induced by policy π
r_π	reward under the environment induced by policy π
π_s^*	performatively stable policy
π_o^*	performatively optimal policy
$\mathbf{P}_{\pi_o^*}$	reward under the environment induced by performatively optimal policy
$r_{\pi_o^*}$	reward under the environment induced by performatively optimal policy
$d_{\pi_o^*}^{\pi_o^*}$	state-action occupancy of optimal policy
$V_{\pi_o^*}^{\pi_o^*}$	value function of optimal policy
$d_{\pi_2}^{\pi_1}$	state-action occupancy of playing policy π_2 in the environment induced by policy π_1
$V_{\pi_1}^{\pi_2}$	value function for playing policy π_2 in the environment induced by policy π_1
$Q_{\pi_1}^{\pi_2}$	Q-value function for playing policy π_2 in the environment induced by policy π_1
$A_{\pi_1}^{\pi_2}$	advantage function for playing policy π_2 in the environment induced by policy π_1
Δ_K	K -dimensional simplex
ρ	Initial state distribution $\in \Delta_{\mathcal{S}}$

B DETAILS OF THE TOY EXAMPLE: LOAN APPROVEMENT PROBLEM

Environment. We consider a population of loan applicants represented by a scalar feature $x \in \mathbb{R}$, distributed as $x \sim \mathcal{N}(\mu, \sigma^2)$, where μ is the population mean and $\sigma > 0$ is fixed.

Bank's Policy. The bank chooses a *threshold policy* parameterized by $\theta \in \mathbb{R}$. A loan is granted to an applicant x if $x \geq \theta$. To smooth analysis, we use a differentiable approximation: $\pi_\theta(x) = \sigma(k(x - \theta))$, where $\sigma(z) = \frac{1}{1+e^{-z}}$ is the logistic sigmoid and $k > 0$ controls smoothness.

Rewards. If a loan is granted to applicant x , the bank receives a random payoff:

$$r(x) = \begin{cases} +R & \text{if applicant repays,} \\ -L & \text{if applicant defaults,} \end{cases}$$

with repayment probability $\mathbb{P}(\text{repay} \mid x) = \sigma(\gamma x - c)$, where $\gamma > 0$ controls sensitivity and c is a calibration constant. The expected reward from granting to x is

$$u(x) = \sigma(\gamma x - c) \cdot R - (1 - \sigma(\gamma x - c)) \cdot L.$$

Expected Utility. Given distribution $x \sim \mathcal{N}(\mu, \sigma^2)$, the bank's expected utility for policy θ is

$$U(\theta, \mu) = \mathbb{E}_{x \sim \mathcal{N}(\mu, \sigma^2)} [\pi_\theta(x) \cdot u(x)].$$

Performative Feedback. The population mean μ depends on the bank's policy, via the grant rate: $g(\theta, \mu) = \mathbb{E}_{x \sim \mathcal{N}(\mu, \sigma^2)} [\pi_\theta(x)]$.

We assume a bounded performative update rule: $\mu_{t+1} = (1 - \beta)\mu_t + \beta \cdot f(g(\theta, \mu_t))$, where $\beta \in [0, 1]$ is the performative strength and $f(g) \in [-M, M]$ maps the grant rate to a feasible population mean.

At equilibrium, the induced feature distribution satisfies the fixed point condition:

$$\mu^*(\theta) = (1 - \beta)\mu^*(\theta) + \beta f(g(\theta, \mu^*(\theta))).$$

Optimization Problems. **ERM Optimum.** Ignoring performative effects (i.e. assuming $\mu = \mu_0$ is fixed), the ERM-optimal policy solves

$$\theta^{\text{ERM}} = \arg \max_{\theta} U(\theta, \mu_0).$$

Performative Optimum. Accounting for performative feedback, the performative-optimal policy solves

$$\theta^{\text{Perf}} = \arg \max_{\theta} U(\theta, \mu^*(\theta)).$$

Learning via Reinforcement Learning

An RL agent plays policies θ_t sequentially. At each round t :

1. Sample $x \sim \mathcal{N}(\mu_t, \sigma^2)$.
2. Grant loan with probability $\pi_{\theta_t}(x)$.
3. Observe reward r_t .
4. Update θ_{t+1} using policy gradient (REINFORCE).
5. Update population mean via performative dynamics:

$$\mu_{t+1} = (1 - \beta)\mu_t + \beta f(g(\theta_t, \mu_t)).$$

C EXTENDED RELATED WORKS

Performative Prediction. The study of performative prediction started with the pioneering work of (Perdomo et al., 2020), where they leveraged repeated retraining with the aim to converge towards a performatively stable point. We see extension of this work trying to achieve performative optimality (Izzo et al., 2021; 2022; Miller et al., 2021). This further opened a plethora of works in various other domains such as Multi-agent systems (Narang et al., 2023; Li et al., 2022; Piliouras & Yu, 2023), control systems (Cai et al., 2024; Barakat et al., 2025), stochastic optimisation (Li & Wai, 2022; Mandler-Dünner et al., 2020), games (Wang et al., 2023; Góis et al., 2024) etc. There has been several attempt of achieve performative optimality or stability for real-life tasks like recommendation (Eilat & Rosenfeld, 2023), to measure the power of firms (Hardt et al., 2022; Mofakhami et al., 2023), in healthcare (Zhang et al., 2022) etc. Another interesting setting is the *stateful* performative prediction i.e. prediction under gradual shifts in the distribution (Brown et al., 2022; Izzo et al., 2022; Ray et al., 2022), that paved the way for incorporating performative prediction in Reinforcement Learning.

Performative Reinforcement Learning. Bell et al. (2021) were the first to propose a setting where the transition and reward of an underlying MDP depend non-deterministically on the deployed policy, thus capturing the essence of performativity to some extent. However, Mandal et al. (2023) can be considered the pioneer in introducing the notion of “*Performative Reinforcement Learning*” and its solution concepts, performatively stable and optimal policy. They propose direct optimization and ascent based techniques which manage to attain performative stability upon repeated retraining. Extensions to this work, Rank et al. (2024) and Mandal & Radanovic (2024) manage to solve the same problem with delayed retraining for linear MDPs. However, there exists no literature that proposes a performative RL algorithm that converges to the performative optimal policy.

Specifically, Mandal et al. (2023) frames the question of using policy gradient to find stable policies as an open problem. The authors further contemplate, as PG functions in the policy space, whether it is possible to converge towards a stable policy. Thus, in this paper, we affirmatively solve an extension (rather a harder problem) of this open problem for tabular MDPs with softmax policies.

Policy Gradient Algorithms. Policy gradient algorithms build a central paradigm in reinforcement learning, directly optimizing parametrised policies by estimating the gradient of expected return. The foundational policy gradient theorem (Sutton et al., 1999) established an expression for this gradient in terms of the score and action-value function, while Williams (1992) introduced the REINFORCE algorithm, providing an unbiased likelihood-ratio estimator. Convergence properties of stochastic gradient ascent in policy space were analysed in these early works. Subsequently, Konda & Tsitsiklis (2000) formalized actor-critic methods via two-timescale stochastic approximation, and Kakade (2002) proposed the natural policy gradient, leveraging the Fisher information geometry to accelerate learning. Extensions to trust region methods (Schulman et al., 2015), proximal policy optimization (Schulman et al., 2017), and entropy-regularized objectives (Mnih et al., 2016) have made policy gradient methods widely practical in high-dimensional settings. Recent theoretical advances provide finite-sample convergence guarantees and complexity analyses (Agarwal et al., 2021; Yuan et al., 2022), as well as robustness to distributional shift and adversarial perturbations (Zhang et al., 2020; Xu et al., 2020). Collectively, this body of work establishes policy gradient methods as both practically effective and theoretically grounded method for solving MDP.

D IMPACT OF POLICY UPDATES ON PEMDPs (SECTION 3.1)

Lemma 1 (Performative Performance Difference Lemma). *The difference in performatice value functions induced by π and $\pi' \in \Pi$ while starting from the initial state distribution ρ is*

$$(1) \quad V_{\pi}(\rho) - V_{\pi'}(\rho) = \frac{1}{1-\gamma} \mathbb{E}_{(s,a) \sim d_{\pi',\rho}} [A_{\pi'}^{\pi'}(s, a)] \\ + \frac{1}{1-\gamma} \mathbb{E}_{(s,a) \sim d_{\pi',\rho}} [(r_{\pi}(s, a) - r_{\pi'}(s, a)) \\ + \gamma (\mathbf{P}_{\pi}(\cdot|s, a) - \mathbf{P}_{\pi'}(\cdot|s, a))^{\top} V_{\pi}(\cdot)]. \quad (16)$$

where $A_{\pi'}^{\pi'}(s, a) \triangleq Q_{\pi'}^{\pi'}(s, a) - V_{\pi'}^{\pi'}(s)$ is the performatice advantage function for any state $s \in \mathcal{S}$ and action $a \in \mathcal{A}$.

$$(2) \quad V_{\pi}(\rho) - V_{\pi'}(\rho) = \frac{1}{1-\gamma} \mathbb{E}_{(s,a) \sim d_{\pi',\rho}} [A_{\pi'}^{\pi'}(s, a)] \\ + \frac{1}{1-\gamma} \mathbb{E}_{(s,a) \sim d_{\pi',\rho}} [(r_{\pi}(s, a) - r_{\pi'}(s, a)) \\ + \gamma (\mathbf{P}_{\pi}(\cdot|s, a) - \mathbf{P}_{\pi'}(\cdot|s, a))^{\top} V_{\pi'}(\cdot)]. \quad (17)$$

where $A_{\pi'}^{\pi'}(s, a) \triangleq Q_{\pi'}^{\pi'}(s, a) - V_{\pi'}^{\pi'}(s)$ is the performatice advantage function for any state $s \in \mathcal{S}$ and action $a \in \mathcal{A}$.

$$(3) \quad V_{\pi}(\rho) - V_{\pi'}(\rho) = \frac{1}{1-\gamma} \mathbb{E}_{(s,a) \sim d_{\pi',\rho}} [A_{\pi'}^{\pi'}(s, a)] \\ + \frac{1}{1-\gamma} \mathbb{E}_{(s,a) \sim d_{\pi',\rho}} [(r_{\pi}(s, a) - r_{\pi'}(s, a)) \\ + \gamma (\mathbf{P}_{\pi}(\cdot|s, a) - \mathbf{P}_{\pi'}(\cdot|s, a))^{\top} V_{\pi'}^{\pi'}(\cdot)]. \quad (18)$$

where $A_{\pi'}^{\pi'}(s, a) \triangleq Q_{\pi'}^{\pi'}(s, a) - V_{\pi'}^{\pi'}(s)$ is the performatice advantage function for any state $s \in \mathcal{S}$ and action $a \in \mathcal{A}$.

We only use the first version of this lemma in the main draft, and also hereafter, for the proofs.

Proof of Lemma 1. We do this proof in two steps. First step involves a decomposition of the difference in value function into two terms : (i) difference in value function after deploying the same policy while agent plays two different policies i.e. the difference that explains stability of the deployed policy, and (ii) difference in value function for deploying two different policies i.e. performance difference for changing the deployed policy. While the second term can be bounded using classic performance difference lemma, in the next and final step, we control the stability inducing term (i).

Part(1) – Step 1: Decomposition. We start by decomposing the performatice performance difference to get a stability and a performance difference terms separately.

$$\begin{aligned} V_{\pi}^{\pi}(s_0) - V_{\pi'}^{\pi'}(s_0) &= \underbrace{V_{\pi}^{\pi}(s_0) - V_{\pi'}^{\pi'}(s_0)}_{\text{performatice shift term}} + \underbrace{V_{\pi'}^{\pi'}(s_0) - V_{\pi'}^{\pi'}(s_0)}_{\text{performance difference term}} \\ &= V_{\pi}^{\pi}(s_0) - V_{\pi'}^{\pi'}(s_0) + \frac{1}{1-\gamma} \mathbb{E}_{(s,a) \sim d_{\pi',\cdot|s_0}} [A_{\pi'}^{\pi'}(s, a)] \end{aligned} \quad (19)$$

The last equality is a consequence of the classical performance difference lemma (Kakade & Langford, 2002b).

Step 2: Controlling the performatice shift term. First, let us define $\mathbf{P}_{\pi}^{\pi}(s', s) \triangleq \sum_{a \in \mathcal{A}} \mathbf{P}_{\pi}(s'|s, a) \pi(a|s)$, and $\langle \mathbf{P}_{\pi}^{\pi}(\cdot, s_0), V_{\pi}^{\pi}(\cdot) \rangle \triangleq \sum_{s \in \mathcal{S}} V_{\pi}^{\pi}(s) \mathbf{P}_{\pi}^{\pi}(s, s_0)$.

We first observe that

$$\begin{aligned}
V_{\pi}^{\pi}(s_0) - V_{\pi'}^{\pi}(s_0) &= \mathbb{E}_{a \sim \pi(\cdot|s_0)} [r_{\pi}(s_0, a) - r_{\pi'}(s_0, a)] + \gamma \mathbb{E}_{s \sim \mathbf{P}_{\pi}^{\pi}(\cdot, s_0)} [V_{\pi}^{\pi}(s)] - \gamma \mathbb{E}_{s \sim \mathbf{P}_{\pi'}^{\pi}(\cdot, s_0)} [V_{\pi'}^{\pi}(s)] \\
&= \mathbb{E}_{a \sim \pi(\cdot|s_0)} [r_{\pi}(s_0, a) - r_{\pi'}(s_0, a)] \\
&\quad + \gamma \sum_s (\mathbf{P}_{\pi}^{\pi}(s, s_0) - \mathbf{P}_{\pi'}^{\pi}(s, s_0)) V_{\pi}^{\pi}(s) + \gamma \sum_s \mathbf{P}_{\pi'}^{\pi}(s, s_0) (V_{\pi}^{\pi}(s) - V_{\pi'}^{\pi}(s)) \\
&= \mathbb{E}_{(s, a) \sim d_{\pi'}^{\pi}(\cdot|s_0)} [r_{\pi}(s, a) - r_{\pi'}(s, a) + \gamma (\mathbf{P}_{\pi}(\cdot|s, a) - \mathbf{P}_{\pi'}(\cdot|s, a))^{\top} V_{\pi}^{\pi}(\cdot)]
\end{aligned}$$

The last equality is obtained by recurring the preceding step iteratively.

Combining steps 1 and 2 and taking expectation over $s_0 \sim \rho$, we get

$$\begin{aligned}
V_{\pi}^{\pi}(\rho) - V_{\pi'}^{\pi}(\rho) &= \frac{1}{1-\gamma} \mathbb{E}_{(s, a) \sim d_{\pi', \rho}^{\pi}} [A_{\pi'}^{\pi}(s, a) + (r_{\pi}(s, a) - r_{\pi'}(s, a))] \\
&\quad + \gamma (\mathbf{P}_{\pi}(\cdot|s, a) - \mathbf{P}_{\pi'}(\cdot|s, a))^{\top} V_{\pi}^{\pi}(\cdot).
\end{aligned}$$

Part(2) – The second equality is obtained by changing the Step 2 as follows:

$$\begin{aligned}
V_{\pi}^{\pi}(s_0) - V_{\pi'}^{\pi}(s_0) &= \mathbb{E}_{a \sim \pi(\cdot|s_0)} [r_{\pi}(s_0, a) - r_{\pi'}(s_0, a)] + \gamma \mathbb{E}_{s \sim \mathbf{P}_{\pi}^{\pi}(\cdot, s_0)} [V_{\pi}^{\pi}(s)] - \gamma \mathbb{E}_{s \sim \mathbf{P}_{\pi'}^{\pi}(\cdot, s_0)} [V_{\pi'}^{\pi}(s)] \\
&= \mathbb{E}_{a \sim \pi(\cdot|s_0)} [r_{\pi}(s_0, a) - r_{\pi'}(s_0, a)] \\
&\quad + \gamma \sum_s (\mathbf{P}_{\pi}^{\pi}(s, s_0) - \mathbf{P}_{\pi'}^{\pi}(s, s_0)) V_{\pi'}^{\pi}(s) + \gamma \sum_s \mathbf{P}_{\pi}^{\pi}(s, s_0) (V_{\pi}^{\pi}(s) - V_{\pi'}^{\pi}(s)) \\
&= \frac{1}{1-\gamma} \mathbb{E}_{(s, a) \sim d_{\pi', \rho}^{\pi}} [A_{\pi'}^{\pi}(s, a)] \\
&\quad + \frac{1}{1-\gamma} \mathbb{E}_{(s, a) \sim d_{\pi', \rho}^{\pi}} [(r_{\pi}(s, a) - r_{\pi'}(s, a)) + \gamma (\mathbf{P}_{\pi}(\cdot|s, a) - \mathbf{P}_{\pi'}(\cdot|s, a))^{\top} V_{\pi'}^{\pi}(\cdot)].
\end{aligned}$$

The last equality is obtained by recurring the preceding step iteratively.

Part(3) – The third equality is obtained through the following steps.

$$\begin{aligned}
V_{\pi}^{\pi}(\rho) - V_{\pi'}^{\pi}(\rho) &= V_{\pi}^{\pi}(s_0) - V_{\pi'}^{\pi}(s_0) + V_{\pi'}^{\pi}(s_0) - V_{\pi'}^{\pi}(\rho) \\
&= \frac{1}{1-\gamma} \mathbb{E}_{(s, a) \sim d_{\pi}^{\pi}(\cdot|s_0)} [A_{\pi}^{\pi}(s, a)] + V_{\pi'}^{\pi}(s_0) - V_{\pi'}^{\pi}(\rho) \\
&= \frac{1}{1-\gamma} \mathbb{E}_{(s, a) \sim d_{\pi}^{\pi}(\cdot|s_0)} [A_{\pi}^{\pi}(s, a)] + \mathbb{E}_{a \sim \pi'(\cdot|s_0)} [r_{\pi}(s_0, a) - r_{\pi'}(s_0, a)] \\
&\quad + \gamma \sum_s (\mathbf{P}_{\pi'}^{\pi}(s, s_0) - \mathbf{P}_{\pi'}^{\pi}(s, s_0)) V_{\pi'}^{\pi}(s) + \gamma \sum_s \mathbf{P}_{\pi'}^{\pi}(s, s_0) (V_{\pi'}^{\pi}(s) - V_{\pi'}^{\pi}(\rho)) \\
&= \frac{1}{1-\gamma} \mathbb{E}_{(s, a) \sim d_{\pi', \rho}^{\pi}} [A_{\pi'}^{\pi}(s, a)] \\
&\quad + \frac{1}{1-\gamma} \mathbb{E}_{(s, a) \sim d_{\pi', \rho}^{\pi}} [(r_{\pi}(s, a) - r_{\pi'}(s, a)) + \gamma (\mathbf{P}_{\pi}(\cdot|s, a) - \mathbf{P}_{\pi'}(\cdot|s, a))^{\top} V_{\pi'}^{\pi}(\cdot)].
\end{aligned}$$

□

Lemma 2 (Bounding Performative Performance Difference for Gradually Shifting Environments). *Let us assume that both rewards and transitions are Lipschitz functions of policy, i.e. $\|r_{\pi} - r_{\pi'}\|_1 \leq L_r \|\pi - \pi'\|_1$ and $\|\mathbf{P}_{\pi} - \mathbf{P}_{\pi'}\|_1 \leq L_{\mathbf{P}} \|\pi - \pi'\|_1$, for some $L_r, L_{\mathbf{P}} \geq 0$. Then, under Assumption 1, the performative shift in the sub-optimality gap of a policy π_{θ} satisfies*

$$\begin{aligned}
&\left| V_{\pi_{\theta}^*}^{\pi_{\theta}}(\rho) - V_{\pi_{\theta}}^{\pi_{\theta}}(\rho) - \frac{1}{1-\gamma} \mathbb{E}_{(s, a) \sim d_{\pi_{\theta}, \rho}^{\pi_{\theta}}} [A_{\pi_{\theta}}^{\pi_{\theta}}(s, a)] \right| \\
&\leq \frac{2\sqrt{2}}{1-\gamma} (L_r + \frac{\gamma}{1-\gamma} L_{\mathbf{P}} R_{\max}) \mathbb{E}_{s_0 \sim \rho} D_{\text{H}}(\pi_{\theta}^*(\cdot|s_0) \| \pi_{\theta}(\cdot|s_0)). \tag{20}
\end{aligned}$$

where $D_{\text{H}}(\mathbf{x} \| \mathbf{y})$ denotes the Hellinger distance between \mathbf{x} and \mathbf{y} .

Proof of Lemma 2. We do this proof in three steps. We start from the final expression in Lemma 1, then in step 2 we impose bounds on reward and transition differences leveraging the Lipschitz assumption. Lastly, we bound the policy difference in first order norm using relation between Total Variation (TV) and Hellinger distance.

Step 1: From Lemma 1, we get

$$\begin{aligned} V_{\pi_o^*}(s_0) - V_{\pi_\theta}(\pi_o^*)(s_0) &= \frac{1}{1-\gamma} \mathbb{E}_{(s,a) \sim d_{\pi_\theta, \rho}^{\pi_o^*}} \left[A_{\pi_\theta}^{\pi_\theta}(s, a) + (r_{\pi_o^*}(s, a) - r_{\pi_\theta}(s, a)) \right. \\ &\quad \left. + \gamma (\mathbf{P}_{\pi_o^*}(\cdot|s, a) - \mathbf{P}_{\pi_\theta}(\cdot|s, a))^{\top} V_{\pi_o^*}^{\pi_o^*}(\cdot) \right]. \end{aligned}$$

Thus,

$$\begin{aligned} & \left| V_{\pi_o^*}(\rho) - V_{\pi_\theta}(\rho) - \frac{1}{1-\gamma} \mathbb{E}_{(s,a) \sim d_{\pi_\theta, \rho}^{\pi_o^*}} [A_{\pi_\theta}^{\pi_\theta}(s, a)] \right| \\ &= \frac{1}{1-\gamma} \left| \mathbb{E}_{(s,a) \sim d_{\pi_\theta, \rho}^{\pi_o^*}} (r_{\pi_o^*}(s, a) - r_{\pi_\theta}(s, a)) + \gamma (\mathbf{P}_{\pi_o^*}(\cdot|s, a) - \mathbf{P}_{\pi_\theta}(\cdot|s, a))^{\top} V_{\pi_o^*}^{\pi_o^*}(\cdot) \right| \quad (21) \end{aligned}$$

Step 2: Using Jensen's inequality together with the fact that $d_{\pi_\theta, \rho}^{\pi_o^*}(s, a|s_0) \leq 1$, for rewards, we get

$$\begin{aligned} \left| \mathbb{E}_{(s,a) \sim d_{\pi_\theta, \rho}^{\pi_o^*}(\cdot, \cdot|s_0)} [r_{\pi_o^*}(s, a) - r_{\pi_\theta}(s, a)] \right| &\leq \mathbb{E}_{(s,a) \sim d_{\pi_\theta, \rho}^{\pi_o^*}(\cdot, \cdot|s_0)} |r_{\pi_o^*}(s, a) - r_{\pi_\theta}(s, a)| \\ &\leq \|r_{\pi_o^*} - r_{\pi_\theta}\|_1 \end{aligned}$$

Similarly for transitions, we get

$$\begin{aligned} \left| \mathbb{E}_{(s,a) \sim d_{\pi_\theta, \rho}^{\pi_o^*}(\cdot, \cdot|s_0)} [(\mathbf{P}_{\pi_o^*} - \mathbf{P}_{\pi_\theta})^{\top} V_{\pi}^{\pi}] \right| &\leq \mathbb{E}_{(s,a) \sim d_{\pi_\theta, \rho}^{\pi_o^*}(\cdot, \cdot|s_0)} |(\mathbf{P}_{\pi_o^*} - \mathbf{P}_{\pi_\theta})^{\top} V_{\pi}^{\pi}| \\ &\stackrel{(a)}{\leq} \mathbb{E}_{(s,a) \sim d_{\pi_\theta, \rho}^{\pi_o^*}(\cdot, \cdot|s_0)} [\|\mathbf{P}_{\pi_o^*} - \mathbf{P}_{\pi_\theta}\|_1 \cdot \|V_{\pi_o^*}^{\pi_o^*}\|_{\infty}] \\ &= \|\mathbf{P}_{\pi_o^*} - \mathbf{P}_{\pi_\theta}\|_1 \cdot \|V_{\pi_o^*}^{\pi_o^*}\|_{\infty}, \end{aligned}$$

(a) holds due to Hölder's inequality.

Now, leveraging the triangle inequality and Lipschitzness assumption on reward and transitions, we further get

$$\begin{aligned} & \left| \mathbb{E}_{(s,a) \sim d_{\pi_\theta, \rho}^{\pi_o^*}(\cdot, \cdot|s_0)} [r_{\pi_o^*}(s, a) - r_{\pi_\theta}(s, a) + \gamma (\mathbf{P}_{\pi_o^*} - \mathbf{P}_{\pi_\theta})^{\top} V_{\pi}^{\pi}] \right| \\ &\leq L_r \|\pi_o^* - \pi_\theta\|_1 + \gamma L_{\mathbf{P}} \left\| \mathbf{V}_{\pi_o^*}^{\pi_o^*} \right\|_{\infty} \|\pi_o^* - \pi_\theta\|_1 \end{aligned}$$

Finally, due to Assumption 1, we get $\left\| \mathbf{V}_{\pi_o^*}^{\pi_o^*} \right\|_{\infty} \leq \frac{R_{\max}}{1-\gamma}$, and thus,

$$\begin{aligned} & \left| \mathbb{E}_{(s,a) \sim d_{\pi_\theta, \rho}^{\pi_o^*}(\cdot, \cdot|s_0)} [r_{\pi_o^*}(s, a) - r_{\pi_\theta}(s, a) + \gamma (\mathbf{P}_{\pi_o^*} - \mathbf{P}_{\pi_\theta})^{\top} V_{\pi_o^*}^{\pi_o^*}] \right| \\ &\leq L_r \|\pi_o^* - \pi_\theta\|_1 + \frac{\gamma}{1-\gamma} L_{\mathbf{P}} R_{\max} \|\pi_o^* - \pi_\theta\|_1 \end{aligned}$$

Step 3: We know $\|\pi_o^* - \pi_\theta\|_1 = 2\text{TV}(\pi_o^* \parallel \pi_\theta) \leq 2\sqrt{2}D_H(\pi_o^* \parallel \pi_\theta)$. Thus,

$$\left| \mathbb{E}_{(s,a) \sim d_{\pi_\theta, \rho}^{\pi_o^*}(\cdot, \cdot|s_0)} [r_{\pi_o^*}(s, a) - r_{\pi_\theta}(s, a) + \gamma (\mathbf{P}_{\pi_o^*} - \mathbf{P}_{\pi_\theta})^{\top} V_{\pi_o^*}^{\pi_o^*}] \right|$$

$$\leq 2\sqrt{2} \left(L_r + \frac{\gamma}{1-\gamma} L_{\mathbf{P}} R_{\max} \right) D_{\text{H}}(\pi_o^*(\cdot | s_0) \| \pi_{\theta}(\cdot | s_0)) \quad (22)$$

We conclude this proof by putting the upper bound in Equation (22) in Equation (21) and taking expectation over $s_0 \sim \rho$ to get the desired expression. \square

E SMOOTHNESS OF PERFORMATIVE VALUE FUNCTION AND ENTROPY REGULARISER

Lemma 4 (Performative Smoothness Lemma). *Let $\pi_{\alpha} \triangleq \pi_{\theta+\alpha u}$, and let $V_{\alpha}^{\alpha}(s_0)$ be the corresponding value at a fixed state s_0 , i.e., $V_{\alpha}^{\alpha}(s_0) \triangleq V_{\pi_{\alpha}}^{\alpha}(s_0)$. If the following conditions hold true,*

$$\begin{aligned} \sum_{a \in \mathcal{A}} \left| \frac{d\pi_{\alpha}(a | s_0)}{da} \right|_{\alpha=0} &\leq C_1, \quad \sum_{a \in \mathcal{A}} \left| \frac{d^2\pi_{\alpha}(a | s_0)}{da^2} \right|_{\alpha=0} \leq C_2, \quad \sum_{s \in \mathcal{S}} \left| \frac{d\mathbf{P}_{\alpha}(s | s_0, a_0)}{da} \right|_{\alpha=0} \leq T_1, \\ \sum_{s \in \mathcal{S}} \left| \frac{d^2\mathbf{P}_{\alpha}(s | s_0, a_0)}{da^2} \right|_{\alpha=0} &\leq T_2, \quad \sum_{a \in \mathcal{A}} \left| \frac{dr_{\alpha}(s_0, a)}{da} \right|_{\alpha=0} \leq R_1, \quad \sum_{a \in \mathcal{A}} \left| \frac{d^2r_{\alpha}(s_0, a)}{da^2} \right|_{\alpha=0} \leq R_2, \end{aligned}$$

we get

$$\max_{\|u\|_2=1} \left\| \frac{d^2V_{\alpha}^{\alpha}(s_0)}{da^2} \right\|_{\alpha=0} \leq \frac{C_2}{1-\gamma} + 2C_1\beta_1 + C_2\beta_2 \triangleq L,$$

where $\beta_1 = \frac{\gamma}{(1-\gamma)^2}(C_1 + T_1) + \frac{R_1}{1-\gamma}$ and $\beta_2 = \frac{2\gamma^2}{(1-\gamma)^3}(C_1 + T_1)^2 + \frac{\gamma}{(1-\gamma)^2}(C_2 + 2C_1T_1 + T_2) + \frac{2\gamma R_1}{(1-\gamma)^2}(C_2 + 2C_1T_1 + T_2) + \frac{R_2}{1-\gamma} + \frac{\gamma C_1 R_1}{(1-\gamma)^2}$.

Proof. **Step 1:** To prove the second order smoothness of the value function we start by taking its second derivative. Consider the expected return under policy π_{α} :

$$V_{\alpha}^{\alpha}(s_0) = \sum_a \pi_{\alpha}(a | s_0) Q_{\alpha}^{\alpha}(s_0, a)$$

Differentiating twice with respect to α , we obtain:

$$\frac{d^2V_{\alpha}^{\alpha}(s_0)}{da^2} = \sum_a \frac{d^2\pi_{\alpha}(a | s_0)}{da^2} Q_{\alpha}^{\alpha}(s_0, a) + 2 \sum_a \frac{d\pi_{\alpha}(a | s_0)}{da} \frac{dQ_{\alpha}^{\alpha}(s_0, a)}{da} + \sum_a \pi_{\alpha}(a | s_0) \frac{d^2Q_{\alpha}^{\alpha}(s_0, a)}{da^2}$$

$Q_{\alpha}^{\alpha}(s_0, a_0)$ is the Q-function corresponding to the policy π_{α} at state s_0 and action a_0 . Observe that $Q_{\alpha}^{\alpha}(s_0, a_0)$ can further be written as:

$$Q_{\alpha}^{\alpha}(s_0, a_0) = e_{(s_0, a_0)}^{\top} (I - \gamma \tilde{\mathbf{P}}(\alpha))^{-1} r_{\alpha} = e_{(s_0, a_0)}^{\top} M(\alpha) r_{\alpha}$$

where $M(\alpha) \triangleq (I - \gamma \mathbf{P}(\alpha))^{-1}$ and $\tilde{\mathbf{P}}(\alpha)$ is the state-action transition matrix under policy π_{α} , defined as:

$$[\tilde{\mathbf{P}}(\alpha)](s', a' | s, a) \triangleq \pi_{\alpha}(a' | s') \mathbf{P}_{\alpha}(s' | s, a)$$

Differentiating $Q_{\alpha}^{\alpha}(s, a)$ with respect to α gives:

$$\frac{dQ_{\alpha}^{\alpha}(s_0, a_0)}{da} = \gamma e_{(s_0, a_0)}^{\top} M(\alpha) \frac{d\tilde{\mathbf{P}}(\alpha)}{da} M(\alpha) r_{\alpha} + e_{(s_0, a_0)}^{\top} M(\alpha) \frac{dr_{\alpha}}{da}$$

And correspondingly,

$$\frac{d^2Q_{\alpha}^{\alpha}(s_0, a_0)}{da^2} = 2\gamma^2 e_{(s_0, a_0)}^{\top} M(\alpha) \frac{d\tilde{\mathbf{P}}(\alpha)}{da} M(\alpha) \frac{d\tilde{\mathbf{P}}(\alpha)}{da} M(\alpha) r_{\alpha} + \gamma e_{(s_0, a_0)}^{\top} M(\alpha) \frac{d^2\tilde{\mathbf{P}}(\alpha)}{da^2} M(\alpha) r_{\alpha}$$

$$\begin{aligned}
& + \gamma e_{(s_0, a_0)}^\top M(\alpha) \frac{d\tilde{\mathbf{P}}(\alpha)}{d\alpha} M(\alpha) \frac{dr_\alpha}{d\alpha} + e_{(s_0, a_0)}^\top M(\alpha) \frac{d^2 r_\alpha}{d\alpha^2} \\
& + \gamma e_{(s_0, a_0)}^\top M(\alpha) \frac{d\tilde{\mathbf{P}}(\alpha)}{d\alpha} M(\alpha) \frac{dr_\alpha}{d\alpha}
\end{aligned} \tag{23}$$

Step 2: Now we need to find the derivative of $\tilde{\mathbf{P}}(\alpha)$ w.r.t α in order to substitute in (23). Hence, we can differentiate $\tilde{\mathbf{P}}(\alpha)$ with respect to α to obtain:

$$\left. \frac{d\tilde{\mathbf{P}}(\alpha)}{d\alpha} \right|_{\alpha=0} (s', a' | s, a) = \left. \frac{d\pi_\alpha(a' | s')}{d\alpha} \right|_{\alpha=0} \mathbf{P}_\alpha(s' | s, a) + \left. \frac{d\mathbf{P}_\alpha(s' | s, a)}{d\alpha} \right|_{\alpha=0} \pi_\alpha(a' | s')$$

Now, for an arbitrary vector \mathbf{x} , we have:

$$\begin{aligned}
\left[\left. \frac{d\tilde{\mathbf{P}}(\alpha)}{d\alpha} \right|_{\alpha=0} \mathbf{x} \right]_{(s, a)} &= \sum_{s', a'} \left. \frac{d\pi_\alpha(a' | s')}{d\alpha} \right|_{\alpha=0} \mathbf{P}_\alpha(s' | s, a) \mathbf{x}_{s', a'} \\
&+ \sum_{s', a'} \left. \frac{d\mathbf{P}_\alpha(s' | s, a)}{d\alpha} \right|_{\alpha=0} \pi_\alpha(a' | s') \mathbf{x}_{s', a'}
\end{aligned}$$

Taking the maximum over unit vectors \mathbf{u} in ℓ_2 -norm:

$$\begin{aligned}
\max_{\|\mathbf{u}\|_2=1} \left\| \left. \frac{d\tilde{\mathbf{P}}(\alpha)}{d\alpha} \right|_{\alpha=0} \mathbf{x} \right\|_\infty &\leq \max_{\|\mathbf{u}\|_2=1} \left| \sum_{s', a'} \left. \frac{d\pi_\alpha(a' | s')}{d\alpha} \right|_{\alpha=0} \mathbf{P}_\alpha(s' | s, a) \mathbf{x}_{s', a'} \right| \\
&+ \max_{\|\mathbf{u}\|_2=1} \left| \sum_{s', a'} \left. \frac{d\mathbf{P}_\alpha(s' | s, a)}{d\alpha} \right|_{\alpha=0} \pi_\alpha(a' | s') \mathbf{x}_{s', a'} \right| \\
&\leq \max_{s, a} \sum_{s'} \mathbf{P}_\alpha(s' | s, a) \sum_{a'} \left| \left. \frac{d\pi_\alpha(a' | s')}{d\alpha} \right|_{\alpha=0} \right| \cdot \|\mathbf{x}\|_\infty \\
&+ \max_{s, a} \sum_{a'} \pi_\alpha(a' | s') \sum_{s'} \left| \left. \frac{d\mathbf{P}_\alpha(s' | s, a)}{d\alpha} \right|_{\alpha=0} \right| \cdot \|\mathbf{x}\|_\infty \\
&\leq \max_{s, a} \sum_{s'} \mathbf{P}_\alpha(s' | s, a) \|\mathbf{x}\|_\infty C_1 + \max_{s, a} \sum_{a'} \pi_\alpha(a' | s') \|\mathbf{x}\|_\infty T_1 \\
&\leq C_1 \|\mathbf{x}\|_\infty + T_1 \|\mathbf{x}\|_\infty = (C_1 + T_1) \|\mathbf{x}\|_\infty
\end{aligned}$$

By the definition of the ℓ_∞ -norm, we conclude:

$$\max_{\|\mathbf{u}\|_2=1} \left\| \left. \frac{d\mathbf{P}_\alpha}{d\alpha} \right|_{\alpha=0} \mathbf{x} \right\|_\infty \leq (C_1 + T_1) \|\mathbf{x}\|_\infty \tag{24}$$

Similarly, differentiating $\tilde{\mathbf{P}}(\alpha)$ twice w.r.t. α , we get

$$\begin{aligned}
\left[\left. \frac{d^2 \tilde{\mathbf{P}}(\alpha)}{d\alpha^2} \right|_{\alpha=0} \right]_{(s, a) \rightarrow (s', a')} &= \left. \frac{d^2 \pi_\alpha(a' | s')}{(d\alpha)^2} \right|_{\alpha=0} \mathbf{P}_\alpha(s' | s, a) + \left. \frac{d^2 \mathbf{P}_\alpha(s' | s, a)}{d\alpha^2} \right|_{\alpha=0} \pi_\alpha(a' | s') \\
&+ 2 \left. \frac{d\pi_\alpha(a' | s')}{d\alpha} \right|_{\alpha=0} \left. \frac{d\mathbf{P}_\alpha(s' | s, a)}{d\alpha} \right|_{\alpha=0}
\end{aligned}$$

Hence, we can consider the following norm bound:

$$\max_{\|\mathbf{u}\|_2=1} \left\| \left. \frac{d^2 \tilde{\mathbf{P}}(\alpha)}{d\alpha^2} \right|_{\alpha=0} \mathbf{x} \right\|_\infty \leq C_2 \|\mathbf{x}\|_\infty + 2C_1 T_1 \|\mathbf{x}\|_\infty + T_2 \|\mathbf{x}\|_\infty = (C_2 + 2C_1 T_1 + T_2) \|\mathbf{x}\|_\infty \tag{25}$$

Step 3: Now we need to put the pieces back together in order to calculate the second derivative of V_α^α w.r.t α . Let us recall $M(\alpha)$. Using the power series expansion of the matrix inverse, we can write $M(\alpha)$ as:

$$M(\alpha) = (I - \gamma \tilde{\mathbf{P}}(\alpha))^{-1} = \sum_{n=0}^{\infty} \gamma^n \tilde{\mathbf{P}}(\alpha)^n$$

which implies that $M(\alpha) \geq 0$ (component-wise), and

$$M(\alpha)\mathbf{1} = \frac{1}{1-\gamma}\mathbf{1},$$

i.e., each row of $M(\alpha)$ is positive and sums to $\frac{1}{1-\gamma}$.

This implies:

$$\max_{\|\mathbf{u}\|_2=1} \|M(\alpha)\mathbf{x}\|_\infty \leq \frac{1}{1-\gamma} \|\mathbf{x}\|_\infty.$$

This gives, using the expressions for $\frac{d^2 Q_\alpha^\alpha(s_0, a_0)}{d\alpha^2}$ and $\frac{d Q_\alpha^\alpha(s_0, a_0)}{d\alpha}$, an upper bound on their magnitudes based on $\|\mathbf{x}\|_\infty$ and constants arising from bounds on the derivatives of $\tilde{\mathbf{P}}(\alpha)$ and r_α .

$$\begin{aligned} & \max_{\|\mathbf{u}\|_2=1} \left\| \frac{d^2 Q_\alpha^\alpha(s_0, a_0)}{d\alpha^2} \right\|_\infty \\ & \leq 2\gamma^2 \left\| M(\alpha) \frac{d\tilde{\mathbf{P}}(\alpha)}{d\alpha} M(\alpha) \frac{d\tilde{\mathbf{P}}(\alpha)}{d\alpha} M(\alpha) r_\alpha \right\|_\infty + \gamma \left\| M(\alpha) \frac{d^2 \tilde{\mathbf{P}}(\alpha)}{d\alpha^2} M(\alpha) r_\alpha \right\|_\infty \\ & \quad + \gamma \left\| M(\alpha) \frac{d^2 \tilde{\mathbf{P}}(\alpha)}{d\alpha^2} M(\alpha) \frac{dr_\alpha}{d\alpha} \right\|_\infty + \left\| M(\alpha) \frac{d^2 r_\alpha}{d\alpha^2} \right\|_\infty + 2\gamma \left\| M(\alpha) \frac{d\tilde{\mathbf{P}}(\alpha)}{d\alpha} M(\alpha) \frac{dr_\alpha}{d\alpha} \right\|_\infty \end{aligned}$$

Bounding using known bounds on transitions and rewards:

$$\begin{aligned} \max_{\|\mathbf{u}\|_2=1} \left\| \frac{d^2 Q_\alpha^\alpha(s_0, a_0)}{d\alpha^2} \right\|_\infty & \leq \frac{2\gamma^2}{(1-\gamma)^3} (C_1 + T_1)^2 + \frac{\gamma}{(1-\gamma)^2} (C_2 + 2C_1T_1 + T_2) \\ & \quad + \frac{2\gamma R_1}{(1-\gamma)^2} (C_2 + 2C_1T_1 + T_2) + \frac{R_2}{1-\gamma} + \frac{\gamma C_1 R_1}{(1-\gamma)^2} = \beta_2 \end{aligned}$$

Corresponding bound on the first derivative is:

$$\begin{aligned} \max_{\|\mathbf{u}\|_2=1} \left\| \frac{d Q_\alpha^\alpha(s_0, a_0)}{d\alpha} \right\|_\infty & \leq \gamma \left\| M(\alpha) \frac{d\tilde{\mathbf{P}}(\alpha)}{d\alpha} M(\alpha) \frac{dr_\alpha}{d\alpha} \right\|_\infty + \left\| M(\alpha) \frac{dr_\alpha}{d\alpha} \right\|_\infty \\ & \leq \frac{\gamma}{(1-\gamma)^2} (C_1 + T_1) + \frac{R_1}{1-\gamma} = \beta_1 \end{aligned}$$

Step 4: Finally, putting all the bounds together to evaluate the upper bound of the desired quantity, we get,

$$\max_{\|\mathbf{u}\|_2=1} \left\| \frac{d^2 V_\alpha^\alpha(s_0)}{d\alpha^2} \right\|_\infty \leq \frac{C_2}{1-\gamma} + 2C_1\beta_1 + \beta_2 \quad (26)$$

□

Corollary 1. For softmax PeMDPs, we characterise

$$C_1 = 2, \quad C_2 = 6, \quad T_1 = \max_s |\psi(s)| \triangleq \psi_{\max}, \quad T_2 = \max_s |\psi(s)|^2, \quad R_1 = \xi|\mathcal{A}|, \quad R_2 = 0$$

Thus,

$$\max_{\|u\|_2=1} \left\| \frac{d^2 V_\alpha^\alpha(s_0)}{d\alpha^2} \Big|_{\alpha=0} \right\| \leq \mathcal{O} \left(\max \left\{ \frac{\gamma R_{\max} |\mathcal{A}|}{(1-\gamma)^2}, \frac{\gamma^2}{(1-\gamma)^3} \right\} \right) \triangleq \mathcal{O}(L). \quad (27)$$

Proof. We use the expressions already found in (33) to state the following:

$$\sum_{a \in \mathcal{A}} \left| \frac{d}{d\alpha} \pi_{\theta+\alpha u}(a | s) \Big|_{\alpha=0} \right| \leq \sum_{a \in \mathcal{A}} \pi_\theta(a | s) |\mathbf{u}_s^\top (\mathbf{e}_a - \pi(\cdot | s))| \leq \max_{a \in \mathcal{A}} (\mathbf{u}_s^\top \mathbf{e}_a + \mathbf{u}_s^\top \pi(\cdot | s)) \leq 2.$$

Similarly, differentiating once again w.r.t. α , we get

$$\begin{aligned} \sum_{a \in \mathcal{A}} \left| \frac{d^2}{d\alpha^2} \pi_{\theta+\alpha u}(a | s) \Big|_{\alpha=0} \right| &\leq \max_{a \in \mathcal{A}} (\mathbf{u}_s^\top \mathbf{e}_a \mathbf{e}_a^\top \mathbf{u}_s + \mathbf{u}_s^\top \mathbf{e}_a \pi(\cdot | s)^\top \mathbf{u}_s + \mathbf{u}_s^\top \pi(\cdot | s) \mathbf{e}_a^\top \mathbf{u}_s \\ &\quad + 2 \mathbf{u}_s^\top \pi(\cdot | s) \pi(\cdot | s)^\top \mathbf{u}_s + \mathbf{u}_s^\top \text{diag}(\pi(\cdot | s)) \mathbf{u}_s) \leq 6. \end{aligned}$$

And hence for transition we get,

$$\begin{aligned} \sum_{s' \in \mathcal{S}} \left| \frac{d}{d\alpha} \mathbf{P}_{\pi_{\theta+\alpha u}}(a | s) \Big|_{\alpha=0} \right| &\leq \sum_{s' \in \mathcal{S}} |\psi(s')| \mathbf{P}_{\pi_\theta}(s' | s, a) |\mathbf{u}_{s,a}(1 - \mathbf{P}_{\pi_\theta}(\cdot | s, a))| \\ &\leq |\mathbf{u}_{s,a}| \max_s |\psi(s)| \leq \max_s |\psi(s)| \end{aligned}$$

And similarly, it can be shown that:

$$\sum_{a \in \mathcal{A}} \left| \frac{d^2}{d\alpha^2} \mathbf{P}_{\pi_{\theta+\alpha u}}(a | s) \Big|_{\alpha=0} \right| \leq |\mathbf{u}_{s,a}|^2 \max_s |\psi(s)|^2 \leq \max_s |\psi(s)|^2$$

Similarly for rewards we get:

$$\sum_{a \in \mathcal{A}} \left| \frac{d}{d\alpha} r_{\pi_{\theta+\alpha u}}(a | s) \Big|_{\alpha=0} \right| \leq \xi |\mathcal{A}|, \quad \sum_{a \in \mathcal{A}} \left| \frac{d^2}{d\alpha^2} r_{\pi_{\theta+\alpha u}}(a | s) \Big|_{\alpha=0} \right| = 0$$

Hence, we can use the following choice of constants for softmax parametrization,

$$\begin{aligned} C_1 &= 2, \quad C_2 = 6 \\ T_1 &= \max_s |\psi(s)|, \quad T_2 = \max_s |\psi(s)|^2 \\ R_1 &= \xi |\mathcal{A}|, \quad R_2 = 0 \end{aligned}$$

to get the desired order of $\max_{\|u\|_2=1} \left\| \frac{d^2 V_\alpha^\alpha(s_0)}{d\alpha^2} \Big|_{\alpha=0} \right\|$.

□

Lemma 5 (Smoothness of Entropy Regularizer). Define the discounted entropy regularizer as:

$$\mathcal{H}_{\pi_{\theta_\alpha}}^\pi(s) = \mathbb{E}_{\tau \sim \mathbf{P}_\pi} \left[\sum_{t=0}^{\infty} -\gamma^t \log \pi_{\theta_\alpha}(a_t | s_t) \right]$$

Under the same assumptions as 4, the following holds:

$$\max_{\|u\|_2=1} \left\| \frac{\partial^2 \mathcal{H}_{\pi_{\theta_\alpha}}(s)}{\partial \alpha^2} \Big|_{\alpha=0} \right\|_\infty \leq \beta_\lambda$$

where

$$\beta_\lambda = 2\gamma^2 \frac{3(1 + \log |\mathcal{A}|)}{1 - \gamma} + \gamma \frac{2 \log |\mathcal{A}|}{(1 - \gamma)^2} (C_1 + T_1) + 2\gamma \frac{\log |\mathcal{A}|}{(1 - \gamma)^2} (C_2 + 2C_1T_1 + T_2) + \frac{\log |\mathcal{A}|}{(1 - \gamma)^3} (C_1 + T_1)^2.$$

Proof. **Step 1:** Define the state-wise entropy term:

$$h_{\theta_\alpha}(s) = - \sum_a \pi_{\theta_\alpha}(a | s) \log \pi_{\theta_\alpha}(a | s).$$

From [Mei et al. \(2020\)](#) (Lemma 7) we report that,

$$\left\| \frac{\partial h_{\theta_\alpha}}{\partial \alpha} \right\|_\infty \leq 2 \cdot \log |\mathcal{A}| \cdot \|u\|_2, \quad \left\| \frac{\partial^2 h_{\theta_\alpha}}{\partial \alpha^2} \right\|_\infty \leq 3 \cdot (1 + \log |\mathcal{A}|) \cdot \|\mathbf{u}\|_2^2. \quad (28)$$

Additionally, [Mei et al. \(2020\)](#) also presents a second result expressing the second derivative of the entropy w.r.t α ,

$$\begin{aligned} \frac{\partial^2 \mathcal{H}_{\pi_{\theta_\alpha}}(s)}{\partial \alpha^2} &= 2\gamma^2 \mathbf{e}_s^\top M(\alpha) \frac{\partial \mathbf{P}(\alpha)}{\partial \alpha} M(\alpha) \frac{\partial \mathbf{P}(\alpha)}{\partial \alpha} M(\alpha) h_{\theta_\alpha} \\ &\quad + \gamma \mathbf{e}_s^\top M(\alpha) \frac{\partial^2 \mathbf{P}(\alpha)}{\partial \alpha^2} M(\alpha) h_{\theta_\alpha} + 2\gamma \mathbf{e}_s^\top M(\alpha) \frac{\partial \mathbf{P}(\alpha)}{\partial \alpha} M(\alpha) \frac{\partial h_{\theta_\alpha}}{\partial \alpha} + \mathbf{e}_s^\top M(\alpha) \frac{\partial^2 h_{\theta_\alpha}}{\partial \alpha^2}. \end{aligned}$$

Step 2: Now we proceed with bounding the absolute value of each term which will contribute towards bounding the overall second derivative of the regulariser.

For the last term,

$$\begin{aligned} \left| \mathbf{e}_s^\top M(\alpha) \frac{\partial^2 h_{\theta_\alpha}}{\partial \alpha^2} \Big|_{\alpha=0} \right| &\leq \|\mathbf{e}_s^\top\|_1 \cdot \left\| M(\alpha) \frac{\partial^2 h_{\theta_\alpha}}{\partial \alpha^2} \Big|_{\alpha=0} \right\|_\infty \\ &\leq \frac{1}{1 - \gamma} \cdot \left\| \frac{\partial^2 h_{\theta_\alpha}}{\partial \alpha^2} \Big|_{\alpha=0} \right\|_\infty \\ &\leq \frac{3 \cdot (1 + \log |\mathcal{A}|)}{1 - \gamma} \cdot \|\mathbf{u}\|_2^2. \end{aligned}$$

For the second last term,

$$\begin{aligned} \left| \mathbf{e}_s^\top M(\alpha) \frac{\partial \mathbf{P}(\alpha)}{\partial \alpha} M(\alpha) \frac{\partial h_{\theta_\alpha}}{\partial \alpha} \Big|_{\alpha=0} \right| &\leq \left\| M(\alpha) \frac{\partial \mathbf{P}(\alpha)}{\partial \alpha} M(\alpha) \frac{\partial h_{\theta_\alpha}}{\partial \alpha} \Big|_{\alpha=0} \right\|_\infty \\ &\leq \frac{1}{1 - \gamma} \cdot \left\| \frac{\partial \mathbf{P}(\alpha)}{\partial \alpha} M(\alpha) \frac{\partial h_{\theta_\alpha}}{\partial \alpha} \Big|_{\alpha=0} \right\|_\infty \\ &\leq \frac{(C_1 + T_1) \cdot \|u\|_2}{1 - \gamma} \cdot \left\| M(\alpha) \frac{\partial h_{\theta_\alpha}}{\partial \alpha} \Big|_{\alpha=0} \right\|_\infty \\ &\leq \frac{(C_1 + T_1) \cdot \|\mathbf{u}\|_2}{(1 - \gamma)^2} \cdot \left\| \frac{\partial h_{\theta_\alpha}}{\partial \alpha} \Big|_{\alpha=0} \right\|_\infty \\ &\leq \frac{2 \cdot \log |\mathcal{A}|}{(1 - \gamma)^2} (C_1 + T_1) \cdot \|\mathbf{u}\|_2^2. \end{aligned}$$

For the second term,

$$\begin{aligned}
\left| \mathbf{e}_s^\top M(\alpha) \frac{\partial^2 \mathbf{P}(\alpha)}{\partial \alpha^2} M(\alpha) h_{\theta_\alpha} \Big|_{\alpha=0} \right| &\leq \left\| M(\alpha) \frac{\partial^2 \mathbf{P}(\alpha)}{\partial \alpha^2} M(\alpha) h_{\theta_\alpha} \Big|_{\alpha=0} \right\|_\infty \\
&\leq \frac{1}{1-\gamma} \cdot \left\| \frac{\partial^2 \mathbf{P}(\alpha)}{\partial \alpha^2} M(\alpha) h_{\theta_\alpha} \Big|_{\alpha=0} \right\|_\infty \\
&\leq \frac{\|\mathbf{u}\|_2^2}{1-\gamma} \cdot \left\| M(\alpha) h_{\theta_\alpha} \Big|_{\alpha=0} \right\|_\infty (C_2 + 2C_1 T_1 + T_2) \\
&\leq \frac{\|\mathbf{u}\|_2^2}{(1-\gamma)^2} \cdot \left\| h_{\theta_\alpha} \Big|_{\alpha=0} \right\|_\infty (C_2 + 2C_1 T_1 + T_2) \\
&\leq \frac{\log |\mathcal{A}|}{(1-\gamma)^2} (C_2 + 2C_1 T_1 + T_2) \cdot \|\mathbf{u}\|_2^2.
\end{aligned}$$

For the first term,

$$\begin{aligned}
\left| \mathbf{e}_s^\top M(\alpha) \frac{\partial \mathbf{P}(\alpha)}{\partial \alpha} M(\alpha) \frac{\partial \mathbf{P}(\alpha)}{\partial \alpha} M(\alpha) h_{\theta_\alpha} \Big|_{\alpha=0} \right| &\leq \left\| M(\alpha) \frac{\partial \mathbf{P}(\alpha)}{\partial \alpha} M(\alpha) \frac{\partial \mathbf{P}(\alpha)}{\partial \alpha} M(\alpha) h_{\theta_\alpha} \Big|_{\alpha=0} \right\|_\infty \\
&\leq \frac{1}{1-\gamma} \cdot \|\mathbf{u}\|_2 \cdot \frac{1}{1-\gamma} \cdot \|\mathbf{u}\|_2 \cdot \frac{1}{1-\gamma} \cdot \log |\mathcal{A}| \\
&\quad \cdot (C_1 + T_1)^2 \\
&= \frac{\log |\mathcal{A}|}{(1-\gamma)^3} (C_1 + T_1)^2 \cdot \|\mathbf{u}\|_2^2.
\end{aligned}$$

Step 3: Now combining all the above equations, we get the final expression,

$$\max_{\|\mathbf{u}\|_2=1} \left\| \frac{\partial^2 \mathcal{H}_{\pi_{\theta_\alpha}}^{\pi_{\theta_\alpha}}(s)}{\partial \alpha^2} \Big|_{\alpha=0} \right\|_\infty \leq \beta_\lambda$$

where

$$\begin{aligned}
\beta_\lambda = & 2\gamma^2 \cdot \frac{3 \cdot (1 + \log |\mathcal{A}|)}{1-\gamma} + \gamma \cdot \frac{2 \cdot \log |\mathcal{A}|}{(1-\gamma)^2} (C_1 + T_1) \\
& + 2\gamma \cdot \frac{\log |\mathcal{A}|}{(1-\gamma)^2} (C_2 + 2C_1 T_1 + T_2) + \frac{\log |\mathcal{A}|}{(1-\gamma)^3} (C_1 + T_1)^2
\end{aligned}$$

□

By definition of smoothness, the “soft performatice value function” \tilde{V}_π^π is Lipschitz smooth with Lipschitz constant L_λ where $L_\lambda \triangleq L + \lambda\beta_\lambda$. Once again, we can choose C_1, C_2, T_1, T_2 according to Corollary 1 for simplification to get the order $\beta_\lambda = \mathcal{O}\left(\frac{\log |\mathcal{A}|}{(1-\gamma)^3} \psi_{\max}^2\right)$. Thus, the final bound for L_λ as

$$L_\lambda = \mathcal{O}(\max\{L, \lambda\beta_\lambda\}) = \mathcal{O}\left(\max\left\{\frac{\gamma R_{\max} |\mathcal{A}|}{(1-\gamma)^2}, \frac{\lambda \log |\mathcal{A}| \psi_{\max}^2}{(1-\gamma)^3}\right\}\right). \quad (29)$$

F DERIVATION OF PERFORMATIVE POLICY GRADIENTS

Theorem 2 (Performative Policy Gradient Theorem). *The gradient of the performative value function w.r.t θ is as follows:*

(a) *For the unregularised objective,*

$$\begin{aligned} \nabla_{\theta} V_{\pi_{\theta}}^{\pi_{\theta}}(\tau) = \\ \mathbb{E}_{\tau \sim \mathbb{P}_{\pi_{\theta}}^{\pi_{\theta}}} \left[\sum_{t=0}^{\infty} \gamma^t (A_{\pi_{\theta}}^{\pi_{\theta}}(s_t, a_t) (\nabla_{\theta} \log \pi_{\theta}(a_t | s_t) + \nabla_{\theta} \log P_{\pi_{\theta}}(s_{t+1} | s_t, a_t)) + \nabla_{\theta} r_{\pi_{\theta}}(s_t, a_t)) \right]. \end{aligned} \quad (30)$$

(b) *For the entropy-regularised objective, we define the soft advantage, soft Q, and soft value functions with respect to the soft rewards $\tilde{r}_{\pi_{\theta}}$ satisfying $\tilde{A}_{\pi_{\theta}}^{\pi_{\theta}}(s, a) = \tilde{Q}_{\pi_{\theta}}^{\pi_{\theta}}(s, a) - \tilde{V}_{\pi_{\theta}}^{\pi_{\theta}}(s)$ that further yields*

$$\begin{aligned} \nabla_{\theta} \tilde{V}_{\pi_{\theta}}^{\pi_{\theta}}(\tau) = \\ \mathbb{E}_{\tau \sim \mathbb{P}_{\pi_{\theta}}^{\pi_{\theta}}} \left[\sum_{t=0}^{\infty} \gamma^t (\tilde{A}_{\pi_{\theta}}^{\pi_{\theta}}(s_t, a_t) (\nabla_{\theta} \log \pi_{\theta}(a_t | s_t) + \nabla_{\theta} \log P_{\pi_{\theta}}(s_{t+1} | s_t, a_t)) + \nabla_{\theta} \tilde{r}_{\pi_{\theta}}(s_t, a_t | \theta)) \right]. \end{aligned} \quad (31)$$

Proof of Theorem 2. We prove each part of this theorem separately.

Proof of part (a). First, we derive explicit closed form gradient for unregularised performative value function.

Step 1. Given a trajectory $\tau = \{s_0, a_0, \dots, s_t, a_t, \dots\}$, let us denote the unregularised objective function as

$$f_{\theta}(\tau) = \sum_{t=0}^{\infty} \gamma^t r_{\pi_{\theta}}(s_t, a_t)$$

Thus,

$$\begin{aligned} \nabla_{\theta} V_{\pi_{\theta}}^{\pi_{\theta}}(\tau) &= \nabla_{\theta} \mathbb{E}_{\tau \sim \mathbb{P}_{\pi_{\theta}}^{\pi_{\theta}}} [f_{\theta}(\tau)] = \nabla_{\theta} \sum_{\tau} \mathbb{P}_{\pi_{\theta}}^{\pi_{\theta}}(\tau) f_{\theta}(\tau) \\ &= \sum_{\tau} \nabla_{\theta} (\mathbb{P}_{\pi_{\theta}}^{\pi_{\theta}}(\tau) f_{\theta}(\tau)) \\ &= \sum_{\tau} (\nabla_{\theta} \mathbb{P}_{\pi_{\theta}}^{\pi_{\theta}}(\tau)) f_{\theta}(\tau) + \sum_{\tau} \mathbb{P}_{\pi_{\theta}}^{\pi_{\theta}}(\tau) (\nabla_{\theta} f_{\theta}(\tau)) \\ &\stackrel{(a)}{=} \sum_{\tau} \mathbb{P}_{\pi_{\theta}}^{\pi_{\theta}}(\tau) (\nabla_{\theta} \log \mathbb{P}_{\pi_{\theta}}^{\pi_{\theta}}(\tau)) f_{\theta}(\tau) + \mathbb{E}_{\tau \sim \mathbb{P}_{\pi_{\theta}}^{\pi_{\theta}}} [\nabla_{\theta} f_{\theta}(\tau)] \\ &= \mathbb{E}_{\tau \sim \mathbb{P}_{\pi_{\theta}}^{\pi_{\theta}}} [(\nabla_{\theta} \log \mathbb{P}_{\pi_{\theta}}^{\pi_{\theta}}(\tau)) f_{\theta}(\tau)] + \mathbb{E}_{\tau \sim \mathbb{P}_{\pi_{\theta}}^{\pi_{\theta}}} [\nabla_{\theta} f_{\theta}(\tau)]. \end{aligned}$$

(a) holds since $\nabla_{\theta} \log \mathbb{P}_{\pi_{\theta}}^{\pi_{\theta}}(\tau) = \frac{\nabla_{\theta} \mathbb{P}_{\pi_{\theta}}^{\pi_{\theta}}(\tau)}{\mathbb{P}_{\pi_{\theta}}^{\pi_{\theta}}(\tau)}$.

Step 2. Given the initial state distribution ρ , we further have

$$\log \mathbb{P}_{\pi_{\theta}}^{\pi_{\theta}}(\tau) = \log \rho(s_0) + \sum_{t=0}^{\infty} \log \pi_{\theta}(a_t | s_t) + \sum_{t=0}^{\infty} \log P_{\pi_{\theta}}(s_{t+1} | s_t, a_t)$$

Taking the gradient with respect to θ , we obtain

$$\nabla_{\theta} \log \mathbb{P}_{\pi_{\theta}}^{\pi_{\theta}}(\tau) = \sum_{t=0}^{\infty} \nabla_{\theta} \log \pi_{\theta}(a_t | s_t) + \sum_{t=0}^{\infty} \nabla_{\theta} \log P_{\pi_{\theta}}(s_{t+1} | s_t, a_t)$$

Step 3. Now, by substituting the value of $\nabla_{\boldsymbol{\theta}} \log(\mathbb{P}_{\pi_{\boldsymbol{\theta}}}^{\pi_{\boldsymbol{\theta}}})$ in $\nabla_{\boldsymbol{\theta}} V_{\pi_{\boldsymbol{\theta}}}^{\pi_{\boldsymbol{\theta}}}(\tau)$, we get,

$$\begin{aligned}
\nabla_{\boldsymbol{\theta}} V_{\pi_{\boldsymbol{\theta}}}^{\pi_{\boldsymbol{\theta}}}(\tau) &= \nabla_{\boldsymbol{\theta}} \mathbb{E}_{\tau \sim \mathbb{P}_{\pi_{\boldsymbol{\theta}}}^{\pi_{\boldsymbol{\theta}}}} [f_{\boldsymbol{\theta}}(\tau)] \\
&= \mathbb{E}_{\tau \sim \mathbb{P}_{\pi_{\boldsymbol{\theta}}}^{\pi_{\boldsymbol{\theta}}}} \left[\left(\sum_{t=0}^{\infty} \nabla_{\boldsymbol{\theta}} \log \pi_{\boldsymbol{\theta}}(a_t | s_t) \right) \cdot \left(\sum_{t=0}^{\infty} \gamma^t r_{\pi_{\boldsymbol{\theta}}}(s_t, a_t) \right) \right] \\
&\quad + \mathbb{E}_{\tau \sim \mathbb{P}_{\pi_{\boldsymbol{\theta}}}^{\pi_{\boldsymbol{\theta}}}} \left[\left(\sum_{t=1}^{\infty} \nabla_{\boldsymbol{\theta}} \log \mathbf{P}_{\pi_{\boldsymbol{\theta}}}(s_t | s_{t-1}, a_{t-1}) \right) \cdot \left(\sum_{t=0}^{\infty} \gamma^t r_{\pi_{\boldsymbol{\theta}}}(s_t, a_t) \right) \right] \\
&\quad + \mathbb{E}_{\tau \sim \mathbb{P}_{\pi_{\boldsymbol{\theta}}}^{\pi_{\boldsymbol{\theta}}}} \left[\sum_{t=0}^{\infty} \gamma^t \nabla_{\boldsymbol{\theta}} r_{\pi_{\boldsymbol{\theta}}}(s_t, a_t) \right] \\
&= \mathbb{E}_{\tau \sim \mathbb{P}_{\pi_{\boldsymbol{\theta}}}^{\pi_{\boldsymbol{\theta}}}} \left[\sum_{t=0}^{\infty} \gamma^t A_{\pi_{\boldsymbol{\theta}}}(s_t, a_t) \nabla_{\boldsymbol{\theta}} \log \pi_{\boldsymbol{\theta}}(a_t | s_t) \right] \\
&\quad + \mathbb{E}_{\tau \sim \mathbb{P}_{\pi_{\boldsymbol{\theta}}}^{\pi_{\boldsymbol{\theta}}}} \left[\sum_{t=1}^{\infty} \gamma^t A_{\pi_{\boldsymbol{\theta}}}(s_t, a_t) \nabla_{\boldsymbol{\theta}} \log \mathbf{P}_{\pi_{\boldsymbol{\theta}}}(s_t | s_{t-1}, a_{t-1}) \right] \\
&\quad + \mathbb{E}_{\tau \sim \mathbb{P}_{\pi_{\boldsymbol{\theta}}}^{\pi_{\boldsymbol{\theta}}}} \left[\sum_{t=0}^{\infty} \gamma^t \nabla_{\boldsymbol{\theta}} r_{\pi_{\boldsymbol{\theta}}}(s_t, a_t) \right].
\end{aligned}$$

The last equality is due to the definition of advantage function

$$\begin{aligned}
A_{\pi_{\boldsymbol{\theta}}}(s_t, a_t) &\triangleq \sum_{i=t+1}^{\infty} \gamma^{t-i} r_{\pi_{\boldsymbol{\theta}}}(s_i, \pi_{\boldsymbol{\theta}}(s_i)) - \mathbb{E}_{\substack{s_{t'+1} \sim \mathbf{P}_{\pi_{\boldsymbol{\theta}}}(s_{t'}, a_{t'}) \\ \forall t' \in [t, \infty)}} \left[\sum_{i=t+1}^{\infty} \gamma^{t-i} r_{\pi_{\boldsymbol{\theta}}}(s_i, \pi_{\boldsymbol{\theta}}(s_i)) | (s_t, a_t) \right] \\
&\triangleq Q_{\pi_{\boldsymbol{\theta}}}(s_t, a_t) - V_{\pi_{\boldsymbol{\theta}}}(s_t)
\end{aligned}$$

as in classical policy gradient theorem. Hence, we conclude the proof for part (a) of the theorem.

Proof of part (b). Now, we derive explicit gradient form for entropy-regularised value function.

Let us define the soft reward as $\tilde{r}_{\pi_{\boldsymbol{\theta}}}(s_t, a_t) \triangleq r_{\pi_{\boldsymbol{\theta}}}(s_t, a_t) - \lambda \log \pi_{\boldsymbol{\theta}}(a_t | s_t)$. Again, we start by defining regularised objective function

$$\tilde{f}_{\boldsymbol{\theta}}(\tau) = \sum_{t=0}^{\infty} \gamma^t \tilde{r}_{\pi_{\boldsymbol{\theta}}}(s_t, a_t)$$

Following the same steps as that of *Part (a)*, we get

$$\begin{aligned}
\nabla_{\boldsymbol{\theta}} \tilde{V}_{\pi_{\boldsymbol{\theta}}}^{\pi_{\boldsymbol{\theta}}}(\tau) &= \nabla_{\boldsymbol{\theta}} \mathbb{E}_{\tau \sim \mathbb{P}_{\pi_{\boldsymbol{\theta}}}^{\pi_{\boldsymbol{\theta}}}} [\tilde{f}_{\boldsymbol{\theta}}(\tau)] \\
&= \mathbb{E}_{\tau \sim \mathbb{P}_{\pi_{\boldsymbol{\theta}}}^{\pi_{\boldsymbol{\theta}}}} \left[\sum_{t=0}^{\infty} \gamma^t \tilde{A}_{\pi_{\boldsymbol{\theta}}}(s_t, a_t) \nabla_{\boldsymbol{\theta}} \log \pi_{\boldsymbol{\theta}}(a_t | s_t) \right] \\
&\quad + \mathbb{E}_{\tau \sim \mathbb{P}_{\pi_{\boldsymbol{\theta}}}^{\pi_{\boldsymbol{\theta}}}} \left[\sum_{t=1}^{\infty} \gamma^t \tilde{A}_{\pi_{\boldsymbol{\theta}}}(s_t, a_t) \nabla_{\boldsymbol{\theta}} \log \mathbf{P}_{\pi_{\boldsymbol{\theta}}}(s_t | s_{t-1}, a_{t-1}) \right] \\
&\quad + \mathbb{E}_{\tau \sim \mathbb{P}_{\pi_{\boldsymbol{\theta}}}^{\pi_{\boldsymbol{\theta}}}} \left[\sum_{t=0}^{\infty} \gamma^t \nabla_{\boldsymbol{\theta}} \tilde{r}_{\pi_{\boldsymbol{\theta}}}(s_t, a_t) \right] \\
&= \mathbb{E}_{\tau \sim \mathbb{P}_{\pi_{\boldsymbol{\theta}}}^{\pi_{\boldsymbol{\theta}}}} \left[\sum_{t=0}^{\infty} \gamma^t \tilde{A}_{\pi_{\boldsymbol{\theta}}}(s_t, a_t) \nabla_{\boldsymbol{\theta}} \log \pi_{\boldsymbol{\theta}}(a_t | s_t) \right] \\
&\quad + \mathbb{E}_{\tau \sim \mathbb{P}_{\pi_{\boldsymbol{\theta}}}^{\pi_{\boldsymbol{\theta}}}} \left[\sum_{t=1}^{\infty} \gamma^t \tilde{A}_{\pi_{\boldsymbol{\theta}}}(s_t, a_t) \nabla_{\boldsymbol{\theta}} \log \mathbf{P}_{\pi_{\boldsymbol{\theta}}}(s_t | s_{t-1}, a_{t-1}) \right] \\
&\quad + \mathbb{E}_{\tau \sim \mathbb{P}_{\pi_{\boldsymbol{\theta}}}^{\pi_{\boldsymbol{\theta}}}} \left[\sum_{t=0}^{\infty} \gamma^t \nabla_{\boldsymbol{\theta}} \tilde{r}_{\pi_{\boldsymbol{\theta}}}(s_t, a_t) \right] - \lambda \mathbb{E}_{\tau \sim \mathbb{P}_{\pi_{\boldsymbol{\theta}}}^{\pi_{\boldsymbol{\theta}}}} \left[\sum_{t=0}^{\infty} \gamma^t \nabla_{\boldsymbol{\theta}} \log \pi_{\boldsymbol{\theta}}(a_t | s_t) \right]
\end{aligned}$$

Here,

$$\begin{aligned}\tilde{A}_{\pi_{\theta}}^{\pi_{\theta}}(s_t, a_t) &\triangleq \sum_{i=t+1}^{\infty} \gamma^{t-i} \tilde{r}_{\pi_{\theta}}(s_i, \pi_{\theta}(s_i)) - \mathbb{E}_{\substack{s_{t'+1} \sim \mathbb{P}_{\pi_{\theta}}^{\pi_{\theta}}(\cdot | s_{t'}, a_{t'}) \\ \forall t' \in [t, \infty)}} \left[\sum_{i=t}^{\infty} \gamma^{t-i} \tilde{r}_{\pi_{\theta}}(s_i, \pi_{\theta}(s_i)) | (s_t, a_t) \right] \\ &\triangleq \tilde{Q}_{\pi_{\theta}}^{\pi_{\theta}}(s_t, a_t) - \tilde{V}_{\pi_{\theta}}^{\pi_{\theta}}(s_t)\end{aligned}$$

denotes the advantage function with soft rewards, or in brief, the soft advantage function. Hence, we conclude proof of part (b). \square

G CONVERGENCE OF PePG : PROOFS OF SECTION 4

G.1 PROOFS FOR UNREGULARISED VALUE FUNCTION

Lemma 6 (Performative Policy Gradient for Softmax PeMDPs). *Given softmax PeMDPs defined by (12), for all $(s, a, s') \in (\mathcal{S}, \mathcal{A}, \mathcal{S})$, derivative of the performative value function w.r.t $\theta_{s,a}$ satisfies:*

$$\frac{\partial V_{\pi_{\theta}}^{\pi_{\theta}}(\rho)}{\partial \theta_{s,a}} \geq \frac{1}{1-\gamma} d_{\pi_{\theta}}^{\pi_{\theta}}(s, a | \rho) (A_{\pi_{\theta}}^{\pi_{\theta}}(s, a) + \xi) . \quad (32)$$

Proof. First, we note that

$$\begin{aligned}\frac{\partial}{\partial \theta_{s',a'}} \log \pi_{\theta}(a | s) &= \mathbb{1}[s = s', a = a'] - \pi_{\theta}(a | s) \mathbb{1}[s = s'] \\ \frac{\partial}{\partial \theta_{s',a'}} \log \mathbb{P}_{\pi_{\theta}}(s'' | s, a) &= \psi(s'') \mathbb{1}[s = s', a = a'] (1 - \mathbb{P}_{\pi_{\theta}}(s'' | s, a)) \\ \frac{\partial}{\partial \theta_{s',a'}} r_{\pi_{\theta}}(s, a) &= \xi \mathbb{1}[s = s', a = a'] .\end{aligned} \quad (33)$$

In this proof, we further substitute the expressions of individual gradients in Equation (33) into Equation (9).

Therefore, for a given initial state distribution ρ , we get

$$\begin{aligned}\frac{\partial}{\partial \theta_{s,a}} V_{\pi_{\theta}}^{\pi_{\theta}}(\rho) &= \mathbb{E}_{\tau \sim \mathbb{P}_{\pi_{\theta}}^{\pi_{\theta}}} \left[\sum_{t=0}^{\infty} \gamma^t \left(A_{\pi_{\theta}}^{\pi_{\theta}}(s_t, a_t) \frac{\partial}{\partial \theta_{s,a}} \log \pi_{\theta}(a_t | s_t) \right. \right. \\ &\quad \left. \left. + A_{\pi_{\theta}}^{\pi_{\theta}}(s_t, a_t) \frac{\partial}{\partial \theta_{s,a}} \log P_{\pi_{\theta}}(s_{t+1} | s_t, a_t) \right. \right. \\ &\quad \left. \left. + \frac{\partial}{\partial \theta_{s,a}} r_{\pi_{\theta}}(s_t, a_t) \right) \right] \\ &= \mathbb{E}_{\tau \sim \mathbb{P}_{\pi_{\theta}}^{\pi_{\theta}}} \left[\sum_{t=0}^{\infty} \gamma^t \left(A_{\pi_{\theta}}^{\pi_{\theta}}(s_t, a_t) (\mathbb{1}[s_t = s, a_t = a] - \pi_{\theta}(a | s) \mathbb{1}[s_t = s]) \right. \right. \\ &\quad \left. \left. + A_{\pi_{\theta}}^{\pi_{\theta}}(s_t, a_t) \psi(s_{t+1}) \mathbb{1}[s_t = s, a_t = a] (1 - \mathbb{P}_{\pi_{\theta}}(s_{t+1} | s, a)) \right. \right. \\ &\quad \left. \left. + \xi \mathbb{1}[s_t = s, a_t = a] \right) \right] \\ &\stackrel{(a)}{\geq} \mathbb{E}_{\tau \sim \mathbb{P}_{\pi_{\theta}}^{\pi_{\theta}}} \left[\sum_{t=0}^{\infty} \gamma^t A_{\pi_{\theta}}^{\pi_{\theta}}(s_t, a_t) \mathbb{1}[s_t = s, a_t = a] \right] \\ &\quad - \mathbb{E}_{\tau \sim \mathbb{P}_{\pi_{\theta}}^{\pi_{\theta}}} \left[\sum_{t=0}^{\infty} \gamma^t \pi_{\theta}(a | s) \mathbb{1}[s_t = s] A_{\pi_{\theta}}^{\pi_{\theta}}(s_t, a_t) \right] \\ &\quad + \mathbb{E}_{\tau \sim \mathbb{P}_{\pi_{\theta}}^{\pi_{\theta}}} \left[\sum_{t=0}^{\infty} \gamma^t \xi \mathbb{1}[s_t = s, a_t = a] \right] \\ &\stackrel{(b)}{=} \frac{1}{1-\gamma} d_{\pi_{\theta}, \rho}^{\pi_{\theta}}(s, a) A_{\pi_{\theta}}^{\pi_{\theta}}(s, a) + \frac{1}{1-\gamma} \xi d_{\pi_{\theta}, \rho}^{\pi_{\theta}}(s, a)\end{aligned}$$

(a) is due to the fact that $1 - \mathbf{P}_{\pi_\theta}(s, a) \geq 0$ for all s, a . (b) is due to $\mathbb{E}_{\tau \sim \mathbb{P}_{\pi_\theta}^{\pi_\theta}} \left[\sum_{t=0}^{\infty} \gamma^t \pi_\theta(a|s) \mathbb{1}[s_t = s] A_{\pi_\theta}^{\pi_\theta}(s_t, a_t) \right] = 0$.

□

Lemma 3. *Performative Gradient Domination for Softmax PeMDPs* Let us consider PeMDPs defined in (12).

(a) For unregularised value function,

$$V_{\pi_o^*}^{\pi_o^*}(\rho) - V_{\pi_\theta}^{\pi_\theta}(\rho) \leq \sqrt{|\mathcal{S}||\mathcal{A}|} \left\| \frac{d_{\pi_\theta, \rho}^{\pi_o^*}}{d_{\pi_\theta, \nu}^{\pi_\theta}} \right\|_\infty \|\nabla_\theta V_{\pi_\theta}^{\pi_\theta}(\nu)\|_2 + \frac{R_{\max}}{1-\gamma} \left(1 + \frac{2\gamma}{1-\gamma} \psi_{\max} \right). \quad (34)$$

Proof of Lemma 3–Part (a). This proof is divided into two parts. In the first part we bound the expected advantage term from Lemma 2 with the norm of the gradient of value function. During this step, we need to express the expected advantage as a linear combination of the advantage itself and the occupancy measure over all states and actions like in equation (32). The expectation however is taken w.r.t the occupancy measure $d_{\pi_\theta}^{\pi_o^*}$, thus we need to perform a change of measure which introduces a coverage term as shown below. In the second step we directly use the bound of rewards and transitions obtained from their Lipchitzness in lemma 2. We know by Lemma 1 that

$$\begin{aligned} V_{\pi_o^*}^{\pi_o^*}(\rho) - V_{\pi_\theta}^{\pi_\theta}(\rho) &= \frac{1}{1-\gamma} \mathbb{E}_{(s,a) \sim d_{\pi_\theta, \rho}^{\pi_o^*}(\cdot | \rho)} [A_{\pi_\theta}^{\pi_\theta}(s, a)] \\ &\quad + \frac{1}{1-\gamma} \mathbb{E}_{(s,a) \sim d_{\pi_\theta, \rho}^{\pi_o^*}} [(r_{\pi_o^*}(s, a) - r_{\pi_\theta}(s, a)) \\ &\quad \quad \quad + \gamma (\mathbf{P}_{\pi_o^*}(\cdot | s, a) - \mathbf{P}_{\pi_\theta}(\cdot | s, a))^\top V_{\pi_o^*}^{\pi_o^*}(\cdot)]. \end{aligned}$$

Step 1: Upper bounding Term 1.

$$\begin{aligned} \mathbb{E}_{(s,a) \sim d_{\pi_\theta, \rho}^{\pi_o^*}} [A_{\pi_\theta}^{\pi_\theta}(s, a)] &= \sum_{s,a} d_{\pi_\theta}^{\pi_o^*}(s, a | \rho) A_{\pi_\theta}^{\pi_\theta}(s, a) = \sum_{s,a} \frac{d_{\pi_\theta}^{\pi_o^*}(s, a | \rho)}{d_{\pi_\theta}^{\pi_\theta}(s, a | \nu)} d_{\pi_\theta}^{\pi_\theta}(s, a | \nu) A_{\pi_\theta}^{\pi_\theta}(s, a) \\ &\leq \left\| \frac{d_{\pi_\theta, \rho}^{\pi_o^*}}{d_{\pi_\theta, \nu}^{\pi_\theta}} \right\|_\infty \sum_{s,a} d_{\pi_\theta}^{\pi_\theta}(s, a | \nu) A_{\pi_\theta}^{\pi_\theta}(s, a) \end{aligned} \quad (35)$$

Now, we leverage the gradient of softmax performative MDPs to obtain

$$\begin{aligned} \sum_{s,a} d_{\pi_\theta}^{\pi_\theta}(s, a | \nu) A_{\pi_\theta}^{\pi_\theta}(s, a) &\leq (1-\gamma) \sum_{s,a} \frac{\partial V_{\pi_\theta}^{\pi_\theta}(\nu)}{\partial \theta_{s,a}} - \xi \\ &= (1-\gamma) \mathbf{1}^\top \nabla_\theta V_{\pi_\theta}^{\pi_\theta}(\nu) - \xi \\ &\leq (1-\gamma) \sqrt{|\mathcal{S}||\mathcal{A}|} \|\nabla_\theta V_{\pi_\theta}^{\pi_\theta}(\nu)\|_2 - \xi \end{aligned}$$

The last inequality is obtained by applying Cauchy-Schwarz inequality.

Now, substituting the above result back in Equation (35), we get

$$\frac{1}{1-\gamma} \mathbb{E}_{(s,a) \sim d_{\pi_\theta, \rho}^{\pi_o^*}(\cdot | s_0)} [A_{\pi_\theta}^{\pi_\theta}(s, a)] \leq \sqrt{|\mathcal{S}||\mathcal{A}|} \left\| \frac{d_{\pi_\theta, \rho}^{\pi_o^*}}{d_{\pi_\theta, \nu}^{\pi_\theta}} \right\|_\infty \|\nabla_\theta V_{\pi_\theta}^{\pi_\theta}(\nu)\|_2 - \left\| \frac{d_{\pi_\theta, \rho}^{\pi_o^*}}{d_{\pi_\theta, \nu}^{\pi_\theta}} \right\|_\infty \frac{\xi}{1-\gamma} \quad (36)$$

Step 2: Upper bounding Term 2. For softmax rewards and transitions, we further obtain from Lemma 2,

$$\text{Term 2} = \frac{1}{1-\gamma} \mathbb{E}_{(s,a) \sim d_{\pi_\theta, \rho}^{\pi_o^*}} [(r_{\pi_o^*}(s, a) - r_{\pi_\theta}(s, a)) + \gamma (\mathbf{P}_{\pi_o^*}(\cdot | s, a) - \mathbf{P}_{\pi_\theta}(\cdot | s, a))^\top V_{\pi_o^*}^{\pi_o^*}(\cdot)]$$

$$\leq \frac{1}{1-\gamma}(\xi + \frac{\gamma}{1-\gamma}R_{\max}\psi_{\max})\|\boldsymbol{\pi}_o^*(\cdot|s_0) - \boldsymbol{\pi}_{\theta}(\cdot|s_0)\|_1 \quad (37)$$

$$\leq \frac{2}{1-\gamma}(\xi + \frac{\gamma}{1-\gamma}R_{\max}\psi_{\max}). \quad (38)$$

Step 3: Now, if we use Equation (36) and (38) together, we get

$$\begin{aligned} V_{\boldsymbol{\pi}_o^*}(\rho) - V_{\boldsymbol{\pi}_{\theta}}(\rho) &\leq \sqrt{|\mathcal{S}||\mathcal{A}|} \left\| \frac{d_{\boldsymbol{\pi}_{\theta}, \rho}^{\boldsymbol{\pi}_o^*}}{d_{\boldsymbol{\pi}_{\theta}, \nu}^{\boldsymbol{\pi}_o^*}} \right\|_{\infty} \|\nabla_{\theta} V_{\boldsymbol{\pi}_{\theta}}(\nu)\|_2 + \left(2 - \left\| \frac{d_{\boldsymbol{\pi}_{\theta}, \rho}^{\boldsymbol{\pi}_o^*}}{d_{\boldsymbol{\pi}_{\theta}, \nu}^{\boldsymbol{\pi}_o^*}} \right\|_{\infty} \right) \frac{\xi}{1-\gamma} \\ &\quad + \frac{2\gamma}{(1-\gamma)^2} R_{\max} \psi_{\max} \\ &\leq \sqrt{|\mathcal{S}||\mathcal{A}|} \left\| \frac{d_{\boldsymbol{\pi}_{\theta}, \rho}^{\boldsymbol{\pi}_o^*}}{d_{\boldsymbol{\pi}_{\theta}, \nu}^{\boldsymbol{\pi}_o^*}} \right\|_{\infty} \|\nabla_{\theta} V_{\boldsymbol{\pi}_{\theta}}(\nu)\|_2 + \frac{R_{\max}}{1-\gamma} + \frac{2\gamma}{(1-\gamma)^2} R_{\max} \psi_{\max} \\ &= \sqrt{|\mathcal{S}||\mathcal{A}|} \left\| \frac{d_{\boldsymbol{\pi}_{\theta}, \rho}^{\boldsymbol{\pi}_o^*}}{d_{\boldsymbol{\pi}_{\theta}, \nu}^{\boldsymbol{\pi}_o^*}} \right\|_{\infty} \|\nabla_{\theta} V_{\boldsymbol{\pi}_{\theta}}(\nu)\|_2 + \frac{R_{\max}}{1-\gamma} \left(1 + \frac{2\gamma}{1-\gamma} \psi_{\max} \right) \end{aligned}$$

The last inequality is true since $\left\| \frac{d_{\boldsymbol{\pi}_{\theta}, \rho}^{\boldsymbol{\pi}_o^*}}{d_{\boldsymbol{\pi}_{\theta}, \nu}^{\boldsymbol{\pi}_o^*}} \right\|_{\infty} \geq 1$ (Lemma 9) and $\xi \leq R_{\max}$. \square

Theorem 3 (Convergence of PePG in softmax PeMDPs – Part (a)). *Let Cov $\triangleq \max_{\theta, \nu} \left\| \frac{d_{\boldsymbol{\pi}_{\theta}, \rho}^{\boldsymbol{\pi}_o^*}}{d_{\boldsymbol{\pi}_{\theta}, \nu}^{\boldsymbol{\pi}_o^*}} \right\|_{\infty}$. The gradient ascent algorithm on $V_{\boldsymbol{\pi}_{\theta}}(\rho)$ (Equation (8)) with step size $\eta = \Omega(\min\{\frac{(1-\gamma)^2}{\gamma|\mathcal{A}|}, \frac{(1-\gamma)^3}{\gamma^2}\})$ satisfies, for all distributions $\rho \in \Delta(\mathcal{S})$.*

(a) For unregularised case,

$$\min_{t < T} \left\{ V_{\boldsymbol{\pi}_o^*}(\rho) - V_{\boldsymbol{\pi}_{\theta_t}}(\rho) \right\} \leq \epsilon + \mathcal{O}\left(\frac{1}{1-\gamma}\right)$$

when

$$T = \Omega\left(\frac{|\mathcal{S}||\mathcal{A}| \text{Cov}^2}{\epsilon^2} \max\left\{\frac{\gamma R_{\max} |\mathcal{A}|}{(1-\gamma)^3}, \frac{\gamma^2}{(1-\gamma)^4}\right\}\right)$$

Proof of Theorem 3–Part (a). We proceed with this proof by dividing it in four steps. In the first step, we use the smoothness of the value function to prove an upper bound for the minimum squared gradient norm of the value over time which is a constant times $1/T$. In the second step, we derive a lower bound on the norm of gradient of value function using Lemma 3. In the final two steps, we combine the bounds obtained from the first two steps to derive lower bounds for T and ϵ , i.e. the error threshold.

Step 1: As $V_{\boldsymbol{\pi}_{\theta}}$ is L -smooth (Lemma 4), it satisfies

$$\left| V_{\boldsymbol{\pi}_{\theta}}(\rho) - V_{\boldsymbol{\pi}'_{\theta}}(\rho) - \langle \nabla_{\theta} V_{\boldsymbol{\pi}_{\theta}}(\rho), \boldsymbol{\theta} - \boldsymbol{\theta}' \rangle \right| \leq \frac{L}{2} \|\boldsymbol{\theta} - \boldsymbol{\theta}'\|^2$$

Thus, taking $\boldsymbol{\theta}$ as $\boldsymbol{\theta}_{t+1}$ and $\boldsymbol{\theta}'$ as $\boldsymbol{\theta}_t$ and using the gradient ascent expression (Equation (8)) yields

$$\begin{aligned} \left| V_{\boldsymbol{\pi}_{\theta}^{(t+1)}}(\rho) - V_{\boldsymbol{\pi}_{\theta}^{(t)}}(\rho) - \eta \|\nabla_{\theta} V_{\boldsymbol{\pi}_{\theta}^{(t)}}(\rho)\|^2 \right| &\leq \frac{L}{2} \|\boldsymbol{\theta}_{t+1} - \boldsymbol{\theta}_t\|^2 \\ \implies V_{\boldsymbol{\pi}_{\theta}^{(t+1)}}(\rho) - V_{\boldsymbol{\pi}_{\theta}^{(t)}}(\rho) &\geq \eta \|\nabla_{\theta} V_{\boldsymbol{\pi}_{\theta}^{(t)}}(\rho)\|^2 - \frac{L}{2} \|\boldsymbol{\theta}_{t+1} - \boldsymbol{\theta}_t\|^2 \end{aligned}$$

This further implies that

$$V_{\boldsymbol{\pi}_{\theta}^{(t+1)}}(\rho) - V_{\boldsymbol{\pi}_o^*}(\rho) \geq V_{\boldsymbol{\pi}_{\theta}^{(t)}}(\rho) - V_{\boldsymbol{\pi}_o^*}(\rho) + \eta \|\nabla_{\theta} V_{\boldsymbol{\pi}_{\theta}^{(t)}}(\rho)\|^2 - \frac{L}{2} \|\boldsymbol{\theta}_{t+1} - \boldsymbol{\theta}_t\|^2$$

$$= V_{\pi_\theta^{(t)}}^{\pi_\theta^{(t)}}(\rho) - V_{\pi_\theta^{(t)}}^{\pi_\theta^*}(\rho) + \eta(1 - \frac{L\eta}{2}) \|\nabla V_{\pi_\theta^{(t)}}^{\pi_\theta^{(t)}}(\rho)\|^2 \quad (39)$$

The last equality is due to Equation (8).

Now, telescoping Equation (39) leads to

$$\eta(1 - \frac{L\eta}{2}) \sum_{t=0}^{T-1} \|\nabla V_{\pi_\theta^{(t)}}^{\pi_\theta^{(t)}}(\rho)\|^2 \leq \left(V_{\pi_\theta^*}^{\pi_\theta^*}(\rho) - V_{\pi_\theta^0}^{\pi_\theta^0}(\rho) \right) - \left(V_{\pi_\theta^*}^{\pi_\theta^*}(\rho) - V_{\pi_\theta^T}^{\pi_\theta^T}(\rho) \right) \quad (40)$$

$$\leq \left(V_{\pi_\theta^*}^{\pi_\theta^*}(\rho) - V_{\pi_\theta^0}^{\pi_\theta^0}(\rho) \right) \quad (41)$$

Since $\sum_{t=0}^{T-1} \|\nabla V_{\pi_\theta^{(t)}}^{\pi_\theta^{(t)}}(\rho)\|^2 \geq T \min_{t \in [T-1]} \|\nabla V_{\pi_\theta^{(t)}}^{\pi_\theta^{(t)}}(\rho)\|^2$, we obtain

$$\min_{t \in [T-1]} \|\nabla V_{\pi_\theta^{(t)}}^{\pi_\theta^{(t)}}(\rho)\|^2 \leq \frac{1}{T\eta \left(1 - \frac{L\eta}{2}\right)} \left(V_{\pi_\theta^*}^{\pi_\theta^*}(\rho) - V_{\pi_\theta^0}^{\pi_\theta^0}(\rho) \right) \leq \frac{R_{\max}}{T\eta \left(1 - \frac{L\eta}{2}\right) (1 - \gamma)}.$$

The last inequality comes from $V_{\pi_\theta^*}^{\pi_\theta^*}(\rho) \leq \frac{R_{\max}}{1-\gamma}$ (Assumption 1).

Step 2: We derive from Equation (14) that

$$\begin{aligned} (V_{\pi_\theta^*}^{\pi_\theta^*}(\rho) - V_{\pi_\theta}^{\pi_\theta}(\rho))^2 &\leq \left(\sqrt{|\mathcal{S}||\mathcal{A}|} \left\| \frac{\mathbf{d}_{\pi_\theta, \rho}^{\pi_\theta^*}}{\mathbf{d}_{\pi_\theta, \nu}^{\pi_\theta}} \right\|_\infty \|\nabla_{\pi_\theta} V_{\pi_\theta}^{\pi_\theta}(\nu)\|_2 + \frac{2R_{\max}}{1-\gamma} \left(\frac{1}{2} + \frac{\gamma}{1-\gamma} \psi_{\max} \right) \right)^2 \\ &\leq 2|\mathcal{S}||\mathcal{A}| \left\| \frac{\mathbf{d}_{\pi_\theta, \rho}^{\pi_\theta^*}}{\mathbf{d}_{\pi_\theta, \nu}^{\pi_\theta}} \right\|_\infty^2 \|\nabla_{\pi_\theta} V_{\pi_\theta}^{\pi_\theta}(\nu)\|_2^2 + \frac{8R_{\max}^2}{(1-\gamma)^2} \left(\frac{1}{2} + \frac{\gamma}{1-\gamma} \psi_{\max} \right)^2. \end{aligned}$$

Thus, we further get

$$\begin{aligned} \min_{t \in [T]} (V_{\pi_\theta^*}^{\pi_\theta^*}(\rho) - V_{\pi_\theta^{(t)}}^{\pi_\theta^{(t)}}(\rho))^2 &\leq 2|\mathcal{S}||\mathcal{A}| \min_{t \in [T]} \left\| \frac{\mathbf{d}_{\pi_\theta^{(t)}, \rho}^{\pi_\theta^*}}{\mathbf{d}_{\pi_\theta^{(t)}, \nu}^{\pi_\theta}} \right\|_\infty^2 \|\nabla_{\pi_\theta} V_{\pi_\theta^{(t)}}^{\pi_\theta^{(t)}}(\nu)\|_2^2 + \frac{8R_{\max}^2}{(1-\gamma)^2} \left(\frac{1}{2} + \frac{\gamma}{1-\gamma} \psi_{\max} \right)^2 \\ &\leq 2|\mathcal{S}||\mathcal{A}| \text{Cov}^2 \min_{t \in [T]} \|\nabla_{\pi_\theta} V_{\pi_\theta^{(t)}}^{\pi_\theta^{(t)}}(\nu)\|_2^2 + \frac{8R_{\max}^2}{(1-\gamma)^2} \left(\frac{1}{2} + \frac{\gamma}{1-\gamma} \psi_{\max} \right)^2 \\ &\leq 2|\mathcal{S}||\mathcal{A}| \text{Cov}^2 \frac{R_{\max}}{T\eta \left(1 - \frac{L\eta}{2}\right) (1 - \gamma)} + \frac{8R_{\max}^2}{(1-\gamma)^2} \left(\frac{1}{2} + \frac{\gamma}{1-\gamma} \psi_{\max} \right)^2. \end{aligned}$$

Step 3: Now, we set

$$\begin{aligned} \min_{t \in [T]} (V_{\pi_\theta^*}^{\pi_\theta^*}(\rho) - V_{\pi_\theta^{(t)}}^{\pi_\theta^{(t)}}(\rho))^2 &\leq 2|\mathcal{S}||\mathcal{A}| \text{Cov}^2 \frac{R_{\max}}{T\eta \left(1 - \frac{L\eta}{2}\right) (1 - \gamma)} + \frac{8R_{\max}^2}{(1-\gamma)^2} \left(\frac{1}{2} + \frac{\gamma}{1-\gamma} \psi_{\max} \right)^2 \\ &\leq \left(\sqrt{2|\mathcal{S}||\mathcal{A}| \frac{R_{\max}}{T\eta \left(1 - \frac{L\eta}{2}\right) (1 - \gamma)}} \text{Cov} + \frac{2\sqrt{2}R_{\max}}{(1-\gamma)} \left(\frac{1}{2} + \frac{\gamma}{1-\gamma} \psi_{\max} \right) \right)^2 \\ &\leq \left(\epsilon + \frac{\sqrt{2}R_{\max}}{(1-\gamma)} \left(1 + \frac{2\gamma}{1-\gamma} \psi_{\max} \right) \right)^2, \end{aligned}$$

and solve for T to get

$$T \geq \frac{2|\mathcal{S}||\mathcal{A}| \text{Cov}^2 R_{\max}}{\eta(1 - \frac{L\eta}{2})(1 - \gamma)\epsilon^2} \quad (42)$$

Choosing $\eta = \frac{1}{L}$, we get the final expression

$$T \geq \frac{4L|\mathcal{S}||\mathcal{A}| \text{Cov}^2 R_{\max}}{\epsilon^2(1 - \gamma)}. \quad (43)$$

for any $\epsilon > 0$ and the smoothness constant $L = \mathcal{O} \left(\max \left\{ \frac{\gamma R_{\max} |\mathcal{A}|}{(1-\gamma)^2}, \frac{\gamma^2}{(1-\gamma)^3} \right\} \right)$.

Hence, we conclude that for $T = \Omega \left(\frac{|\mathcal{S}||\mathcal{A}|}{\epsilon^2} \max \left\{ \frac{\gamma R_{\max} |\mathcal{A}|}{(1-\gamma)^3}, \frac{\gamma^2}{(1-\gamma)^4} \right\} \right)$ and $\psi_{\max} = \mathcal{O}(\frac{1-\gamma}{\gamma})$,

$$\min_{t \in [T]} (V_{\pi_o^*}^*(\rho) - V_{\pi_\theta^{(t)}}^*(\rho)) \leq \epsilon + \mathcal{O} \left(\frac{1}{1-\gamma} \right).$$

□

G.2 PROOFS FOR ENTROPY-REGULARISED OR SOFT VALUE FUNCTION

Definition 6. The discounted state occupancy measure $d_{\pi'}^\pi(s|s_0)$ induced by a policy π and an MDP environment defined by π' is defined as

$$d_{\pi'}^\pi(s|s_0) \triangleq \sum_{a \in \mathcal{A}} d_{\pi'}^\pi(s, a|s_0) = (1-\gamma) \sum_{a \in \mathcal{A}} \mathbb{E}_{\tau \sim \mathbb{P}_{\pi'}} \left[\sum_{t=0}^{\infty} \gamma^t \mathbb{1}\{s_t = s, a_t = a\} \right].$$

Lemma 7 (Regularized Performative Policy Difference: Generic Upper Bound). *Under Assumption 1, the sub-optimality gap of a policy π_θ is*

$$\begin{aligned} \tilde{V}_{\pi_o^*}^*(s_0) - \tilde{V}_{\pi_\theta}^*(s_0) &\leq \frac{1}{1-\gamma} \mathbb{E}_{(s,a) \sim d_{\pi_\theta}^{\pi_o^*}(\cdot|s_0)} [\tilde{A}_{\pi_\theta}(s, a)] \\ &\quad + \frac{2}{1-\gamma} \left(\xi + \frac{\gamma}{1-\gamma} \psi_{\max}(R_{\max} + \lambda \log |\mathcal{A}|) \right) \\ &\quad - \frac{\lambda}{1-\gamma} \sum_s d_{\pi_\theta}^{\pi_o^*}(s|s_0) D_{\text{KL}}(\pi_o^*(\cdot|s) \parallel \pi_\theta(\cdot|s)) \end{aligned} \quad (44)$$

Proof. This lemma follows the same sketch as Lemma 2 with an exception in the way the soft rewards are handled. The difference in the soft rewards equals the difference of the original rewards with a lagrange dependent term. This term is the expected KL divergence over the state visitation distribution. Lemma 1 for regularized rewards reduces to,

$$\begin{aligned} \tilde{V}_\pi(s_0) - \tilde{V}_{\pi'}(s_0) &= \frac{1}{1-\gamma} \mathbb{E}_{(s,a) \sim d_{\pi'}^{\pi}(\cdot|s_0)} [\tilde{A}_{\pi'}(s, a)] \\ &\quad + \frac{1}{1-\gamma} \mathbb{E}_{(s,a) \sim d_{\pi'}^{\pi}(\cdot|s_0)} \left([\tilde{r}_\pi(s, a) - \tilde{r}_{\pi'}(s, a)] + \gamma (\mathbf{P}_\pi - \mathbf{P}_{\pi'})^\top \tilde{V}_\pi(s_0) \right). \end{aligned} \quad (45)$$

Therefore,

$$\tilde{r}_{\pi_o^*}(s, a) - \tilde{r}_{\pi_\theta}(s, a) = r_{\pi_o^*}(s, a) - r_{\pi_\theta}(s, a) + \lambda \left(\log \pi_\theta(a|s) - \log \pi_o^*(a|s) \right)$$

Therefore, we can write (45) in the following way,

$$\begin{aligned} \tilde{V}_{\pi_o^*}^*(s_0) - \tilde{V}_{\pi_\theta}^*(s_0) &= \frac{1}{1-\gamma} \mathbb{E}_{(s,a) \sim d_{\pi_\theta}^{\pi_o^*}(\cdot|s_0)} [\tilde{A}_{\pi_\theta}(s, a)] \\ &\quad + \frac{1}{1-\gamma} \mathbb{E}_{(s,a) \sim d_{\pi_\theta}^{\pi_o^*}(\cdot|s_0)} \left([\tilde{r}_{\pi_o^*}(s, a) - \tilde{r}_{\pi_\theta}(s, a)] + \gamma (\mathbf{P}_{\pi_o^*} - \mathbf{P}_{\pi_\theta})^\top \tilde{V}_{\pi_o^*}^*(s_0) \right) \\ &\quad + \frac{\lambda}{1-\gamma} \sum_{s,a} [\log \pi_\theta(a|s) - \log \pi_o^*(a|s)] d_{\pi_\theta}^{\pi_o^*}(s, a|s_0) \\ &= \frac{1}{1-\gamma} \mathbb{E}_{(s,a) \sim d_{\pi_\theta}^{\pi_o^*}(\cdot|s_0)} [\tilde{A}_{\pi_\theta}(s, a)] \\ &\quad + \frac{1}{1-\gamma} \mathbb{E}_{(s,a) \sim d_{\pi_\theta}^{\pi_o^*}(\cdot|s_0)} \left([\tilde{r}_{\pi_o^*}(s, a) - \tilde{r}_{\pi_\theta}(s, a)] + \gamma (\mathbf{P}_{\pi_o^*} - \mathbf{P}_{\pi_\theta})^\top \tilde{V}_{\pi_o^*}^*(s_0) \right) \end{aligned}$$

$$\begin{aligned}
& + \frac{\lambda}{1-\gamma} \sum_{s,a} d_{\pi_\theta}^{\pi_o^*}(s|s_0) \pi_o^*(a|s) [\log \pi_\theta(a|s) - \log \pi_o^*(a|s)] \\
& \stackrel{(a)}{=} \frac{1}{1-\gamma} \mathbb{E}_{(s,a) \sim d_{\pi_\theta}^{\pi_o^*}(\cdot|s_0)} [\tilde{A}_{\pi_\theta}^{\pi_\theta}(s,a)] \\
& \quad + \frac{1}{1-\gamma} \mathbb{E}_{(s,a) \sim d_{\pi_\theta}^{\pi_o^*}(\cdot|s_0)} \left([\tilde{r}_{\pi_o^*}(s,a) - \tilde{r}_{\pi_\theta}(s,a)] + \gamma (\mathbf{P}_{\pi_o^*} - \mathbf{P}_{\pi_\theta})^\top \tilde{V}_{\pi_o^*}^{\pi_o^*}(s_0) \right) \\
& \quad - \frac{\lambda}{1-\gamma} \sum_s d_{\pi_\theta}^{\pi_o^*}(s|s_0) D_{\text{KL}}(\pi_o^*(\cdot|s) \parallel \pi_\theta(\cdot|s)) \\
& \stackrel{\text{Holder's ineq.}}{\leq} \frac{1}{1-\gamma} \mathbb{E}_{(s,a) \sim d_{\pi_\theta}^{\pi_o^*}(\cdot|s_0)} [\tilde{A}_{\pi_\theta}^{\pi_\theta}(s,a)] \\
& \quad + \frac{1}{1-\gamma} \mathbb{E}_{(s,a) \sim d_{\pi_\theta}^{\pi_o^*}(\cdot|s_0)} \left([\tilde{r}_{\pi_o^*}(s,a) - \tilde{r}_{\pi_\theta}(s,a)] \right. \\
& \quad \left. + \gamma \|\mathbf{P}_{\pi_o^*} - \mathbf{P}_{\pi_\theta}\|_1 \|\tilde{V}_{\pi_o^*}^{\pi_o^*}(s_0)\|_\infty \right) \\
& \quad - \frac{\lambda}{1-\gamma} \sum_s d_{\pi_\theta}^{\pi_o^*}(s|s_0) D_{\text{KL}}(\pi_o^*(\cdot|s) \parallel \pi_\theta(\cdot|s)) \\
& \stackrel{(b)}{\leq} \frac{1}{1-\gamma} \mathbb{E}_{(s,a) \sim d_{\pi_\theta}^{\pi_o^*}(\cdot|s_0)} [\tilde{A}_{\pi_\theta}^{\pi_\theta}(s,a)] \\
& \quad + \frac{1}{1-\gamma} \mathbb{E}_{(s,a) \sim d_{\pi_\theta}^{\pi_o^*}(\cdot|s_0)} \left([\tilde{r}_{\pi_o^*}(s,a) - \tilde{r}_{\pi_\theta}(s,a)] \right. \\
& \quad \left. + \gamma \|\mathbf{P}_{\pi_o^*} - \mathbf{P}_{\pi_\theta}\|_1 \frac{R_{\max} + \lambda \log |\mathcal{A}|}{1-\gamma} \right) \\
& \quad - \frac{\lambda}{1-\gamma} \sum_s d_{\pi_\theta}^{\pi_o^*}(s|s_0) D_{\text{KL}}(\pi_o^*(\cdot|s) \parallel \pi_\theta(\cdot|s)) \\
& \stackrel{\text{Lipschitz } r \text{ & } \mathbf{P}}{\leq} \frac{1}{1-\gamma} \mathbb{E}_{(s,a) \sim d_{\pi_\theta}^{\pi_o^*}(\cdot|s_0)} [\tilde{A}_{\pi_\theta}^{\pi_\theta}(s,a)] \\
& \quad + \frac{1}{1-\gamma} \mathbb{E}_{(s,a) \sim d_{\pi_\theta}^{\pi_o^*}(\cdot|s_0)} \left(L_r + L_{\mathbf{P}} \frac{\gamma(R_{\max} + \lambda \log |\mathcal{A}|)}{1-\gamma} \right) \|\pi_o^* - \pi_\theta\|_1 \\
& \quad - \frac{\lambda}{1-\gamma} \sum_s d_{\pi_\theta}^{\pi_o^*}(s|s_0) D_{\text{KL}}(\pi_o^*(\cdot|s) \parallel \pi_\theta(\cdot|s)) \\
& \stackrel{(c)}{\leq} \frac{1}{1-\gamma} \mathbb{E}_{(s,a) \sim d_{\pi_\theta}^{\pi_o^*}(\cdot|s_0)} [\tilde{A}_{\pi_\theta}^{\pi_\theta}(s,a)] \\
& \quad + \frac{2}{1-\gamma} \mathbb{E}_{(s,a) \sim d_{\pi_\theta}^{\pi_o^*}(\cdot|s_0)} \left(L_r + L_{\mathbf{P}} \frac{\gamma(R_{\max} + \lambda \log |\mathcal{A}|)}{1-\gamma} \right) \\
& \quad - \frac{\lambda}{1-\gamma} \sum_s d_{\pi_\theta}^{\pi_o^*}(s|s_0) D_{\text{KL}}(\pi_o^*(\cdot|s) \parallel \pi_\theta(\cdot|s))
\end{aligned}$$

The equality (a) holds since,

$$\mathbb{E}_{a \sim \pi_o^*(\cdot|s)} [\log \pi_\theta(a|s) - \log \pi_o^*(a|s)] = -D_{\text{KL}}(\pi_o^*(\cdot|s) \parallel \pi_\theta(\cdot|s))$$

The inequality (b) holds due to the result of [Mei et al. \(2020\)](#), i.e.

$$\|\tilde{V}_{\pi_o^*}^{\pi_o^*}\|_\infty \leq \frac{R_{\max} + \lambda \log |\mathcal{A}|}{1-\gamma} \tag{46}$$

Finally, (c) is due to the fact that $\|\pi_o^* - \pi_\theta\|_1 \leq 2$.

□

Lemma 8 (Regularized Performative Policy gradient for softmax policies and softmax MDPs). *For a class of PeMDPs $\mathcal{M} \triangleq (\mathcal{S}, \mathcal{A}, \boldsymbol{\pi}, \mathbf{P}_\pi, r_\pi, \boldsymbol{\theta}, \rho)$ consider softmax parametrization for policy $\pi_\theta \in \Delta(\boldsymbol{\theta} \in \Theta)$ and transition dynamics \mathbf{P}_{π_θ} and linear parametrization for reward r_{π_θ} . For*

all $(s, a, s') \in (\mathcal{S}, \mathcal{A}, \mathcal{S})$, derivative of the expected return w.r.t $\theta_{s,a}$ satisfies:

$$\frac{\partial \tilde{V}_{\pi_\theta}(\rho)}{\partial \theta_{s,a}} \geq \frac{1}{1-\gamma} d_{\pi_\theta}^{\pi_\theta}(s, a | \rho) \left(\tilde{A}_{\pi_\theta}^{\pi_\theta}(s, a) + \xi \right) - \frac{\lambda}{1-\gamma} (1 + \log |\mathcal{A}|). \quad (47)$$

Proof. This proof follows the same sketch as the proof of Theorem 3. However, we get two additional λ -dependent terms— (a) one from the log policy term in the soft advantage, and (b) the other from the log policy term in the soft rewards. We then simplify these terms to obtain the final expression.

First, let us note that

$$\begin{aligned} \frac{\partial}{\partial \theta_{s',a'}} \log \pi_\theta(a|s) &= \mathbb{1}[s = s', a = a'] - \pi_\theta(a'|s) \mathbb{1}[s = s'] \\ \frac{\partial}{\partial \theta_{s',a'}} \log \mathbf{P}_{\pi_\theta}(s''|s, a) &= \psi(s'') \mathbb{1}[s = s', a = a'] (1 - \mathbf{P}_{\pi_\theta}(s''|s, a)) \\ \frac{\partial}{\partial \theta_{s',a'}} r_{\pi_\theta}(s, a) &= \xi \mathbb{1}[s = s', a = a']. \end{aligned} \quad (48)$$

Now, we get from Theorem 2,

$$\begin{aligned} \frac{\partial}{\partial \theta_{s,a}} \tilde{V}_{\pi_\theta}(\rho) &= \mathbb{E}_{\tau \sim \mathbb{P}_{\pi_\theta}^{\pi_\theta}} \left[\sum_{t=0}^{\infty} \gamma^t \left(\tilde{A}_{\pi_\theta}^{\pi_\theta}(s_t, a_t) \frac{\partial}{\partial \theta_{s,a}} \log \pi_\theta(a_t | s_t) \right. \right. \\ &\quad \left. \left. + \tilde{A}_{\pi_\theta}^{\pi_\theta}(s_t, a_t) \frac{\partial}{\partial \theta_{s,a}} \log P_{\pi_\theta}(s_{t+1} | s_t, a_t) \right. \right. \\ &\quad \left. \left. + \frac{\partial}{\partial \theta_{s,a}} r_{\pi_\theta}(s_t, a_t) - \lambda \frac{\partial}{\partial \theta_{s,a}} \log \pi_\theta(a_t | s_t) \right) \right] \\ &= \mathbb{E}_{\tau \sim \mathbb{P}_{\pi_\theta}^{\pi_\theta}} \left[\sum_{t=0}^{\infty} \gamma^t \left(\tilde{A}_{\pi_\theta}^{\pi_\theta}(s_t, a_t) (\mathbb{1}[s_t = s, a_t = a] - \pi_\theta(a|s) \mathbb{1}[s_t = s]) \right. \right. \\ &\quad \left. \left. + \tilde{A}_{\pi_\theta}^{\pi_\theta}(s_t, a_t) \psi(s_{t+1}) \mathbb{1}[s_t = s, a_t = a] (1 - \mathbf{P}_{\pi_\theta}(s_{t+1}|s, a)) \right. \right. \\ &\quad \left. \left. + \xi \mathbb{1}[s_t = s, a_t = a] - \lambda \mathbb{1}[s_t = s, a_t = a] + \lambda \pi_\theta(a|s) \mathbb{1}[s_t = s] \right) \right] \\ &\stackrel{(a)}{\geq} \mathbb{E}_{\tau \sim \mathbb{P}_{\pi_\theta}^{\pi_\theta}} \left[\sum_{t=0}^{\infty} \gamma^t \tilde{A}_{\pi_\theta}^{\pi_\theta}(s_t, a_t) \mathbb{1}[s_t = s, a_t = a] \right] \\ &\quad - \mathbb{E}_{\tau \sim \mathbb{P}_{\pi_\theta}^{\pi_\theta}} \left[\sum_{t=0}^{\infty} \gamma^t \pi_\theta(a|s) \mathbb{1}[s_t = s] \tilde{A}_{\pi_\theta}^{\pi_\theta}(s_t, a_t) \right] \\ &\quad + \mathbb{E}_{\tau \sim \mathbb{P}_{\pi_\theta}^{\pi_\theta}} \left[\sum_{t=0}^{\infty} \gamma^t \xi \mathbb{1}[s_t = s, a_t = a] \right] - \lambda \mathbb{E}_{\tau \sim \mathbb{P}_{\pi_\theta}^{\pi_\theta}} \left[\sum_{t=0}^{\infty} \gamma^t \mathbb{1}[s_t = s, a_t = a] \right] \\ &\quad + \lambda \mathbb{E}_{\tau \sim \mathbb{P}_{\pi_\theta}^{\pi_\theta}} \left[\sum_{t=0}^{\infty} \gamma^t \pi_\theta(a_t | s_t) \mathbb{1}[s_t = s] \right] \\ &= \frac{1}{1-\gamma} d_{\pi_\theta, \rho}^{\pi_\theta}(s, a) \tilde{A}_{\pi_\theta}^{\pi_\theta}(s, a) + \lambda \mathbb{E}_{\tau \sim \mathbb{P}_{\pi_\theta}^{\pi_\theta}} \left[\sum_{t=0}^{\infty} \gamma^t \pi_\theta(a_t | s_t) \log \pi_\theta(a_t | s_t) \mathbb{1}[s_t = s] \right] \\ &\quad + \frac{1}{1-\gamma} \xi d_{\pi_\theta, \rho}^{\pi_\theta}(s, a) - \frac{\lambda}{1-\gamma} d_{\pi_\theta, \rho}^{\pi_\theta}(s, a | s_0) \\ &\quad + \lambda \mathbb{E}_{\tau \sim \mathbb{P}_{\pi_\theta}^{\pi_\theta}} \left[\sum_{t=0}^{\infty} \gamma^t \pi_\theta(a_t | s_t) \sum_a \mathbb{1}[s_t = s, a_t = a] \right] \\ &= \frac{1}{1-\gamma} d_{\pi_\theta, \rho}^{\pi_\theta}(s, a) \tilde{A}_{\pi_\theta}^{\pi_\theta}(s, a) \\ &\quad + \lambda \mathbb{E}_{\tau \sim \mathbb{P}_{\pi_\theta}^{\pi_\theta}} \left[\sum_{t=0}^{\infty} \gamma^t \pi_\theta(a_t | s_t) \log \pi_\theta(a_t | s_t) \sum_a \mathbb{1}[s_t = s, a_t = a] \right] \end{aligned}$$

$$\begin{aligned}
& + \frac{1}{1-\gamma} \xi d_{\pi_\theta, \rho}^{\pi_\theta}(s, a) - \frac{\lambda}{1-\gamma} d_{\pi_\theta, \rho}^{\pi_\theta}(s, a) \\
& + \lambda \mathbb{E}_{\tau \sim \mathbb{P}_{\pi_\theta}^{\pi_\theta}} \left[\sum_a \pi_\theta(a|s) \sum_{t=0}^{\infty} \gamma^t \mathbf{1}[s_t = s, a_t = a] \right] \\
& = \frac{1}{1-\gamma} d_{\pi_\theta, \rho}^{\pi_\theta}(s, a) \tilde{A}_{\pi_\theta}^{\pi_\theta}(s, a) \\
& + \lambda \mathbb{E}_{\tau \sim \mathbb{P}_{\pi_\theta}^{\pi_\theta}} \left[\sum_a \pi_\theta(a|s) \log \pi_\theta(a|s) \sum_{t=0}^{\infty} \gamma^t \mathbf{1}[s_t = s, a_t = a] \right] \\
& + \frac{1}{1-\gamma} \xi d_{\pi_\theta, \rho}^{\pi_\theta}(s, a) - \frac{\lambda}{1-\gamma} d_{\pi_\theta, \rho}^{\pi_\theta}(s, a) + \frac{\lambda}{1-\gamma} \sum_a d_{\pi_\theta, \rho}^{\pi_\theta}(s, a) \pi_\theta(a|s) \\
& \geq \frac{1}{1-\gamma} d_{\pi_\theta, \rho}^{\pi_\theta}(s, a) \left(\tilde{A}_{\pi_\theta}^{\pi_\theta}(s, a) + \xi \right) - \frac{\lambda}{1-\gamma} d_{\pi_\theta, \rho}^{\pi_\theta}(s, a) \\
& - \frac{\lambda}{1-\gamma} \sum_a d_{\pi_\theta, \rho}^{\pi_\theta}(s, a) \pi_\theta(a|s) \log \frac{1}{\pi_\theta(a|s)} \\
& \stackrel{(b)}{\geq} \frac{1}{1-\gamma} d_{\pi_\theta, \rho}^{\pi_\theta}(s, a) \left(\tilde{A}_{\pi_\theta}^{\pi_\theta}(s, a) + \xi \right) - \frac{\lambda}{1-\gamma} d_{\pi_\theta, \rho}^{\pi_\theta}(s, a) (1 + \log |\mathcal{A}|).
\end{aligned}$$

(b) holds from the following:

$$\begin{aligned}
-\sum_a d_{\pi_\theta}^{\pi_\theta}(s, a|s_0) \log \pi_\theta(a|s) &= d_{\pi_\theta}^{\pi_\theta}(s|s_0) \left(-\sum_a \pi_\theta(a|s) \log \pi_\theta(a|s) \right) \\
&\stackrel{(c)}{\leq} d_{\pi_\theta}^{\pi_\theta}(s|s_0) \log |\mathcal{A}| \leq \log |\mathcal{A}|
\end{aligned}$$

and (c) holds as entropy is upper bounded by $\log |\mathcal{A}|$ (Cover & Thomas, 2006, Theorem 2.6.4). \square

Lemma 3 (Regularized Performative Gradient Domination: Part(b) of Lemma 3). *For regularized PeMDPs the following inequality holds:*

$$\begin{aligned}
& \tilde{V}_{\pi_o^*}^{\pi_\theta^*}(\rho) - \tilde{V}_{\pi_\theta}^{\pi_\theta}(\rho) \\
& \leq \sqrt{|\mathcal{S}||\mathcal{A}|} \left\| \frac{d_{\pi_\theta, \rho}^{\pi_\theta^*}}{d_{\pi_\theta, \nu}^{\pi_\theta}} \right\|_\infty \|\nabla_\theta \tilde{V}_{\pi_\theta}^{\pi_\theta}(\nu)\|_2 + \frac{R_{\max}}{1-\gamma} \left(1 + \frac{2\gamma}{1-\gamma} \psi_{\max} \left(1 + \frac{\lambda}{R_{\max}} \log |\mathcal{A}| \right) \right) \\
& + \frac{\lambda}{1-\gamma} (1 + \log |\mathcal{A}|).
\end{aligned} \tag{49}$$

Proof. **Step 1.** First, we observe that

$$-D_{\text{KL}}(\pi_o^*(\cdot|s) \parallel \pi_\theta(\cdot|s)) \leq -\sum_{a \in \mathcal{A}} \pi_o^*(a|s) \log \pi_o^*(a|s) \leq \log |\mathcal{A}|$$

Hence, we get

$$-\sum_s d_{\pi_\theta}^{\pi_\theta^*}(s|s_0) D_{\text{KL}}(\pi_o^*(\cdot|s) \parallel \pi_\theta(\cdot|s)) \leq \log |\mathcal{A}| \tag{50}$$

Step 2. Using Lemma 8 and applying Cauchy-Schwarz inequality, we get

$$\sum_{s,a} d_{\pi_\theta}^{\pi_\theta}(s, a) \tilde{A}_{\pi_\theta}^{\pi_\theta}(s, a) \leq \sqrt{|\mathcal{S}||\mathcal{A}|} (1-\gamma) \|\nabla_\theta \tilde{V}_{\pi_\theta}^{\pi_\theta}(\nu)\|_2 - \xi + \lambda (\log |\mathcal{A}| + 1) \tag{51}$$

Step 3. Now, substituting Equation (50) and (51) in Equation (44), we finally get

$$\tilde{V}_{\pi_o^*}^{\pi_\theta^*}(\rho) - \tilde{V}_{\pi_\theta}^{\pi_\theta}(\rho) \leq \sqrt{|\mathcal{S}||\mathcal{A}|} \left\| \frac{d_{\pi_\theta, \rho}^{\pi_\theta^*}}{d_{\pi_\theta, \nu}^{\pi_\theta}} \right\|_\infty \|\nabla_\theta \tilde{V}_{\pi_\theta}^{\pi_\theta}(\nu)\|_2 - \left\| \frac{d_{\pi_\theta, \rho}^{\pi_\theta^*}}{d_{\pi_\theta, \nu}^{\pi_\theta}} \right\|_\infty \frac{\xi}{1-\gamma} + \frac{\lambda}{1-\gamma} (\log |\mathcal{A}| + 1)$$

$$\begin{aligned}
& + \frac{2}{1-\gamma} \left(L_r + \frac{\gamma}{1-\gamma} L_{\mathbf{P}} (R_{\max} + \lambda \log |\mathcal{A}|) \right) \\
& \stackrel{(a)}{=} \sqrt{|\mathcal{S}||\mathcal{A}|} \left\| \frac{d_{\pi_\theta^*, \rho}^{\pi_\theta^*}}{d_{\pi_\theta, \nu}^{\pi_\theta^*}} \right\|_\infty \|\nabla_\theta \tilde{V}_{\pi_\theta}^{\pi_\theta}(\nu)\|_2 - \left\| \frac{d_{\pi_\theta^*, \rho}^{\pi_\theta^*}}{d_{\pi_\theta, \nu}^{\pi_\theta^*}} \right\|_\infty \frac{\xi}{1-\gamma} + \frac{\lambda}{1-\gamma} (1 + \log |\mathcal{A}|) \\
& + \frac{2}{1-\gamma} \left(\xi + \frac{\gamma}{1-\gamma} \psi_{\max}(R_{\max} + \lambda \log |\mathcal{A}|) \right) \\
& \stackrel{(b)}{\leq} \sqrt{|\mathcal{S}||\mathcal{A}|} \left\| \frac{d_{\pi_\theta^*, \rho}^{\pi_\theta^*}}{d_{\pi_\theta, \nu}^{\pi_\theta^*}} \right\|_\infty \|\nabla_\theta \tilde{V}_{\pi_\theta}^{\pi_\theta}(\nu)\|_2 + \frac{\xi}{1-\gamma} \\
& + \frac{2\gamma}{1-\gamma} \psi_{\max}(R_{\max} + \lambda \log |\mathcal{A}|) + \frac{\lambda}{1-\gamma} (1 + \log |\mathcal{A}|) \\
& = \sqrt{|\mathcal{S}||\mathcal{A}|} \left\| \frac{d_{\pi_\theta^*, \rho}^{\pi_\theta^*}}{d_{\pi_\theta, \nu}^{\pi_\theta^*}} \right\|_\infty \|\nabla_\theta \tilde{V}_{\pi_\theta}^{\pi_\theta}(\nu)\|_2 \\
& + \frac{2}{1-\gamma} \left(\frac{\xi}{2} + \frac{\gamma}{1-\gamma} \psi_{\max}(R_{\max} + \lambda \log |\mathcal{A}|) \right) + \frac{\lambda}{1-\gamma} (1 + \log |\mathcal{A}|) \\
& \leq \sqrt{|\mathcal{S}||\mathcal{A}|} \left\| \frac{d_{\pi_\theta^*, \rho}^{\pi_\theta^*}}{d_{\pi_\theta, \nu}^{\pi_\theta^*}} \right\|_\infty \|\nabla_\theta \tilde{V}_{\pi_\theta}^{\pi_\theta}(\nu)\|_2 \\
& + \frac{R_{\max}}{1-\gamma} \left(1 + \frac{2\gamma}{1-\gamma} \psi_{\max} \left(1 + \frac{\lambda}{R_{\max}} \log |\mathcal{A}| \right) \right) + \frac{\lambda}{1-\gamma} (1 + \log |\mathcal{A}|)
\end{aligned}$$

In (a), we substitute the values of L_r and $L_{\mathbf{P}}$ for softmax PeMDPs, and in (b), we use $\left\| \frac{d_{\pi_\theta^*, \rho}^{\pi_\theta^*}}{d_{\pi_\theta, \nu}^{\pi_\theta^*}} \right\|_\infty \geq 1$ (Lemma 9). \square

Theorem 3 (Convergence of **PePG** in softmax PeMDPs – Part (b)). *Let $\text{Cov} \triangleq \max_{\theta, \nu} \left\| \frac{d_{\pi_\theta^*, \rho}^{\pi_\theta^*}}{d_{\pi_\theta, \nu}^{\pi_\theta^*}} \right\|_\infty$. The gradient ascent algorithm on $V_{\pi_\theta}^{\pi_\theta}(\rho)$ (Equation (8)) with step size $\eta = \Omega \left(\frac{(1-\gamma)^2}{\gamma |\mathcal{A}|} \right)$ satisfies, for all distributions $\rho \in \Delta(\mathcal{S})$.*

(b) For entropy regularised case, if we set $\lambda = \frac{(1-\gamma)R_{\max}}{1+\log |\mathcal{A}|}$, we get

$$\min_{t < T} \left\{ \tilde{V}_{\pi_\theta^*}^{\pi_\theta^*}(\rho) - \tilde{V}_{\pi_\theta^{(t)}}^{\pi_\theta^{(t)}}(\rho) \right\} \leq \epsilon + \mathcal{O} \left(\frac{1}{1-\gamma} \right) \text{ when } T = \Omega \left(\frac{R_{\max} |\mathcal{S}||\mathcal{A}|^2}{\epsilon^2 (1-\gamma)^3} \text{Cov}^2 \right),$$

Proof. This proof follows similar steps as part (a) of Theorem 3 with two additional changes: (i) We have a λ , i.e. regularisation coefficient, dependent term due to the entropy regulariser. (ii) The maximum value of the soft value function is $\frac{R_{\max} + \lambda \log |\mathcal{A}|}{1-\gamma}$ instead of $\frac{R_{\max}}{1-\gamma}$ for the unregularised value function.

Step 1: From Equation (29), we observe that the soft-value function $\tilde{V}_{\pi_\theta}^{\pi_\theta}$ is L_λ -smooth.

Thus, following the Step 1 of Theorem 3, we get

$$\begin{aligned}
\min_{t \in [T-1]} \|\nabla \tilde{V}_{\pi_\theta^{(t)}}^{\pi_\theta^{(t)}}(\rho)\|^2 & \leq \frac{1}{T\eta \left(1 - \frac{L_\lambda \eta}{2} \right)} \left(\tilde{V}_{\pi_\theta^*}^{\pi_\theta^*}(\rho) - \tilde{V}_{\pi_\theta^0}^{\pi_\theta^0}(\rho) \right) \\
& \leq \frac{R_{\max} + \lambda \log |\mathcal{A}|}{T\eta \left(1 - \frac{L_\lambda \eta}{2} \right) (1-\gamma)}. \tag{52}
\end{aligned}$$

The last inequality is true due to the fact that $\tilde{V}_{\pi_\theta^*}^{\pi_\theta^*}(\rho) - \tilde{V}_{\pi_\theta^0}^{\pi_\theta^0}(\rho) \leq \tilde{V}_{\pi_\theta^*}^{\pi_\theta^*}(\rho) \leq \frac{R_{\max} + \lambda \log |\mathcal{A}|}{1-\gamma}$.

Step 2: Now, from Part (b) of Lemma 3, we obtain that

$$\begin{aligned}
& \min_{t \in [T-1]} \left(\tilde{V}_{\pi_o^*}^{\pi_o^*}(\rho) - \tilde{V}_{\pi_\theta^{(t)}}^{\pi_\theta^{(t)}}(\rho) \right)^2 \\
& \leq \min_{t \in [T-1]} \left(\sqrt{|\mathcal{S}||\mathcal{A}|} \left\| \frac{d^{\pi_o^*}}{d^{\pi_\theta^{(t)}, \rho}} \right\|_\infty \|\nabla_{\theta} \tilde{V}_{\pi_\theta^{(t)}}^{\pi_\theta^{(t)}}(\nu)\|_2 + \frac{R_{\max}}{1-\gamma} \left(1 + \frac{2\gamma}{1-\gamma} \psi_{\max} \left(1 + \frac{\lambda}{R_{\max}} \log |\mathcal{A}| \right) \right) \right. \\
& \quad \left. + \frac{\lambda}{1-\gamma} (1 + \log |\mathcal{A}|) \right)^2 \\
& \leq 2|\mathcal{S}||\mathcal{A}| \min_{t \in [T-1]} \left\| \frac{d^{\pi_o^*}}{d^{\pi_\theta^{(t)}, \nu}} \right\|_\infty^2 \|\nabla_{\theta} \tilde{V}_{\pi_\theta^{(t)}}^{\pi_\theta^{(t)}}(\nu)\|_2^2 \\
& \quad + 2 \left(\frac{R_{\max}}{1-\gamma} \left(1 + \frac{2\gamma}{1-\gamma} \psi_{\max} \left(1 + \frac{\lambda}{R_{\max}} \log |\mathcal{A}| \right) \right) + \frac{\lambda}{1-\gamma} (1 + \log |\mathcal{A}|) \right)^2 \\
& \leq \frac{2|\mathcal{S}||\mathcal{A}| \text{Cov}^2(R_{\max} + \lambda \log |\mathcal{A}|)}{T\eta \left(1 - \frac{L_\lambda \eta}{2} \right) (1-\gamma)} \\
& \quad + 2 \left(\frac{R_{\max}}{1-\gamma} \left(1 + \frac{2\gamma}{1-\gamma} \psi_{\max} \left(1 + \frac{\lambda}{R_{\max}} \log |\mathcal{A}| \right) \right) + \frac{\lambda}{1-\gamma} (1 + \log |\mathcal{A}|) \right)^2.
\end{aligned}$$

The last inequality is due to the upper bound on the minimum gradient norm as in Equation (52) and by definition of the coverage parameter Cov.

Thus, we conclude that

$$\begin{aligned}
& \min_{t \in [T-1]} \tilde{V}_{\pi_o^*}^{\pi_o^*}(\rho) - \tilde{V}_{\pi_\theta^{(t)}}^{\pi_\theta^{(t)}}(\rho) \\
& \leq \sqrt{\frac{2|\mathcal{S}||\mathcal{A}| \text{Cov}^2(R_{\max} + \lambda \log |\mathcal{A}|)}{T\eta \left(1 - \frac{L_\lambda \eta}{2} \right) (1-\gamma)}} \tag{53}
\end{aligned}$$

$$+ \sqrt{2} \left(\frac{R_{\max}}{1-\gamma} \left(1 + \frac{2\gamma}{1-\gamma} \psi_{\max} \left(1 + \frac{\lambda}{R_{\max}} \log |\mathcal{A}| \right) \right) + \frac{\lambda}{1-\gamma} (1 + \log |\mathcal{A}|) \right). \tag{54}$$

Step 4: Now, by setting the T -dependent term in Equation (54) to ϵ , we get $T \geq \frac{2|\mathcal{S}||\mathcal{A}| \text{Cov}^2(R_{\max} + \lambda \log |\mathcal{A}|)}{\eta \left(1 - \frac{L_\lambda \eta}{2} \right) (1-\gamma) \epsilon^2}$.

Choosing $\eta = \frac{1}{L_\lambda}$, $\lambda = \frac{(1-\gamma)R_{\max}}{(1+\log |\mathcal{A}|)}$, and $\psi_{\max} = \mathcal{O}(\frac{1-\gamma}{\gamma})$, we get the final expression $T \geq \frac{8|\mathcal{S}||\mathcal{A}| \text{Cov}^2 L_\lambda R_{\max}}{(1-\gamma) \epsilon^2}$, and

$$\min_{t \in [T-1]} \tilde{V}_{\pi_o^*}^{\pi_o^*}(\rho) - \tilde{V}_{\pi_\theta^{(t)}}^{\pi_\theta^{(t)}}(\rho) \leq \epsilon + \mathcal{O}\left(\frac{1}{1-\gamma}\right).$$

Finally, noting that $L_\lambda = \mathcal{O}\left(\max\left\{\frac{\gamma R_{\max} |\mathcal{A}| \psi_{\max}^2}{(1-\gamma)^2}, \frac{R_{\max} \psi_{\max}^2}{(1-\gamma)^2}\right\}\right)$, we get

$$T = \Omega\left(\frac{|\mathcal{S}||\mathcal{A}|}{\epsilon^2 (1-\gamma)^3} \max\{1, \gamma |\mathcal{A}|\}\right).$$

□

H ABLATION STUDIES

H.1 ENTROPY REGULARISATION

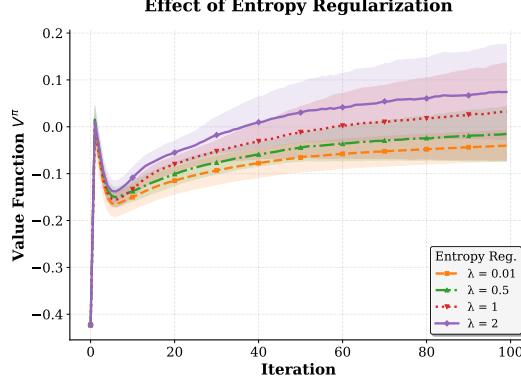


Figure 3: Ablation study for PePG for different values of regularised λ with 20 random seeds, each for 100 iterations

We conducted an ablation study across four entropy regularization strengths ($\lambda \in \{0.01, 0.5, 1, 2\}$) to determine the optimal balance between exploration and convergence stability in entropy regularised PePG. The results demonstrate that $\lambda = 2$ achieves the highest final performance (0.05), while smaller values ($\lambda \leq 1$) converge to similar suboptimal levels around -0.01 to 0, indicating that stronger entropy regularization enables more effective exploration of the policy space in performative settings.

H.2 LEARNING RATE

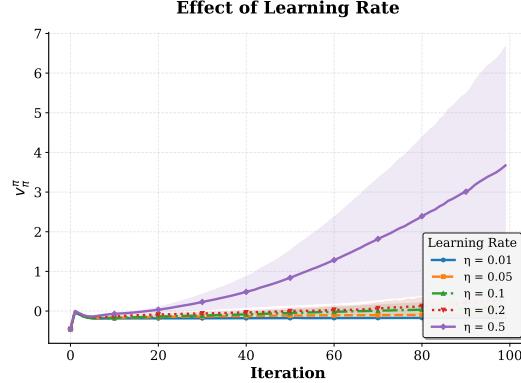


Figure 4: Ablation study for PePG for different values of η with 20 random seeds across 100 iterations

We additionally performed an ablation study on the learning rate, considering $\eta \in \{0.01, 0.05, 0.1, 0.2, 0.5\}$, to examine its effect on convergence behavior and performance stability in PePG. The results indicate that the largest learning rate ($\eta = 0.5$) attains the highest final performance (4); however, it is also accompanied by substantially higher variance across runs. In contrast, smaller learning rates yield more stable learning dynamics but converge to comparatively lower performance levels. This highlights the classical trade-off between fast convergence and stability in policy optimization.

I TECHNICAL LEMMAS

Lemma 9 (Lower Bound of Coverage). *For any $\pi, \pi' \in \Pi(\Theta)$, the following non-trivial lower bound holds,*

$$\left\| \frac{\mathbf{d}_{\pi'}}{\mathbf{d}_{\pi}} \right\|_{\infty} \geq 1$$

Proof.

$$\left\| \frac{\mathbf{d}_{\pi'}}{\mathbf{d}_{\pi}} \right\|_{\infty} = \max_{s,a} \frac{\mathbf{d}_{\pi'}(s,a)}{\mathbf{d}_{\pi}(s,a)} \geq \frac{1}{\sum_{s,a} w_{s,a}} \sum_{s,a} \frac{\mathbf{d}_{\pi'}(s,a)}{\mathbf{d}_{\pi}(s,a)} \cdot w_{s,a}$$

Choose $w_{s,a} = \mathbf{d}_{\pi}(s,a)$. Hence, we get,

$$\max_{s,a} \frac{\mathbf{d}_{\pi'}(s,a)}{\mathbf{d}_{\pi}(s,a)} \geq \frac{\sum_{s,a} \mathbf{d}_{\pi'}(s,a)}{\sum_{s,a} \mathbf{d}_{\pi}(s,a)} = 1$$

The last equality holds from the fact that the state-action occupancy measure is a distribution over $\mathcal{S} \times \mathcal{A}$. Hence, $\sum_{s,a} \mathbf{d}_{\pi'}(s,a) = \sum_{s,a} \mathbf{d}_{\pi}(s,a)$ \square

Lemma 10. *The discounted state occupancy measure*

$$\mathbf{d}_{\pi'}^{\pi}(s|s_0) \triangleq (1 - \gamma) \mathbb{E}_{\tau \sim \mathbb{P}_{\pi'}^{\pi}} \left[\sum_{t=0}^{\infty} \gamma^t \mathbf{1}\{s_t = s\} \right]$$

is a probability mass function over the state-space \mathcal{S} .

Proof. For each fixed s the integrand $\sum_{t=0}^{\infty} \gamma^t \mathbf{1}\{s_t = s\} \geq 0$, hence $\mathbf{d}_{\pi'}^{\pi}(s|s_0) \geq 0$.

To check normalization, we sum over all states and use Tonelli/Fubini (permitted because the summand is non-negative) to exchange sums and expectation:

$$\begin{aligned} \sum_{s \in \mathcal{S}} \mathbf{d}_{\pi'}^{\pi}(s|s_0) &= (1 - \gamma) \mathbb{E}_{\tau \sim \mathbb{P}_{\pi'}^{\pi}(\cdot|s_0)} \left[\sum_{t=0}^{\infty} \gamma^t \sum_{s \in \mathcal{S}} \mathbf{1}\{s_t = s\} \right] \\ &= (1 - \gamma) \mathbb{E}_{\tau \sim \mathbb{P}_{\pi'}^{\pi}(\cdot|s_0)} \left[\sum_{t=0}^{\infty} \gamma^t \cdot 1 \right] = (1 - \gamma) \sum_{t=0}^{\infty} \gamma^t = 1. \end{aligned}$$

Therefore ρ is a probability mass function on \mathcal{S} . \square

A similar argument holds for the discounted state-action occupancy measure $\mathbf{d}_{\pi'}^{\pi}(s,a|s_0)$ as well.

J CONVERGENCE RESULTS UNDER MINIMAL ASSUMPTIONS

Assumption 2 (Minimal Structural Assumption: PeMDPs with bounded sensitivity). (a) *Rewards and transitions are Lipschitz functions of policy, i.e.*

$$\|r_{\pi} - r_{\pi'}\|_1 \leq L_r \|\pi - \pi'\|_1 \text{ and } \|\mathbf{P}_{\pi} - \mathbf{P}_{\pi'}\|_1 \leq L_{\mathbf{P}} \|\pi - \pi'\|_1,$$

for some $L_r, L_{\mathbf{P}} \geq 0$.

(b) *Rewards and transitions are smooth functions of policy, i.e.*

$$\|\nabla_{\pi} r_{\pi} - \nabla_{\pi'} r_{\pi'}\|_1 \leq R_2 \|\pi - \pi'\|_1 \text{ and } \|\nabla_{\pi} \mathbf{P}_{\pi} - \nabla_{\pi'} \mathbf{P}_{\pi'}\|_1 \leq T_2 \|\pi - \pi'\|_1,$$

for some $R_2, T_2 \geq 0$. In control theory, this is also known as bounded sensitivity assumption on the underlying dynamics.

Assumption 2 (a) is enough to prove the gradient domination lemma, whereas Assumption 2 (b) is required to ensure smoothness of the value function (Lemma 4).

Lemma 11 (Generic Gradient Domination Lemma for Softmax Policies under Assumption 2 (a)).

$$V_{\pi_\theta^*}^*(\rho) - V_{\pi_\theta}^*(\rho) \leq \sqrt{|\mathcal{S}||\mathcal{A}|} \text{Cov} \|\nabla_{\theta} V_{\pi_\theta}^*(\nu)\|_2 + \frac{2 + \text{Cov}}{(1-\gamma)^2} \left(L_r + L_{\mathbf{P}} R_{\max} \right)$$

Proof.

$$\begin{aligned} \frac{\partial}{\partial \theta_{s,a}} V_{\pi_\theta}^*(\nu) &= \mathbb{E}_{\tau \sim \mathbb{P}_{\pi_\theta, \nu}} \left[\sum_{t=0}^{\infty} \gamma^t \left(A_{\pi_\theta}^*(s_t, a_t) \frac{\partial}{\partial \theta_{s,a}} \log \pi_\theta(a_t | s_t) \right. \right. \\ &\quad \left. \left. + A_{\pi_\theta}^*(s_t, a_t) \frac{\partial}{\partial \theta_{s,a}} \log P_{\pi_\theta}(s_{t+1} | s_t, a_t) \right. \right. \\ &\quad \left. \left. + \frac{\partial}{\partial \theta_{s,a}} r_{\pi_\theta}(s_t, a_t) \right) \right] \\ &\stackrel{(a)}{\geq} \mathbb{E}_{\tau \sim \mathbb{P}_{\pi_\theta, \nu}} \left[\sum_{t=0}^{\infty} \gamma^t A_{\pi_\theta}^*(s_t, a_t) \mathbb{1}[s_t = s, a_t = a] \right] \\ &\quad - \mathbb{E}_{\tau \sim \mathbb{P}_{\pi_\theta, \nu}} \left[\sum_{t=0}^{\infty} \gamma^t \pi_\theta(a | s) \mathbb{1}[s_t = s] A_{\pi_\theta}^*(s_t, a_t) \right] \\ &\quad - \mathbb{E}_{\tau \sim \mathbb{P}_{\pi_\theta, \nu}} \left[\sum_{t=0}^{\infty} \gamma^t L_r \mathbb{1}[s_t = s, a_t = a] \right] \\ &\quad - \mathbb{E}_{\tau \sim \mathbb{P}_{\pi_\theta, \nu}} \left[\sum_{t=0}^{\infty} \gamma^t A_{\pi_\theta}^*(s_t, a_t) \mathbb{1}[s_t = s, a_t = a] L_{\mathbf{P}} \right] \\ &= \frac{1}{1-\gamma} d_{\pi_\theta, \nu}^*(s, a) A_{\pi_\theta}^*(s, a) - \frac{1}{(1-\gamma)^2} d_{\pi_\theta, \nu}^*(s, a) R_{\max} L_{\mathbf{P}} - \frac{1}{1-\gamma} L_r d_{\pi_\theta, \nu}^*(s, a) \end{aligned}$$

(a) holds due to Lipchitzness of rewards and transitions. Hence,

$$\begin{aligned} \frac{1}{1-\gamma} \sum_{s,a} d_{\pi_\theta}^*(s, a | \nu) A_{\pi_\theta}^*(s, a) &\leq \sum_{s,a} \frac{\partial V_{\pi_\theta}^*(\nu)}{\partial \theta_{s,a}} + \frac{L_r}{1-\gamma} + \frac{R_{\max} L_{\mathbf{P}}}{(1-\gamma)^2} \\ &\leq \sqrt{|\mathcal{S}||\mathcal{A}|} \|\nabla_{\theta} V_{\pi_\theta}^*(\nu)\|_2 + \frac{L_r}{1-\gamma} + \frac{R_{\max} L_{\mathbf{P}}}{(1-\gamma)^2} \end{aligned}$$

Now, following the steps in the proof of Lemma 3, we obtain the final gradient domination lemma. \square

Thus, as a consequence, we obtain convergence of PePG following the steps of Theorem 3.

Theorem 4 (Convergence of PePG in softmax policies and PeMDPs with Assumption 2). *Let $\text{Cov} \triangleq \max_{\theta, \nu} \left\| \frac{d_{\pi_\theta^*, \rho}^*}{d_{\pi_\theta, \nu}^*} \right\|^\infty$. The gradient ascent algorithm on $V_{\pi_\theta}^*(\rho)$ (Equation (8)) satisfies, for all distributions $\rho \in \Delta(\mathcal{S})$.*

(a) For unregularised case, if we set step size $\eta = \frac{1}{L}$

$$\min_{t < T} \left\{ V_{\pi_\theta^*}^*(\rho) - V_{\pi_{\theta_t}}^*(\rho) \right\} \leq \epsilon + \mathcal{O} \left(\frac{\text{Cov}}{(1-\gamma)^2} \right)$$

when $T = \Omega \left(\frac{|\mathcal{S}||\mathcal{A}|L \text{Cov}^2}{\epsilon^2(1-\gamma)} \right)$, and L is defined in Theorem 3.

(b) For entropy regularised case, if we set $\lambda = \frac{(1-\gamma)R_{\max}}{1+\log |\mathcal{A}|}$ and $\eta = \frac{1}{L_\lambda}$, we get

$$\min_{t < T} \left\{ \tilde{V}_{\pi_\theta^*}^*(\rho) - \tilde{V}_{\pi_{\theta_t}}^*(\rho) \right\} \leq \epsilon + \mathcal{O} \left(\frac{\text{Cov}}{(1-\gamma)^2} \right)$$

when $T = \Omega \left(\frac{|\mathcal{S}||\mathcal{A}| \text{Cov}^2 L_\lambda}{(1-\gamma)\epsilon^2} \right)$. Here, $L_\lambda = \mathcal{O}(\max\{L, \lambda\beta_\lambda\})$ and β_λ is defined in Lemma 5.