

Q) Given N array elements & Q queries on same array.  
for each query calculate sum of all elements in Given Range  
 $[L-R]$

	0	1	2	3	4	5	6	7	8	9
<u>ar[10]</u> :	-3	6	2	4	5	2	8	<u>-9</u>	3	1

Q = 5

Idea: For every query iterate in given range & print sum.

L - R      Sum

$$\left\{ \begin{array}{l} 4-8 : 9 \\ 2-7 : 12 \\ 1-3 : 12 \\ 0-4 : 14 \\ 7-7 : -9 \end{array} \right\}$$

Q,

Read Q

$$i = 1; q \leftarrow Q; Q \leftarrow \{ \}$$

$L, R; S=0$

read L, R

$$i = L; \left. \begin{array}{l} q \leftarrow R; Q \leftarrow \{ \} \\ S = S + ar[i] \end{array} \right\} \text{N Iterations}$$

$$\left. \begin{array}{l} q \\ S = S + ar[i] \end{array} \right\} O(N)$$

print(S)

$$\underline{TC: Q \times N \rightarrow O(Q+N)}$$

$$\underline{SC: O(1)}$$

// Given Indian Team Score for first 10 Overs of Batting

<u>Overs:</u>	1	2	3	4	5	6	7	8	9	10	
<u>Scores:</u>	-	8	14	29	31	49	65	79	88	97	

Q1: Runs scored in last Over:  $97 - 88 = 9$   
 $\underline{[10 - 10]} \rightarrow \text{scr}[10] - \text{scr}[9]$

Q2: Runs scored in last 5 Over:  $97 - 31 = 66$   
 $\underline{[6 - 10]} \rightarrow \text{scr}[10] - \text{scr}[5]$

Q3: Runs scored in 7<sup>th</sup> Over:  $65 - 49 = 16$   
 $\underline{[7 - 7]} \rightarrow \text{scr}[7] - \text{scr}[6]$

Q4: Runs scored from [3 - 6] :  $\text{scr}[6] - \text{scr}[2]$   
 $\rightarrow 49 - 8 = 41$

Q5: Runs scored [4 - 9] :  $\text{scr}[9] - \text{scr}[3]$   
 $\rightarrow 88 - 14 = 74$

$$ar[10] : \begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ -3 & 6 & 2 & 4 & 5 & 2 & 8 & -9 & 3 & 1 \end{matrix}$$

$$Pf[10] : -3 \ 3 \ 5 \ 9 \ 14 \ 16 \ 24 \ 15 \ 18 \ 19$$

$Pf[i] \Rightarrow$  Sum of all array elements from  $[0-i]$

Succes:

$$\underbrace{4-8}_{4, 5, 6, 7, 8} : \frac{Pf[8]}{0, 1, 2, 3, 4, 5, 6, 7, 8}$$

$$\textcircled{Pf[8]} = \underbrace{Pf[3]}_{0 \ 1 \ 2 \ 3} + \underbrace{\text{sum}[4-8]}_{4 \ 5 \ 6 \ 7 \ 8}$$

$$3-7 : Pf[7] - Pf[2]$$

$$4-9 : Pf[9] - Pf[3]$$

$$\text{sum}[4-8] = Pf[8] - Pf[3]$$

$$(i-j) : Pf[j] - Pf[i-1]$$

$$\underbrace{[0 \ i]}_{\substack{\nearrow \\ \rightarrow}} = Pf[i] - Pf[-1] \} \text{ Array out bounds}$$

$$\left\{ \begin{array}{l} C(i, j) : \\ \text{if } i == 0 = Pf[j] \\ \text{else} = Pf[j] - Pf[i-1] \end{array} \right\}$$

→ Construct  $Pf[N]$

Given  $ar[N]$

$$Pf[0] = ar[0]$$

$$Pf[1] = ar[0] + ar[1]$$

$$Pf[2] = ar[0] + ar[1] + ar[2]$$

$$Pf[p] = ar[0] + ar[1] + ar[2] + \dots + ar[p-1] + ar[p]$$

$$Pf[i] = Pf[i-1] + ar[i]$$

If  $p=20$ , error

$TC: O(N)$

$\left\{ \begin{array}{l} Pf[N] \\ Pf[0] = ar[0] \\ p = 1; p \leq N; p++ \\ Pf[i] = Pf[i-1] + ar[i] \end{array} \right.$

$TC: O(Q)$

$p = 1; p \leq Q; p++$

Read  $L, R;$

if ( $L == 0$ ) {

print( $Pf[R]$ )

else {

print( $Pf[R] - Pf[L-1]$ )

Overall  $TC: O(N+Q)$

Sc:  $O(N)$

## 28) Equilibrium Index [ ]

Given N array elements, count no. of equilibrium index

An index is said to be equilibrium index if

$$\text{Sum of all elements} = \text{Sum of all Elements after } i^{\text{th}} \text{ index}$$

Before  $i^{\text{th}}$  index

$$\text{Sum } [0, i-1] = \text{Sum } [i+1, N-1]$$

Note1:  $i=0$ ,  $\text{leftsum} = 0$

Note2:  $i=N-1$ ,  $\text{rightsum} = 0$

$$\text{arr: } -2 -7 1 5 \quad \begin{array}{|c|} \hline 3 \\ \hline \end{array} \quad -4 3 0 \quad \begin{array}{|c|} \hline 6 \\ \hline \end{array} \quad \left. \begin{array}{l} \text{leftsum} = 0 \\ \text{rightsum} = 0 \end{array} \right\} \text{cnt} = 2$$

$$\text{left: } 0 -7 -6$$

$$\text{right: } 7 6 1$$

$$\text{arr: } 3 -1 2 \quad \begin{array}{|c|} \hline 3 \\ \hline \end{array} \quad 1 2 \quad \begin{array}{|c|} \hline 5 \\ \hline \end{array} \quad 6 \quad \left. \begin{array}{l} \text{leftsum} = 0 \\ \text{rightsum} = 0 \end{array} \right\} \text{cnt} = 2$$

$$\text{left: } 0 3 2$$

$$\text{right: } 4 5 3$$

Pseudo Code

→ Every index checks if its equilibrium index is not //

$\text{Pf}[N]$

// Construct  $\text{Pf}[N]$  } → N

int c = 0

i = 0; i <= N; i++ {

{ if ( $i == 0$ ) { left = 0 }

{ else { left = sum [0,  $i-1$ ] = Pf [ $i-1$ ] }

} right = sum [ $i+1, N-1$ ] = Pf [ $N-1$ ] - Pf [ $i$ ]

if (left == right) {

| c = c + 1

}

return c;

10: 35 PM break

TC:  $(N \times N) \rightarrow O(N)$

SC:  $O(N)$

TODO:

→ Can be done with  $\text{Pf}()$



Q8) Given N array elements & Q queries, for each query

Task Calculate sum of all even indices in given range



O is even number

$\text{arr}[8] : \quad \underline{3} \quad 4 \quad -2 \quad 8 \quad 6 \quad \textcircled{2} \quad \textcircled{1} \quad 3$

$$\text{Pf}_{\text{even}}[N]: \quad 3 \quad 3 \quad 1 \quad 1 \quad 7 \quad 7 \quad 8 \quad 8$$

## 5 Quincas      Sun

Pferen [i] = Summe aller crwenden [0 i]

$$[2 \underline{5}] : 4$$

Pf even [0] = sum even [0 o] = ar[0]

$$[3 \underline{-} 7] : 7$$

$$Pf\{even[1]\} = \text{Sum even } [0, 1] = \text{arctan}[1]$$

[0-7] : R

Pferen  $[N]$ ;

$$[\begin{smallmatrix} 7 & 7 \\ 1 & 1 \end{smallmatrix}] : \quad 0$$

$$Pf\ even[0] = ar[0]$$

$$[4 \quad -4] : \quad 6$$

$i = 1; i < N; i + 1\}$

(3) (7) :

$$\text{Pferen}[7] - \text{Pferen}[2]$$

if ( $\%_2 = 1$ ) {

$$Pfeilen[p] = \underline{Pfeilen[p-1]}$$

dn { }

$$Pferen[T] = Pferen[P_{-i}] + ar[\vec{q}_i]$$

Sumeren [07] - sumeren [02]

Sumeren [3 7]

→ Questions ✓

Similar manner  $\Rightarrow$

$Pf_{odd}[i] = \text{Sum of all } \underline{\text{odd indices}} \text{ from } [0-i]$

$Pf_{odd}[0] = 0$

$ar[] = \begin{matrix} 0 & 1 & 2 & 3 & 4 \\ 2 & 4 & 3 & 1 & 5 \end{matrix}$

$Pf_{odd} = 0 \ 4 \ 4 \ 5 \ 5$

// Construct  $Pf_{odd}[N]$  in your own (TODD) ✓

$ar[10] : \begin{array}{cccc|ccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ -1 & 2 & 3 & 8 & \cancel{7} & \cancel{5} & \cancel{6} & 7 & 8 & 9 \end{array}$

$ar[9] : \begin{array}{cccc|ccccc} -1 & 2 & 3 & 8 & 6 & 4 & 2 & 7 & 10 \\ -1 & 2 & 3 & 8 & 6 & 4 & 2 & 7 & 10 \end{array}$

$ar[10] : \begin{array}{cccc|ccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ -1 & 2 & \cancel{3} & \cancel{3} & 4 & 5 & 6 & 7 & 8 & 9 \end{array}$

$ar[9] : \begin{array}{cccc|ccccc} -1 & 2 & \cancel{3} & \cancel{3} & 4 & 5 & 6 & 7 & 10 \\ -1 & 2 & 8 & 7 & 6 & 4 & 2 & 7 & 10 \end{array}$

Group 1

Special Index : HARD

An index is said to be special index, if after deleting that index

Sum of even = Sum of all Odd  
Index Value Index Values

Count how many special index are there?

$\text{arr}:$	4	1	2	3	4	5	
$=$	4	3	2	7	6	-2	

Delete 0 ✓

$\text{arr}[0]:$	3	2	7	6	-2	
------------------	---	---	---	---	----	--

Sumeven index = 8

Sumodd index = 8

Delete 1 ✗

$\text{arr}[1]:$	4	2	7	6	-2	
------------------	---	---	---	---	----	--

Sumeven index = 9

Sumodd index = 8

Delete 2 ✓

$\text{arr}[2]:$	4	3	7	6	-2	
------------------	---	---	---	---	----	--

Sumeven index = 9

Sumodd index = 7

Delete 3 ✗

$\text{arr}[3]:$	4	3	2	6	-2	
------------------	---	---	---	---	----	--

Sumeven index = 4

Sumodd index = 9

Delete 4 ✗

$\text{arr}[4]:$	4	3	2	7	-2	
------------------	---	---	---	---	----	--

Sumeven index = 4

Sumodd index = 10

Delete 5 ✗

$\text{arr}[5]:$	4	3	2	7	6	
------------------	---	---	---	---	---	--

Sumeven index = 12

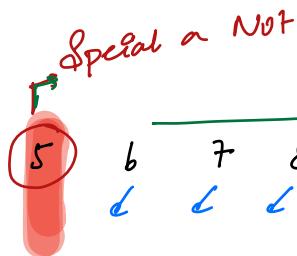
Sumodd index = 10

Hint:  
 $ar[4]$

Indices: 0 1 2 3 4

Sumeven [0 4]

Sumodd [0 4]

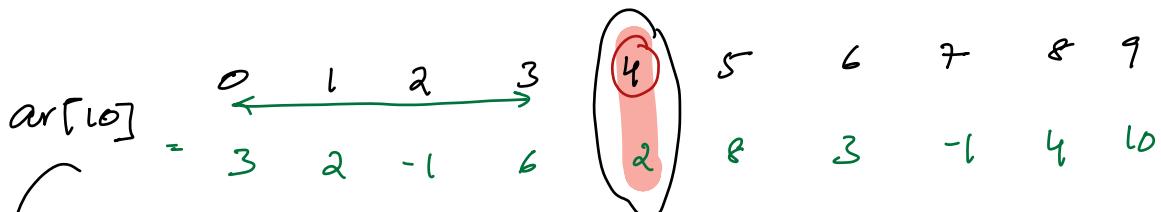


Sumeven [6 13]

Sumodd [6 13]

$$\text{Total sumeven} = \text{sumeven}[0_4] + \text{sumeven}[6_{13}]$$

$$\text{Total sumodd} = \text{sumodd}[0_4] + \text{sumodd}[6_{13}]$$

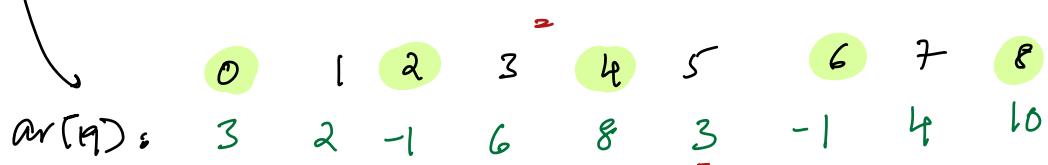


$$\text{sumeven}[0_3] = 2$$

$$\text{sumodd}[0_3] = 8$$

$$\text{sumeven}[5_9] = 7$$

$$\text{sumodd}[5_9] = 17$$

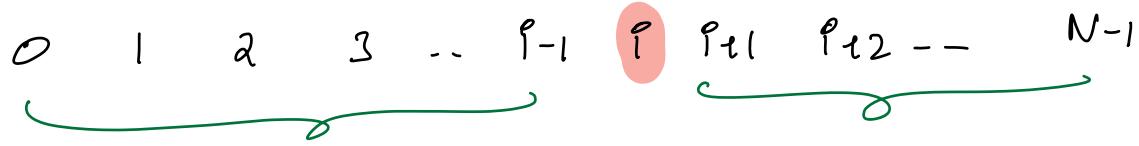


$$\text{Total sumeven} = 19 = \text{sumeven}[0_3] + \text{sumeven}[5_9]$$

$$\text{Total odd sum} = 15 = \text{sumodd}[0_3] + \text{sumeven}[5_9]$$

// Generalize:

↳ Special or Not



{sumeven [0,  $i-1$ ] }

sum odd [0,  $i-1$ ]

sumeven [ $i+1$ ,  $N-1$ ]

sum odd [ $i+1$ ,  $N-1$ ]

$$\text{Total Even sum} = \text{sumeven}[0, i-1] + \text{sum odd}[i+1, N-1]$$

$$\text{Total Odd sum} = \text{sum odd}[0, i-1] + \text{sum even}[i+1, N-1]$$

Pseudocode

1)  $\rightarrow \text{PfEven}[] \Rightarrow O(N) \rightarrow \text{Loop}$

2)  $\rightarrow \text{PfOdd}[] \Rightarrow O(N) \rightarrow \text{Loop}$

$\text{cnt} = 0$

$i = 0; i < N; i++ \{$

// check if  $q$  is special index

Total Even sum =

Solve PfEven[] query

SumEven[0,  $q-1$ ]

SumOdd[ $q+1, N-1$ ]

Total Odd sum =

SumOdd[0,  $q-1$ ]

CumEven[ $q+1, N-1$ ]

PfEven[]

if (TotalEvenSum == TotalOddSum) {

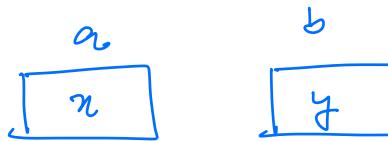
    cnt++

Note: If  $q=0$ ,  $q=N-1$ , handle Edge Cases.

TC:  $O(N + N + N) \Rightarrow O(N)$  SC:  $O(N \times N) \Rightarrow O(N^2)$

Doubts:

int  $a = x$



int  $b = y$

$a = a + b$



b = a - b



a = a - b



TODO:

$\text{man} = \text{ar}[0]$ ,  $\text{cnt} = 0$

$i = 0; i < N; i++ \{$

```

if (man == ar[i]) {
    cnt = cnt + 1
}
else if (man < ar[i]) {
    man = ar[i]
    cnt = 1
}
}

```

(84))

→ Notes :  $\text{ar}[\underline{n}] : [O, \underline{\underline{N}}]$

↳ Working hard → Talent