Deep learning and Artificial Neural Network for Object recognition

Dr. Sebastien Ambellouis Ingénieur de recherche Université Gustave Eiffel Associé à IMT-Lille-Douai

sebastien.ambellouis@univ-eiffel.fr sebastien.ambellouis@imt-lille-douai.fr

The perceptron

- Supervised learning: regression/classification
- Single neuron model
 - Linear regression
 - Logistic regression
 - Multi-output and softmax regression
- Multi-layer perception?

Types of machine learning

We can categorize three types of learning procedures:

Supervised Learning:

$$\mathbf{y} = f(\mathbf{x})$$

Unsupervised Learning:

$$f(\mathbf{x})$$

Reinforcement Learning:

$$\mathbf{y} = f(\mathbf{x})$$

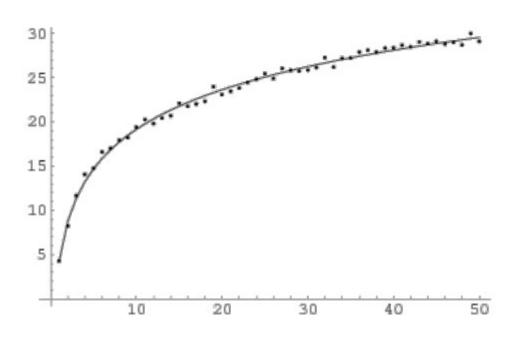
Z

We have a labeled dataset with pairs (**x**, **y**), e.g. classify a signal window as containing speech or not:

```
\mathbf{x}_{1} = [x(1), x(2), ..., x(T)] \quad \mathbf{y}_{1} = \text{"no"} \\ \mathbf{x}_{2} = [x(T+1), ..., x(2T)] \quad \mathbf{y}_{2} = \text{"yes"} \\ \mathbf{x}_{3} = [x(2T+1), ..., x(3T)] \quad \mathbf{y}_{3} = \text{"yes"} \\ ...
```

Supervised learning

Fit a function: y = f(x), $x \in \mathbb{R}^m$



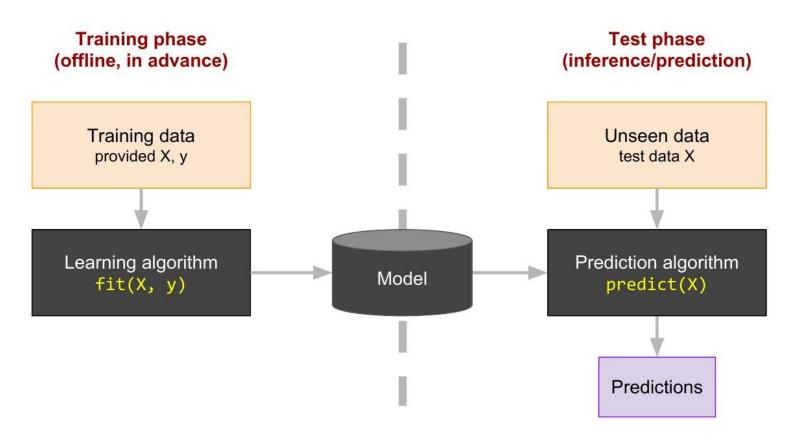
Supervised learning

Fit a function: y = f(x), $x \in \mathbb{R}^m$

Given paired training examples $\{(\mathbf{x}_i, \mathbf{y}_i)\}$



Black box abstraction of supervised learning



Supervised learning

Fit a function: y = f(x), $x \in \mathbb{R}^m$

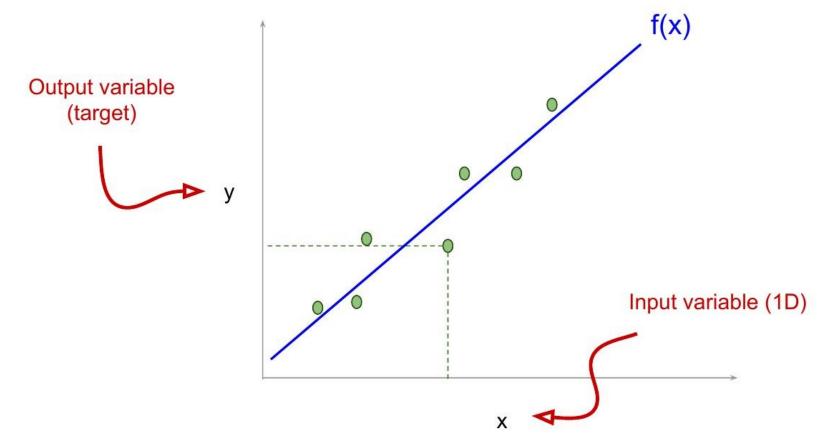
Given paired training examples {(x, y)}

Key point: generalize well to unseen examples

Depending on the type of target y we get:

- Regression: $y \in \mathbb{R}^N$ is continuous (e.g. temperatures $y = \{19^\circ, 23^\circ, 22^\circ\}$)
- Classification: y is discrete (e.g. y = {1, 2, 5, 2, 2}).
 - Beware! These are unordered categories, not numerically meaningful outputs: e.g. code[1] = "dog", code[2] = "cat", code[5] = "ostrich", ...

Linear Regression (1D input – 1D output)



Linear Regression (Multi-D input)

Input data can also be M-dimensional with vector x:

$$y = \mathbf{w}^{T} \cdot \mathbf{x} + b = w1 \cdot x1 + w2 \cdot x2 + w3 \cdot x3 + ... + wM \cdot xM + b$$

e.g. we want to predict the **price of a house (y)** based on:

```
x1 = square-meters (sqm)
```

$$x2,3 = location (lat, lon)$$

$$y = price = w1 \cdot (sqm) + w2 \cdot (lat) + w3 \cdot (lon) + w4 \cdot (yoc) + b$$



Supervised learning

Fit a function: y = f(x), $x \in \mathbb{R}^m$

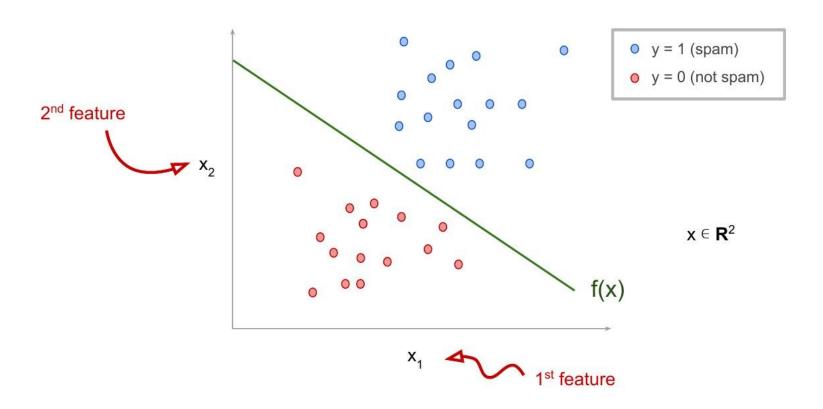
Given paired training examples $\{(\mathbf{x}_i, \mathbf{y}_i)\}$

Key point: generalize well to unseen examples

Depending on the type of target y we get:

- Regression: $y \in \mathbb{R}^N$ is continuous (e.g. temperatures $y = \{19^\circ, 23^\circ, 22^\circ\}$)
- Classification: \mathbf{y} is discrete (e.g. $\mathbf{y} = \{1, 2, 5, 2, 2\}$).
 - Beware! These are unordered categories, not numerically meaningful outputs: e.g. code[1] = "dog", code[2] = "cat", code[5] = "ostrich", ...

Classification (spam/not spam classifier)



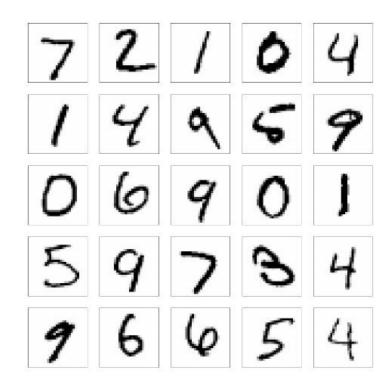
Classification (digit classifier)

NMIST dataset

Produce a classifier to map from pixels to the digit.

- ▶ If images are grayscale and 28×28 pixels in size, then $\mathbf{x}_i \in \mathbb{R}^{784}$
- $y_i \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

Example of a multi-class classification task.



A network for classification

Data & Labels

The perceptron

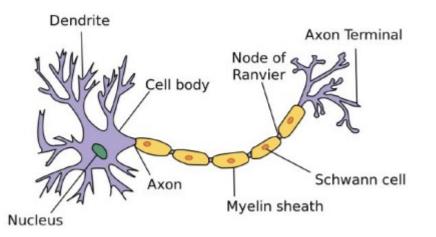
- Supervised learning: regression/classification
- Single neuron model
 - Linear regression
 - Logistic regression
 - Multi-output and softmax regression
- Multi-layer perception?

Biological inspiration

The Perceptron is seen as an analogy to a biological neuron.

Biological neurons fire an impulse once the sum of all inputs is over a threshold.

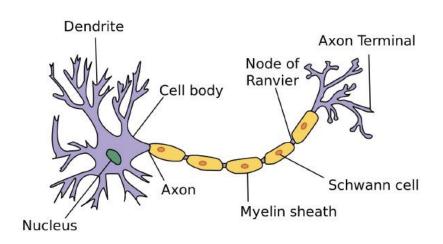
The perceptron acts like a switch (learn how in the next slides...).



McCullough and Pitts model (1943)

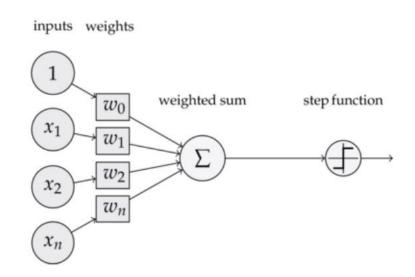


Biological inspiration



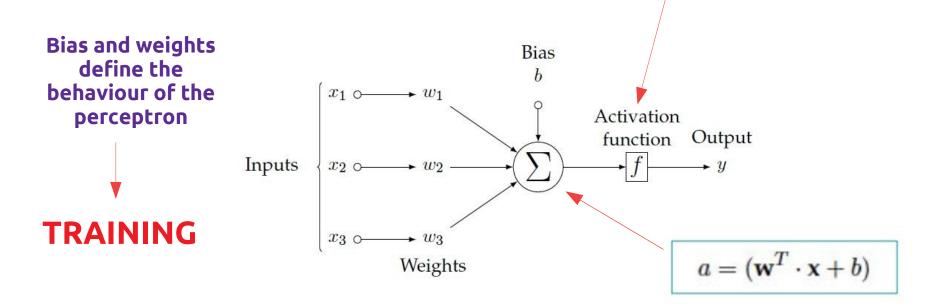


Rosenblatt's Perceptron (1958)



Single neuron model (perceptron)

The perceptron can address both <u>regression</u> or <u>classification</u> problems, depending on the chosen <u>activation</u> function.

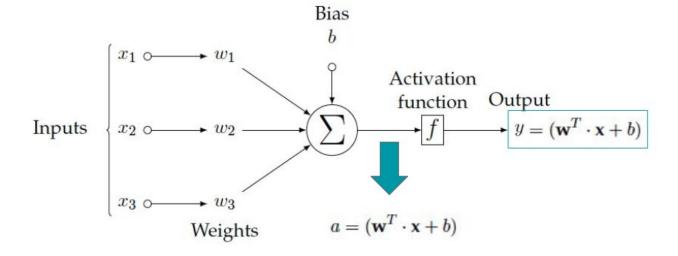


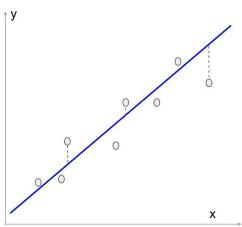
The perceptron

- Supervised learning: regression/classification
- Single neuron model
 - Linear regression
 - Logistic regression
 - Multi-output and softmax regression
- Multi-layer perception?

Linear regression

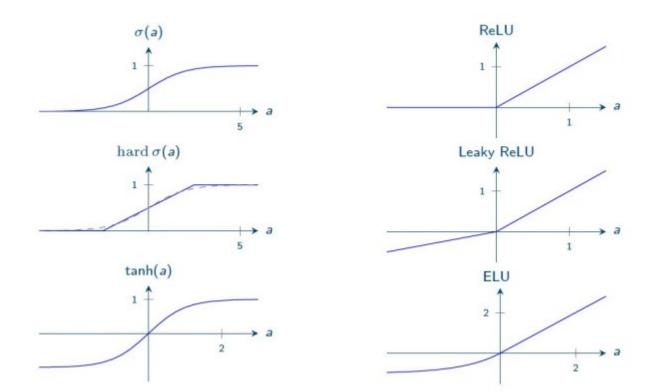
The perceptron can solve <u>linear regression</u> problems when f(a)=a. [identity]





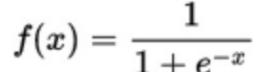
Other activation function

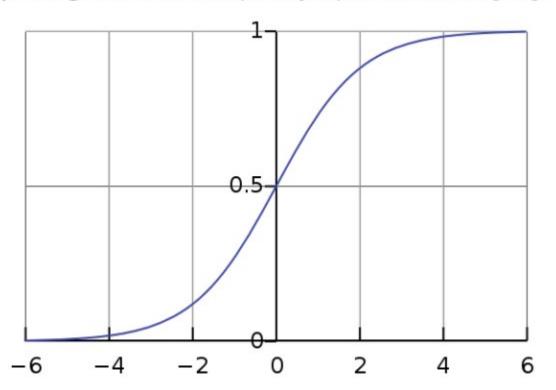
More interesting when the <u>activation function</u> f(a) is not the identity, but:



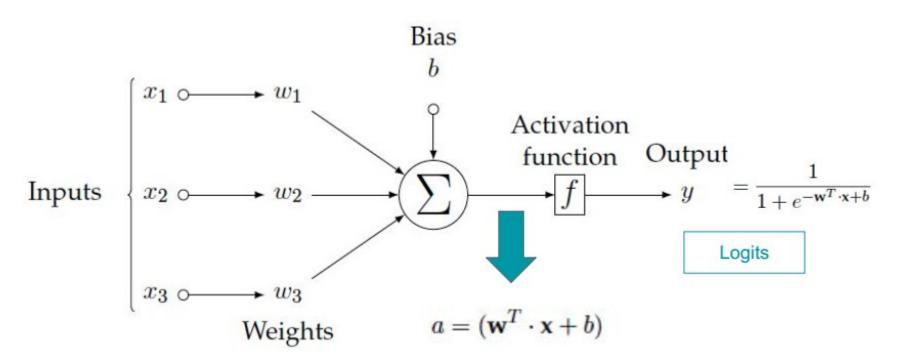
Other activation function

The **sigmoid function** $\sigma(x)$ or **logistic curve** maps any input x between [0,1]:



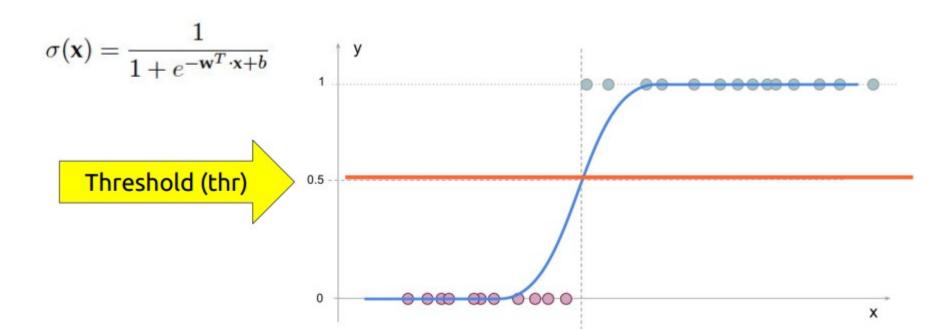


Logistic regression



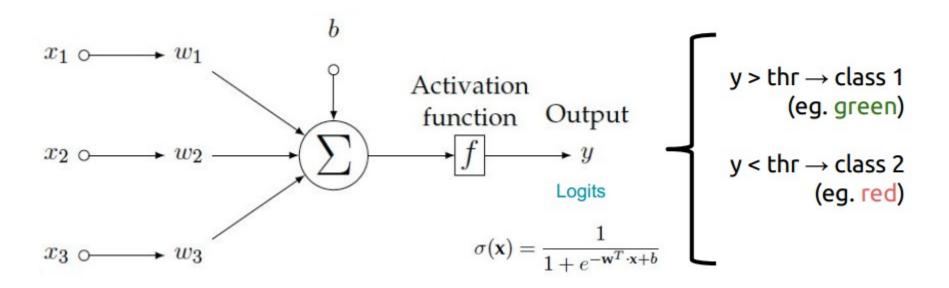
Binary classification

For classification, regressed values should b collapsed into 0 and 1 to quantize the confidence of the predictions ("probabilities").



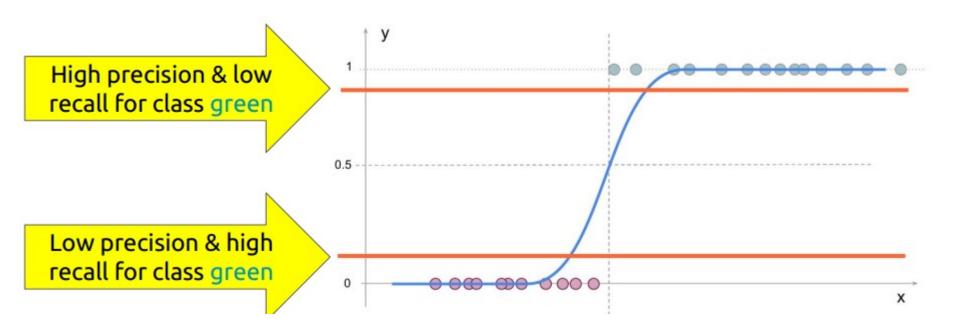
Binary classification

Setting a **threshold (thr)** at the output of the perceptron allows solving classification problems between two classes (binary):



Binary classification

The classification threshold can be adjusted based on the desired precision - recall trade-off:



Softmax regression (more than 2 classes)

Ideally we would like to predict the probability that y takes a particular value given x,

$$P(y = j | \mathbf{x}), \quad j \in \{1, \dots, K\}$$

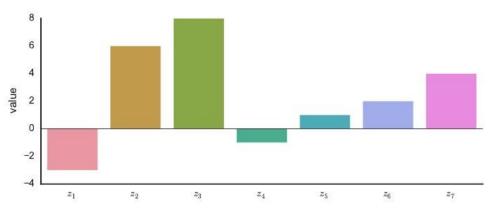
with:

$$\sum_{i=1}^K P(y=j|x) = 1$$

The logistic regression classifier does this for the binary case. The **softmax classifier** extends it to the multiclass case.

$$p(y=j|\mathbf{x}) = rac{e^{(\mathbf{w}_j^T\mathbf{x}+b_j)}}{\sum_{k\in K}e^{(\mathbf{w}_k^T\mathbf{x}+b_k)}}$$

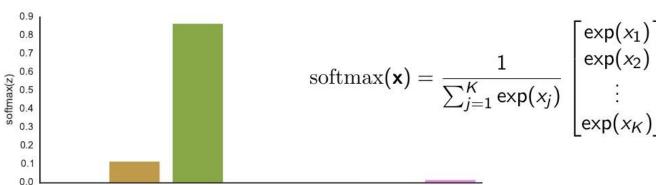
Effect of the softmax



 z_2

 z_3

 z_4

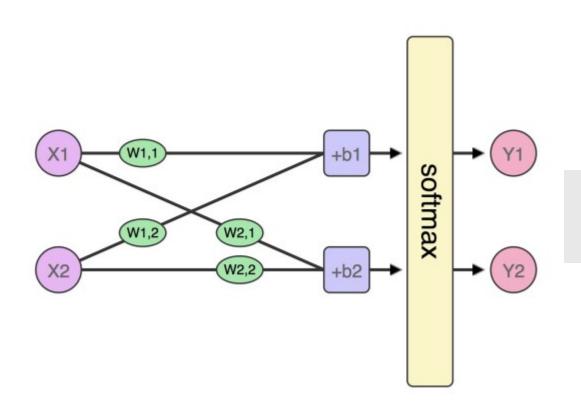


 z_5

 z_6

 z_7

Softmax regression: two classes

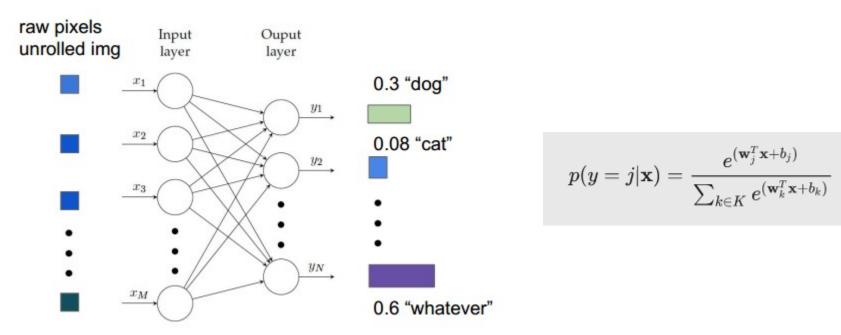


2 perceptrons are necessary

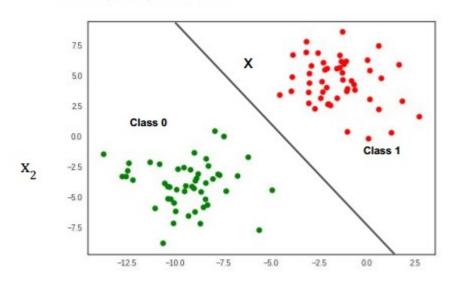
$$p(y=j|\mathbf{x}) = rac{e^{(\mathbf{w}_j^T\mathbf{x}+b_j)}}{\sum_{k\in K}e^{(\mathbf{w}_k^T\mathbf{x}+b_k)}}$$

Softmax regression: more than 2 classes

Multiple classes can be predicted by putting many neurons in parallel, each processing its binary output out of N possible classes.



2D input space data

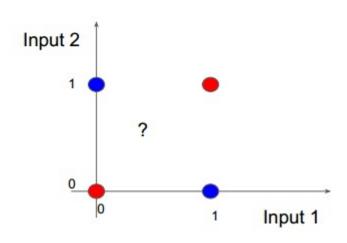


XOR logic table

Input 1	Input 2	Desired Output
0	0	0
0	1	1
1	0	1
1	1	0

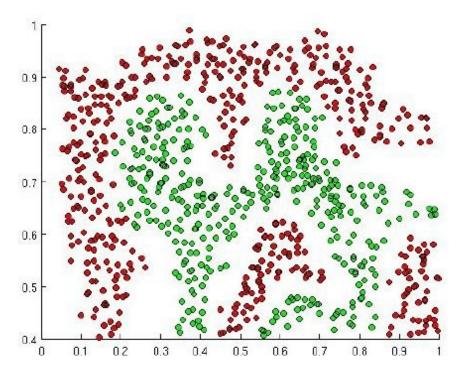
Data might be non linearly separable

→ One single neuron is not enough



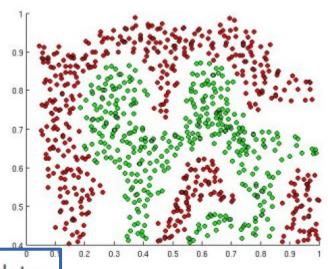
Real world problems often need non-linear boundaries

- Images
- Audio
- Text



What can we do?

- Use a non-linear classifier
 - Decision trees (and forests)
 - K nearest neighbours
- 2. Engineer a suitable representation
 - One in which features are more linearly separable
 - Then use a linear model
- Engineer a kernel
 - Design a kernel K(x₁, x₂)
 - Use kernel methods (e.g. SVM)
- 4. Learn a suitable representation space from the data
 - Deep learning, deep neural networks
 - Boosted cascade classifiers like Viola Jones also take this approach



The perceptron

- Supervised learning: regression/classification
- Single neuron model
 - Linear regression
 - Logistic regression
 - Multi-output and softmax regression
- Multi-layer perception

Multi layer perceptron

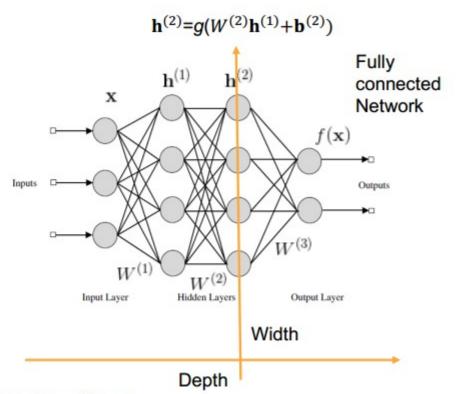
When each node in each layer is a linear combination of all inputs from the previous layer then the network is called a multilayer perceptron (MLP)

Weights can be organized into matrices.

Forward pass computes

$$\mathbf{h}_0 = \mathbf{x}$$

 $\mathbf{h}^{(t)} = g(W^{(t)}\mathbf{h}^{(t-1)} + \mathbf{b}^{(t)})$
 $f(\mathbf{x}) = \mathbf{h}^{(L)}$
 $\mathbf{y} = \mathbf{f}(\mathbf{x})$



g: activation function. i.e. sigmoid f: target function. i.e. softmax

Multi layer perceptron (matrix)

 W_1

W ₁₁	W ₁₂	W ₁₃	W ₁₄
W ₂₁	W ₂₂	W ₂₃	W ₂₄
W ₃₁	W ₃₂	W ₃₃	W ₃₄
W ₄₁	W ₄₂	W ₄₃	W ₄₄

 h_0

X₁

 x_2

X3

 X_4

 b_1

b ₁	
h.	
b ₂	
b ₃	
b ₄	

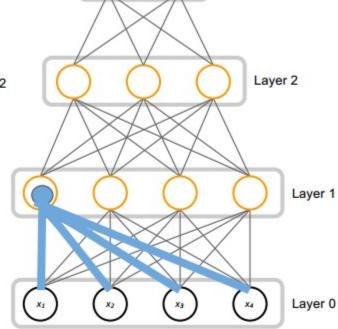
 $h_{11} = g(wx + b)$

 h_2

 h_3

h₁

h₀



Layer 3

Forward pass computes

$$\begin{aligned} \mathbf{h}_0 &= \mathbf{x} \\ \mathbf{h}^{(t)} &= g(W^{(t)}\mathbf{h}^{(t-1)} + \mathbf{b}^{(t)}) \\ f(\mathbf{x}) &= \mathbf{h}^{(L)} \end{aligned}$$

Multi layer perceptron (matrix)

 W_1

W ₁₁	W ₁₂	W ₁₃	W ₁₄
W ₂₁	W ₂₂	W ₂₃	W ₂₄
W ₃₁	W ₃₂	W ₃₃	W ₃₄
W ₄₁	W ₄₂	W ₄₃	W ₄₄

h₀

 X_4

 b_1

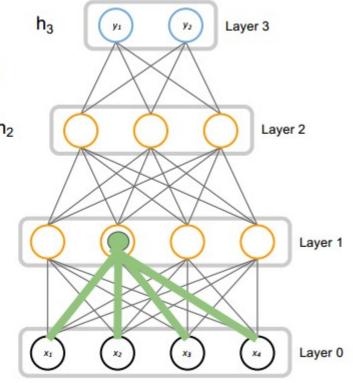
b₁
b₂
b₃

 $h_{11}=g(wx+b)$

 $h_{12} = g(wx + b) h_2$

h₁

 h_0



Forward pass computes

$$\mathbf{h}_0 = \mathbf{x}$$

 $\mathbf{h}^{(t)} = g(W^{(t)}\mathbf{h}^{(t-1)} + \mathbf{b}^{(t)})$
 $f(\mathbf{x}) = \mathbf{h}^{(L)}$

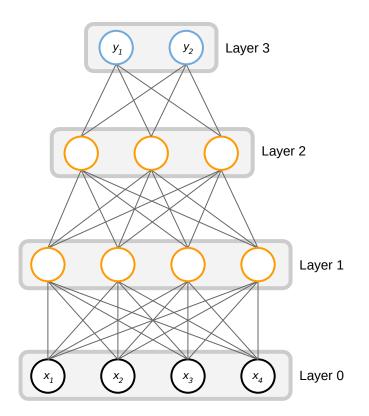
Deep Neural Network

The i-th layer is defined by a matrix **Wi** and a vector **bi**, and the activation is simply a dot product plus **bi**:

$$h_i = f(W_i \cdot h_{i-1} + b_i)$$

Num parameters to learn at i-th layer:

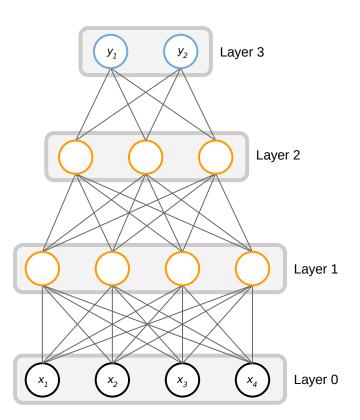
$$N_{params}^i = N_{inputs}^i \times N_{units}^i + N_{units}^i$$



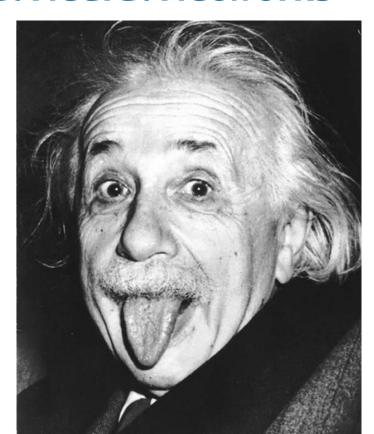
Deep nets

Just a neural network with several hidden layers

- Often uses different types of layers, especially fully connected, convolution, and pooling layers
- Output of each layer can be thought of as a representation of the input
- Outputs of the lower layers are more closely related to the input features
- Outputs of the higher layers contain more abstract features closer in semantics to the target variable



What if the Input is an image?



MNIST Example

Handwritten digits

- 60.000 training examples
- 10.000 test examples
- 10 classes (digits 0-9)
- 28x28 grayscale images(784 pixels)
- http://yann.lecun.com/exdb/mnist/

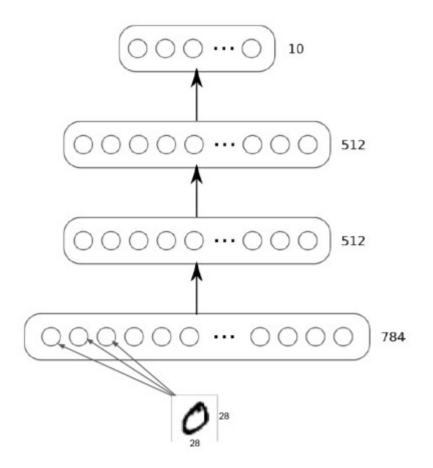
The objective is to learn a function that predicts the digit from the image

MNIST Example

Model

- 3 layer neural-network (2 hidden layers)
- Tanh units (activation function)
- 512-512-10
- Softmax on top layer
- Cross entropy Loss

Layer	#Weights	#Biases	Total
1	784 x 512	512	401,920
2	512 x 512	512	262,656
3	512 x 10	10	5,130
			669,706



MNIST Example

Training

- 40 epochs using min-batch SGD
- Size of the mini-batch: 128
- Leaning Rate: 0.1 (fixed)
- Takes 5 minutes to train on GPU

Metrics

$$Accuracy = \frac{TP + TN}{TP + TN + FP + FN}$$

there are other metrics....

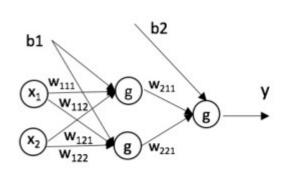
Accuracy Results

• 98.12% (188 errors in 10.000 test examples)

there are ways to improve accuracy...

Exercise...

Given the following network to obtain a XNOR operation, Indicate which parameters are correct:

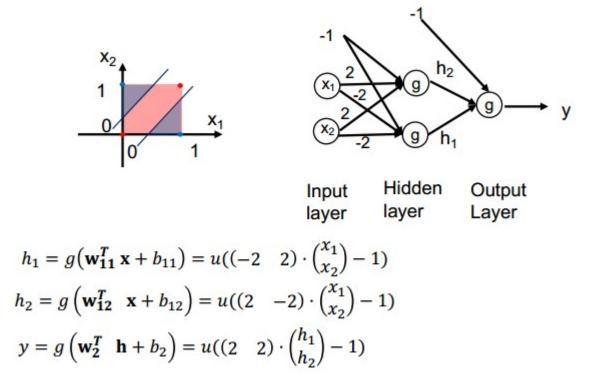


Input vector (x1,x2)	Class XNOR
(0,0)	1
(0,1)	0
(1,0)	0
(1,1)	1

- w₁₁₁=-2, w₁₁₂=2, w₁₂₁=2,w₁₂₂=-2,b1=-1,w₂₁₁=2,w₂₂₁=2,b2=-1
- w_{111} =-2, w_{112} =2, w_{121} =2, w_{122} =-2, b1=-1, w_{211} =2, w_{221} =2, b2=1
- w₁₁₁=-2, w₁₁₂=2, w₁₂₁=2,w₁₂₂=-2,b1=-1,w₂₁₁=-2,w₂₂₁=-2,b2=1
- w₁₁₁=-2, w₁₁₂=2, w₁₂₁=2,w₁₂₂=-2,b1=-1,w₂₁₁=-2,w₂₂₁=-2,b2=-1

Exercise...

Given the following network to obtain a XNOR operation, Indicate which parameters are correct:



Training ...

With Multiple layers we need to minimize the **loss function** $\mathcal{L}(f_{\theta}(x), y)$ with respect to all the parameters of the model $\theta(W^{(k)}, b^{(k)})$:

$$W^* = argmin_{\theta} \mathcal{L}(f_{\theta}(x), y)$$

Gradient Descent: Move the parameter θ_j in small steps in the direction opposite sign of the derivative of the loss with respect j:

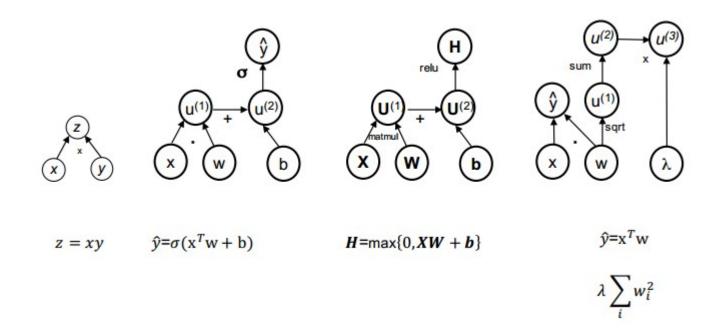
$$\theta_j^{(n)} = \theta_j^{(n-1)} - \alpha^{(n-1)} \cdot \nabla_{\theta_j} \mathcal{L}(y, f(x))$$

Stochastic gradient descent (SGD): estimate the gradient with one sample, or better, with a **minibatch** of examples.

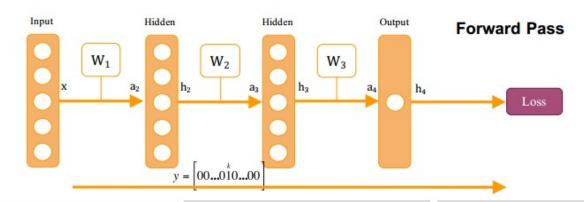
For MLP gradients can be found using the **chain rule** of differentiation.

The calculations reveal that the gradient wrt. the parameters in layer k only depends on the error from the above layer and the output from the layer below. This means that the gradients for each layer can be computed iteratively, starting at the last layer and propagating the error back through the network. This is known as the **backpropagation** algorithm.

Training: based on computational graphs



Training: two pass ...



Probability Class given an input (softmax)

$$p(c_k = 1|\mathbf{x}) = \frac{\exp(a_k)}{\sum_c \exp(a_c)}$$

Loss function; e.g., negative log-likelihood (good for classification)

$$L(\mathbf{x}, y; \mathbf{W}) = -\sum_{j} y_{j} \log(p(c_{j}|\mathbf{x}))$$

Regularization term (L2 Norm) aka as weight decay

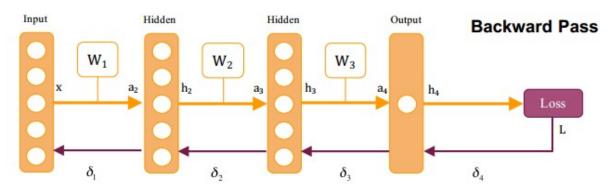
$$L(\mathbf{x}, y; \mathbf{W}) = -\sum_{j} y_{j} \log(p(c_{j}|\mathbf{x})) + \frac{\lambda}{2} ||\mathbf{W}||_{2}^{2}$$

Minimize the loss (plus some regularization term) w.r.t. Parameters over the whole training set.

$$\mathbf{W}^* = argmin_{\theta} \sum\nolimits_{j} L(\mathbf{x}^n, y^n; \mathbf{W})$$

Figure Credit: Kevin McGuiness

Training: two pass ...



1. Find the error in the top layer:

$$\delta_{\scriptscriptstyle K} = \frac{\partial L}{\partial a_{\scriptscriptstyle K}}$$

$$\delta_K = \frac{\partial L}{\partial h_K} \frac{\partial h_K}{\partial a_K}$$

$$\delta_{\scriptscriptstyle K} = \frac{\partial L}{\partial h_{\scriptscriptstyle K}} \bullet g^{\scriptscriptstyle \dagger}(a_{\scriptscriptstyle K})$$

Figure Credit: Kevin McGuiness

2. Compute weight updates

$$\frac{\partial L}{\partial W_k} = \frac{\partial L}{\partial a_{k+1}} \frac{\partial a_{k+1}}{\partial W_k}$$

$$\frac{\partial L}{\partial W_k} = \frac{\partial L}{\partial a_{k+1}} \bullet h_k$$

$$\frac{\partial L}{\partial W_k} = \delta_{k+1} \bullet h_k$$

To simplify we don't consider the biass

3. Backpropagate error to layer below

$$\delta_k = \frac{\partial L}{\partial a}$$

$$\delta_k = \frac{\partial L}{\partial a_{k+1}} \frac{\partial a_{k+1}}{\partial h_k} \frac{\partial h_k}{\partial a_k}$$

$$\delta_k = W_k^T \frac{\partial L}{\partial a_{k+1}} \bullet g'(a_k)$$

$$\delta_k = W_k^T \delta_{k+1} \bullet g^!(a_k)$$

Training: iterative estimation

Gradient Descent: Move the parameter θ_j in small steps in the direction opposite sign of the derivative of the loss with respect j.

$$\theta^{(n)} = \theta^{(n-1)} - \alpha^{(n-1)} \cdot \nabla_{\theta} \mathcal{L}(y, f(x)) - \lambda \theta^{(n-1)}$$

Weight Decay: Penalizes large weights, distributes values among all the parameters

Stochastic gradient descent (SGD): estimate the gradient with one sample, or better, with a **minibatch** of examples.

Momentum: the movement direction of parameters averages the gradient estimation with previous ones.

Several strategies have been proposed to update the weights: optimizers

Training: weight initialization

Need to pick a starting point for gradient descent: an initial set of weights

Zero is a very **bad idea!** Zero is a **critical point**

Error signal will not propagate

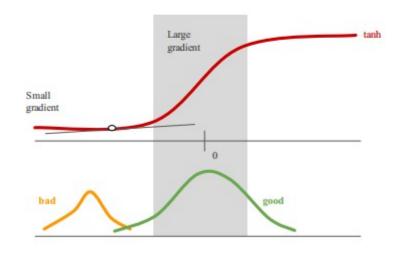
Gradients will be zero: no progress

Constant value also bad idea: Need to break symmetry

Use small random values:

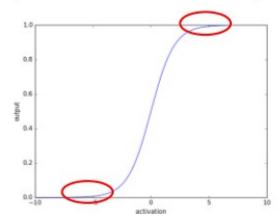
E.g. zero mean Gaussian noise with constant variance

Ideally we want inputs to activation functions (e.g. sigmoid, tanh, ReLU) to be mostly in the linear area to allow larger gradients to propagate and converge faster.



Training: vanishing gradients

In the backward pass you might be in the flat part of the sigmoid (or any other activation function like tanh) so derivative tends to zero and your training loss will not go down

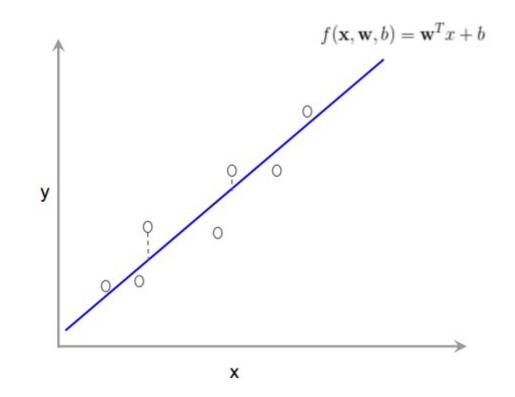


Training: loss function (linear regression)

$$X = \begin{bmatrix} \mathbf{x}_1^T \\ \mathbf{x}_2^T \\ \vdots \\ \mathbf{x}_N^T \end{bmatrix} \in \mathbb{R}^{N \times D} \quad \mathbf{y} \in \mathbb{R}^N$$

Loss function is square (Euclidean) loss

$$L(X, y, \mathbf{w}, b) = \frac{1}{2N} \sum_{i=1}^{N} (y_i - f(x_i, \mathbf{w}, b))^2$$



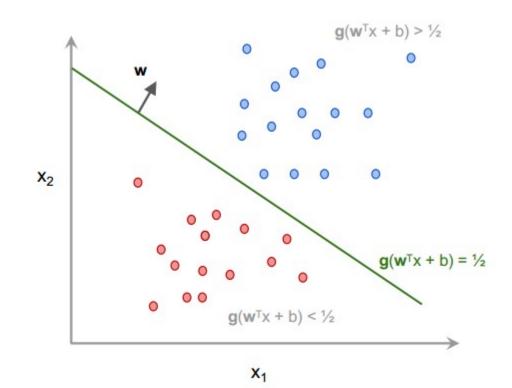
Training: loss function (logistic regression)

Activation function is the sigmoid

$$g(x) = \frac{1}{1 + e^{-x}}$$

Loss function is cross entropy

$$L = -\frac{1}{N} \sum_{i=1}^{N} y_i \log f(\mathbf{x}_i) + (1 - y_i) \log(1 - f(\mathbf{x}_i))$$



Training: gradient descent

E.g. linear regression

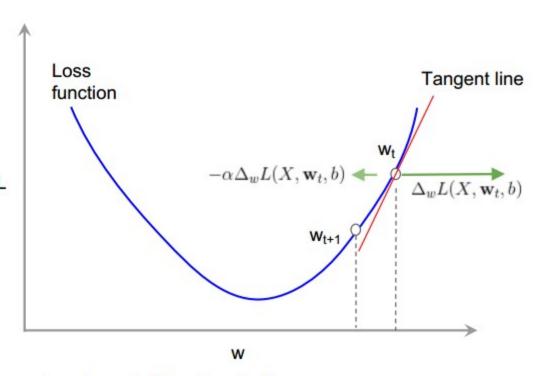
$$L(X, y, \mathbf{w}, b) = \frac{1}{2N} \sum_{i=1}^{N} (y_i - f(x_i, \mathbf{w}, b))^2$$

Need to optimize L

Gradient descent

$$\Delta_w L = \frac{1}{N} \sum_{i=1}^{N} (f(x_i) - y_i) x_i$$

$$\mathbf{w}_{t+1} = \mathbf{w}_t - \alpha \Delta_w L(X, \mathbf{w}_t, b)$$



α : learning rate (aka step size)

Training: all the hyperparameters

So far we have lots of hyperparameters to choose:

- 1. Learning rate (α)
- 2. Regularization constant (λ)
- 3. Number of epochs
- Number of hidden layers
- 5. Nodes in each hidden layer
- Weight initialization strategy
- 7. Loss function
- 8. Activation functions
- 9. ...

For a 200x200 image, we have 4x10⁴ neurons each one with 4x10⁴ inputs, that is 16x10⁸ parameters, only for one layer (not counting the bias)!!!

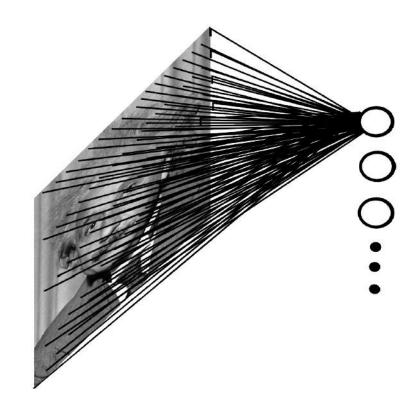


Figure Credit: Ranzatto

For a 200x200 image, we have $4x10^4$ neurons each one with 10x10 "local connections" (also called receptive field) inputs, that is $4x10^6$

What else can we do to reduce the number of parameters?

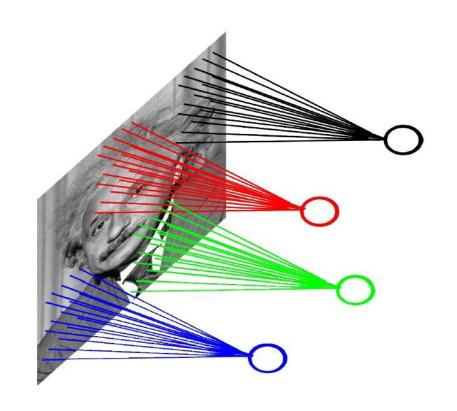
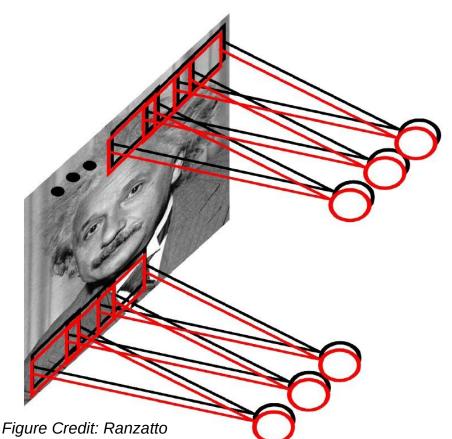


Figure Credit: Ranzatto

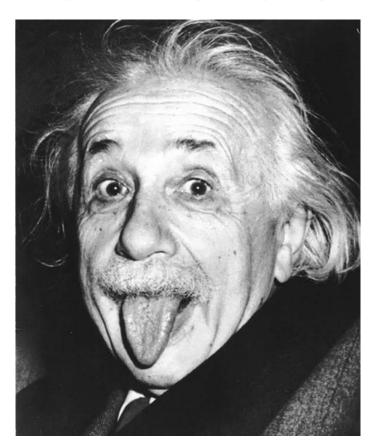


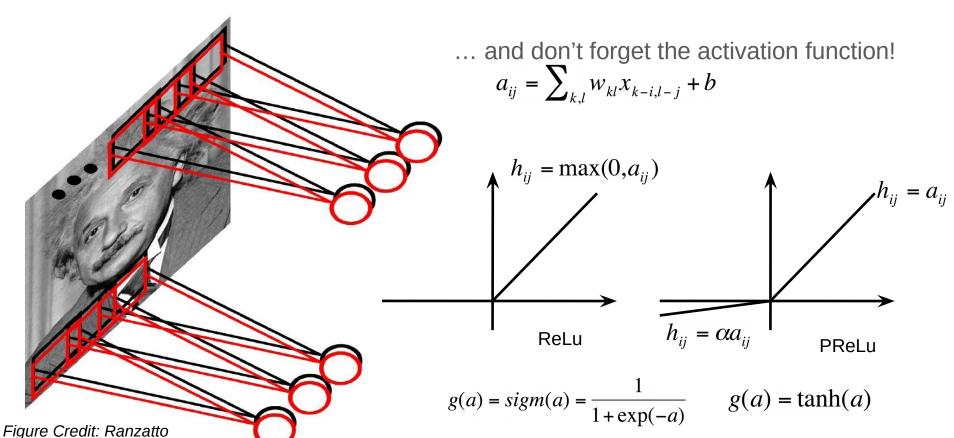
Translation invariance: we can use same parameters to capture a specific "feature" in any area of the image. We can try different sets of parameters to capture different features.

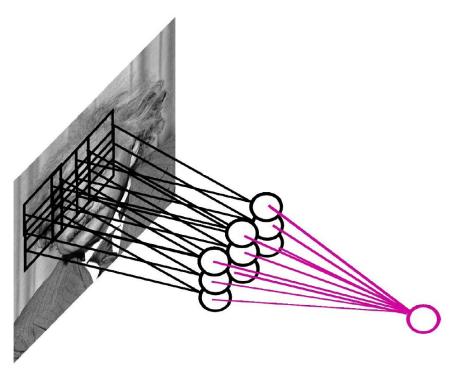
These operations are equivalent to perform **convolutions** with different filters.

Ex: With100 different filters (or feature extractors) of size 10x10, the number of parameters is 10⁴

- sparse interactions
- parameter sharing
- equivariant representations





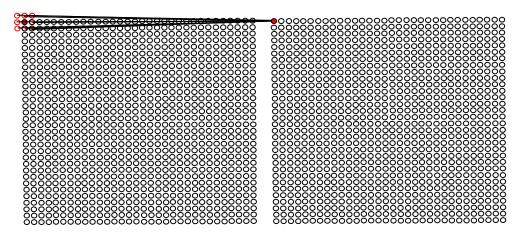


Most ConvNets use **Pooling** (or subsampling) to reduce dimensionality and provide invariance to small local changes.

Pooling options:

- Max
- Average
- Stochastic pooling

Figure Credit: Ranzatto



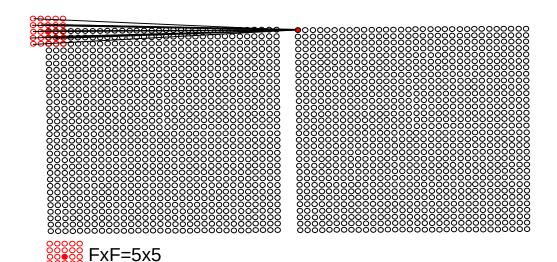
Padding (P): When doing the convolution in the borders, you may add values to compute the convolution.

When the values are zero, that is quite common, the technique is called zero-padding.

When padding is not used the output size is reduced.



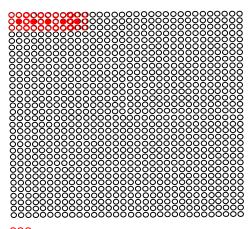
FxF=3x3

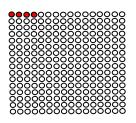


Padding (P): When doing the convolution in the borders, you may add values to compute the convolution.

When the values are zero, that is quite common, the technique is called zero-padding.

When padding is not used the output size is reduced.





Stride (S): When doing the convolution or another operation, like pooling, we may decide to slide not pixel by pixel but every 2 or more pixels. The number of pixels that we skip is the value of the stride. It might be used to reduce the dimensionality of the output

•

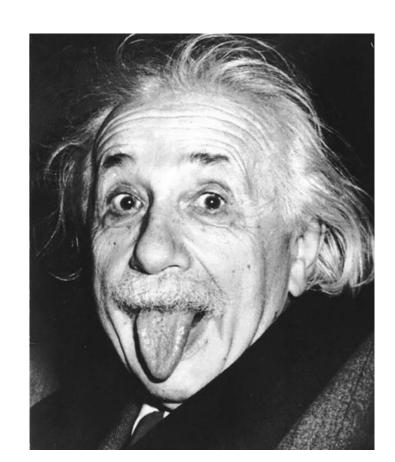
References

Neocognitron: A Self-organizing Neural Network Model for a Mechanism of Pattern Recognition Unaffected by Shift in Position, Kunihiko Fukushima NHK BroadcastingScienceResearchLaboratories,Kinuta, Setagaya, Tokyo, Japan, Bio Cybernetics, (4) 1980, 193-202

Deep Learning

An MIT Press book Ian Goodfellow and Yoshua Bengio and Aaron Courville http://www.deeplearningbook.org/contents/convnets.html

Stanford Course in Convolutional NN for Visual Representation (2017) http://cs231n.github.io/convolutional-networks/
2016 course: https://youtu.be/LxfUGhug-iQ



Questions?