The Project Gutenberg EBook of A Short Account of the History of Mathematics, by W. W. Rouse Ball

A SHORT ACCOUNT

OF THE

HISTORY OF MATHEMATICS

BY

W. W. ROUSE BALL

FELLOW OF TRINITY COLLEGE, CAMBRIDGE

Preface

The subject-matter of this book is a historical summary of the development of mathematics, illustrated by the lives and discoveries of those to whom the progress of the science is mainly due. It may serve as an introduction to more elaborate works on the subject, but primarily it is intended to give a short and popular account of those leading facts in the history of mathematics which many who are unwilling, or have not the time, to study it systematically may yet desire to know.

The first edition was substantially a transcript of some lectures which I delivered in the year 1888 with the object of giving a sketch of the history, previous to the nineteenth century, that should be intelligible to any one acquainted with the elements of mathematics. In the second edition, issued in 1893, I rearranged parts of it, and introduced a good deal of additional matter.

The scheme of arrangement will be gathered from the table of contents at the end of this preface. Shortly it is as follows. The first chapter contains a brief statement of what is known concerning the mathematics of the Egyptians and Phoenicians; that is introductory to the history of mathematics under Greek influence. The subsequent history is divided into three periods: first, that under Greek influence, chapters II to VII; second, that of the middle ages and renaissance, chapters VIII to XIII; and lastly that of modern times, chapters XIV to XIX.

In discussing the mathematics of these periods I have confined myself to giving the leading events in the history, and frequently have passed in silence over men or works whose influence was comparatively unimportant. Doubtless an exaggerated view of the discoveries of those mathematicians who are mentioned may be caused by the non-allusion to minor writers who preceded and prepared the way for them, but in all historical sketches this is to some extent inevitable, and I have done my best to guard against it by interpolating remarks on the progress of the science at different times. Perhaps also I should here state that generally I

have not referred to the results obtained by practical astronomers and physicists unless there was some mathematical interest in them. In quoting results I have commonly made use of modern notation; the reader must therefore recollect that, while the matter is the same as that of any writer to whom allusion is made, his proof is sometimes translated into a more convenient and familiar language.

The greater part of my account is a compilation from existing histories or memoirs, as indeed must be necessarily the case where the works discussed are so numerous and cover so much ground. When authorities disagree I have generally stated only that view which seems to me to be the most probable; but if the question be one of importance, I believe that I have always indicated that there is a difference of opinion about it.

I think that it is undesirable to overload a popular account with a mass of detailed references or the authority for every particular fact mentioned. For the history previous to 1758, I need only refer, once for all, to the closely printed pages of M. Cantor's monumental *Vorlesungen über die Geschichte der Mathematik* (hereafter alluded to as Cantor), which may be regarded as the standard treatise on the subject, but usually I have given references to the other leading authorities on which I have relied or with which I am acquainted. My account for the period subsequent to 1758 is generally based on the memoirs or monographs referred to in the footnotes, but the main facts to 1799 have been also enumerated in a supplementary volume issued by Prof. Cantor last year. I hope that my footnotes will supply the means of studying in detail the history of mathematics at any specified period should the reader desire to do so.

My thanks are due to various friends and correspondents who have called my attention to points in the previous editions. I shall be grateful for notices of additions or corrections which may occur to any of my readers.

<div align="right">

W. W. ROUSE BALL

</div>

TRINITY COLLEGE, CAMBRIDGE

Note

The fourth edition was stereotyped in 1908, but no material changes have been made since the issue of the second edition in 1893, other duties having, for a few years, rendered it impossible for me to find time for any extensive revision. Such revision and incorporation of recent researches on the subject have now to be postponed till the cost of printing has fallen, though advantage has been taken of reprints to make trivial corrections and additions.

<div align="right">

W. W. R. B.

</div>

TRINITY COLLEGE, CAMBRIDGE

Contents

Chapter 1

Egyptian and Phoenician Mathematics

The history of mathematics cannot with certainty be traced back to any school or period before that of the Ionian Greeks. The subsequent history may be divided into three periods, the distinctions between which are tolerably well marked. The first period is that of the history of mathematics under Greek influence, that is discussed in chapters II to VII; the second is that of the mathematics of the middle ages and the renaissance, this is discussed in chapters VIII to XIII; the third is that of modern mathematics, and this is discussed in chapters XIV to XIX.

Although the history of mathematics commences with that of the Ionian schools, there is no doubt that those Greeks who first paid attention to the subject were largely indebted to the previous investigations of the Egyptians and Phoenicians. Our knowledge of the mathematical attainments of those races is imperfect and partly conjectural, but, such as it is, it is here briefly summarized. The definite history begins with the next chapter.

On the subject of prehistoric mathematics, we may observe in the first place that, though all early races which have left records behind them knew something of numeration and mechanics, and though the majority were also acquainted with the elements of land-surveying, yet the rules which they possessed were in general founded only on the results of observation and experiment, and were neither deduced from nor did they form part of any science. The fact then that various nations in the vicinity of Greece had reached a high state of civilization does not justify us in assuming that they had studied mathematics.

The only races with whom the Greeks of Asia Minor (amongst whom our history begins) were likely to have come into frequent contact were those inhabiting the eastern littoral of the Mediterranean; and Greek tradition uniformly assigned the special development of geometry to the Egyptians, and that of the science of numbers either to the Egyptians or to the Phoenicians. I discuss these subjects separately.

Part I

First Period
Mathematics under Greek Influence

This period begins with the teaching of Tales, circ. 600 B.C., *and ends with the capture of Alexandria by the Mohammedans in or about* 641 A.D. *The characteristic feature of this period is the development of Geometry.*

It will be remembered that I commenced the last chapter by saying that the history of mathematics might be divided into three periods, namely, that of mathematics under Greek influence, that of the mathematics of the middle ages and of the renaissance, and lastly that of modern mathematics. The next four chapters (chapters II, III, IV, and V) deal with the history of mathematics under Greek influence: to these it will be convenient to add one (chapter VI) on the Byzantine school, since through it the results of Greek mathematics were transmitted to western Europe; and another (chapter VII) on the systems of numeration which were ultimately displaced by the system introduced by the Arabs. I should add that many of the dates mentioned in these chapters are not known with certainty, and must be regarded as only approximately correct.

There appeared in December 1921, just before this reprint was struck off, Sir T. L. Heath's work in 2 volumes on the History of Greek Mathematics. This may now be taken as the standard authority for this period.

Chapter 2

The Ionian and Pythagorean Schools Circ. 600 B.C.–400 B.C.

1

With the foundation of the Ionian and Pythagorean schools we emerge from the region of antiquarian research and conjecture into the light of history. The materials at our disposal for estimating the knowledge of the philosophers of these schools previous to about the year 430 B.C. are, however, very scanty. Not only have all but fragments of the different mathematical treatises then written been lost, but we possess no copy of the history of mathematics written about 325 B.C. by Eudemus (who was a pupil of Aristotle). Luckily Proclus, who about 450 A.D. wrote a commentary on the earlier part of Euclid's *Elements*, was familiar with Eudemus's work, and freely utilized it in his historical references. We have also a fragment of the *General View of Mathematics* written by Geminus about 50 B.C., in which the methods of proof used by the early Greek geometricians are compared with those current at a later date. In addition to these general statements we have biographies of a few of the leading mathematicians, and some scattered notes in various writers in which allusions are made to the lives and works of others. The original authorities are criticized and discussed at length in the works mentioned in the footnote to the heading of the chapter.

Thales.[2] The founder of the earliest Greek school of mathematics and philosophy was

---

[1]The history of these schools has been discussed by G. Loria in his *Le Scienze Esatte nell' Antica Grecia*, Moderna, 1893-1900; by Cantor, chaps. v-viii; by G.J. Allman in his *Greek Geometry from Thales to Euclid*, Dublin, 1889; by J. Gow, in his *Greek Mathematics*, Cambridge, 1884; by C.A. Bretschneider in his *Die Geometrie und die Geometer vor Eukleides*, Leipzig, 1870; and partially by H. Hankel in his posthumous *Geschichte der Mathematik*, Leipzig, 1874.

Thales, one of the seven sages of Greece, who was born about 640 B.C. at Miletus, and died in the same town about 550 B.C. The materials for an account of his life consist of little more than a few anecdotes which have been handed down by tradition.

During the early part of his life Thales was engaged partly in commerce and partly in public affairs; and to judge by two stories that have been preserved, he was then as distinguished for shrewdness in business and readiness in resource as he was subsequently celebrated in science. It is said that once when transporting some salt which was loaded on mules, one of the animals slipping in a stream got its load wet and so caused some of the salt to be dissolved, and finding its burden thus lightened it rolled over at the next ford to which it came; to break it of this trick Thales loaded it with rags and sponges which, by absorbing the water, made the load heavier and soon effectually cured it of its troublesome habit. At another time, according to Aristotle, when there was a prospect of an unusually abundant crop of olives Thales got possession of all the olive-presses of the district; and, having thus "cornered" them, he was able to make his own terms for lending them out, or buying the olives, and thus realized a large sum. These tales may be apocryphal, but it is certain that he must have had considerable reputation as a man of affairs and as a good engineer, since he was employed to construct an embankment so as to diver the river Halys in such a way as to permit the construction of a ford.

Probably it was as a merchant that Thales first went to Egypt, but during his leisure there he studied astronomy and geometry. He was middle-aged when he returned to Miletus; he seems then to have abandoned business and public life, and to have devoted himself to the study of philosophy and science—subjects which in the Ionian, Pythagorean, and perhaps also the Athenian schools, were closely connected: his views on philosophy do not here concern us. He continued to live at Miletus till his death circ. 550 B.C.

We cannot form any exact idea as to how Thales presented his geometrical teaching. We infer, however, from Proclus that it consisted of a number of isolated propositions which were not arranged in a logical sequence, but that the proofs were deductive, so that the theorems were not a mere statement of an induction from a large number of special instances, as probably was the case with the Egyptian geometricians. The deductive character which he thus gave to the science is his chief claim to distinction.

The following comprise the chief propositions that can now with reasonable probability be attributed to him; they are concerned with the geometry of angles and straight lines.

---

[2]See Loria, book I, chap. ii; Cantor, chap. v; Allman, chap. i.

(i) The angles at the base of an isosceles triangle are equal (Euc. I, 5). Proclus seems to imply that this was proved by taking another exactly equal isosceles triangle, turing it over, and then superposing it on the first—a sort of experimental demonstration.

(ii) If two straight lines cut one another, the vertically opposite angles are equal (Euc. I, 15). Thales may have regarded this as obvious, for Proclus adds that Euclid was the first to give a strict proof of it.

(iii) A triangle is determined if its base and base angles be given (*cf.* Euc. I, 26). Apparently this was applied to find the distance of a ship at sea—the base being a tower, and the base angles being obtained by observation.

(iv) The sides of equiangular triangles are proportionals (Euc. VI, 4, or perhaps rather Euc. VI, 2). This is said to have been used by Thales when in Egypt to find the height of a pyramid. In a dialogue given by Plutarch, the speaker, addressing Thales, says, "Placing your stick at the end of the shadow of the pyramid, you made by the sun's rays two triangles, and so proved that the [height of the] pyramid was to the [length of the] stick as the shadow of the pyramid to the shadow of the stick." It would seem that the theorem was unknown to the Egyptians, and we are told that the king Amasis, who was present, was astonished at this application of abstract science.

(v) A circle is bisected by any diameter. This may have been enunciated by Thales, but it must have been recognized as an obvious fact from the earlier times.

(vi) The angle subtended by a diameter of a circle at any point in the circumference is a right angle (Euc. III, 31). This appears to have been regarded as the most remarkable of the geometrical achievements of Thales, and it is stated that on inscribing a right-angled triangle in a circle he sacrificed an ox to the immortal gods. It has been conjectured that he may have come to this conclusion by noting that the diagonals of a rectangle are equal and bisect one another, and that therefore a rectangle can be inscribed in a circle. If so, and if he went on to apply proposition (i), he would have discovered that the sum of the angles of a right-angled triangle is equal to two right angles, a fact with which it is believed that he was acquainted. It has been remarked that the shape of the tiles used in paving floors may have suggested these results.

On the whole it seems unlikely that he knew how to draw a perpendicular from a point to a line; but if he possessed this knowledge, it is possible he was also aware, as suggested by some modern commentators, that the sum of the angles of any triangle is equal to two right angles.

As far as equilateral and right-angled triangles are concerned, we know from Eudemus that the first geometers proved the general property separately for three species of triangles, and it is not unlikely that they proved it thus. The area about a point can be filled by the angles of six equilateral triangles or tiles, hence the proposition is true for an equilateral triangle. Again, any two equal right-angled triangles can be placed in juxtaposition so as to form a rectangle, the sum of whose angles is four right angles; hence the proposition is true for a right-angled triangle. Lastly, any triangle can by split into the sum of two right-angled triangles by drawing a perpendicular from the biggest angle on the opposite side, and therefore again the proposition is true. The first of these proofs is evidently included in the last, but there is nothing improbable in the suggestion that the early Greek geometers continued to teach the first proposition in the form above given.

Thales wrote on astronomy, and among his contemporaries was more famous as an astronomer than as a geometrician. A story runs that one night, when walking out, he was looking so intently at the stars that he tumbled into a ditch, on which an old woman exclaimed, "How can you tell what is going on in the sky when you can't see what is lying at your own feet?"—an anecdote which was often quoted to illustrate the unpractical character of philosophers.

Without going in to astronomical details, it may be mentioned that he taught that a year contained about 365 days, and not (as is said to have been previously reckoned) twelve months of thirty days each. It is said that his predecessors occasionally intercalated a month to keep the seasons in their customary places, and if so they must have realized that the year contains, on the average, more than 360 days. There is some reason to think that he believed the earth to be a disc-like body floating on water. He predicted a solar eclipse which took place at or about the time he foretold; the actual date was either May 28, 585 B.C., or September 30, 609 B.C But though this prophecy and its fulfillment gave extraordinary prestige to his teaching, and secured him the name of one of the seven sages of Greece, it is most likely that he only made use of one of the Egyptian or Chaldaean registers which stated that solar eclipses recur at intervals of about 18 years 11 days.