**Problem 2: A mixture problem**

Part A

i)

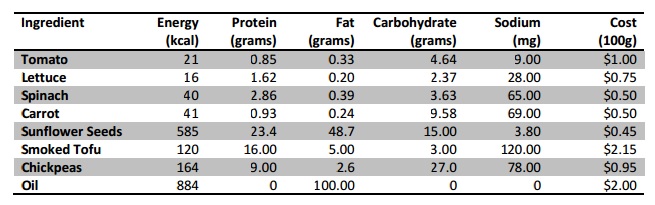
Each ingredient of the 8 ingredients in the salad will be assigned as if it is an array. The optimal solution will be a combination of the ingredients. Variables are:

* T = Tomato
* L = Lettuce
* S = Spinach
* C = Carrot
* SS = Sunflower Seeds
* ST = Smoked Tofu
* CP = Chickpeas
* O = Oil

Additionally, each ingredient will have an assigned variable for nutritional value as follows:

* P = Protein
* F = Fat
* C = Carbohydrates
* S = Sodium
* LG = Leafy Greens

Given nutritional content of the ingredients:



Our goal is to minimize the amount of calories but still meet a specific nutritional requirement. Each salad must contain:

* At least 2 but no more than 8 ingredients.
* At least 15 grams of protein
* At least 2 and at most 8 grams of fat
* At least 4 grams of carbohydrates
* At most 200mg of sodium
* At least 40% leafy greens by mass

Given the provided requirements for a salad we can derive some constraints for our linear program:

* P >= 15
* 2 <= F <=8
* C >= 4
* S <= .2
* LG >= .4(mass)

ii)

Using LINDO, we can determine the optimal solution for this program with the following code:

MIN 21T + 16L + 40S + 41C + 585SS + 120ST + 164CP + 884O

ST

0.85T + 1.62L + 2.86S + 0.93C + 23.4SS + 16ST + 9CP >= 15

0.33T + 0.20L + 0.39S + 0.24C + 48.7SS + 5ST + 2.6CP + 100O >= 2

0.33T + 0.20L + 0.39S + 0.24C + 48.7SS + 5ST + 2.6CP + 100O <= 8

4.64T + 2.37L + 3.63S + 9.58C + 15SS + 3ST + 27CP >= 4

9T + 28L + 65S + 69C + 3.8SS + 120ST + 78CP <= 200

0.4L + 0.4S 0.6T

0.6C

0.6SS

0.6ST

0.6CP

0.6O

>= 0

T >= 0

L >= 0

S >= 0

C >= 0

SS >= 0

ST >= 0

CP >= 0

O >= 0

END

Results of the code will be attached to this report titled Problem 2-A Lindo Results.txt.

iii)

The lowest calorie salad solution:

|  |  |  |  |
| --- | --- | --- | --- |
| Ingredient | Calories | Cost ($) | Nutrition |
| Lettuce (1.220836 units) | 20 | $0.92 | 1.98g Protein  .24g Fat  2.89g Carbohydrates  34mg Sodium |
| Smoked Tofu (.813890 units) | 98 | $1.75 | 13.02g Protein  4.07g Fat  2.44g Carbohydrates  98mg Sodium |
| Totals: |  |  |  |
| Lettuce and Smoked Tofu | 118 | $2.67 | 15g Protein  4.31g Fat  5.33g Carbohydates  132mg Sodum |

As you can see, this combination meets all of our requirements outlined above.

Part B

i)

The goal for this problem is to minimize cost associated with the salad, while still meeting the minimum nutritional requirements. We will reuse the same variables and nutritional table as in Part A for this problem, so the constraints will be:

* P >= 15
* 2 <= F <=8
* C >= 4
* S <= .2
* LG >= .4(mass)

ii)

Again we will be using LINDO for this solution:

MIN 1T + 0.75L + 0.5S + 0.5C + 0.45SS + 2.15ST + .95CP + 2O

ST

0.85T + 1.62L + 2.86S + 0.93C + 23.4SS + 16ST + 9CP >= 15

0.33T + 0.20L + 0.39S + 0.24C + 48.7SS + 5ST + 2.6CP + 100O >= 2

0.33T + 0.20L + 0.39S + 0.24C + 48.7SS + 5ST + 2.6CP + 100O <= 8

4.64T + 2.37L + 3.63S + 9.58C + 15SS + 3ST + 27CP >= 4

9T + 28L + 65S + 69C + 3.8SS + 120ST + 78CP <= 200

0.4L + 0.4S - 0.6T - 0.6C - 0.6SS - 0.6ST - 0.6CP - 0.6O >= 0

T >= 0

L >= 0

S >= 0

C >= 0

SS >= 0

ST >= 0

CP >= 0

O >= 0

END

Results will be attached to this report as Problem 2-B Lindo Results.txt.

iii)

Given the results, the following is the solution for a low cost salad:

|  |  |  |  |
| --- | --- | --- | --- |
| Ingredient | Calories | Cost ($) | Nutrition |
| Spinach (1.525128 units) | 61 | $0.76 | 4.36g Protein  0.60g Fat  5.54g Carbohydrates  99mg Sodium |
| Sunflower Seeds (0.103289 units) | 60 | $0.05 | 2.42g Protein  5.02g Fat  1.55g Carbohydrates  < 1mg Sodium |
| Chickpeas (0.913462 units) | 150 | $0.87 | 8.22g Protein  2.38g Fat  24.66g Carbohydrates  71mg Sodium |
| Totals: |  |  |  |
| Spinach, Sunflower Seeds, and Chickpeas | 270 | $1.68 | 15g Protein  8g Fat  31.75g Carbohydrates  171mg Sodium (rounded up) |

As we can see, these results meet our requirements for being low cost and also meeting the specified nutritional values.

Part C

i)

Suppose you want to sell a salad that is both low calorie and low cost. In this case, we want to sell a salad for $5.00, and make a profit of at least $3.00. However, you can advertise the salad as low calorie and potentially sell more if it is under 250 calories.

The objective for this problem is to optimize both cost and minimize calories. We believe advertising a salad as low calorie will sell more and therefore generate more revenue. We will use Lindo again for this:

MIN 1T + 0.75L + 0.5S + 0.5C + 0.45SS + 2.15ST + .95CP + 2O

ST

21T + 16L + 40S + 41C + 585SS + 120ST + 164CP + 884O <= 250

0.85T + 1.62L + 2.86S + 0.93C + 23.4SS + 16ST + 9CP >= 15

0.33T + 0.20L + 0.39S + 0.24C + 48.7SS + 5ST + 2.6CP + 100O >= 2

0.33T + 0.20L + 0.39S + 0.24C + 48.7SS + 5ST + 2.6CP + 100O <= 8

4.64T + 2.37L + 3.63S + 9.58C + 15SS + 3ST + 27CP >= 4

9T + 28L + 65S + 69C + 3.8SS + 120ST + 78CP <= 200

0.4L + 0.4S - 0.6T - 0.6C - 0.6SS - 0.6ST - 0.6CP - 0.6O >= 0

T >= 0

L >= 0

S >= 0

C >= 0

SS >= 0

ST >= 0

CP >= 0

O >= 0

END

Results will be attached to this report as Problem 2-C Lindo Results.txt.

ii)

The low cost, 250 calorie salad solution:

|  |  |  |  |
| --- | --- | --- | --- |
| Ingredient | Calories | Cost ($) | Nutrition |
| Spinach (1.426983 units) | 57 | $0.71 | 4.08g Protein  0.56g Fat  5.18g Carbohydrates  93mg Sodium |
| Sunflower Seeds (0.101109 units) | 59 | $0.05 | 2.37g Protein  4.92g Fat  1.52g Carbohydrates  <1mg Sodium |
| Smoked Tofu (0.128708 units) | 15 | $0.28 | 2.06g Protein  0.64g Fat  0.39g Carbohydrates  15mg Sodium |
| Chickpeas (0.721505 units) | 118 | $0.69 | 6.49g Protein  1.88g Fat  19.48g Carbohydrates  56mg Sodium |
| Totals |  |  |  |
| Spinach, Sunflower Seeds, Smoked Tofu, and Chickpeas | 249 | $1.73 | 15g Protein  8g Fat  26.57g Carbohydrates  165mg Sodium |

iii)

For this solution we emphasized the 250 calorie cap so that the salad can be marketed as low calorie to achieve more sales. However, you could also emphasize the $3.00 profit and get a different answer, but the salad would likely not be able to be sold as "low calorie". However, if you were to sell the salad in our solution, you could still market it as a low calorie salad and make greater than then $3.00 profit so we believe this is the best solution.