

# COURSE "AUTOMATED PLANNING: THEORY AND PRACTICE"

## CHAPTER 06: THE PARTIAL ORDER CAUSAL LINK SEARCH SPACE

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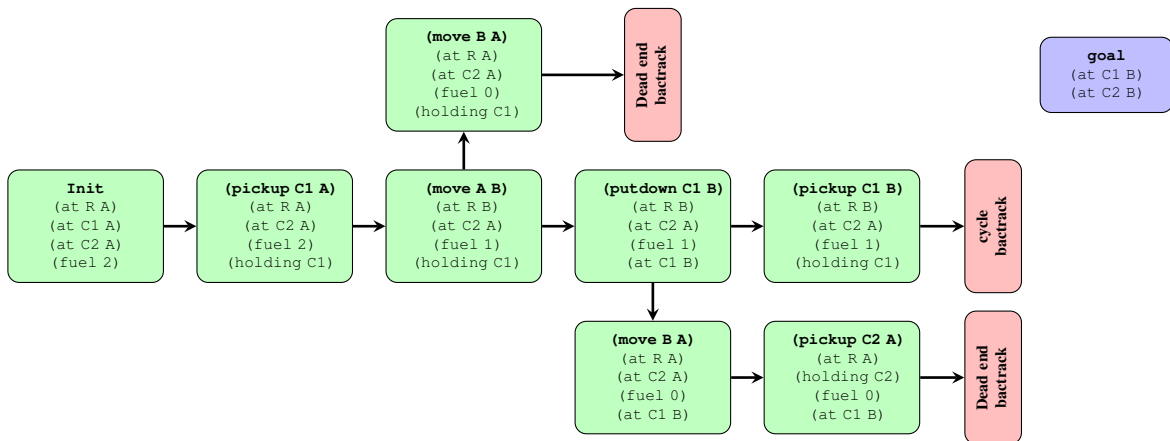
# MOTIVATING PROBLEM

- Let's consider a simple planning problem:
  - Two crates C1, C2, two positions A, B, and one robot R
  - The robot:
    - can carry up to two crates
    - can move between locations, consuming one unit of fuel
  - Initially crates and robot are at A, and the robot has 2 unit of fuel
  - Both crates shall be moved to B

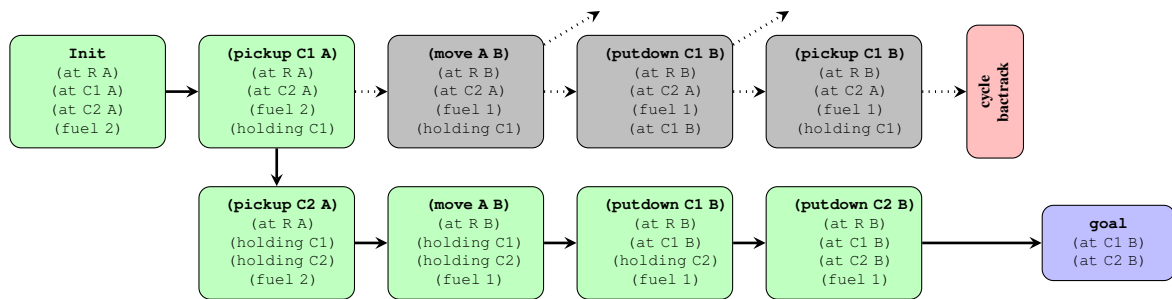


Let's see what forward-chaining planning might do (depending on heuristics)...

# MOTIVATING PROBLEM: FORWARD SEARCH



# MOTIVATING PROBLEM: FORWARD SEARCH

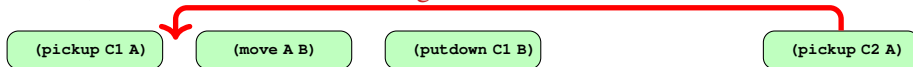


# MOTIVATING PROBLEM: OBSERVATIONS

- Most of the actions added before backtracking were **useful** and **necessary**



- At first, we added them in the **wrong order**



- Forward** and **backward** planning **commit** immediately to action order!
  - Puts each action in its **final place** in the plan!
- State space **heuristics** must be "smart enough" to tell us:
  - Which actions are **useful**
  - When** to add them in the plan

What if we could **"rearrange"** actions?

# FIRST STEP: INSERTION

- Sequences with arbitrary insertion: Useful?

- Most of the actions added before backtracking were **useful** and **necessary**

(pickup C1 A)

(move A B)

(putdown C1 B)

- Realize you need another one...

(pickup C2 A)

How to decide which action to *insert*, if not at the end?

- Make a space...

(pickup C1 A)

(move A B)

(putdown C1 B)

How to decide where to insert it?

- ... and place the action there

(pickup C1 A)

(pickup C2 A)

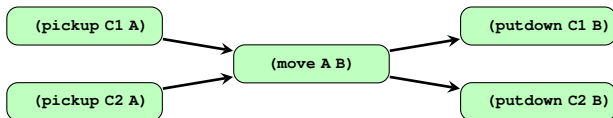
(move A B)

(putdown C1 B)

How to check that the "**old**" preconditions remain satisfied?

## SECOND STEP: PARTIAL ORDER

- If we must deal with this complexity:
  - We can "get more from the same price"
- Let's skip sequences completely - a plan could be **partially** ordered



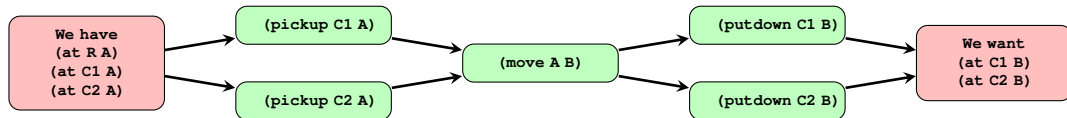
- A set of **actions**  $A = \{a_1, a_2, a_3, \dots\}$
- A set of **precedence constraints**  $\{a_1 < a_2, a_1 < a_3, \dots\}$ 
  - $a_1$  must finish before  $a_2$  starts,  $a_1$  must finish before  $a_3$  starts
  - We represent them graphically with **solid arrows**

How do we generate such plans?



# POCL: INTRODUCTION

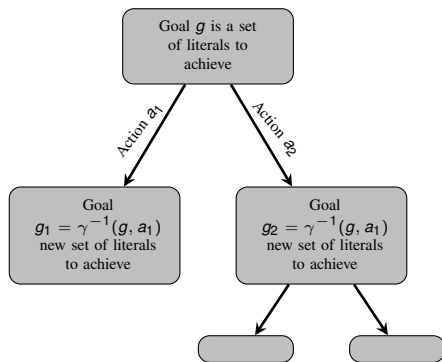
- **Partial Order Causal Link (POCL) planning**
  - Use a partial order as described before
    - Not when executing the plan
    - Only to **delay commitment** to action ordering
  - As in backward search:
    - Add **useful** actions to achieve necessary conditions
    - Keep track of what **remains** to be achieved
    - Insert actions "**at any point**" in the plan



More sophisticated "bookkeeping" required!

# POCL: COMPARISON TO BACKWARD SEARCH

- Search tree for backward search (as seen earlier)



The goal is a set of literals – simple!

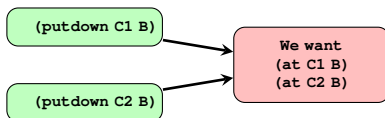
Every step takes you to a new set of literals to achieve

From a search node, you know how to reach the goal using a sequence of actions!

A search node [2] can simply be a goal set!

# POCL: COMPARISON WITH BACKWARD SEARCH

- In POCL planning there is no sequence – and no clear "before" relation!



The goal is a set of literals – simple!

But no set of literals can describe what must be true **before** e.g. (putdown C1, B) ...

... because we could add a new action "in parallel" ...

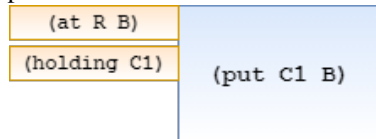
... or even **between** (putdown C1 B) and the goal!

- There are consequences for the POCL plan structure **and** the node plan structure...

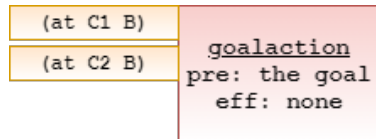
# POCL: CONDITIONS, GOAL ACTION

- ... must keep track of **individual propositions to be achieved**
  - Throughout the plan – not a single state  $g_1 = \gamma^{-1}(g, a_1)$
  - May come from **preconditions** of every action in the plan

Notation chosen:  
Preconditions on the left/top side



- May come from **problem goal** as in backward search
  - Trick: Use a **uniform representation**
  - Add a "fake" **goal action** to every plan with the goals as preconditions!

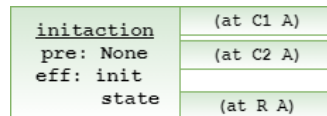
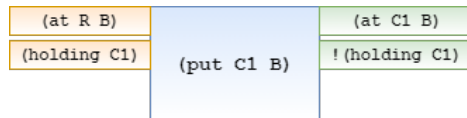


# POCL: EFFECTS, INITIAL ACTION

- Must keep track of **individual propositions that are achieved**
  - Throughout the plan - not from a single *relevant* action
  - May come from **effects** of every action in the plan

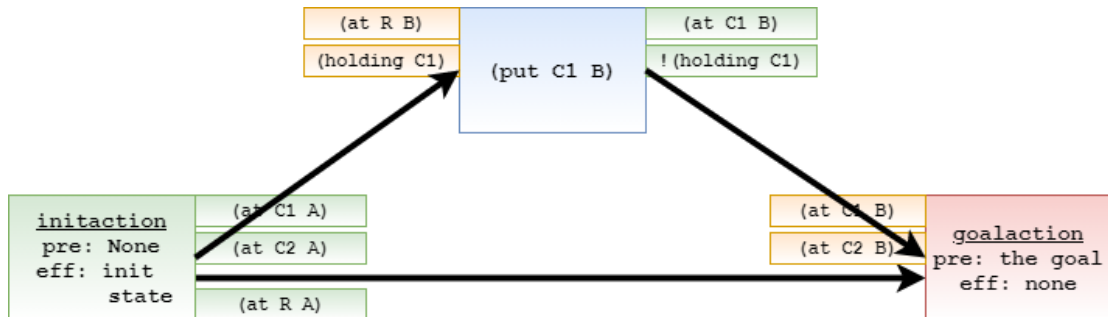
Notation chosen:  
Effects on the right/bottom side

- May come from **initial state**
  - Trick: Use a **uniform representation**
  - Add a "fake" **initial action** with the initial state as effect



# POCL: PRECEDENCE CONSTRAINTS

- Plan structure so far



# POCL: CAUSAL LINKS

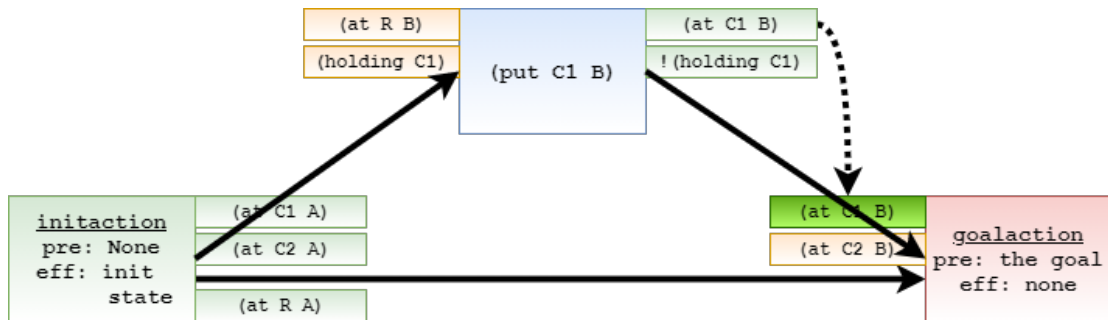
- Let's keep track of **which** actions achieves **which** precondition: **Causal links**

Causal links (dashed):

(at C1 B) must **remain true** between end of (put C1 B) and the beginning of goalaction.

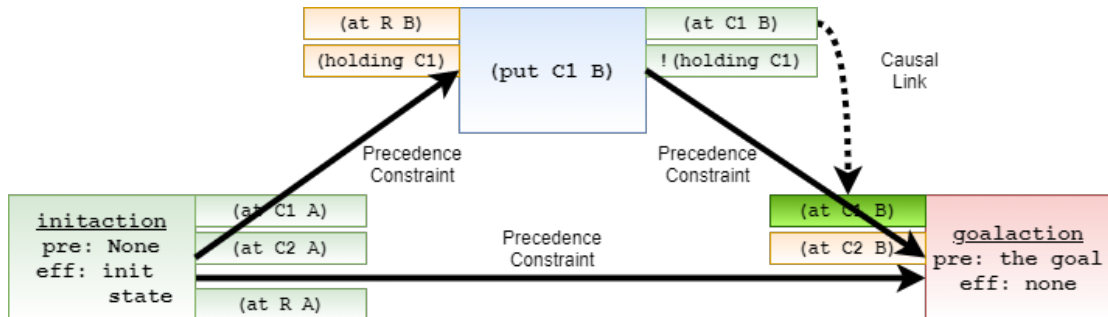
No one must delete it!

Important for *threat management* (later)



# POCL: PARTIAL-ORDER PLANS

- A ground **partial-order plan** consists of:
  - A set of **actions**
  - A set of **precedence constraints**  $a \rightarrow b$  ( $a$  must precede  $b$ )
  - A set of **causal links**  $a \xrightarrow{p} b$  - action  $a$  establishes the preconditions  $p$  needed by  $b$

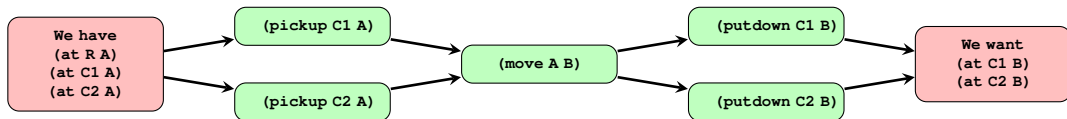




# PARTIAL-ORDER SOLUTIONS

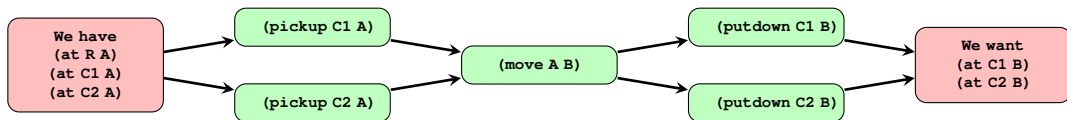
- Original motivation: **performance**

- A partial-order plan is a **solution** iff **all sequential** plans satisfying the ordering are solutions
- A partial-order plan is a **executable** iff **all sequential** plans satisfying the ordering are solutions
  - (pickup C1 A); (pickup C2 A); (move A B); (putdown C1 B); (putdown C2 B)
  - (pickup C2 A); (pickup C1 A); (move A B); (putdown C1 B); (putdown C2 B)
  - (pickup C1 A); (pickup C2 A); (move A B); (putdown C2 B); (putdown C1 B)
  - (pickup C2 A); (pickup C1 A); (move A B); (putdown C2 B); (putdown C1 B)



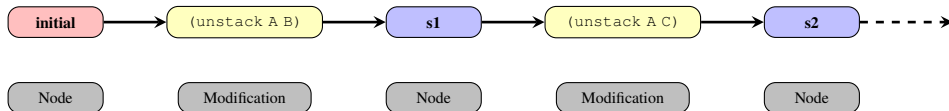
# PARTIAL-ORDER SOLUTIONS

- Can be **extended** to allow **concurrent action execution**
  - Requires a **new** formal model!
    - The so far considered transition model *does not define* what happens if C1 and C2 are picked up simultaneously

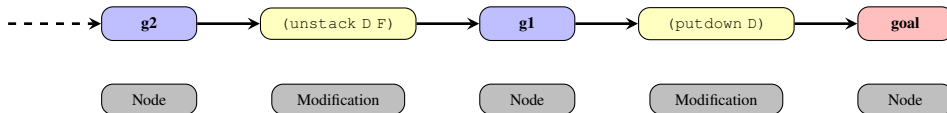


# CONTEXT: FORWARD, BACKWARD

- **Forward Search:** a search node is a "current state"

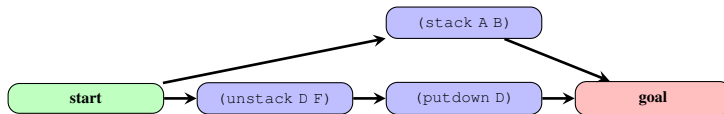


- **Backward Search:** a search node is a "current goal"



# NO CURRENT STATE DURING SEARCH!

- With **partial-order plans**: No "current" state or goal!
  - What's true after (stack A B) in example below?
    - **Depends** on the order in which the **other** actions are executed!
    - **Changes** if we insert **new** actions **before** (stack A B)!

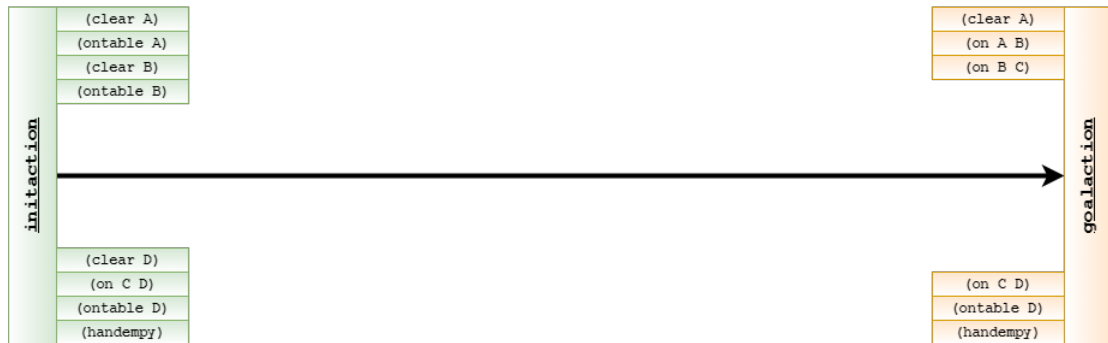


A search node cannot correspond to a state or to a goal!

# SEARCH NODES ARE PARTIAL ORDER PLANS

- [1] Each node must contain *more information: the entire plan!*
  - [2] The **initial** search node contains the **initial plan**
    - The special **initial action** and **goal action**
    - A single **precedence constraint**

This is one form of  
"plan-space" planning!



# BRANCHING RULE

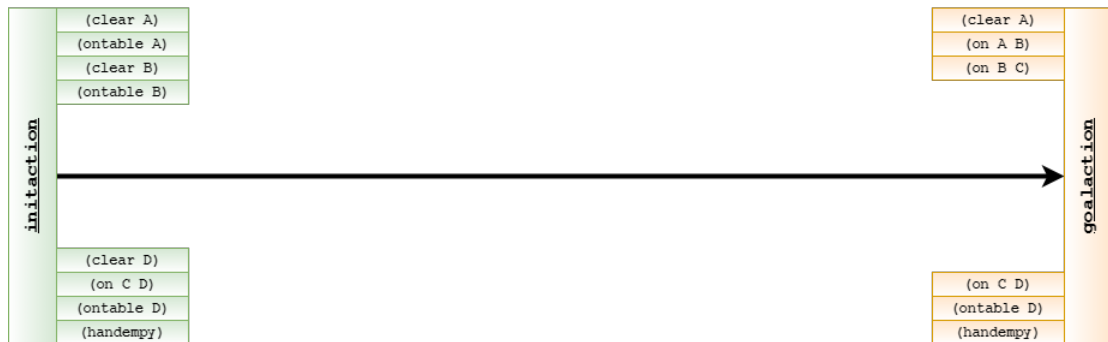
- [3] We need a **branching rule**!

- Forward planning:
- Backward planning:
- POCL:

One successor per action **applicable** is  $s$

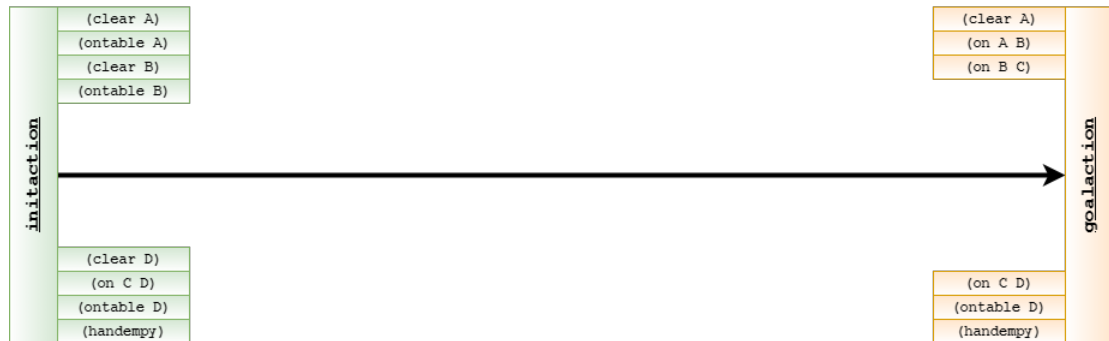
One successor per action **relevant** to  $g$

???



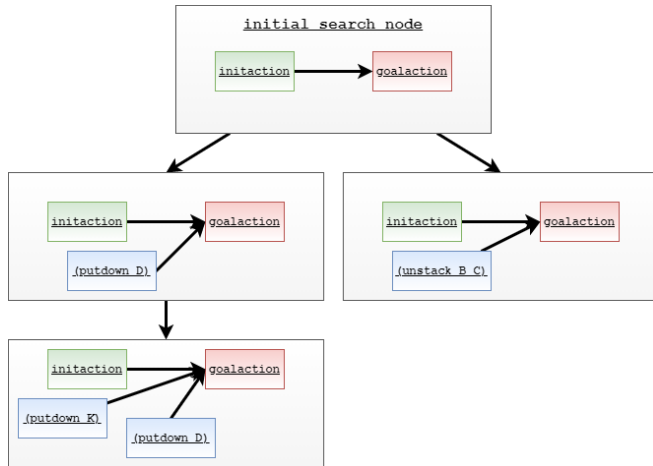
### [3] BRANCHING RULE

- Identify specific **reasons for modifying the plan**, called **flaws** (i.e. **todos**)
  - 1) **Open goal**: we have not decided *how* to achieve a precondition (E.g. (clear A))
  - 2) **Threat**: An action may *interfere* with another
- One successor for each different way of **repairing a flaw**



# SEARCH SPACE

- [6] Use search strategies, backtracking, heuristics, ... to search [this](#) space!



[4] Solution iff there are no flaws  
(We will see later how to do it)

[5] Plan extraction: pick any sequential  
order consistent with the precedence  
constraints

(putdown D); (putdown K)  
(putdown K); (putdown D)



# FLAWS

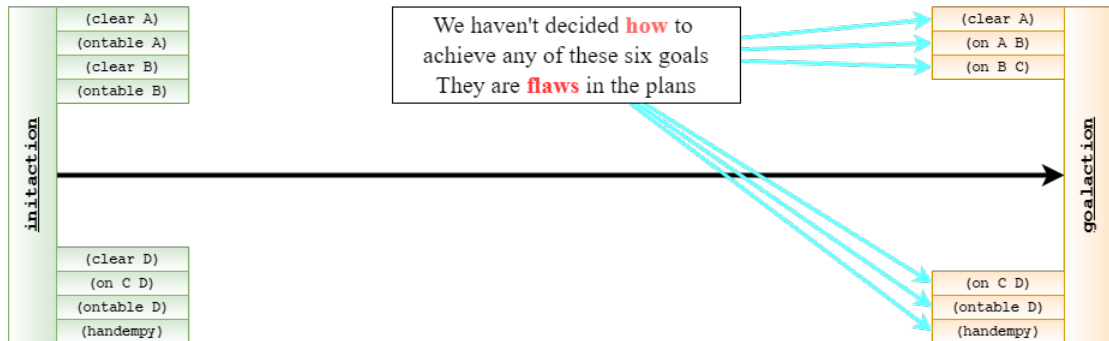
- Flaw, *noun*:
  - a feature that mars the perfection of something; defect; fault: beauty without flaw; the flaws in our plan.
  - a defect impairing legal soundness or validity.
  - a crack, break, breach, or rent.
- Flaw, in POCL planning:
  - Something we **need to take care of to complete the plan**
  - Technical definition: An *open goal* or a *threat*
- Not:
  - Something that has "gone wrong"!
  - A problem during planning
  - A mistake in the final solution
  - ...

# FLAW TYPES

- Open Goals
- Threats

# FLAW TYPE 1: OPEN GOALS


- An action  $a$  has precondition  $p$  with no incoming causal link



(clear A) is already true in  $s_0$ , but there is no causal link...  
 Adding one causal link from  $s_0$  means (clear A) **must never be deleted**!  
 We need other alternatives: delete (clear A), then re-achieve it for goalaction..

# FLAW TYPE 1: OPEN GOALS

- To **resolve** an open goal
  - Find an action  $b$  that cause  $p$ 
    - Can be a **new** action
    - Can be an action **already** in the plan, if we can **make** it precede  $a$
  - Add a **causal link**



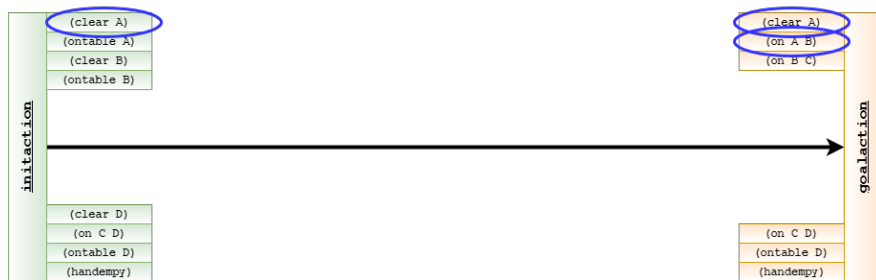
Partial order! This was not possible in backward search...

## Essential:

Even if there **is already** an action that causes  $p$ ,  
you can still add a **new** action that **also** causes  $p$ !

# RESOLVING OPEN GOALS

- We can chose to **find support for (clear A)** 8 successors!
  - From initaction; from a new (unstack B A), (unstack C A), or (unstack D A); from a new (stack A B), (stack A C), (stack A D), or (putdown A)
- We can chose to **find support for (on A B)** +1 successor
  - Only from a new instance of (stack A B)
- ...

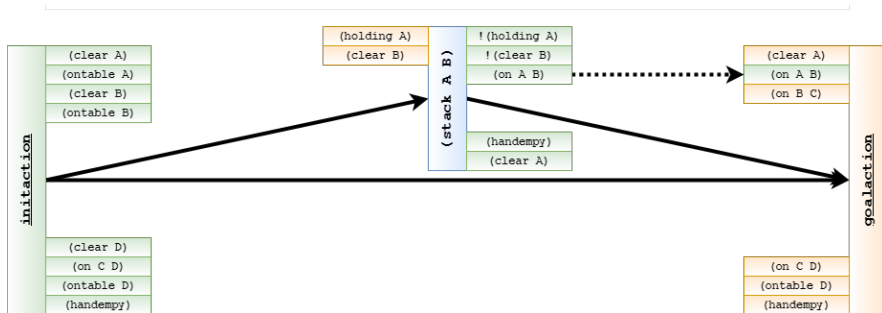


## RESOLVING OPEN GOALS (CONT.)

- Suppose we add (stack A B) to achieve (on A B)
  - Must add a causal link for (on A B) (dashed lines)

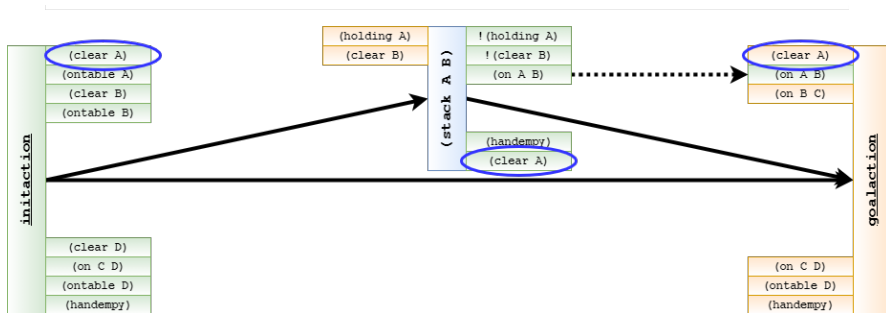
This instance of (stack A B) is responsible for achieving (on A B) for the goalaction

- Must also add precedence constraints
- Looks totally ordered: we actually have only one "real" action!



## RESOLVING OPEN GOALS (CONT.)

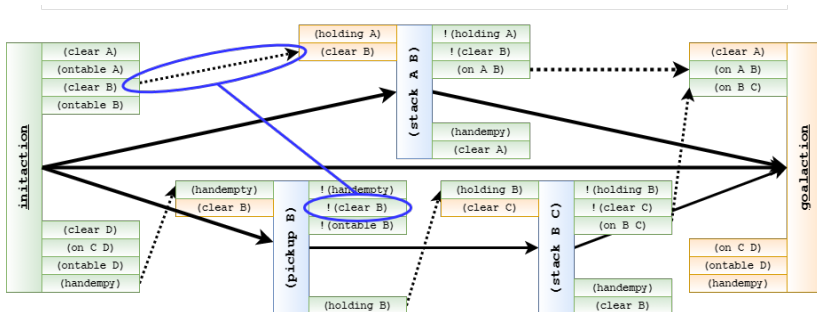
- Now we have 7 open goals
  - We can choose to find support for (clear A):
    - From `initaction`; from the instance of (stack A B) just added; from a new instance of (stack A B), (stack A, C), (stack A D), or (putdown A); from a new instance of (unstack B A), (unstack C A), (unstack D A)
  - ...



## FLAW TYPE 2: THREATS

### • A threat against a causal link

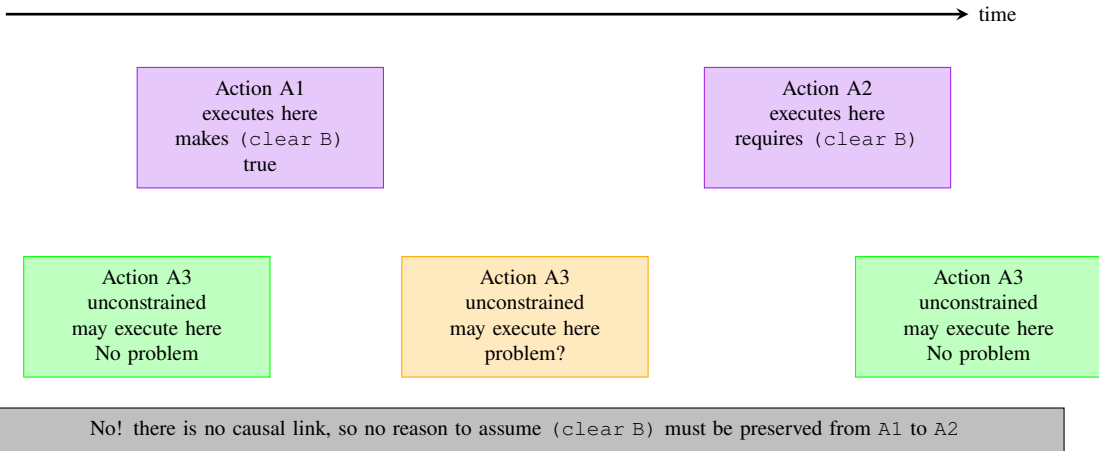
- initiation **should achieve** (clear B) for (stack A B) - there is a **causal link**
- (pickup B) **deletes** (clear B), and may occur **between** initiation and (stack A B)
- We cannot be certain that (clear B) still holds when (stack A B) starts!





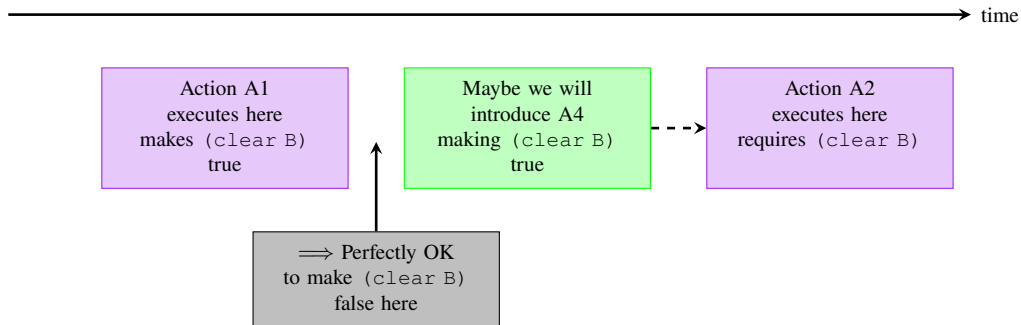
## FLAW TYPE 2: THREATS

- A threat against a causal link



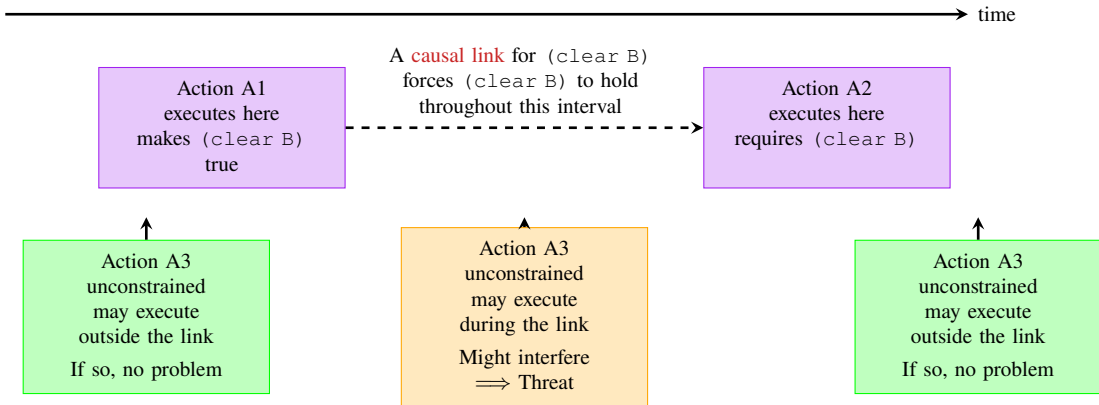
## FLAW TYPE 2: THREATS

- Why no threats without causal links?



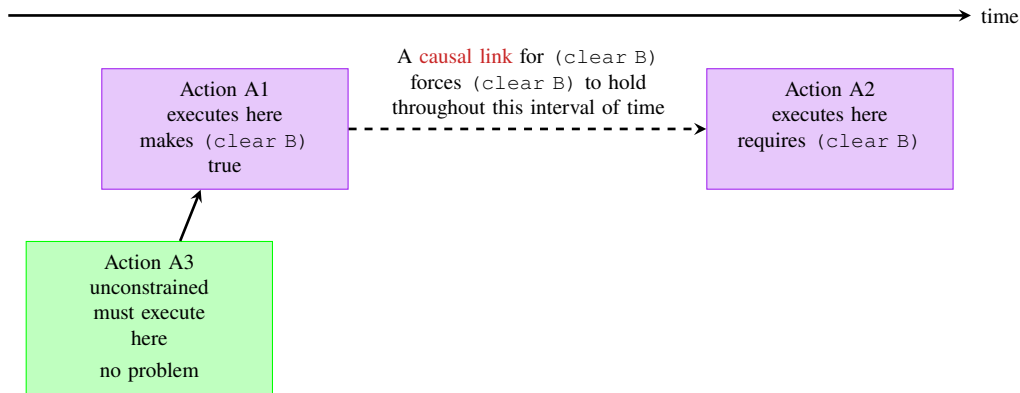
## FLAW TYPE 2: THREATS

- But when we have a causal link:

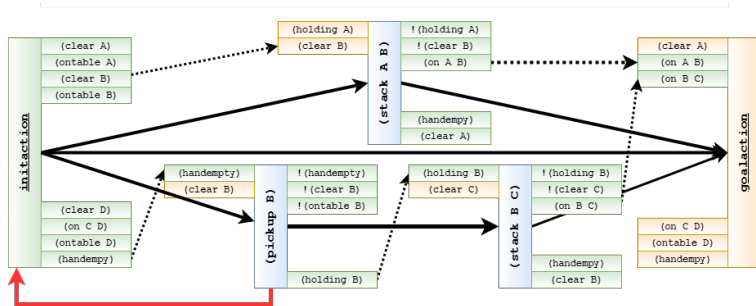


# RESOLVING THREATS: RULE 1

- The action that **disturbs** the causal link is placed **before** the action that **support/achieves** the precondition
  - Only possible if the resulting partial order is consistent (acyclic)!



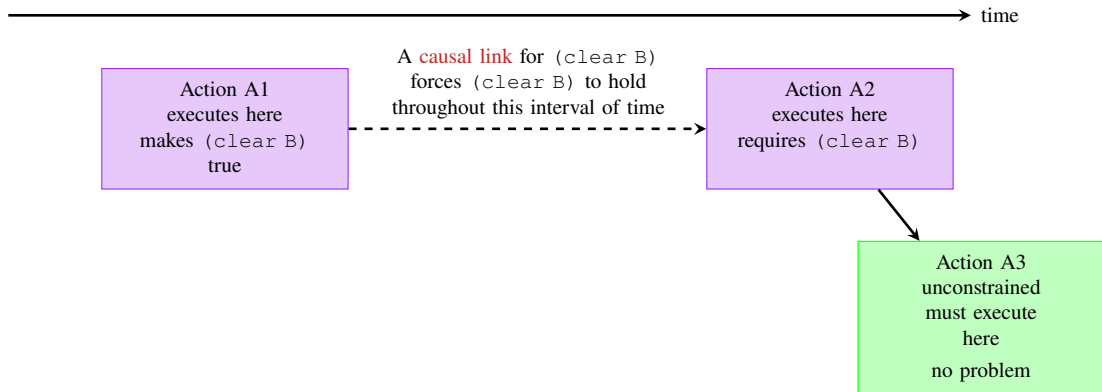
# RESOLVING THREATS



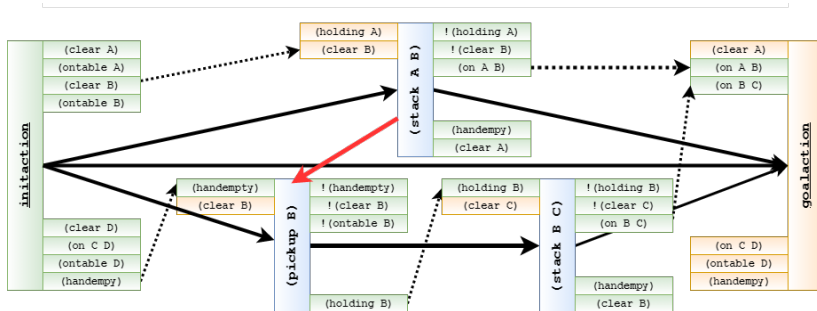
In this case not consistent! (a cycle is created)

## RESOLVING THREATS: RULE 2

- The action that **disturbs** the causal link is placed **after** the action that **requires** the precondition
  - Only possible if the the resulting partial order is consistent (acyclic)!

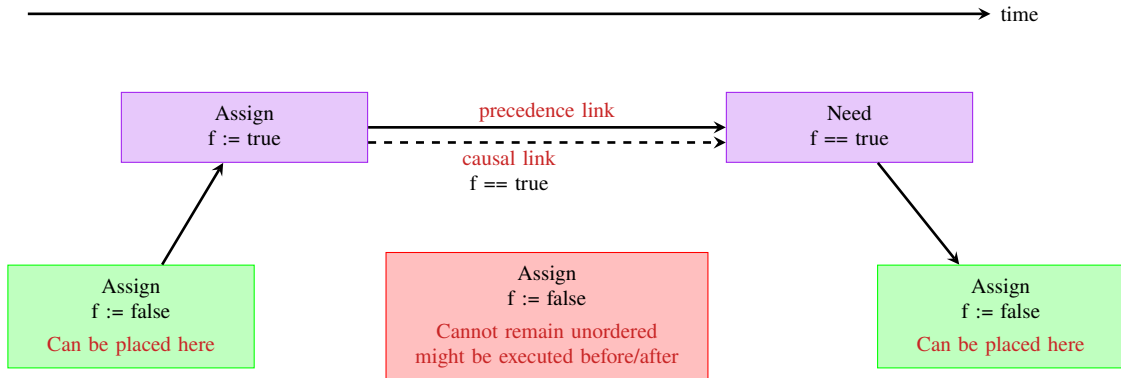


# RESOLVING THREATS



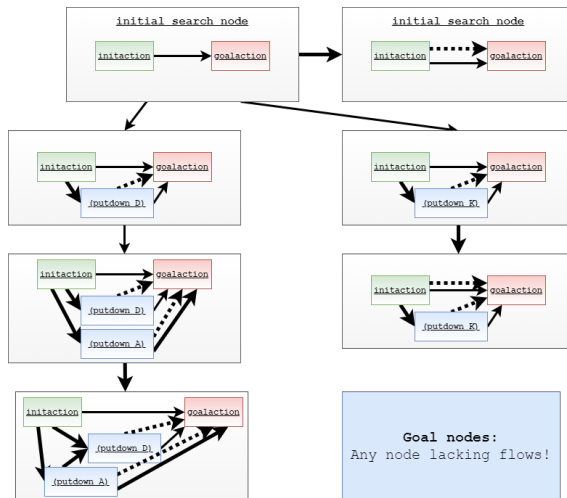
In this case it works! (There are no cycles)

# RESOLVING THREATS: SUMMARY





# POCL SEARCH SPACE



# PLANNING AS SEARCH

```

function SEARCH(problem)
  initial-node  $\leftarrow$  MAKE-INITIAL-NODE(problem)
  open  $\leftarrow$  {initial-node}
  while (open  $\neq \emptyset$ ) do
    node  $\leftarrow$  SEARCH-STRATEGY-REMOVE-FROM(open)
    if IS-SOLUTION(node) then
      return EXTRACT-PLAN-FROM(node)
    end if
    for each newnode  $\in$  SUCCESSORS(node) do
      open  $\leftarrow$  open  $\cup$  {newnode}
    end for
  end while
  return Failure
end function

```

→ [2]  
 → [6]  
 → [4]  
 → [5]  
 → [3] *All ways of resolving some flaw*  
 → *Expanded the entire search space without finding a solution*

# POCL PLANNING: POSSIBLE FORMULATION (SOUND/COMPLETE)

**function** SEARCH(problem)

initial-node  $\leftarrow$  MAKE-INITIAL-NODE(problem.init,problem.goal)

$\rightarrow$  [2]

open  $\leftarrow$  {initial-node}

**while** (open  $\neq \emptyset$ ) **do**

$\pi \leftarrow$  SEARCH-STRATEGY-REMOVE-FROM(open)

$\rightarrow$  [6]

flaws  $\leftarrow$  OPENGOALS( $\pi$ )  $\cup$  THREATS( $\pi$ )

**if** flaws =  $\emptyset$  **then**

$\rightarrow$  Can prove:  $\pi$  is a solution if there are no remaining flaws

**return**  $\pi$

$\rightarrow$  [5] Returns a *partially ordered* solution plan. Any *total ordering* is a plan achieving the goal!

**end if**

select any flaw  $\varphi \in$  flaws

$\rightarrow$  One flaw chosen!

resolvers  $\leftarrow$  FINDRSOLVERS( $\varphi, \pi$ )

$\rightarrow$  May be the empty set

**for each** r  $\in$  resolvers **do**

$\pi' \leftarrow$  REFINER(r,  $\pi$ )

$\rightarrow$  Actually apply the resolver

open  $\leftarrow$  open  $\cup$  { $\pi'$ }

$\rightarrow$  But all resolvers must be tested...

**end for**

**end while**

**return** Failure

$\rightarrow$  Expanded the entire search space without finding a solution

**end function**

# POCL: SUCCESSORS

## WE SAID: "EVERY FLAW LEADS TO SUCCESSORS"

- It is **sufficient** to try **one** (**any**) flaw to resolve!
- Testing other flaw will be redundant!
  - Every flaw has to be resolved
  - Choosing the flaw *later* cannot help us resolve it: all possibilities already exists
  - Choosing the flaw *later* cannot help us resolve some other flaw

**select any** flaw  $\varphi \in \text{flaws}$   
 resolvers  $\leftarrow \text{FINDRSOLVERS}(\varphi, \pi)$

Enables the use of  
 heuristics to **select flaws**  
 and to **prioritize open nodes**

## WE MUST "TEST" DIFFERENT RESOLVERS

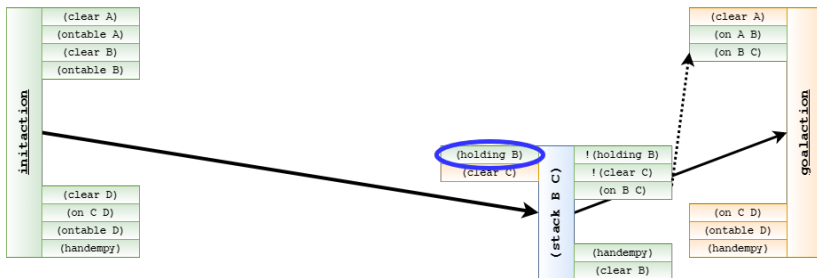
- Choosing one resolver can prevent other problems resolutions
- Open goal: Use action A or action B?
- Threat: Which order to choose?

**for each**  $r \in \text{resolvers}$  **do**  
      $\pi' \leftarrow \text{REFINE}(r, \pi)$   
      $\text{open} \leftarrow \text{open} \cup \{\pi'\}$   
**end for**

# PARTIAL INSTANTIATION

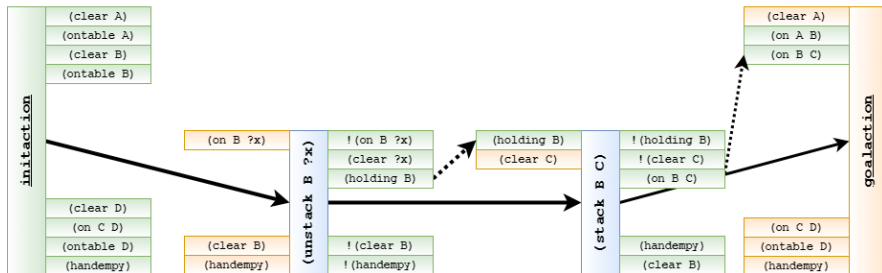
Suppose in the example we want to achieve (holding B)

- **Ground** search generates **many** alternatives
  - Add (unstack B A), (unstack B F), (unstack B G), ...
  - Add (pickup B)
- **Lifted** search generates two **partially instantiated** alternatives
  - Add (unstack B ?x)
  - Add (pickup B)



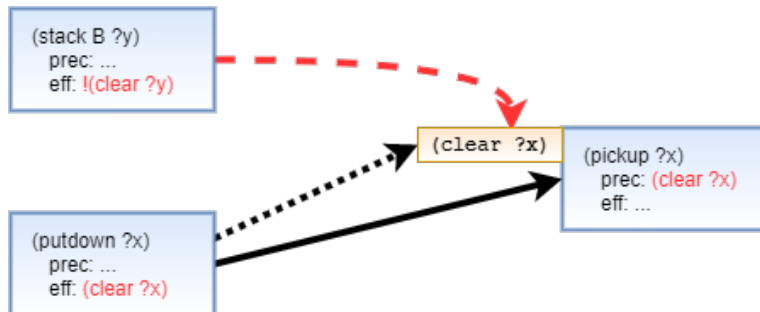
# LIFTED PARTIAL-ORDER PLANS

- A set of **possibly unground actions**
- A set of **precedence constraints**:  $a$  must precede  $b$
- A set of **causal links**:  $a$  establishes precondition  $p$  needed by  $b$
- A set of **binding constraints**
  - equality constraints:  $v_1 = v_2$  or  $v_1 = c$
  - inequality constraints:  $v_1 \neq v_2$  or  $v_1 \neq c$



## RESOLVING THREATS: ALTERNATIVE APPROACH

- For partially uninstantiated actions, we may find **potential** threats
  - $(\text{stack } B \ ?y)$  **may threaten** the causal link, but only if  $?x = ?y$
  - Can be resolved adding a constraint:  $?x \neq ?y$



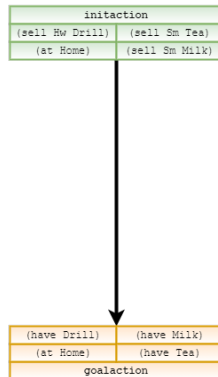
# EXAMPLE

- Taken and adapted from Russell and Norvig [4]
  - **Operator:** (go ?from ?to)  
pre: (at ?from)  
eff: (at ?to),  $\neg$  (at ?from)
  - **Operator:** (buy ?product ?store)  
pre: (at ?store), (sell ?store ?product)  
eff: (have ?product)
  - **Initial state**  
(at Home), (sell Hws Drill), (sell Sm Milk), (sell Sm Tea)
  - **Goal**  
(at Home), (have Drill), (have Milk), (have Tea)



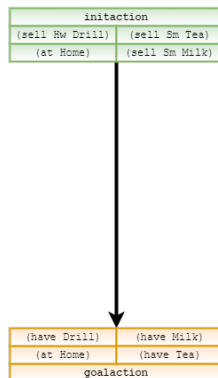
# EXAMPLE (1)

- Initial plan: `initaction`, `goalaction`, and a precedence constraint



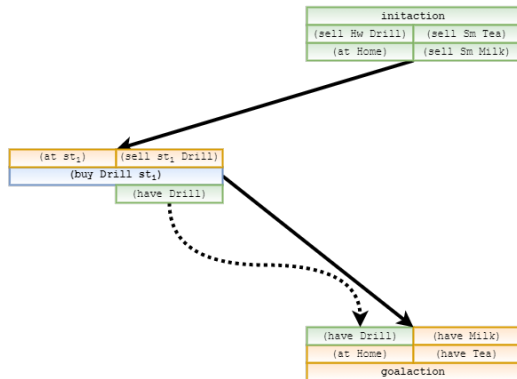
## EXAMPLE (2)

- Four **flaws** exists: open goals
  - The heuristics suggests to resolve (have Drill) first



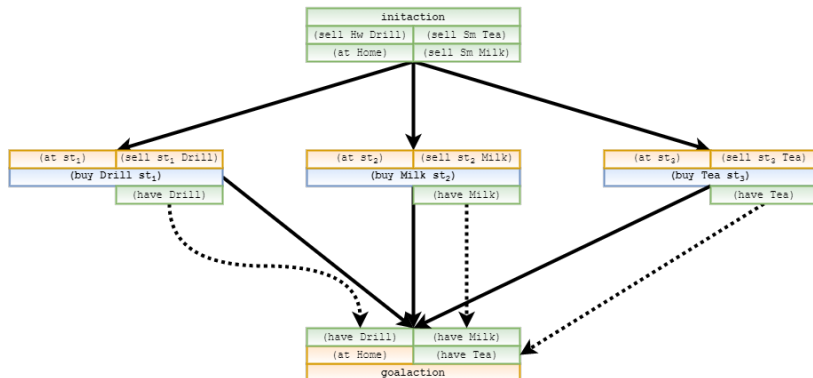
## EXAMPLE (3)

- No action in plan achieves (have Drill), but (buy ?p ?s) achieves (have ?p)
  - Partially instantiate (buy Drill ?s) (ignoring where to buy)



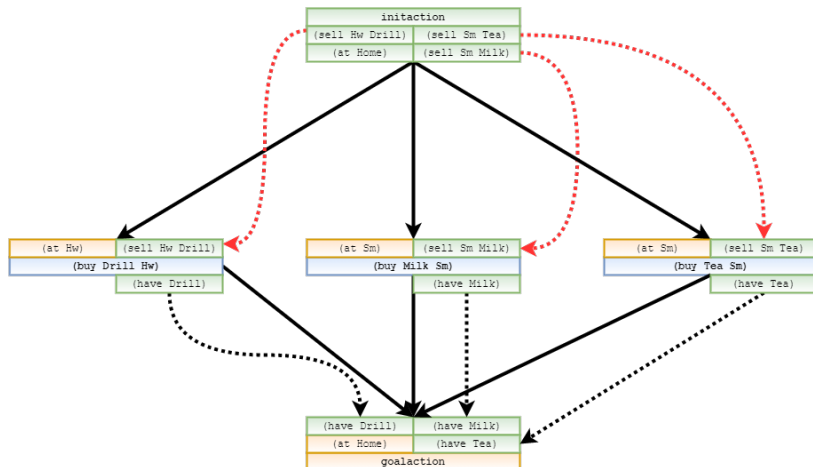
## EXAMPLE (4)

- First three refinements: the possible ways to achieve (have ?p) preconditions
  - We do not care the order in which to buy things!



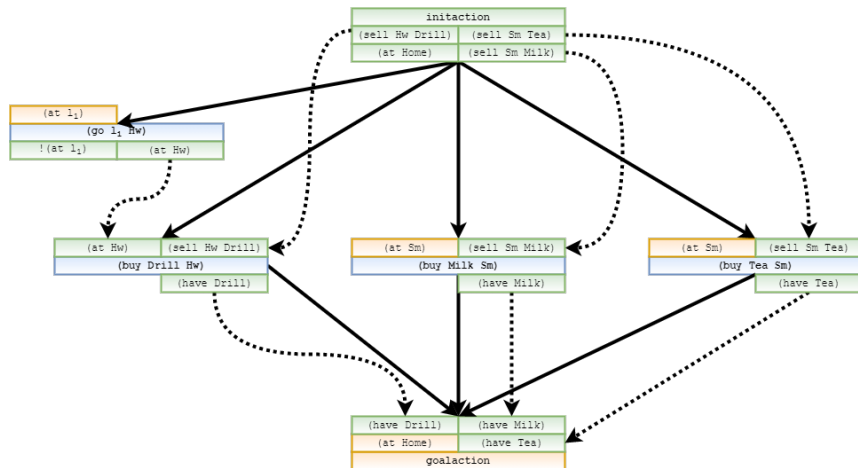
## EXAMPLE (5)

- Three more refinements: no action causes (sell ?p) - except initaction
  - $\Rightarrow$  use it for support



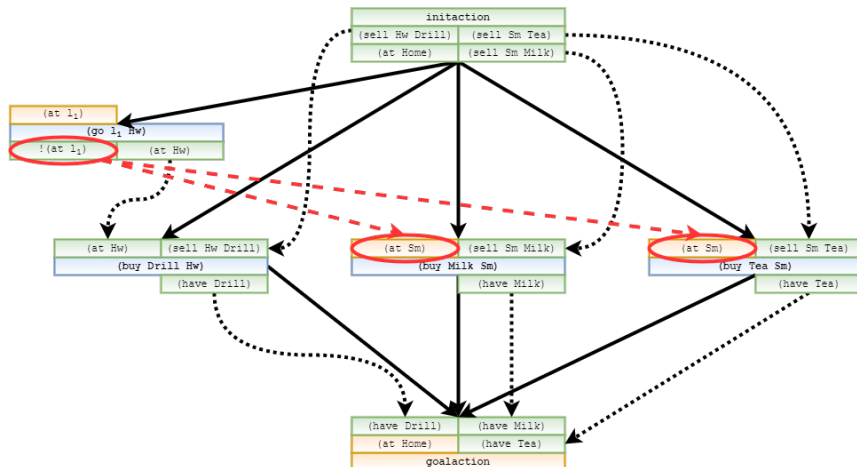
## EXAMPLE (6)

- To establish (at Hws): must go there from somewhere



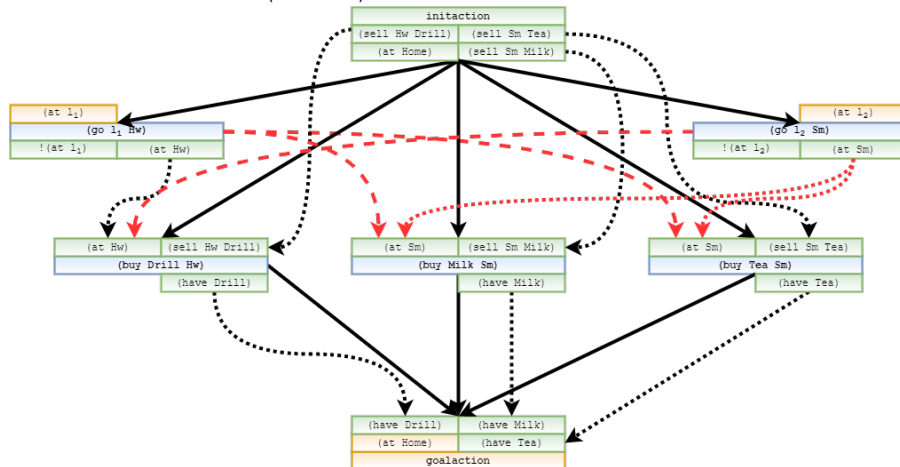
## EXAMPLE (7)

- Does  $\neg(\text{at } l_1)$  threaten  $(\text{at } S_m)$ ?
  - No! Only a causal link to  $(\text{at } S_m)$  can be threatened!



## EXAMPLE (8)

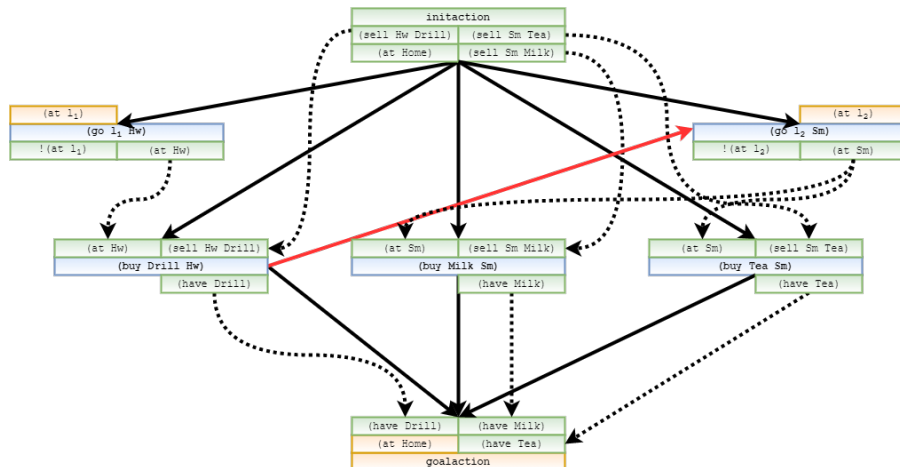
- To establish  $(at\ Sm)$ : must go there from somewhere  $\implies$  mutual threats...
- Let's use same action for both  $(at\ Sm) \implies$  even more threats – deal with them now or wait!





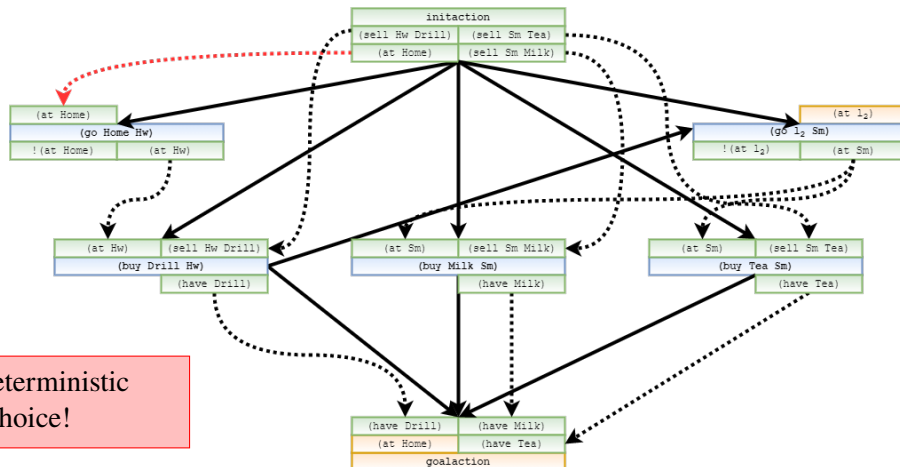
## EXAMPLE (9)

- How to resolve the threat to (at Hw)?: Make (buy Drill) precede the (go  $l_2$  Sm)
- Also happens to resolve the other two threats!



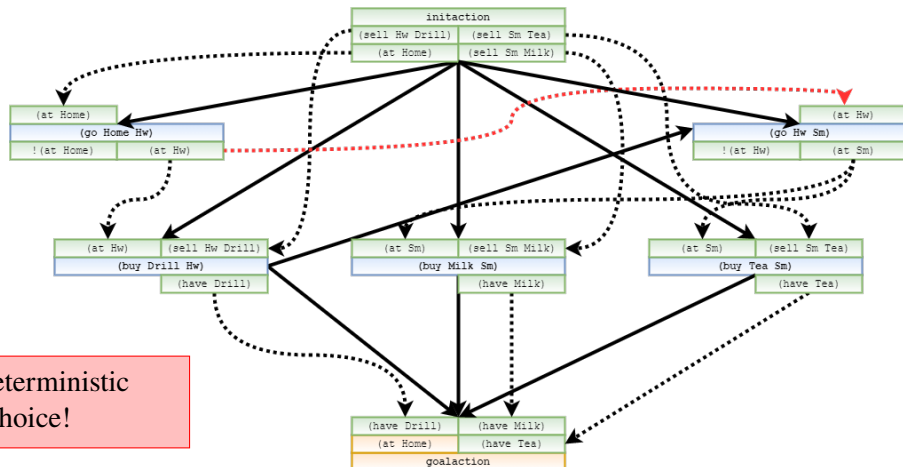
## EXAMPLE (10)

- How to establish  $(at\ l_1)$ ?
- We do it from initaction forcing  $l_1 = \text{Home}$



## EXAMPLE (11)

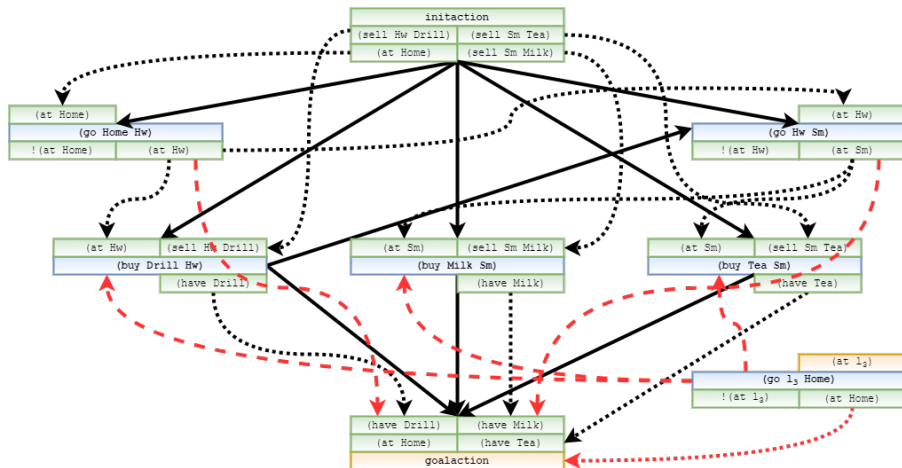
- How to establish  $(at\ l_2)$  ?
  - We do it from  $(go\ Home\ Hw)$  forcing  $l_2 = Hw$



Nondeterministic  
choice!

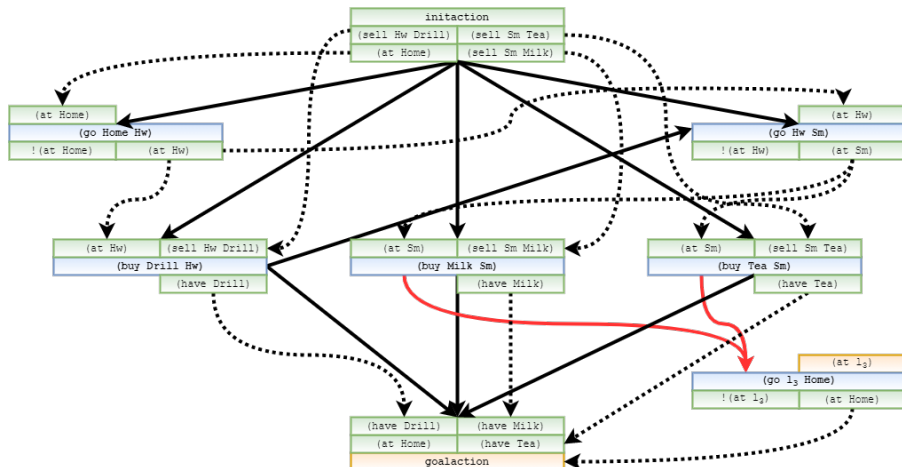
## EXAMPLE (12)

- The only possible way to establish (at Home) for goalaction
- Creates several threats



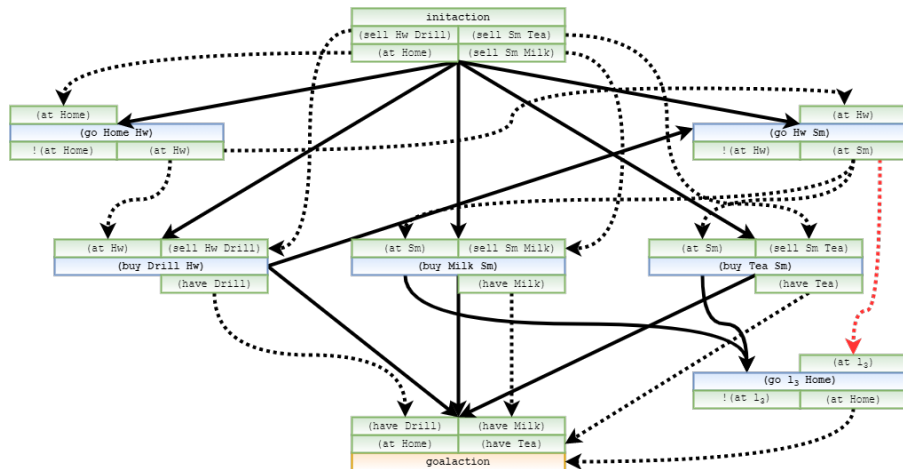
## EXAMPLE (13)

- To remove threats to (at Sm) and (at Hw)
  - Make (go Hw Sm) and (go Home Hw) precede (go  $l_3$  Home): removes other threats

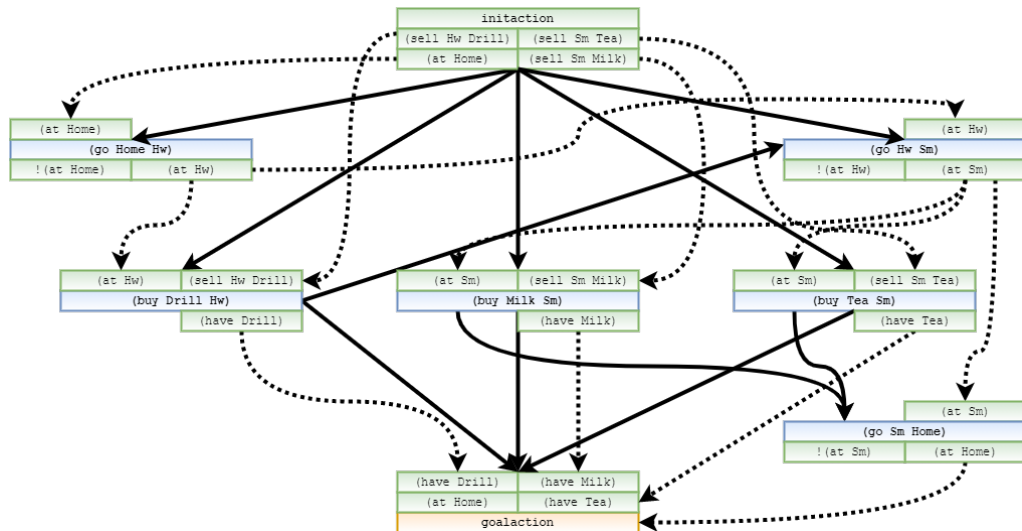


## EXAMPLE (14)

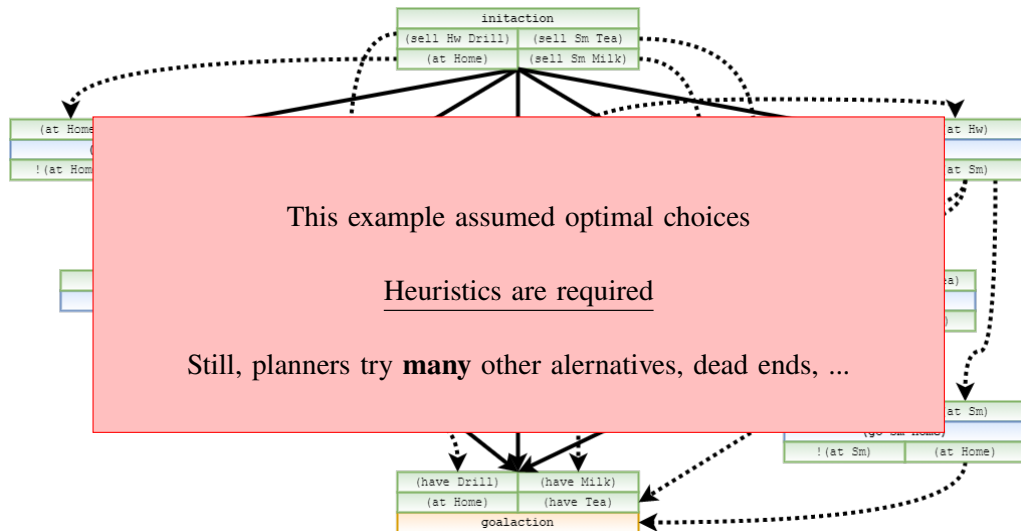
- Establish (at  $l_3$ ) with  $l_3 = \text{Sm}$



## EXAMPLE (15): FINAL PLAN



# EXAMPLE (16): FINAL PLAN





# REFERENCES I

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- [2] Malik Ghallab, Dana S. Nau, and Paolo Traverso. *Automated planning - theory and practice*. Elsevier, 2004. ISBN 978-1-55860-856-6.
- [3] Malik Ghallab, Dana S. Nau, and Paolo Traverso. *Automated Planning and Acting*. Cambridge University Press, 2016. ISBN 978-1-107-03727-4. URL <http://www.cambridge.org/de/academic/subjects/computer-science/artificial-intelligence-and-natural-language-processing/automated-planning-and-acting?format=HB>.
- [4] Stuart J. Russell and Peter Norvig. *Artificial Intelligence: A Modern Approach (4th Edition)*. Pearson, 2020. ISBN 9780134610993. URL <http://aima.cs.berkeley.edu/>. 48