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Lab 1

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Exercise 1

Taylor approximations you have to know.
1. Compute the Taylor approximation of e^x of degree n about $a = 0$.
2. Compute the Taylor approximation of $sin(x)$ of degree $2n1$ about $a = 0$.
3. Compute the Taylor approximation of $cos(x)$ of degree $2n$ about $a = 0$.
4. Compute the Taylor approximation of $log(1 + x)$ of degree n about $a = 0$.
5. Compute the Taylor approximation of $\frac{1}{1-x}$ of degree n about $a = 0$.

Solutions

$$e^x \approx P_n(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ...\frac{x^n}{n!}$$
$$sin(x) \approx P_n(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} + ...\frac{-1^n x^{2n+1}}{(2n+1)!}$$
$$cos(x) \approx P_n(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + ...\frac{-1^n x^{2n}}{2n!}$$
$$log(1 + x) \approx P_n(x) = x - \frac{x^2}{2!} + \frac{x^3}{3!} + ...\frac{-1^n x^{n+1}}{n!}$$
$$\frac{1}{1-x} \approx P_n(x) = 1 + x + x^2 + ...x^n$$

Exercise 2

Compute the taylor polynomial of degree 2 center at $x = 1$ or $f(x) = \frac{1}{1+x}$.

Work

$$f'(x) = -\frac{1}{(1+x)^2},\ f''(x) = \frac{2}{(1+x)^3},$$
$$f(1) = 1,\ f'(1) = -\frac{1}{4},\ f''(1) = \frac{1}{4}$$
$$P_n(x) = \frac{(x-1)^0}{0!} * 1 + \frac{(x-1)^1}{1!} * -\frac{1}{4} + \frac{(x-1)^2}{2!} * \frac{1}{4}$$

Solution

$$\frac{x^2+4x+11}{8}$$

Exercise 3

Does $f(x) = \sqrt[3]{x}$ have a Taylor polynomial approximation of degree 1 based on expanding about $x = 0$? $x = 1$? Explain and justify your answers.

Solution

When computing $P_n(0)$ you find that there becomes a 0 in the denominator, while when doing $P_n(1)$, there is not.

Exercise 4

The quotient

$$g(x) = \frac{log(1+x)}{x}$$

is undefined for $x = 0$. Approximate $log(1 + x)$ using Taylor polynomials of degrees 1, 2, and 3, in turn, top determine a natural definition of $g(0)$.

Work

$$g(x) \approx \frac{x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}}{x} = 1 - \frac{x}{2} + \frac{x^2}{3} - \frac{x^3}{4}$$

Solution

$$g(0) = 1$$

Exercise 7

What happens when x is negative?
What can you conclude?
If you where to approximate $e10$ with a polynomial of degree 4, how would you do it?
How could you improve the previous approximation?

Solution

1. It becomes less accurate the deeper it gets into negative numbers.
2. They're the same thing.
3. I would use $\frac{1}{P_4(10)}$
4. I would let $P_n(5)$ be the approximation for e^5 and do $P_n(5)^2$

Exercise 8

1.

Work

$$\frac{1-cos(x)}{x^2} = \frac{1-(1-\frac{x^2}{2!})}{x^2} = \frac{\frac{x^2}{2}}{x^2}$$

Solution

$$\frac{1}{2}$$

2.

Work

Since $log(1+x^2) \approx x^2 - \frac{x^4}{2}$ and we're dividing by 2x, then all terms at the top will have at least one x, making the limit equal to zero.

Solution

$$\frac{1}{2}$$

3.

Work

$$log(1-x) \approx P_n(x) = -x - \frac{x^2}{2} - \frac{x^3}{3} + ... + \frac{-x^n}{n}$$
$$\mathrm{e}^{x/2} \approx P_n(x) = 1 + \frac{x}{2} + \frac{x^2}{8} + ... + \frac{x^n}{2^n n!}$$
$$\lim_{x \to 0} \frac{log(1-x)+xe^x}{x^3} \approx \frac{(-x-\frac{x^2}{2}-\frac{x^3}{3})+x(1+\frac{x}{2}+\frac{x^2}{8})}{x^3}$$

Solution

$$\lim_{x \to 0} \frac{log(1-x)+xe^x}{x^3} \approx \frac{5}{24}$$

For Exercises 9, 10, and 11, please see the attached images. I could no longer keep up writing equations at 1:30am on Latex. This is probably one of the most annoying things I have ever done in my life.

If you want me to keep this is, please let me know before it's too late. Thanks.