Leonardo Carrico
CSCI/MATH 240

Lab 4

Exercise 1

1. The code for the Trapezoidal Rule is inside the lib.py python file.

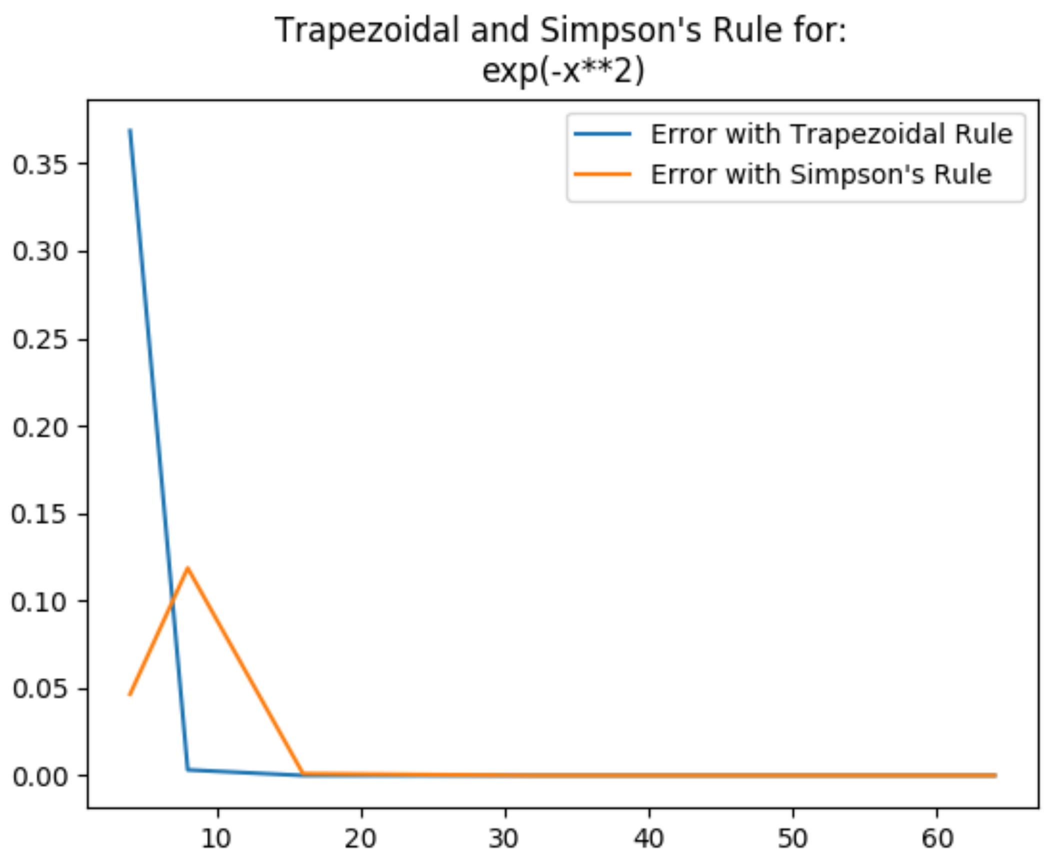
Exercise 2

1. The code for the Simpson's Rule is inside the lib.py python file.

Exercise 3

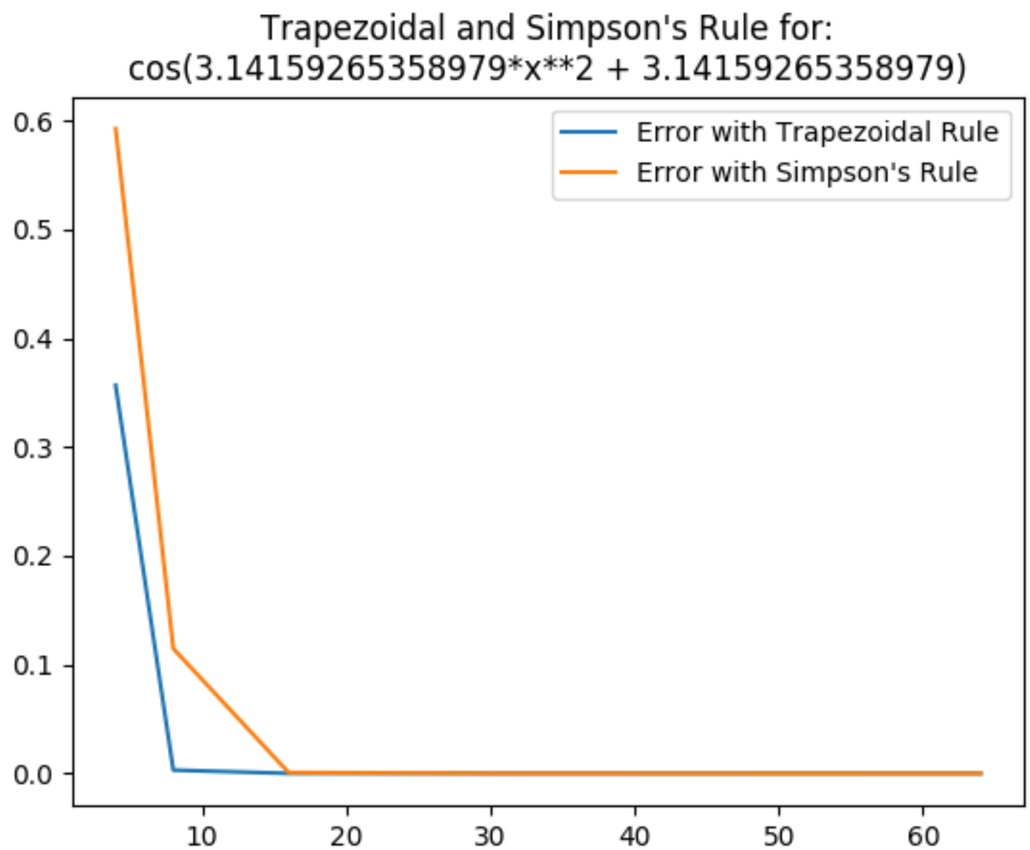
1. For the Integral: $\int_0^{10} e^{-x^2} dx$

n	4	8	16	32	64
Trapezoidal	1.254826135375289	0.8894282780626929	0.886226925471621	0.8862269254527579	0.8862269254527578
Simpson's	0.8397681804772389	0.7676289922918275	0.8851598079412636	0.8862269254464703	0.886226925452758



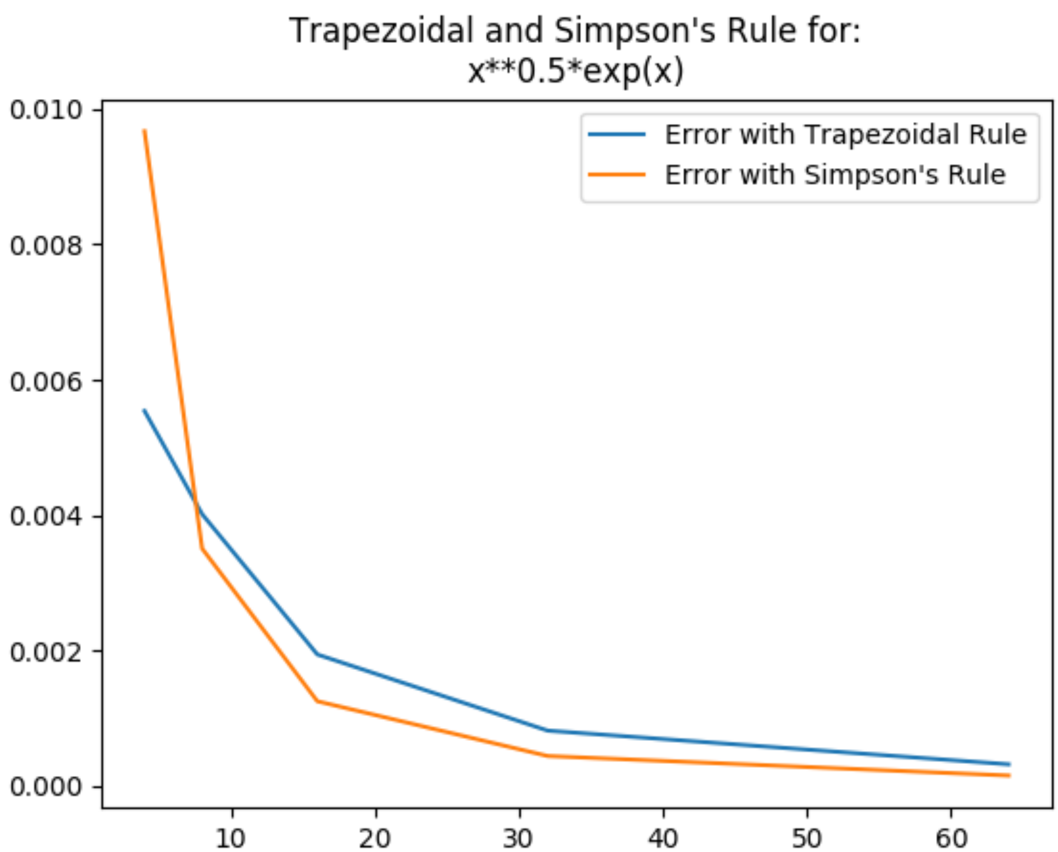
2. For the Integral: $\int_0^2 \cos(\pi(1+x^2))dx$

n	4	8	16	32	64
Trapezoidal	−0.7071067811865526	−0.35355339059327506	−0.3505543413715142	−0.3504613232121879	−0.35045636703036487
Simpson’s	−0.9428090415820701	−0.23570226039551598	−0.34955465829759397	−0.35043031715907924	−0.3504547149697574



3. For the Integral: $\int_0^2 \sqrt{x}e^x dx$

n	4	8	16	32	64
Trapezoidal	1.2500878518926526	1.2516121251639316	1.2536843987528026	1.2548090999163364	1.2553062339580383
Simpson's	1.2459566516855227	1.2521202162543579	1.2543751566157595	1.2551840003041808	1.2554719453052723



Exercise 4

The formula written was a bit difficult to understand due to what I believe are typos, and my understanding is that the length of the curve is equal to: $\int_a^b \sqrt{1+f'(x)^2}dx$. With n = 50, we get the following values:

1. With $f(x)$ as $sin(\pi x)$ where $0 \leq x \leq 5$, $\int_a^b \sqrt{1+f'(x)^2}dx \cong 11.524133760288862$
2. With $f(x)$ as e^x where $0 \leq x \leq 2$, $\int_a^b \sqrt{1+f'(x)^2}dx \cong 6.789533205034732$
3. With $f(x)$ as e^{x^2} where $0 \leq x \leq 2$, $\int_a^b \sqrt{1+f'(x)^2}dx \cong 54.16529789515147$

Exercise 5

Below are the numerical derivatives at the indicated points with the backward, forward, and centered formula, all with h = 0.05.

Values from Each Numerical Derivative

	Backward	Center	Forward
e^x at $x = 0$	0.9754115099857197	1.000416718753101	1.0254219275204823
$tan^{-1}(x^2 - x + 1)$ at $x = 1$	0.48645974400530534	0.4989597374948951	0.5114597309844848
$tan^{-1}(100x^2 - 199x + 100)$ at $x = 0$	-1.8131977440150426	0.3904264995515372	2.594050743118117

Errors from Each Numerical Derivative

	Backward	Center	Forward
e^x at $x = 0$	0.024588490014280318	0.0004167187531010086	0.025421927520482335
$tan^{-1}(x^2 - x + 1)$ at $x = 1$	0.013540255994694661	0.0010402625051049164	0.011459730984484828
$tan^{-1}(100x^2 - 199x + 100)$ at $x = 0$	2.3131977440150426	0.10957350044846281	2.094050743118117

From this we notice that the lowest errors are all from using the Center Numerical Derivative. Thus, we can determine that center will typically be most resourceful at getting us the most accurate value.

Exercise 6

For calculating the number of points necessary for the accuracy of 10^{-6}, I created the functions called simpsonError and trapRuleError. They respectively give us the values for n: 24 and 816. Finally, I get the calculated values for $\int_1^3 x ln(x)\, dx$ as:

Simpson's: 2.9437555358624157
Trapezoidal: 2.943755848981364

Exercise 8

For this problem, I found that the area of the function $2\sqrt{1 - x^2}$ returns to us the approx. value of π. I created the monteCarlo function, which is stored inside of the lib.py file, to calculate our approximation. We get a value of: 3.1417083194143074.