# Construction of High Performance Block and Convolutional Multi-Edge Type QC-LDPC codes

Vasiliy Usatyuk,
South-West State University,
Kursk, L@Lcrypto.com

Ilya Vorobyev,
Skolkovo Institute of Science and Technology,
Moscow, vorobyev.i.v@yandex.ru

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# Quasi-Cyclic LDPC codes

$$H = \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} & \\ \left[\begin{array}{cc|cc|cc} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ \hline 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right] & \begin{array}{c} c_1 \\ c_2 \\ c_3 \\ c_4 \end{array} \end{array}$$

$$H_{QC} = \begin{bmatrix} I^0 & I^1 & I^1 \\ I^0 & I^{-1} & I^0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

Quasi-Cyclic LDPC(QC-LDPC codes) - LDPC-codes with parity-check matrix defined by structured block submatrix – Circulant Permutation matrix.

$$Circulant\ \ Permutation\ Matrix\ (CPM)\ of\ \ size\ \ 2\times 2:\ I^0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, I^1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, I^{-1} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

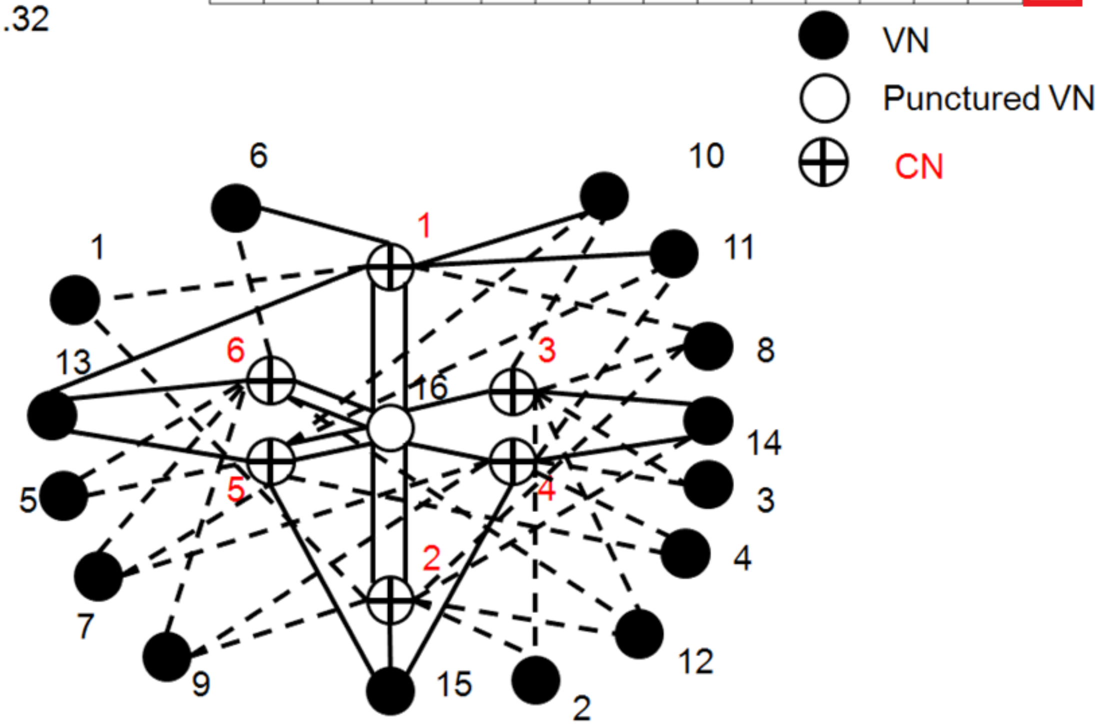
R. M. Tanner, D. Sridhara, T. Fuja, ”A class of group structured LDPC codes”, Proc. ICSTA 2001, Ambleside, England, 2001

# MET LDPC

R=2/3
Capacity 1.059
Our Threshold 1.32
Gap 0.261
AR4JA 1.414

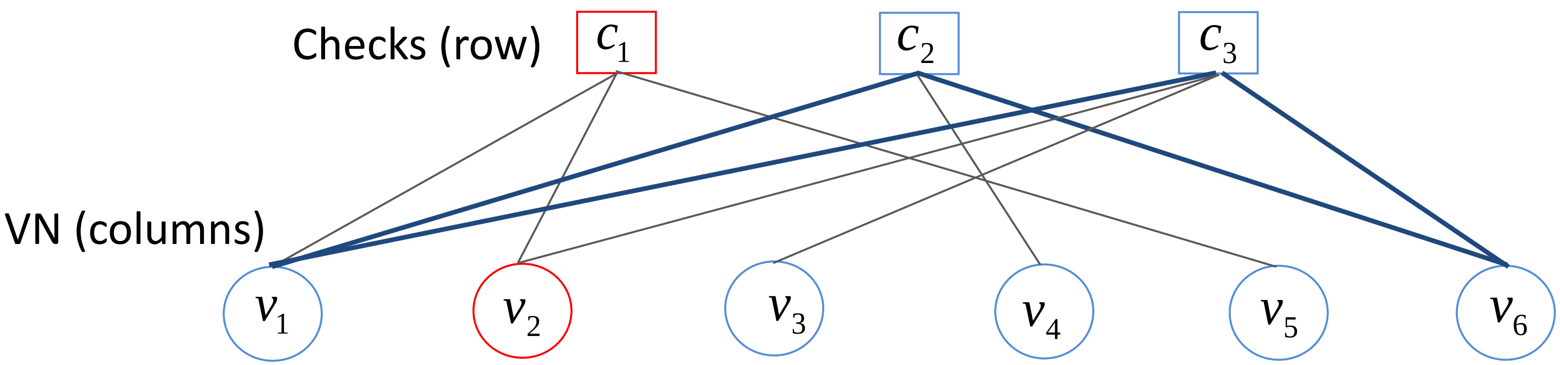
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 2 |
| 2 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 2 |
| 3 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 |
| 4 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 |
| 5 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 2 |
| 6 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 2 |

MET QC-LDPC codes, by cost of the relative small, increasing numbers of decoder iterations to recover puncture VNs (converge), improve iterative decoding threshold compare to irregular codes with the same size of protograph.
It allow to have even high EMD cycles 4 in punctured bit to improve waterfall properties like 5G eMBB




• J. Richardson and R. L. Urbanke, “Multi-edge type LDPC codes,” in Workshop honoring Prof. Bob McEliece on his 60th birthday, California Institute of Technology, Pasadena, California, 2002.
• Mattoussi F., Roca V., Sayadi B. GLDPC-Staircase AL-FEC codes: A Fundamental study and New results. EURASIP Journal on Wireless Communications and Networking, SpringerOpen, 2016
• Sarah J. Johnson Reported Thresholds and BER Performance for LDPC and LDPC-Like Codes, 2011

Tanner Graph– equivalent bipartite graph for the parity check matrix *H*




Cycle - closed simple way in Tanner-graph.

Example of cycle 4: $c_2 \rightarrow v_1 \rightarrow c_3 \rightarrow v_6 \rightarrow c_2$

*Girth – shortest cycles in Tanner-graph.*

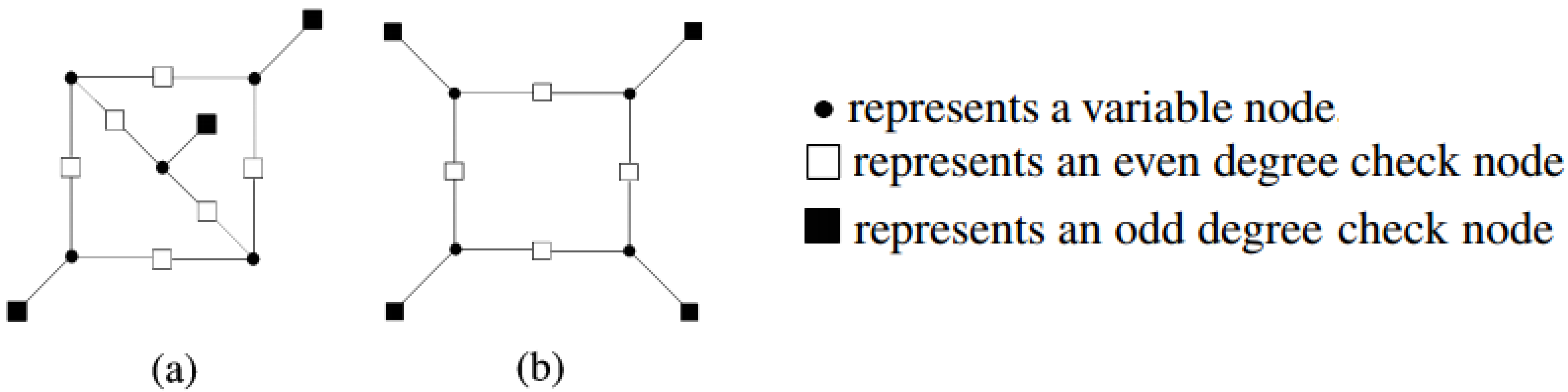
Tanner R.M., "A recursive approach to low complexity codes", IEEE Trans. Inform. Theory, IT-27, pp. 533-547, September 1981.

# Trapping sets - subgraphs formed by cycles or it's union:

After $m$ decoding iterations Belief Propagation algorithm under the tanner graph with girth $g$ produce wrong decoding result due Trapping sets:

$$m < \frac{g}{4} \leq m+1.$$

**Trapping set $(a,b)$ is a sub-graph with $a$ variable nodes and $b$ odd degree checks.**




Trapping sets: (a) $(5,3)$ and (b) $(4,4)$

**For example, TS(5,3) produced by three 8-cycles;**
**TS(a,0) is most harmfulness pseudocodeword of weight a formed by cycles 2*a.**

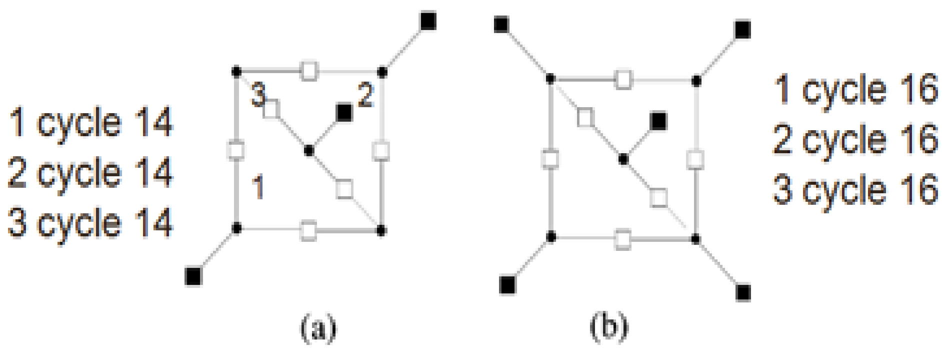
# Simple way to broke harmful TS improve EMD/ACE:

**Approximate cycle extrinsic message degree** metric for Codes graph:

$$ACE(C) = \sum_{v \in C} \left( d(v) - 2 \right)$$

$d(v)$ – degree of node in cycles

**Trapping set $(a, b)$ is a sub-graph with $a$ variable nodes and $b$ odd degree checks.**

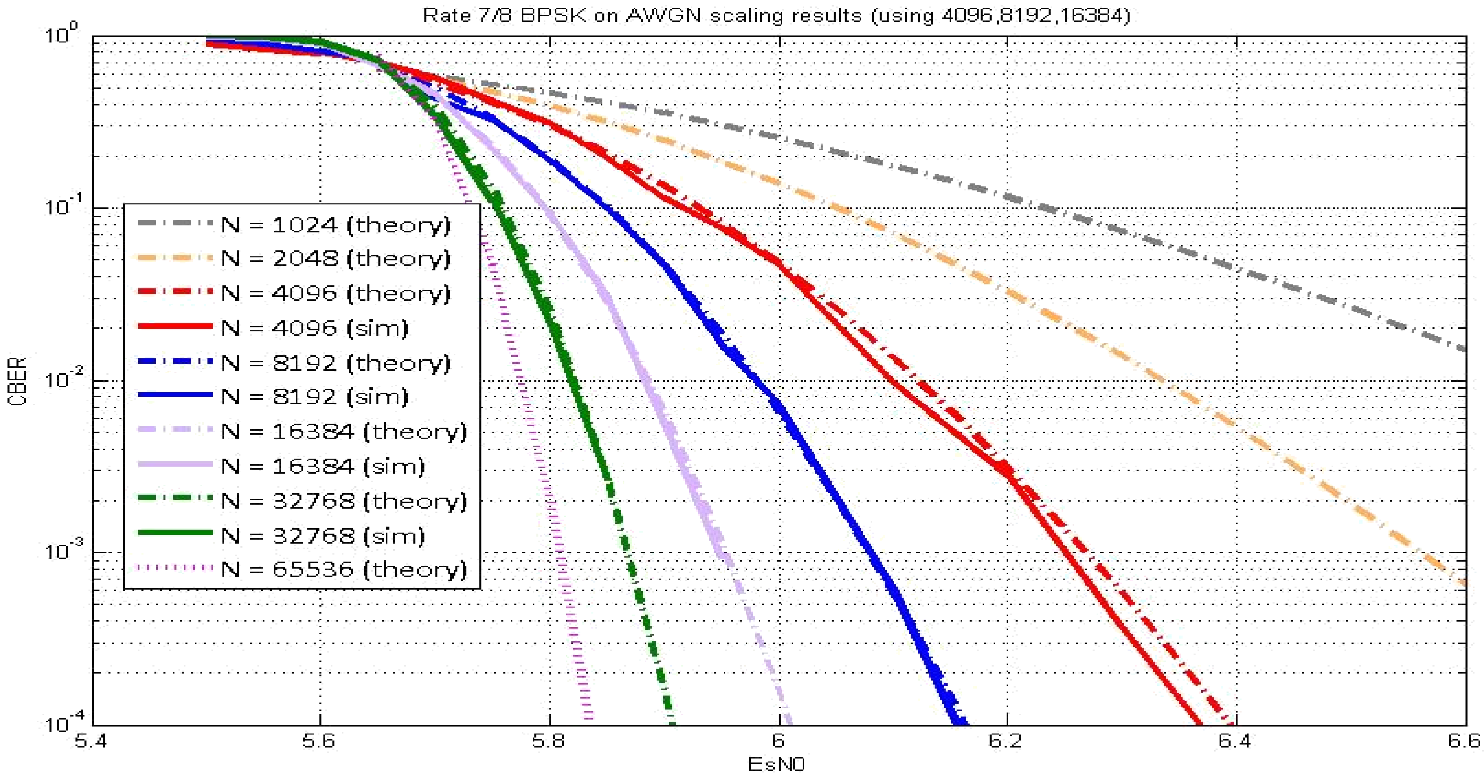



• represents a variable node
□ represents an even degree check node
■ represents an odd degree check node

Trapping sets: (a) $(5,3)$ , (b) $(5,5)$

**For example, TS(5,3) produced by 3 cycle of ACE 14 and TS(5,5) by 3 cycles of ACE 16.**
**Improve of this metrics improve TS structures.**
**Decoder properties like quantization, normalize and offset values change harmfulness of TS.**
**This is why solution of TS elimination under LDPC code construction and choice properties of decoder need doing simultaneously.**

# Linear size TS ('big cycles') affect on waterfall performance*




**$Th_p$ –Penalty from Iterative decoding threshold**

**Covariance Evolution( Finite-Length scale) estimate $Th_p$ by Poisson TS distribution assumption**

$$P_{BLOCK}(n,\varepsilon) \cong Q\left(\sqrt{n}\left(\varepsilon^{*}-\varepsilon\right)\alpha\right)$$

$n - code\ length, a - scale\ \ parameter$

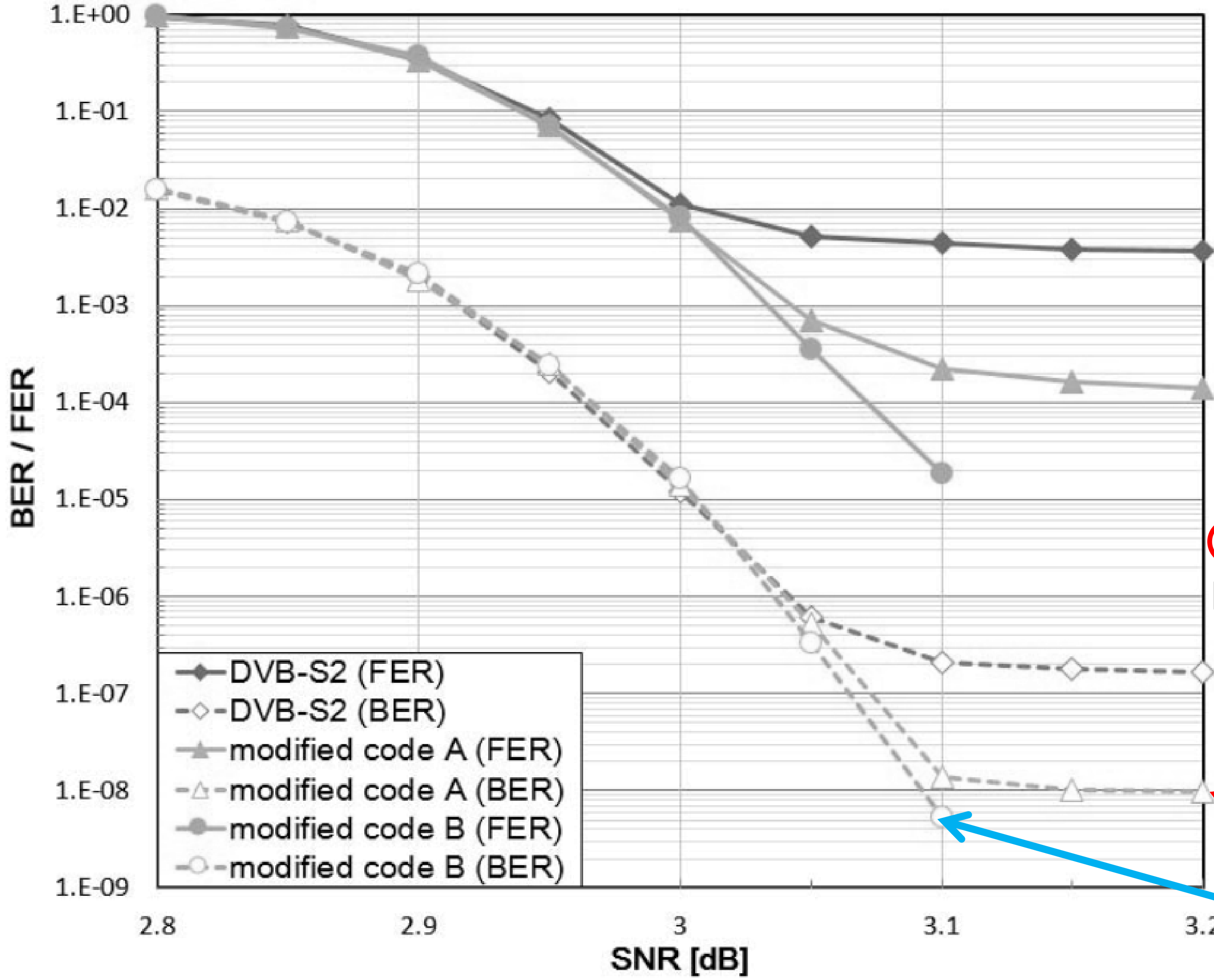
$\varepsilon^{*} - treshold\ \ from\ density\ evolution$

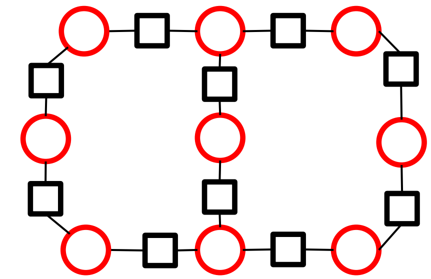
$\varepsilon - channel\ \ probability\ of\ \ error$

$a - scale\ characteristics\ of\ \ code\ ensemble.$

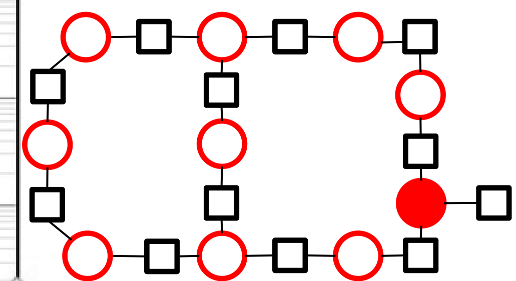
**It reason why regular code have so bad waterfall under BP***
**A lot of long cycles make penalty From iterative decoding threshold**

# Sublinear size TS (‘small cycles’) affect on error-floor performance



TS(9,0)



TS(10,1)

Without TS(9,0).

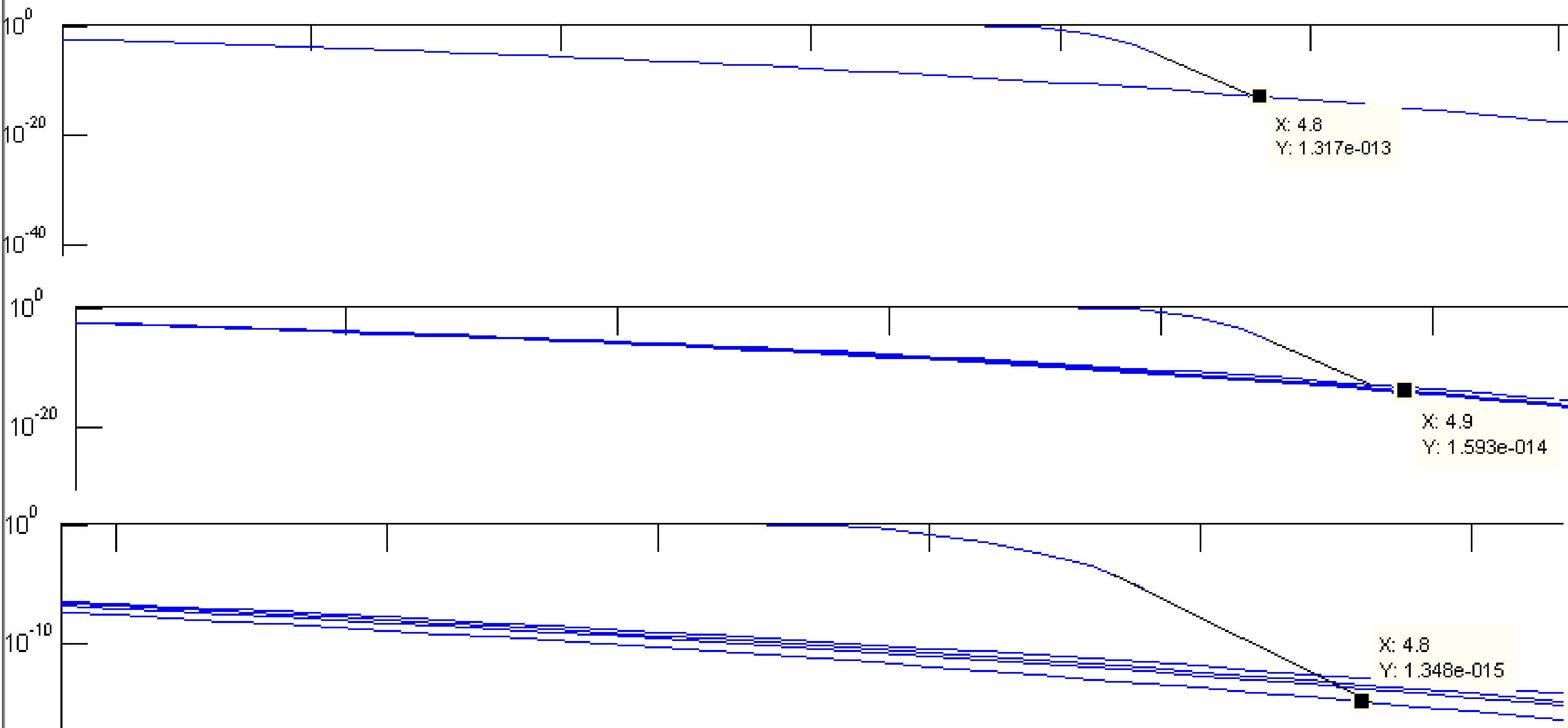
Without TS(9,0)
And TS(10,1).

BER and FER performance of the original and modified DVB-S2 QC-LDPC codes of information length K=43200 and rate 2/3.

# Low weight TS enumerating with Importance Sampling allow to predict error-floor

| TS(a,b) | pseudo weight of TS | Volume nodes |
|---|---|---|
| (7, 6) | [11.953960] | 164 1846 2741 2751 3410 4429 8627 |
| (6, 6) | [12.928841] | 1001 2213 6379 6646 8176 9495 |
| (6, 6) | [13.009602] | 251 1640 5006 5745 8085 8347 |
| (7, 6) | [13.077987] | 1289 1616 1856 2501 2761 4439 8637 |
| (7, 6) | [13.120068] | 3362 3751 5558 6785 7224 7495 8370 |
| (8, 8) | [13.164738] | 1 1545 3839 5179 6719 6873 7144 7970 |
| (6, 6) | [13.207715] | 1846 2741 2751 3410 4429 8627 |
| (9, 6) | [13.267264] | 419 769 1275 1352 1990 2896 4222 5501 9739 |
| (6, 6) | [13.295882] | 209 1501 2968 3233 7738 9995 |
| (6, 6) | [13.335162] | 1 831 1933 2838 3001 8714 |
| (6, 6) | [13.585278] | 501 2414 3774 4263 6038 9212 |
| (7, 6) | [13.737830] | 210 2251 4348 6890 7552 7775 8597 |
| (6, 6) | [13.871107] | 3294 3901 4042 4303 5001 7477 |
| (6, 6) | [13.891160] | 3501 5086 5610 5999 6953 7297 |
| (7, 6) | [14.179450] | 1001 1294 1773 2038 2899 7499 9897 |
| (8, 8) | [14.199011] | 1351 1678 1918 2563 2823 3482 4251 8699 |
| (7, 6) | [14.259830] | 454 624 2763 3163 3751 7642 8218 |
| (6, 6) | [14.295253] | 3251 4260 4671 5208 5344 5402 |
| (6, 6) | [14.363166] | 1412 2094 3001 5719 7706 8978 |
| … | … | … |
| (12, 12) | [32.320236] | 155 2231 2781 5001 5953 7174 7554 8921 9004 9374 9414 9909 |

# TS analysis (importance Sampling for predict Error-floor):




FER error-floor

Error-floor performance Fig. Second line without TS(7, 6) [11.953960]. Third line without first 5 TS: (7, 6) [11.953960], (6, 6) [12.928841], (6, 6) [13.009602], (7, 6) [13.077987], (7, 6) [13.120068]. The fifth line without 15 first TS.

**Algorithm 1** Codes Sieving Method For Construction High Performance Long Length Block MET QC-LDPC Codes

**Require:** $m \times n-$size of mother matrix (protograph), $p$-number of punctured variable nodes, $L-$ circulant size, $\delta-$require level of Bit Error Rate, $Iter-$ maximal number of iterations, $Card_m$-number of mother matrix for sieving, $Card_c$-number of code candidate lifting per mother matrix, $Th_p-$ minimal allowed threshold (EB/No), $seed$-a seed to be used in a pseudo-random number generator

1: $Th_p = 0,\ E_{final}(\mathbf{H}) = \varnothing$

2:

$$M(\mathbf{H}) = PEXIT(m, n, p, Card_m, iter, Th_p, seed),$$

where $M(\mathbf{H})$- total order set of base matrix (protograph)

$$Th(M(\mathbf{H})_0) \leq Th(M(\mathbf{H})_1) \leq \ldots \leq Th(M(\mathbf{H})_{Card_m})$$

with minimized iterative decoding threshold $(Eb/No)$:

$$\boxed{(Th(M(\mathbf{H})_0) - Th_{Shannon}) \to Th_p.}$$

3: **for** $i = 0; i < Card_m; i = i + 1$ **do** Penalty

4: **for** $j = 0; j < Card_c; j = j + 1$ **do**

5: $E(\mathbf{H})_{i,j} = SALift(M(\mathbf{H}), L),$

where $E(\mathbf{H})_{i,j}$ - set of codes constructed using $Card_c$ different simulation annealing lifting (CPMs extension) of mother matrix (protograph) $E(\mathbf{H}) = \max\limits_{girth,EMD} M(\mathbf{H})$.

6: **end for**

7: **end for**

8: **for** $i = 0; i < Card_m; i = i + 1$ **do**

9: **for** $j = Card_c; j > 0; j = j - 1$ **do**

10: **if** $IS_{RCBP}(E(\mathbf{H})_{i,j}) < \delta$ **then** Sieved code

11:

$$\boxed{E_{final}(\mathbf{H}) = [E_{final}(\mathbf{H}) \cup E(\mathbf{H})_{i,j}]}$$

where $IS_{RCBP}(E(\mathbf{H})_{i,j})$ - importance sampling or simulation show that code candidate $E(\mathbf{H})_{i,j}$ have error-floor level below required Bit/Frame error rate $\delta$.

12: **end for**

13: **end for**

14: **if** $E_{final}(\mathbf{H}) = \varnothing$ **then**

$$Th_p = Th\Big(\max_{0 \leq j < Card_c} M(\mathbf{H})_{Card_m}\Big), \textbf{goto 2.}$$

15: **else Return** $E_{final}(\mathbf{H})$

We combine idea taking into account the influence of parameters:

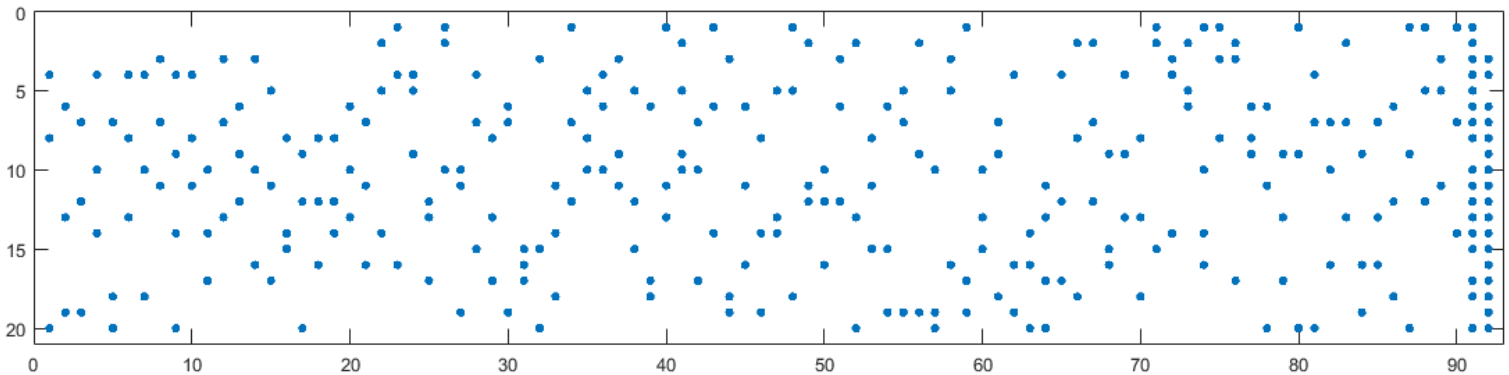
- sublinear/linear TS to threshold Penalty
- non-perfectness of QC lifting

In such case to **control error-floor and waterfall of code simultaneous**.

Proposed method Sieved code candidates to improve waterfall in a such way to have error-floor at desire level **according to property of lifting method to broke TS**.

# Results: MET QC-LDPC Protograph Rate=4/5

Our constructed codes 92x20 Protograph, last 2 VNs punctured, threshold 2.333 (200 it)




**Gap to capacity: 0.293**

AR4JA Protograph (after twice lifting 66x18), last 3 VNs punctured, threshold 2.386 (200 it)

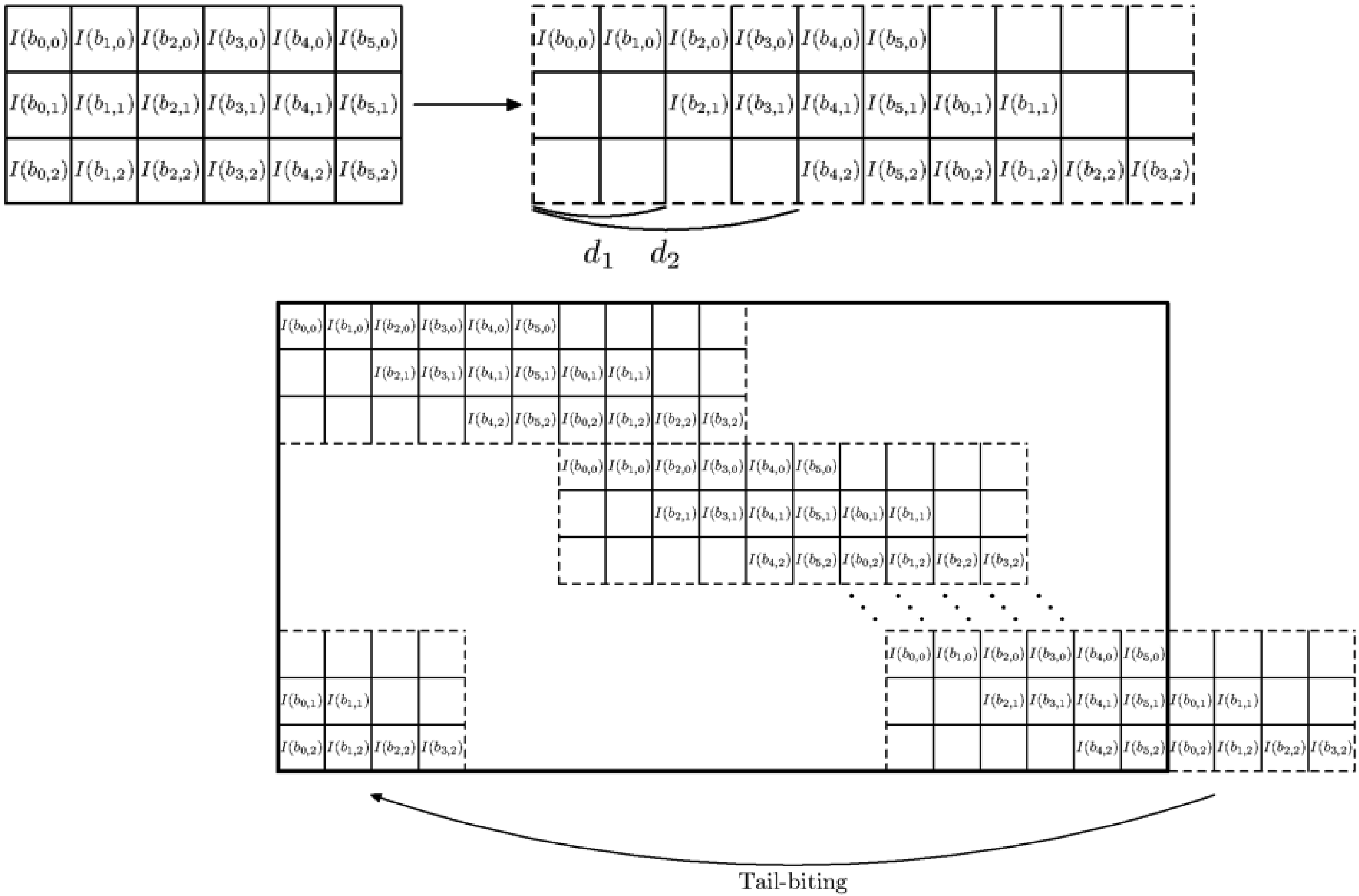
$$H_{4/5} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 1 & 1 & 1 & 3 & 1 & 3 & 1 & 3 & 1 & 3 \\ 0 & 1 & 2 & 2 & 1 & 3 & 1 & 3 & 1 & 3 & 1 \end{pmatrix}$$

**Gap to capacity: 0.346**

$\boldsymbol{BER < 10^{-15}}$

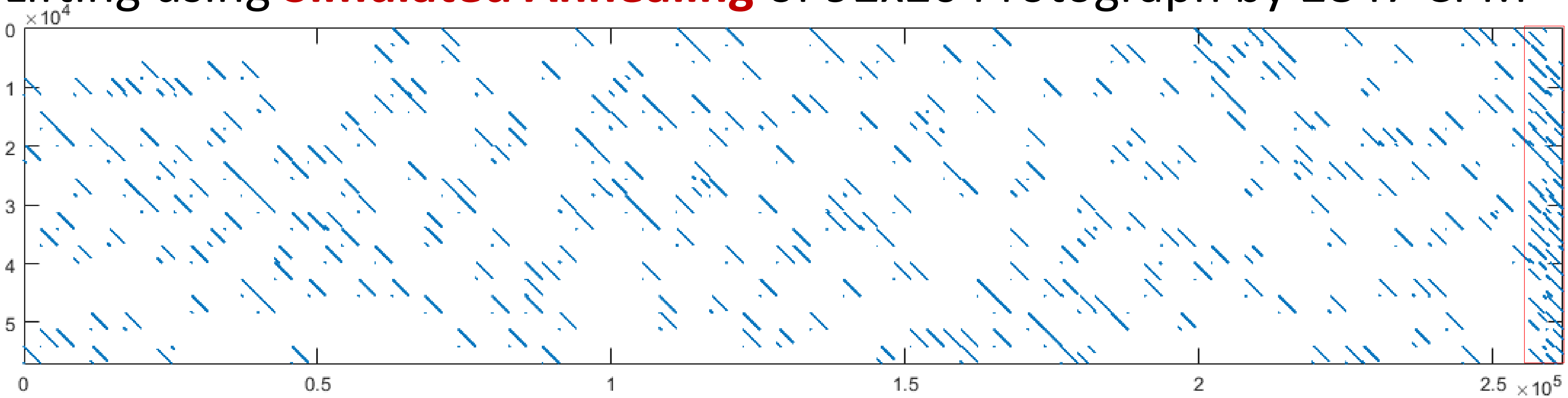
Shannon limit 2.040

# Construction of Tail-Biting Spatially-Couples Codes from Block MET QC-LDPC Codes

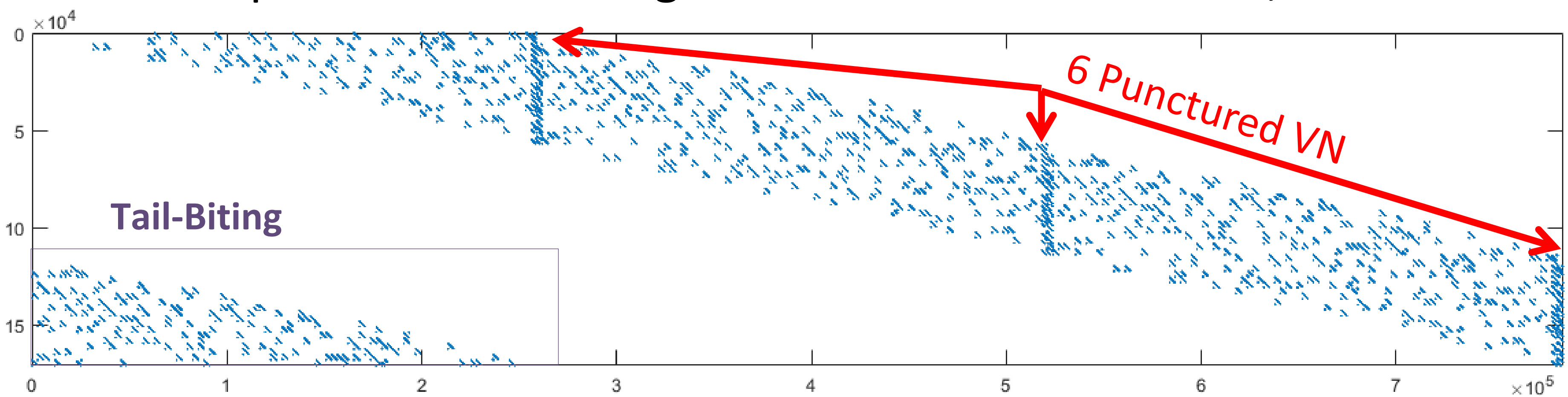


# Tail-Biting Spatially-Coupled MET QC-LDPC Codes

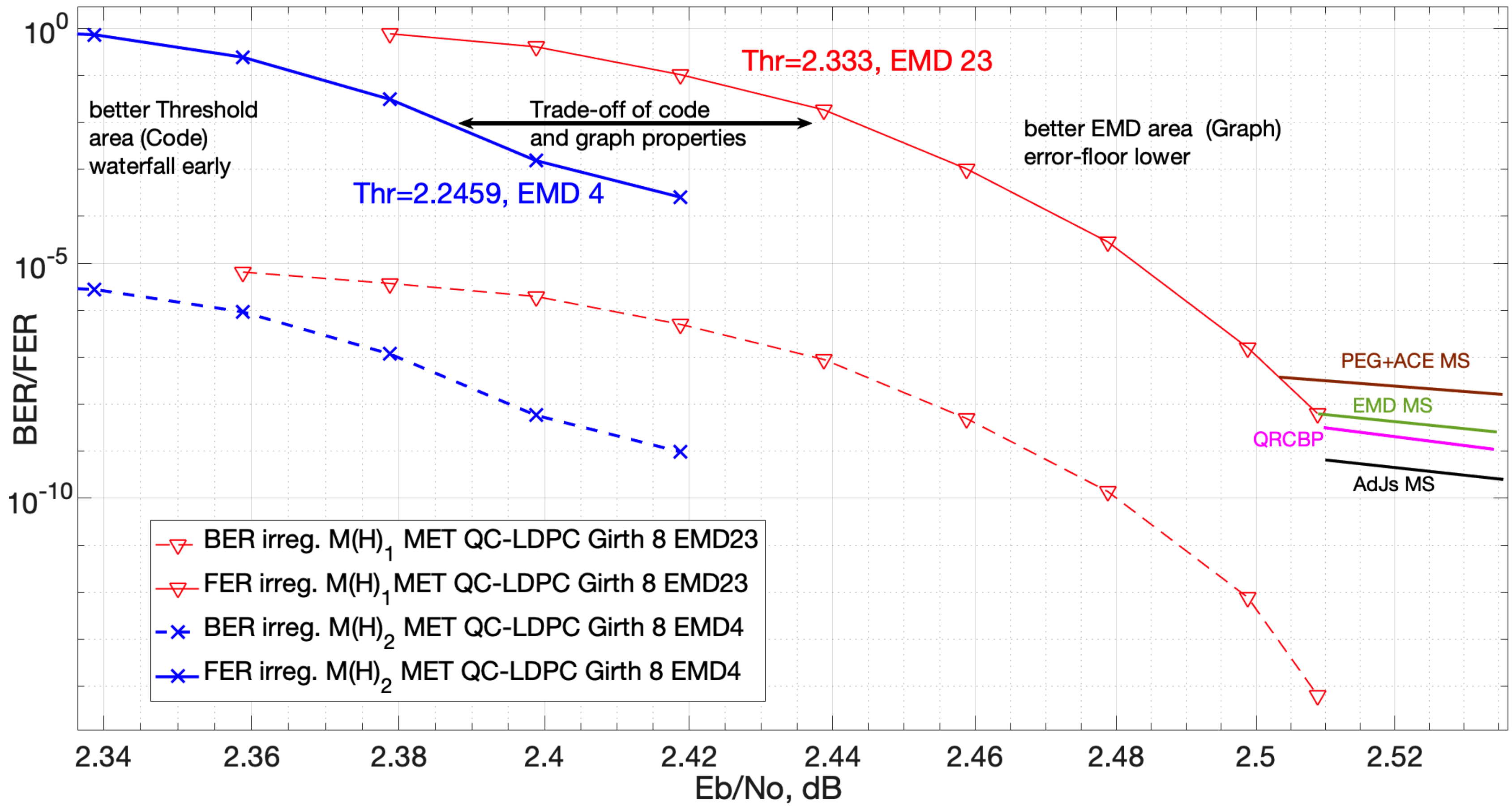
Lifting using **Simulated Annealing** of 92x20 Protograph by 2847 CPM




3 times couple from 92x20 to get 276x60 with 2847 CPMs, 6 VN Punctur.

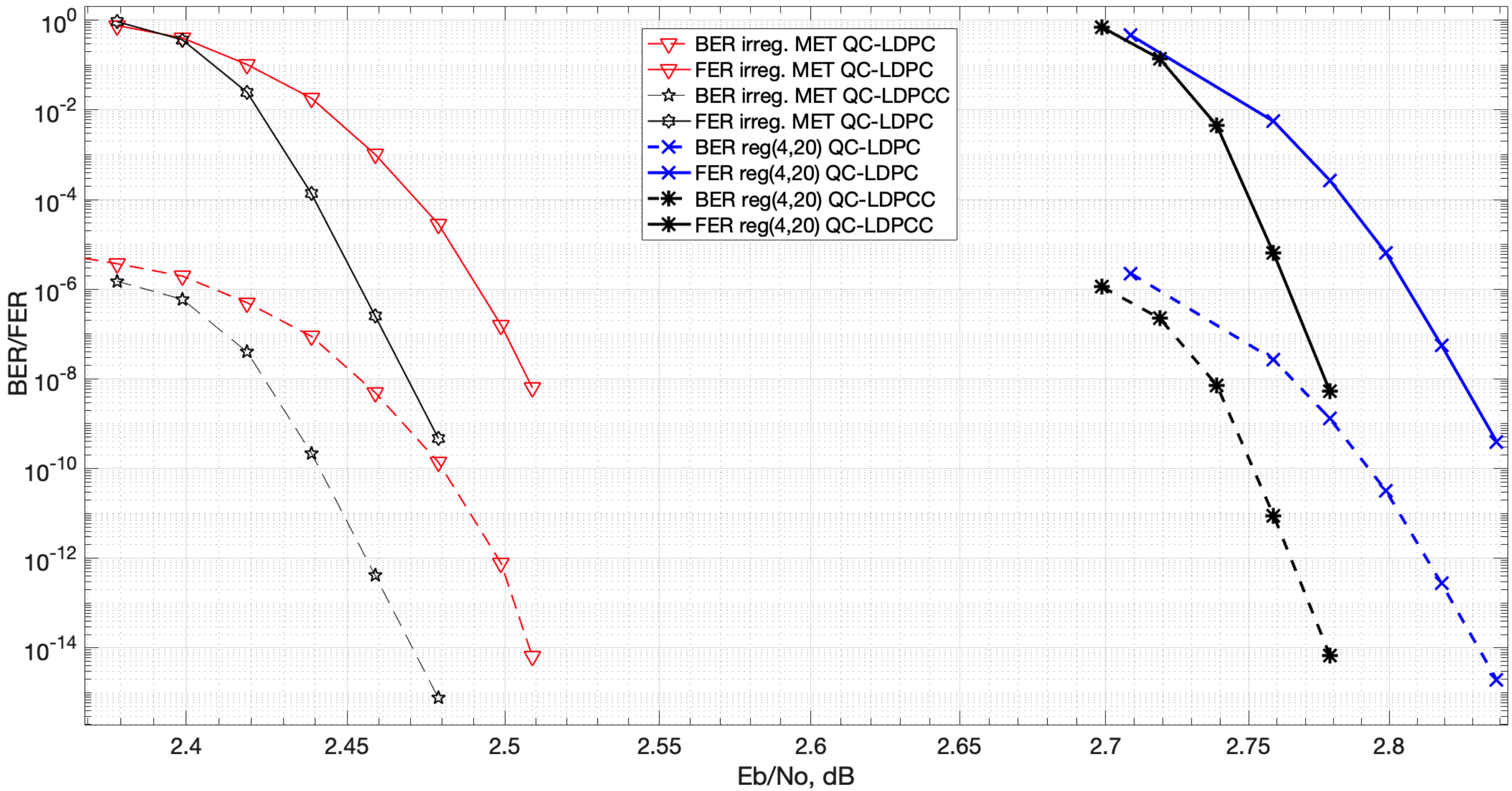


# Results




https://github.com/Lcrypto/trapping-sets-enumeration

# Results




https://github.com/Lcrypto/Construct-Long-Length-Block-and-Convolutional-MET-QC-LDPC-

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(71) **Applicant: HUAWEI TECHNOLOGIES CO., LTD.** [CN/CN]; Huawei Administration Building, Bantian, Longgang District, Shenzhen, Guangdong, 518129 (CN).

(72) **Inventor; and**
(71) **Applicant** *(for RU only)*: **USATYUK, Vasily Stanislavovich** [RU/CN]; Huawei Administration Building, Bantian, Longgang District, Shenzhen, Guangdong, 518129 (CN).

(74) **Agent: POPOVA, Elizaveta Vitalievna** et al.; LAW FIRM "GORODISSKY & PARTNERS" LTD., ul. B. Spasskaya, 25, bldg. 3, Moscow, 129090 (RU).

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(54) **Title:** GENERATION OF SPATIALLY-COUPLED QUASI-CYCLIC LDPC CODES

https://github.com/Lcrypto/Patents

# Thank You!