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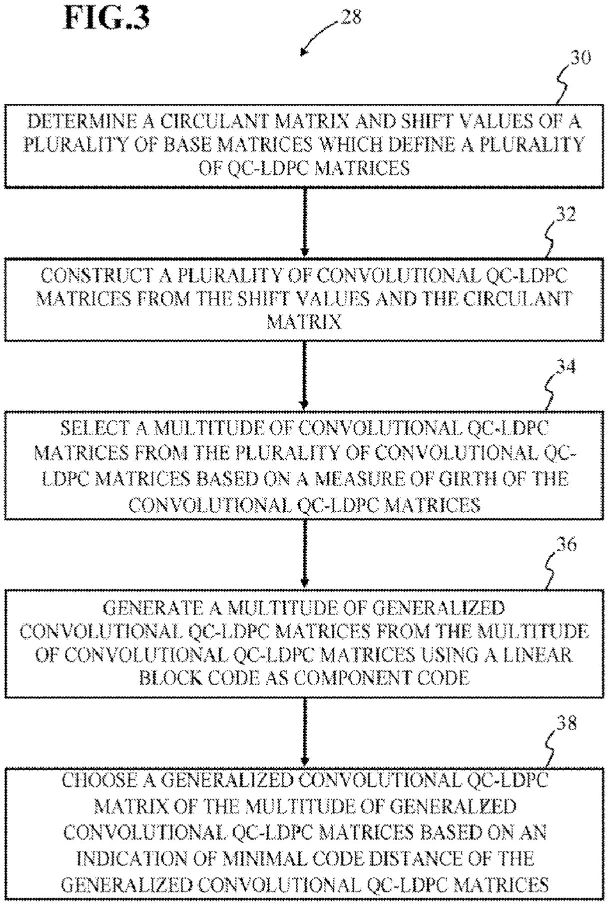
(54) Title: GENERALIZED QUASI-CYCLIC LDPC CONVOLUTIONAL CODES FOR DIGITAL COMMUNICATION SYSTEMS

FIG.3



(57) **Abstract**: Provided is an encoder, a decoder and corresponding methods of forward error correction channel encoding based on a generalized low-density parity-check convolutional code.

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GENERALIZED QUASI-CYCLIC LDPC CONVOLUTIONAL CODES FOR DIGITAL COMMUNICATION SYSTEMS

FIELD

The present disclosure relates to forward error correction. In particular, the present disclosure relates to Low-Density Parity-Check (LDPC) codes for channel coding in digital communication systems.

BACKGROUND

Fig. 1 shows a block diagram illustrating a generic digital communications system 10 in which the present disclosure may be practiced. The system includes a transmitting side comprising a generic encoder 12 and a receiving side comprising a generic decoder 14. The input of the generic encoder 12 at the transmitting side may be an information sequence IS_1 of k bits to which a redundancy sequence of r bits is added in an encoding operation performed by the generic encoder 12, thereby producing an encoded information sequence IS_2 of $k+r=n$ bits which may be forwarded to a modulator 16.

The modulator 16 may transform the encoded sequence IS_2 into a modulated signal vector CH_IN which is in turn transmitted through a channel 18 such as, for example, a radio channel or an optical channel. Since the channel 18 is usually subject to noisy disturbances, the channel output CH_OUT may differ from the channel input CH_IN.

At the receiving side, the channel output vector CH_OUT may be processed by a demodulator 20, which produces some likelihood ratio. The generic decoder 14 may use the redundancy in the received information sequence IS_3 in a decoding operation to correct errors in the received information sequence IS_3 and produce a decoded information sequence IS_4 which is an estimate of the information sequence IS_2 from which the information sequence IS_1 can be extracted.

The encoding operation and the decoding operation may be governed by an LDPC code. In the general formulation of channel coding, an LDPC code may employ a generator matrix G for the encoding operation in the generic encoder 12 and a parity-check matrix H for the decoding operation in the generic decoder 14.

For a LDPC code with an information sequence IS_1 of size $1xk$, a code word IS_2 of size $1xn$ and a redundancy (parity) sequence of $r=(n-k)$ bits, the generator matrix G

has size *kxn*, and the parity-check matrix *H* has size *rxn=(n-k)xn*. The parity-check matrix H_{rxn} and the generator matrix G_{kxn} enjoy the orthogonality property, which states that for any generator matrix G_{kxn} with *k* linearly independent rows there exists a parity-check matrix H_{rxn} with *r=(n-k)* linearly independent rows. Thus, any row of the generator matrix G_{kxn} is orthogonal to the rows of the parity-check matrix H_{rxn} such that the following equation is satisfied:

$$G_{kxn} \cdot H_{nxr}^{T} = 0 \qquad \textbf{(1)}$$

The encoding operation can be performed by means of a multiplication between the information sequence IS_1 and the generator matrix G_{kxn}, wherein the result of the multiplication provides the encoded output sequence IS_2 as follows:

$$IS_2 = IS_1 \cdot G_{kxn} \qquad \textbf{(2)}$$

At the receiving side, due to the orthogonality property between the generator matrix G_{kxn} and the parity-check matrix H_{rxn}, the following equation should be satisfied:

$$H_{rxn} \cdot IS_4^{T} = 0 \qquad \textbf{(3)}$$

where IS_4 is the decoded received information sequence of size *1xn*. If the above equation is verified, the information signal estimate IS_4 is correct.

Once the parity-check matrix H_{rxn} is generated, it is possible to obtain the generator matrix G_{kxn} and vice versa. Accordingly, any process of determining a parity-check matrix H_{rxn} may be mapped to an equivalent process of obtaining a generator matrix G_{kxn}, so that any process disclosed throughout the description and claims in relation to determining a parity-check matrix H_{rxn} shall be understood as encompassing the equivalent process of obtaining a generator matrix G_{kxn} and vice versa.

A particular form of the parity-check matrix H_{rxn} is a regular QC-LDPC matrix ${}^{reg}H_{rxn}^{QC}$ which can be divided into quadratic submatrices $I(p_{j,l})$, i.e. circulant matrices (or "circulants" for short), which may, for example, be obtained from cyclically right-shifting an *NxN* identity matrix $I(0)$ by $p_{j,l}$ positions:

$$ {}^{reg}H_{rxn}^{QC} = \begin{bmatrix} I(p_{0,0}) & I(p_{0,1}) & \cdots & I(p_{0,L-1}) \\ I(p_{1,0}) & I(p_{1,1}) & & I(p_{1,L-1}) \\ \vdots & \vdots & \ddots & \vdots \\ I(p_{J-1,0}) & I(p_{J-1,1}) & \cdots & I(p_{J-1,L-1}) \end{bmatrix} \quad \textbf{(4)} $$

with $N=n/L$ (cf. M. P. C. Fossorier, "*Quasi-Cyclic Low-Density Parity-Check Codes from Circulant Permutation Matrices*", IEEE TRANSACTIONS ON INFORMATION THEORY, Volume 50, Issue 8, Pages 1788–1793, August 2004). Thus, a regular QC-LDPC matrix ${}^{reg}H_{rxn}^{QC}$ may be defined by a base matrix B which satisfies:

$$ B = \begin{bmatrix} p_{0,0} & p_{0,1} & \cdots & p_{0,L-1} \\ p_{1,0} & p_{1,1} & & p_{1,L-1} \\ \vdots & \vdots & \ddots & \vdots \\ p_{J-1,0} & p_{J-1,1} & \cdots & p_{J-1,L-1} \end{bmatrix} \quad \textbf{(5)} $$

Moreover, an irregular QC-LDPC matrix ${}^{irreg}H_{rxn}^{QC}$ may be obtained by ${}^{irreg}H_{rxn}^{QC} = {}^{reg}H_{rxn}^{QC} \circ M_{mask}$ where "$\circ$" denotes the Hadamard product and

$$ M_{mask} = \begin{bmatrix} m_{0,0} & m_{0,1} & \cdots & m_{0,L-1} \\ m_{1,0} & m_{1,1} & & m_{1,L-1} \\ \vdots & \vdots & \ddots & \vdots \\ m_{J-1,0} & m_{J-1,1} & \cdots & m_{J-1,L-1} \end{bmatrix} \quad \textbf{(6)} $$

denotes a mask matrix with $m_{j,l} \in \{0,1\}$.

Thus, for employing a QC-LDPC code in the generic encoder 12 and the generic decoder 14, the generic encoder 12 and the generic decoder 14 may be provided with a circulant, shift values, i.e., values corresponding to the entries of the base matrix B, and (optionally) a mask matrix M_{mask}. For instance, an apparatus configured to choose shift values for determining a QC-LDPC matrix H_{rxn}^{QC} (or the corresponding generator matrix) may be integrated in (or connected to) the generic encoder 12 and/or the generic decoder 14. Moreover, the generic encoder 12 and the generic decoder 14 may also be provided with a mask matrix M_{mask} to generate an irregular QC-LDPC matrix ${}^{irreg}H_{rxn}^{QC}$.

While the above indicated QC-LDPC code is referred to as QC-LDPC block code, a QC-LDPC convolutional code may be derived from a QC-LDPC block code by forming a semi-infinite parity-check matrix H_{conv}^{QC} of the form

$$H_{conv}^{QC}=\begin{bmatrix} H_0 & & & & & \\ H_1 & H_0 & & & & \\ \vdots & \vdots & \ddots & & & \\ H_{m_s} & H_{m_s-1} & \cdots & H_0 & & \\ & H_{m_s} & H_{m_s-1} & \cdots & H_0 & \\ & \ddots & \ddots & & & \ddots \end{bmatrix} \quad (7)$$

such as, for instance,

$$H_{conv}^{QC}=\begin{bmatrix} I(p_{0,0}) & & & & & \\ I(p_{1,0}) & I(p_{1,1}) & & & & \\ \vdots & \vdots & \ddots & & & \\ I(p_{J-1,0}) & I(p_{J-1,1}) & \cdots & I(p_{J-1,L-1}) & & \\ & I(p_{0,1}) & \cdots & I(p_{0,L-1}) & I(p_{0,0}) & \\ & & \ddots & \ddots & \ddots & \ddots \end{bmatrix} \quad (8)$$

(cf. A. E. Pusane et al., "*Deriving Good LDPC Convolutional Codes from LDPC Block Codes*", IEEE TRANSACTIONS IN INFORMATION THEORY, Volume 57, Issue 2, Pages 835-857, February 2011).

Moreover, a QC-LDPC matrix H^{QC} can be described by its equivalent bipartite graph ("Tanner graph"), wherein each edge of the Tanner graph connects one variable node of a plurality of variable nodes (which from the first set of the bipartite graph) to one check node of a plurality of check nodes (which form the second set of the bipartite graph). For example, a QC-LDPC matrix H_{rxn}^{QC} of r rows and n columns can be represented by its equivalent bipartite graph with r check nodes and n variable nodes which has edges between the check nodes and the variable nodes if there are corresponding "1s" in the QC-LDPC matrix H_{rxn}^{QC} (cf. R. Tanner, "*A Recursive Approach to Low Complexity Codes*", IEEE TRANSACTIONS IN INFORMATION THEORY, Volume 27, Issue 5, Pages 533-547, September 1981). Thus, the variable nodes represent code-word bits and the check nodes represent parity-check equations.

In the Tanner graph of an LDPC code, any degree-s check node may be interpreted as a length-s single parity-check code, i.e., as an *(s, s−1)* linear block code. Thus, for generalizing an LDPC code, check nodes of the LDPC code may be replaced with a linear block code to enhance the overall minimum distance (cf. M. Lentmaier et al., "*On Generalized Low-Density Parity-Check Codes based on Hamming Component*

Codes", IEEE COMMUNICATIONS LETTERS, Volume 3, Issue 8, Pages 248-250, August 1999).

A finite set of connected edges in the Tanner graph, wherein the set starts and ends at the same node and satisfies the condition that no node (except the initial and final node) appears more than once forms a "cycle". The "length" of the cycle is the number of edges of the set. The "girth" of the Tanner graph (or "girth" in short) is the length of the shortest cycle(s) in the graph. In this regard, it is noted that short cycles in a Tanner graph of an LDPC code may prevent decoding algorithms from converging. Furthermore, short cycles may degrade the performance of the generic decoder 14, because they affect the independence of the extrinsic information exchanged in the iterative decoding. Accordingly, shift values are to be chosen that achieve a high girth of the Tanner graph representation of the respective LDPC matrix.

Moreover, a LDPC code may contain Trapping Sets (TSs). More particularly, a (a,b) TS contains b check nodes which have an odd number of connections to a variable nodes. Accordingly, when the a variable nodes are wrong, only the b check nodes will be unsatisfied which may lead to a high error floor, as a belief propagation algorithm employed in the decoder 14 may be "trapped" in a false minimum.

While the above approaches to channel coding such as regular or irregular QC-LDPC block or convolutional codes or generalized QC-LDPC block codes have proven to perform well for a wide variety of scenarios, the urge for higher data throughput requires even more sophisticated solutions that achieve high data throughput with decent encoding/decoding resources. It is thus the object of the present invention to provide for a more efficient forward error correction channel coding technique applicable to the generic digital communications system 10.

SUMMARY

According to a first aspect of the present invention, there is provided an encoder for forward error correction channel encoding, the encoder configured to determine a circulant matrix and shift values of a plurality of base matrices which define a plurality of quasi-cyclic low-density parity-check, QC-LDPC, matrices, construct a plurality of convolutional QC-LDPC matrices from the shift values and the circulant matrix, select a multitude of convolutional QC-LDPC matrices from the plurality of convolutional QC-LDPC matrices based on a measure of girth of the convolutional QC-LDPC matrices, generate a multitude of generalized convolutional QC-LDPC matrices from the multitude of convolutional QC-LDPC matrices using a linear block code as component code, choose a generalized convolutional QC-LDPC matrix of the multitude of generalized convolutional QC-LDPC matrices based on an indication of minimal code distance of the generalized convolutional QC-LDPC matrices, and encode a binary data stream using a generalized low-density parity-check convolutional code, GLDPC-CC, based on the chosen generalized convolutional QC-LDPC matrix.

In this regard, the term “circulant matrix” as used throughout the description and claims particularly refers to a matrix, e.g., the identity matrix, where each row vector is shifted one element to the right relative to the preceding row vector. Furthermore, the term “base matrix” as used throughout the description and claims in particular refers to an array of shift values defining a QC-LDPC matrix. The QC-LDPC matrix is divided into circulant submatrices with each circulant submatrix being generated by cyclically shifting the rows of a common circulant matrix by a number of places given by a corresponding shift value. In other words, each shift value of a base matrix gives the number of times by which the rows of a common circulant matrix, e.g., the identity matrix, are to be cyclically (right-)shifted to generate a corresponding submatrix, wherein the array of submatrices forms the QC-LDPC matrix.

It is to be noted that values forming a matrix do not necessarily have to be physically stored, presented in matrix- (or array-) form, or used in matrix algebra throughout a process involving the matrix. Rather the term “matrix” as used throughout the description and claims in particular refers to a set of (integer) values with assigned row and column indices or which are stored in a (logical) memory array. Moreover, if not involving matrix algebra or if respective matrix algebra routines are suitably redefined, the notion of rows and columns may even be changed or freely chosen. However, throughout the description and claims it is adhered to the mathematical concepts and notations regularly used in the art and they shall be understood as encompassing equivalent mathematical concepts and notations.

Moreover, the term “convolutional QC-LDPC matrix” as used throughout the description and claims in particular refers to a matrix having rows which contain vectors derived from the rows of a corresponding QC-LDPC matrix and zeros. The vectors in consecutive rows may only partially overlap in the vertical (column) direction while vectors fully overlapping in the vertical (column) direction form submatrices of the corresponding QC-LDPC matrix. A base matrix of a convolutional QC-LDPC matrix can be derived from the base matrix of a QC-LDPC matrix by inserting the rows of the base matrix of the QC-LDPC matrix as vectors into the rows of the base matrix of the convolutional QC-LDPC matrix and repeating the rows of the base matrix of the convolutional QC-LDPC matrix (a predefined number of times or ad infinitum) with each repeated vector being shifted to the right. Further, the term “generalized convolutional QC-LDPC matrix” as used throughout the description and claims in particular refers to a matrix having column vectors that correspond to columns of a (linear) block code control matrix.

By evolving the QC-LDP matrices to generalized convolutional QC-LDPC matrices, a low bit error rate (BER) and frame error rate (FER) can be achieved.

In a first possible implementation form of the encoder according to the first aspect, to generate the multitude of generalized convolutional QC-LDPC matrices from the multitude of convolutional QC-LDPC matrices, the encoder is further configured to expand each convolutional QC-LDPC matrix, wherein to expand a convolutional QC-LDPC matrix comprises to copy each row of the convolutional QC-LDPC matrix a predetermined number of times and to adjust a submatrix formed by the copied rows to the linear block code.

For example, the convolutional QC-LDPC matrix may be expanded by copying each row a given number of times and replacing “1-column-vectors” (i.e. vectors in a column filled with “1s”) of the copied rows with columns of a (linear) block code control matrix, thereby effectively replacing some of the ones in the vectors with zeros. By expanding the convolutional QC-LDPC matrices with linear block codes, the code distance can be increased.

In a second possible implementation form of the encoder according to the first aspect as such or according to the first implementation form of the first aspect, the linear block code is a Hamming code.

Using a Hamming code as the linear block code simplifies encoding of the convolutional code as it does, for instance, not require a zero information word for initialization.

In a third possible implementation form of the encoder according to the first aspect as such or according to the first or second implementation form of the first aspect, the shift values are equally divided onto a first row and a second row of the plurality of base matrices and wherein to construct the plurality of convolutional QC-LDPC matrices from the shift values, the encoder is further configured to construct convolutional base matrices of the plurality of convolutional QC-LDPC matrices having a third row inheriting the shift values of the first row and a fourth row inheriting the shift values of the second row, wherein the shift values of the fourth row are vertically displaced relative to the shift values of the third row.

Using base matrices with only two rows allows for a small number of cycles in the corresponding Tanner graph and thus simplifies achieving a high girth of the QC-LDPC code.

In a fourth possible implementation form of the encoder according to the first aspect as such or according to any of the preceding implementation forms of the first aspect, to encode the binary data stream using the GLDPC-CC, the encoder is further configured to group the binary data stream into multiple groups, wherein the size of each group corresponds to the size of the circulant, and to cyclically shift each group based on the circulant and a shift value of the shift values and to calculate parity data based on an output of a plurality of Hamming encoders fed with the cyclically shifted groups.

Using a serial encoding process based on cyclically shifting groups of bits of the binary data stream (i.e., applying the correspondingly shifted circulant) and feeding the cyclically shifted groups to the Hamming encoders allows for a reduction in processing complexity and thus an increase in processing speed.

In a fifth possible implementation form of the encoder according to the fourth implementation form of the first aspect, to calculate parity data based on an output of a plurality of Hamming encoders fed with the cyclically shifted groups, the encoder is further configured to cyclically shift the output of the plurality of Hamming encoders based on the circulant and shift values not applied to the multiple groups.

Applying matrices derived from inversely (left-)shifting the circulant by the remaining shift values to the output of the Hamming encoders allows to calculate the parity check bits with low computational effort.

In a sixth possible implementation form of the encoder according to the first aspect as such or according to any of the preceding implementation forms of the first aspect, to determine shift values of the plurality of QC-LDPC matrices, the encoder is configured to choose the shift values in a row-by-row order of a respective base matrix of a plurality of base matrices, and wherein shift values corresponding to entries in a row of the base matrix are chosen in an order that is at least in part based on a measure of a number of cycles in which check nodes and variable nodes of node groups corresponding to the entries, participate.

By choosing the shift-values in a row-by-row order, it is possible to create stronger connected row-code which is advantageous for layered belief-propagation decoding.

In a seventh possible implementation form of the encoder according to the sixth implementation form of the first aspect, the measure defines a cycle density and wherein the encoder is configured to choose values corresponding to entries in a row of the base matrix in order of decreasing cycle density of the corresponding node groups, given the already chosen values.

Filling the entries of the shift matrix in an order of decreasing cycle density of the corresponding Tanner graph representation gives greater freedom of choosing shift values for those node groups which do already participate in many cycles and for which the generation of further cycles is hence more probable. By this, cycle density of all parts of the Tanner graph becomes more balanced.

In an eighth possible implementation form of the encoder according to the first aspect as such or according to any of the preceding implementation forms of the first aspect, the encoder is configured to transmit the encoded binary data stream via an optical channel.

According to a second aspect of the present invention, there is provided a decoder for forward error correction channel decoding of a generalized quasi-cyclic low-density parity-check convolutional code, QC-LDPC-CC, the decoder including an interleaver and a plurality of Hamming decoders, wherein the interleaver is based on a circulant matrix and shift values of a first row of a base matrix of a quasi-cyclic low-density parity-check, QC-LDPC, matrix and is configured to pass groups of reordered bits of a bitstream to the Hamming decoders.

In this regard, the term "generalized quasi-cyclic low-density parity-check convolutional code" as used throughout the description and claims in particular refers to a code defined by a generalized convolutional QC-LDPC matrix. Moreover, the term "interleaver" as used throughout the description and claims in particular refers to an entity which reorders bits of a binary data stream received at the decoder into groups which are passed to the Hamming decoders. Notably, the Hamming decoders, although having favorable properties, may be replaced by other linear block code decoders.

By evolving the QC-LDP code to a generalized QC-LDPC convolutional code, a low bit error rate (BER) can be achieved.

In a first possible implementation form of the decoder according to the second aspect, the decoder further includes a de-interleaver and a second layer interleaver, the de-interleaver configured to receive an output of the Hamming decoders, de-interleave the output and pass the de-interleaved output to the second layer interleaver, wherein the second layer interleaver is based on the circulant matrix and shift values of a second row of the base matrix of the QC-LDPC matrix.

In a second possible implementation form of the decoder according to the second aspect as such or according to the first implementation form of the second aspect, the Hamming decoders are max-log-map decoders and the decoder applies an a-posteriori probability, APP, based algorithm.

According to a third aspect of the present invention, there is provided a system for forward error correction channel encoding comprising an encoder according to the first aspect as such or according to any of the preceding implementation forms of the first aspect and a decoder according to the second aspect as such or according to any of the preceding implementation forms of the second aspect.

According to a fourth aspect of the present invention, there is provided a method of forward error correction channel encoding, comprising determining a circulant and shift values of a plurality of quasi-cyclic low-density parity-check, QC-LDPC, matrices, constructing a plurality of convolutional QC-LDPC matrices from the circulant and the shift values, selecting a multitude of convolutional QC-LDPC matrices of the plurality of convolutional QC-LDPC matrices based on an indication of girth of the convolutional QC-LDPC matrices, generating a multitude of generalized convolutional QC-LDPC matrices from the selected convolutional QC-LDPC matrices, and encoding a binary data stream using a generalized low-density parity-check convolutional code, GLDPC-CC, based on a generalized convolutional QC-LDPC matrix chosen from the multitude of generalized convolutional QC-LDPC matrices.

According to a fifth aspect of the present invention, there is provided a method of forward error correction channel decoding, comprising passing messages between Hamming decoders, wherein the passing is governed by a convolutional quasi-cyclic low-density parity-check, QC-LDPC, matrix.

BRIEF DESCRIPTION OF THE DRAWINGS

Fig. 1 is a schematic illustration of a digital communication system;

Fig. 2 is a flow chart of a process of determining a QC-LDPC code;

Fig. 3 is a flow chart of a process of determining a generalized QC-LDPC convolutional code from a plurality of QC-LDPC codes;

Fig. 4 is a schematic illustration of an encoder;

Fig. 5 is a schematic illustration of a decoder; and

Fig. 6 is a schematic illustration of a first layer interleaver and a plurality of Hamming decoders.

DETAILED DESCRIPTION

As shown in Fig. 2, a process 22 of determining a QC-LDPC code may start at step 24 with choosing shift values in a row-by-row order of a base matrix. In an example, a base matrix B_{LxJ} may consist of $L=2$ rows and $J=15$ columns, wherein each entry of the base matrix, i.e., each shift value, is to define a corresponding shift of a circulant which, in the example, may be an identity matrix of size NxN. If seen from a Tanner graph perspective, choosing shift values in a row-by-row order corresponds to defining edges between all variable nodes and a first subset of the check nodes before defining edges between all variable nodes and a second subset of the check nodes. However, instead of choosing shift values in a row-by-row order, it is also contemplated that shift values may be chosen in a column-by-column order or in a random order, i.e. a row and column agnostic order.

As indicated at step 26, the entries within one row of the base matrix are consecutively chosen in an order that is at least in part based on a measure of a number of cycles in which variable nodes and check nodes of node groups corresponding to the row entries, participate. In the example, each row entry involves $N=500$ variable nodes and $N=500$ check nodes due to the circulant being an identity matrix of size $500x500$. However, because a single-layer QC-LDPC code derived from a circulant of weight one is represented by a tree-like graph structure in which no cycles exist, no cycle-density-based order can be established for the row that is labeled first. Thus, the shift values of the first row of the base matrix may be chosen in any order.

After labeling the first row of the base matrix with shift values "S_0" to "S_{14}", the base matrix may look like:

$$B_{2x15} = \begin{bmatrix} S_0 & S_1 & S_2 & S_3 & S_4 & S_5 & S_6 & S_7 & S_8 & S_9 & S_{10} & S_{11} & S_{12} & S_{13} & S_{14} \\ & & & & & & & & & & & & & & \end{bmatrix}$$

Due to the absence of cycles in a single-layer QC-LDPC code derived from a circulant of weight one, the second layer can be labelled starting at any entry. For example, the 9th entry in the second row may be labelled with shift value "S_{23}", so that the base matrix looks like:

$$B_{2x15} = \begin{bmatrix} S_0 & S_1 & S_2 & S_3 & S_4 & S_5 & S_6 & S_7 & S_8 & S_9 & S_{10} & S_{11} & S_{12} & S_{13} & S_{14} \\ & & & & & & & & S_{23} & & & & & & \end{bmatrix}$$

From this point onwards, entries in the second row of the base matrix are labelled in order of decreasing cycle-density of the corresponding node groups, taking into account the already chosen shift values, i.e., all entries of the first row and the one or more shift values in the second row that have already been chosen, e.g., the 9th entry. Accordingly, the selection of shift values in the second row may continue with labeling an entry that defines edges between a set of variable nodes and a set of check nodes that are involved in a larger number of cycles than variable nodes and check nodes corresponding to other entries. As each entry in a row corresponds to the same check nodes, the selection of shift values in the second row may continue with labeling an entry that, by being labelled, defines edges of a set of variable nodes that are involved in more cycles than any other set of variable nodes.

Each entry that is labelled in order of decreasing cycle-density may be labelled with a shift value which is selected from a plurality of possible shift values that all produce code that meets one or more quality criteria. For example, the shift values that are available for labelling an entry may all produce code that satisfies a pre-defined girth condition regarding the girth of the corresponding Tanner graph, which may, for example, be calculated using the equation in M. P. C. Fossorier, "*Quasi-Cyclic Low-Density Parity-Check Codes from Circulant Permutation Matrices*", IEEE TRANSACTIONS ON INFORMATION THEORY, Volume 50, Issue 8, Pages 1788–1793, August 2004.

Furthermore, a shift value may be selected from the available shift values based on a measure of connectivity of cycles in the corresponding Tanner graph representation that would result from the selection. For example, a shift value may be selected from the available shift values based on enumerating the TSs involved in the selection. In this regard, it may be made use of the fact that under certain conditions, e.g., that check nodes connected to the variable nodes in the cycle but not involved in the cycle are singly connected to the cycle, the ACE spectrum is equal to the extrinsic message degree (EMD) as described in Deka et al., "*On the Equivalence of the ACE and the EMD of a Cycle for the ACE Spectrum Constrained LDPC Codes*", 8th INTERNATIONAL SYMPOSIUM ON TURBO CODES AND ITERATIVE INFORMATION PROCESSING (ISTC), Pages 55-59, August 2014. Thus, a TS enumerator can be obtained by the ACE spectrum, because the cycle of length l contains $l/2$ variable nodes. For the ACE value η, the subgraph induced by the $l/2$ variable nodes constrains exactly η number of odd-degree check nodes. Hence, the cycle can be treated as $\mathrm{TS}\,(l/2,\eta)$. Moreover, the reliability of messages coming from check nodes singly connected to a cycle can be increased relative to messages coming from check nodes involved in the cycle in order to enhance the benefit of the messages coming from the rest of the Tanner graph.

In this way, better connectivity can be ensured for isolated small cycles and the performance of iterative decoders may be improved which may result in a gain in the waterfall region. The process 22 may then be continued until all shift values of the base matrix are chosen or, if only shift values exist for an entry that would produce code which violates a requirement such as a required minimal girth of the QC-LDPC code, the process 22 may be aborted. After choosing all shift values of the second row, the base matrix in the example looks like:

$$B_{2x15}\begin{bmatrix} S_0 & S_1 & S_2 & S_3 & S_4 & S_5 & S_6 & S_7 & S_8 & S_9 & S_{10} & S_{11} & S_{12} & S_{13} & S_{14} \\ S_{15} & S_{16} & S_{17} & S_{18} & S_{19} & S_{20} & S_{21} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} & S_{27} & S_{28} & S_{29} \end{bmatrix}$$

The process 22 may then be repeated a defined number of times to generate a plurality of base matrices. Furthermore, heuristics may be used to filter the generated base matrices in view of favorable properties of the corresponding QC-LDPC codes.

For example, the generated base matrices may be filtered based on parameters which measure how fast messages in a corresponding QC-LDPC code disperse to all nodes. More particularly, there exist $\lfloor (g-1)/4 \rfloor$ independent iterations, with g denoting the girth of the corresponding Tanner graph. Thus, after $diameter/2$ iterations, information from any variable nodes shall have reached any other variable node. In this case, if $diameter \geq \lfloor g-1 \rfloor /2$, all vertexes shall have statistically independent information. These considerations motivate a heuristic which is directed at selecting a graph design with large girth and smallest diameter for the degrees of the vertexes as described in Tanner et al., "*A Class of Group-Structured LDPC Codes*", PROCEEDINGS OF THE INTERNATIONAL SYMPOSIUM ON COMMUNICATION THEORY AND APPLICATIONS, 2001, Ambleside, England, 2001. Thus, one goal of filtering the base matrices may be to obtain QC-LDPC codes with maximal girth and minimum graph diameter.

Another possible filtering parameter is described in Wang et al., "*Hierarchical and High-Girth QC LDPC Codes*", IEEE TRANSACTIONS ON INFORMATION THEORY, Volume 59, Issue 7, pages 4553-4583, July 2013. Therein, a bipartite graph defined by a QC-LDPC code is taken and the spectral graph (connective) property of the corresponding graph is considered by the real-value adjacency (connectivity) matrix:

$$A^2 = \begin{pmatrix} HH^T & 0 \\ 0 & H^T H \end{pmatrix}$$

This representation is equvalent to

$$A = \begin{pmatrix} 0 & H \\ H^T & 0 \end{pmatrix} \quad \textbf{(8)}$$

because a change of column (VN) positions does not change code and eigenvalues monotonically. Thus, a similar order for spectral graph filtering of base matrices with a simplification of the calculation may be obtained. After diagonalizing and taking real eigenvalues $\mu_1 > \mu_2 > \cdots > \mu_x$, QC-LDPC codes may be filtered which achieve a QC-

LDPC code having a ratio of the second eigenvalue to the first eigenvalue $\frac{\mu_2}{\mu_1}$ which is below a predetermined threshold.

Thus, the base matrices may be filtered based on parameters which point at the quality of the corresponding QC-LDPC codes. In particular, the base matrices may be filtered based on a weight spectrum enumerator or a TS spectrum enumerator. Furthermore, the base matrices may be filtered based on Tanner spectral graph properties. After filtering some (or all) base matrices whose parameters are below or above pre-determined thresholds, the remaining base matrices may be further filtered based on code distance.

After filtering the base matrices, the performance of the QC-LDPC codes defined by the (sieved) base matrices may be simulated and the base matrices corresponding to the QC-LDPC codes that perform best may be determined. Hence, QC-LDPC codes with favorable ACE spectrum, Tanner spectral graph properties and large code distance may be simulated at one or more working points. For example, an *En/No* of 2 dB may be used and QC-LDPC codes exhibiting the probability of error (BER, FER, etc.) below threshold may be determined.

The determined base matrices may be used in a process 28 of determining a generalized QC-LDPC convolutional code from a plurality of QC-LDPC codes as shown in Fig. 3 which starts at step 30 with determining a circulant matrix and shift values of a plurality of base matrices which define a plurality of QC-LDPC matrices. For instance, the plurality of base matrices may be determined following the process 22 described above.

At step 32 of the process 28, the determined plurality of base matrices may be used to construct a plurality of convolutional QC-LDPC matrices. I.e., continuing with the above example, the determined base matrices may define a plurality of QC-LDPC block codes with QC-LDPC matrices of size $JN \times LN$ with $J=2$, $L=15$, and $N=500$.

A convolutional QC-LDPC matrix H_{conv}^{QC} may be constructed from a two-layer QC-LDPC matrix H_{LNxJN}^{QC} by copying M times each circulant $I(p_{j,l})$ of the QC-LDPC matrix H_{LNxJN}^{QC} (with $0 \le j < J$ and $0 < K < L-1$, K defining the overlap) while adhering to the following set of rules:

- $H_{conv}^{QC}\left(N(2m+a),N(l+Lm)+b\right)=I\left(p_{0,l}\right)(a,b)$ for $j=0$, $0\le m<M$, $0\le 1<L$ and $0\le a,b<N$.
- $H_{conv}^{QC}\left(N(1+2m+a),N(l+Lm)+b\right)=I\left(p_{1,l}\right)(a,b)$ for $j=1$, $0\le m<M$, $K\le 1<L$ and $0\le a,b<N$.
- $H_{conv}^{QC}\left(N(1+2m+a),N(l+L(m+1))+b\right)=I\left(p_{1,l}\right)(a,b)$ for $j=1$, $0\le m<M$, $0\le 1<K$ and $0\le a,b<N$.
- All remaining entries of H_{conv}^{QC} are filled with zeros.

For *K=7*, the first three rows of a base matrix B_{2Mx15M} of the resulting H_{conv}^{QC} will look like:

$$B_{2Mx15M}=\begin{bmatrix} S_0 & S_1 & S_2 & S_3 & S_4 & S_5 & S_6 & S_7 & S_8 & S_9 & S_{10} & S_{11} & S_{12} & S_{13} & S_{14} & & & \\ & & & & & & & S_{22} & S_{23} & S_{24} & S_{35} & S_{26} & S_{27} & S_{28} & S_{29} & S_{15} & S_{16} & \cdots \\ & & & & & & & & & & & & & & & S_0 & S_1 & \cdots \\ & & & & & & & & & & & & & & & & & \end{bmatrix}$$

After cutting the first *KN* columns of H_{conv}^{QC}, the base matrix $B_{2Mx15M-K}$ of the resulting ${}^{cut}H_{conv}^{QC}$ of size *JNM* ×*LNM-KN* will look like:

$$B_{2Mx15M-K}=\begin{bmatrix} S_7 & S_8 & S_9 & S_{10} & S_{11} & S_{12} & S_{13} & S_{14} & & & \\ S_{22} & S_{23} & S_{24} & S_{25} & S_{26} & S_{27} & S_{28} & S_{29} & S_{15} & S_{16} & \cdots \\ & & & & & & & & S_0 & S_1 & \cdots \\ & & & & & & & & & & \end{bmatrix}$$

Repeating the above procedure for some or all of the plurality of determined QC-LDPC block codes with QC-LDPC matrices of size *JN*×*LN*, a plurality of corresponding QC-LDPC convolutional codes may be constructed, wherein the QC-LDPC convolutional codes are defined by convolutional QC-LDPC matrices H_{conv}^{QC} of size *JNM* ×*LNM-KN*.

After selecting a multitude of convolutional QC-LDPC matrices from the plurality of convolutional QC-LDPC matrices based on a measure of girth of the convolutional QC-LDPC matrices as indicated by step 34, the process 28 is continued with generating a multitude of generalized convolutional QC-LDPC matrices from the multitude of convolutional QC-LDPC matrices using a linear block code as component code.

Continuing with the above example, the linear block code may be a Hamming code of size *4x15* such as:

$$H_{Ham}=\begin{bmatrix}1&1&0&1&1&0&1&0&1&0&1&1&0&0&0\\1&0&1&1&0&1&1&0&0&1&1&0&1&0&0\\0&1&1&1&0&0&0&1&1&1&1&0&0&1&0\\0&0&0&0&1&1&1&1&1&1&1&0&0&0&1\end{bmatrix}$$

However, other linear block codes such as Hadamard codes, Reed-Muller codes or Reed-Salomon codes are also contemplated.

To generate generalized convolutional QC-LDPC matrices $H^{QC}_{gener,conv}$ from the convolutional QC-LDPC matrices ${}^{cut}H^{QC}_{conv}$, each row of the convolutional QC-LDPC matrices ${}^{cut}H^{QC}_{conv}$ may be copied $P=4$ times (i.e., the number of rows of the linear block code) and the "1s" in each row may be changed to conform to entries of the corresponding row of the linear block code.

For example, let $b_{a,i}$ denote the column in which the i[th] "1" of the a[th] row (from left to right) of a convolutional QC-LDPC matrix ${}^{cut}H^{QC}_{conv}$ is placed (with $0\le i<L$ and $0\le a<JNM$). Then, $b_{a,i}$ may be determined according to the following set of rules:

- $b_{a,i}=(i-K)N+\operatorname{mod}(\operatorname{mod}(a,N)+p_{0,i},N)$ for $K\le i<L$ and $a<N$.
- $b_{a,i}=\left(i+L\left\lfloor\frac{\left\lfloor\frac{a}{N}\right\rfloor}{2}\right\rfloor\right)N+\operatorname{mod}\left(\operatorname{mod}(a,N)+p_{\left\lfloor\frac{a}{N}\right\rfloor,i+K},N\right)$ for $0\le i<L-K$, $a\ge N$ and $\left\lfloor\frac{a}{N}\right\rfloor$ odd.
- $b_{a,i}=\left(i+L\left\lfloor\frac{\left\lfloor\frac{a}{N}\right\rfloor}{2}\right\rfloor\right)N+\operatorname{mod}\left(\operatorname{mod}(a,N)+p_{\left\lfloor\frac{a}{N}\right\rfloor,i-(L-K)},N\right)$ for $L-K\le i<L$, $a\ge N$ and $\left\lfloor\frac{a}{N}\right\rfloor$ odd.

- $$b_{a,i} = \left(\left(i - K + L \left\lfloor \frac{\left\lfloor \frac{a}{N} \right\rfloor}{2} \right\rfloor \right) N + \mathrm{mod}\left(\mathrm{mod}(a,N) + p_{\left\lfloor \frac{a}{N} \right\rfloor, i}, N \right) \right) \quad \text{for } 0 \le i < L, \quad a \ge N$$

  and $\left\lfloor \frac{a}{N} \right\rfloor$ even.

Based on $b_{a,i}$ which represents another sparse representation of a convolutional QC-LDPC matrix ${}^{cut}H_{conv}^{QC}$, a generalized convolutional QC-LDPC matrix $H_{gener,conv}^{QC}$ can be generated by adhering to the following rule:

- $H_{gener,conv}^{QC}(a+c, b_{a,i}) = H_{Ham}(c,i)$ for $0 \le a < JNM$, $0 \le c < P$ and $0 \le i < L$.

By applying this procedure to some or all of the convolutional QC-LDPC matrices H_{conv}^{QC} selected in step 34, a multitude of generalized convolutional QC-LDPC matrices $H_{gener,conv}^{QC}$ can be generated.

After generating a multitude of generalized convolutional QC-LDPC matrices $H_{gener,conv}^{QC}$, the process 28 is continued at step 38 with choosing a generalized convolutional QC-LDPC matrix of the multitude of generalized convolutional QC-LDPC matrices based on an indication of minimal code distance of the generalized convolutional QC-LDPC matrices. In addition, choosing a generalized convolutional QC-LDPC matrix of the multitude of generalized convolutional QC-LDPC matrices may be based on enumerating a TS spectrum of the generalized convolutional QC-LDPC matrices or one or more other code quality criteria of the code quality criteria discussed above.

A GLDPC-CC based on the chosen generalized convolutional QC-LDPC matrix $H_{gener,conv}^{QC}$ may then be used to encode a binary data stream IS_1. Continuing with the above example, a codeword E of a binary data stream IS_1 encoded with the GLDPC-CC has length *LNM-KN* with a repeat structure of *(L-K)N-PN* information bits, *PN* check bits, *KN-PN* information bits and *PN* check bits. For example, the first *N* rows connect *(L-K)N* variable nodes to *PN* check nodes. The next *N* rows connect *LN* variable nodes to *PN* check nodes. However, the first *(L-K)N* variable nodes are already given, leaving *KN* free variable nodes

Due to the structure of the Hamming code H_{Ham}, the first *PN* check bits can be directly calculated by multiplying the first *(L-K)N-PN* information bits with the corresponding submatrix of size *PNx(L-K)N-PN* of the first layer of $H^{QC}_{gener,conv}$. The vector of the first *(L-K)N-PN* information bits and the appended calculated *PN* check bits and *KN-PN* information bits is then multiplied with the corresponding submatrix of size *PNxLN-PN* of the second layer of $H^{QC}_{gener,conv}$ to calculate the next *PN* check bits.

The codeword *E* may also be determined by shifting the information bits in accordance with the shift values S_1, S_2, S_3, S_5, S_6, S_7, S_8, S_9, S_{10}, and S_{11} for even layers or in accordance with the shift values S_{16}, S_{17}, S_{22}, S_{23}, S_{24}, S_{25}, S_{26}, S_{27}, S_{28}, and S_{29} for odd layers of the base matrix $B_{2Mx15M-K}$ and feeding *N* identical Hamming encoders (or equivalently *N* times the same Hamming encoder) with the shifted information bits and shifting the check bits by S_{12}, S_{13}, S_{14}, and S_{15} for even layers or by S_{18}, S_{19}, S_{20}, and S_{21} for odd layers.

Fig. 4 shows an encoder 40 which can be used instead of the generic encoder 12 in the generic system 10 of Fig. 1. The encoder 40 is configured to receive the binary data stream *B1*, carry out some or all steps of the above processes 22 and 28 and to transmit the codeword *E*. The encoder 40 comprises a processor 42 and a memory 44 coupled to the processor 42. The memory 44 may be a computer-readable medium such as an optic or a magnetic storage, which comprises persistently stored computer-readable instructions that when carried out by the processor 42 implement some or all steps of the above processes 22 and 28. Moreover, although the encoder 40 of Fig. 4 is shown as a single entity, the encoder 40 may cooperate with one or more devices connected to the encoder 40 via a wireless or wired link to carry out some or all steps of the above processes 22 and 28 cooperatively. Furthermore, the encoder 40 may be connected to the modulator 16. The modulator 16 may transform the codeword *E* into a modulated signal vector CH_IN which is in turn transmitted through the channel 18.

At the receiving end, the channel output vector CH_OUT may be processed by the demodulator 20 which produces some likelihood ratio. However, instead of being connected to the generic decoder 14, the demodulator 20 may be connected to a decoder 46 as shown in Fig. 5. The decoder 46 comprises a processor 48 and a memory 50 coupled to the processor 48 and is configured to use the redundancy in the received information sequence IS_3 in a decoding operation, e.g., a BCJR, Chase

Pyndiah, or Fast MAP decoding algorithm, to correct errors in the information sequence of the received information sequence IS_3 and to produce a decoded signal IS_4 which is an information sequence estimate.

For example, the memory 50 may be a computer-readable medium such as an optic or a magnetic storage, which comprises persistently stored computer-readable instructions that when carried out by the processor 48 implement a first-layer interleaver 52 and a plurality of (identical) Hamming decoders 54 as shown in Fig. 6. Continuing with the above example, the first-layer interleaver 52 may be based on the circulant of size NxN and the shift values of the first row of the base matrix B_{LxJ} defining the QC-LDPC matrix H^{QC}_{LNxJN} corresponding to the GLDPC-CC used for encoding the binary data stream IS_1. After interleaving the bits of the received information sequence IS_3, the interleaver 52 passes groups of the reordered bits of the bitstream IS_1 to N identical Hamming decoders 54 (or equivalently consecutively N times to the same Hamming decoder 54).

The Hamming decoders 54 may be implemented as soft-input soft-output (SISO) decoders, e.g., max-log-map decoders, and pass a posteriori probability values and extrinsic values to a de-interleaver which de-interleaves the values. The de-interleaved a posteriori probability values and extrinsic values and the extrinsic values may then be passed by the de-interleaver to a second layer interleaver which is based on the circulant matrix and shift values of a second row of the base matrix B_{LxJ} defining the QC-LDPC matrix H^{QC}_{LNxJN} corresponding to the GLDPC-CC used for encoding the binary data stream IS_1.

After a predetermined number of iterations, a hard decision may be calculated based on the determined log-likelihood ratios and a corresponding estimate for the binary data stream IS_1 may be determined.

CLAIMS

1. An encoder for forward error correction channel encoding, the encoder configured to:

determine a circulant matrix and shift values of a plurality of base matrices which define a plurality of quasi-cyclic low-density parity-check, QC-LDPC, matrices;

construct a plurality of convolutional QC-LDPC matrices from the shift values and the circulant matrix;

select a multitude of convolutional QC-LDPC matrices from the plurality of convolutional QC-LDPC matrices based on a measure of girth of the convolutional QC-LDPC matrices;

generate a multitude of generalized convolutional QC-LDPC matrices from the multitude of convolutional QC-LDPC matrices using a linear block code as component code;

choose a generalized convolutional QC-LDPC matrix of the multitude of generalized convolutional QC-LDPC matrices based on an indication of minimal code distance of the generalized convolutional QC-LDPC matrices; and

encode a binary data stream using a generalized low-density parity-check convolutional code, GLDPC-CC, based on the chosen generalized convolutional QC-LDPC matrix.

2. The encoder of claim 1, wherein to generate the multitude of generalized convolutional QC-LDPC matrices from the multitude of convolutional QC-LDPC matrices, the encoder is further configured to expand each convolutional QC-LDPC matrix, wherein to expand a convolutional QC-LDPC matrix comprises to copy each row of the convolutional QC-LDPC matrix a predetermined number of times and to adjust a submatrix formed by the copied rows to the linear block code.

3. The encoder of claim 1 or 2, wherein the linear block code is a Hamming code.

4. The encoder of any one of claims 1 to 3, wherein the shift values are equally divided onto a first row and a second row of the plurality of base matrices and wherein to construct the plurality of convolutional QC-LDPC matrices from the shift values, the encoder is further configured to construct convolutional base matrices of the plurality of convolutional QC-LDPC matrices having a third row inheriting the shift

values of the first row and a fourth row inheriting the shift values of the second row, wherein the shift values of the fourth row are vertically displaced relative to the shift values of the third row.

5. The encoder of any one of claims 1 to 4, wherein to encode the binary data stream using the GLDPC-CC, the encoder is further configured to group the binary data stream into multiple groups, wherein the size of each group corresponds to the size of the circulant, and to cyclically shift each group based on the circulant and a shift value of the shift values and to calculate parity data based on an output of a plurality of Hamming encoders fed with the cyclically shifted groups.

6. The encoder of claim 5, wherein to calculate parity data based on an output of a plurality of hamming encoders fed with the cyclically shifted groups, the encoder is further configured to cyclically shift the output of the plurality of Hamming encoders based on the circulant and shift values not applied to the multiple groups.

7. The encoder of any one of claims 1 to 6, wherein to determine shift values of the plurality of QC-LDPC matrices, the encoder is configured to choose the shift values in a row-by-row order of a respective base matrix of a plurality of base matrices, and wherein shift values corresponding to entries in a row of the base matrix are chosen in an order that is at least in part based on a measure of a number of cycles in which check nodes and variable nodes of node groups corresponding to the entries, participate.

8. The encoder of claim 7, wherein the measure defines a cycle density and wherein the encoder is configured to choose values corresponding to entries in a row of the base matrix in order of decreasing cycle density of the corresponding node groups, given the already chosen values.

9. The encoder of any one of claims 1 to 8, wherein the encoder is configured to transmit the encoded binary data stream via an optical channel.

10. A decoder for forward error correction channel decoding of a generalized quasi-cyclic low-density parity-check convolutional code, QC-LDPC-CC, the decoder including an interleaver and a plurality of Hamming decoders, wherein the interleaver is based on a circulant matrix and shift values of a first row of a base matrix of a quasi-cyclic low-density parity-check, QC-LDPC, matrix and is configured to pass groups of reordered bits of a bitstream to the Hamming decoders.

11. The decoder of claim 10, wherein the decoder further includes a de-interleaver and a second layer interleaver, the de-interleaver configured to receive an output of the Hamming decoders, de-interleave the output and pass the de-interleaved output to the second layer interleaver, wherein the second layer interleaver is based on the circulant matrix and shift values of a second row of the base matrix of the QC-LDPC matrix.

12. The decoder of claim 10 or 11, wherein the Hamming decoders are max-log-map decoders and the decoder applies an a-posteriori probability, APP, based algorithm.

13. A system for forward error correction channel encoding comprising an encoder according to any one of claims 1 to 9 and a decoder according to any one of claims 10 to 12.

14. A method of forward error correction channel encoding, comprising:

determining a circulant and shift values of a plurality of quasi-cyclic low-density parity-check, QC-LDPC, matrices;

constructing a plurality of convolutional QC-LDPC matrices from the circulant and the shift values;

selecting a multitude of convolutional QC-LDPC matrices of the plurality of convolutional QC-LDPC matrices based on an indication of girth of the convolutional QC-LDPC matrices;

generating a multitude of generalized convolutional QC-LDPC matrices from the selected convolutional QC-LDPC matrices; and

encoding a binary data stream using a generalized low-density parity-check convolutional code, GLDPC-CC, based on a generalized convolutional QC-LDPC matrix chosen from the multitude of generalized convolutional QC-LDPC matrices.

15. A method of forward error correction channel decoding, comprising:

passing messages between Hamming decoders, wherein the passing is governed by a convolutional quasi-cyclic low-density parity-check, QC-LDPC, matrix.

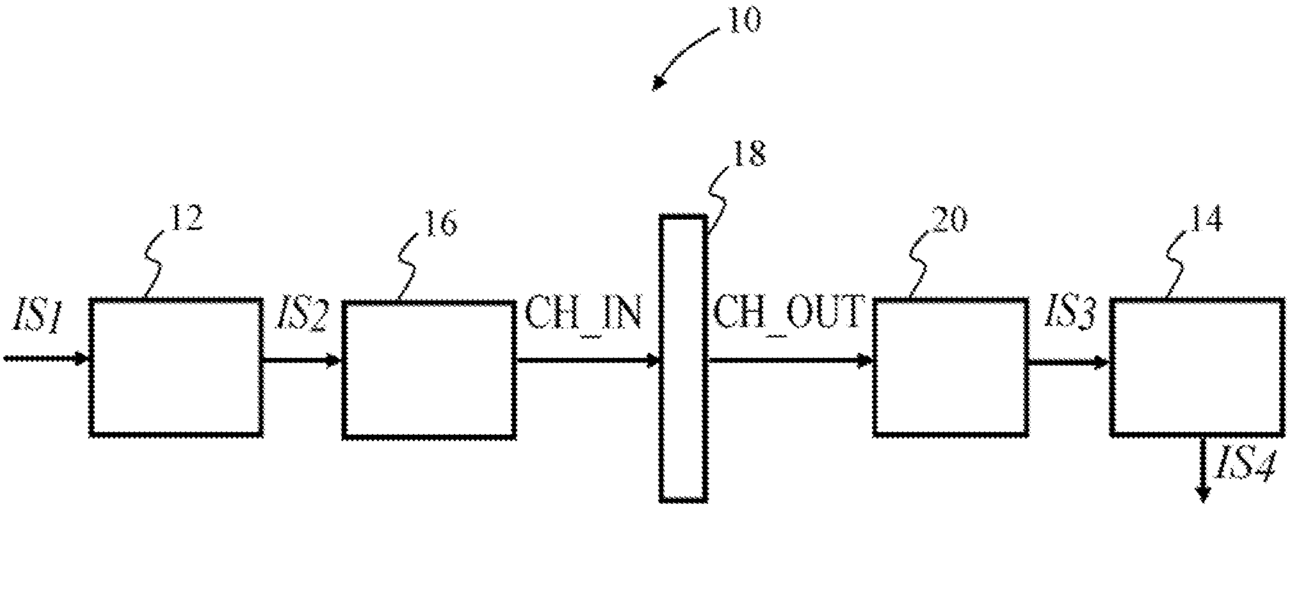
10
12
16
18
20
14
IS1
IS2
CH_IN
CH_OUT
IS3
IS4

FIG.1

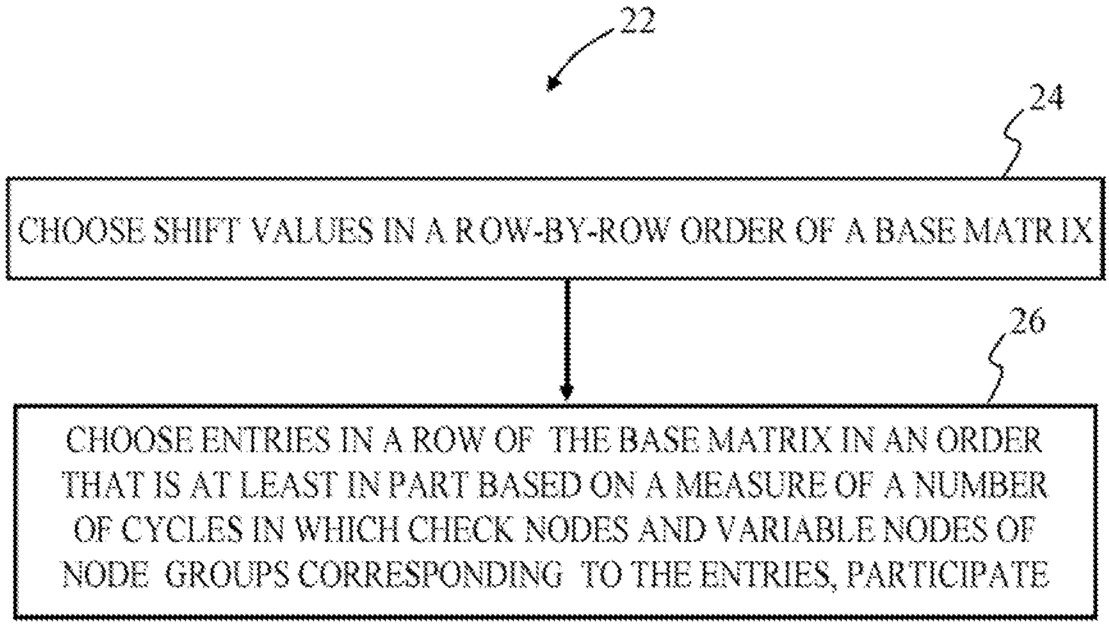
22
24
CHOOSE SHIFT VALUES IN A ROW-BY-ROW ORDER OF A BASE MATRIX
26
CHOOSE ENTRIES IN A ROW OF THE BASE MATRIX IN AN ORDER THAT IS AT LEAST IN PART BASED ON A MEASURE OF A NUMBER OF CYCLES IN WHICH CHECK NODES AND VARIABLE NODES OF NODE GROUPS CORRESPONDING TO THE ENTRIES, PARTICIPATE

FIG.2

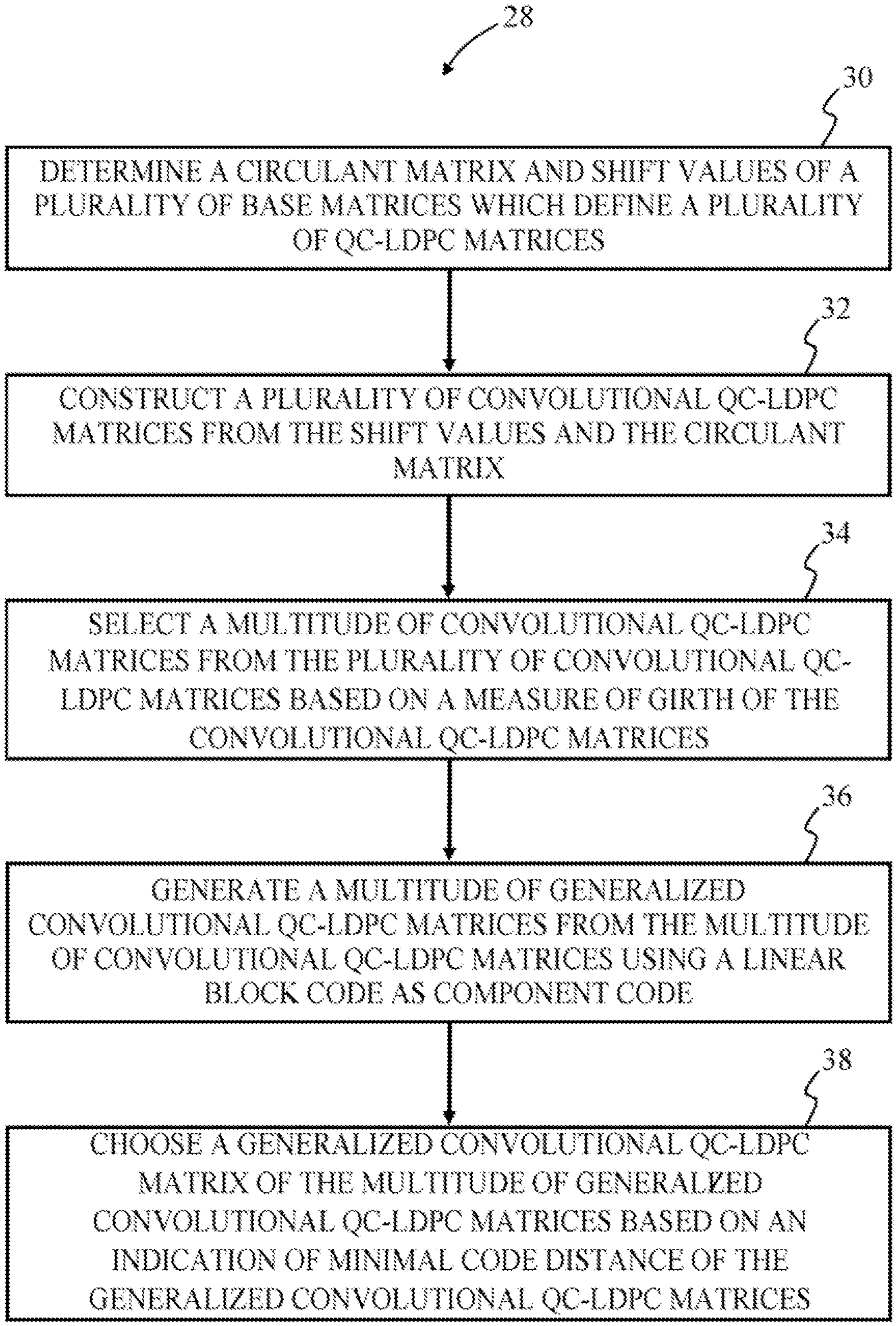
28
30
DETERMINE A CIRCULANT MATRIX AND SHIFT VALUES OF A PLURALITY OF BASE MATRICES WHICH DEFINE A PLURALITY OF QC-LDPC MATRICES
32
CONSTRUCT A PLURALITY OF CONVOLUTIONAL QC-LDPC MATRICES FROM THE SHIFT VALUES AND THE CIRCULANT MATRIX
34
SELECT A MULTITUDE OF CONVOLUTIONAL QC-LDPC MATRICES FROM THE PLURALITY OF CONVOLUTIONAL QC-LDPC MATRICES BASED ON A MEASURE OF GIRTH OF THE CONVOLUTIONAL QC-LDPC MATRICES
36
GENERATE A MULTITUDE OF GENERALIZED CONVOLUTIONAL QC-LDPC MATRICES FROM THE MULTITUDE OF CONVOLUTIONAL QC-LDPC MATRICES USING A LINEAR BLOCK CODE AS COMPONENT CODE
38
CHOOSE A GENERALIZED CONVOLUTIONAL QC-LDPC MATRIX OF THE MULTITUDE OF GENERALIZED CONVOLUTIONAL QC-LDPC MATRICES BASED ON AN INDICATION OF MINIMAL CODE DISTANCE OF THE GENERALIZED CONVOLUTIONAL QC-LDPC MATRICES

FIG.3

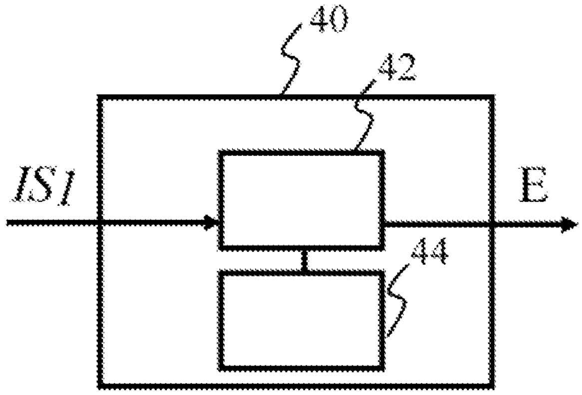

FIG.4

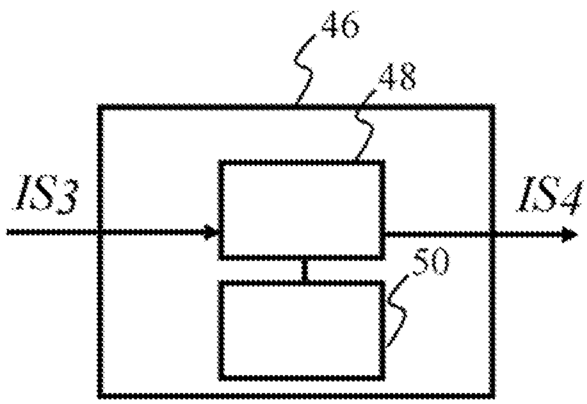

FIG.5

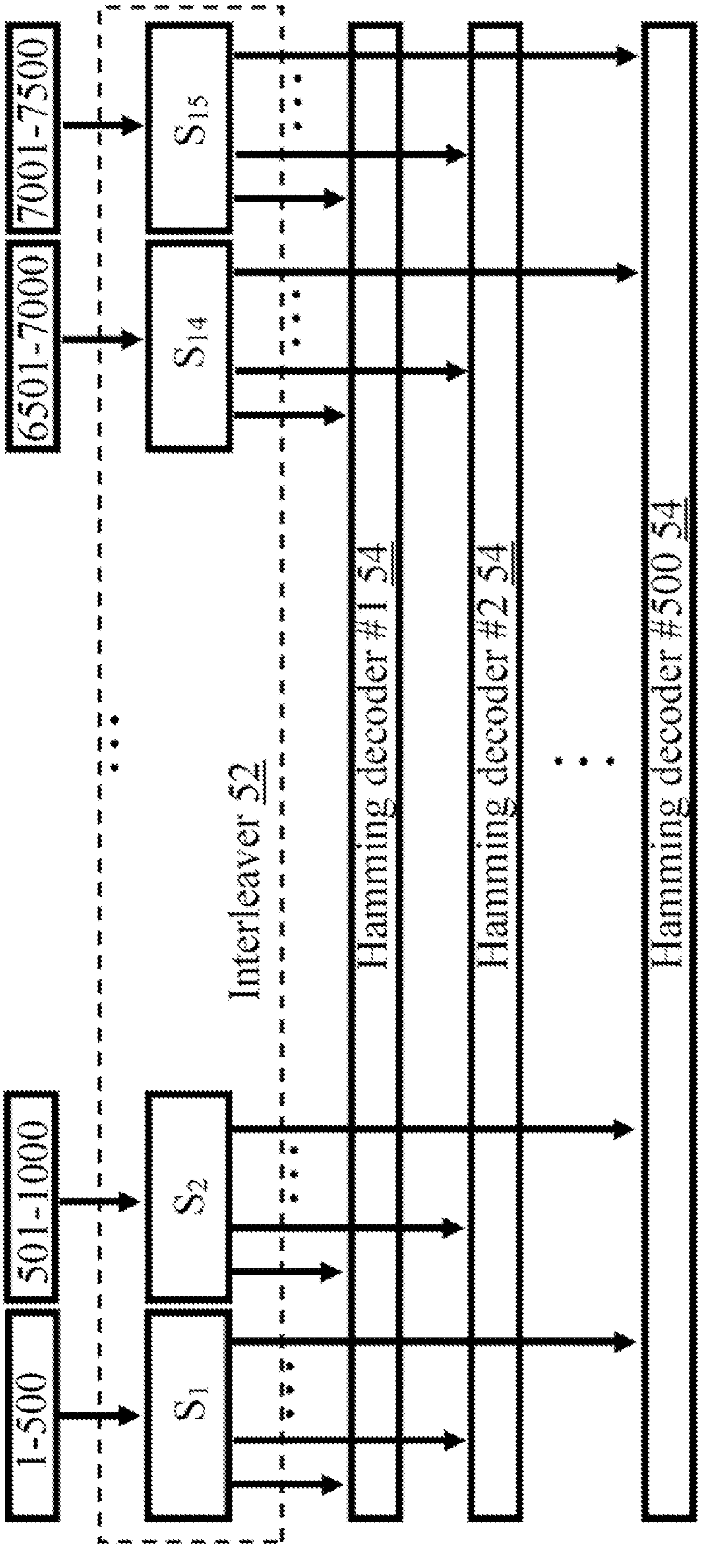
1-500
501-1000
6501-7000
7001-7500
S_1
S_2
S_{14}
S_{15}
Interleaver 52
Hamming decoder #1 54
Hamming decoder #2 54
Hamming decoder #500 54

FIG.6

INTERNATIONAL SEARCH REPORT

International application No
PCT/RU2016/050027

A. CLASSIFICATION OF SUBJECT MATTER
INV. H03M13/11 H03M13/03
ADD. H03M13/19

According to International Patent Classification (IPC) or to both national classification and IPC

B. FIELDS SEARCHED

Minimum documentation searched (classification system followed by classification symbols)
H03M

Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched

Electronic data base consulted during the international search (name of data base and, where practicable, search terms used)

EPO-Internal, WPI Data

C. DOCUMENTS CONSIDERED TO BE RELEVANT

Category*	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
Y A	MITCHELL DAVID G M ET AL: "On the minimum distance of generalized spatially coupled LDPC codes", PROC., IEEE INTERNATIONAL SYMPOSIUM ON INFORMATION THEORY, IEEE, 7 July 2013 (2013-07-07), pages 1874-1878, XP032496913, ISSN: 2157-8095, DOI: 10.1109/ISIT.2013.6620551 [retrieved on 2013-10-03] the whole document ----- -/--	1-6,9-15 7,8

[X] Further documents are listed in the continuation of Box C. [X] See patent family annex.

* Special categories of cited documents :

"A" document defining the general state of the art which is not considered to be of particular relevance
"E" earlier application or patent but published on or after the international filing date
"L" document which may throw doubts on priority claim(s) or which is cited to establish the publication date of another citation or other special reason (as specified)
"O" document referring to an oral disclosure, use, exhibition or other means
"P" document published prior to the international filing date but later than the priority date claimed

"T" later document published after the international filing date or priority date and not in conflict with the application but cited to understand the principle or theory underlying the invention
"X" document of particular relevance; the claimed invention cannot be considered novel or cannot be considered to involve an inventive step when the document is taken alone
"Y" document of particular relevance; the claimed invention cannot be considered to involve an inventive step when the document is combined with one or more other such documents, such combination being obvious to a person skilled in the art
"&" document member of the same patent family

Date of the actual completion of the international search	Date of mailing of the international search report
21 December 2016	05/01/2017

Name and mailing address of the ISA/	Authorized officer
European Patent Office, P.B. 5818 Patentlaan 2 NL - 2280 HV Rijswijk Tel. (+31-70) 340-2040, Fax: (+31-70) 340-3016	Offer, Elke

2

INTERNATIONAL SEARCH REPORT

International application No
PCT/RU2016/050027

C(Continuation). DOCUMENTS CONSIDERED TO BE RELEVANT

Category*	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
Y	LIVA G ET AL: "Quasi-cyclic generalized LDPC codes with low error floors", IEEE TRANSACTIONS ON COMMUNICATIONS, IEEE SERVICE CENTER, PISCATAWAY, NJ. USA, vol. 56, no. 1, 1 January 2008 (2008-01-01), pages 49-57, XP011224721, ISSN: 0090-6778, DOI: 10.1109/TCOMM.2008.050600	1-6,9-15
A	the whole document	7,8

Y	MICHAEL LENTMAIER ET AL: "Density evolution analysis of Protograph-based braided block codes on the erasure channel", PROC., INTERNATIONAL ITG CONFERENCE ON SOURCE AND CHANNEL CODING (SCC), IEEE, PISCATAWAY, NJ, USA, 18 January 2010 (2010-01-18), pages 1-6, XP031657414, ISBN: 978-1-4244-6872-0	1-6,9-15
A	the whole document	7,8

A	US 2010/257425 A1 (YUE GUOSEN [US] ET AL) 7 October 2010 (2010-10-07) claim 1 paragraph [0051] - paragraph [0086]	1-15

A	TAO TIAN ET AL: "Construction of irregular LDPC codes with low error floors", NEW FRONTIERS IN TELECOMMUNICATIONS : 2003 IEEE INTERNATIONAL CONFERENCE ON COMMUNICATIONS ; ICC 2003 ; 11 - 15 MAY 2003, ANCHORAGE, ALASKA, USA; [IEEE INTERNATIONAL CONFERENCE ON COMMUNICATIONS], IEEE OPERATIONS CENTER, PISCATAWAY, NJ, vol. 5, 11 May 2003 (2003-05-11), pages 3125-3129, XP010643022, DOI: 10.1109/ICC.2003.1203996 ISBN: 978-0-7803-7802-5 the whole document	1-15

A	VUKOBRATOVIC D ET AL: "Generalized ACE Constrained Progressive Edge-Growth LDPC Code Design", IEEE COMMUNICATIONS LETTERS, IEEE SERVICE CENTER, PISCATAWAY, NJ, US, vol. 10, no. 1, 1 January 2008 (2008-01-01), pages 32-34, XP011224659, ISSN: 1089-7798 the whole document	1-15

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2

INTERNATIONAL SEARCH REPORT

International application No
PCT/RU2016/050027

C(Continuation). DOCUMENTS CONSIDERED TO BE RELEVANT

Category*	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
A	Vasily  Usatyuk: "Some problems of Graph Based Codes for Belief Propagation decoding Quasi-cyclic Low Density Parity-check code (QC-LDPC)", , 24 November 2015 (2015-11-24), XP055302971, Retrieved from the Internet: URL:https://arxiv.org/ftp/arxiv/papers/1511/1511.00133.pdf [retrieved on 2016-09-15] the whole document -----	1-15
A	LENTMAIER M ET AL: "On generalized low-density parity-check codes based on Hamming component codes", IEEE COMMUNICATIONS LETTERS, IEEE SERVICE CENTER, PISCATAWAY, NJ, US, vol. 3, no. 8, 1 August 1999 (1999-08-01), pages 248-250, XP011423422, ISSN: 1089-7798, DOI: 10.1109/4234.781010 cited in the application the whole document -----	1-15
A	IVAN B. DJORDJEVIC ET AL: "Multiple component codes based generalized LDPC codes for high-speed optical transport", OPTICS EXPRESS, vol. 22, no. 14, 30 June 2014 (2014-06-30), page 16694, XP055173086, DOI: 10.1364/OE.22.016694 the whole document -----	1-15

2

INTERNATIONAL SEARCH REPORT

International application No.
PCT/RU2016/050027

Box No. II Observations where certain claims were found unsearchable (Continuation of item 2 of first sheet)

This international search report has not been established in respect of certain claims under Article 17(2)(a) for the following reasons:

1. ☐ Claims Nos.:
because they relate to subject matter not required to be searched by this Authority, namely:

2. ☐ Claims Nos.:
because they relate to parts of the international application that do not comply with the prescribed requirements to such an extent that no meaningful international search can be carried out, specifically:

3. ☐ Claims Nos.:
because they are dependent claims and are not drafted in accordance with the second and third sentences of Rule 6.4(a).

Box No. III Observations where unity of invention is lacking (Continuation of item 3 of first sheet)

This International Searching Authority found multiple inventions in this international application, as follows:

see additional sheet

1. ☐ As all required additional search fees were timely paid by the applicant, this international search report covers all searchable claims.

2. ☒ As all searchable claims could be searched without effort justifying an additional fees, this Authority did not invite payment of additional fees.

3. ☐ As only some of the required additional search fees were timely paid by the applicant, this international search report covers only those claims for which fees were paid, specifically claims Nos.:

4. ☐ No required additional search fees were timely paid by the applicant. Consequently, this international search report is restricted to the invention first mentioned in the claims; it is covered by claims Nos.:

Remark on Protest

☐ The additional search fees were accompanied by the applicant's protest and, where applicable, the payment of a protest fee.

☐ The additional search fees were accompanied by the applicant's protest but the applicable protest fee was not paid within the time limit specified in the invitation.

☐ No protest accompanied the payment of additional search fees.

FURTHER INFORMATION CONTINUED FROM PCT/ISA/ 210

This International Searching Authority found multiple (groups of) inventions in this international application, as follows:

1. claims: 1-15

Encoder/system comprising encoder/encoding method configured to/comprising code generating and choosing steps for a generalized convolutional QC-LDPC code and an encoding step using the chosen code.

1.1. claims: 10-12, 15

Decoder for a generalized convolutional QC-LDPC code including an interleaver and Hamming decoders, wherein the interleaver is based on a circulant matrix and shift values of a first row of a base matrix of a QC LDPC matrix and is configured to pass groups of reordered bits of a bitstream to the Hamming decoders. Decoding method for a generalized convolutional QC-LDPC code including passing messages between Hamming decoders wherein the passing is governed by a convolutional QC-LDPC matrix.

INTERNATIONAL SEARCH REPORT

Information on patent family members

International application No
PCT/RU2016/050027

Patent document cited in search report	Publication date	Patent family member(s)	Publication date
US 2010257425 A1	07-10-2010	NONE	