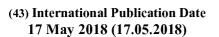


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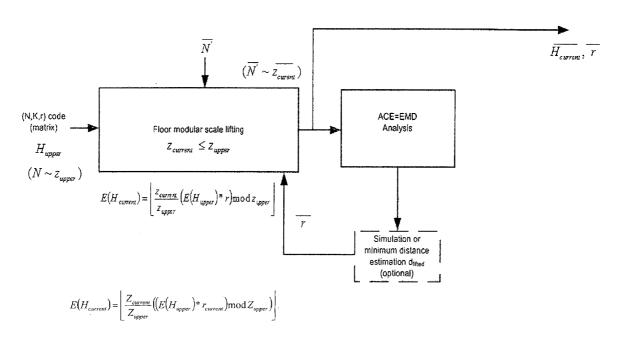


Fig. 2

(57) Abstract: A method for quasi-cyclic low-density parity-check (QC-LDPC) encoding and decoding of a data packet by a lifted matrix is provided, the method comprising: lifting the QC-LDPC code for maximal code length Nmax and maximal circulant size Z upper of the base matrix; generating a plurality of optimal values r, for a plurality of circulants  $Z_1, Z_2,...,Z_{upper}$  based on the QC-LDPC code lifted for maximal length  $N_{max}$ ,  $0 \le r \le Z_{upper} - 1$ ; saving the generated plurality of optimal values r; corresponding to the plurality of circulants  $Z_1, Z_2,..., Z_{upper}$  and a matrix for the QC-LDPC code lifted for maximal length  $N_{max}$  in the memory unit; receiving a current circulant  $Z_1, Z_2,..., Z_n$  current from the plurality of circulants  $Z_1, Z_2,..., Z_n$  selecting a current optimal

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value r <sub>current</sub> from the plurality of optimal values r; stored in the memory unit corresponding to the current circulant Z <sub>current</sub>; and lifting the base matrix based on the current optimal value r <sub>current</sub>, wherein a floor scale modular lifting of the base matrix is calculated as: where  $E(H_{upper})$  is a value of circulant shift in the base matrix for maximal circulant size; wherein  $0 \le r$  <sub>current</sub>  $\le z$  <sub>upper</sub> 1 and r <sub>current</sub> =1 is excluded. The apparatus for QC-LDPC encoding and decoding of a data packet by a lifted matrix is further provided.

METHOD AND APPARATUS FOR ENCODING AND DECODING OF VARIABLE LENGTH QUASI-CYCLIC LOW-DENSITY PARITY-CHECK, QC-LDPC, CODES

## TECHNICAL FIELD

The present invention relates to a method for quasi-cyclic low-density parity-check (QC-LDPC) encoding and decoding and an apparatus for quasi-cyclic low-density parity-check encoding and decoding.

The present invention also relates to a computer-readable storage medium storing program code, the program code comprising instructions for carrying out such a method.

### **BACKGROUND**

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Error-correcting coding is an efficient method to improve capacity of a communication system. Wireless systems require employing a large set of code with different length and rate. For example LTE provides more than several thousand of different code lengths and rates using a hardware friendly interleaver and simply puncturing pattern, but the sequential nature of the BCJR decoder of Turbo code significantly limits parallelism – decoder throughput. Hence, it is thus a problem how to create a compact representation of QC-LDPC codes, which supports sets of QC-LDPC codes with different lengths and rates. Other problems to be solved include getting additive increase of circulant size to minimize gap between several lengths of code; to define some block-structured memory efficient puncture pattern with minimal performance lost; and to maximize number of variable node in block-structured which recover under practical number iteration.

A problem of existing floor lifting methods is the possibility of appearing of short cycles in parity check matrices and bad weight spectrum of codewords. This leads to lower code gain.

### SUMMARY OF THE INVENTION

The objective of the present invention is to provide a method for quasi-cyclic low-density parity-check encoding and decoding and an apparatus for quasi-cyclic low-density parity-

check encoding and decoding, wherein the method for QC-LDPC encoding and decoding and the apparatus for QC-LDPC encoding and decoding overcome one or more of the above-mentioned problems of the prior art. Aspects of the invention provide error correction, especially to channel coding for wireless communication, such as WI-FI or 5G communication.

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The foregoing and other objects are achieved by the features of the independent claims. Further implementation forms are apparent from the dependent claims, the description and the figures.

A first aspect of the invention provides a method for quasi-cyclic low-density parity-check (QC-LDPC) encoding and decoding of a data packet by a lifted matrix, obtained by floor scale modular lifting of a base matrix of QC-LDPC code, the method comprising: lifting the QC-LDPC code for maximal code length N<sub>max</sub> and maximal circulant size Z<sub>upper</sub> of the base matrix, N<sub>max</sub> = Z<sub>upper</sub> \* L, wherein L is a column in the base matrix; generating a plurality of optimal values r<sub>i</sub> for a plurality of circulants Z<sub>1</sub>, Z<sub>2</sub>,..., Z<sub>upper</sub> based on the QC-LDPC code lifted for maximal length N<sub>max</sub>, 0 ≤ r<sub>i</sub> ≤ Z<sub>upper</sub> − 1; saving the generated plurality of optimal values r<sub>i</sub> corresponding to the plurality of circulants Z<sub>1</sub>, Z<sub>2</sub>,..., Z<sub>upper</sub> and a matrix for the QC-LDPC code lifted for maximal length N<sub>max</sub> in the memory unit. These steps may be made offline only once. The method further comprises receiving a current circulant Z<sub>current</sub> from the plurality of circulants Z<sub>1</sub>, Z<sub>2</sub>,..., Z<sub>upper</sub>; selecting a current optimal value r<sub>current</sub> from the plurality of optimal values r<sub>i</sub> stored in the memory unit corresponding to the current circulant

lifting of the base matrix is calculated as:  $E(H_{current}) = \left\lfloor \frac{Z_{current}}{Z_{upper}} ((E(H_{upper}) * r_{current}) \mod Z_{upper}) \right\rfloor$ , where  $E(H_{upper})$  is a value of circulant shift in the base matrix for maximal circulant size;

 $Z_{current}$ ; and lifting the base matrix based on the current optimal value  $r_{current}$ , wherein a floor

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wherein  $0 \le r_{current} \le Z_{upper} - 1$  and  $r_{current} = 1$  is excluded. Therefore, a QC-LDPC mother code lifting method with flexible length and rate is provided to be used for encoding and decoding of data packets. This method provides memory efficient QC-LDPC code representation with maximal flexibility of length and rate. The overall code performance is also therefore increased due to providing memory consumption and processing speed increase.

The methods according to the first aspect of the invention can be performed by a computer-readable storage medium according to the second aspect of the invention. Further features or implementations of the method according to the first aspect of the invention can perform the functionality of an apparatus for QC-LDPC encoding and decoding according to the third aspect of the invention and its different implementation forms.

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In a first implementation of the method for QC-LDPC encoding and decoding of a data packet by a lifted matrix according to the first aspect, generating the plurality of optimal values  $r_i$  for the plurality of circulants  $Z_1, Z_2, ..., Z_{upper}$  further comprises: constructing a plurality of families of parity-check matrixes, each family corresponds to value r in a plurality of values  $r_1, r_2, ..., r_k$  corresponding to code lengths  $N_1, N_2, N_3, ..., N_k$ ; and based on the plurality of the families of the parity-check matrixes, selecting the plurality of optimal values  $r_i$  for the plurality of circulants  $Z_1, Z_2, ..., Z_{upper}$  by multi-parameter filtering. Using at least one parity-check matrix a set of code can be represented with minimal performance degradation and high memory efficiency.

In a second implementation of the method for QC-LDPC encoding and decoding of a data packet by a lifted matrix according to the first implementation of the first aspect, the multiparameter filtering includes at least on of: Extrinsic Message Degree, ACE Spectrum, Tanner Spectral Bound, Code Distance, Codeword's weight spectrum enumerator, Trapping Set Weight Enumerator, simulations result. All these ways of choosing best r value provide improved filtering quality due to better consideration of multiple parameters and enable choosing the optimal r value to be used in a lifting procedure.

In a third implementation of the method for QC-LDPC encoding and decoding of a data packet by a lifted matrix according to any of the first or second implementations of the first aspect, constructing the plurality of the families of the parity-check matrixes is performed using formula:  $E_r(H_{upper}) = E(H_{upper}) \cdot r \mod Z_{upper}$ . Using  $E_r(H_{upper})$  provides additional flexibility due to possibility to choose r value to avoid critical points.

A second aspect of the invention refers to a a computer-readable storage medium storing program code, the program code comprising instructions for carrying out the method of the first aspect or one of the implementations of the first aspect.

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A third aspect of the invention refers to an apparatus for quasi-cyclic low-density parity-check (QC-LDPC) encoding and decoding of a data packet by a lifted matrix, obtained by floor scale modular lifting of a base matrix of QC-LDPC code, the apparatus comprising a processing unit and a memory unit, the memory unit storing: a maximal length  $N_{\mathrm{max}}$  and a maximal circulant size  $Z_{upper}$  of the base matrix, a matrix for the QC-LDPC code lifted for maximal length  $N_{\text{max}}$ ; and a plurality of optimal values  $r_i$  corresponding to a plurality of circulants  $Z_1, Z_2, ..., Z_{upper}$ , the plurality of optimal values  $r_i$  is generated based on the QC-LDPC code lifted for maximal length  $N_{\text{max}}$  and maximal circulant size  $Z_{\text{upper}}$  of the base matrix, wherein  $N_{\text{max}} = Z_{upper} * L$ , L is a column in the base matrix and  $0 \le r_i \le Z_{upper} - 1$ . The processing unit is configured to: receive a current circulant  $Z_{current}$  from the plurality of circulants  $Z_1, Z_2, ..., Z_{upper}$ ; select a current optimal value  $r_{current}$  from the plurality of optimal values  $r_i$  stored in the memory unit corresponding to the current circulant  $Z_{current}$ ; and lift the base matrix based on the current optimal value  $r_{current}$ , wherein a floor lifting of the base matrix is calculated as:  $E(H_{current}) = \left| \frac{Z_{current}}{Z_{......}} ((E(H_{upper}) * r_{current}) \mod Z_{upper}) \right|$ , where  $E(H_{upper})$  is a value of circulant shift in the base matrix for maximal circulant size; wherein  $0 \le$  $r_{current} \le Z_{upper} - 1$  and  $r_{current} = 1$  is excluded.

In a first implementation of the apparatus for QC-LDPC encoding and decoding of a data packet by a lifted matrix of the third aspect, generating the plurality of optimal values  $r_i$  for the plurality of circulants  $Z_1, Z_2, ..., Z_{upper}$  comprises: constructing a plurality of families of parity-check matrixes, each family corresponds to value r in a plurality of values  $r_1, r_2, ..., r_k$  corresponding to code lengths  $N_1, N_2, N_3, ..., N_k$ ; and based on the plurality of the families of the parity-check matrixes, selecting the plurality of optimal values  $r_i$  for the plurality of circulants  $Z_1, Z_2, ..., Z_{upper}$  by multi-parameter filtering.

In a second implementation of the apparatus for QC-LDPC encoding and decoding of a data packet by a lifted matrix according to the first implementation of the third aspect, the multiparameter filtering includes at least on of: Extrinsic Message Degree, ACE Spectrum, Tanner

Spectral Bound, Code Distance, Codeword's weight spectrum enumerator, Trapping Set Weight Enumerator, simulations result.

In a third implementation of the apparatus for QC-LDPC encoding and decoding of a data packet by a lifted matrix according to any of the first or second implementations of the third aspect, the processing unit is further configured to construct the plurality of the families of the parity-check matrixes using formula:  $E_r(H_{upper}) = E(H_{upper}) \cdot r \mod Z_{upper}$ .

All the implementations of the first aspect may be easily combined and used together with all the implementations of the third aspect.

These and other aspects of the invention will be apparent from the embodiments described below.

## **BRIEF DESCRIPTION OF THE DRAWINGS**

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To illustrate the technical features of embodiments of the present invention more clearly, the accompanying drawings provided for describing the embodiments are introduced briefly in the following. The accompanying drawings in the following description are merely some embodiments of the present invention, modifications on these embodiments are possible without departing from the scope of the present invention as defined in the claims.

- FIG. 1 is a flow chart of a method for QC-LDPC encoding and decoding of a data packet by a lifted matrix in accordance with an embodiment of the present invention,
- FIG. 2 shows generating a lifting value r for floor-scale modular lifting method in accordance with the present invention,
- 30 FIG. 3 shows an example of critical points elimination due to changing r values,

- FIG. 4 is a block diagram illustrating an apparatus for QC-LDPC encoding and decoding of a data packet by a lifted matrix in accordance with an embodiment of the present invention,
- 5 FIGS. 5-7 show comparison of the floor lifting in accordance with the present invention with traditional floor lifting.

## Detailed Description of the Embodiments

10 FIG. 1 illustrates a method 100 for QC-LDPC encoding and decoding of a data packet by a lifted matrix in accordance with the first aspect of the invention. The lifted matrix is obtained by floor scale modular lifting of a base matrix of QC-LDPC code. The method starts at block 101, where the QC-LDPC code for maximal code length  $N_{\mathrm{max}}$  and maximal circulant size  $Z_{upper}$  of the base matrix is lifted.

$$N_{\max} = Z_{upper} * L, \tag{1}$$

wherein L is a column in the base matrix.

At step 102 a plurality of optimal values  $r_i$  for a plurality of circulants  $Z_1, Z_2, ..., Z_{upper}$  is generated based on the QC-LDPC code lifted for maximal length  $N_{\text{max}}$ ,  $0 \le r_i \le Z_{upper} - 1$ .

The generated plurality of optimal values  $r_i$  corresponding to the plurality of circulants  $Z_1, Z_2, ..., Z_{upper}$  and a matrix for the QC-LDPC code lifted for maximal length  $N_{\max}$  are saved in the memory unit at step 103. At step 104 a current circulant  $Z_{current}$  from the plurality of circulants  $Z_1, Z_2, ..., Z_{upper}$  is received. Then a current optimal value  $r_{current}$  is selected from the plurality of optimal values  $r_i$  stored in the memory unit corresponding to the current circulant  $Z_{current}$  (step 105). Finally at step 106 the base matrix is lifted based on the current opti-25 mal value  $r_{current}$ . A floor lifting of the base matrix is calculated as:

$$E(H_{current}) = \left[ \frac{Z_{current}}{Z_{upper}} \left( \left( E(H_{upper}) * r_{current} \right) \mod Z_{upper} \right) \right], \tag{2}$$

where  $E(H_{upper})$  is a value of circulant shift in the base matrix for maximal circulant size; wherein  $0 \le r_{current} \le Z_{upper} - 1$  and  $r_{current} = 1$  is excluded.

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The method for QC-LDPC encoding and decoding in accordance with the present invention may be widely used, for example in cryptography, in data transfer and for data storage.

A (J,L) regular QC-LDPC code of length N is usually defined by a parity-check matrix

$$H = \begin{bmatrix} I(p_{0,0}) & I(p_{0,1}) & \dots & I(p_{0,L-1}) \\ I(p_{1,0}) & I(p_{1,1}) & & I(p_{1,L-1}) \\ \vdots & \vdots & \ddots & \vdots \\ I(p_{J-1,0}) & I(p_{J-1,1}) & \dots & I(p_{J-1,L-1}) \end{bmatrix}$$
(3)

where  $1 \le j \le J-1, 1 \le l \le L-1$  and  $I(p_{j,l})$  represents the  $p \times p$  circulant permutation matrix obtained by cyclically right-shifting the  $p \times p$  identity matrix I(0) by  $p_{j,l}$  positions, with p = N/L.

For a specific QC-LDPC code the corresponding "base matrix" ("mother matrix" or protograph) is defined as the matrix of circulant shift that defines the QC-LDPC code:

$$B = \begin{bmatrix} p_{0,0} & p_{0,1} & \cdots & p_{0,L-1} \\ p_{1,0} & p_{1,1} & p_{1,L-1} \\ \vdots & \vdots & \ddots & \vdots \\ p_{J-1,0} & p_{J-1,1} & \cdots & p_{J-1,L-1} \end{bmatrix}$$
(4).

Mask matrix for which regular QC-LDPC code can become irregular for different column weight case or QC-LDPC regular code with zero block circulant may be defined as:

$$M = \begin{bmatrix} m_{0,0} & m_{0,1} & \dots & m_{0,L-1} \\ m_{1,0} & m_{1,1} & m_{1,L-1} \\ \vdots & \vdots & \ddots & \vdots \\ m_{J-1,0} & m_{J-1,1} & \dots & m_{J-1,L-1} \end{bmatrix}.$$
 (5)

$$H = H \otimes M, \tag{6}$$

where  $\otimes$  is Hadamard product.

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Lifting is operation under base matrix (protograph), by using of which code with different authomorphism or circulant size from a similar base matrix can be obtained.

Normally floor-lifting of base matrix is calculated by formula:

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$$E(H_{current}) = \left| \frac{z_{current}}{z_{upper}} E(H_{upper}) \right| , \qquad (7)$$

where  $z_{current}$  – lifting size of circulant,

 $z_{upper}$  – maximal circulant size of base matrix,

 $E(H_{upper})$  – value of circulant shift in base matrix for maximal size of circulant.

Length of code N from  $z_{current} * VN_{protograph}$  to  $z_{upper} * VN_{protograph}$  with some additive step between  $z_{current}$ :  $step: z_{upper}$ , where  $VN_{protograph}$  is number of variable nodes in base matrix.

The method according to the present invention uses random matrix design approach lifting QC-LDPC directly from mask matrix (base matrix or protograph).

A cycle of even length 2K in H is defined by 2K positions such that:

- 1) Two consecutive positions are obtained by changing alternatively column of row only;
- 2) All positions are distinct except first and last one;

Two consecutive elements of the path belong to different circulant permutation matrices. So a chain of circulant permutation matrices can be defined:

$$I(p_{i_0,j_0}), I(p_{i_0,j_1}), I(p_{i_1,j_1}), I(p_{i_1,j_2}), \dots, I(p_{i_{K-1},j_{K-1}}), I(p_{i_{K-1},j_0}), I(p_{i_0,j_0})$$
 (8) where  $i_a \neq i_{a+1}, j_a \neq j_{a+1}$ , for all  $0 \leq a \leq K-1$ .

As each part of cycle is one, the circulant permutation matrix  $I(p_{i,j})$  participating in cycle cannot be empty. Using these shifts of identity matrix necessary and sufficient conditions of existing of the cycle can be defined as:

$$\sum_{a=0}^{K-1} p_{i_a, j_a} - p_{i_a, j_{a+1}} \equiv 0 \pmod{N}$$
(9)

Fig. 2 illustrates generating a lifting value r for floor-scale modular lifting method. To generate a lifting value (code for flexible length  $N_1 < N_2 < N_3 < N_4 < ... < N_k < N_{max}$ ) using floor scale modular approach QC-LDPC lifted for maximal length  $N_{max}$  is used. This matrix can be lifted using simulation annulling, hill-climbing, guest-and-search, PEG, ACE+PEG or any another algorithms.

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$$M = \begin{bmatrix} m_{0,0} & m_{0,1} & \dots & m_{0,L-1} \\ m_{1,0} & m_{1,1} & m_{1,L-1} \\ \vdots & \vdots & \ddots & \vdots \\ m_{J-1,0} & m_{J-1,1} & \dots & m_{J-1,L-1} \end{bmatrix}$$

$$(10)$$

The input for floor modular scale lifting represents a QC-LDPC code lifted for maximal circulant size  $Z_{upper}$  with L variable nodes (columns in base matrix) and J parity-check (rows in base matrix):

$$Z_{upper} * L = N_{\text{max}}. \tag{11}$$

Circulant sizes for which lifting this base matrix is desired:  $Z_1 < Z_2 < ... < Z_k < Z_{upper}$ , to get lengths  $Z_1 * L = N_1 < Z_2 * L = N_2 < Z_3 * L = N_3 < Z_4 * L = N_4 < ... < Z_{upper} * L = N_{max}$ . The output of the floor modular scale lifting represents scale values  $r_1, r_2, ..., r_k$  for every circulant sizes  $Z_1, Z_2, ..., Z_k$ . By using these values it is possible to generate code for every code lengths  $N_1, N_2, N_3, ..., N_k$  in a fast manner using formula (12).

QC-LDPC code lifted for maximal code length is received and for every value  $r_1, r_2, ..., r_k$  (related to circulants size  $Z_1, Z_2, ..., Z_k$ ) which corresponds to codes with lengths  $N_1, N_2, N_3, ..., N_k$  using formula (12) i parity-check matrixes are determined. Every  $r_{current}$  can be in the range  $1...Z_i-1$ . After using multi-parameter sieving, best value r is chosen based on: Extrinsic Message Degree, ACE Spectrum, Tanner Spectral Bound, Code Distance, Codeword's weight spectrum enumerator, Trapping Set Weight Spectrum Enumerator, simulation result, as shown in Fig. 2 by ACE=EMD Analysis.

This procedure may be made offline only once, then a matrix lifted for maximal length is saved along with r values.

After value  $r_i$  is got for every circulant  $Z_1, Z_2, ..., Z_{upper}$  parity-check matrix for every length  $N_1, N_2, N_3, ..., N_k$  can be constructed using formula:

$$E_r(H_{upper}) = E(H_{upper}) \cdot r \mod z_{upper}$$
where  $r$  is integer  $1 \le r \le z_{upper} - 1$  and  $GCD(r, z_{upper}) = 1$ . (12)

For any path P shift  $d'_P$  in the  $E_r(H_{upper})$  is equal to r times of shift  $d_P$  by same path in  $E(H_{upper})$ :

$$d'_{P} \equiv \sum_{a=0}^{K-1} r p_{i_{a}, j_{a}} - r p_{i_{a}, j_{a+1}} \left( mod \ z_{upper} \right) \equiv$$

$$\equiv r \sum_{a=0}^{K-1} p_{i_{a}, j_{a}} - p_{i_{a}, j_{a+1}} \left( mod \ z_{upper} \right) \equiv r \ d_{P}$$
(13)

When  $GCD(r, z_{upper}) = 1$ ,  $d_P' \equiv 0 \pmod{z_{upper}}$  in the same time with  $d_P \equiv$ 

So, structure of cycles of  $E(H_{upper})$  and  $E_r(H_{upper})$  are equivalent. In comparison with classical floor lifting approach, this method provides additional freedom and flexibility. Such r can be chosen as to avoid catastrophic (critical) points and to improve quality of graph in general, example of improving using change of r is presented in Fig 3.

Combining formula (12) with formula (7) of classical floor-lifting of base matrix the following formula for a floor lifting can be obtained:

$$E(H_{current}) = \left[ \frac{z_{current}}{z_{upper}} \left( \left( E(H_{upper}) * r_{current} \right) \mod z_{upper} \right) \right], \tag{14}$$

where  $r_{current}$  - scale factor, being an integer value from 0...  $z_{upper}$ -1

20  $CGD(r_{current}, z_{upper}) = 1$ .

 $0 \pmod{z_{unner}}$ .

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This increases freedom and flexibility of floor lifting.

For each  $z_{current}$  we can find such  $r_{current}$  that bring best possible quality of  $E(H_{current})$ . This method can be applied to any QC-LDPC codes to get flexible length properties.

Fig. 4 illustrates an apparatus 200 for QC-LDPC encoding and decoding of a data packet by a lifted matrix comprising a processing unit 201 and a memory unit 202. The memory unit 202 stores: a maximal length  $N_{\rm max}$  and a maximal circulant size  $Z_{upper}$  of the base matrix, a matrix for the QC-LDPC code lifted for maximal length  $N_{\rm max}$ ; and a plurality of optimal values  $r_i$  corresponding to a plurality of circulants  $Z_1, Z_2, ..., Z_{upper}$ . The plurality of optimal values  $r_i$  is

generated based on the QC-LDPC code lifted for maximal length  $N_{\rm max}$  and maximal circulant size  $Z_{\it upper}$  of the base matrix. The processing unit 201 is configured to: receive a current circulant  $Z_{\it current}$  from the plurality of circulants  $Z_1, Z_2, ..., Z_{\it upper}$ ; select a current optimal value  $r_{\it current}$  from the plurality of optimal values  $r_i$  stored in the memory unit corresponding to the current circulant  $Z_{\it current}$ ; and lift the base matrix based on the current optimal value  $r_{\it current}$ .

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Comparison of the floor lifting in accordance with the present invention with traditional floor lifting is further provided with the reference to Figs. 5-7. For simplicity of the comparison, the extended irregular repeat-accumulate (EIRA) QC-LDPC base matrix is lifted. Designed QC-LDPC is:

| 6  | 5  | 7  | -1 | 6  | 3  | -1 | 9  | 8 | 4  | 8 | 3 | 3 | 6 | 9 | 7 | 1 | 5 | 1 | 0 | 7 | 0 | - 1 | -1 |
|----|----|----|----|----|----|----|----|---|----|---|---|---|---|---|---|---|---|---|---|---|---|-----|----|
| 0  | 4  | 5  |    | 9  | 8  |    |    | 4 |    |   | 9 | 2 | 4 | 2 | 9 |   | 6 | 7 |   |   |   | 1   |    |
| -1 | 7  | -1 | 3  | 1  | 7  | 2  | 3  | 4 | -1 | 4 | 4 | 4 | 5 | 5 | 8 | 4 | 5 | 6 | 5 | 0 | 0 | 0   | -1 |
|    | 3  |    |    | 0  | 0  | 5  | 7  | 6 |    | 7 | 6 | 4 | 6 | 5 | 1 | 3 | 9 | 2 | 3 |   |   |     |    |
| 9  | 8  | 8  | 5  | 4  | -1 | 6  | -1 | 1 | 8  | 9 | 5 | 4 | 9 | 2 | 2 | 4 | 2 | 2 | 2 | - | - | 0   | 0  |
| 2  | 1  | 6  | 8  |    |    | 6  |    | 3 | 1  | 2 | 6 | 8 | 4 | 0 | 9 | 4 | 2 |   | 1 | 1 | 1 |     |    |
| 3  | -1 | 8  | 7  | -1 | 8  | 1  | 4  | 2 | 4  | 6 | 3 | 8 | 7 | 8 | 2 | 4 | 1 | 8 | 7 | 7 | - | -   | 0  |
| 1  |    | 3  | 1  |    | 9  | 1  | 2  | 3 | 0  | 2 | 1 | 1 | 4 | 2 | 5 | 2 | 3 | 6 | 0 |   | 1 | 1   |    |

Table 1 contains a comparison of the lifting approach in accordance with the provided method and a traditional floor lifting approach based on the number of cycles.

|                      |        | r lifting in ac-<br>vided method | Traditional floor lifting |        |             |  |  |  |  |
|----------------------|--------|----------------------------------|---------------------------|--------|-------------|--|--|--|--|
| Z <sub>current</sub> | cycles | Number of                        | Z <sub>current</sub>      | cycles | Number of   |  |  |  |  |
|                      |        | cycles; r                        |                           |        | cycles; r   |  |  |  |  |
| 24                   | 6      | 669 ; r = 4                      | 24                        | 4      | 2;r=1       |  |  |  |  |
| 28                   | 6      | 583 ; r = 28                     | 28                        | 4      | 2; r = 1    |  |  |  |  |
| 32                   | 6      | 508 ; r = 52                     | 32                        | 4      | 1;r=1       |  |  |  |  |
| 36                   | 6      | 439 ; r = 20                     | 36                        | 4      | 1;r=1       |  |  |  |  |
| 40                   | 6      | 420 ; r = 28                     | 40                        | 4      | 1; r = 1    |  |  |  |  |
| 44                   | 6      | 364 ; r = 76                     | 44                        | 4      | 1; r = 1    |  |  |  |  |
| 48                   | 6      | 238 ; r = 2                      | 48                        | 4      | 1;r=1       |  |  |  |  |
| 52                   | 6      | 341 ; r = 20                     | 52                        | 4      | 1;r=1       |  |  |  |  |
| 56                   | 6      | 217 ; r = 14                     | 56                        | 4      | 1;r=1       |  |  |  |  |
| 60                   | 6      | 195 ; r = 10                     | 60                        | 4      | 1;r=1       |  |  |  |  |
| 64                   | 6      | 179 ; r = <b>74</b>              | 64                        | 6      | 263 ; r = 1 |  |  |  |  |
| 68                   | 6      | 177 ; r = 86                     | 68                        | 6      | 223 ; r = 1 |  |  |  |  |
| 72                   | 6      | 151 ; r = 58                     | 72                        | 6      | 228 ; r = 1 |  |  |  |  |
| 76                   | 6      | 153 ; r = 94                     | 76                        | 6      | 223;r=1     |  |  |  |  |
| 80                   | 6      | 144 ; r = 14                     | 80                        | 6      | 205 ; r = 1 |  |  |  |  |

| 84               | 6 | 139 ; r = 82 | 84               | 6 | 201 ; r = 1 |
|------------------|---|--------------|------------------|---|-------------|
| 88               | 6 | 130 ; r = 38 | 88               | 6 | 197;r=1     |
| 92               | 6 | 132 ; r = 86 | 92               | 6 | 178 ; r = 1 |
| $z_{upper} = 96$ | 6 | 68 ; r = 1   | $z_{upper} = 96$ | 6 | 173 ; r = 1 |

Table 1

Comparison based on the ACE Spectrum for lifting with circulant size  $z_{current} = 60$ , N=1440 is provided in Fig. 5, where ACE Spectrum of floor scale modular lifting in accordance with the provided method (left) and traditional floor lifting (right) under similar base matrix are provided.

BER performance comparison of traditional floor-lifting QC-LDPC and QC-LDPC lifted using the provided approach of same base-matrix under min-sum decoder 15 iterations under AWGN channel is illustrated in Fig. 6. FER performance comparison of traditional floor-lifting QC-LDPC and QC-LDPC lifted using the provided approach of same base-matrix under min-sum decoder 15 iterations under AWGN channel is illustrated in Fig. 7.

Using the floor scale modular lifting method described in the present description the following two parity-check matrix of Repeat Accumulate QC-LDPC code may be designed:

12x24 circulant from 28 to 2304 with step 4, length 672 to 55296 with step 96, rate 0.5

| 0 -1 -1  | -1 -1 -1 -1 -1                                   |
|----------|--|
| <u> </u> | <del>                                     </del> |
| -        | -1 -1 -1 -1 -1                                   |
| 0 0 -1   | -1 -1 -1 -1 -1                                   |
| 1 0 0    | -1 -1 -1 -1 -1                                   |
| 1 -1 0   | 0 -1 -1 -1 -1                                    |
| 1 -1 -1  | 0 0 -1 -1 -1                                     |
| 1 -1 -1  | -1 0 0 -1 -1                                     |
| 1 -1 -1  | -1 -1 0 0 -1                                     |
| 1 -1 -1  | -1 -1 -1 0 0                                     |
| 1 -1 -1  | -1 -1 -1 -1 0                                    |
|          | -1 -1 -1 -1 -1                                   |
| 1 -1     | -1   |

and

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6x24 circulant from 4 to 2304 with step 4 length 96 to 55296 with step 96, rate 0.75

| 984  | 581  | 2108 | 942  | 855  | 1987 | 1404 | -1  | 1365 | -1   | 2025 | - 1  | -1   | 667 | 719  | 804 | -1   | -1  | 899 | 0  | -1 | -1 | -1 | -1  |
|------|------|------|------|------|------|------|-----|------|------|------|------|------|-----|------|-----|------|-----|-----|----|----|----|----|-----|
| 682  | 737  | 1893 | 932  | 2126 | 185  | 1472 | 522 | -1   | 377  | 1122 | 1161 | -1   | - 1 | -1   | -1  | 934  | 212 | -1  | 0  | 0  | -1 | -1 | -1  |
| 83   | 1971 | 342  | 858  | 1726 | 2205 | 815  | - 1 | -1   | 109  | 671  | -1   | 1876 | 442 | -1   | 447 | 1576 | -1  | ō   | -1 | 0  | 0  | -1 | - 1 |
| 201  | 907  | 1490 | 191  | 272  | 1986 | 970  | 616 | 1393 | -1   | -1   | 646  | -1   | -1  | 930  | 270 | -1   | 629 | -1  | -1 | -1 | 0  | 0  | -1  |
| 2158 | 2244 | 1820 | 390  | 1445 | 2051 | 861  | -1  | 1454 | 1022 | 1163 | - 1  | 139  | 742 | -1   | -1  | -1   | -1  | - 1 | -1 | -1 | -1 | 0  | ō   |
| 679  | 421  | 874  | 2035 | 1806 | 723  | 2097 | 884 | -1   | -1   | -1   | 19   | 1449 | -1  | 1793 | - 1 | 1081 | 275 | 899 | -1 | -1 | -1 | -1 | 0   |

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The foregoing descriptions are only implementation manners of the present invention, the scope of the present invention is not limited to this. Any variations or replacements can be easily made through person skilled in the art. Therefore, the protection scope of the present invention should be subject to the protection scope of the attached claims.

## **CLAIMS**

1. A method (100) for quasi-cyclic low-density parity-check, QC-LDPC, encoding and decoding of a data packet by a lifted matrix, obtained by floor scale modular lifting of a base matrix of a QC-LDPC code,

the method (100) comprising:

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lifting (101) the QC-LDPC code for a maximal code length  $N_{\rm max}$  and a maximal circulant size  $Z_{upper}$  of the base matrix,  $N_{\rm max}=Z_{upper}*L$ , wherein L is the number of columns of the base matrix;

generating (102) a plurality of optimal values  $r_i$  for a plurality of circulants  $Z_1, Z_2, ..., Z_{upper}$  based on the QC-LDPC code lifted for maximal length  $N_{\max}$ ,  $0 \le r_i \le Z_{upper} - 1$ ;

saving (103) the generated plurality of optimal values  $r_i$  corresponding to the plurality of circulants  $Z_1, Z_2, ..., Z_{upper}$  and a matrix for the QC-LDPC code lifted for maximal length  $N_{\max}$  in a memory unit; and

receiving (104) a current circulant  $Z_{current}$  from the plurality of circulants  $Z_1, Z_2, ..., Z_{upper}$ ;

selecting (105) a current optimal value  $r_{current}$  from the plurality of optimal values  $r_i$  stored in the memory unit corresponding to the current circulant  $Z_{current}$ ; and

lifting (106) the base matrix based on the current optimal value  $r_{current}$ , wherein a floor lifting of the base matrix is calculated as:

$$E(H_{current}) = \left| \frac{Z_{current}}{Z_{upper}} ((E(H_{upper}) * r_{current}) \mod Z_{upper}) \right|,$$

where  $E(H_{upper})$  is a value of circulant shift in the base matrix for maximal circulant size;

wherein  $0 \le r_{current} \le Z_{upper} - 1$  and  $r_{current} = 1$  is excluded.

2. The method of claim 1, wherein generating the plurality of optimal values  $r_i$  for the plurality of circulants  $Z_1, Z_2, ..., Z_{upper}$  further comprises:

constructing a plurality of families of parity-check matrixes, each family corresponds to value r in a plurality of values  $r_1, r_2, ..., r_k$  corresponding to code lengths  $N_1, N_2, N_3, ..., N_k$ ; and

based on the plurality of the families of the parity-check matrixes, selecting the plurality of optimal values  $r_i$  for the plurality of circulants  $Z_1, Z_2, ..., Z_{upper}$  by multi-parameter filtering.

- 3. The method of claim 2, wherein the multi-parameter filtering includes at least one of: Extrinsic Message Degree, ACE Spectrum, Tanner Spectral Bound, Code Distance, Codeword's weight spectrum enumerator, Trapping Set Weight Enumerator, simulations result.
- 4. The method of any of claims 2-3, wherein constructing the plurality of the families of the parity-check matrixes is performed using equation:

$$E_r(H_{upper}) = E(H_{upper}) \cdot r \mod Z_{upper}$$

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- 5. A computer readable storage medium storing program code, the program code comprising instructions, which when performed on a computer cause the computer to perform the method according to any of claims 1-4.
- 6. An apparatus (200) for quasi-cyclic low-density parity-check, QC-LDPC, encoding and decoding of a data packet by a lifted matrix, obtained by floor scale modular lifting of a base matrix of QC-LDPC code,

the apparatus (200) comprising a processing unit (201) and a memory unit (202), the memory unit (202) storing:

- a maximal length  $N_{\text{max}}$  and a maximal circulant size  $Z_{\text{upper}}$  of the base matrix,
  - a matrix for the QC-LDPC code lifted for maximal length  $N_{\mathrm{max}}$ ; and
  - a plurality of optimal values  $r_i$  corresponding to a plurality of circulants

 $Z_1, Z_2, ..., Z_{upper}$ , the plurality of optimal values  $r_i$  is generated based on the QC-LDPC code lifted for maximal length  $N_{\max}$  and maximal circulant size  $Z_{upper}$  of the base matrix,

30 wherein  $N_{\text{max}} = Z_{upper} * L$ , L is a column in the base matrix and  $0 \le r_i \le Z_{upper} - 1$ ; the processing unit (201) configured to:

receive a current circulant  $Z_{current}$  from the plurality of circulants  $Z_1, Z_2, ..., Z_{upper}$ ;

select a current optimal value  $r_{current}$  from the plurality of optimal values  $r_i$  stored in the memory unit corresponding to the current circulant  $Z_{current}$ ; and

lift the base matrix based on the current optimal value  $r_{current}$ , wherein a floor lifting of the base matrix is calculated as:

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$$E(H_{current}) = \left| \frac{Z_{current}}{Z_{upper}} ((E(H_{upper}) * r_{current}) \mod Z_{upper}) \right|,$$

where  $E(H_{upper})$  is a value of circulant shift in the base matrix for maximal circulant size;

wherein  $0 \le r_{current} \le Z_{upper} - 1$  and  $r_{current} = 1$  is excluded.

7. The apparatus of claim 6, wherein, generating the plurality of optimal values  $r_i$  for the plurality of circulants  $Z_1, Z_2, ..., Z_{upper}$  comprises:

constructing a plurality of families of parity-check matrixes, each family corresponds to value r in a plurality of values  $r_1, r_2, ..., r_k$  corresponding to code lengths  $N_1, N_2, N_3, ..., N_k$ ; and

- based on the plurality of the families of the parity-check matrixes, selecting the plurality of optimal values  $r_i$  for the plurality of circulants  $Z_1, Z_2, ..., Z_{upper}$  by multi-parameter filtering.
- 8. The apparatus of claim 7, wherein the multi-parameter filtering includes at least one of:
  Extrinsic Message Degree, ACE Spectrum, Tanner Spectral Bound, Code Distance, Codeword's weight spectrum enumerator, Trapping Set Weight Enumerator, simulations result.
  - 9. The apparatus of any of claims 7-8, wherein the processing unit is further configured to construct the plurality of the families of the parity-check matrixes using equation:
- 25  $E_r(H_{upper}) = E(H_{upper}) \cdot r \mod Z_{upper}$ .

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100

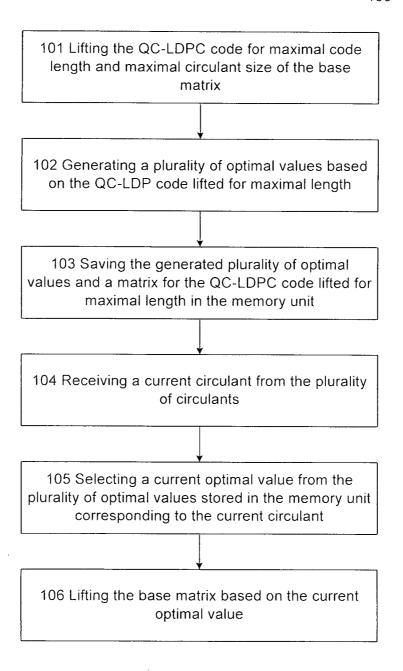


Fig. 1

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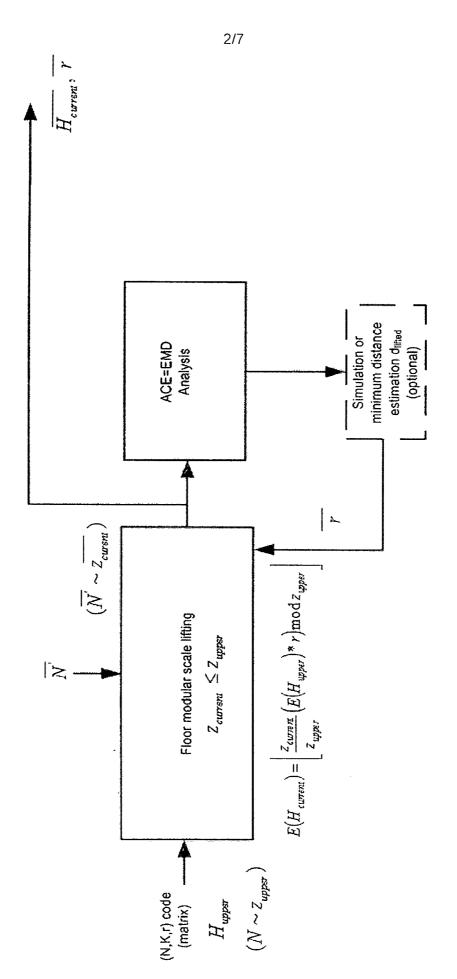
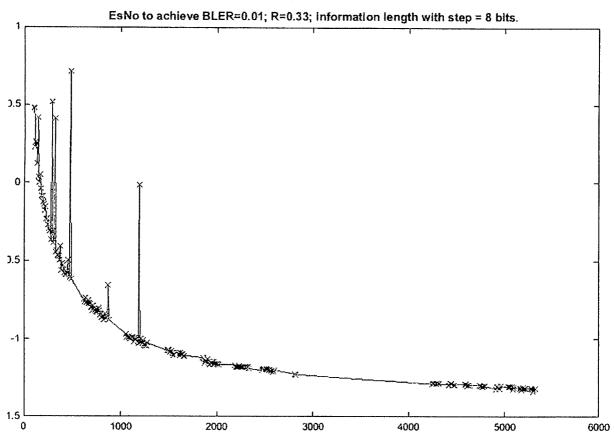


Fig. 2

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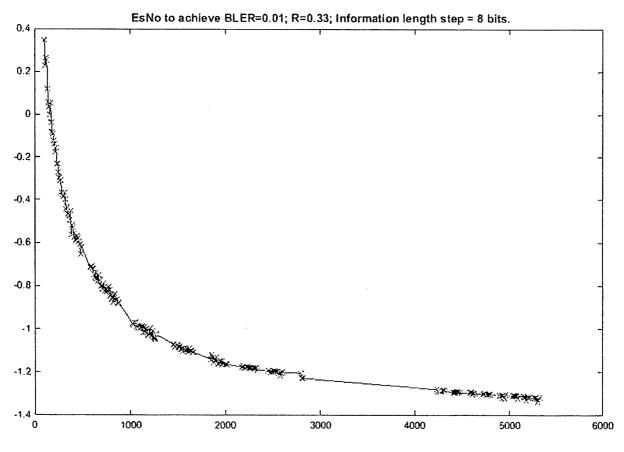
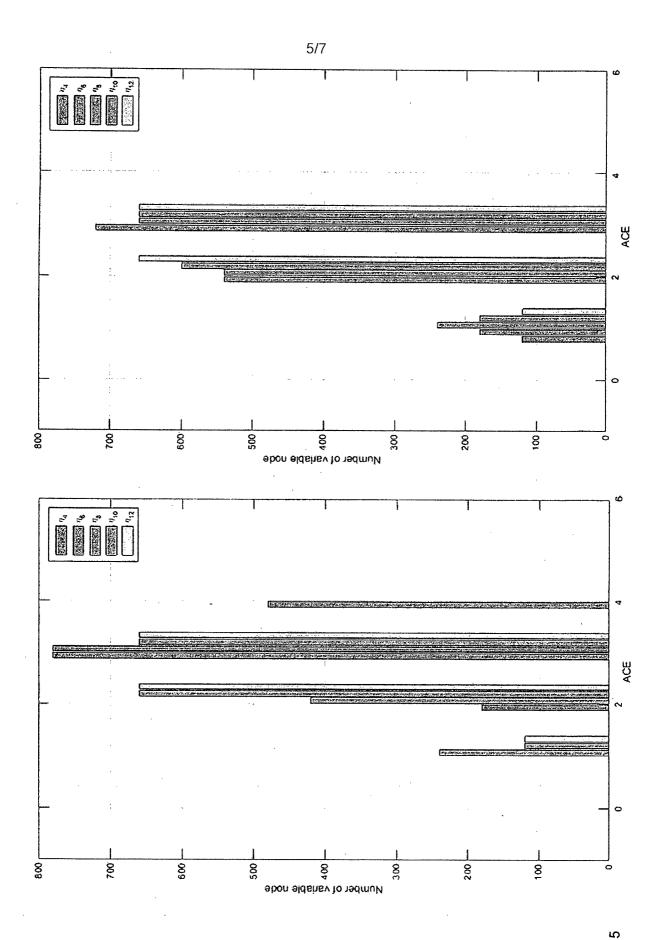
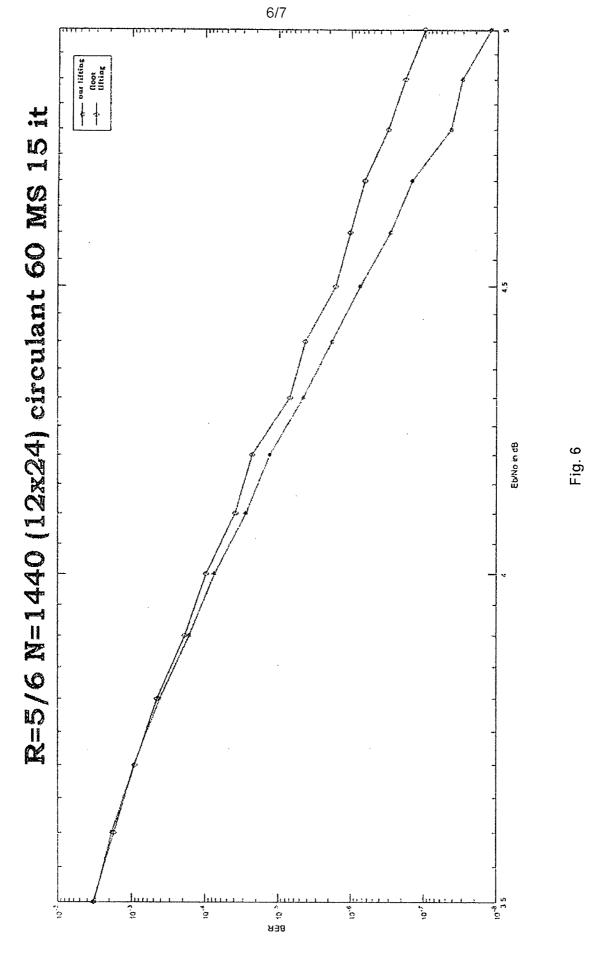


Fig. 3

202 Memory unit

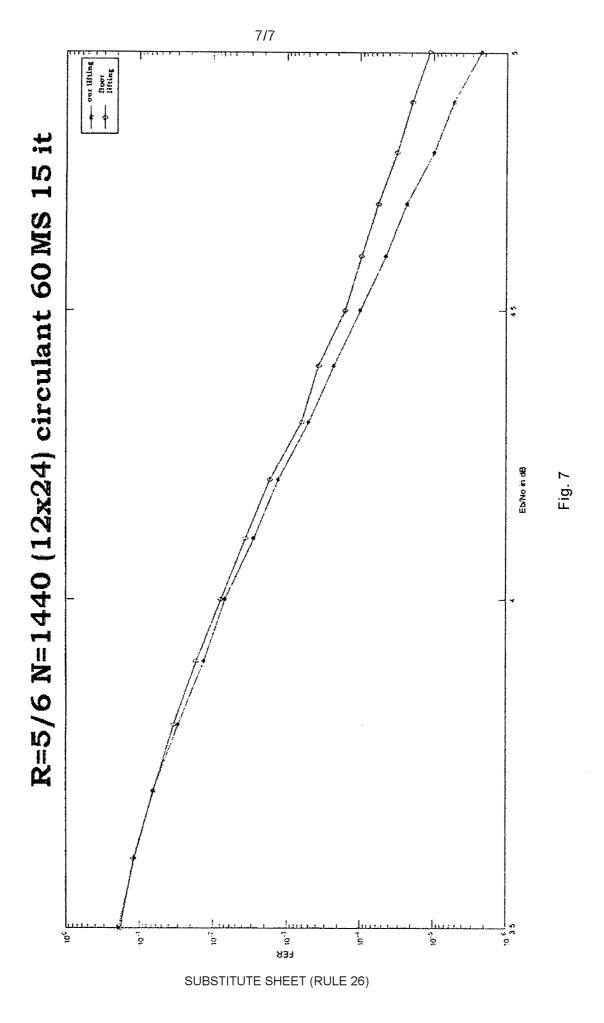
-ig. 4





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## INTERNATIONAL SEARCH REPORT

International application No PCT/RU2016/000777

A. CLASSIFICATION OF SUBJECT MATTER INV. H03M13/11 H03M13/03 ADD.

According to International Patent Classification (IPC) or to both national classification and IPC

#### B. FIELDS SEARCHED

 $\label{lem:minimum} \mbox{Minimum documentation searched (classification system followed by classification symbols)} \\ \mbox{H03M}$ 

Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched

Electronic data base consulted during the international search (name of data base and, where practicable, search terms used)

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| С. ДОСИМІ | DOCUMENTS CONSIDERED TO BE RELEVANT  |                       |  |  |  |  |  |  |  |  |
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| X Further documents are listed in the continuation of Box C.  | X See patent family annex.   |
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|   |  |
| 20 July 2017  | 01/08/2017   |
| Name and mailing address of the ISA/  | Authorized officer   |

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# **INTERNATIONAL SEARCH REPORT**

International application No
PCT/RU2016/000777

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PCT/RU2016/000777

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