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(54) Title: DECODING OF POLAR CODES AND POLAR SUBCODES

(I) $c_0^{n-1} = u_0^{n-1}A$

(II) $u_0^{n-1} = (u_0, \dots, u_{n-1})$

(III) $A = B_m \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}^{\otimes m}$

(IV) u_0^t

(V) $M^{(t)}(u_0^t | y_0^{n-1}),\ 0 \le t \le n-1,$

(VI) $M^{(t)}(u_0^t | y_0^{n-1}) = \hat{R}(u_0^t, y_0^{n-1}) - \Psi(t),$

(VII) $y_0^{n-1} = (y_0, \dots, y_{n-1})$

(VIII) c_0^{n-1}

(IX) $\hat{R}(u_0^t, y_0^{n-1})$

(X) u_0^t

(XI) $\hat{R}(u_0^t, y_0^{n-1})$

(XII) $u_0^t,$

(XIII) u_0^{t+v}

(XIV) $u_0^t,$

(XV) $u_0^{t+v},\ M^{(t+v)}(u_0^{t+v} | y_0^{n-1}),\ 0 \le v \le n-1$

(XVI) $c_0^{n-1},$

(XVII) u_0^{n-1}

(XVIII) $M^{(n-1)}(u_0^{n-1} | y_0^{n-1}).$

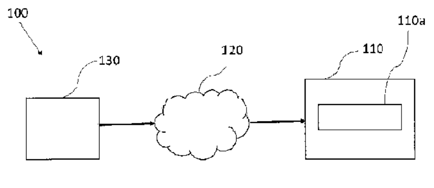

FIG 1

(57) Abstract: The invention relates to a decoding apparatus (110) for decoding a codeword (I) of length n using a ($n = 2^m$, k) polar code or polar subcode C having a set of frozen bit indices F, wherein m is a positive integer number, wherein (II) denotes a vector containing k information bits and $n - k$ frozen bits, wherein (III), wherein B_m is a bit-reversal permutation matrix, wherein $V^{\otimes m}$ denotes the m-times Kronecker product of the matrix V with itself, wherein the decoding apparatus (110) comprises a processor (110a), which is configured to select at each iteration a path (IV) in a code tree having a highest score, wherein the score (V), is defined by the following equation: (VI) wherein (VII) denotes noisy symbols of the codeword (VIII) as received at the decoding apparatus (110), after a transmission over a communication channel (120), wherein (IX) denotes a log-likelihood

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function of a most probable path in a subtree of the code tree starting from (X), wherein $\Psi(t)$ is an expectation value of (XI) assuming that (XII) is the correct path, to construct one or more possible continuations (XIII) of the path (XIV), to compute scores of (XV), and to decode the codeword (XVI), which corresponds to the path (XVII) with the highest score (XVIII).

DECODING OF POLAR CODES AND POLAR SUBCODES

TECHNICAL FIELD

The present invention relates to decoding in communication systems. More specifically, the present invention relates to an apparatus and method for decoding data using polar codes or subcodes.

BACKGROUND

Reliable transmission of data over noisy communication channels requires some kind of error correction coding to be used. Polar codes were shown to achieve the Shannon capacity of many channels see E. Arikan, "Channel polarization: A method for) constructing capacity achieving codes for symmetric binary-input memoryless channels", IEEE Trans. on Inf. Theory, vol. 55, no. 7, pp. 3051–3073, July 2009). However, the performance of polar codes with practical parameters is often unsatisfactory.

Recently, polar subcodes were shown to have higher minimum distance than classical polar codes, and provide substantially better performance under list, sequential and block sequential decodingsee P. Trifonov and V. Miloslavskaya, "Polar subcodes",) IEEE Journal on Selected Areas in Communications, 34(2):254-266, February 2016).

Generally, a $(n = 2^m, k)$ polar subcode C over $GF(2)$ is a set of vectors $c = zWA$, where W is a $k \times n$ matrix, $A = B_m \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}^{\otimes m}$, B_m is the bit reversal permutation matrix, and $F^{\otimes m}$ denotes the m-times Kronecker product of the matrix F with itself. Classical polar codes are obtained by choosing the matrix W such that each column of W has at most a weight of 1 and each row has a weight of 1. Polar subcodes are obtained by choosing the matrix W such that the vectors c are also codewords of some parent code with sufficiently high minimum distance, i.e.,$cH^T = 0$, where H is a check matrix of the parent code. Extended Bose-Chaudhuri-Hocquenghem (BCH) codes were shown to be good parent codes.

After the transmission over a memoryless binary input channel, a codeword $c_0^{n-1} = (c_0, \dots, c_{n-1})$ results in noisy symbols $y_0^{n-1} = (y_0, \dots, y_{n-1})$. In particular, the objective of a maximum likelihood decoding apparatus is to find $\hat{c} = \arg\max_{c \in C} \prod_{i=0}^{n-1} P(y_i|c_i)$, where $P(y_i|c_i)$ is a channel output conditional probability density function. In the prior art, there are several algorithms for decoding polar codes and subcodes, such as the successive cancellation decoding algorithm with complexity $O(n \log n)$, as described in US 20150295593. However, this method does not provide maximum likelihood decoding and has high latency.

Another method is the so-called list decoding (as described by Tal and Vardy in "List decoding of polar codes", in Proc. of IEEE Int. Symp. on Inf. Theory, 2011, pp. 1–5) which can achieve near-ML performance with complexity $O(Ln \log n)$, wherein L is the size of a list constructed by the decoding apparatus and wherein L should be sufficiently large. This algorithm iteratively constructs at most L vectors u_0^i (paths within a code tree) with the highest probability:

$$P\left(u_0^i \middle| y_0^{n-1}\right) = \sum_{u_{i+1}^{n-1}} \prod_{j=0}^{n-1} P((u_0^{n-1}A)_j|y_j)\,, 0 \le i < n,$$

wherein $a_0^i = (a_0, \dots, a_{n-1})$. The average decoding complexity of this decoding algorithm can slightly be reduced by employing the so-called stack decoding algorithm (e.g., K. Niu and K. Chen, “Stack decoding of polar codes,” Electronics Letters, vol. 48, no. 12, pp. 695–697, June 2012) i.e., by considering at each step only one path in the code tree with the highest probability.

Even more complexity reduction can be achieved by employing a sequential decoding algorithm (e.g., V. Miloslavskaya and P. Trifonov, "Sequential decoding of polar codes", IEEE Communications Letters, vol. 18, no. 7, pp. 1127–1130, 2014). The sequential decoding algorithm takes into account not only $P\left(u_0^i \middle| y_0^{n-1}\right)$, but also probabilities of error $P(\hat{u}_i \neq u_i|y_0^{n-1})$, wherein $\hat{u}_i$ represents a decision on a symbol

u_i of a successive cancellation decoding apparatus. The latency and complexity of list and sequential decoding algorithms can be reduced by joint processing of a number of symbols $u_i, t_s \leq i < t_s + 2^{n_s}, t_{s+1} = t_s + n_s$ (e.g., G. Sarkis et al. "Fast List Decoders For Polar Codes", IEEE Journal On Selected Areas In Communications, 34(2), 2016 and G. Trofimiuk, P. Trifonov "Block Sequential Decoding Of Polar Codes", Proceedings Of International Symposium On Wireless Communication Systems, 2015). Joint processing decoding corresponds to a maximum likelihood decoding of some very simple outer polar codes C_s of length n_s, wherein each outer polar code C_s has a set of frozen symbols $F_s = \{j|(t_{s-1} + j) \in F, 0 \leq j < n_s\}$. In the aforementioned work by Sarkis et al., it is suggested to use only rate-0, rate-1, repetition and single-parity check outer codes in conjunction with this approach. Furthermore, in the aforementioned work by Trofimiuk and Trifonov, it is suggested that, in the case of outer codes of length $n_s \leq 8$, the generic Viterbi list decoding algorithm can provide some complexity reduction with respect to straightforward implementation.

The stack, sequential and block sequential decoding algorithms employ data structures as introduced in the aforementioned work by Tal et al. However, employing these data structures results in high memory consumption and significant overhead on data copying and latency.

Therefore, there is a need for an improved apparatus and method for decoding data using polar codes or subcodes.

SUMMARY

It is an object of the invention to provide for an improved apparatus and method for decoding data using polar codes and subcodes.

The foregoing and other objects are achieved by the subject matter of the independent claims. Further implementation forms are apparent from the dependent claims, the description and the figures.

According to a first aspect the invention relates to a decoding apparatus for decoding a codeword $c_0^{n-1} = u_0^{n-1}A$ of length n using a $(n = 2^m, k)$ polar code or polar subcode C having a set of frozen bit indices F, wherein m is a positive integer number, wherein $u_0^{n-1} = (u_0, \dots, u_{n-1})$ denotes a vector containing k information bits and $n - k$ frozen bits, wherein $A = B_m \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}^{\otimes m}$, wherein B_m is a bit-reversal permutation matrix, wherein $\mathrm{V}^{\otimes \mathrm{m}}$ denotes the m-times Kronecker product of the matrix V with itself, wherein the decoding apparatus comprises a processor, which is configured to select at each iteration a path u_0^t in a code tree having a highest score, wherein the score $M^{(t)}(u_0^t|y_0^{n-1})$, $0 \leq t \leq n - 1$, is defined by the following equation:

$$M^{(t)}(u_0^t|y_0^{n-1}) = \hat{R}(u_0^t, y_0^{n-1}) - \Psi(t),$$

wherein $y_0^{n-1} = (y_0, \dots, y_{n-1})$ denotes noisy symbols of the codeword c_0^{n-1} as received at the decoding apparatus, after a transmission over a communication channel, wherein $\hat{R}(u_0^t, y_0^{n-1})$ denotes a log-likelihood function of a most probable path in a subtree of the code tree starting from u_0^t, wherein $\Psi(t)$ is an expectation value of $\hat{R}(u_0^t, y_0^{n-1})$ assuming that u_0^t is the correct path, to construct one or more possible continuations u_0^{t+v} of the path u_0^t, to compute scores of u_0^{t+v}, $M^{(t+v)}(u_0^{t+v}|y_0^{n-1})$, $0 \leq v \leq n - 1$ and to decode the codeword c_0^{n-1}, which corresponds to the path u_0^{n-1} with the highest score $M^{(n-1)}(u_0^{n-1}|y_0^{n-1})$.

In a first possible implementation form of the decoding apparatus according to the first aspect as such, the processor is configured to compute the log-likelihood function $\hat{R}(u_0^t, y_0^{n-1})$ of the most probable path in a subtree of the code tree starting from u_0^t, wherein $\hat{R}(u_0^t, y_0^{n-1})$ is given by:

$$\hat{R}(u_0^t, y_0^{n-1}) = \hat{R}(u_0^{t-1}, y_0^{n-1}) + \tau(S_0^{(t)}, u_t),$$

wherein $\tau(S,u) = \begin{cases} 0, & \operatorname{sgn}(S) = (-1)^u \\ -|S|, & \text{otherwise} \end{cases}$, wherein the function $S_0^{(t)} = \log\frac{\hat{R}(u_0^{t-1}\circ 0, y_0^{n-1})}{\hat{R}(u_0^{t-1}\circ 1, y_0^{n-1})}$ denotes log-likelihood ratios, and the processor is further configured to recursively computed the log-likelihood ratios by means of the expression:

$$S_\lambda^{(2t)}(u_0^{2t-1}, y_0^{n-1}) = \operatorname{sgn}(a)\operatorname{sgn}(b)\min(|a|,|b|)$$

and $S_\lambda^{(2t+1)}(u_0^{2t}, y_0^{n-1}) = (-1)^{u_{2t}}a + b$, wherein $0 \le \lambda \le m$, $u_0^{t-1} \circ x$ denotes the vector obtained by appending x to u_0^{t-1} , $a = S_{\lambda+1}^{(t)}\left(u_{0,e}^{2t-1} + u_{0,o}^{2t-1}, y_0^{\frac{n}{2}-1}\right)$, $b = S_{\lambda+1}^{(t)}\left(u_{0,o}^{2t-1}, y_{\frac{n}{2}}^{n-1}\right)$, $u_{0,e}^i$ and $u_{0,o}^i$ denote subvectors of u_0^i with even e and odd o indices, respectively, and wherein $S_m^{(0)}(y_i)$ denotes log-likelihood ratios of the noisy symbols y_i, $i = 0, \ldots, n-1$.

In a second possible implementation form of the decoding apparatus according to the first aspect as such or the first implementation form thereof, the processor is further configured to compute the expectation value of $\hat{R}(u_0^t, y_0^{n-1})$, which is given by:

$$\Psi(t) = \Psi(t-1) + \int_{-\infty}^{0} f_0^{(t)}(x)x\,dx\,,$$

assuming that u_0^t is the correct path, wherein $f_0^{(t)}(x)$ is a probability density function of $S_0^{(t)}$ under the assumption that a zero codeword was transmitted.

In a third possible implementation form of the decoding apparatus according to the first aspect as such, the processor is further configured to decompose the polar code or subcode C into child codes by applying a recursive generalized Plotkin decomposition, wherein, at a first step of the generalized Plotkin decomposition of the polar code or subcode, two child codes C_0 and $C_1$of length $n/2$ are obtained, wherein $C = \{u \circ (u+v) | u \in C_0, v \in C_1\}$, wherein the symbol $\circ$ denotes a concatenation operator and wherein a set of frozen bit indices of C_0 and C_1 is defined by $F_0 = F \cap \left\{0, \ldots, \frac{n}{2} - 1\right\}$, $F_1 = \{\, j \mid j + n/2 \in F\}$, respectively.

In a fourth possible implementation form of the decoding apparatus according to the first aspect as such or the first or third implementation form thereof, the processor is configured to construct the possible continuations u_0^{t+v} of the path u_0^t by means of a list soft decision decoding algorithm of a vector of values $S_\lambda^{(i)}$ in the child codes, wherein v is the length of the corresponding child code.

In a fifth possible implementation form of the decoding apparatus according to the first aspect as such or the third or fourth implementation form thereof, the processor is further configured to construct a most probable codeword of a child code immediately upon obtaining the vector of values $S_\lambda^{(i)}$, and to construct less probable codewords later on-demand.

In a sixth possible implementation form of the decoding apparatus according to the third implementation form of the first aspect, the processor is further configured to decompose the child codes C_0 and C_1 by means of the recursive generalized Plotkin decomposition until one of the following outer codes is obtained: a $(n_s, 0)$ rate-zero code, wherein $n_s = 2^r, r < m$, a (n_s, n_s) rate-one code, a $(n_s, 1)$ repetition code, a $(n_s, n_s - 1)$ single parity check code, a $(n_s, 2)$ interleaved repetition code, a $(n_s, n_s - 2)$ interleaved single parity check code, a $(tn_s, 1 + \log_2 n_s)$ first order Reed-Muller code serially concatenated with a $(t, 1)$ code, wherein t is a power of 2, a $(n_s, c + \log_2 n_s), c \in \{2,3\}$, supercode of a first order Reed-Muller code or a $(16, f), f \in \{11,10,12\}$ extended Hamming code, its subcode or its supercode.

In a seventh possible implementation form of the decoding apparatus according to the sixth implementation form of the first aspect, the processor is further configured to decode the first order Reed-Muller code serially concatenated with the $(t, 1)$ code and the supercode of the first order Reed-Muller code by means of a Walsh-Hadamard transform, to decode the Hamming code, its subcode and supercode using a Plotkin concatenation of a first Reed-Muller code and a (8,7,2) or a (8,8,1) code, to decode the interleaved single parity check codes using two decoders of a single parity check code, to decode the interleaved repetition code by means of an exhaustive enumeration of all codewords.

In an eighth possible implementation form of the decoding apparatus according to the fourth or sixth implementation form of the first aspect, the processor is further configured to skip a soft-decision decoding of a child code if

$$\left(z_0, \ldots, z_{n_s-1}\right) H_s^T = 0,$$

wherein H_s is a check matrix of the child code and wherein the vector $\left(z_0, \ldots, z_{n_s-1}\right)$ represents a hard decision vector corresponding to log-likelihood ratios passed to a decoder of an outer code.

According to a second aspect the invention relates to a method for decoding a codeword$c_0^{n-1} = u_0^{n-1} A$ of length n using a $(n = 2^m, k)$ polar code or polar subcode C having a set of frozen bit indices F, wherein m is a positive integer number, wherein $u_0^{n-1} = (u_0, \ldots, u_{n-1})$ denotes a vector containing k information bits and $n - k$ frozen bits, wherein $A = B_m \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}^{\otimes m}$, wherein B_m is a bit-reversal permutation matrix, wherein $V^{\otimes m}$ denotes the m-times Kronecker product of the matrix V with itself. The method comprises the steps of selecting at each iteration a path u_0^t in a code tree having a highest score, wherein the score $M^{(t)}(u_0^t | y_0^{n-1})$, $0 \leq t \leq n - 1$, is defined by the following equation:

$$M^{(t)}(u_0^t | y_0^{n-1}) = \hat{R}(u_0^t, y_0^{n-1}) - \Psi(t),$$

wherein $y_0^{n-1} = (y_0, \ldots, y_{n-1})$ denotes noisy symbols of the codeword c_0^{n-1} as received at the decoding apparatus, after a transmission over a communication channel, wherein $\hat{R}(u_0^t, y_0^{n-1})$ denotes a log-likelihood function of the most probable path u_0^t in a subtree of the code tree starting from u_0^t, wherein $\Psi(t)$ is an expectation value of $\hat{R}(u_0^t, y_0^{n-1})$ assuming that u_0^t is the correct path, constructing one or more possible continuations u_0^{t+v} of the path u_0^t, computing scores of u_0^{t+v},

$M^{(t+v)}(u_0^{t+v}|y_0^{n-1})$, $0 \le v \le n-1$, and decoding the codeword c_0^{n-1}, which corresponds to the path u_0^{n-1} with the highest score $M^{(n-1)}(u_0^{n-1}|y_0^{n-1})$.

According to a third aspect, the invention relates to a computer program comprising a program code for performing the method according to the second aspect when executed on a computer.

The invention can be implemented in hardware and/or software.

BRIEF DESCRIPTION OF THE DRAWINGS

Further embodiments of the invention will be described with respect to the following figures, wherein:

Fig. 1 shows a schematic diagram of a communication system comprising an encoding apparatus and a decoding apparatus communicating via a communication channel according to an embodiment;

Fig. 2 shows an example of a recursive generalized Plotkin decomposition of a polar code or subcode according to an embodiment;

Fig. 3a shows a possible implementation of various parts of a decoding algorithm implemented in a decoding apparatus according to an embodiment;

Fig. 3b shows a possible implementation of various parts of a decoding algorithm implemented in a decoding apparatus according to an embodiment;

Fig. 3c shows a possible implementation of various parts of a decoding algorithm implemented in a decoding apparatus according to an embodiment;

Fig. 3d shows a possible implementation of various parts of a decoding algorithm implemented in a decoding apparatus according to an embodiment;

Fig. 4 shows an average number of iterations of different decoding algorithms implemented in a decoding apparatus using a polar code as function of a signal-to noise ratio according to an embodiment;

Fig. 5 shows a frame error rate of different decoding algorithms implemented in a decoding apparatus using different codes as a function of a signal-to-noise ratio according to an embodiment;

Fig. 6 shows addition-equivalent operations of different decoding algorithms implemented in a decoding apparatus using different codes as a function of a signal-to-noise ratio according to an embodiment;

Fig. 7 shows a frame error rate of different decoding algorithms implemented in a decoding apparatus using different polar codes as a function of a signal-to-noise ratio according to an embodiment;

Fig. 8 shows throughputs of different decoding algorithms implemented in a software using different polar codes as a function of a signal-to-noise ratio according to an embodiment; and

Fig. 9 shows a schematic diagram of a method for decoding a codeword using a polar code or subcode according to an embodiment.

In the figures, identical reference signs will be used for identical or functionally equivalent features.

DETAILED DESCRIPTION OF EMBODIMENTS

In the following description, reference is made to the accompanying drawings, which form part of the disclosure, and in which are shown, by way of illustration, specific aspects in which the invention may be placed. It will be appreciated that the invention may be placed in other aspects and that structural or logical changes may be made without departing from the scope of the invention. The following detailed description, therefore, is not to be taken in a limiting sense, as the scope of the invention is defined by the appended claims.

For instance, it will be appreciated that a disclosure in connection with a described method will generally also hold true for a corresponding device or system configured to perform the method and vice versa. For example, if a specific method step is described, a corresponding device may include a unit to perform the described method step, even if such unit is not explicitly described or illustrated in the figures.

Moreover, in the following detailed description as well as in the claims, embodiments with functional blocks or processing units are described, which are connected with each other or exchange signals. It will be appreciated that the invention also covers embodiments which include additional functional blocks or processing units that are arranged between the functional blocks or processing units of the embodiments described below.

Finally, it is understood that the features of the various exemplary aspects described herein may be combined with each other, unless specifically noted otherwise.

Figure 1 shows a schematic diagram of a communication system 100 comprising an encoding apparatus 130 and a decoding apparatus 110 communicating via a communication channel 120 according to an embodiment. A codeword c_0^{n-1} is encoded by the encoding apparatus 130 and is sent via the communication channel 120 to the decoding apparatus 110. In order to decode the codeword $c_0^{n-1} = u_0^{n-1}A$ of length n, the decoding apparatus 110 can use a $(n = 2^m, k)$ polar code or polar subcode C having a set of frozen bit indices F, wherein m is a positive integer number, wherein $u_0^{n-1} = (u_0, \dots, u_{n-1})$ denotes a vector containing k information bits and $n - k$ frozen bits, wherein $A = B_m \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}^{\otimes m}$, wherein B_m is a bit-reversal permutation matrix, wherein $\mathrm{V}^{\otimes \mathrm{m}}$ denotes the m-times Kronecker product of the matrix V with itself. The decoding apparatus 110 comprises a processor 110a, which is configured to select at each iteration a path u_0^t in a code tree having a highest score, wherein the score $M^{(t)}(u_0^t|y_0^{n-1})$, $0 \leq t \leq n-1$, is defined by the following equation:

$$M^{(t)}(u_0^t|y_0^{n-1}) = \hat{R}(u_0^t, y_0^{n-1}) - \Psi(t), \qquad (1)$$

wherein $y_0^{n-1} = (y_0, \ldots, y_{n-1})$ denotes noisy symbols of the codeword c_0^{n-1} as received at the decoding apparatus 110, after the transmission over the communication channel 120, wherein $\hat{R}(u_0^t, y_0^{n-1})$ denotes a log-likelihood function of a most probable path in a subtree of the code tree starting from u_0^t, wherein $\Psi(t)$ is an expectation value of $\hat{R}(u_0^t, y_0^{n-1})$ assuming that u_0^t is the correct path, to construct one or more possible continuations u_0^{t+v} of the path u_0^t , to compute scores of u_0^{t+v}, $M^{(t+v)}(u_0^{t+v}|y_0^{n-1})$, $0 \leq v \leq n-1$, and to decode the codeword c_0^{n-1}, which corresponds to the path u_0^{n-1} with the highest score $M^{(n-1)}(u_0^{n-1}|y_0^{n-1})$.

The definition of the path score as given in equation (1) significantly reduces the average number of iterations performed by the processor 110a of the decoding apparatus 110, compared to, for example, the algorithms presented in the aforementioned works by Niu and Chen, Miloslavskaya and Trifonov, Trofimiuk and Trifonov.

The log-likelihood function of the most probable path u_0^t in a subtree of the code tree is given by:

$$\hat{R}(u_0^t, y_0^{n-1}) = \hat{R}(u_0^{t-1}, y_0^{n-1}) + \tau(S_0^{(t)}, u_t), \qquad (2)$$

wherein $\tau(S,u) = \begin{cases} 0, & \mathrm{sgn}(S) = (-1)^u \\ -|S|, & \text{otherwise} \end{cases}$, and wherein the function $S_0^{(t)} = \log\frac{\hat{R}(u_0^{t-1}\circ 0, y_0^{n-1})}{\hat{R}(u_0^{t-1}\circ 1, y_0^{n-1})}$ denotes log-likelihood ratios, which are recursively computed by the processor 110a as:

$$S_\lambda^{(2t)}(u_0^{2t-1}, y_0^{n-1}) = \operatorname{sgn}(a)\operatorname{sgn}(b)\min(|a|,|b|)$$

and $S_\lambda^{(2t+1)}(u_0^{2t}, y_0^{n-1}) = (-1)^{u_{2t}}a + b$, wherein $0 \le \lambda \le m, u_0^{t-1} \circ x$ denotes the vector obtained by appending x to u_0^{t-1} , $a = S_{\lambda+1}^{(t)}\left(u_{0,e}^{2t-1} + u_{0,o}^{2t-1}, y_0^{\frac{n}{2}-1}\right)$, $b = S_{\lambda+1}^{(t)}\left(u_{0,o}^{2t-1}, y_{\frac{n}{2}}^{n-1}\right)$, $u_{0,e}^i$ and $u_{0,o}^i$ denote the subvectors of u_0^i with even e and odd o indices, respectively, and wherein $S_m^{(0)}(y_i)$ denotes log-likelihood ratios of the noisy symbols y_i, $i = 0, \dots, n-1$.

Furthermore, the expectation value of $\hat{R}(u_0^t, y_0^{n-1})$ assuming that u_0^t is the correct path is given by:

$$\Psi(t) = \Psi(t-1) + \int_{-\infty}^{0} f_0^{(t)}(x)x\,dx\,,$$

wherein $f_0^{(t)}(x)$ is a probability density function of $S_0^{(t)}$ under the assumption that a zero codeword was transmitted.

In the following, a summary of some steps of the block sequential decoding algorithm presented in the aforementioned work by Trofimiuk and Trifonov is given, since some of those steps can be implemented in embodiments of the invention:

First step: Let l_0 be the index of the initial path. Let $S_{l_0,0}[j]$ be the received symbol LLRs. Let $J[l_0] = 0, \mathrm{R}_{\mathrm{l}_0} = 0$. Put $(0, l_0)$ into the priority queue (PQ). Let $\chi_{l_0,i} = 0, q_i = 0, i \in$?, $P = 1$.

Second step: Extract (M, l) with the highest M from the PQ. If $J[l] \ge \mathcal{V}$, call $IterativelyUpdateC(l, m - m_{\mathcal{V}-1}, (N-1)/n_{\mathcal{V}-1})$, return $CGetArrayPointerC(l, 0,0)$, and terminate. Otherwise, let $P = P - 1$.

Third step: If $J[l] > 0$

(a) Let $i' = I[J[l] - 1]$ and $z = \lfloor \phi_{i'}/n_{i'} \rfloor$

(b) If $\chi_{l,i'} < \xi_{i'}$, get the next most probable codeword $c^{(\chi_{l,i'})}$ of $C_{i'}$, together with the corresponding ellipsoidal weight $F^{(\chi_{l,i'})}$, using the state variable $Z_{l,i'}$. Clone the current path. Let l' be the identifier of the cloned path, and let $pC = GetArrayPointerC(l', \lambda - m_{i'}, z)$, $pC[j] = c_j^{(\chi_{l,i'})}, 0 \leq j < n_{i'}$ $R_{l'} = R_l$, and put $(R_{l'} - F^{(\chi_{l,i'})} - \Psi(\phi_{i'}), l')$ into the PQ. Let $P = P + 1, \chi_{l,i'} = \chi_{l,i'} + 1$.

(c) If z is even, call $IterativelyUpdateC(l, \lambda - m_i, \lfloor \phi/n_i \rfloor)$

Fourth step: Let $i = I[J[l]]$, $z_i = \lfloor \phi_i/n_i \rfloor$. Let $q_i = q_i + 1$. Compute $pS[j], 0 \leq j < n_i$, by calling $IterativelyCalcS(l, m - m_i, z_i)$.

Fifth step: Preprocess LLR values $pS[j]$ using a decoder of the outer code C_i. Save the decoder state (the results of pre-processing) in variable $Z_{l,i}$. Use it to obtain the most probable codeword $c^{(0)}$ together with the corresponding ellipsoidal weight $F^{(0)}$. Let $\chi_{l,i} = \chi_{l,i} + 1$.

Sixth step: If $P + 1 > \Theta$, kill $P + 1 - \Theta$ paths with the smallest metrics, remove them from the PQ and decrease P appropriately.

Seventh step: Let $pC = GetArrayPointerC(l, \lambda - m_i, \lfloor \phi/n_i \rfloor)$, $pC[j] = c_j^{(0)}, 0 \leq j < n_i$. Put $(R_l - F^{(0)} - \Psi(\phi_i), l)$ into the PQ. Let $J[l] = J[l] + 1$.

Eighth step: Let $P = P + 1, \chi_i = \chi_i + 1$.

Ninth step: If $q_i > L$, kill all paths $l': J[l'] < J[l]$, remove them from the PQ and decrease P appropriately.

Tenth step: Go to the second step.

In one embodiment of the invention, the decoding algorithm implemented in the decoding apparatus 110 comprises a priority queue (see the first and second step of the above decoding algorithm), which contains tuples (M_l, l), wherein $M_l = M^{(t)}(u_0^t | y_0^{n-1})$ is the score of the l-th path u_0^t, in the code tree considered by the decoding apparatus 110. At each iteration, the processor 110a extracts from the priority queue a tuple with the highest score, and constructs possible extensions of the corresponding path.

The communication channel 120 can be wired, wireless or it can be an optical fiber communication channel.

Figure 2 shows an example of a recursive generalized Plotkin decomposition of a polar code or subcode according to an embodiment. The polar code or subcode C can be decomposed into child codes by applying the recursive generalized Plotkin decomposition, wherein, at a first step of the generalized Plotkin decomposition of the polar code or subcode, two child codes C_0 and $C_1$of length $n/2$ are obtained, wherein $C = \{u \circ (u + v) | u \in C_0, v \in C_1\}$, wherein the symbol $\circ$ denotes a concatenation operator and wherein a set of frozen bit indices of C_0 and C_1 is defined by $F_0 = F \cap \left\{0, \dots, \frac{n}{2} - 1\right\}, F_1 = \{ j \mid j + n/2 \in F\}$, respectively.

For example, a (8,4,4) polar code or subcode with generator matrix:

$$G = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

can be represented as a Plotkin concatenation of a (4,1) code with generator matrices $G_0 = (1 \quad 1 \quad 1 \quad 1)$ and a (4,3) code with generator matrix:

$$G_1 = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}.$$

The latter code can be further decomposed into a (2,1) code with generator matrix $G_{10} = (1 \quad 1)$ and (2,2) code generated by $G_{11} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. This recursive Plotkin decomposition is illustrated by means of a recursive decomposition tree in figure 2.

Furthermore, the recursive generalized Plotkin decomposition may lead to one of the following child codes C_s: a $(n_s, 0)$ rate-zero code, wherein $n_s = 2^r, r < m$, a (n_s, n_s) rate-one code, a $(n_s, 1)$ repetition code, a $(n_s, n_s - 1)$ single parity check code, a $(n_s, 2)$ interleaved repetition code, a $(n_s, n_s - 2)$ interleaved single parity check code, a $(tn_s, 1 + \log_2 n_s)$ first order Reed-Muller code serially concatenated with a $(t, 1)$ code, wherein t is a power of 2, a $(n_s, c + \log_2 n_s), c \in \{2,3\}$, supercode of a first order Reed-Muller code or a $(16, f), f \in \{11,10,12\}$ extended Hamming code, its subcode or its supercode.

The first order Reed-Muller code serially concatenated with the $(t, 1)$ code and the supercode of the first order Reed-Muller code can be decoded by means of a Walsh-Hadamard transform, the Hamming code, its subcode and supercode can be decoded using a Plotkin concatenation of a first Reed-Muller code and a (8,7,2) or a (8,8,1) code, the interleaved single parity check codes can be decoded using two decoders of a single parity check code and the interleaved repetition code can be decoded by means of an exhaustive enumeration of all codewords.

The above list containing the child codes could be further extended if efficient decoding algorithms are developed for other codes. In one embodiment of the invention the recursive generalized Plotkin decomposition of a polar code or subcode can be stopped as soon as any of the above codes is obtained.

Furthermore, it can be shown that $\hat{R}(u_0^{n-1}, y_0^{n-1})$, as defined in equation (2), is equal to the ellipsoidal weight of vector $c_0^{n-1} = u_0^{n-1}A$, which is defined as $E(c_0^{n-1}, y_0^{n-1}) = \sum_{i=0}^{n-1} \tau(L(y_i), c_i)$, where $L(y) = \log \frac{W(y|0)}{W(y|1)}$. This has the advantage that the complexity of the computation of $\hat{R}(u_0^t, y_0^{n-1})$ by employing soft decision decoding algorithms for the above-mentioned outer codes is reduced, since the outer codes naturally compute the ellipsoidal weight of the corresponding codewords.

In one embodiment of the proposed block sequential decoding algorithm implemented in the decoding apparatus 110, the processor 110a is configured to first perform maximum likelihood decoding of a vector of intermediate log-likelihood-ratio (LLR) values in an outer code in order to identify the most probable codeword $c^{(0)}$, and then to construct on demand less probable codewords of this code.

The on-demand construction of less probable codewords has the advantage of providing significant complexity savings with respect to other algorithms used in the prior art.

Furthermore, in most cases, the hard decision vector $(z_0, \dots, z_{n_s-1})$ corresponding to the LLR values is error free. This has the advantage that computationally expensive maximum likelihood soft decision decoding of the outer code can be avoided. Therefore, in one embodiment of the invention, the soft decoding of an outer code can be skipped, if:

$$(z_0, \dots, z_{n_s-1})H_s^T = 0, \qquad (3)$$

wherein H_s is a check matrix of the s-th outer code, and $c^{(0)} = (z_0, \dots, z_{n_s-1})$.

However, even if (3) holds, it still may happen that $(z_0, \dots, z_{n_s-1})$ is still not correct. In order to ensure that less probable codewords $c^{(t)}, t > 0$, can still be recovered, in one embodiment of the invention, the path l can be cloned in order to obtain a path l', and the tuple $(R_l - \Psi(\phi_s) - \mathrm{d_i} \min_{0 \le j < \mathrm{n_i}} |\mathrm{pS[j]}|, l')$ can be pushed into the priority queue, wherein $\mathrm{d_i}$ is the minimum distance of C_s. The value $\mathrm{d_i} \min_{0 \le j < \mathrm{n_i}} |\mathrm{pS[j]}|$ is a lower bound on the ellipsoidal weight of $c^{(1)}$. The corresponding codeword $c^{(1)}$ is not constructed at the preprocessing step (see the algorithm presented in the description of figure 1). If the path l' is extracted from the priority queue, then the processor 110a can further be configured to perform a preprocessing (which was skipped initially), to construct $c^{(1)}$ and to compute its exact ellipsoidal weight.

The performance of the proposed decoding algorithm implemented in the decoding apparatus 110 can further be improved if the processor 110a is not configured to kill the paths permanently at the ninth step of the algorithm shown in the description of figure 1, but it is configured to remove all paths $l': J[l'] < J[l]$ from the priority queue and to save them. If the number of path deletion events exceeds some threshold, then the processor 110a can further be configured to increase the value of L and to reintroduce the saved paths into the priority queue. Moreover, the complexity of the proposed decoding algorithm implemented in the decoding apparatus 110 can still be reduced if the processor 110a is configured to not push the path into the priority queue if its score exceeds the score of the best path stored there.

The decoding algorithm implemented in the decoding apparatus 110 reduces the decoding complexity and latency compared to the classical stack, sequential and block sequential algorithms according to the prior art.

Furthermore, the proposed decoding algorithm implemented in the decoding apparatus 110 can be implemented in a receiver of a communication system employing polar codes or polar subcodes. It can also be implemented in software, FPGA or ASIC.

Figure 3a shows a possible implementation of various parts of a decoding algorithm implemented in a decoding apparatus 110 according to an embodiment. The standard implementation of the block sequential decoding algorithm makes use of the Tal-Vardy data structures suggested for list decoding of polar codes (see the above mentioned work of Tal and Vardy). Although lazy copying mechanism is provided in the work of Tal and Vardy, some data still need to be copied. This results in a performance degradation. In order to avoid the performance degradation and to reduce the amount of memory used by the decoding apparatus 110, the processor 110a can be configured to store partial sums array C[*,1] on array C[*,0] by means of aliasing. In this case, the data does not need to be copied at all while accessing the log-likelihoods and partial sums arrays. Furthermore, the processor 110a can be configured to allocate a pool of memory for storing various arrays of log-likelihood ratios S and partial sums C. As soon as the decoding apparatus 110 needs to write to some subarray S or C, the processor 110a can be configured to allocate for it the appropriate memory block either from the set of released memory blocks or from the memory pool.

Figure 3b shows a possible implementation of various parts of a decoding algorithm implemented in a decoding apparatus 110 according to an embodiment, analogously to figure 3a.

Figure 3c shows a possible implementation of various parts of a decoding algorithm implemented in a decoding apparatus 110 according to an embodiment, analogously to figure 3a.

Figure 3d shows a possible implementation of various parts of a decoding algorithm implemented in a decoding apparatus 110 according to an embodiment, analogously to figure 3a.

Figure 4 shows an average number of iterations of different decoding algorithms implemented in a decoding apparatus 110 using a polar code as function of a signal-to noise ratio according to an embodiment. In this embodiment, the average number of iterations is performed by various sequential-type decoding algorithms for a (1024, 512, 28) polar subcode with child codes of length 1. In figure 4, the results of the decoding algorithm of this invention, which makes use of the path score according to equations (1) and (2), are compared to the results of the Niu-Chen stack decoding algorithm presented in the above mentioned work by Niu and Chen and the Niu-Chen algorithm with min-sum approximation for thc path score function. This latter path score can be obtained from equation (1) by setting $\Psi(t) = 0$. As is shown in figure 4, employing the proposed path score function according to equations (1) and (2) results in a substantial reduction of the average number of iterations performed by the processor 110a of decoding apparatus 110 compared to other prior art algorithms.

Figure 5 shows a frame error rate (FER) of different decoding algorithms implemented in a decoding apparatus 110 using different codes as a function of a signal-to-noise ratio according to an embodiment. In this embodiment, the (1024, 512, 28) subcode and its six decoding algorithms are the same as in figure 4. Furthermore, in figure 5, the performance (namely the FER) of a WiMAX low-density-parity-check (LDPC) (1032, 516) code is shown as well. As it can be seen from figure 5, the performances of the different decoding algorithms presented in figure 4 turn out to be very similar and better than the one of the WiMAX LDPC code.

Figure 6 shows addition-equivalent operations of different decoding algorithms implemented in a decoding apparatus 110 using different codes as a function of a signal-to-noise ratio according to an embodiment. In this embodiment, various sequential (with child codes having a length $n_s = 1$) and block sequential decoding algorithms are used to decode a (1024, 512, 28) polar code or subcode and a (1032, 516) WiMAX LDPC code. As it can be seen in figure 6, the solid line in the case of a block sequential decoding algorithm with $L = 32$, which corresponds to an embodiment of the invention, has the lowest addition-equivalent operations, namely the lowest decoding complexity. As a comparison, the Tal-Vardy list decoding algorithm for $L = 32$, $n = 1024$ requires 32*1024*10=327680 summations and comparisons, which is 46 times more than the result given by proposed algorithm implemented in the decoding apparatus 110. The complexity of the proposed algorithms implemented in the decoding apparatus 110 turns out to be less than that of a belief propagation algorithm used to decode an LDPC code with similar parameters. Furthermore, the performance of the LDPC code turns out to be worse than the performance of the polar subcode decoded using the decoding algorithm implemented in the decoding apparatus 110 according to an embodiment of this invention.

Figure 7 shows a frame error rate of different decoding algorithms implemented in a decoding apparatus 110 using different polar codes as a function of a signal-to-noise ratio according to an embodiment. The curves corresponding to the improved block sequential decoding algorithm of the invention with $L = 32$provide better performance compared to the curves corresponding to the results reproduced using the decoding algorithm suggested in the aforementioned work by Sarakis et al. with $L = 2$.

Figure 8 shows throughputs of different decoding algorithms implemented in a software using different polar codes as a function of a signal-to-noise ratio according to an embodiment. Analogously to figure 7, in this case as well, the throughput in bps of the software implementation of the improved block sequential decoding algorithm of the invention with $L = 32$ are generally lower than the ones given by the decoding algorithm suggested in the aforementioned work by Sarakis et al. with $L = 2$ and

$L = 32$. Therefore, the block sequential decoding algorithm of the invention proves for a better performance than the one suggested by Sarakis et al.

Figure 9 shows a schematic diagram of a method 900 for decoding a codeword using a polar code or subcode according to an embodiment. The method 900 for decoding the codeword $c_0^{n-1} = u_0^{n-1}A$ of length n using the $(n = 2^m, k)$ polar code or polar subcode C having a set of frozen bit indices F, wherein m is a positive integer number, wherein $u_0^{n-1} = (u_0, \dots, u_{n-1})$ denotes a vector containing k information bits and $n - k$ frozen bits, wherein $A = B_m \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}^{\otimes m}$, wherein B_m is a bit-reversal permutation matrix, wherein $\mathrm{V}^{\otimes \mathrm{m}}$ denotes the m-times Kronecker product of the matrix V with itself, comprises the steps of: selecting 902 at each iteration a path u_0^t in a code tree having a highest score, wherein the score $M^{(t)}(u_0^t|y_0^{n-1})$, $0 \le t \le n - 1$, is defined by the following equation:

$$M^{(t)}(u_0^t|y_0^{n-1}) = \hat{R}(u_0^t, y_0^{n-1}) - \Psi(t),$$

wherein $y_0^{n-1} = (y_0, \dots, y_{n-1})$ denotes noisy symbols of the codeword c_0^{n-1} as received at the decoding apparatus, after a transmission over a communication channel, wherein $\hat{R}(u_0^t, y_0^{n-1})$ denotes a log-likelihood function of the most probable path in a subtree of the code tree starting from u_0^t, wherein $\Psi(t)$ is an expectation value of $\hat{R}(u_0^t, y_0^{n-1})$ assuming that u_0^t is the correct path, constructing 904 one or more possible continuations u_0^{t+v} of the path u_0^t, computing 906 scores of u_0^{t+v}, $M^{(t+v)}(u_0^{t+v}|y_0^{n-1})$, $0 \le v \le n - 1$, pushing them into the priority queue, and returning 908 the codeword c_0^{n-1}, which corresponds to the path u_0^{n-1} of length n.

While a particular feature or aspect of the disclosure may have been disclosed with respect to only one of several implementations or embodiments, such feature or aspect may be combined with one or more other features or aspects of the other implementations or embodiments as may be desired and advantageous for any given or particular application. Furthermore, to the extent that the terms "include", "have", "with", or other variants thereof are used in either the detailed description or the claims, such terms are intended to be inclusive in a manner similar to the term "comprise". Also, the terms "exemplary", "for example" and "e.g." are merely meant as an example, rather than the best or optimal. The terms “coupled” and “connected”, along with derivatives may have been used. It should be understood that these terms may have been used to indicate that two elements cooperate or interact with each other regardless whether they are in direct physical or electrical contact, or they are not in direct contact with each other.

Although specific aspects have been illustrated and described herein, it will be appreciated by those of ordinary skill in the art that a variety of alternate and/or equivalent implementations may be substituted for the specific aspects shown and described without departing from the scope of the present disclosure. This application is intended to cover any adaptations or variations of the specific aspects discussed herein.

Although the elements in the following claims are recited in a particular sequence with corresponding labeling, unless the claim recitations otherwise imply a particular sequence for implementing some or all of those elements, those elements are not necessarily intended to be limited to being implemented in that particular sequence.

Many alternatives, modifications, and variations will be apparent to those skilled in the art in light of the above teachings. Of course, those skilled in the art will readily recognize that there are numerous applications of the invention beyond those described herein. While the invention has been described with reference to one or more particular embodiments, those skilled in the art will recognize that many changes may be made thereto without departing from the scope of the invention. It is therefore to be understood that within the scope of the appended claims and their equivalents, the invention may be practiced otherwise than as specifically described herein.

CLAIMS

1. A decoding apparatus (110) for decoding a codeword $c_0^{n-1} = u_0^{n-1}A$ of length n using a $(n = 2^m, k)$ polar code or polar subcode C having a set of frozen bit indices F, wherein m is a positive integer number, wherein $u_0^{n-1} = (u_0, \dots, u_{n-1})$ denotes a vector containing k information bits and $n - k$ frozen bits, wherein $A = B_m \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}^{\otimes m}$, wherein B_m is a bit-reversal permutation matrix, wherein $V^{\otimes m}$ denotes the m-times Kronecker product of the matrix V with itself, wherein the decoding apparatus (110) comprises a processor (110a), which is configured to select at each iteration a path u_0^t in a code tree having a highest score, wherein the score $M^{(t)}(u_0^t|y_0^{n-1})$, $0 \le t \le n - 1$, is defined by the following equation:

$$M^{(t)}(u_0^t|y_0^{n-1}) = \hat{R}(u_0^t, y_0^{n-1}) - \Psi(t),$$

wherein $y_0^{n-1} = (y_0, \dots, y_{n-1})$ denotes noisy symbols of the codeword c_0^{n-1} as received at the decoding apparatus (110), after a transmission over a communication channel (120), wherein $\hat{R}(u_0^t, y_0^{n-1})$ denotes a log-likelihood function of a most probable path in a subtree of the code tree starting from u_0^t, wherein $\Psi(t)$ is an expectation value of $\hat{R}(u_0^t, y_0^{n-1})$ assuming that u_0^t is the correct path, to construct one or more possible continuations u_0^{t+v} of the path u_0^t, to compute scores of u_0^{t+v}, $M^{(t+v)}(u_0^{t+v}|y_0^{n-1})$, $0 \le v \le n - 1$ and to decode the codeword c_0^{n-1}, which corresponds to the path u_0^{n-1} with the highest score $M^{(n-1)}(u_0^{n-1}|y_0^{n-1})$.

2. The decoding apparatus (110) of claim 1, wherein the processor (110a) is further configured to compute the log-likelihood function $\hat{R}(u_0^t, y_0^{n-1})$ of the most probable path in a subtree of the code tree starting from u_0^t, wherein $\hat{R}(u_0^t, y_0^{n-1})$ is given by:

$$\hat{R}(u_0^t, y_0^{n-1}) = \hat{R}(u_0^{t-1}, y_0^{n-1}) + \tau(S_0^{(t)}, u_t),$$

wherein $\tau(S,u) = \begin{cases} 0, & \operatorname{sgn}(S) = (-1)^u \\ -|S|, & \text{otherwise} \end{cases}$, wherein the function $S_0^{(t)} = \log\frac{\hat{R}(u_0^{t-1}\circ 0, y_0^{n-1})}{\hat{R}(u_0^{t-1}\circ 1, y_0^{n-1})}$ denotes log-likelihood ratios, and wherein the processor (110a) is further configured to recursively computed the log-likelihood ratios by means of the expression:

$$S_\lambda^{(2t)}(u_0^{2t-1}, y_0^{n-1}) = \operatorname{sgn}(a)\operatorname{sgn}(b)\min(|a|,|b|)$$

and $S_\lambda^{(2t+1)}(u_0^{2t}, y_0^{n-1}) = (-1)^{u_{2t}}a + b$, wherein $0 \le \lambda \le m, u_0^{t-1}\circ x$ denotes the vector obtained by appending x to u_0^{t-1} , $a = S_{\lambda+1}^{(t)}\left(u_{0,e}^{2t-1} + u_{0,o}^{2t-1}, y_0^{\frac{n}{2}-1}\right)$, $b = S_{\lambda+1}^{(t)}\left(u_{0,o}^{2t-1}, y_{\frac{n}{2}}^{n-1}\right)$, $u_{0,e}^i$ and $u_{0,o}^i$ denote subvectors of u_0^i with even e and odd o indices, respectively, and wherein $S_m^{(0)}(y_i)$ denotes log-likelihood ratios of the noisy symbols y_i, $i = 0,\ldots,n-1$.

3. The decoding apparatus (110) of claim 1 or 2, wherein the processor (110a) is further configured to compute the expectation value of $\hat{R}(u_0^t, y_0^{n-1})$, which is given by:

$$\Psi(t) = \Psi(t-1) + \int_{-\infty}^{0} f_0^{(t)}(x)x\,dx\,,$$

assuming that u_0^t is the correct path, wherein $f_0^{(t)}(x)$ is a probability density function of $S_0^{(t)}$ under the assumption that a zero codeword was transmitted.

4. The decoding apparatus (110) of claim 1, wherein the processor (110a) is further configured to decompose the polar code or subcode C into child codes by applying a recursive generalized Plotkin decomposition, wherein, at a first step of the generalized Plotkin decomposition of the polar code or subcode, two child codes C_0 and $C_1$of length $n/2$ are obtained, wherein $C = \{u\circ(u+v)|u\in C_0, v\in C_1\}$, wherein the symbol $\circ$ denotes a concatenation operator and wherein a set of frozen bit indices of C_0 and C_1 is defined by $F_0 = F\cap\left\{0,\ldots,\frac{n}{2}-1\right\}, F_1 = \{\ j\,|\,j+n/2\in F\}$, respectively.

5. The decoding apparatus (110) of claim 1, 2 or 4, wherein the processor (110a) is configured to construct the possible continuations $u_0^{t+\nu}$ of the path u_0^t by means of a list soft decision decoding algorithm of a vector of values $S_\lambda^{(i)}$ in the child codes, wherein ν is the length of the corresponding child code.

6. The decoding apparatus (110) of claims 1, 4 or 5, wherein the processor (110a) is further configured to construct a most probable codeword of a child code immediately upon obtaining the vector of values $S_\lambda^{(i)}$, and to construct less probable codewords later on-demand.

7. The decoding apparatus (110) of claim 4, wherein the processor (110a) is further configured to decompose the child codes C_0 and C_1 by means of the recursive generalized Plotkin decomposition until one of the following outer codes is obtained: a $(n_s, 0)$ rate-zero code, wherein $n_s = 2^r, r < m$, a (n_s, n_s) rate-one code, a $(n_s, 1)$ repetition code, a $(n_s, n_s - 1)$ single parity check code, a $(n_s, 2)$ interleaved repetition code, a $(n_s, n_s - 2)$ interleaved single parity check code, a $(tn_s, 1 + \log_2 n_s)$ first order Reed-Muller code serially concatenated with a $(t, 1)$ code, wherein t is a power of 2, a $(n_s, c + \log_2 n_s), c \in \{2,3\}$, supercode of a first order Reed-Muller code or a $(16, f), f \in \{11,10,12\}$ extended Hamming code, its subcode or its supercode.

8. The decoding apparatus (110) of claim 7, wherein the processor (110a) is further configured to decode the first order Reed-Muller code serially concatenated with the $(t, 1)$ code and the supercode of the first order Reed-Muller code by means of a Walsh-Hadamard transform, to decode the Hamming code, its subcode and supercode using a Plotkin concatenation of a first Reed-Muller code and a (8,7,2) or a (8,8,1) code, to decode the interleaved single parity check codes using two decoders of a single parity check code, to decode the interleaved repetition code by means of an exhaustive enumeration of all codewords.

9. The decoding apparatus (110) of claim 5 or 7, wherein the processor (110a) is further configured to skip a soft-decision decoding of a child code if

$$\left(z_0, \dots, z_{n_s-1}\right) H_s^T = 0,$$

wherein H_s is a check matrix of the child code and wherein the vector $\left(z_0, \dots, z_{n_s-1}\right)$ represents a hard decision vector corresponding to log-likelihood ratios passed to a decoder of an outer code.

10. A method (900) for decoding a codeword $c_0^{n-1} = u_0^{n-1} A$ of length n using a $(n = 2^m, k)$ polar code or polar subcode C having a set of frozen bit indices F, wherein m is a positive integer number, wherein $u_0^{n-1} = (u_0, \dots, u_{n-1})$ denotes a vector containing k information bits and $n - k$ frozen bits, wherein $A = B_m \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}^{\otimes m}$, wherein B_m is a bit-reversal permutation matrix, wherein $\mathrm{V}^{\otimes \mathrm{m}}$ denotes the m-times Kronecker product of the matrix V with itself, the method comprising the steps of:

selecting (902) at each iteration a path u_0^t in a code tree having a highest score, wherein the score $M^{(t)}(u_0^t | y_0^{n-1})$, $0 \leq t \leq n-1$, is defined by the following equation:

$$M^{(t)}(u_0^t | y_0^{n-1}) = \hat{R}(u_0^t, y_0^{n-1}) - \Psi(t),$$

wherein $y_0^{n-1} = (y_0, \dots, y_{n-1})$ denotes noisy symbols of the codeword c_0^{n-1} as received at the decoding apparatus, after a transmission over a communication channel, wherein $\hat{R}(u_0^t, y_0^{n-1})$ denotes a log-likelihood function of the most probable path u_0^t in a subtree of the code tree starting from u_0^t, wherein $\Psi(t)$ is an expectation value of $\hat{R}(u_0^t, y_0^{n-1})$ assuming that u_0^t is the correct path;

constructing (904) one or more possible continuations u_0^{t+v} of the path u_0^t;

computing (906) scores of u_0^{t+v}, $M^{(t+v)}(u_0^{t+v} | y_0^{n-1})$, $0 \leq v \leq n-1$; and decoding the codeword c_0^{n-1}, which corresponds to the path u_0^{n-1} with the highest score $M^{(n-1)}(u_0^{n-1}|y_0^{n-1})$.

11. A computer program comprising a program code for performing the method of claim 10 when executed on a computer.

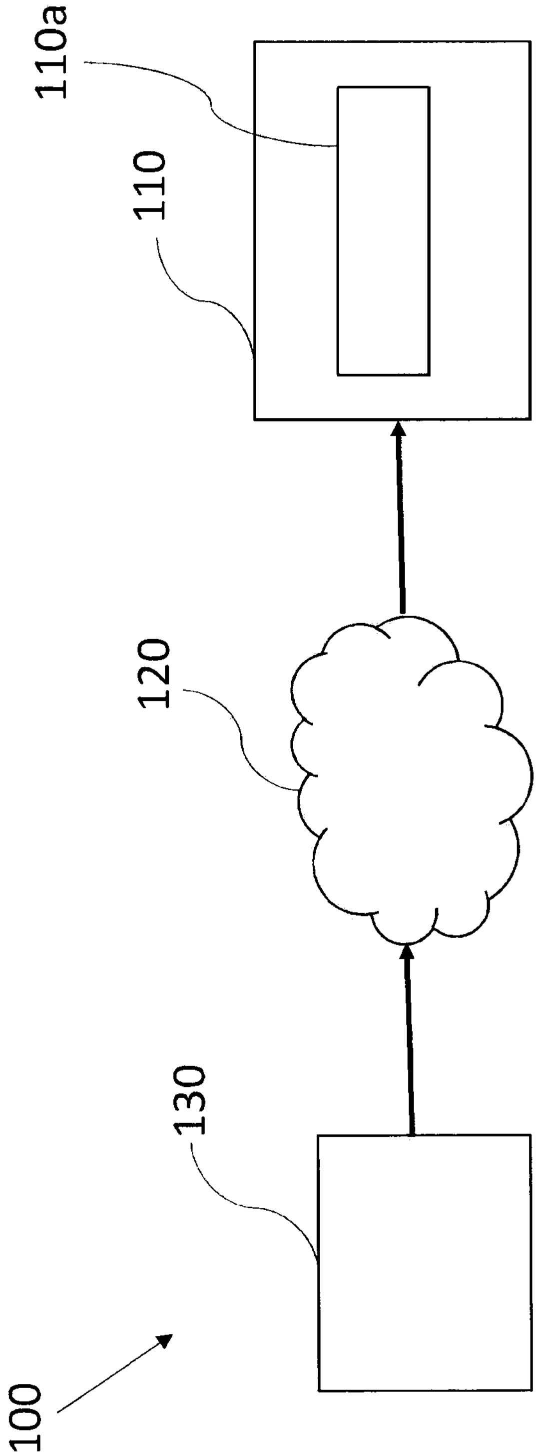


FIG.1

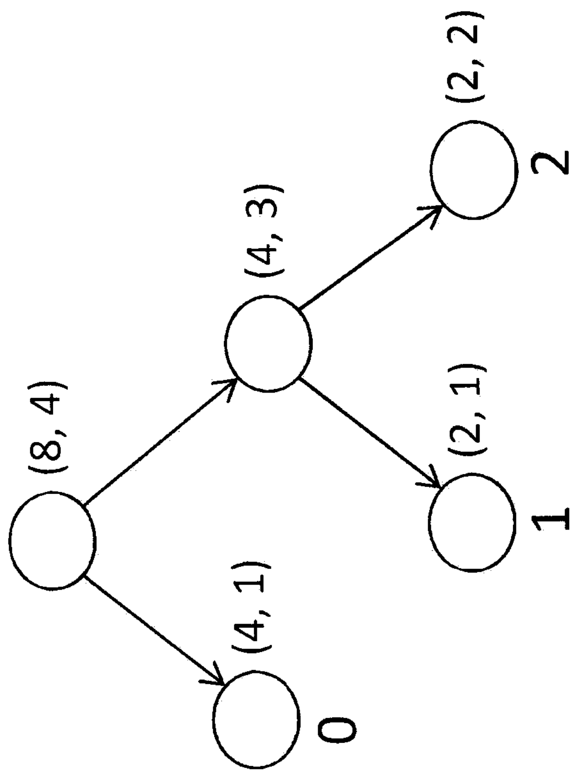


FIG.2

ITERATIVELYCALCS(l, λ, ϕ)

Let d be the maximal integer $\leq \lambda - 1$, s.t. ϕ is divisible by 2^d
$S' = CGetArrayPointerS(l, \lambda - d - 1)$
if $\phi 2^{-d}$ is odd
  then $C_{l,\lambda} \leftarrow CGetArrayPointerC(l, \lambda - d)$
    $S'' = GetArrayPointerS(l, \lambda - d - 1)$
    $S''[\beta] = S'[\beta + N] + (-1)^{C_{l,\lambda-d-1}[\beta]} S'[\beta], 0 \leq \beta < N$
    $S' \leftarrow S''; d \leftarrow d - 1$
while $d \geq 0$
do $\lambda' = \lambda - d$
  $N = 2^{m-\lambda'}$
  $S'' = GetArrayPointerS(l, \lambda')$
  $S''[\beta] = \mathrm{sgn}(S'[\beta + N]) \, \mathrm{sgn}(S'[\beta]) \min(|S'[\beta + N]|, |S'[\beta]|), 0 \leq \beta < N$
  $S' \leftarrow S''; d \leftarrow d - 1$

FIG.3a

ITERATIVELYUPDATEC(l, λ, ϕ)

Let d be the maximal integer, such that $\phi + 1$ is divisible by 2^d
$C' \leftarrow CGetArrayPointerC(l, \lambda, 0)$
$C'' \leftarrow CGetArrayPointerC(l, \lambda, 1)$
$\tilde{C} \leftarrow GetArrayPointerC(l, \lambda - d, 0)$
$N = 2^{m-\lambda}$
$\tilde{C} += 2^d N - 2N$
$\tilde{C}[\beta] = C'[\beta] \oplus C''[\beta], 0 \leq \beta < N$
$\tilde{C}[\beta + N] = C''[\beta], 0 \leq \beta < N$
$\lambda --$
while $\lambda > \lambda_0$
do $N = 2N$
    $C'' = \tilde{C}$
    $\tilde{C} -= N$
    $C' = CGetArrayPointerC(l, \lambda, 0)$
    $\tilde{C}[\beta] = C'[\beta] \oplus C''[\beta], 0 \leq \beta < N$
    $\lambda --$

FIG.3b

```
GetArrayPointerS(l, λ)
  return GetWritablePointer('S', PathIndex2ArrayIndex[l, λ].s, λ)

GetArrayPointerC(l, λ, φ)
  return GetWritablePointer('C', PathIndex2ArrayIndex[l, λ].c[φ], λ)

GetWritablePointer(Pool, Index, λ)
  if Index = −1
     then Index ← Allocate(Pool, λ)
     else  if ArrayReferenceCount[Pool][Index] > 1
              then ArrayReferenceCount[Pool][Index] − −
                   Index ← Allocate(Pool, λ)
  return ArrayPointer[Pool][Index]
```

FIG.3c

```
CGetArrayPointerS(l, λ)
  return ArrayPointer[S][PathIndex2ArrayIndex[l, λ].s]

CGetArrayPointerC(l, λ, φ)
  return ArrayPointer[C][PathIndex2ArrayIndex[l, λ].c[φ]]

Allocate(Pool, λ)
  [t, q] ← Pop(InactiveArrayIndices[Pool][λ])
  t ← t(m + 1) + λ
  ArrayReferenceCount[Pool][t] = 1
  if q = 1
     then if Free[Pool] > Λ
               then Abort
             ArrayPointer[Pool][t] = Free[Pool]
             Free[Pool]+ = 2^(m−λ)
  return t
```

FIG.3d

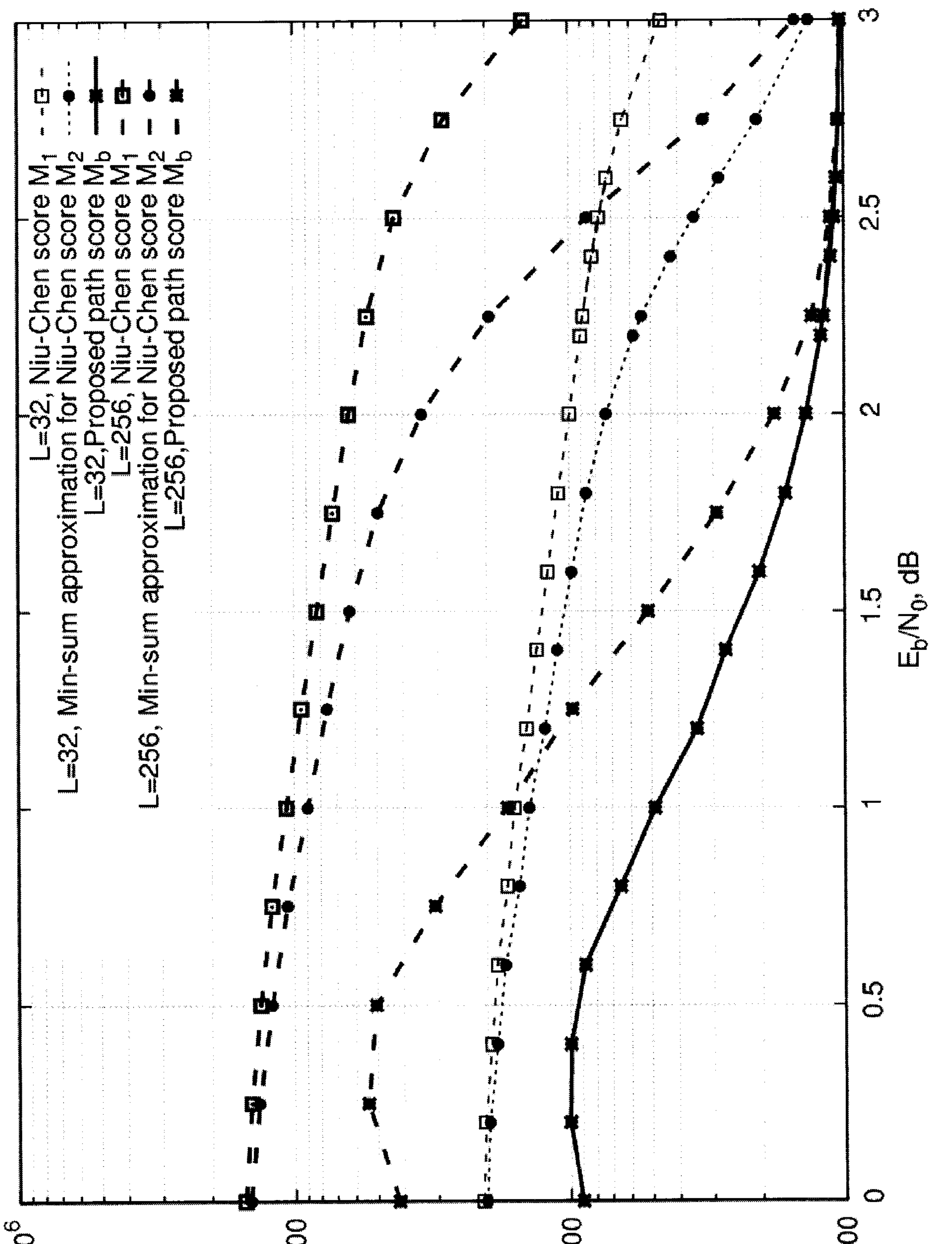

L=32, Niu-Chen score M_1
L=32, Min-sum approximation for Niu-Chen score M_2
L=32,Proposed path score M_b
L=256, Niu-Chen score M_1
L=256, Min-sum approximation for Niu-Chen score M_2
L=256,Proposed path score M_b
Average number of iterations
$1x10^6$
100000
10000
1000
0
0.5
1
1.5
2
2.5
3
E_b/N_0, dB

FIG.4

8/12

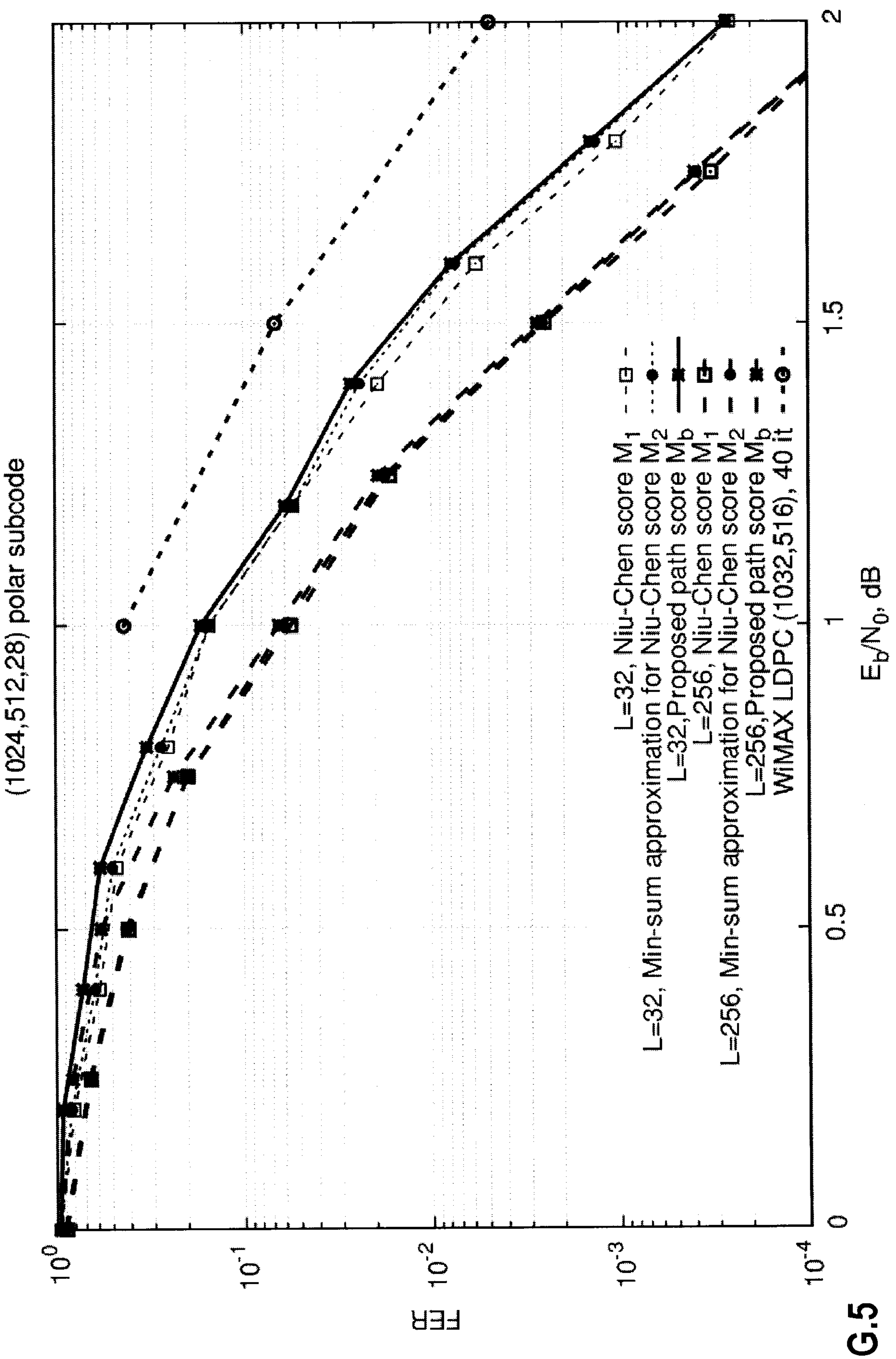
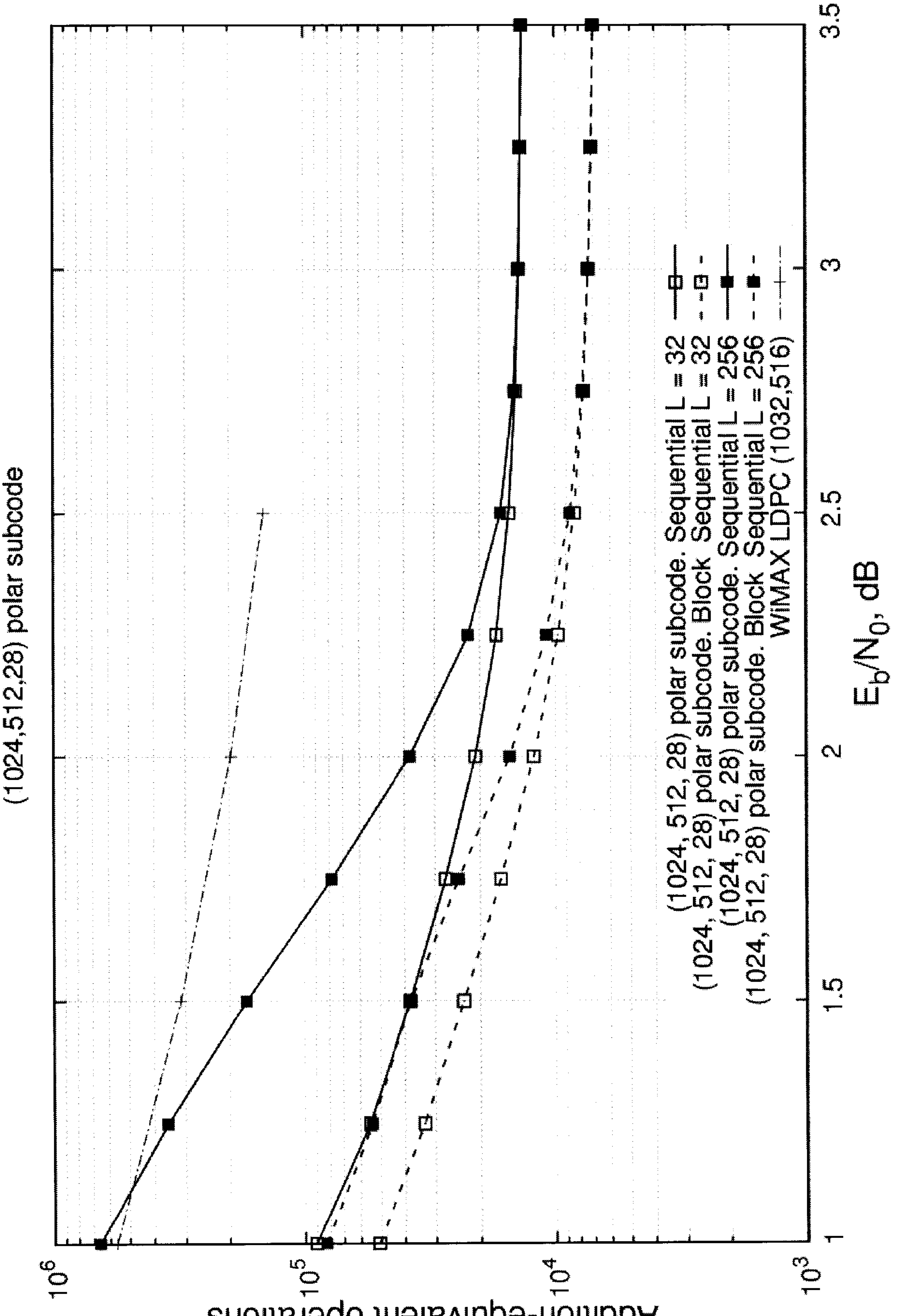

(1024,512,28) polar subcode
FER
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0
0.5
1
1.5
2
E_b/N_0, dB
L=32, Niu-Chen score M_1
L=32, Min-sum approximation for Niu-Chen score M_2
L=32,Proposed path score M_b
L=256, Niu-Chen score M_1
L=256, Min-sum approximation for Niu-Chen score M_2
L=256,Proposed path score M_b
WiMAX LDPC (1032,516), 40 it

FIG.5



(1024,512,28) polar subcode
Addition-equivalent operations
10^3
10^4
10^5
10^6
1
1.5
2
2.5
3
3.5
E_b/N_0, dB
(1024, 512, 28) polar subcode. Sequential L = 32
(1024, 512, 28) polar subcode. Block Sequential L = 32
(1024, 512, 28) polar subcode. Sequential L = 256
(1024, 512, 28) polar subcode. Block Sequential L = 256
WiMAX LDPC (1032,516)

FIG.6

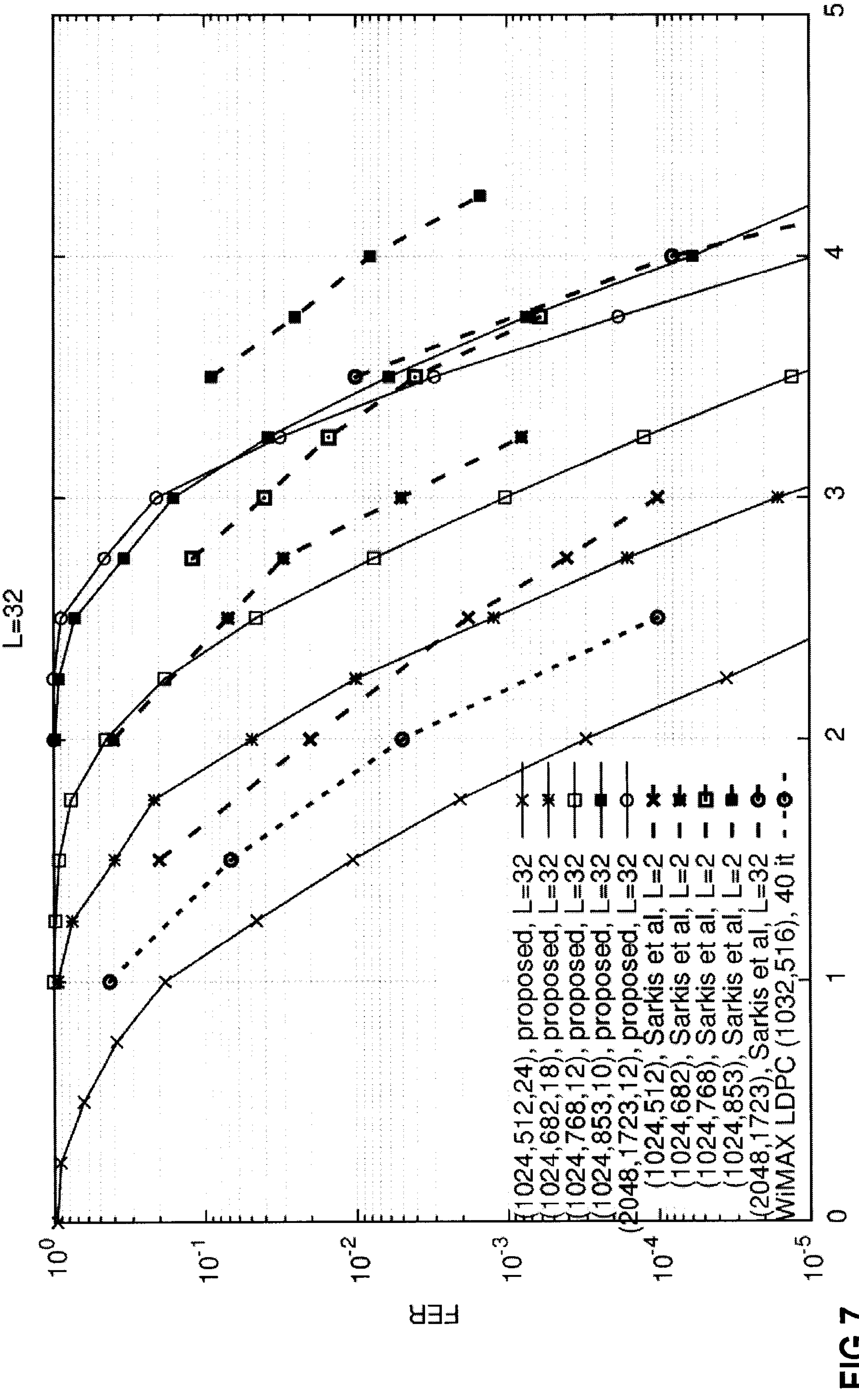

L=32
FER
E_b/N_0, dB
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
0
1
2
3
4
5
(1024,512,24), proposed, L=32
(1024,682,18), proposed, L=32
(1024,768,12), proposed, L=32
(1024,853,10), proposed, L=32
(2048,1723,12), proposed, L=32
(1024,512), Sarkis et al, L=2
(1024,682), Sarkis et al, L=2
(1024,768), Sarkis et al, L=2
(1024,853), Sarkis et al, L=2
(2048,1723), Sarkis et al, L=32
WiMAX LDPC (1032,516), 40 it

FIG.7

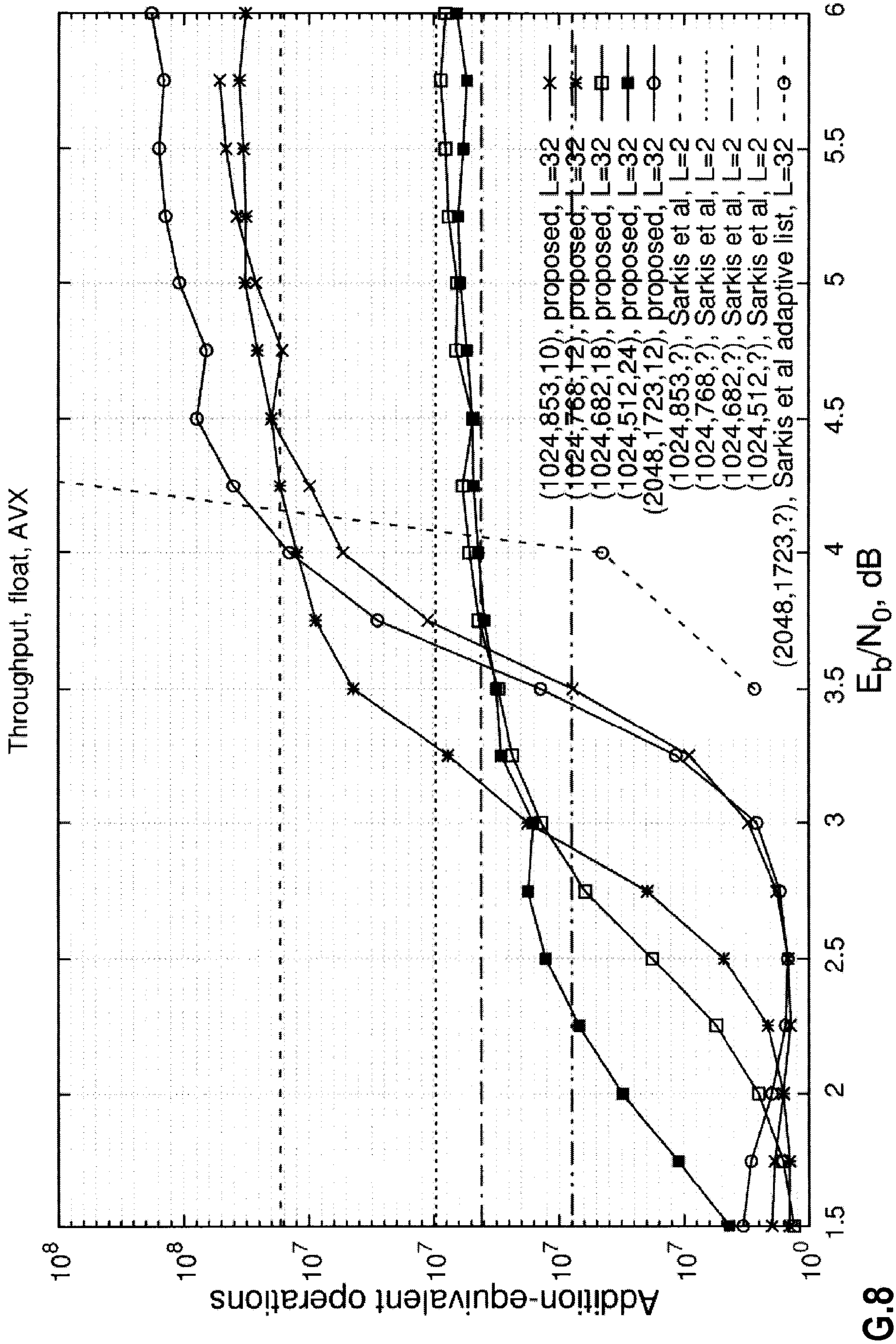

Throughput, float, AVX
Addition-equivalent operations
E_b/N_0, dB
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
(1024,853,10), proposed, L=32
(1024,768,12), proposed, L=32
(1024,682,18), proposed, L=32
(1024,512,24), proposed, L=32
(2048,1723,12), proposed, L=32
(1024,853,?), Sarkis et al, L=2
(1024,768,?), Sarkis et al, L=2
(1024,682,?), Sarkis et al, L=2
(1024,512,?), Sarkis et al, L=2
(2048,1723,?), Sarkis et al adaptive list, L=32

FIG.8

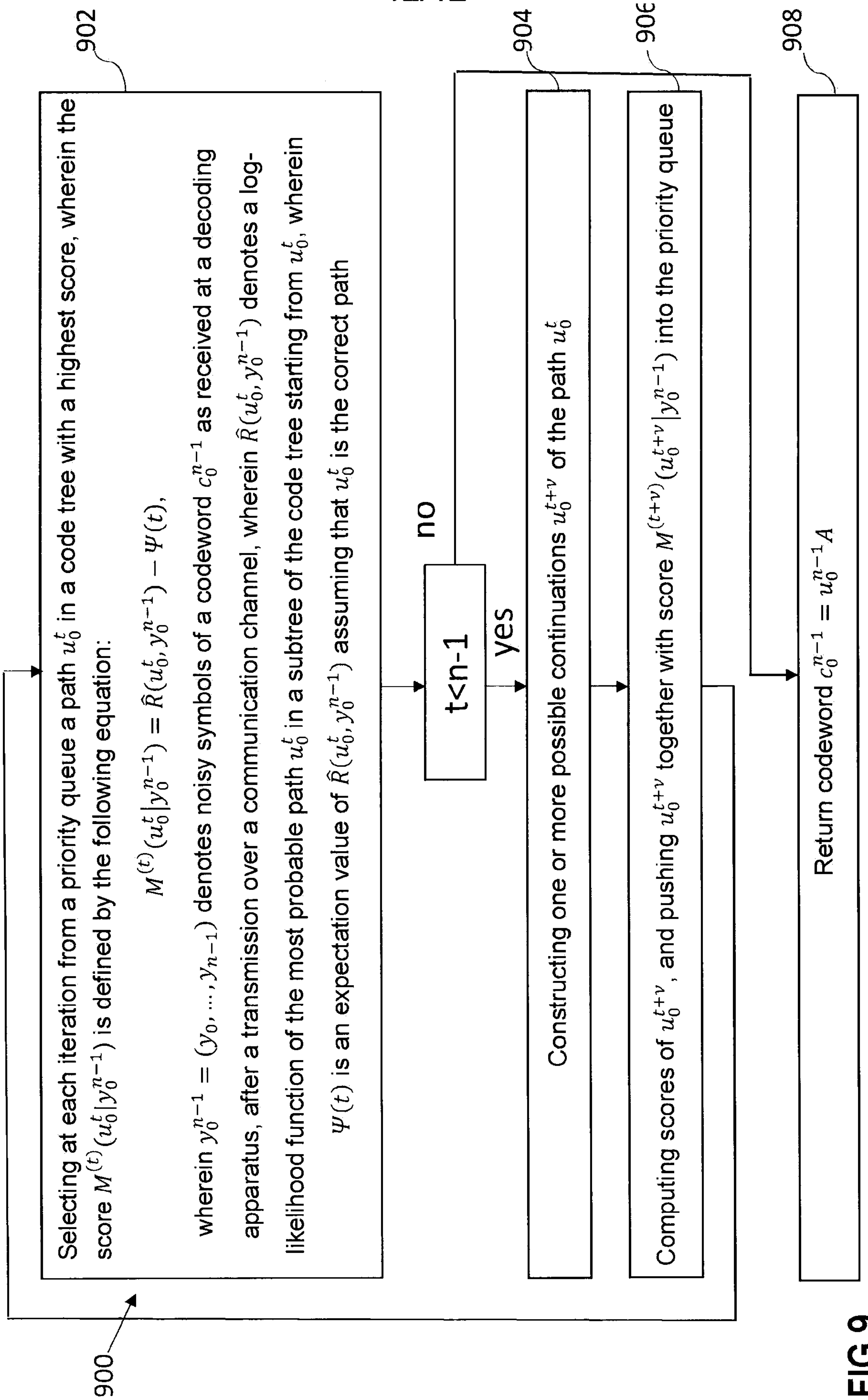

902
Selecting at each iteration from a priority queue a path u_0^t in a code tree with a highest score, wherein the score $M^{(t)}(u_0^t|y_0^{n-1})$ is defined by the following equation:
$M^{(t)}(u_0^t|y_0^{n-1}) = \hat{R}(u_0^t, y_0^{n-1}) - \Psi(t)$,
wherein $y_0^{n-1} = (y_0, \ldots, y_{n-1})$ denotes noisy symbols of a codeword c_0^{n-1} as received at a decoding apparatus, after a transmission over a communication channel, wherein $\hat{R}(u_0^t, y_0^{n-1})$ denotes a log-likelihood function of the most probable path u_0^t in a subtree of the code tree starting from u_0^t, wherein $\Psi(t)$ is an expectation value of $\hat{R}(u_0^t, y_0^{n-1})$ assuming that u_0^t is the correct path
t<n-1
yes
no
904
Constructing one or more possible continuations $u_0^{t+\nu}$ of the path u_0^t
906
Computing scores of $u_0^{t+\nu}$, and pushing $u_0^{t+\nu}$ together with score $M^{(t+\nu)}(u_0^{t+\nu}|y_0^{n-1})$ into the priority queue
908
Return codeword $c_0^{n-1} = u_0^{n-1}A$
900

FIG.9

INTERNATIONAL SEARCH REPORT

International application No
PCT/RU2016/000489

A. CLASSIFICATION OF SUBJECT MATTER
INV. H03M13/13
ADD.

According to International Patent Classification (IPC) or to both national classification and IPC

B. FIELDS SEARCHED

Minimum documentation searched (classification system followed by classification symbols)
H03M

Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched

Electronic data base consulted during the international search (name of data base and, where practicable, search terms used)

EPO-Internal

C. DOCUMENTS CONSIDERED TO BE RELEVANT

Category*	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
A	NIU K ET AL: "Stack decoding of polar codes", ELECTRONICS LETTERS, IEE STEVENAGE, GB, vol. 48, no. 12, 7 June 2012 (2012-06-07), pages 695-697, XP006040947, ISSN: 0013-5194, DOI: 10.1049/EL.2012.1459 cited in the application the whole document -----	1-11
A	V. MILOSLAVSKAYA; P. TRIFONOV: "Sequential decoding of polar codes", IEEE COMMUNICATIONS LETTERS, vol. 18, no. 7, July 2014 (2014-07), pages 1127-1130, XP11553003, cited in the application the whole document -----	1-11
	-/--	

[X] Further documents are listed in the continuation of Box C.

[ ] See patent family annex.

* Special categories of cited documents :

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"X" document of particular relevance; the claimed invention cannot be considered novel or cannot be considered to involve an inventive step when the document is taken alone
"Y" document of particular relevance; the claimed invention cannot be considered to involve an inventive step when the document is combined with one or more other such documents, such combination being obvious to a person skilled in the art
"&" document member of the same patent family

Date of the actual completion of the international search	Date of mailing of the international search report
12 April 2017	21/04/2017

Name and mailing address of the ISA/	Authorized officer
European Patent Office, P.B. 5818 Patentlaan 2 NL - 2280 HV Rijswijk Tel. (+31-70) 340-2040, Fax: (+31-70) 340-3016	Farman, Thomas

2

C(Continuation). DOCUMENTS CONSIDERED TO BE RELEVANT

Category*	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
A	TROFIMIUK GRIGORII ET AL: "Block sequential decoding of polar codes", 2015 INTERNATIONAL SYMPOSIUM ON WIRELESS COMMUNICATION SYSTEMS (ISWCS), IEEE, 25 August 2015 (2015-08-25), pages 326-330, XP032891923, DOI: 10.1109/ISWCS.2015.7454356 [retrieved on 2016-04-15] cited in the application the whole document -----	1,4,10

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