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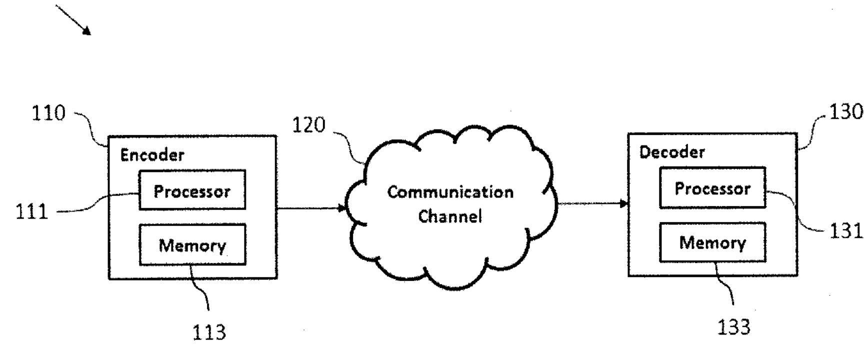
Fig. 1



$$A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}^{\otimes m} \quad \text{(I)}$$

(57) Abstract: The invention relates to devices and methods implementing generalized polar codes with improved Hamming distance. For instance, the invention relates to an encoder (110) for encoding a data vector z, wherein the encoder (100) comprises a processor (111) configured to encode data z into a codeword c such that $c_0^{n-1} = u_0^{n-1}A$ subject to the constraints $u_0^{n-1}V^T = 0$, so that c=z. W.A where W is a matrix such that $W.V^T=0$, wherein Formula (I) wherein $F^{\otimes m}$ denotes the m-times Kronecker product of the matrix F with itself and wherein the constraint matrix V comprises in addition to the constraint matrix V_0 of a parent polar code the constraint matrix V_1 of a first helper code and the constraint matrix V_2 of a second helper code C_2.

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DESCRIPTION

Devices and methods implementing polar codes

TECHNICAL FIELD

In general, the present invention relates to data encoding and decoding in communication systems. More specifically, the present invention relates to devices and methods for encoding and decoding data using polar codes. Moreover, the invention relates to a method for generating a compact specification of a polar code.

BACKGROUND

Reliable transmission of data over noisy communication channels requires some kind of error correction coding to be used. Polar codes were shown to achieve the Shannon capacity of many channels (see E. Arikan, "Channel polarization: A method for constructing capacity achieving codes for symmetric binary-input memoryless channels", IEEE Trans. on Inf. Theory, vol. 55, no. 7, pp. 3051–3073, July 2009). However, the performance of polar codes with practical parameters is often unsatisfactory.

Recently, polar subcodes were shown to have higher minimum distance than classical polar codes, and provide substantially better performance under list, sequential and block sequential decoding (see P. Trifonov and V. Miloslavskaya, "Polar subcodes", IEEE Journal on Selected Areas in Communications, 34(2):254-266, February 2016). However, the performance of polar subcodes still can be improved. Moreover, in the low SNR (signal-to-noise ratio) region a block sequential decoding algorithm may require a lot of iterations for near-ML (maximum likelihood) decoding.

Another problem arising in the practical implementation of polar subcodes is the complexity of their specification, i.e. defining these codes so that they can be executed by a processor. Typically, communication systems need to implement a number of codes with different parameters. The description of all these codes, i.e. their specification, has to be stored in a compact form both at the transmitter and at the receiver.

Generally, a $(n = 2^m, k)$ polar subcode C over $GF(2)$ is a set of vectors $c = zWA$, where W is a $k \times n$ matrix, $A = \begin{pmatrix}1 & 0\\ 1 & 1\end{pmatrix}^{\otimes m}$, and $\mathrm{F}^{\otimes \mathrm{m}}$ denotes the m-times Kronecker product of the matrix F with itself. Classical polar codes are obtained by choosing the matrix W such that each column of W has at most a weight of 1 and each row has a weight of 1. Polar subcodes are obtained by choosing the matrix W such that the vectors c are also codewords of some parent code with sufficiently high minimum distance, i.e. $cH^T = 0$, where H is a check matrix of the parent code. Extended BCH codes were shown to be good parent codes.

An equivalent way to define a polar subcode is to consider it as a set of vectors $c = uA$, where $uV^T = 0$ and V is a $(n-k) \times n$ constraint matrix, such that $WV^T = 0$. By employing Gaussian elimination, the matrix V can be transformed into a form such that at most one row ends in each column. Doing so, one obtains the following set of constraints for the input symbols u_i of the polarizing transformation A: $u_{j_i} = \sum_{s<j_i:V_{is}=1} u_s$, $0 \leq i < n-k$, wherein j_i denotes the position of the last non-zero entry in the i-th row of the matrix V. The symbols j_i are referred to as dynamic frozen symbols, which can be considered as a generalization of the concept of static frozen symbols used in the construction of classical polar codes. The standard way to construct a polar subcode is to construct the constraint matrix $V_0 = HA^T$, where, as already mentioned above, H is the check matrix of a suitable parent code, and to introduce additional constraints $u_{j_i} = 0$ (so-called static freezing constraints) for symbols with the highest error probability P_{j_i} under the successive cancellation decoding for some specific communication channel. Hence, the matrix V can be expressed as:

$$V = \binom{V_0}{E},$$

wherein E is the matrix consisting of rows with weight 1.

The existing techniques for specifying polar codes essentially reduce to specifying the parameters of the channel, and a particular method for computing error probabilities P_i so that one can re-construct the set $\mathcal{F}$ of frozen symbols j_i. The following three methods for solving this problem are known in the prior art:

Firstly, density evolution (see I. Tal, A. Vardy, "How to construct polar codes", IEEE Transactions on Information Theory, 59(10):6562–6582, October 2013) is an optimal and precise method for computing the error probabilities $P_i, 0 \leq i < n$ with complexity

$O(n\mu^2 \log \mu)$, where μ is of the order of several hundreds, which is impractical for online code construction.

Secondly, the Gaussian approximation method described in P. Trifonov, "Efficient design and decoding of polar codes", IEEE Transactions on Communications, 60(11):3221 – 3227, November 2012 has a complexity $O(n)$. However, this method requires evaluation of transcendental functions and, therefore, is very difficult to implement in hardware.

Thirdly, in the binary erasure channel (BEC) approximation method the P_ϕ are taken as erasure probabilities $P_\phi = Z_{m,\phi}$ under successive cancellation decoding in the case of transmission of a codeword of a polar code over the binary erasure channel with erasure probability $Z_{0,0}$, where $Z_{m,2\phi} = 2Z_{m-1,\phi} - \left(Z_{m-1,\phi}\right)^2, Z_{m,2\phi+1} = \left(Z_{m-1,\phi}\right)^2$ (see E. Arikan, "Channel polarization: A method for constructing capacity achieving codes for symmetric binary-input memoryless channels", IEEE Trans. on Inf. Theory, vol. 55, no. 7, pp. 3051–3073, July 2009). The complexity of this method is $O(n)$, but it produces suboptimal codes for channels other than a BEC.

Thus, there is a need for improved devices and methods for encoding and decoding data using polar codes, in particular polar subcodes, as well as specifying these codes in an efficient way.

SUMMARY

It is an object of the invention to provide for improved devices and methods for encoding and decoding data using polar codes. Moreover, it is an object of the invention to provide for a method for generating a specification of a polar code in an efficient way.

The foregoing and other objects are achieved by the subject matter of the independent claims. Further implementation forms are apparent from the dependent claims, the description and the figures.

According to a first aspect the invention relates to an encoder for encoding data. The encoder comprises a processor configured to encode the data using a (n, k, d) parent polar code C into codewords $c_0^{n-1} = u_0^{n-1}A$ subject to the constraints $u_0^{n-1}V^T = 0$, wherein u_0^{n-1} denotes the data, wherein $A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}^{\otimes m}$, wherein $F^{\otimes m}$ denotes the m-times

Kronecker product of the matrix F with itself and wherein the constraint matrix V is defined by the following equation:

$$V = \begin{pmatrix} & V_0 & \\ V_1 & & 0 \\ 0 & & V_2 \\ & E & \end{pmatrix},$$

wherein V_0 denotes the constraint matrix of the parent polar code, V_1 denotes the constraint matrix of a first helper code C_1, V_2 denotes the constraint matrix of a second helper code C_2 and E denotes a matrix consisting of rows of weight 1.

Encoding data by means of the encoder according to the first aspect of the invention allows using a larger number of freezing constraints during the early phases of the sequential/list decoding process. This, in turn, allows reducing the number of high probability paths in the code tree explored by the decoder, thereby reducing the average number of iterations performed and the probability of the decoder losing the correct path. Thus, encoding data by means of the encoder according to the first aspect of the invention provides a peter performance in the high SNR region and substantially reduces the coding complexity compared to encoders using conventional polar subcodes.

In a first possible implementation form of the encoder according to the first aspect as such, the first helper code C_1 defines a minimum distance d_1, which is greater than or equal to the minimum distance d of the parent polar code C.

In a second possible implementation form of the encoder according to the first aspect as such or the first implementation form thereof, the second helper code C_2 defines a minimum distance d_2, which is greater than or equal to half the minimum distance d of the parent polar code C.

In a third possible implementation form of the encoder according to the first aspect as such or the first or second implementation form thereof, the processor is configured to generate the constraint matrix V_1 of the first helper code C_1 and/or the constraint matrix V_2 of the second helper code C_2 recursively.

In a fourth possible implementation form of the encoder according to the third implementation form of the first aspect, the processor is configured to generate the constraint matrix V_1 of the first helper code C_1 and the constraint matrix V_2 of the second helper code C_2 by generating the constraint matrix V_1 of the first helper code C_1 in the same way as the constraint matrix V and by generating the constraint matrix V_2 of the second helper code C_2 in the same way as the constraint matrix V.

In a fifth possible implementation form of the encoder according to the first aspect as such or any one of the first to fourth implementation form thereof, the parent polar code C, the first helper code C_1 and/or the second helper code C_2 is an extended Bose-Chaudhuri-Hocquenghem (BCH) code.

In a sixth possible implementation form of the encoder according to the first aspect as such or any one of the first to fifth implementation form thereof, the encoder further comprises a memory comprising a specification of the parent polar code C, the first helper code C_1 and/or the second helper code C_2, wherein the specification is based on an approximation of the parent polar code C, the first helper code C_1 and/or the second helper code C_2 for a binary erasure channel (BEC).

In a seventh possible implementation form of the encoder according to the sixth implementation form of the first aspect, the specification of the parent polar code C, the first helper code C_1 and/or the second helper code C_2 is generated by the following steps: determining a first frozen set of the code; determining a second frozen set of an approximation of the code using the binary erasure channel (BEC); and storing the differences between the first frozen set and the second frozen set in the memory of the encoder.

According to a second aspect the invention relates to a method for encoding data, wherein the method comprises the step of encoding the data using a (n, k, d) parent polar code C into codewords $c_0^{n-1} = u_0^{n-1}A$ subject to the constraints $u_0^{n-1}V^T = 0$, wherein u_0^{n-1} denotes the data, wherein $A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}^{\otimes m}$, wherein $\mathrm{F}^{\otimes m}$ denotes the m-times Kronecker product of the matrix F with itself and wherein the constraint matrix V is defined by the following equation:

$$V = \begin{pmatrix} & V_0 & \\ V_1 & & 0 \\ 0 & & V_2 \\ & E & \end{pmatrix},$$

wherein V_0 denotes the constraint matrix of the parent polar code, V_1 denotes the constraint matrix of a first helper code C_1, V_2 denotes the constraint matrix of a second helper code C_2 and E denotes a matrix consisting of rows of weight 1.

The method according to the second aspect of the invention can be performed by the encoder according to the first aspect of the invention. Further features of the method according to the second aspect of the invention result directly from the functionality of the encoder according to the first aspect of the invention and its different implementation forms.

According to a third aspect the invention relates to a decoder for decoding codewords, wherein the decoder comprises a processor configured to decode the codewords $c_0^{n-1} = u_0^{n-1}A$ using a (n,k,d) parent polar code C subject to the constraints $u_0^{n-1}V^T = 0$, wherein u_0^{n-1} denotes the data, wherein $A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}^{\otimes m}$, wherein $\mathrm{F}^{\otimes \mathrm{m}}$ denotes the m-times Kronecker product of the matrix F with itself and wherein the constraint matrix V is defined by the following equation:

$$V = \begin{pmatrix} & V_0 & \\ V_1 & & 0 \\ 0 & & V_2 \\ & E & \end{pmatrix},$$

wherein V_0 denotes the constraint matrix of the parent polar code, V_1 denotes the constraint matrix of a first helper code C_1, V_2 denotes the constraint matrix of a second helper code C_2 and E denotes a matrix consisting of rows of weight 1.

In a first possible implementation form of the decoder according to the third aspect, the first helper code C_1 defines a minimum distance d_1, which is greater than or equal to the minimum distance d of the parent polar code C.

In a second possible implementation form of the decoder according to the third aspect as such or the first implementation form thereof, the second helper code C_2 defines a minimum distance d_2, which is greater than or equal to half the minimum distance d of the parent polar code C.

In a third possible implementation form of the decoder according to the third aspect as such or the first or second implementation form thereof, the processor is configured to generate the constraint matrix V_1 of the first helper code C_1 and/or the constraint matrix V_2 of the second helper code C_2 recursively.

In a fourth possible implementation form of the decoder according to the third implementation form of the third aspect, the processor is configured to generate the constraint matrix V_1 of the first helper code C_1 and the constraint matrix V_2 of the second helper code C_2 by generating the constraint matrix V_1 of the first helper code C_1 in the same way as the constraint matrix V and by generating the constraint matrix V_2 of the second helper code C_2 in the same way as the constraint matrix V.

In a fifth possible implementation form of the decoder according to the third aspect as such or any one of the first to fourth implementation form thereof, the parent polar code C, the first helper code C_1 and/or the second helper code C_2 is an extended Bose-Chaudhuri-Hocquenghem (BCH) code.

In a sixth possible implementation form of the decoder according to the third aspect as such or any one of the first to fifth implementation form thereof, the decoder further comprises a memory comprising a specification of the parent polar code C, the first helper code C_1 and/or the second helper code C_2, wherein the specification is based on an approximation of the parent polar code C, the first helper code C_1 and/or the second helper code C_2 for a binary erasure channel (BEC).

In a seventh possible implementation form of the decoder according to the sixth implementation form of the third aspect, the specification of the parent polar code C, the first helper code C_1 and/or the second helper code C_2 is generated by the following steps: determining a first frozen set of the code; determining a second frozen set of an approximation of the code using the binary erasure channel (BEC); and storing the differences between the first frozen set and the second frozen set in the memory of the encoder.

According to a fourth aspect the invention relates to a method for decoding codewords, wherein the method comprises the step of decoding the codewords $c_0^{n-1} = u_0^{n-1}A$ using a (n, k, d) parent polar code C subject to the constraints $u_0^{n-1}V^T = 0$, wherein u_0^{n-1} denotes the data, wherein $A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}^{\otimes m}$, wherein $F^{\otimes m}$ denotes the m-times Kronecker product of the matrix F with itself and wherein the constraint matrix V is defined by the following equation:

$$V = \begin{pmatrix} & V_0 & \\ V_1 & & 0 \\ 0 & & V_2 \\ & E & \end{pmatrix},$$

wherein V_0 denotes the constraint matrix of the parent polar code, V_1 denotes the constraint matrix of a first helper code C_1, V_2 denotes the constraint matrix of a second helper code C_2 and E denotes a matrix consisting of rows of weight 1.

The method according to the fourth aspect of the invention can be performed by the decoder according to the third aspect of the invention. Further features of the method according to the fourth aspect of the invention result directly from the functionality of the decoder according to the third aspect of the invention and its different implementation forms.

According to a fifth aspect the invention relates to a method of providing a compact specification of a polar code, wherein the method comprises the steps of: determining a first frozen set of the polar code; determining a second frozen set of an approximation of the polar code using the binary erasure channel (BEC); and determining the differences between the first frozen set and the second frozen set.

According to a sixth aspect the invention relates to a computer program comprising program code for performing the method according to the second aspect of the invention, the method according to the fourth aspect of the invention or according to the fifth aspect of the invention, when executed on a computer.

The invention can be implemented in hardware and/or software.

BRIEF DESCRIPTION OF THE DRAWINGS

Further embodiments of the invention will be described with respect to the following figures, wherein:

Fig. 1 shows a schematic diagram of a communication system comprising an encoder according to an embodiment and a decoder according to an embodiment;

Fig. 2 shows two exemplary constraint matrices for generating a polar code according to an embodiment;

Fig. 3 shows an exemplary constraint matrix for generating a polar code according to an embodiment;

Fig. 4 shows an exemplary constraint matrix for generating a polar code according to an embodiment;

Fig. 5 shows a diagram for comparing the performance of recursive polar subcodes and conventional non-recursive polar subcodes in the context of sequential and block sequential decoding;

Fig. 6 shows a diagram for comparing the complexity of recursive polar subcodes and conventional non-recursive polar subcodes in the context of sequential decoding; and

Fig. 7 shows a table for comparing the efficiency of the compact code specification implemented in embodiments of the inventions with conventional code specifications.

In the figures, identical reference signs will be used for identical or functionally equivalent features.

DETAILED DESCRIPTION OF EMBODIMENTS

In the following description, reference is made to the accompanying drawings, which form part of the disclosure, and in which are shown, by way of illustration, specific aspects in which the present invention may be placed. It will be appreciated that the invention may be placed in other aspects and that structural or logical changes may be made without departing from the scope of the invention. The following detailed description, therefore, is not to be taken in a limiting sense, as the scope of the invention is defined by the appended claims.

For instance, it will be appreciated that a disclosure in connection with a described method will generally also hold true for a corresponding device or system configured to perform the method and vice versa. For example, if a specific method step is described, a corresponding device may include a unit to perform the described method step, even if such unit is not explicitly described or illustrated in the figures.

Moreover, in the following detailed description as well as in the claims, embodiments with functional blocks or processing units are described, which are connected with each other or exchange signals. It will be appreciated that the invention also covers embodiments which include additional functional blocks or processing units that are arranged between the functional blocks or processing units of the embodiments described below.

Finally, it is understood that the features of the various exemplary aspects described herein may be combined with each other, unless specifically noted otherwise.

Figure 1 shows a schematic diagram illustrating a communication system 100 comprising an encoder 110 and a decoder 130, which can communicate via a communication channel 120.

The encoder 110 comprises a processor 111 and a memory 113 and is configured to encode data. Likewise, the decoder 130 comprises a processor 131 and a memory 133 and is configured to decode data, in particular data encoded by the encoder 110. The encoder 110 and/or the decoder 130 can be implemented as part of a communication device, such as a mobile phone or a base station of a cellular communication network.

In order to reduce the decoding error probability and complexity of sequential decoding, embodiments of the invention provide recursive polar subcodes. The constraint matrix for a recursive polar subcode is given by

$$V = \begin{pmatrix} \multicolumn{2}{c}{V_0} \\ V_1 & 0 \\ 0 & V_2 \\ \multicolumn{2}{c}{E} \end{pmatrix}$$

where V_1 and V_2 are the constraint matrices of some helper $(n/2, k_i, d_i)$ codes C_1 and C_2.

Thus, in an embodiment the processor 111 of the encoder 110 is configured to encode the data using a (n,k,d) parent polar code C into codewords $c_0^{n-1} = u_0^{n-1}A$ subject to the constraints $u_0^{n-1}V^T = 0$, wherein u_0^{n-1} denotes the data, wherein $A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}^{\otimes m}$, wherein $F^{\otimes m}$ denotes the m-times Kronecker product of the matrix F with itself. (n,k,d) defines the length n of the parent polar code C, the number of information bits k, i.e the length of the data, and the minimum distance d of the parent polar code C. The constraint matrix V is defined by the following equation:

$$V = \begin{pmatrix} & V_0 & \\ V_1 & & 0 \\ 0 & & V_2 \\ & E & \end{pmatrix},$$

wherein V_0 denotes the constraint matrix of the parent polar code, V_1 denotes the constraint matrix of a first helper code C_1, V_2 denotes the constraint matrix of a second helper code C_2 and E denotes a matrix consisting of rows of weight 1. In an embodiment, the first helper code C_1 and/or the second helper code C_2 are subcodes of a Reed-Muller code of sufficiently small order. The rows of weight 1 of the matrix E define static freezing constraints, which, in an embodiment, are imposed onto bit subchannels of low capacity by the polarizing transformation.

Likewise, in an embodiment the processor 131 of the decoder 130 is configured to decode the codewords $c_0^{n-1} = u_0^{n-1}A$ using a (n,k,d) parent polar code C subject to the constraints $u_0^{n-1}V^T = 0$, wherein u_0^{n-1} denotes the data, wherein $A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}^{\otimes m}$, wherein $F^{\otimes m}$ denotes the m-times Kronecker product of the matrix F with itself and wherein the constraint matrix V is defined by the following equation:

$$V = \begin{pmatrix} & V_0 & \\ V_1 & & 0 \\ 0 & & V_2 \\ & E & \end{pmatrix}$$

wherein V_0 denotes the constraint matrix of the parent polar code, V_1 denotes the constraint matrix of a first helper code C_1, V_2 denotes the constraint matrix of a second helper code C_2 and E denotes a matrix consisting of rows of weight 1.

In an embodiment, the parameters of the helper codes are selected in such a way that they have sufficiently high minimum distance, i.e.

$$d_1 \geq d,\ d_2 \geq \frac{d}{2}.$$

The constraint matrices V_1 and V_2 of the helper codes can be constructed recursively in the same way as the constraint matrix V_0 of the parent code. Thus, embodiments of the invention allow introducing more freezing constraints to the early phases of the sequential/list decoding process. This allows reducing the number of high probability paths in the code tree explored by the decoder, reducing thus the average number of iterations performed, and the probability of the decoder losing the correct path.

In an embodiment, the code can be constructed in the follow way:

1. Construct dynamic freezing constraints for a parent (n, k', d) extended BCH code;
2. Construct dynamic freezing constraints for the helper (n_i, k_i, d_i) extended BCH codes;
3. Combine the constraint matrices of parent and helper codes to obtain $r' \times n$ matrix V', and let $F' = \{i_j = \max_{i:V_{ji} \neq 0} i \,|0 \leq j < r'\}$;
4. Compute the error probability in bit subchannels $W_m^{(i)}(y_0^{n-1}, u_0^{i-1}|u_\phi)$, $0 \leq \phi < 2^m$ using the techniques described, for example, in I. Tal, A. Vardy, "How to construct polar codes", IEEE Transactions on Information Theory, 59(10):6562–6582, October 2013 or P. Trifonov, "Efficient design and decoding of polar codes", IEEE Transactions on Communications, 60(11):3221 – 3227, November 2012; and
5. Find $n - r' - k$ indices $\phi \notin F'$ with the highest error probability, and define additional constraints $u_\phi = 0$.

In an embodiment, the parent polar code C, the first helper code C_1 and/or the second helper code C_2 is an extended Bose-Chaudhuri-Hocquenghem (BCH) code. The check matrix of an extended BCH parent or helper code is given by

$$\boldsymbol{H} = \begin{pmatrix} x_0^0 & x_1^0 & \dots & x_{n-1}^0 \\ x_0^1 & x_1^1 & \dots & x_{n-1}^1 \\ \vdots & \vdots & \ddots & \vdots \\ x_0^{d-2} & x_1^{d-2} & \dots & x_{n-1}^{d-2} \end{pmatrix},$$

where x_i are distinct values of $GF(2^m)$. Each row of this matrix results in a number of freezing constraints of the following form:

$$\boldsymbol{u}\boldsymbol{A}(x_0^t, \ldots, x_{n-1}^t)^T = 0,\ 0 \leq t \leq d-2 \qquad (1)$$

It is possible to show that after combining with additional constraints $u_i = 0$ corresponding to bit subchannels with high error probability most of the equations defined by (1) become trivial (i.e. $u_{i_j} = 0$).

The rows of check matrices corresponding to the non-trivial constraints $\boldsymbol{u}_{i_j} = \sum_{s=0}^{i_j-1} \boldsymbol{u}_s \boldsymbol{V}_{js}$ are referred to as active rows. In an embodiment, only the active rows (i.e. integers t) of the constraint matrix are specified.

In an embodiment, the remaining static freezing constraints are specified in the following manner. A parameter $Z_{0,0}$ defining the binary erasure channel (BEC) is specified and only the symmetric differences between the set of static frozen symbol indices (i.e. integers $i: u_i = 0$) of the polar code constructed for the BEC and the considered polar subcode are stored as part of the specification. As known to the person skilled in the art, the symmetric difference of two sets A and D is their union excluding their intersection, i.e. $A \triangle B = (A \cup B) \setminus (A \cap B)$.

Thus, the present invention provides a method of generating a compact specification of a polar code, wherein the method comprises the steps of: determining a first frozen set of the polar code; determining a second frozen set of an approximation of the polar code using the binary erasure channel (BEC); and determining the differences between the first frozen set and the second frozen set. Such compact specifications can be stored in the memory 113 of the encoder 110 and/or the memory 133 of the decoder 130.

Let $\mathcal{F}$ be the set of trivial frozen symbol indices for a code of length $n = 2^m, |\mathcal{F}| = f$. Consider a BEC with erasure probability $Z_{0,0} = p$. Denote $B(p)$ as the vector of indices i of subchannels sorted by their erasure probabilities $Z_{m,i}$ in descending order. Let $B(p)_0^{f'-1}$ be the set of frozen symbol indices for a $(n, n-f')$ polar code C' optimized for the BEC. In an embodiment, the size of the difference of the set of indices of frozen symbols between C' and the target code is minimized. Thereafter, $B_{\mathcal{F}} = B(p^*)_0^{f^*-1}$ can be computed, where

$(p^*, f^*) = \arg\min_{p\in[0,1],0\leq f<n}|\mathcal{F}\,\Delta\, B(p)_0^{f-1}|$. Hence, according to an embodiment the code can be specified by a tuple $S(m,\mathcal{F}) = (m, |\mathcal{F}|, p^*, |B_{\mathcal{F}}|, \mathcal{F}\Delta B_F)$, where m is the logarithm to the base 2 of the code length, $|\mathcal{F}|$ is number of static frozen symbols, and p^* is the optimal erasure probability.

In an embodiment, this approach is implemented recursively. In an embodiment, the set $\mathcal{F}$ is split into two subsets $H_L(m,\mathcal{F}) = \{i \in \mathcal{F} | i < 2^{m-1}\}$ and $H_R(m,\mathcal{F}) = \{i - 2^{m-1} | i \in \mathcal{F},\, i \geq 2^{m-1}\}$. In an embodiment, the compact specification procedure $S(m,F)$ is recursively applied to obtain a recursive specification in the following form:

$$S^*(m,F) = \begin{cases} S(m,F), & |S(m,F)| \leq |S^*(m-1,H_L(m,F))| + |S^*(m-1,H_R(m,F))| \\ S^*(m-1,H_L(m,F)).S^*(m-1,H_R(m,F)), & otherwise, \end{cases}$$

where $|S|$ is the length in bytes of the specification and $S_1.S_2$ denotes a concatenation of the two specifications S_1 and S_2.

In the following an example will be described, which illustrate several aspects of the invention.

Consider construction of a (32,17,6) recursive polar subcode. A constraint matrix of the (32,21,6) e-BCH parent code is given by the matrix V_0 shown in figure 2. A constraint matrix of the (16,7,6) e-BCH code is given by the matrix V_1 shown also in figure 2. Combining these matrices, one obtains the matrix V shown in figure 3. Eliminating linearly dependent rows, one obtains the equivalent constraint matrix V shown in figure 4. Since this matrix has already 32-17=15 rows, there is no need to impose any additional constraints, so it can be taken as a constraint matrix of the (32,17,6) recursive polar subcode.

Consider a (32, 16)-polar code with the following set of frozen symbols F:

$$F = \{0,16,17,18,19,20,21,22,23,24,25,26,28,29,30,31\}$$

In an embodiment, the frozen set F can be approximated by the frozen set for the binary erasure channel (BEC) with erasure probabilities of 0.005, 0.01, 0.015, ..., 0.995. In an embodiment, the optimal approximated set is $B_F = \{0\}, p^* = 0.005, f^* = 1$. The non-recursive compact specification is defined as follows:

$$S(5,F) = (m, |F|, p^*, |B_F|, |F\backslash B_F|, F\backslash B_F, B_F\backslash F)$$
$$= (5, 16, 0.005, 1, 15, \{16,17,18,19,20,21,22,23,24,25,26,28,29,30,31\}, \{\})$$

This exemplary specification contains 20 numbers. After the algorithm finishes building this specification, it tries to split the frozen set into two parts and approximate them separately. In the considered case the partitioning is

$$H_L = \{0\}, H_R = \{0,1,2,3,4,5,6,7,8,9,10,12,13,14,15\}$$

The set H_L is approximated by $B_{H_L} = B_F, p^*_{H_L} = p^*, f^*_{H_L} = f^*$. The set H_R is approximated by

$$B_{H_R} = \{0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15\}, f^*_{H_R} = 16, p^*_{H_R} = 0.005,$$
$$|H_R\backslash B_{H_R}| = 0, |B_{H_R}\backslash H_R| = 1, B_{H_R}\backslash H_R = \{11\}$$

Hence, the code can be specified by giving the following numbers:

$$4, 0.005, 1, 1, 0$$
$$4, 0.005, 15, 16, 0$$
$$11$$

This recursive specification requires just 11 numbers, which is much less than is required for the above presented non-recursive specification.

Figure 5 illustrates the performance of polar subcodes with recursive and non-recursive construction. It can be seen that recursive polar subcodes provide substantially better performance. Figure 6 illustrates the average number of operations performed by the block sequential decoding algorithm while decoding recursive and non-recursive polar subcodes. It can be seen that the decoding complexity for the case of recursive polar subcodes is approximately 22% lower compared to the case of non-recursive ones.

The efficiency of the embodiments for compact code specification is illustrated by the table shown in figure 7.

While a particular feature or aspect of the disclosure may have been disclosed with respect to only one of several implementations or embodiments, such feature or aspect may be combined with one or more other features or aspects of the other implementations or embodiments as may be desired and advantageous for any given or particular application. Furthermore, to the extent that the terms "include", "have", "with", or other variants thereof are used in either the detailed description or the claims, such terms are intended to be inclusive in a manner similar to the term "comprise". Also, the terms "exemplary", "for example" and "e.g." are merely meant as an example, rather than the best or optimal. The terms “coupled” and “connected”, along with derivatives may have been used. It should be understood that these terms may have been used to indicate that two elements cooperate or interact with each other regardless whether they are in direct physical or electrical contact, or they are not in direct contact with each other.

Although specific aspects have been illustrated and described herein, it will be appreciated by those of ordinary skill in the art that a variety of alternate and/or equivalent implementations may be substituted for the specific aspects shown and described without departing from the scope of the present disclosure. This application is intended to cover any adaptations or variations of the specific aspects discussed herein.
Although the elements in the following claims are recited in a particular sequence with corresponding labeling, unless the claim recitations otherwise imply a particular sequence for implementing some or all of those elements, those elements are not necessarily intended to be limited to being implemented in that particular sequence.

Many alternatives, modifications, and variations will be apparent to those skilled in the art in light of the above teachings. Of course, those skilled in the art will readily recognize that there are numerous applications of the invention beyond those described herein. While the present invention has been described with reference to one or more particular embodiments, those skilled in the art will recognize that many changes may be made thereto without departing from the scope of the present invention. It is therefore to be understood that within the scope of the appended claims and their equivalents, the invention may be practiced otherwise than as specifically described herein.

CLAIMS

1. An encoder (110) for encoding data, wherein the encoder (100) comprises:

a processor (111) configured to encode the data using a (n, k, d) parent polar code C into codewords $c_0^{n-1} = u_0^{n-1}A$ subject to the constraints $u_0^{n-1}V^T = 0$, wherein u_0^{n-1} denotes the data, wherein $A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}^{\otimes m}$, wherein $\mathrm{F}^{\otimes \mathrm{m}}$ denotes the m-times Kronecker product of the matrix F with itself and wherein the constraint matrix V is defined by the following equation:

$$V = \begin{pmatrix} & V_0 & \\ V_1 & & 0 \\ 0 & & V_2 \\ & E & \end{pmatrix}$$

wherein V_0 denotes the constraint matrix of the parent polar code, V_1 denotes the constraint matrix of a first helper code C_1, V_2 denotes the constraint matrix of a second helper code C_2 and E denotes a matrix consisting of rows of weight 1.

2. The encoder (110) of claim 1, wherein the first helper code C_1 defines a minimum distance d_1, which is greater than or equal to the minimum distance d of the parent polar code C.

3. The encoder (110) of claim 1 or 2, wherein the second helper code C_2 defines a minimum distance d_2, which is greater than or equal to half the minimum distance d of the parent polar code C.

4. The encoder (110) of any one of the preceding claims, wherein the processor (111) is configured to generate the constraint matrix V_1 of the first helper code C_1 and/or the constraint matrix V_2 of the second helper code C_2 recursively.

5. The encoder (110) of claim 4, wherein the processor (111) is configured to generate the constraint matrix V_1 of the first helper code C_1 and the constraint matrix V_2 of the second helper code C_2 by generating the constraint matrix V_1 of the first helper code C_1 in the same way as the constraint matrix V and by generating the constraint matrix V_2 of the second helper code C_2 in the same way as the constraint matrix V.

6. The encoder (110) of any one of the preceding claims, wherein the parent polar code C, the first helper code C_1 and/or the second helper code C_2 is an extended Bose-Chaudhuri-Hocquenghem (BCH) code.

7. The encoder (110) of any one of the preceding claims, wherein the encoder (110) further comprises a memory (113) comprising a specification of the parent polar code C, the first helper code C_1 and/or the second helper code C_2, wherein the specification is based on an approximation of the parent polar code C, the first helper code C_1 and/or the second helper code C_2 for a binary erasure channel (BEC).

8. The encoder (110) of claim 7, wherein the specification of the parent polar code C, the first helper code C_1 and/or the second helper code C_2 is generated by the following steps: determining a first frozen set of the code; determining a second frozen set of an approximation of the code using the binary erasure channel (BEC); and storing the differences between the first frozen set and the second frozen set in the memory (113) of the encoder (110).

9. A method for encoding data, wherein the method comprises the step of:

encoding the data using a (n, k, d) parent polar code C into codewords $c_0^{n-1} = u_0^{n-1}A$ subject to the constraints $u_0^{n-1}V^T = 0$, wherein u_0^{n-1} denotes the data, wherein $A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}^{\otimes m}$, wherein $\mathrm{F}^{\otimes \mathrm{m}}$ denotes the m-times Kronecker product of the matrix F with itself and wherein the constraint matrix V is defined by the following equation:

$$V = \begin{pmatrix} & V_0 & \\ V_1 & & 0 \\ 0 & & V_2 \\ & E & \end{pmatrix}$$

wherein V_0 denotes the constraint matrix of the parent polar code, V_1 denotes the constraint matrix of a first helper code C_1, V_2 denotes the constraint matrix of a second helper code C_2 and E denotes a matrix consisting of rows of weight 1.

10. A decoder (130) for decoding codewords, wherein the decoder (130) comprises:

a processor (131) configured to decode the codewords $c_0^{n-1} = u_0^{n-1}A$ using a (n, k, d) parent polar code C subject to the constraints $u_0^{n-1}V^T = 0$, wherein u_0^{n-1} denotes the data, wherein $A = \begin{pmatrix}1 & 0\\ 1 & 1\end{pmatrix}^{\otimes m}$, wherein $\mathrm{F}^{\otimes m}$ denotes the m-times Kronecker product of the matrix F with itself and wherein the constraint matrix V is defined by the following equation:

$$V = \begin{pmatrix} & V_0 & \\ V_1 & & 0 \\ 0 & & V_2 \\ & E & \end{pmatrix}$$

wherein V_0 denotes the constraint matrix of the parent polar code, V_1 denotes the constraint matrix of a first helper code C_1, V_2 denotes the constraint matrix of a second helper code C_2 and E denotes a matrix consisting of rows of weight 1.

11. The decoder (130) of claim 10, wherein the first helper code C_1 defines a minimum distance d_1, which is greater than or equal to the minimum distance d of the parent polar code C.

12. The decoder (130) of claim 10 or 11, wherein the second helper code C_2 defines a minimum distance d_2, which is greater than or equal to half the minimum distance d of the parent polar code C.

13. The decoder (130) of any one of claims 10 to 12, wherein the processor (111) is configured to generate the constraint matrix V_1 of the first helper code C_1 and/or the constraint matrix V_2 of the second helper code C_2 recursively.

14. The decoder (130) of claim 13, wherein the processor (111) is configured to generate the constraint matrix V_1 of the first helper code C_1 and the constraint matrix V_2 of the second helper code C_2 by generating the constraint matrix V_1 of the first helper code C_1 in the same way as the constraint matrix V and by generating the constraint matrix V_2 of the second helper code C_2 in the same way as the constraint matrix V.

15. The decoder (130) of any one of claims 10 to 14, wherein the parent polar code C, the first helper code C_1 and/or the second helper code C_2 is an extended Bose-Chaudhuri-Hocquenghem (BCH) code.

16. The decoder (130) of any one of claims 10 to 15, wherein the decoder (130) further comprises a memory (133) comprising a specification of the parent polar code C, the first helper code C_1 and/or the second helper code C_2, wherein the specification is based on an approximation of the parent polar code C, the first helper code C_1 and/or the second helper code C_2 for a binary erasure channel (BEC).

17. The decoder (130) of claim 16, wherein the specification of the parent polar code C, the first helper code C_1 and/or the second helper code C_2 is generated by the following steps: determining a first frozen set of the code; determining a second frozen set of an approximation of the code using the binary erasure channel (BEC); and storing the differences between the first frozen set and the second frozen set in the memory (133) of the decoder (130).

18. A method for decoding codewords, wherein the method comprises the step of:

decoding the codewords $c_0^{n-1} = u_0^{n-1}A$ using a (n,k,d) parent polar code C subject to the constraints $u_0^{n-1}V^T = 0$, wherein u_0^{n-1} denotes the data, wherein $A = \begin{pmatrix}1 & 0\\1 & 1\end{pmatrix}^{\otimes m}$, wherein $F^{\otimes m}$ denotes the m-times Kronecker product of the matrix F with itself and wherein the constraint matrix V is defined by the following equation:

$$V = \begin{pmatrix} & V_0 & \\ V_1 & & 0 \\ 0 & & V_2 \\ & E & \end{pmatrix}$$

wherein V_0 denotes the constraint matrix of the parent polar code, V_1 denotes the constraint matrix of a first helper code C_1, V_2 denotes the constraint matrix of a second helper code C_2 and E denotes a matrix consisting of rows of weight 1.

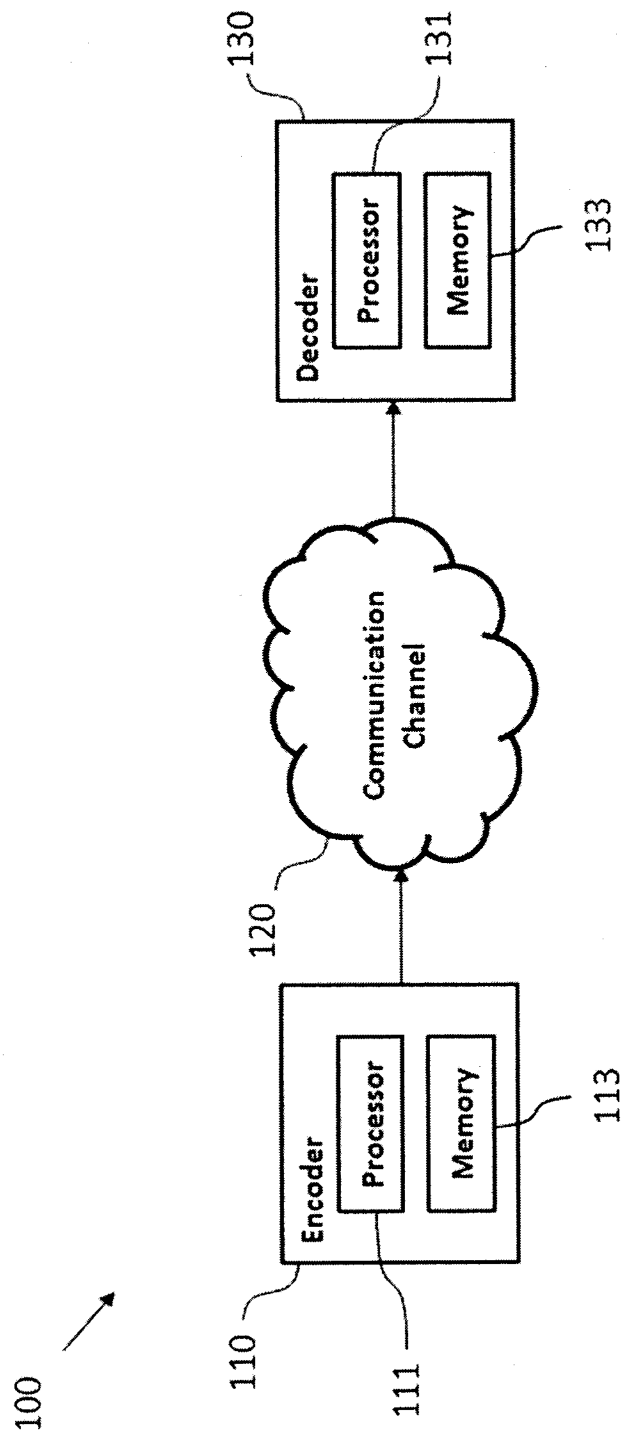
19. A method of generating a compact specification of a polar code, wherein the method comprises the steps of:

determining a first frozen set of the polar code;

determining a second frozen set of an approximation of the polar code using the binary erasure channel (BEC); and

determining the differences between the first frozen set and the second frozen set.

20. A computer program comprising program code for performing the method of claim 9, the method of claim 18 or the method of claim 19 when executed on a computer.



100
110
111
Encoder
Processor
Memory
113
120
Communication Channel
130
131
Decoder
Processor
Memory
133

Fig. 1

$$v_0 = \begin{pmatrix}
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&1&0&0&1&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&1&1&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&1&1&0&0&1&1&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&1&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0
\end{pmatrix}$$

$$V_1 = \begin{pmatrix}
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&1&0&0&0&0&0&0\\
0&0&0&1&0&1&0&0&0&0&1&0&0&0&0&0\\
0&0&0&1&0&1&0&0&0&0&0&0&1&0&0&0
\end{pmatrix}$$

Fig. 2

$$V = \begin{pmatrix}
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&1&0&0&1&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&1&1&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&1&1&0&0&1&1&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&1&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
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0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&1&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&1&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0
\end{pmatrix}$$

Fig. 3

$$V = \begin{pmatrix}
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0
\end{pmatrix}$$

Fig. 4

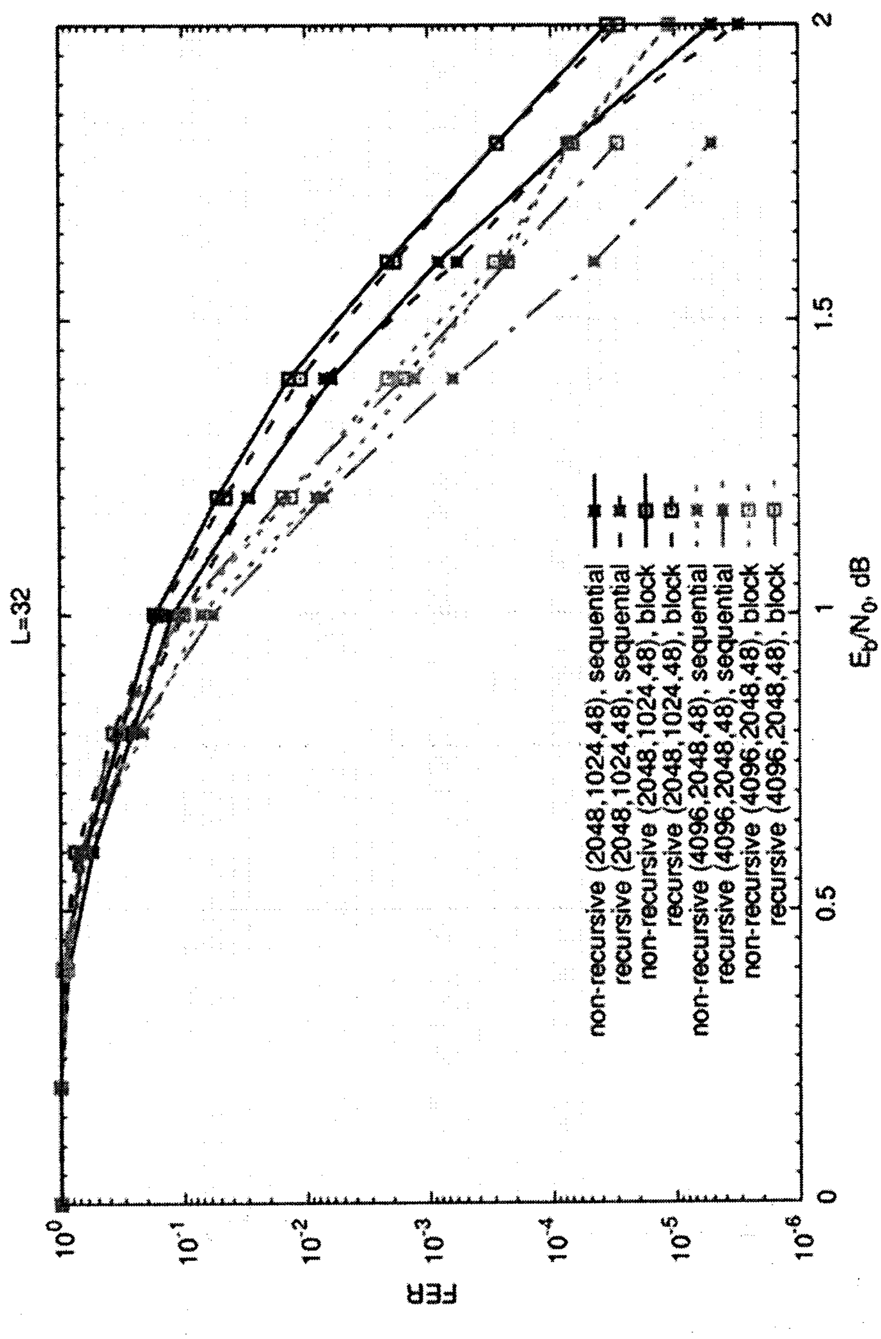
L=32
FER
E_b/N_0, dB
non-recursive (2048,1024,48), sequential
recursive (2048,1024,48), sequential
non-recursive (2048,1024,48), block
recursive (2048,1024,48), block
non-recursive (4096,2048,48), sequential
recursive (4096,2048,48), sequential
non-recursive (4096,2048,48), block
recursive (4096,2048,48), block

Fig. 5

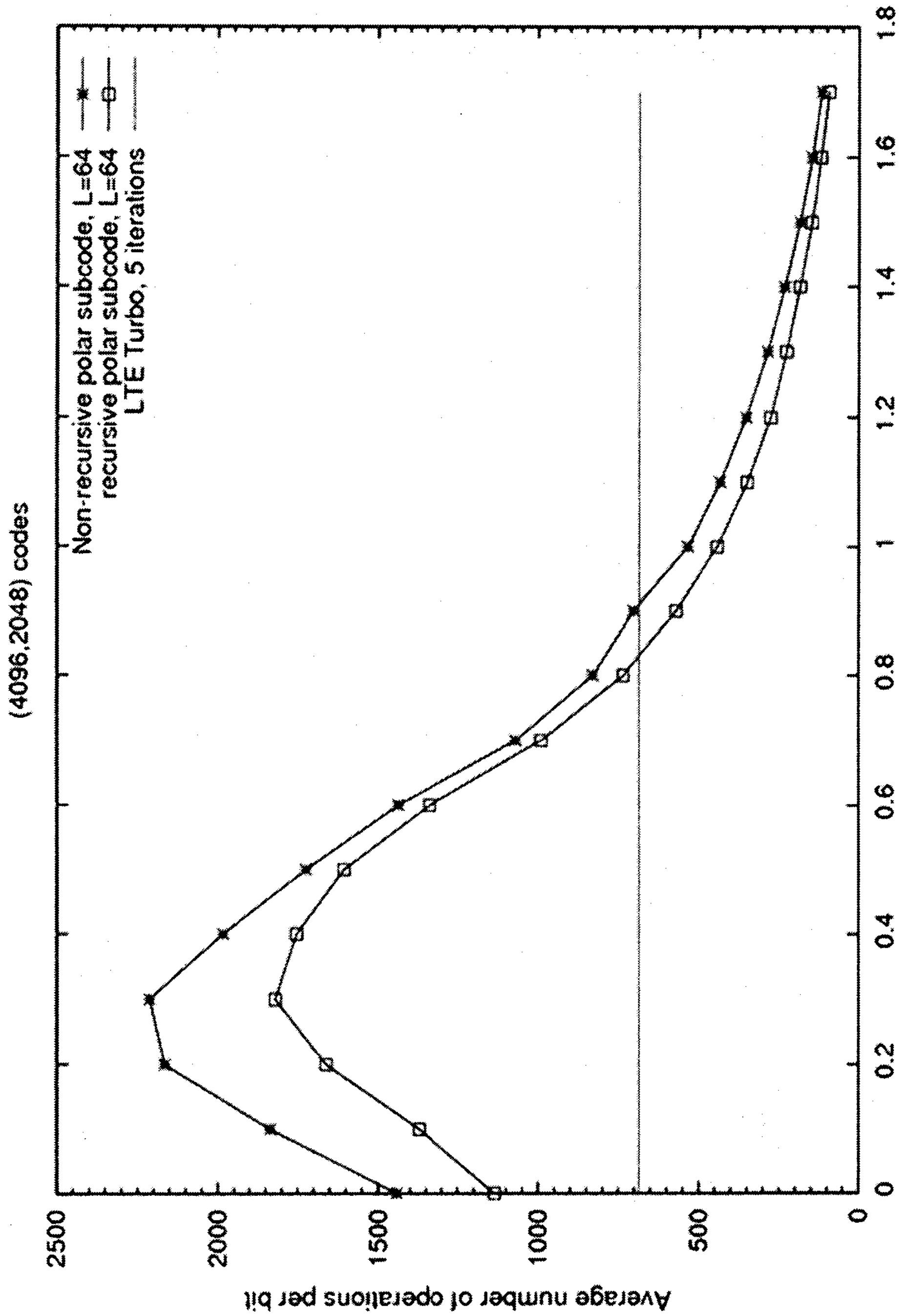
(4096,2048) codes
Non-recursive polar subcode, L=64
recursive polar subcode, L=64
LTE Turbo, 5 iterations
Average number of operations per bit
2500
2000
1500
1000
500
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8

Fig. 6

Code (n, k, d)	Size of an explicit specification, bytes	Compact specification size, bytes
(4096, 3500, 64)	191587	304
(8192, 7000, 100)	586746	518
(16384, 14000, 128)	1414869	583
(65536, 57000, 256)	854111	1765
(4096, 2048, 48)	20507	189
(8192, 4096, 64)	38071	251
(16384, 8192, 96)	97719	454
(32768, 16384, 128)	211598	709
(65536, 32768, 192)	475486	1181

Fig. 7

INTERNATIONAL SEARCH REPORT

International application No
PCT/RU2016/000285

A. CLASSIFICATION OF SUBJECT MATTER
INV. H03M13/13 H03M13/29
ADD.

According to International Patent Classification (IPC) or to both national classification and IPC

B. FIELDS SEARCHED

Minimum documentation searched (classification system followed by classification symbols)
H03M

Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched

Electronic data base consulted during the international search (name of data base and, where practicable, search terms used)

EPO-Internal, WPI Data

C. DOCUMENTS CONSIDERED TO BE RELEVANT

Category*	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
A	TRIFONOV PETER ET AL: "Polar Subcodes", IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS, IEEE SERVICE CENTER, PISCATAWAY, US, vol. 34, no. 2, February 2016 (2016-02), pages 254-266, XP011593857, ISSN: 0733-8716, DOI: 10.1109/JSAC.2015.2504269 [retrieved on 2016-01-15] section IV.A. Section V. "Polar Subcodes" -----	1-18,20
A	US 2015/295593 A1 (TRIFONOV PETER VLADIMIROVICH [RU] ET AL) 15 October 2015 (2015-10-15) paragraph [0042] paragraph [0050] - paragraph [0055] -----	1-18,20

-/--

[X] Further documents are listed in the continuation of Box C.

[X] See patent family annex.

* Special categories of cited documents :

"A" document defining the general state of the art which is not considered to be of particular relevance

"E" earlier application or patent but published on or after the international filing date

"L" document which may throw doubts on priority claim(s) or which is cited to establish the publication date of another citation or other special reason (as specified)

"O" document referring to an oral disclosure, use, exhibition or other means

"P" document published prior to the international filing date but later than the priority date claimed

"T" later document published after the international filing date or priority date and not in conflict with the application but cited to understand the principle or theory underlying the invention

"X" document of particular relevance; the claimed invention cannot be considered novel or cannot be considered to involve an inventive step when the document is taken alone

"Y" document of particular relevance; the claimed invention cannot be considered to involve an inventive step when the document is combined with one or more other such documents, such combination being obvious to a person skilled in the art

"&" document member of the same patent family

Date of the actual completion of the international search	Date of mailing of the international search report
7 February 2017	12/04/2017

Name and mailing address of the ISA/	Authorized officer
European Patent Office, P.B. 5818 Patentlaan 2 NL - 2280 HV Rijswijk Tel. (+31-70) 340-2040, Fax: (+31-70) 340-3016	Farman, Thomas

INTERNATIONAL SEARCH REPORT

International application No
PCT/RU2016/000285

C(Continuation). DOCUMENTS CONSIDERED TO BE RELEVANT

Category*	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
A	EP 2 849 377 A1 (HUAWEI TECH CO LTD [CN]) 18 March 2015 (2015-03-18) figure 2 -----	1-18,20

INTERNATIONAL SEARCH REPORT

International application No.
PCT/RU2016/000285

Box No. II Observations where certain claims were found unsearchable (Continuation of item 2 of first sheet)

This international search report has not been established in respect of certain claims under Article 17(2)(a) for the following reasons:

1. ☐ Claims Nos.:
because they relate to subject matter not required to be searched by this Authority, namely:

2. ☐ Claims Nos.:
because they relate to parts of the international application that do not comply with the prescribed requirements to such an extent that no meaningful international search can be carried out, specifically:

3. ☐ Claims Nos.:
because they are dependent claims and are not drafted in accordance with the second and third sentences of Rule 6.4(a).

Box No. III Observations where unity of invention is lacking (Continuation of item 3 of first sheet)

This International Searching Authority found multiple inventions in this international application, as follows:

see additional sheet

1. ☐ As all required additional search fees were timely paid by the applicant, this international search report covers all searchable claims.

2. ☐ As all searchable claims could be searched without effort justifying an additional fees, this Authority did not invite payment of additional fees.

3. ☐ As only some of the required additional search fees were timely paid by the applicant, this international search report covers only those claims for which fees were paid, specifically claims Nos.:

4. ☒ No required additional search fees were timely paid by the applicant. Consequently, this international search report is restricted to the invention first mentioned in the claims; it is covered by claims Nos.:

1-18(completely); 20(partially)

Remark on Protest

☐ The additional search fees were accompanied by the applicant's protest and, where applicable, the payment of a protest fee.

☐ The additional search fees were accompanied by the applicant's protest but the applicable protest fee was not paid within the time limit specified in the invitation.

☐ No protest accompanied the payment of additional search fees.

FURTHER INFORMATION CONTINUED FROM PCT/ISA/ 210

This International Searching Authority found multiple (groups of) inventions in this international application, as follows:

1. claims: 1-18(completely); 20(partially)

   Encoding and decoding of a polar subcode having a higher Hamming distance than a conventional polar code. Before performing the polarizing transformation, a set of constraints are enforced, a part of which defining frozen bits (i.e. input values being zero, not usable as data bit), another part of which defining linear constraints among the input bits, enforcing a minimum Hamming distance for the code. Further constraints are enforced by using a first and a second helper code.

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2. claims: 19(completely); 20(partially)

   Obtaining a short representation of a polar code by determining a frozen set of an approximation of the polar code using the binary erasure channel (BEC).

INTERNATIONAL SEARCH REPORT

Information on patent family members

International application No
PCT/RU2016/000285

Patent document cited in search report		Publication date	Patent family member(s)			Publication date
US 2015295593	A1	15-10-2015	RU	2014114215	A	20-10-2015
			US	2015295593	A1	15-10-2015
EP 2849377	A1	18-03-2015	CN	103516476	A	15-01-2014
			EP	2849377	A1	18-03-2015
			US	2015103947	A1	16-04-2015
			WO	2014000532	A1	03-01-2014