Interleaved Cyclic Group Decomposition Length Adaptive MET QC-LDPC Codes

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Quasi-Cyclic LDPC codes

$$H = \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \left[\begin{array}{cc|cc|cc} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ \hline 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \end{array} \begin{array}{c} \\ c_1 \\ c_2 \\ c_3 \\ c_4 \end{array}$$

$$H_{QC} = \begin{bmatrix} I^0 & I^1 & I^1 \\ I^0 & I^{-1} & I^0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

Quasi-Cyclic LDPC(QC-LDPC codes) - LDPC-codes with parity-check matrix defined by structured block submatrix – Circulant Permutation matrix.

$$Circulant\ \ Permutation\ Matrix\ (CPM)\ of\ \ size\ \ 2\times 2:\ I^0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, I^1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, I^{-1} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

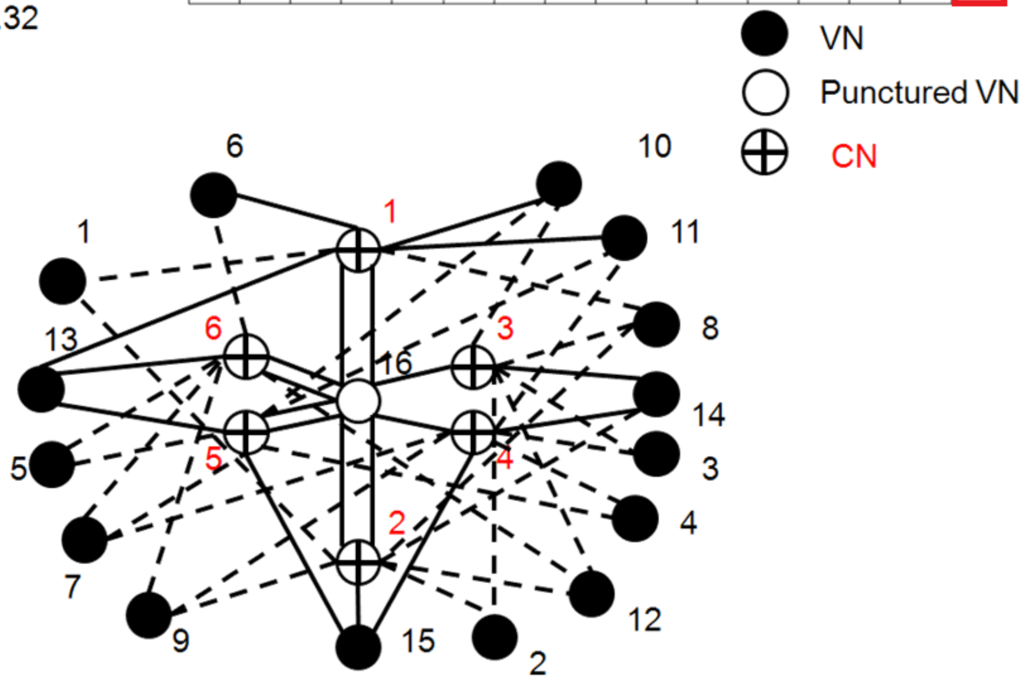
R. M. Tanner, D. Sridhara, T. Fuja, ”A class of group structured LDPC codes”, Proc. ICSTA 2001, Ambleside, England, 2001

MET LDPC

MET QC-LDPC codes in 5G eMBB were constructed to provide the best performance at BLER 10^{-2}

R=2/3
Capacity 1.059
Our Threshold 1.32
Gap 0.261
AR4JA 1.414

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	1	0	0	0	0	1	0	1	0	1	1	0	1	0	0	2
2	1	1	0	0	0	0	0	1	1	0	0	1	0	1	1	2
3	0	1	1	0	0	0	0	1	0	1	0	1	0	1	0	1
4	0	0	1	1	0	0	1	0	1	0	1	0	0	1	1	1
5	0	0	0	1	1	0	1	0	0	1	1	0	1	0	1	2
6	0	0	0	0	1	1	1	0	1	0	0	1	1	0	0	2



- J. Richardson and R. L. Urbanke, “Multi-edge type LDPC codes,” in Workshop honoring Prof. Bob McEliece on his 60th birthday, California Institute of Technology, Pasadena, California, 2002.
- Mattoussi F., Roca V., Sayadi B. GLDPC-Staircase AL-FEC codes: A Fundamental study and New results. EURASIP Journal on Wireless Communications and Networking, SpringerOpen, 2016
- Sarah J. Johnson Reported Thresholds and BER Performance for LDPC and LDPC-Like Codes, 2011

Length adaptation of QC-LDPC Codes

8x16 protograph base matrix

$$H_{proto} = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$$

Circulant size 42

Circulant size 9

$$H = \begin{pmatrix} 33 & 0 & 15 & 0 & 8 & 0 & 28 & 0 & 26 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 14 & 0 & 25 & 0 & 11 & 0 & 0 & 9 & 18 & 33 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 8 & 0 & 11 & 0 & 29 & 0 & 30 & 0 & 20 & 18 & 0 & 0 & 0 & 0 & 0 \\ 0 & 31 & 0 & 39 & 0 & 22 & 31 & 0 & 0 & 0 & 37 & 11 & 0 & 0 & 0 & 0 \\ 33 & 0 & 9 & 0 & 5 & 0 & 24 & 0 & 25 & 0 & 0 & 28 & 23 & 0 & 0 & 0 \\ 20 & 0 & 30 & 0 & 0 & 20 & 0 & 12 & 0 & 30 & 0 & 22 & 0 & 12 & 0 & 0 \\ 2 & 20 & 0 & 11 & 0 & 31 & 0 & 7 & 0 & 0 & 36 & 0 & 0 & 17 & 6 & 0 \\ 0 & 7 & 0 & 32 & 24 & 0 & 39 & 0 & 30 & 0 & 0 & 0 & 26 & 0 & 38 & 28 \end{pmatrix}$$

Code length N=16*42=672

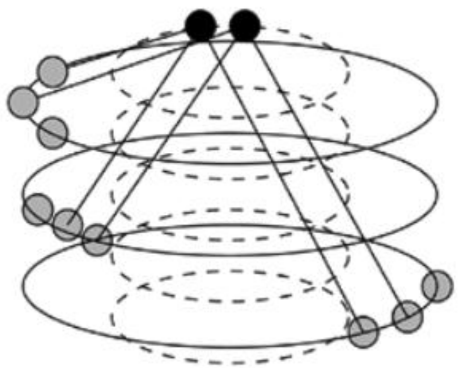
$$H = \begin{matrix} 2 & -1 & 6 & -1 & 7 & -1 & 7 & -1 & 8 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 7 & -1 & 6 & -1 & 1 & -1 & -1 & 3 & 6 & 3 & -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & 2 & -1 & 8 & -1 & 2 & -1 & 8 & -1 & 1 & 8 & -1 & -1 & -1 & -1 & -1 \\ -1 & 5 & -1 & 6 & -1 & 2 & 4 & -1 & -1 & -1 & 0 & 1 & -1 & -1 & -1 & -1 \\ 4 & -1 & 6 & -1 & 3 & -1 & 1 & -1 & 5 & -1 & -1 & 5 & 4 & -1 & -1 & -1 \\ 7 & -1 & 4 & -1 & -1 & 2 & -1 & 0 & -1 & 6 & -1 & 5 & -1 & 4 & -1 & -1 \\ 8 & 1 & -1 & 4 & -1 & 4 & -1 & 5 & -1 & -1 & 8 & -1 & -1 & 2 & 7 & -1 \\ -1 & 1 & -1 & 3 & 4 & -1 & 5 & -1 & 3 & -1 & -1 & -1 & 4 & -1 & 4 & 3 \end{matrix}$$

Code length N=16*9=144

Modular lifting

$$E\left(H_{current}\right) = E\left(H_{upper}\right) \bmod z_{current}$$

$z_{current}$ – *circulant size which you want to get*



For m to n protograph

● code bit nodes (n rings of L_K each)

● constraint nodes (m rings of L_K each)

We can represent 8 cyclic groups of 5G as:

$$L_k = a \times 2^i, \quad i = \{0,1,\dots,7\}$$

$$a = \{2,3,5,7,9,11,13,15\}$$

$$L_{256} = L_2 \times L_2 \times \cdots \times L_2$$

$$L_{384} = L_3 \times L_2 \times \cdots \times L_2$$

$$L_{320} = L_5 \times L_2 \times \cdots \times L_2$$

$$\dots$$

$$L_{240} = L_{15} \times L_2 \times \cdots \times L_2$$

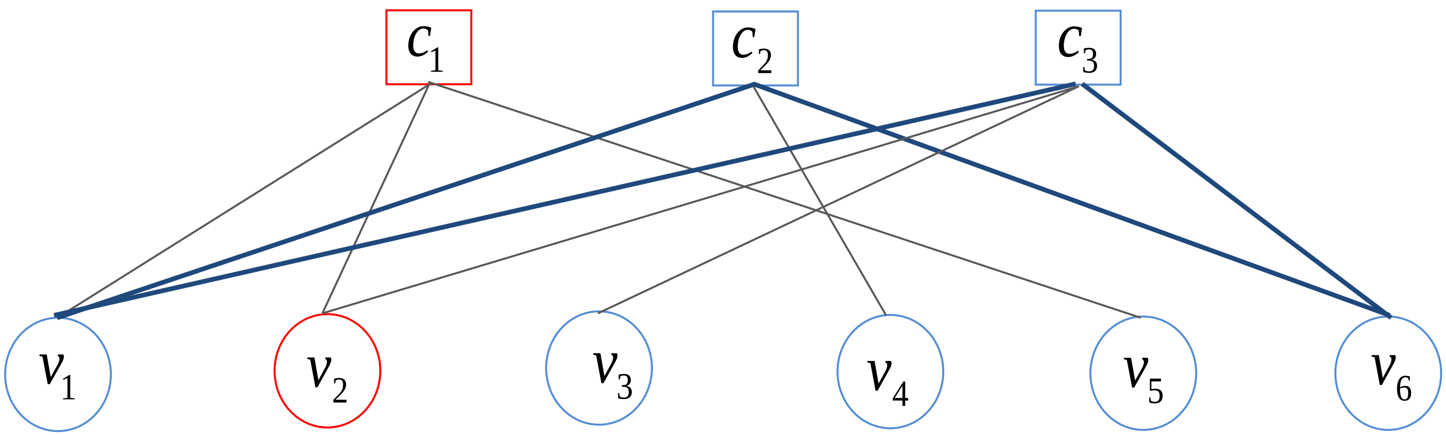
- 3GPP TS38.212V15.4.0:NR;Multiplexing and channel coding (Rel. 15)
- Tanner R.M., Sridhara D., Sridharan A., Fuja T.E., and Costello D.J., “LDPC block and convolutional codes based on circulant matrices”, IEEE Trans. Inf. Theory, vol. 50, no. 12, pp. 2966-2984, Dec. 2004.
- Seho Myung; Kyeongcheol Yang, "Extension of quasi-cyclic LDPC codes by lifting," in Information Theory, 2005. ISIT 2005. Proceedings. International Symposium on, vol., no., pp.2305-2309, 4-9 Sept. 2005
- Seho Myung; Kyeongcheol Yang; Youngkyun Kim, "Lifting methods for quasi-cyclic LDPC codes," in Communications Letters, IEEE , vol.10, no.6, pp.489-491

How to build good MET QC LDPC?

Cycle is a closed simple way in Tanner-graph.

Example of cycle 4: $c_2 \rightarrow v_1 \rightarrow c_3 \rightarrow v_6 \rightarrow c_2$

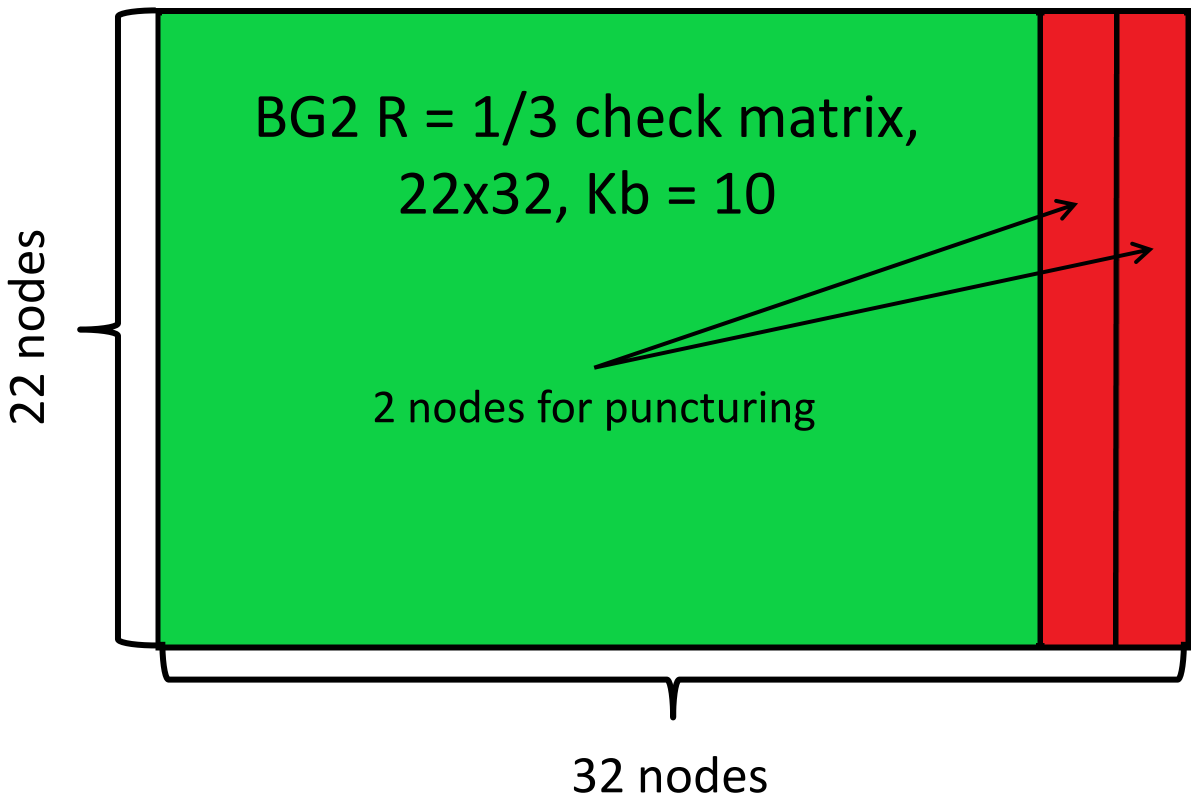
Girth is the length of the shortest cycle in Tanner-graph.



How to build good MET QC LDPC?

The ***Extrinsic Message Degree (EMD)*** metric of the cycle in the Tanner graph is defined as a number of check nodes singly connected to the variable nodes involved in the cycle. The EMD metric estimates how strong the subgraph of the cycle is connected with the rest of the Tanner graph.

How to build good MET QC LDPC?



$\mathit{if}\ \ TBS > 640$

$\quad K_b = 10$

$\mathit{elseif}\ \ TBS > 560$

$\quad K_b = 9$

$\mathit{elseif}\ \ TBS > 192$

$\quad K_b = 8$

else

$\quad K_b = 6$

Result: catastrophically performance behavior for short TBS!
Can we find better way to cut base graph from 10 info symbols to 9, 8 or 6?

3GPP TS38.212V15.4.0:NR;Multiplexing and channel coding (Rel. 15)

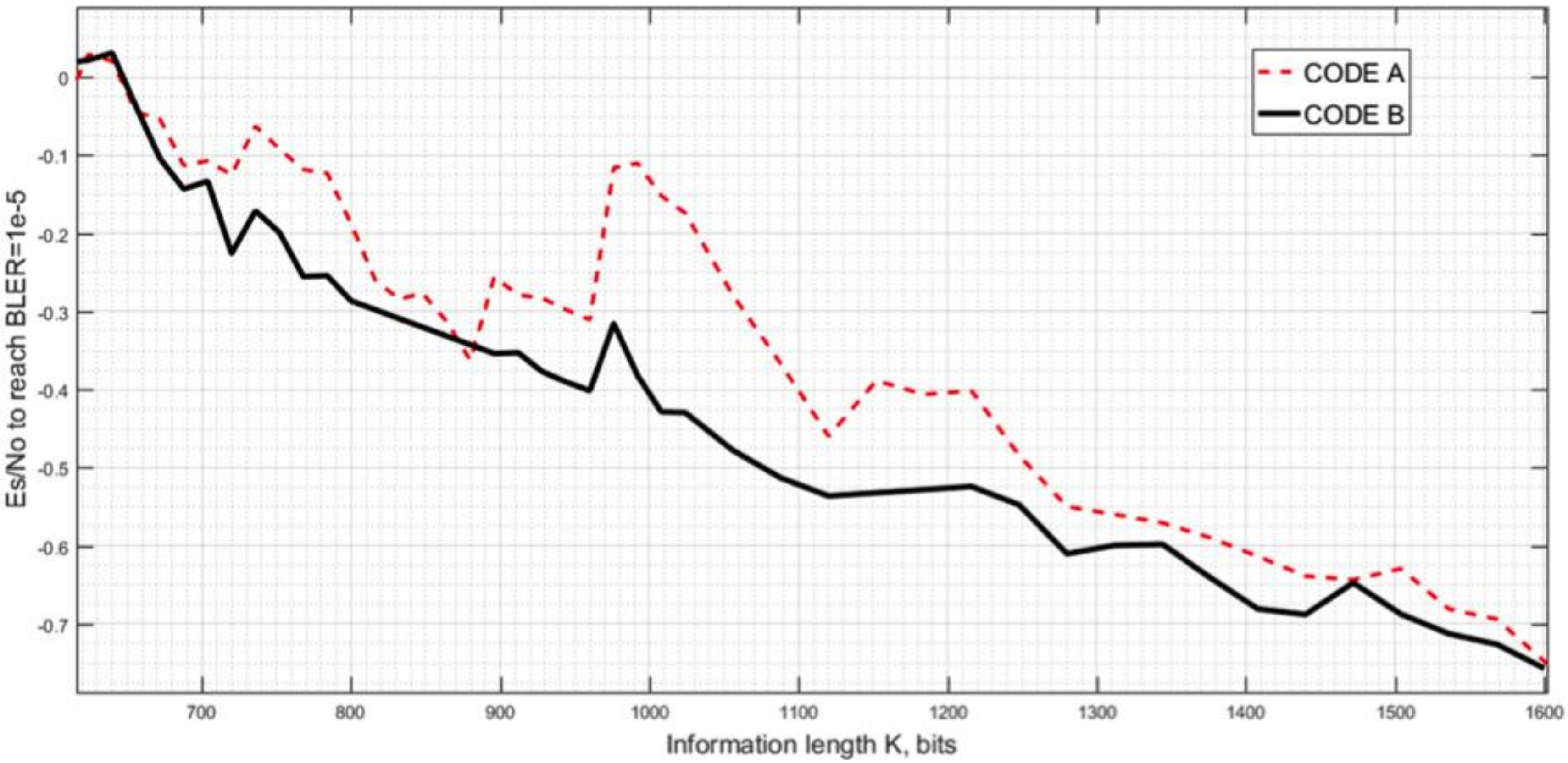
Code construction method

1. Generate 8 exponent matrix for each cyclic group L_k using simulated annealing method with maximization of EMD metric and Tanner graph girth;
2. Use permutation of variable nodes to improve code convergence and prevent catastrophically performance degradation on troublesome range of lengths by optimizing the info symbols choosing and ME puncturing pattern via nodes recoverability classification.

   Recoverability of an arbitrary variable node is a number of message passing steps required for it's recovering. It is assumed that nodes which requires less steps for the recovering (nodes with better *recoverability*) should be punctured first.

• Usatyuk V., Vorobyev I., "Simulated Annealing Method for Construction of High-Girth QC-LDPC Codes," 2018 41st International Confer. on Telecom. and Signal Processing (TSP), Athens, 2018, pp. 1-5.
• Svistunov G., Usatyuk V. et al., "Generalization of Floor Lifting for QC-LDPC Codes: Theoretical Properties and Applications," 2018 IEEE East-West Design & Test Symposium (EWDTS), Kazan, 2018, pp. 1-6.

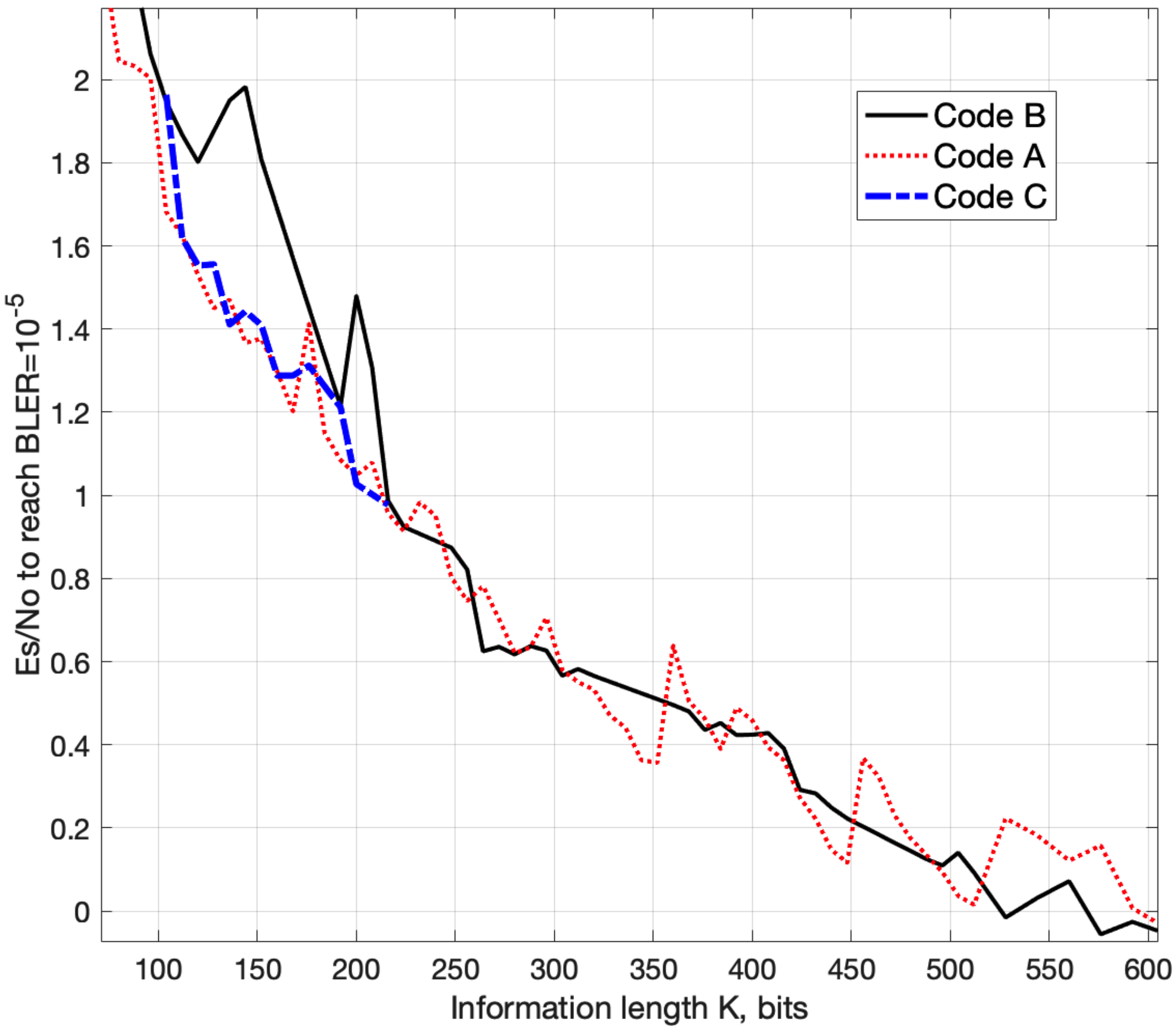
Results



Code A is a simulation result for 5G standard BG2 codes.

Code B is a simulation result of code constructed from BG2 protograph using simulated annealing method with maximization of girth and EMD.

Results



Code A is a simulation result for 5G standard BG2 codes.

Code B is a simulation result of code constructed from BG2 protograph using simulated annealing method with maximization of girth and EMD.

Code C is the Code B with interleaved by proposed method for information length K in range [104, 232]

Thank You!