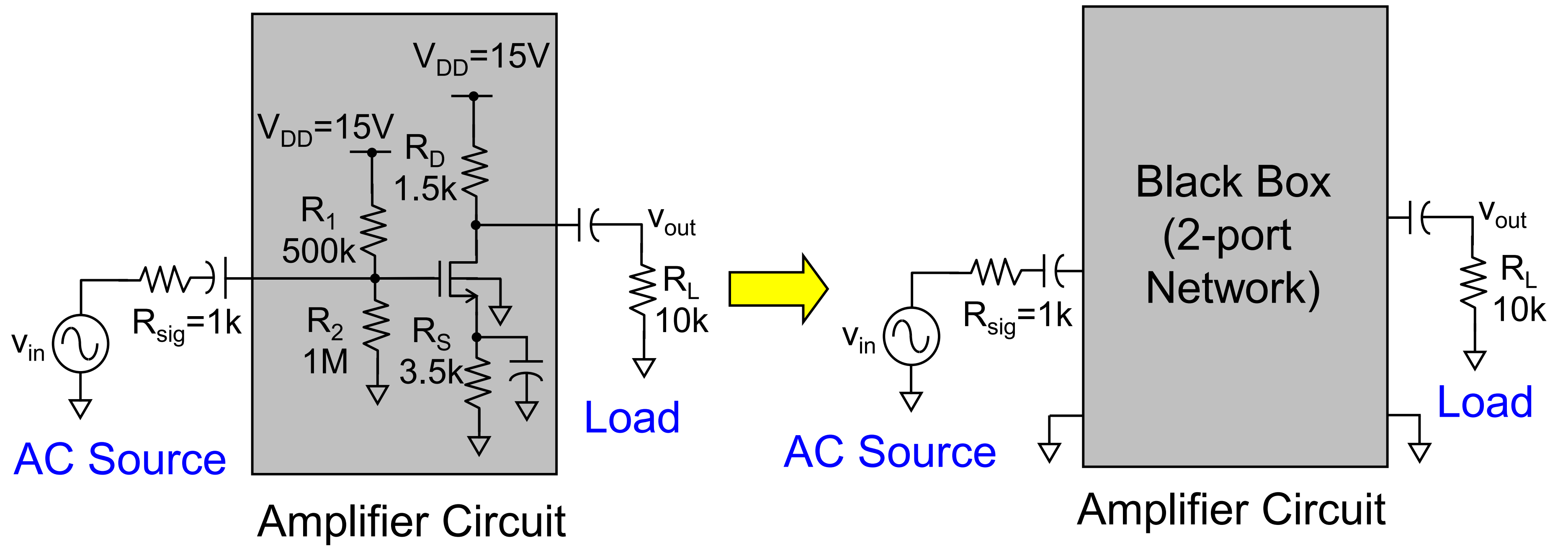
EE2027 Electronic Circuits

Single-Stage Amplifier

Lecture Outline

- **2-Port Network (Voltage Amplifier and Transconductance Amplifier)**
  2-port network parameters: $R_{in,}$ $R_{out,}$ voltage gain (A_V) or transconductance (G_m)
- **Single-Stage Amplifiers**
  CE/CS, CB/CG.

Modeling Amplifier Circuit using 2-Port Network (in small-signal ac operation)



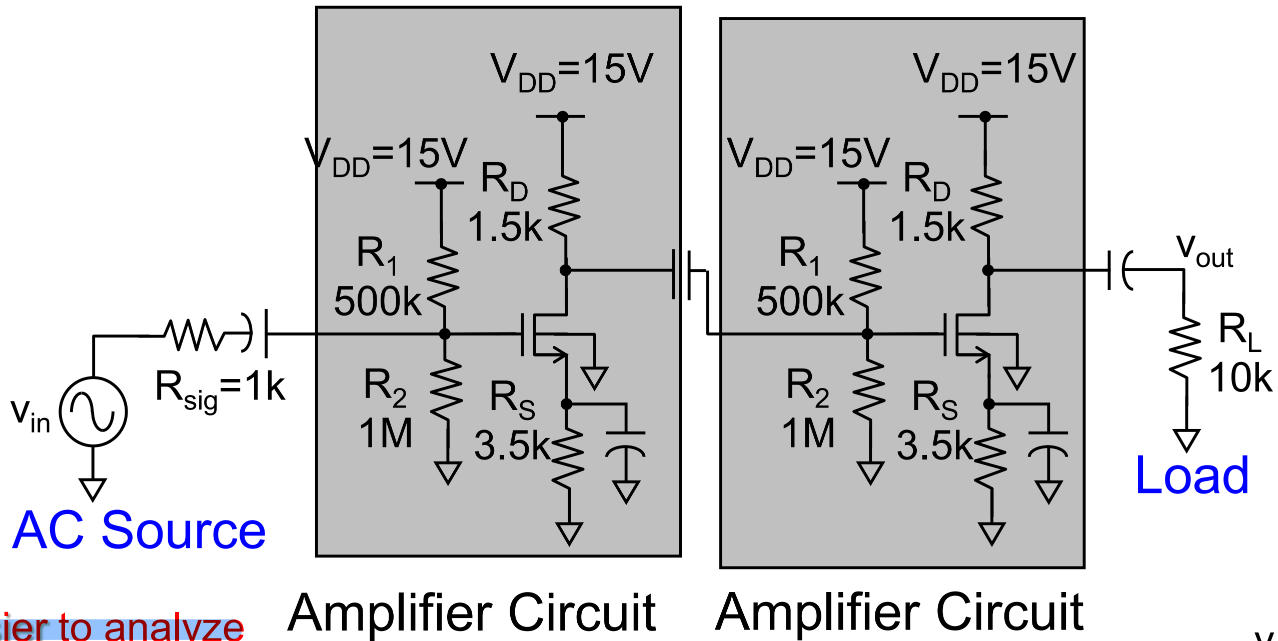
Can we fully characterize the complicated amplifier circuit using a **simple** black box (**2-port network**) with limited set of parameters?

Concept is similar to replacing a single-port complex linear network by Thevenin Equivalent with only one Thevenin voltage source (V_{THV}) in series with one Thevenin equivalent resistor (R_{THV}), i.e., only 2 parameters.

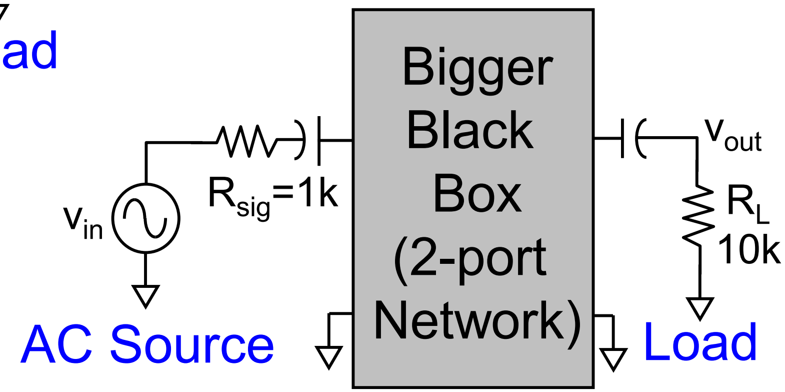
Analogy of 2-Port Network to Investment Scheme

- We are only interested in the relationship between output and input
- We don’t care about what happens in between

Why Use 2-port Network?

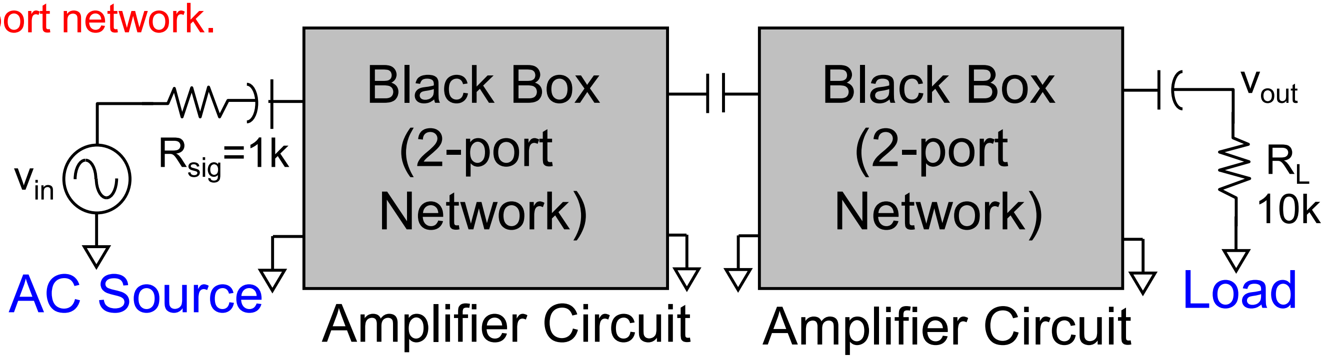


Cascaded 2-port networks can be combined into a 'bigger' 2-port network.

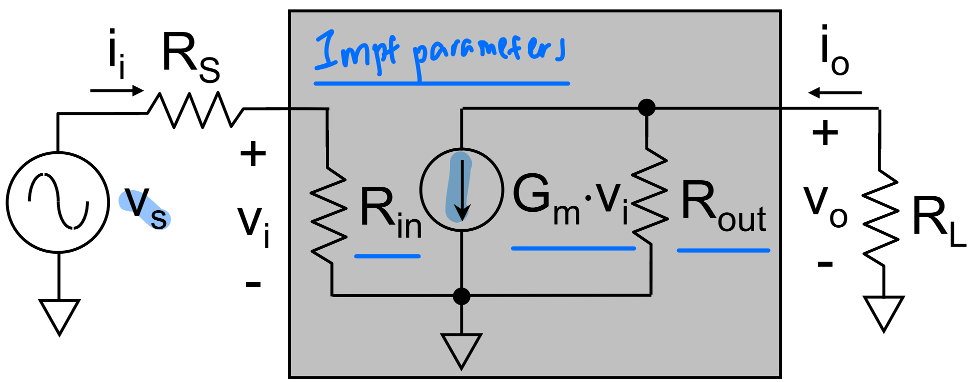


Easier to analyze complicated circuit (e.g., multi-stage amplifier) as each amplifier stage is simplified to its equivalent 2-port network.

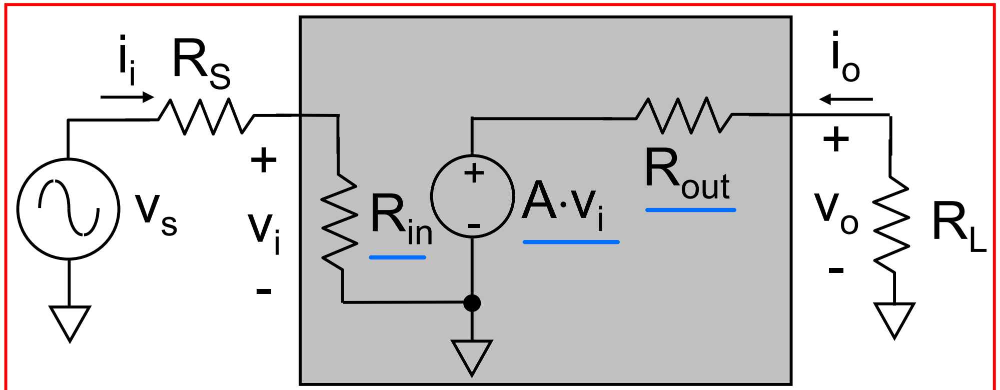
Similar concept as Thevenin equivalent or Norton equivalent.



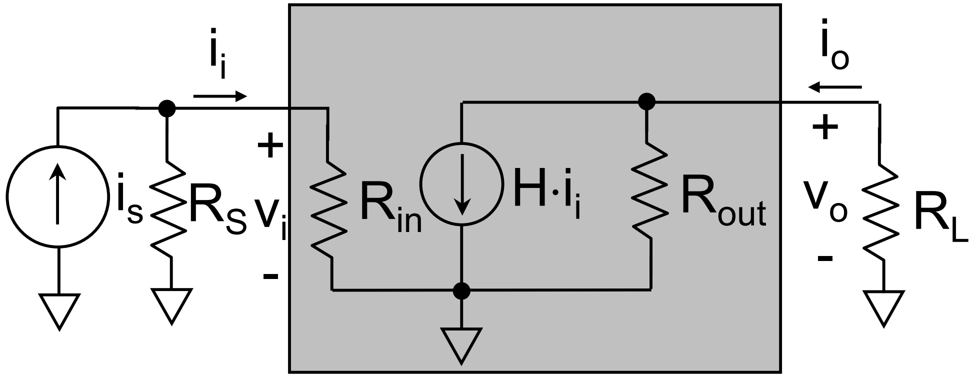
Different 2-Port Amplifier Models



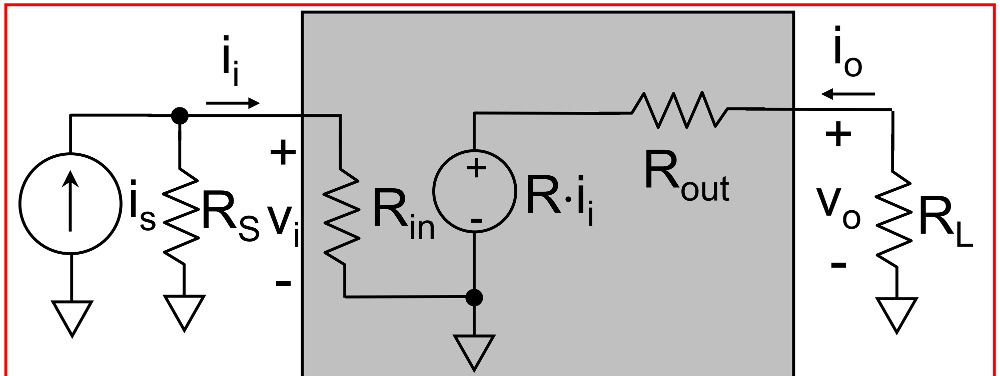
2-Port Transconductance Amplifier



2-Port Voltage Amplifier



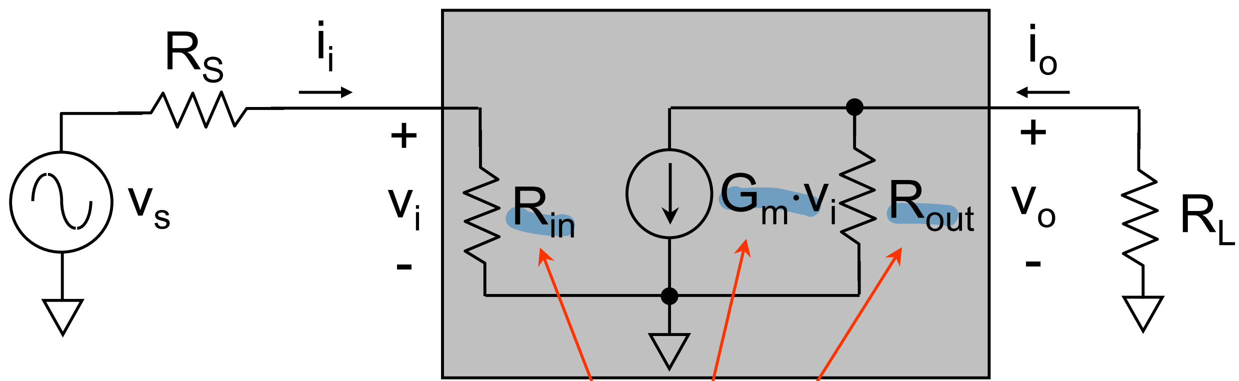
2-Port Current Amplifier (Not tested)



(Not tested) 2-Port Trans-resistance Amplifier

- All four can be used to represent the same amplifier circuit.
  - Same concept as Thevenin equivalent versus Norton equivalent (for a single-port linear network).
- In some scenarios, it is easier to represent the amplifier circuit as 2-Port Transconductance Amplifier to simplify the analysis. In others, it is easier to represent the amplifier circuit as 2-Port Voltage Amplifier to simplify the analysis. Same reasoning applies to 2-Port Current and Trans-resistance amplifier.
- In this module, only 2-Port Transconductance Amplifier and 2-Port Voltage Amplifier are considered as they simplify the analysis (based on experience of circuit designers).

Two-Port Network – Transconductance Amplifier

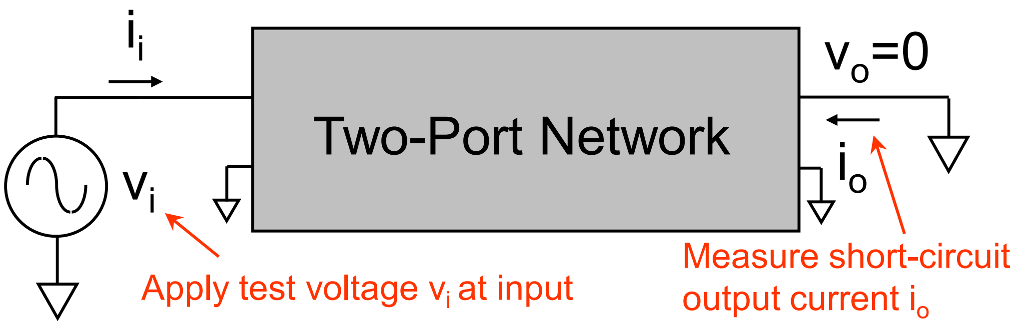


- After converting amplifier circuit to 2-port network, it is easier to cascade multiple of them together and analyze (Refer to slide SSA1-4)

Characteristics:

- High input resistance (R_{in})
- High output resistance (R_{out})
- Transconductance amplifier (Voltage-to-current conversion: $G_m v_i$)

Transconductance Amplifier – Parameter Characterizations



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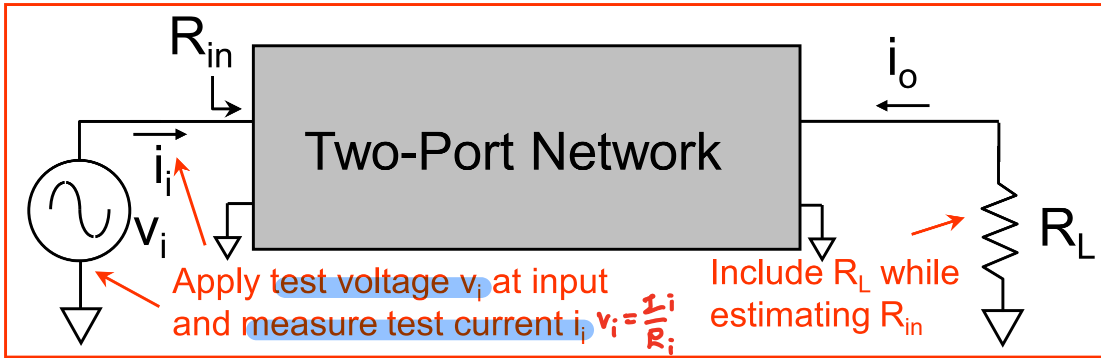
$$\boxed{G_m = \left.\frac{i_o}{v_i}\right|_{v_o=0}}$$

By **short-circuit the output**, current through R_{out} would be zero, and the measured current i_o would be $G_m v_i$.



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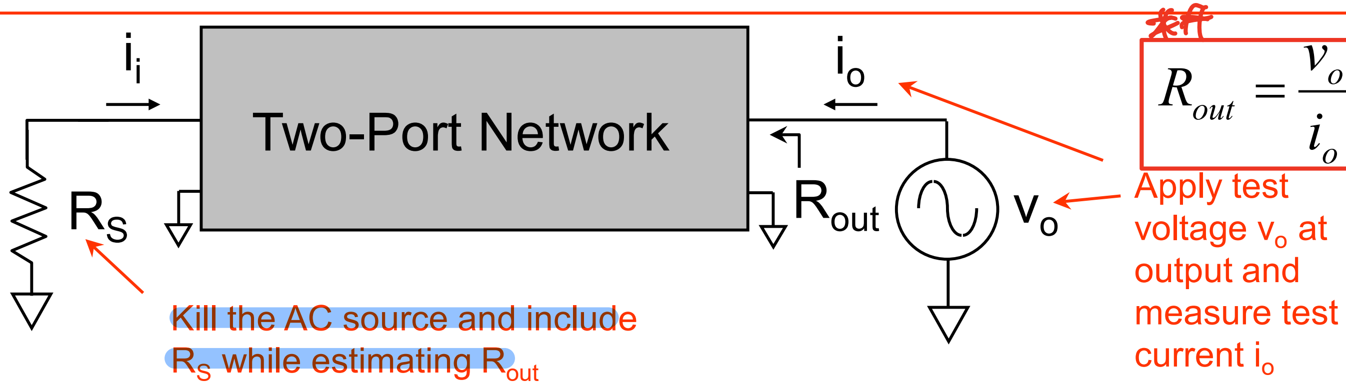
$$\boxed{R_{in} = \frac{v_i}{i_i}}$$

R_{in} is the equivalent resistance **looking into the input port**, R_L needs to be included for the estimation of R_{in}.



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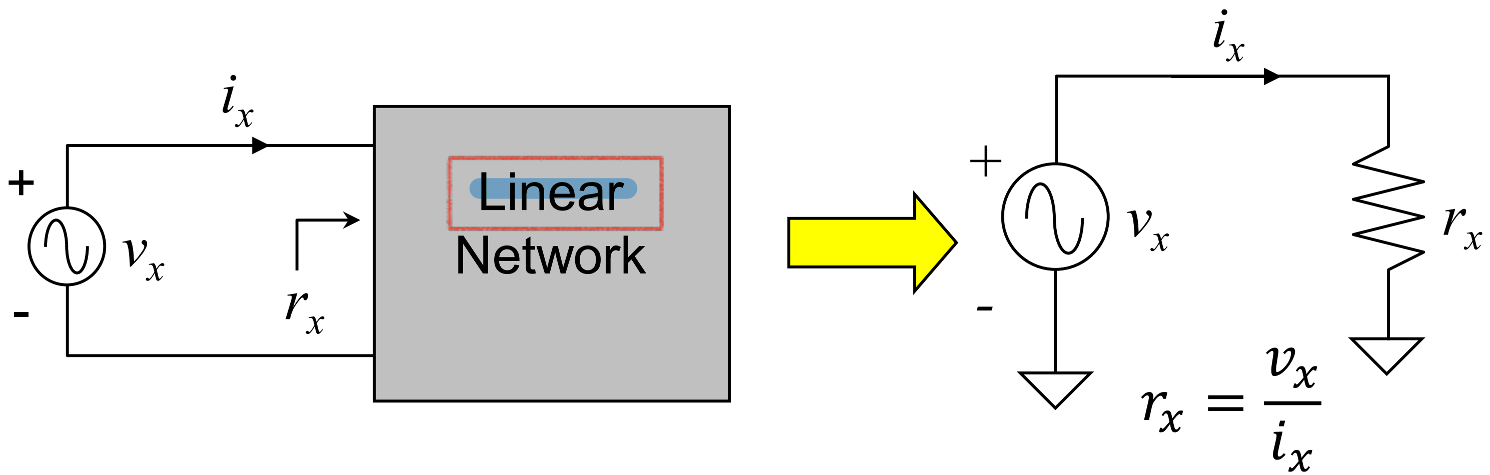
$$\boxed{R_{out} = \frac{v_o}{i_o}}$$

R_{out} is the equivalent resistance **looking into the output port**, R_S (source resistance) needs to be included for the estimation of R_{out}.

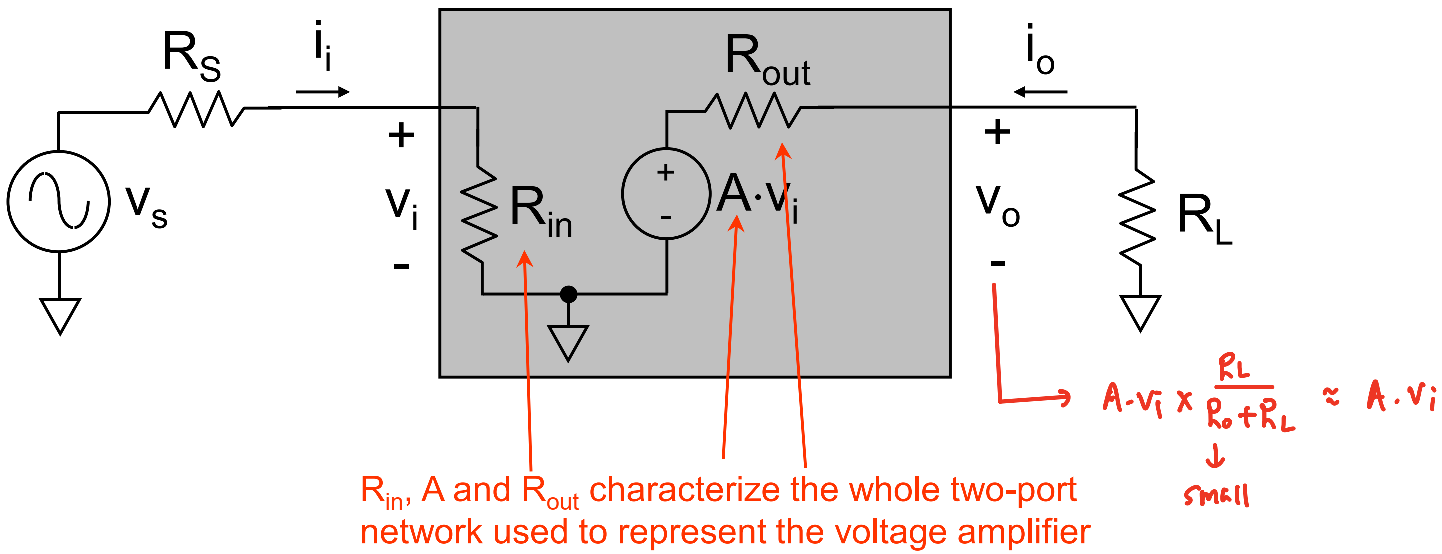
Equivalent Resistance



If the single-port linear network draws a current of i_x under an applied test voltage of v_x, the relation between i_x and v_x is linear. The linear network therefore behaves similarly to a simple resistor r_x, obeying Ohm's law

- Similar concept as Thevenin equivalent
- For a complicated single-port LINEAR network without any dependent voltage/current source, the current-voltage relationship can be modeled as a simple equivalent resistor

Two-Port Network – Voltage Amplifier

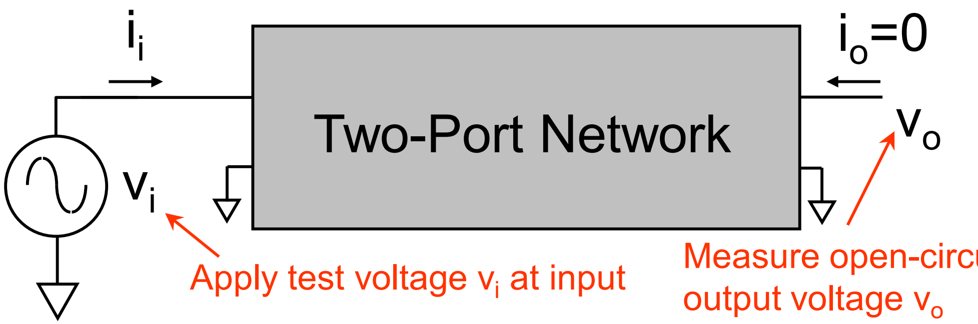


- After converting amplifier circuit to 2-port network, it is easier to cascade multiple of them together and analyze (Refer to slide SSA1-4)

Characteristics:

- High input resistance (R_{in})
- Low output resistance (R_{out})
- Voltage amplifier (Voltage gain: A)

Two-Port Network – Voltage Amplifier



Apply test voltage v_i at input

Measure open-circuit output voltage v_o

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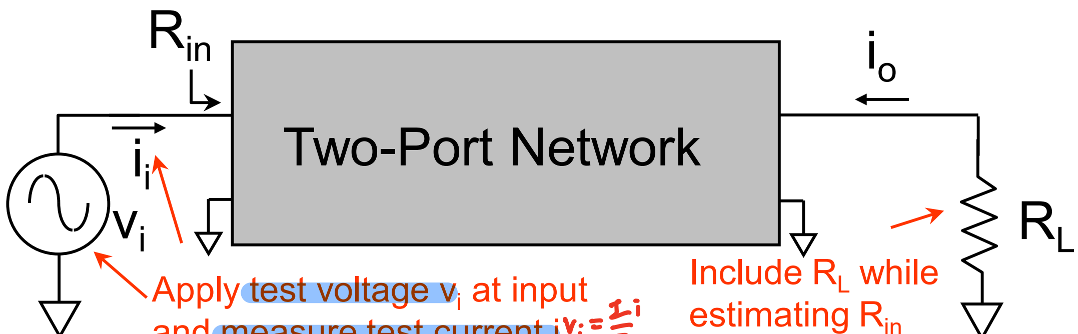
$$\boxed{A = \left.\frac{v_o}{v_i}\right|_{i_o=0}}$$

By **open-circuit the output**, current i_o would be zero. Thus, the voltage drop across R_{out} would be zero, and v_o would be same as Av_i.



Apply test voltage v_i at input and measure test current i_i $v_i = \frac{I_i}{R_i}$

Include R_L while estimating R_{in}

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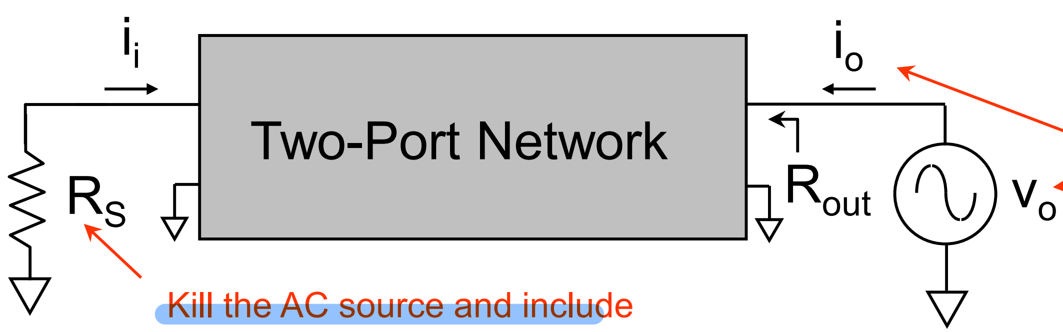
$$\boxed{R_{in} = \frac{v_i}{i_i}}$$

R_{in} is the equivalent resistance **looking into the input port**, R_L needs to be included for the estimation of R_{in}.



Kill the AC source and include R_S while estimating R_{out}

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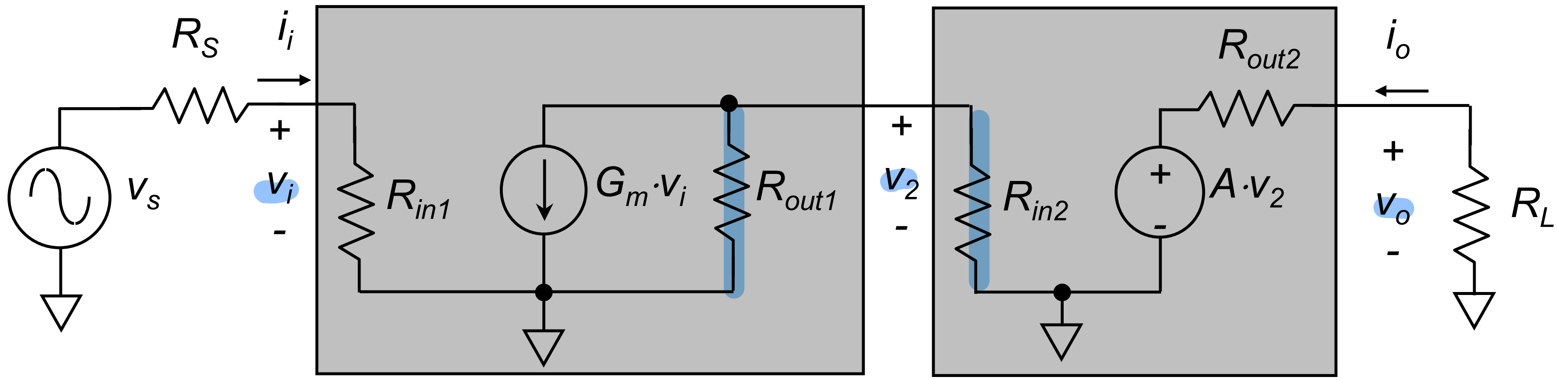
$$\boxed{R_{out} = \frac{v_o}{i_o}}$$

Apply test voltage v_o at output and measure test current i_o

R_{out} is the equivalent resistance **looking into the output port**, R_S (source resistance) needs to be included for the estimation of R_{out}.

Example on Cascade Two-Port



Amplifier Circuit 1

Amplifier Circuit 2

$$v_i = \frac{R_{in1}}{R_{in1}+R_S} \times v_s$$

$$v_2 = -G_m v_i \times (R_{out1} // R_{in2})$$

$$= -G_m \times \frac{R_{in1}}{R_{in1}+R_S} \times v_s \times (R_{out1} // R_{in2})$$

$$v_o = A v_2 \times \frac{R_L}{R_{out2}+R_L}$$

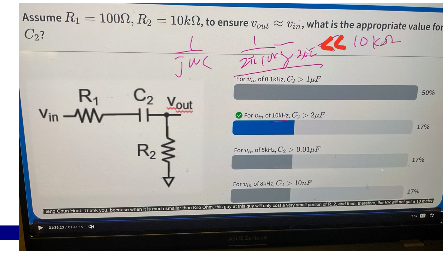
$$= \frac{AR_L}{R_{out2}+R_L}\left[-G_m \times \frac{R_{in1}}{R_{in1}+R_S} \times v_s \times (R_{out1} // R_{in2})\right]$$

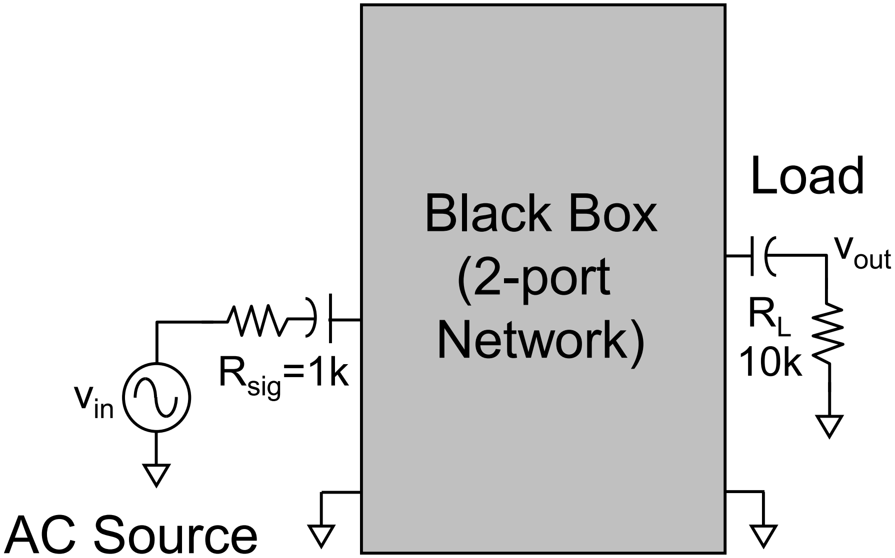
$$Gain = \frac{v_o}{v_s} = \underbrace{\frac{R_{in1}}{R_{in1}+R_S}}_{1^{st}} \times \underbrace{[-G_m \times (R_{out1} // R_{in2})]}_{2^{nd}} \times A \times \underbrace{\frac{R_L}{R_{out2}+R_L}}_{3^{rd}} = \frac{v_i}{v_s} \times \frac{v_2}{v_i} \times \frac{v_o}{v_2}$$

In amplifier design,

- How should you choose R_{in1} with respect to R_S?
- How should you choose R_{out2} with respect to R_L?
- Why?
- What if the source and load are antenna or equipment with long coaxial cable?

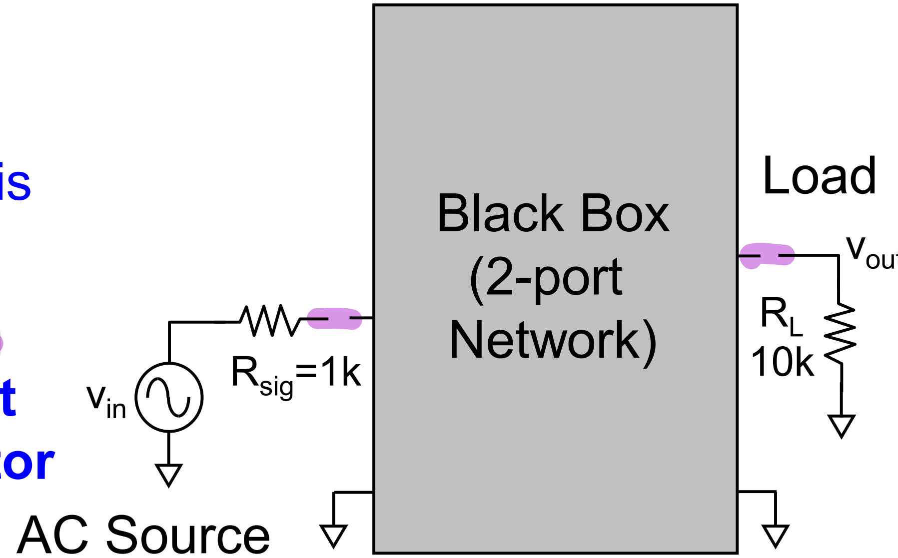
How to Treat Capacitors?





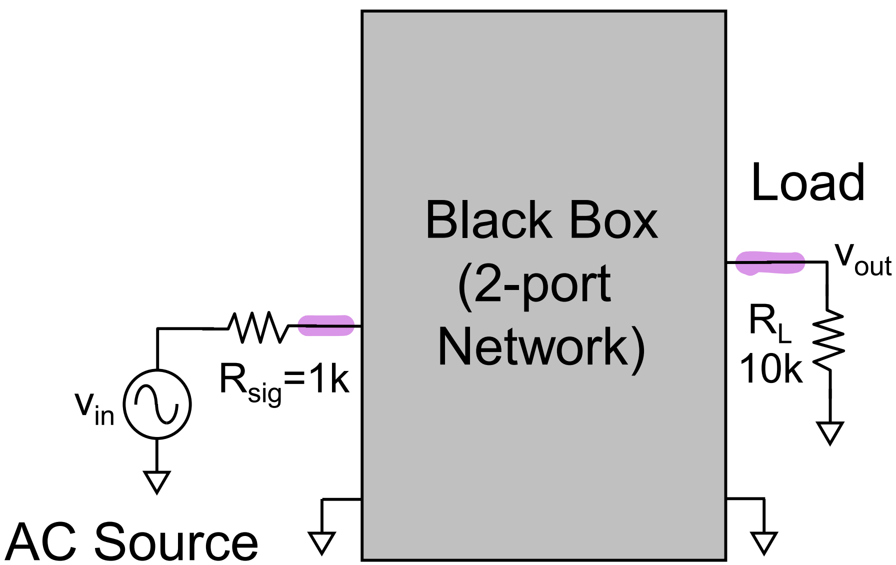
DC Analysis

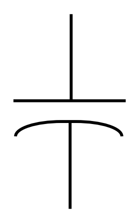
Open Circuit Capacitor



AC Analysis

Short Circuit Capacitor





How to treat capacitors?

$Z_C = 1/j\omega C$

At DC (DC analysis), i.e., $\omega = 0$, $Z_C \rightarrow \infty$ (open circuit)

At frequency of interest (AC analysis), C can always be chosen large enough such that $Z_C \rightarrow 0$ (short circuit)

Steps for Circuit Analysis

DC Analysis

⇩

Remove AC source

⇩

Replace capacitor by open circuit

⇩

Obtain DC biasing of transistor

⇩

Obtain small-signal model parameters of transistor (e.g., g_m, r_π, r_o)

I_C , I_D

BJT: $g_m = \frac{I_C}{V_T}$, $r_o = \frac{V_A}{I_C}$, $r_\pi = \frac{\beta}{g_m}$

MOSFET: $g_m = 2\sqrt{k_n I_D}$, $r_o = \frac{1}{\lambda I_D}$

AC Analysis

⇩

DC voltage source ⇒ short circuit
DC current source ⇒ open circuit

⇩

Replace capacitor by short circuit

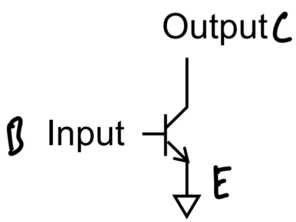
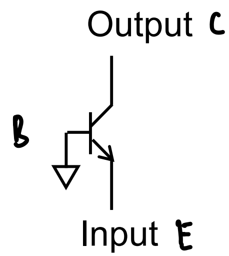
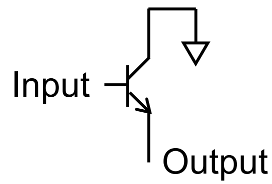
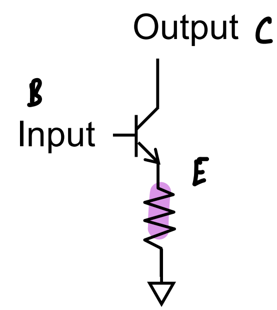
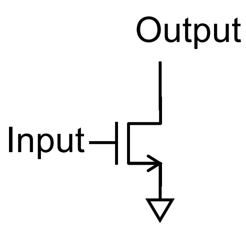
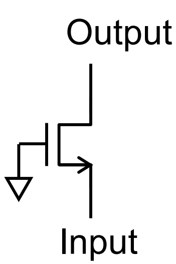
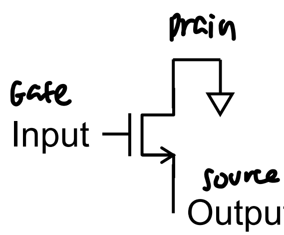
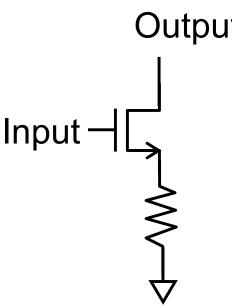
⇩

Replace transistor by small-signal model

⇩

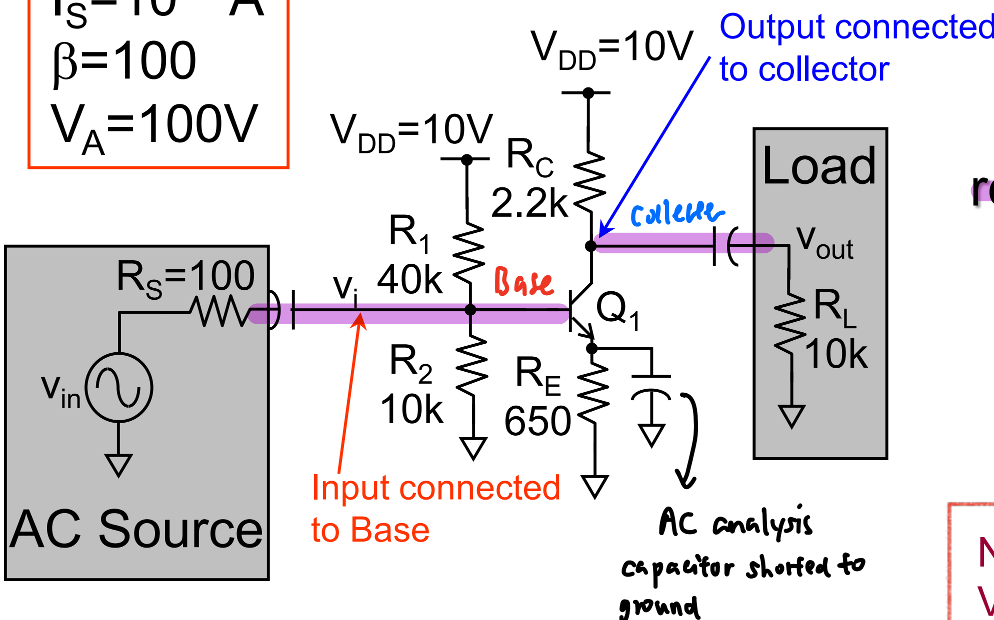
Find the two-port equivalent parameters: R_{in}, R_{out}, and G_m or A

8 Amplifier Configurations

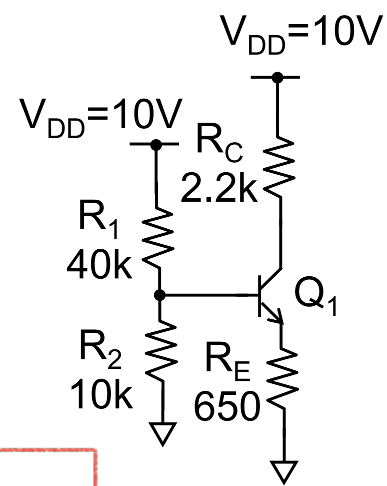
 Input – Base Output – Collector Neither – Emitter ⇒ CE	 Input – Emitter Output – Collector Neither – Base ⇒ CB	 Input – Base Output – Emitter Neither – Collector ⇒ CC	 Input – Base Output – Collector Neither – Emitter (with resistor) ⇒ CE with degeneration
 Input – Gate Output – Drain Neither – Source ⇒ CS	 Input – Source Output – Drain Neither – Gate ⇒ CG	 Input – Gate Output – Source Neither – Drain ⇒ CD	 Input – Gate Output – Drain Neither – Source (with resistor) ⇒ CS with degeneration

Common Emitter (CE) Amplifier

$I_S=10^{-15}A$
$\beta=100$
$V_A=100V$



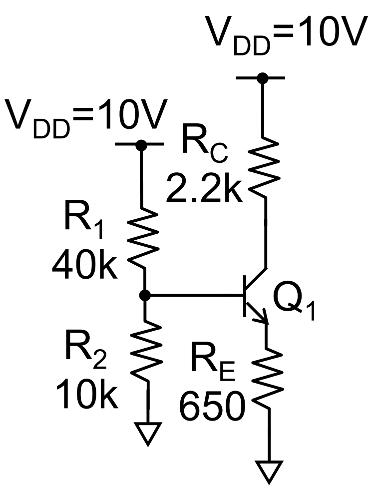
For DC analysis, remove AC source and open circuit capacitors



Notations:
$V_{B,Q1}$: Base voltage of Q_1
$V_{E,Q1}$: Emitter voltage of Q_1
$V_{C,Q1}$: Collector voltage of Q_1

- Identify AC Source and Load
- To identify amplifier configuration, we need to consider AC equivalent circuit, i.e., short circuit capacitors
  - Input connected to Base, output connected to Collector, Emitter connected to neither input nor output ⇒ Common Emitter (CE)
- DC Analysis – refer to "BJT Lecture Notes" slides ***BJT-21 to BJT-28***

DC Analysis for CE Amplifier



$I_S=10^{-15}$A
β=100
V_A=100V

Determine DC biasing

Assume $V_{BE,Q1} = 0.7\,\text{V}$ and

$I_{B,Q1} << I_{R1}, I_{R2}$

Assume forward active operation. Need to verify.

$\Rightarrow$ voltage divider method

$$V_{B,Q1} = \frac{10\text{k}}{40\text{k}+10\text{k}} \times 10 = 2\,\text{V}$$

$$V_{E,Q1} = 2 - 0.7 = 1.3\,\text{V}$$

$$I_{C,Q1} \approx I_{E,Q1} = \frac{1.3}{650} = 2\,\text{mA}$$

$$I_{B,Q1} = \frac{I_{C,Q1}}{100} \approx 20\,\mu A$$

$I_C = \beta I_B$

$$I_{R1} \approx I_{R2} \approx 200\,\mu\text{A} >> I_{B,Q1}$$

$\Rightarrow$ voltage divider valid

$$V_{BC,Q1} = V_{B,Q1} - V_{C,Q1}$$

$$= 2 - (V_{DD} - I_{C,Q1} \times R_C) = -3.6\,\text{V}$$

BJT operates in forward active region.

$V_{BE} > 0$

$V_{BC} < 0$

Determine AC small-signal parameters

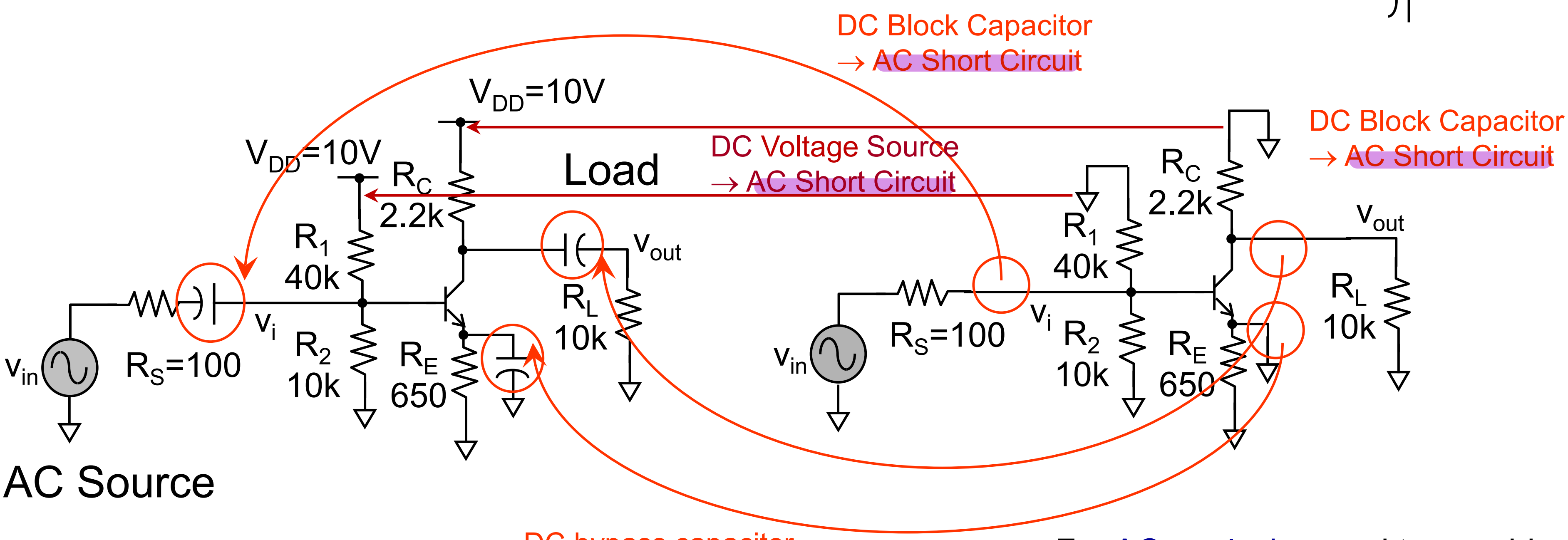
$$g_m = \frac{I_C}{V_T} = 80\,\text{mA/V}$$

$V_T \approx 25\text{m}$

$$r_\pi = \frac{\beta}{g_m} = 1.25\,\text{k}\Omega$$

$$r_o = \frac{V_A}{I_C} = 50\,\text{k}\Omega$$

AC Analysis for CE



For AC analysis, need to consider the AC equivalent circuit -

Step 1 :
Replace capacitor with AC short circuit

Step 2 :
Replace DC voltage source with AC short circuit

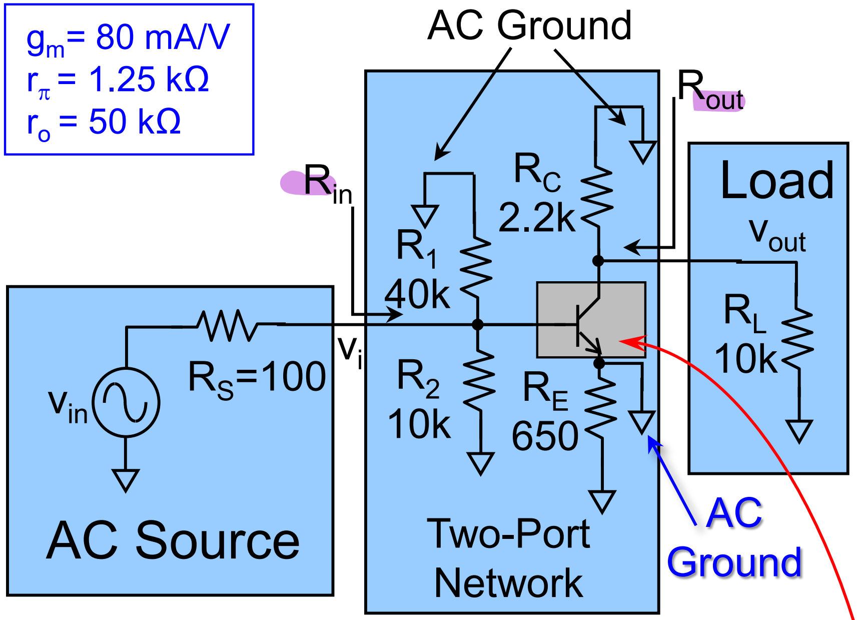
Step 3 :
Replace DC current source with AC open circuit

Continue the analysis with AC equivalent circuit on the right.

g_m=80 mA/V
r_π=1.25 kΩ
r_o = 50 kΩ

AC Analysis for CE

g_m = 80 mA/V
r_π = 1.25 kΩ
r_o = 50 kΩ



Step 1 :

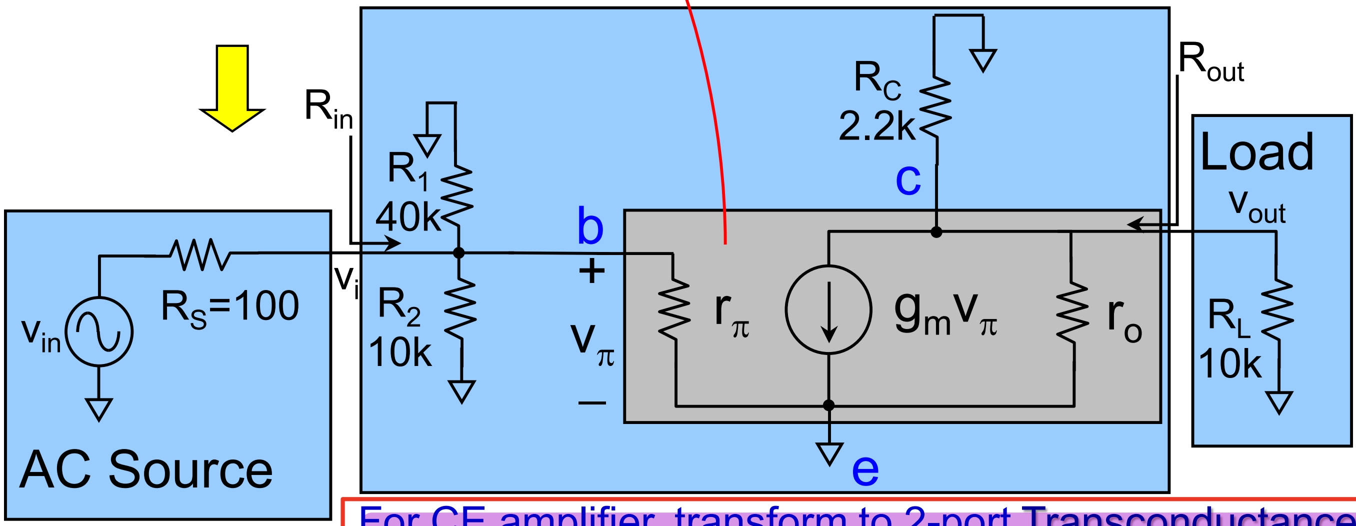
Identify AC source and load

Step 2 :

Group the remaining components (the amplifier circuit) into two-port network

Step 3 :

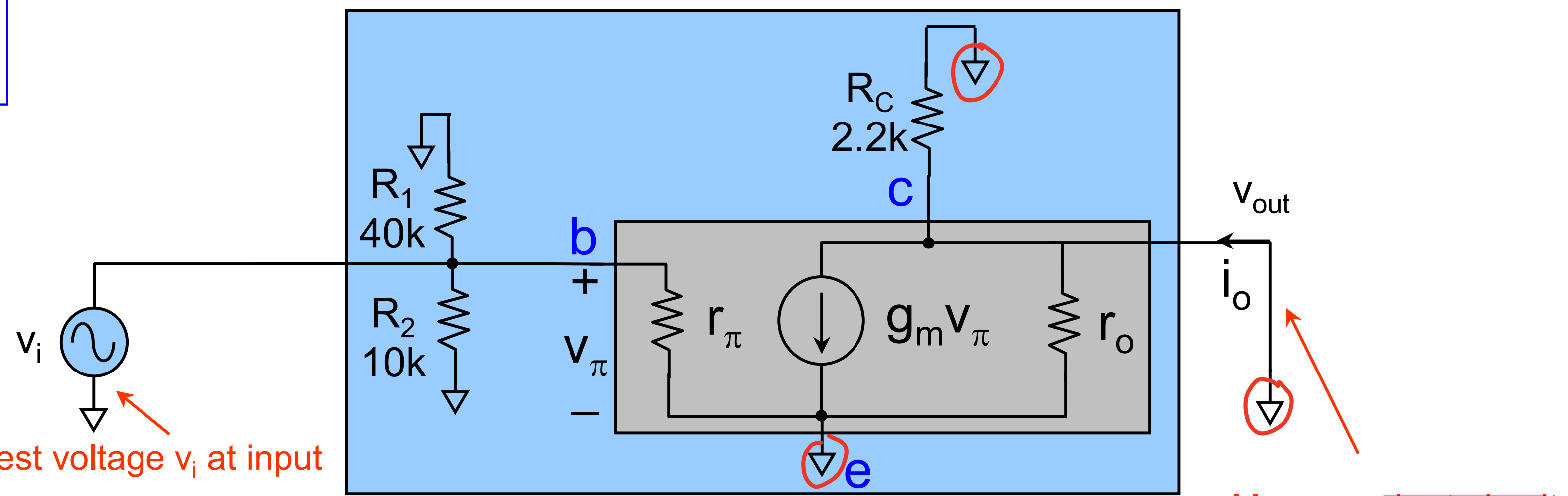
Replace transistor with small-signal AC model



For CE amplifier, transform to 2-port Transconductance Amplifier (based on experience of circuit designers). Need to evaluate G_m, R_{in} and R_{out}.

Two-Port Network (G_m) CE

g_m = 80 mA/V
r_π = 1.25 kΩ
r_o = 50 kΩ



Refer to slide SSA1-7 for the evaluation of G_m.

There is no current through r_o and R_c.

$$\therefore i_o = g_m v_\pi = g_m v_i$$ → $v_\pi = v_i$ (parallel)

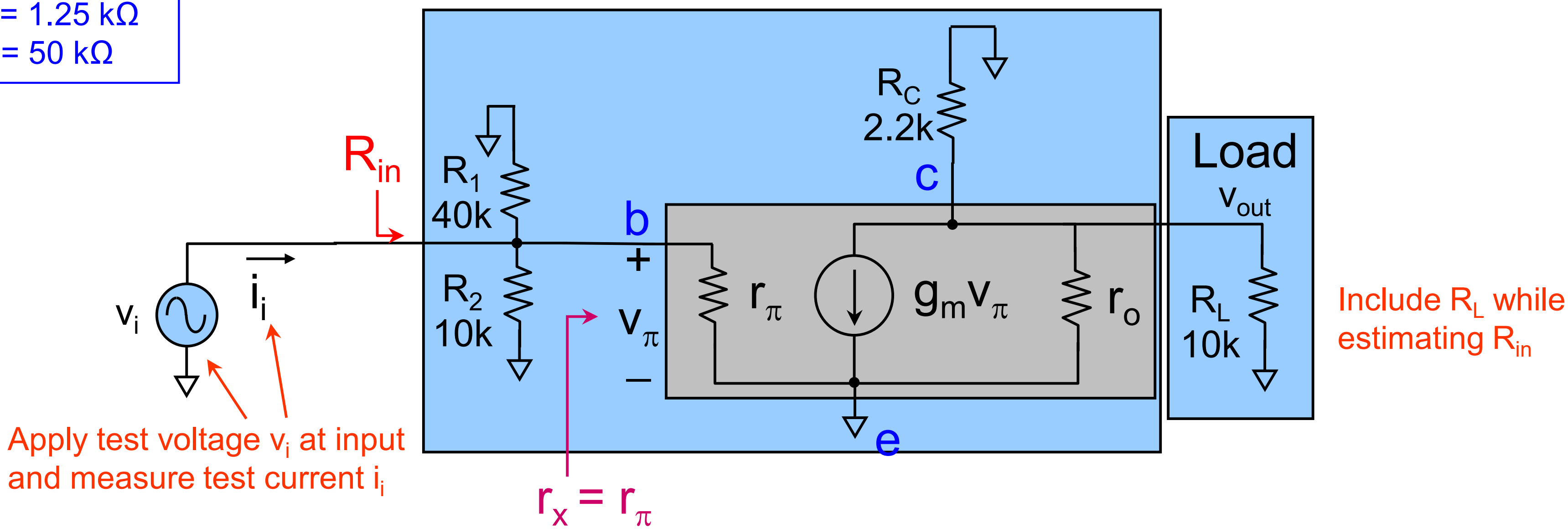
$$\therefore G_m = \left.\frac{i_o}{v_i}\right|_{v_{out}=0} = g_m$$

****

Important Result:
For CE, G_m is directly given by g_m. **No need to rederive.**

Two-Port Network (R_{in}) CE

g_m = 80 mA/V
r_π = 1.25 kΩ
r_o = 50 kΩ



Refer to slide SSA1-7 for evaluation of R_{in}.

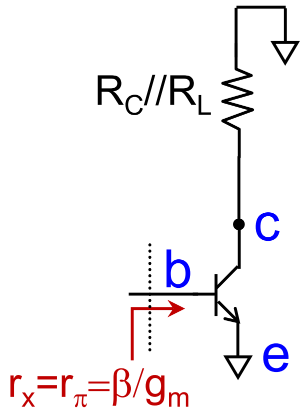
No part of i_i flows to the output circuit.

$$\text{Hence } i_i = \frac{v_i}{R_1 // R_2 // r_\pi}.$$

Two slanted parallel lines mean resistors in parallel

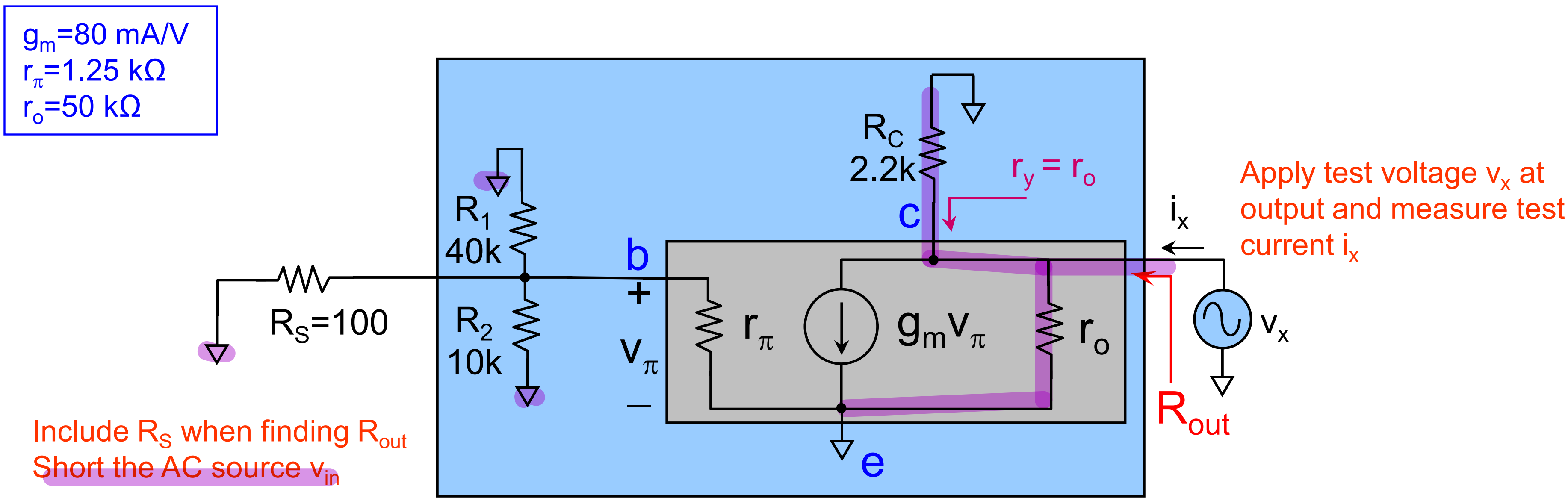
$$\text{Hence } R_{in} = \frac{v_i}{i_i} = R_1 // R_2 // r_\pi.$$

Two-Port Network (R_{in}) CE



Important Result:
If you see a BJT connected in similar fashion, the equivalent resistance looking into the base (r_x) is directly given by r_π. **No need to rederive.** The resistance in the collector branch has no effect on r_x.

Two-Port Network (R_{out}) CE



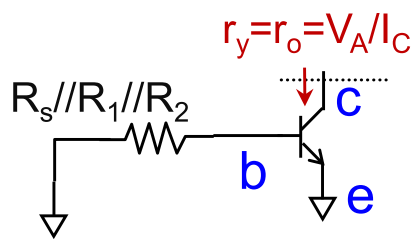
Refer to slide SSA1-7 for evaluation of R_{out}.

As there is no base current, $v_\pi = 0 \Rightarrow g_m v_\pi = 0.$ → (This means that $g_m V_\pi$ is open circuit)

Hence, $i_x = \dfrac{v_x}{R_c // r_o}.$

Hence $R_{out} = \dfrac{v_x}{i_x} = R_c // r_o.$

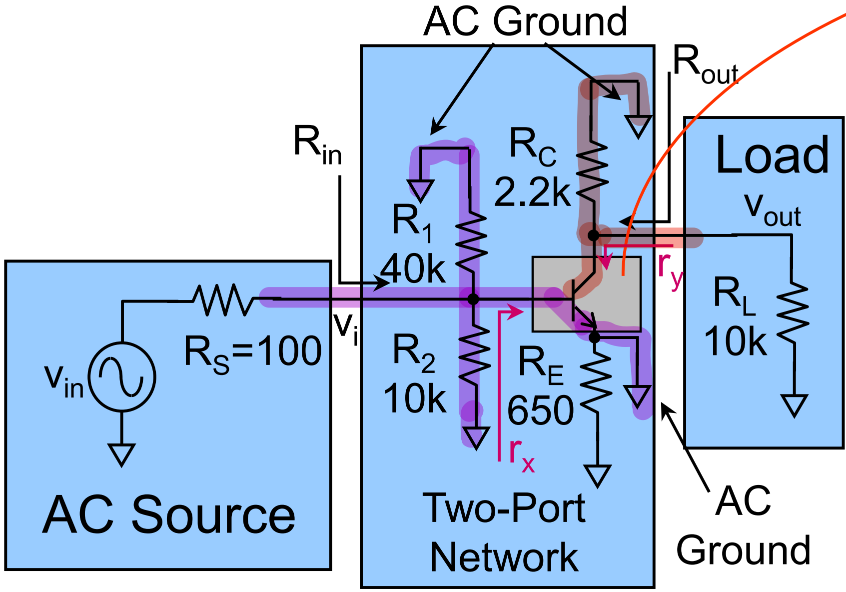
Two-Port Network (R_{out}) CE

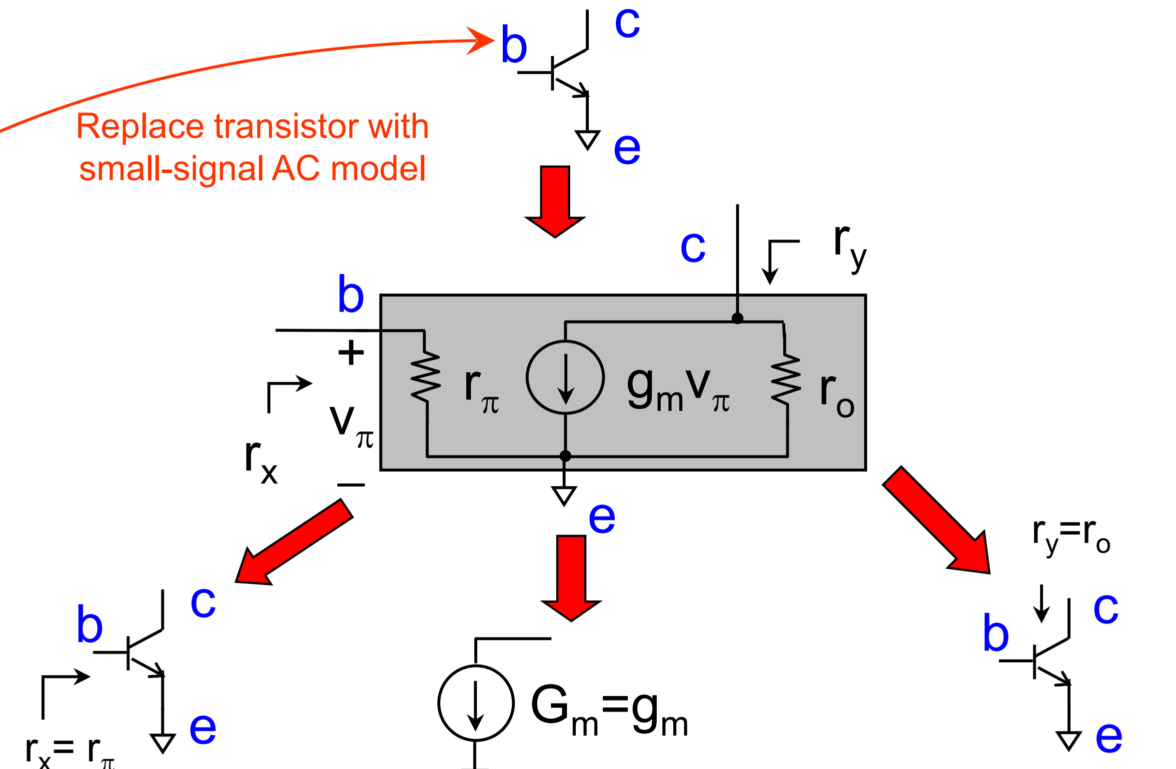


Important Result:
If you see a BJT connected in similar fashion, the equivalent resistance looking into the collector (r_y) is directly given by r_o. **No need to rederive.** The resistance in the base branch has no effect on r_y.

AC Analysis for CE

Short cut AC treatment for quicker analysis.





g_m = 80 mA/V
r_π = 1.25 kΩ
r_o = 50 kΩ

Important Result:
If you see a BJT connected in similar fashion, the **equivalent resistance** looking into the **Base** (r_x) is directly given by r_π. **No need to rederive.** The resistance in the Collector branch has no effect on r_x.

$$R_{in} = R_1 // R_2 // r_\pi.$$

Important Result:
For CE, G_m is directly given by g_m. **No need to rederive.**

CE

$$G_m = g_m$$

↓
BJT g_m

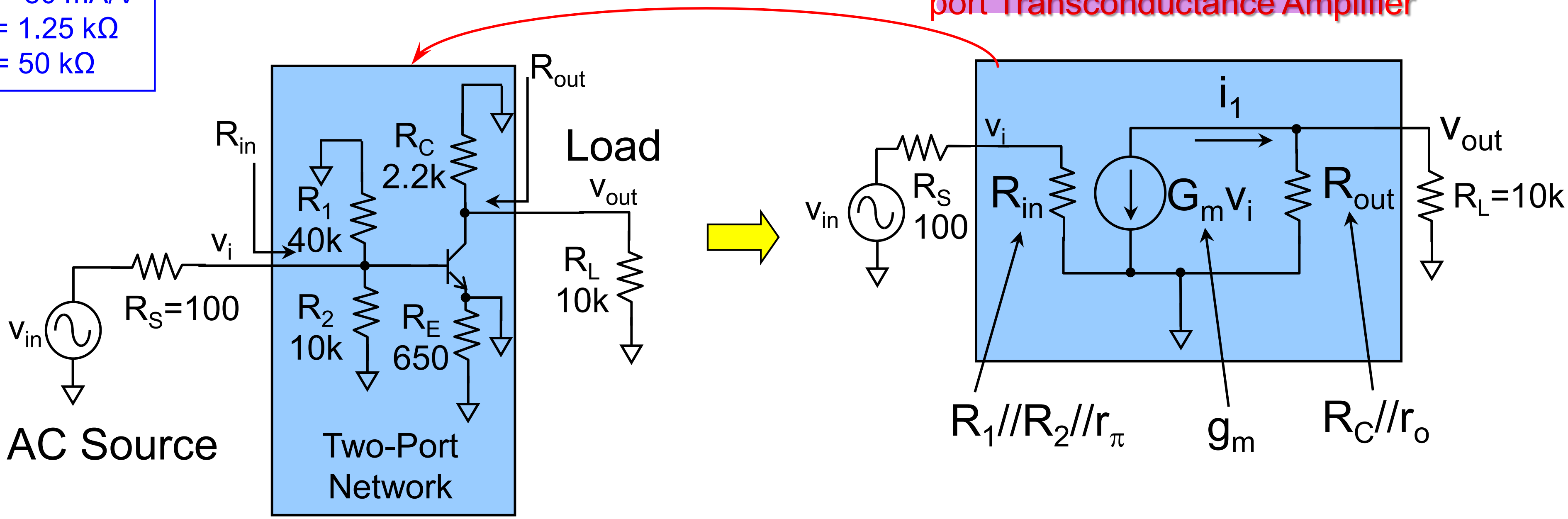
Important Result:
If you see a BJT connected in similar fashion, the **equivalent resistance** looking into the **Collector** (r_y) is directly given by r_o. **No need to rederive.** The resistance in the Base branch has no effect on r_y.

$$R_{out} = R_c // r_o.$$

Two-Port Network (A_V) CE

g_m = 80 mA/V
r_π = 1.25 kΩ
r_o = 50 kΩ

Transform CE amplifier to its equivalent 2-port Transconductance Amplifier



$$v_i = \frac{R_{in}}{R_S + R_{in}} v_{in} = \frac{R_1 // R_2 // r_\pi}{R_S + R_1 // R_2 // r_\pi} v_{in} \approx v_{in}, \text{ as } R_S << R_1 // R_2 // r_\pi$$

** (Don't Assume This unless R_S given)

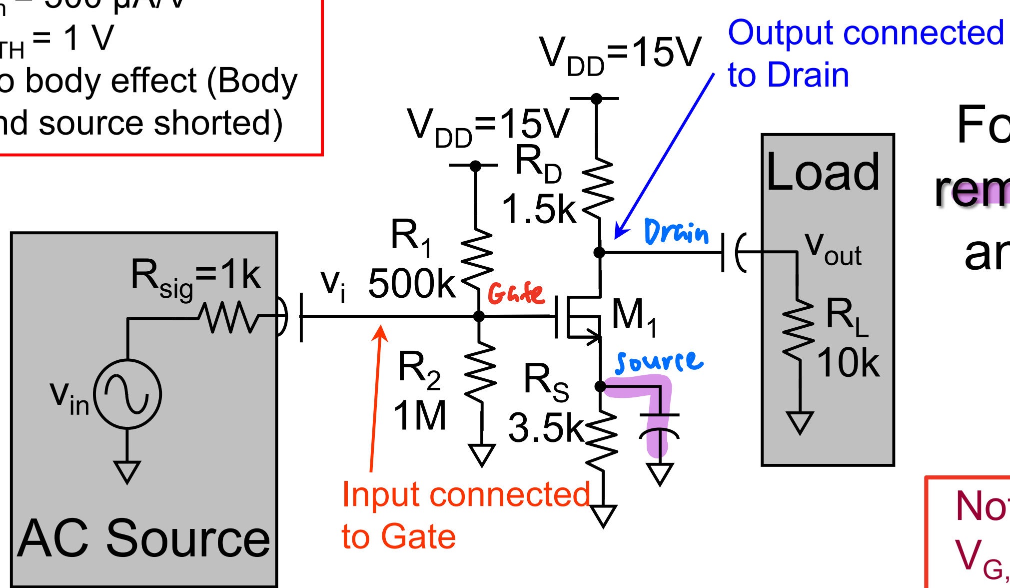
$$i_1 = -g_m v_i \approx -g_m v_{in}$$

$$v_{out} = i_1 \times [R_{out} // R_L] = i_1 \times [(R_C // r_o) // R_L] = -g_m (R_C // r_o // R_L) v_{in}$$

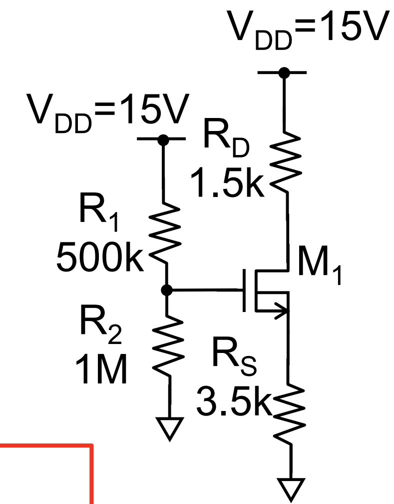
$$\Rightarrow A_V = \frac{v_{out}}{v_{in}} = -g_m (R_C // r_o // R_L) = -144.3$$

Common Source (CS) Amplifier

K_n = 500 μA/V^2
V_{TH} = 1 V
No body effect (Body and source shorted)



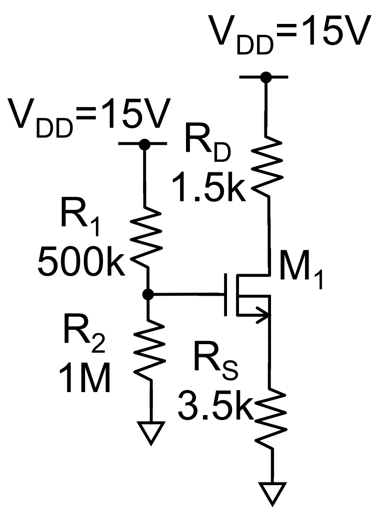
For DC analysis, remove AC source and open circuit capacitors



Notations:
$V_{G,M1}$: Drain voltage of M_1
$V_{S,M1}$: Source voltage of M_1
$V_{D,M1}$: Drain voltage of M_1
$V_{B,M1}$: Body voltage of M1

- Identify AC Source and Load
- To identify amplifier configuration, we need to consider AC equivalent circuit, i.e., short circuit capacitors
  - Input connected to Gate, output connected to Drain, Source connected to neither input nor output ⇒ Common Source (CS)
- DC Analysis - refer to "MOSFET Lecture Notes" slides ***MOSFET-30 to MOSFET-33.***

DC Analysis for CS Amplifier



K_n=500 μA/V^2
V_{TH}=1 V
No body effect (Body and source shorted)

Determine DC biasing

$$V_{G,M1} = \frac{1\text{M}}{500\text{k}+1\text{M}} \times 15 = 10\,\text{V}$$

Square Law :

Square Law is applicable for saturation region operation of MOSFET. Need to verify.

$$I_{D,M1} = K_n (V_{GS} - V_{TH})^2$$
$$= K_n (V_{G,M1} - V_{S,M1} - V_{TH})^2$$
$$= 500\mu (9 - V_{S,M1})^2$$

Ohm's Law :

$$I_{D,M1} = I_{S,M1} = \frac{V_{S,M1}}{R_S} = \frac{V_{S,M1}}{3.5\text{k}}$$

$$\Rightarrow \frac{V_{S,M1}}{3.5\text{k}} = 500\mu (9 - V_{S,M1})^2$$

$$1.75 V_{S,M1}^2 - 32.5 V_{S,M1} + 141.75 = 0$$

$$\Rightarrow V_{S,M1} = 6.94 \text{ or } 11.63\,\text{V}$$

$$11.63\,\text{V} > V_{G,M1} \Rightarrow \text{invalid}$$

$$I_{D,M1} = 2\,\text{mA}$$

$$V_{DS} = V_{DD} - I_{D,M1} R_D - V_{S,M1} = 5.06\,\text{V}$$

$$V_{GS} - V_{TH} = 2.06\ \text{V}$$

$$\Rightarrow V_{DS} > V_{GS} - V_{TH}$$

Device operates in saturation region

Determine AC small-signal parameters

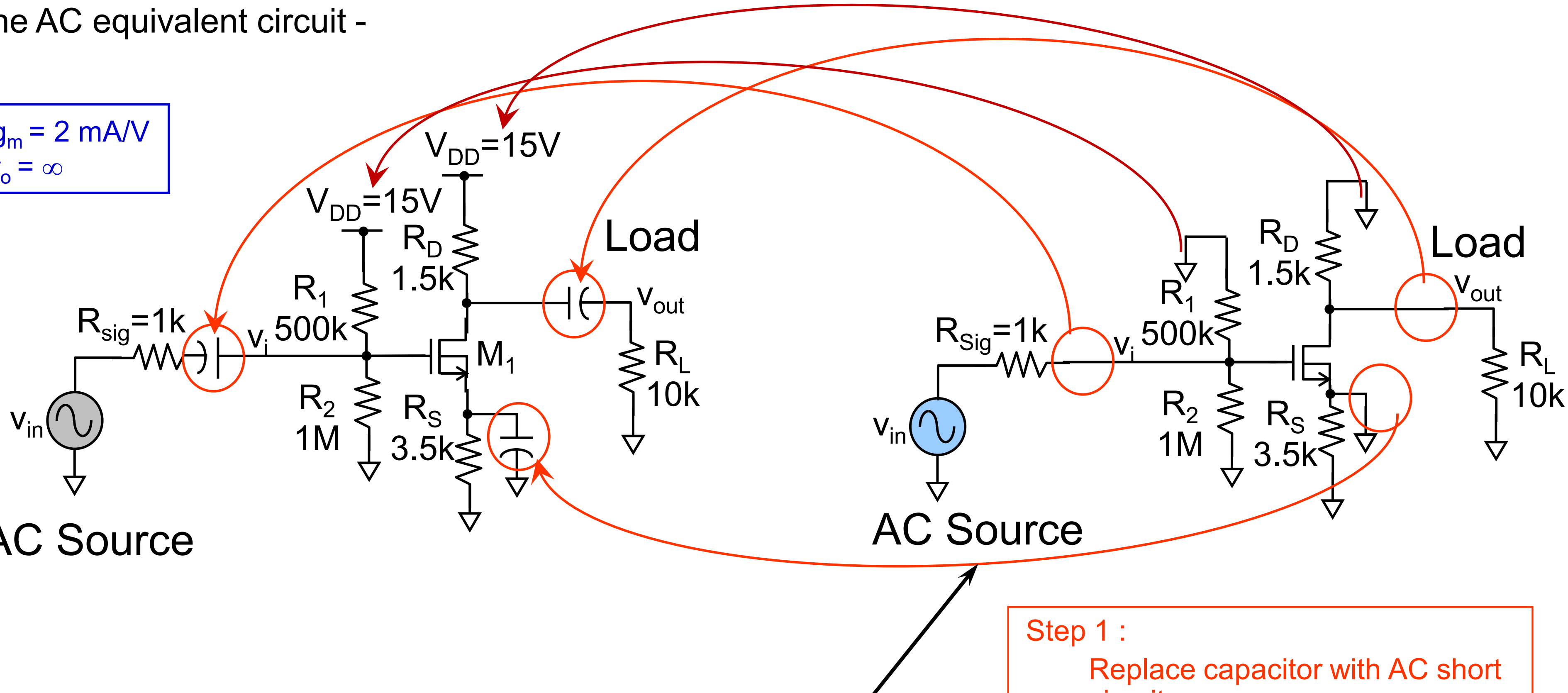
$$g_m = 2\sqrt{K_n I_{D,M1}} = 2\,\text{mA/V}$$

$$r_o = \frac{1}{\lambda I_{D,M1}} = \infty$$

AC Analysis for CS (Self Reading)

For AC analysis, need to consider the AC equivalent circuit -

g_m = 2 mA/V
$r_o = \infty$



Continue the analysis with AC equivalent circuit on the right.

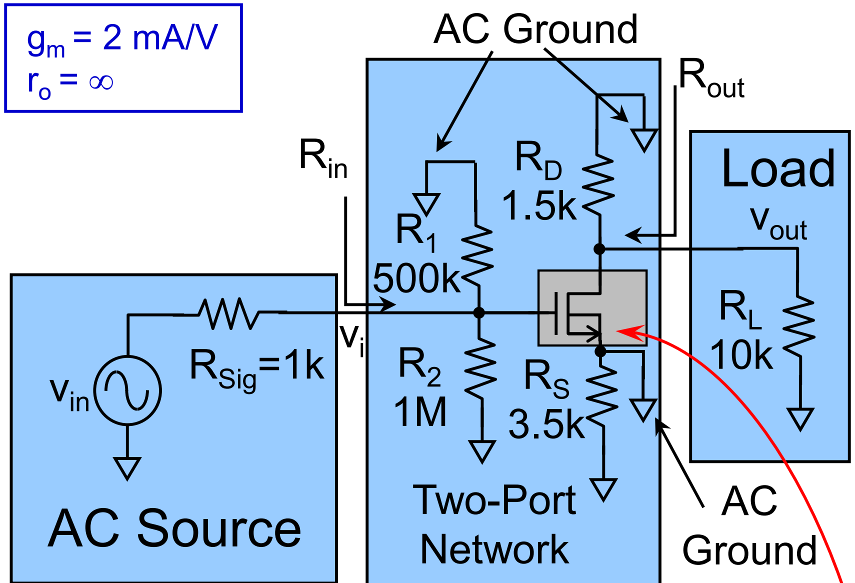
Step 1 :
Replace capacitor with AC short circuit

Step 2 :
Replace DC voltage source with AC short circuit

Step 3 :
Replace DC current source with AC open circuit

AC Analysis for CS (Self Reading)

g_m = 2 mA/V
$r_o = \infty$



Step 1 :

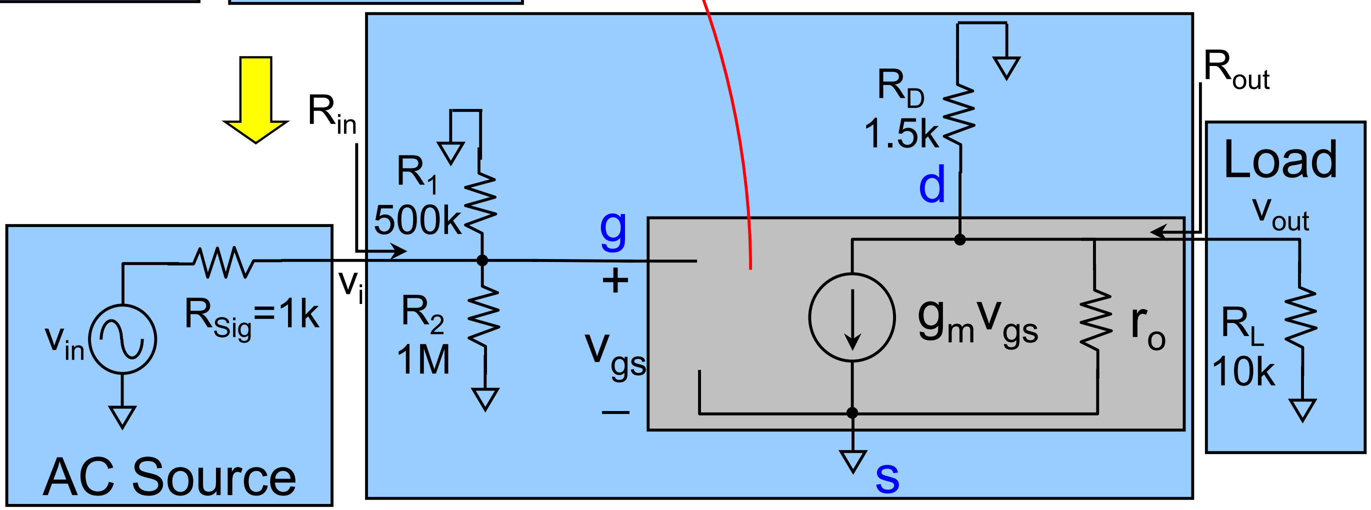
Identify AC source and load

Step 2 :

Group the remaining components (the amplifier circuit) into two-port network

Step 3 :

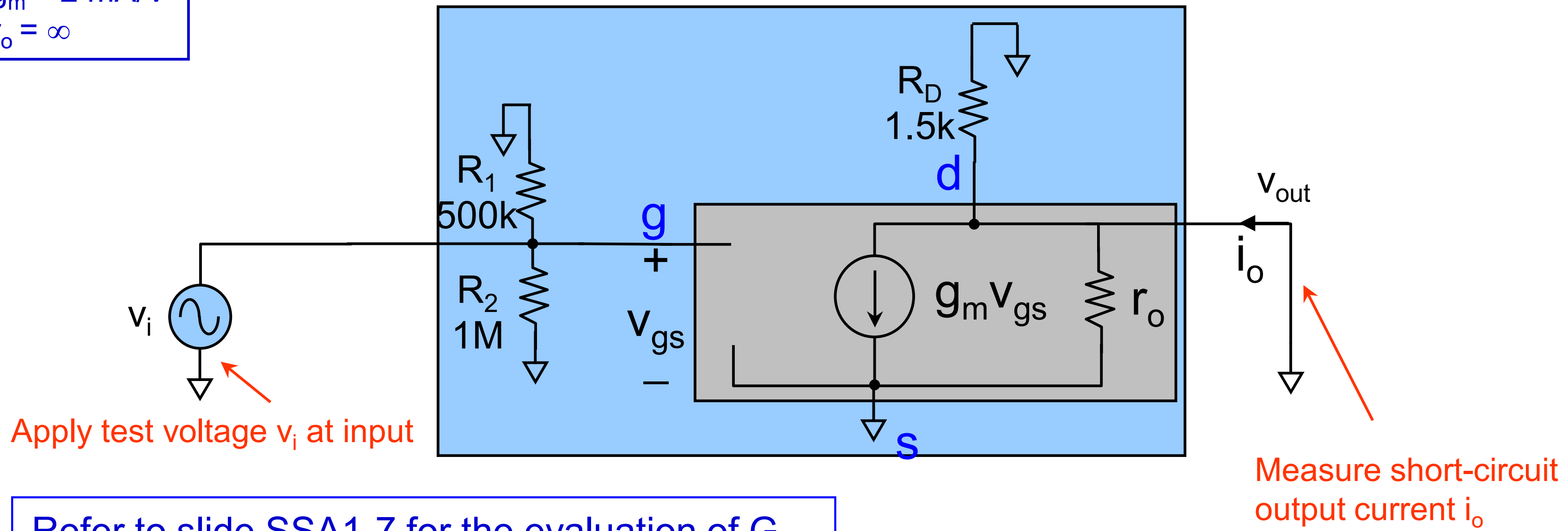
Replace transistor with small-signal AC model



For CS amplifier, transform to 2-port Transconductance Amplifier (based on experience of circuit designers). Need to evaluate G_m, R_{in} and R_{out}.

Two-Port Network(G_m)CS (Self Reading)

g_m = 2 mA/V
$r_o = \infty$



Refer to slide SSA1-7 for the evaluation of G_m.

There is no current through r_o and R_D.

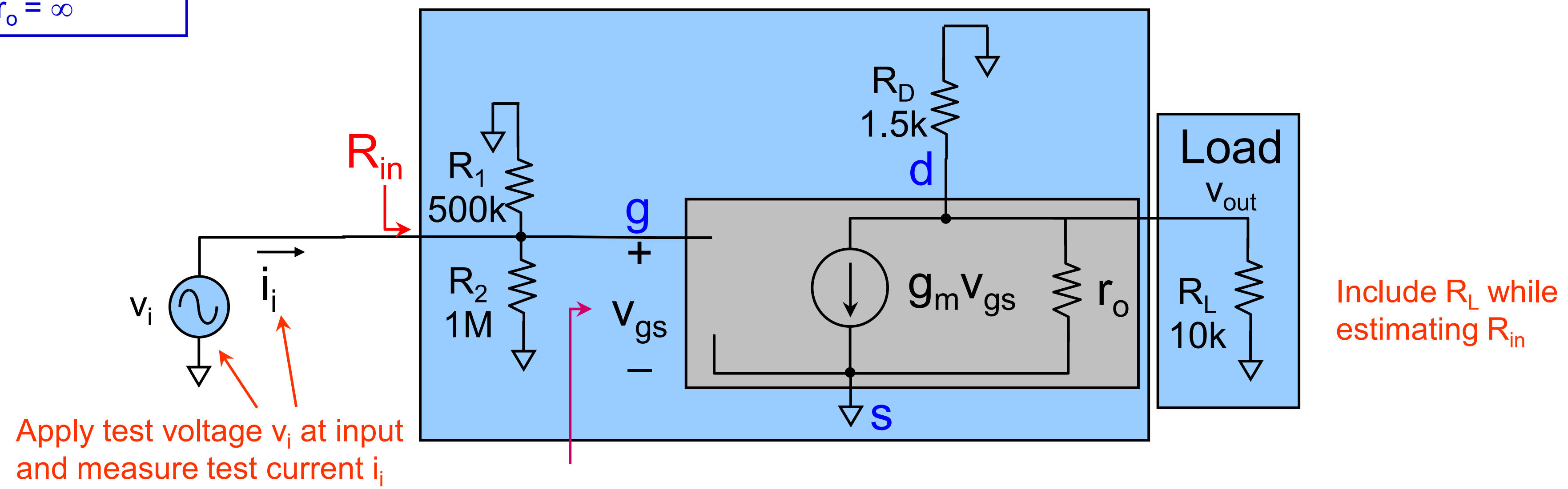
$$\therefore i_o = g_m v_{gs} = g_m v_i.$$

$$\therefore G_m = \left.\frac{i_o}{v_i}\right|_{v_{out}=0} = g_m$$

Important Result:
For CS, G_m is directly given by g_m. **No need to rederive.**

Two-Port Network(R_{in})CS (Self Reading)

g_m = 2 mA/V
$r_o = \infty$



$r_x = r_\pi = \infty$ (looking into the gate is an open circuit)

Refer to slide SSA1-7 for the evaluation of R_{in}.

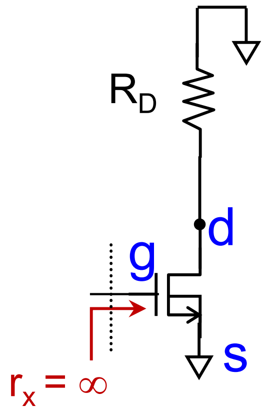
No part of i_i flows to the output circuit.

Hence $i_i = \dfrac{v_i}{R_1 // R_2}$.

Two slanted parallel lines mean resistors in parallel

Hence $R_{in} = \dfrac{v_i}{i_i} = R_1 // R_2$.

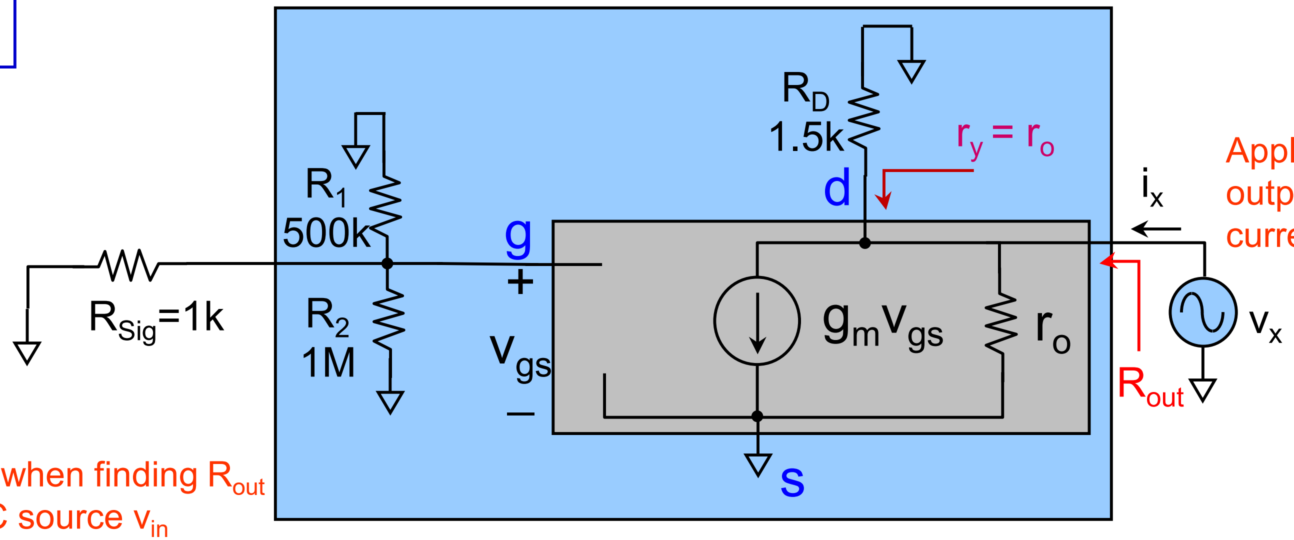
Two-Port Network(R_{in})CS (Self Reading)



Important Result:
If you see a MOSFET connected in similar fashion, the equivalent resistance looking into the gate (r_x) is ∞. **No need to rederive.** The resistance in the drain branch has no effect on r_x.

Two-Port Network(R_{out})CS (Self Reading)

g_m = 2 mA/V
$r_o = \infty$



Apply test voltage v_x at output and measure test current i_x

Include R_{Sig} when finding R_{out}
Short the AC source v_{in}

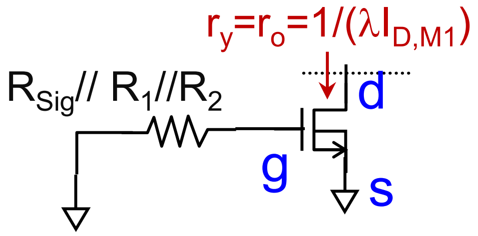
Refer to slide SSA1-7 for the evaluation of R_{out}.

As there is no gate current, $v_{gs} = 0 \Rightarrow g_m v_{gs} = 0$.

Hence, $i_x = \dfrac{v_x}{R_D \,//\, r_o}$.

Hence $R_{out} = \dfrac{v_x}{i_x} = R_D \,//\, r_o$.

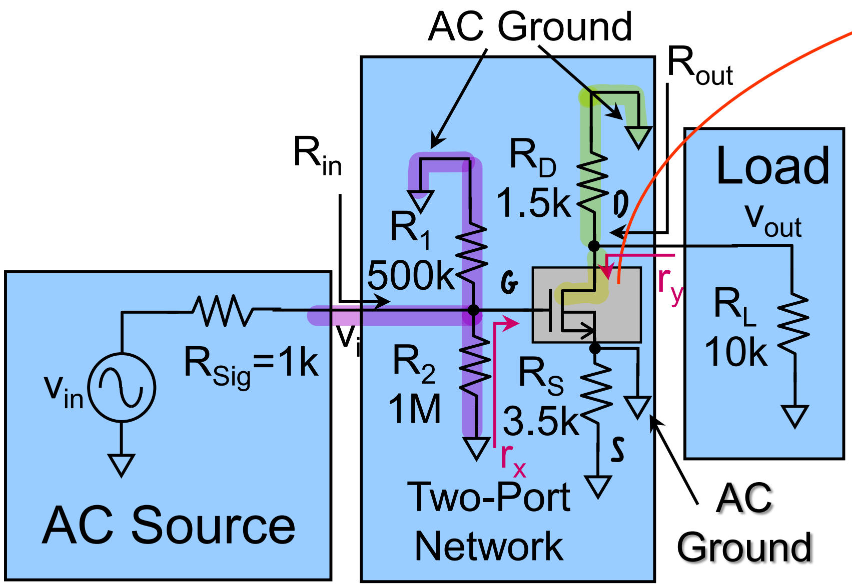
Two-Port Network(R_{out})CS (Self Reading)



Important Result:
If you see a MOSFET connected in the similar fashion, the **equivalent resistance** looking into the **drain** (r_y) is directly given by r_o. **No need to rederive.** The resistance in the gate branch has no effect on r_y.

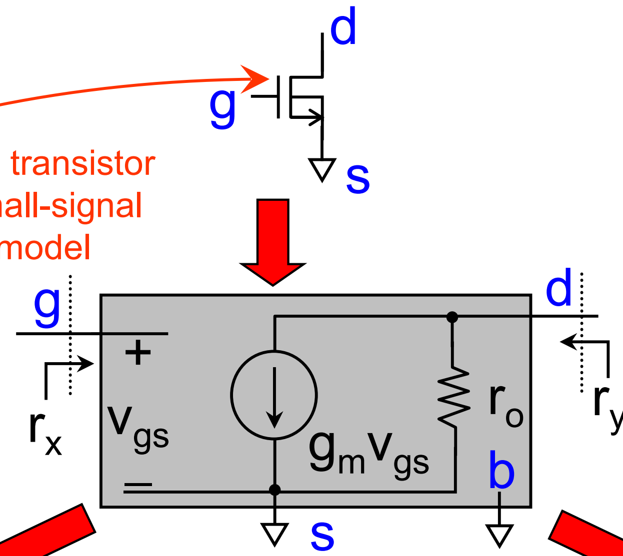
AC Analysis for CS

Short cut AC treatment for quicker analysis.

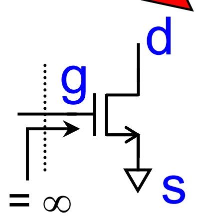


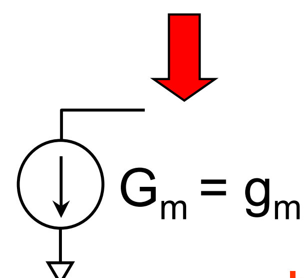
g_m = 2 mA/V
$r_o = \infty$

Replace transistor with small-signal AC model

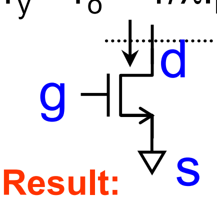


No body effect ⇒ v_{sb}=0 ⇒ dependent current source $g_{mb}v_{sb}$ is removed





$r_y = r_o = 1/\lambda I_{D,M1}$



Important Result:
If you see a MOSFET connected in similar fashion, the equivalent resistance looking into the gate (r_x) is open circuit. **No need to rederive.** The resistance in the drain branch has no effect on r_x.

$$R_{in} = R_1 // R_2$$

Important Result:
For CS, G_m is directly given by g_m. **No need to rederive.**

CS

$$G_m = g_m$$

MOSFET g_m

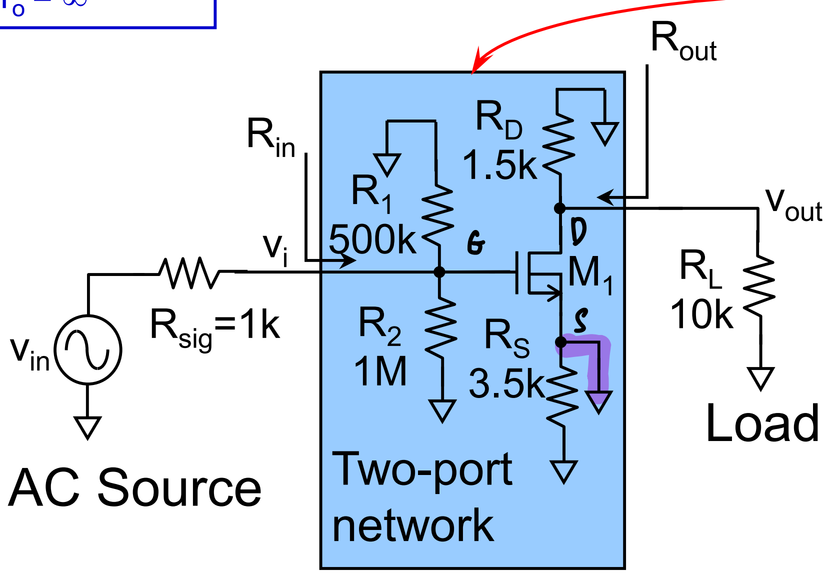
Important Result:
If you see a MOSFET connected in similar fashion, the equivalent resistance looking into the drain (r_y) is directly given by r_o. **No need to rederive.** The resistance in the gate branch has no effect on r_y.

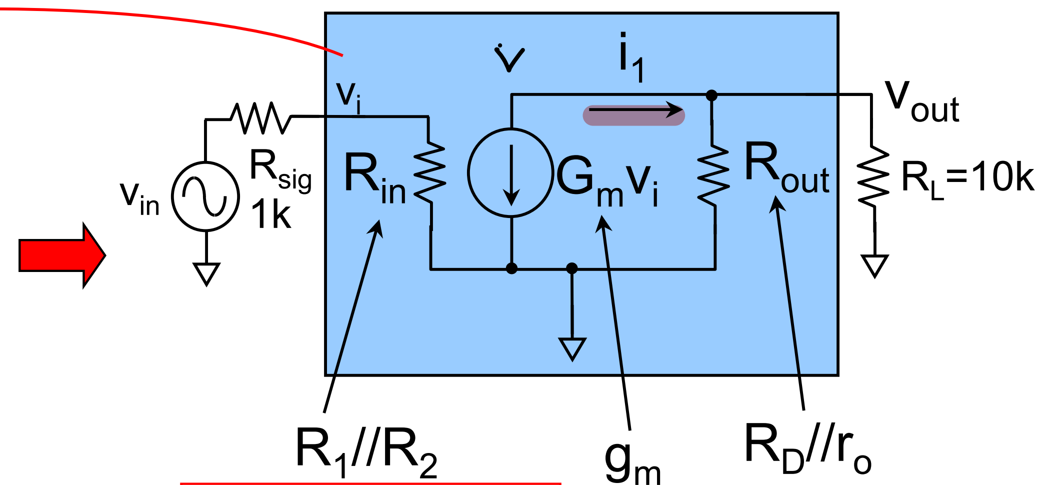
$$R_{out} = R_D // r_o$$

Two-Port Network (A_V) CS

g_m = 2 mA/V
$r_o = \infty$

Transform CS amplifier to its equivalent 2-port
Transconductance Amplifier





R_1, R_2 in parallel at the input as gate is open circuited.

R_D, r_o in parallel at the output as equivalent resistance looking into the drain is r_o.

$$v_i = \frac{R_{in}}{R_{sig}+R_{in}} v_{in} = \frac{R_1 // R_2}{R_{sig}+R_1 // R_2} v_{in} \approx v_{in} \quad \because R_{sig} << R_1, R_2$$

$$i_1 = -g_m v_i \approx -g_m v_{in}$$

$$v_{out} = i_1 \times \left[R_{out} // R_L\right] = i_1 \times \left[\left(R_D // r_o\right) // R_L\right] = -g_m\left(R_D // r_o // R_L\right) v_{in}$$

$$\Rightarrow A_V = \frac{v_{out}}{v_{in}} = -g_m\left(R_D // r_o // R_L\right) = -2.61$$

Characteristics of CE/CS

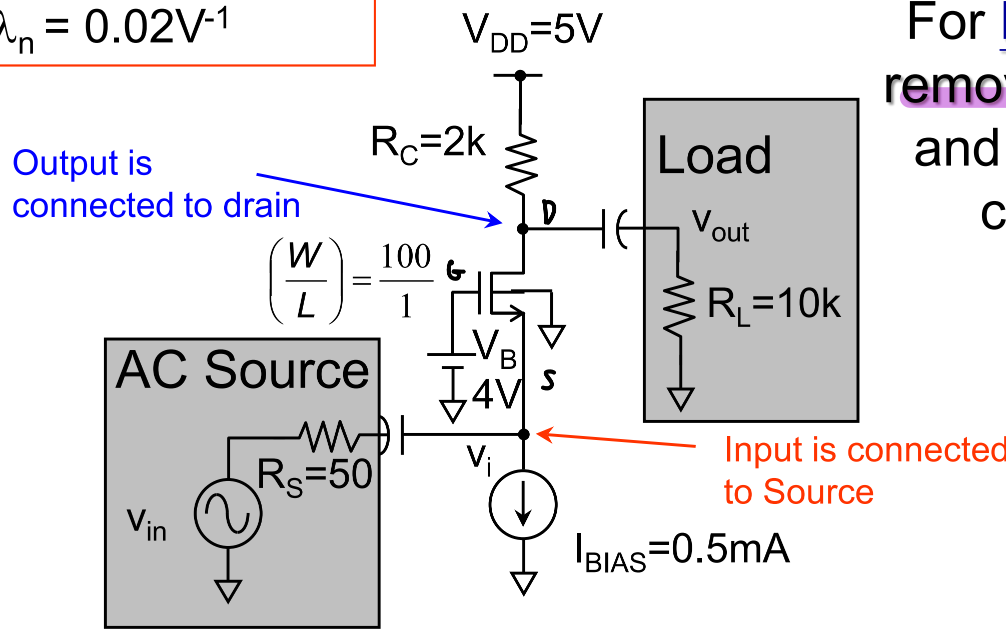
- High input resistance
- High output resistance
- Medium gain
- Polarity inversion, i.e., v_{out} and v_{in} has opposite sign

  if input +ve , output -ve and vice versa
- The higher the G_m and the total output resistance, the higher the gain (A_V)
- BJT provides larger g_m than MOS

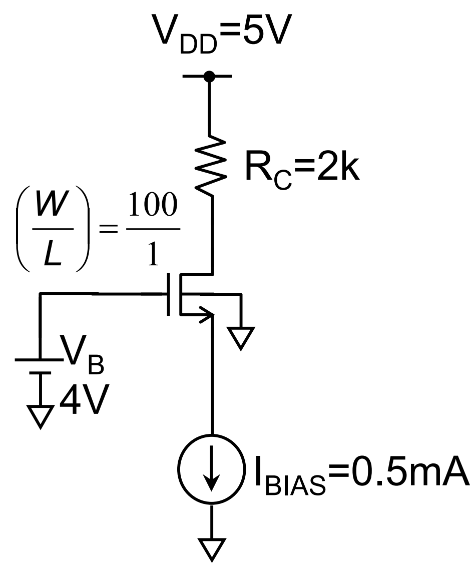
  $g_m = \frac{I_C}{V_T}$ $g_m = 2\sqrt{K_n I_D}$

Common Gate (CG)

$\mu_n C_{ox} = 80\ \mu A/V^2$
$V_{THN} = 0.7V$
$\lambda_n = 0.02V^{-1}$



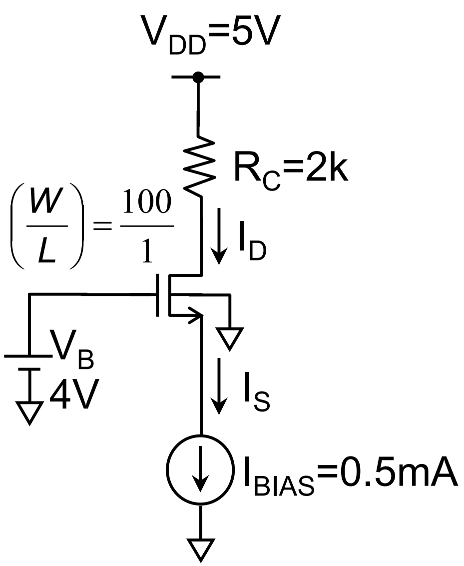
For DC analysis, remove AC source and open circuit capacitors



- Identify AC Source and Load
- To identify amplifier configuration, we need to consider AC equivalent circuit, i.e., short circuit capacitors
  - Input connected to Source, output connected to Drain, Gate connected to neither input nor output $\Rightarrow$ Common Gate (CG)

DC Analysis for CG

- Remove AC source and open circuit capacitors when doing DC analysis



$\mu_n C_{ox}$ = 80 μA/V^2
V_{THN} = 0.7V
λ_n = 0.02V^{-1}

Determine DC biasing

$$I_D = I_S = I_{BIAS} = 0.5\,\text{mA}$$

$$\left(\frac{W}{L}\right) = \frac{100}{1}$$

Good approximation, no need to go through detailed calculations

Determine AC small-signal parameters

$$g_m = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right) I_D}$$

$$= 2.83\,\text{mA/V}$$

$g_m = 2\sqrt{k_n I_D}$
$= \sqrt{4 \times \frac{1}{2} \dots I_D}$
$= \sqrt{2\mu_n \dots I_D}$

MOSFET has body effect (body not shorted to source)

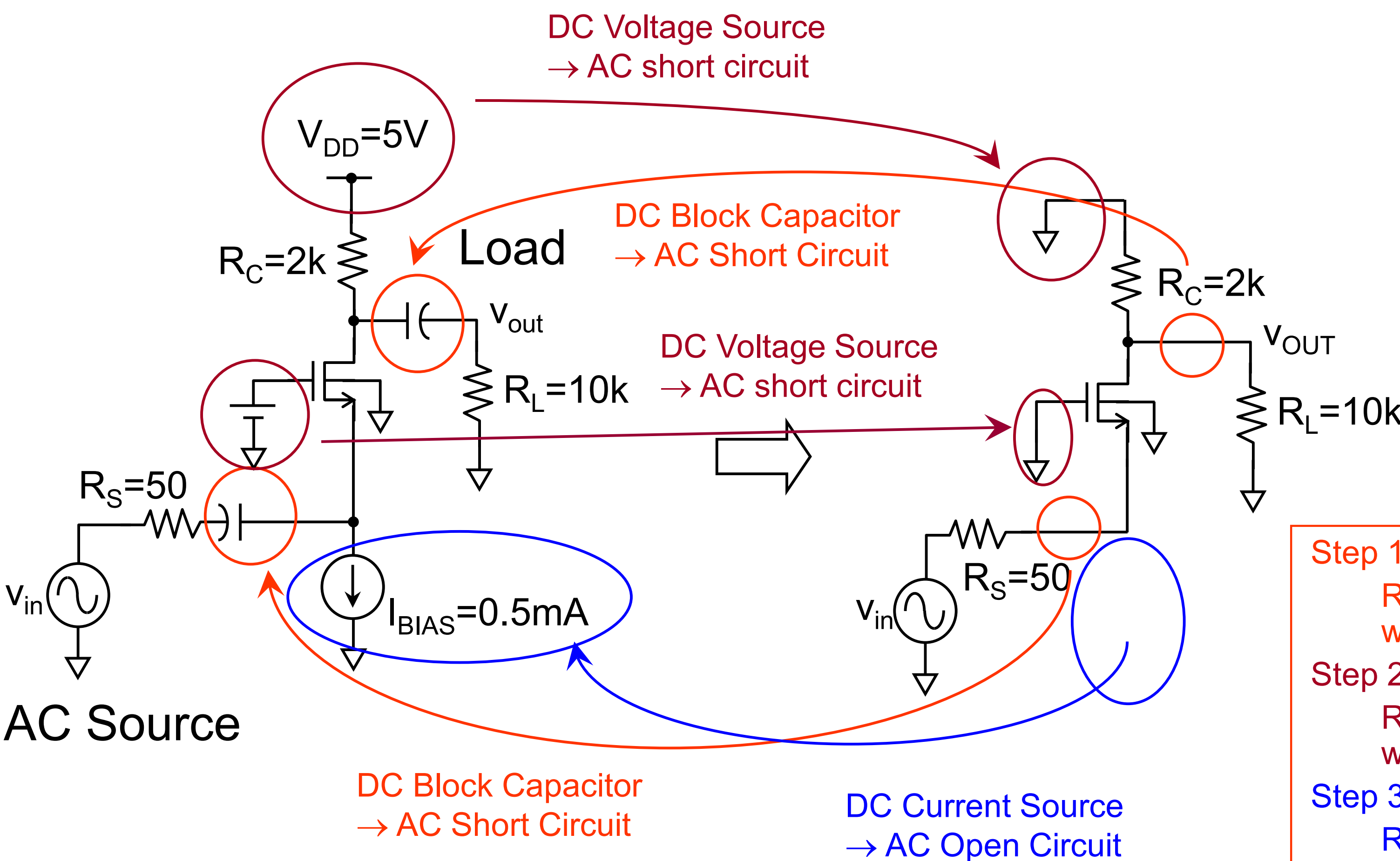
$$\boxed{g_{mb} \approx -\frac{g_m}{4} = -0.71\,\text{mA/V}}$$

$$r_i = \infty$$

$$r_o = \frac{1}{\lambda_n I_D} = 100\,\text{k}\Omega$$

AC Analysis for CG

For AC analysis, need to consider the AC equivalent circuit -



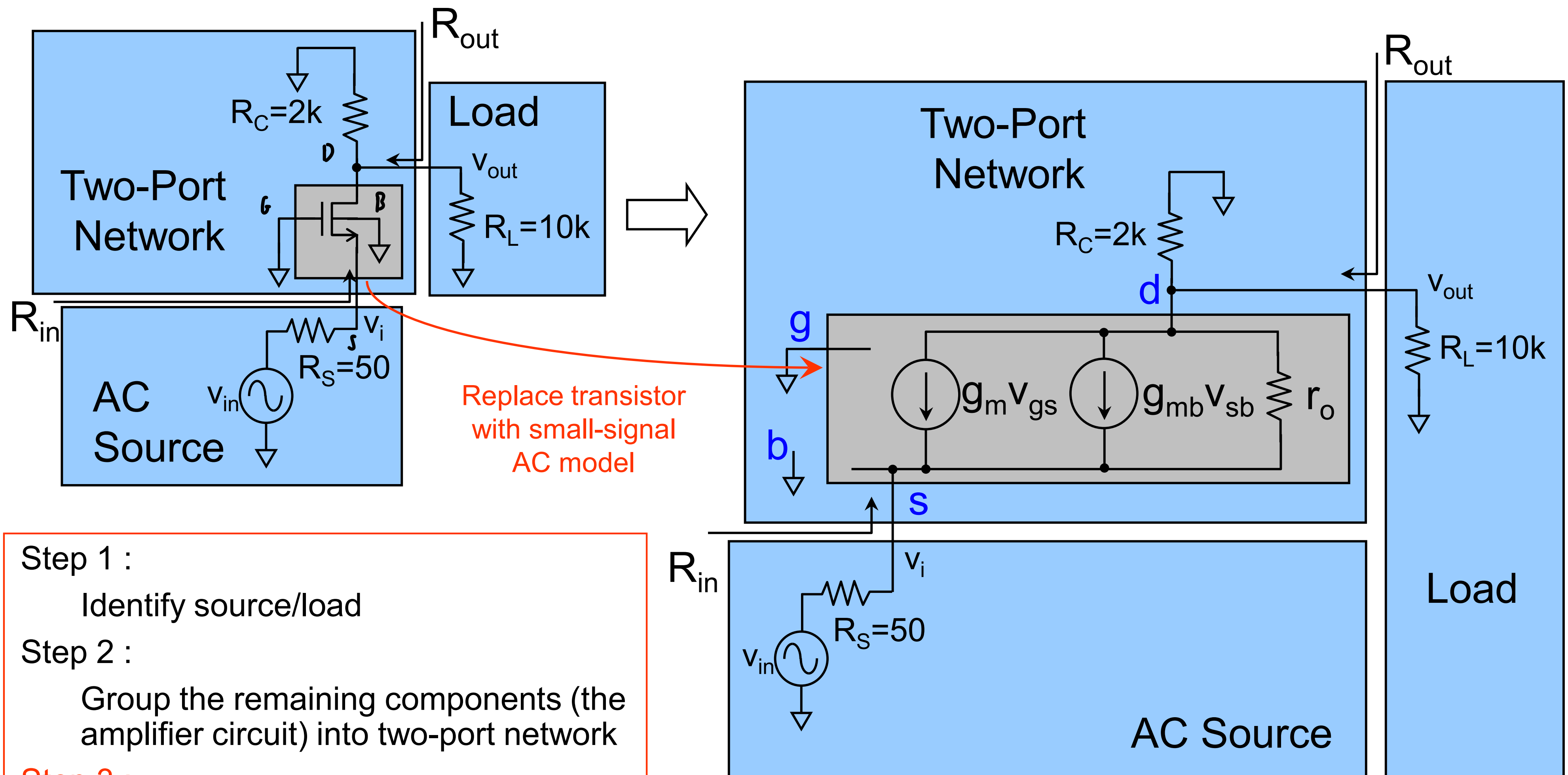
Step 1 :
Replace DC block capacitor with AC short circuit

Step 2 :
Replace DC voltage source with AC short circuit

Step 3 :
Replace DC current source with AC open circuit

g_m = 2.83 mA/V
g_{mb} = -0.71 mA/V
r_o = 100 kΩ

AC Analysis for CG



Step 1 :

Identify source/load

Step 2 :

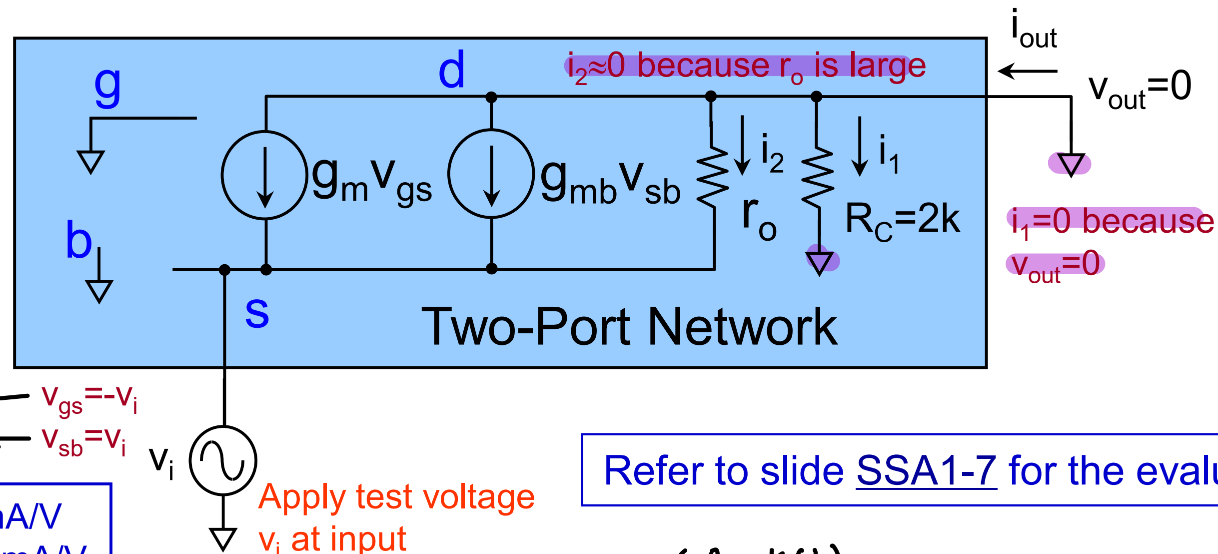
Group the remaining components (the amplifier circuit) into two-port network

Step 3 :

Replace transistor with small-signal AC model

For CG amplifier, transform to 2-port Transconductance Amplifier (based on experience of circuit designers). Need to evaluate G_m, R_{in} and R_{out}.

CG – Two-Port Network (G_m)



Measure short-circuit output current i_{out}

(0) − Vi
Vg − Vs = −Vi ← $v_{gs}=-v_i$
Vs − Vb = Vi ← $v_{sb}=v_i$
(Vi) − 0

g_m = 2.83 mA/V
g_{mb} = -0.71 mA/V
r_o = 100 kΩ

Refer to slide SSA1-7 for the evaluation of G_m.

(By KCL)

$$i_{out}\Big|_{v_{out}=0} = g_m \underbrace{v_{gs}}_{-V_i} + g_{mb} \underbrace{v_{sb}}_{V_i} + \cancel{i_2} \approx -(g_m - g_{mb}) v_i$$

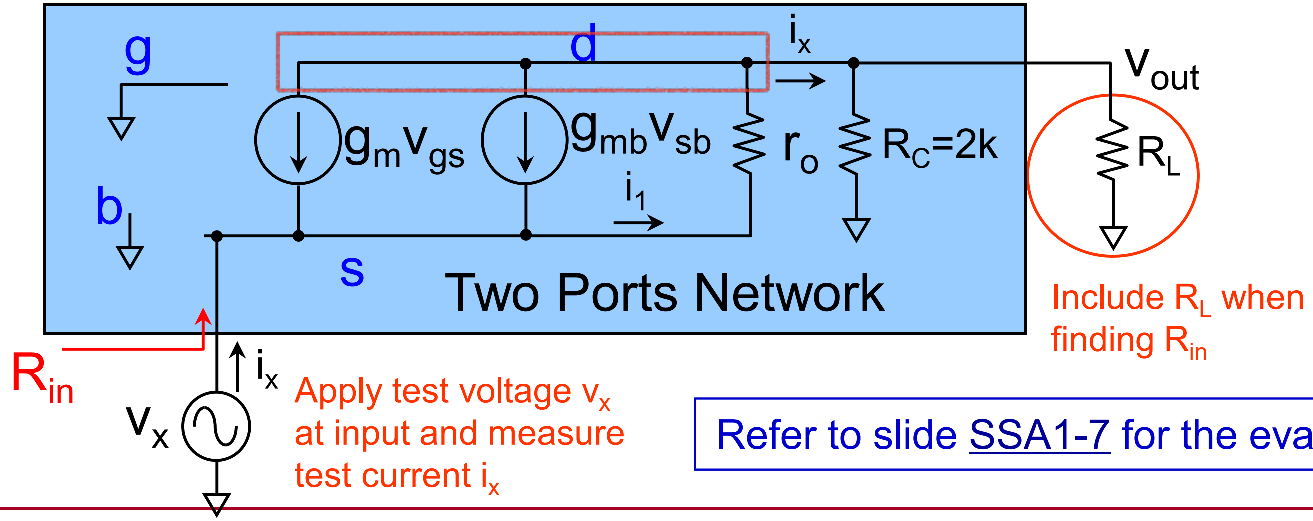
$$\underline{G_m} = \frac{i_{out}}{v_i}\Bigg|_{v_{out}=0} \approx \boxed{-(g_m - g_{mb})} = -3.54\,\text{mA/V}$$

****

Important Result :
Transconductance (G_m) for CG is just $-(g_m - g_{mb})$ → if there is no body effect, (body shorted to source) result is just $-g_m$.

CG – Two-Port Network (R_{in})

KCL



$$v_{gs} = -v_x \quad v_{sb} = v_x$$

$$i_1 = \frac{v_x - v_{out}}{r_o}$$

Refer to slide SSA1-7 for the evaluation of R_{in}.

$g_m - g_{mb} >> 1/r_o$

(By KCL)

$$R_{in} = \frac{v_x}{i_x} \quad \begin{cases} i_x = -g_m v_{gs} - g_{mb} v_{sb} + i_1 \\ \quad = (g_m - g_{mb}) v_x + \dfrac{v_x - v_{out}}{r_o} \\ v_{out} = i_x (R_C // R_L) \end{cases}$$

Eliminate v_{out} and keep v_x and i_x

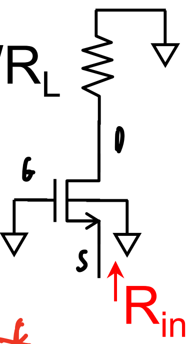
$$\Rightarrow i_x = (g_m - g_{mb}) v_x + \frac{v_x}{r_o} - i_x \frac{(R_C // R_L)}{r_o}$$

$$\Rightarrow R_{in} = \frac{1 + \dfrac{(R_C // R_L)}{r_o}}{(g_m - g_{mb}) + \cancel{\dfrac{1}{r_o}}}$$

$$\approx \frac{1}{g_m - g_{mb}} \cdot \frac{r_o + (R_C // R_L)}{r_o}$$

CG – Two-Port Network (R_{in})

$R_T = R_C // R_L$



$$R_{in} \approx \frac{1}{g_m - g_{mb}} \cdot \frac{r_o + R_T}{r_o}$$

$$\approx \frac{1}{g_m - g_{mb}} \quad [If \quad R_T << r_o]$$

Important Result:
If you see a MOSFET connected in similar fashion, the equivalent resistance looking into the source (R_{in}) is directly given by the formula. **No need to rederive.**

Example :

$$R_C = 2k \quad R_L = 10k \quad r_o = 100k$$

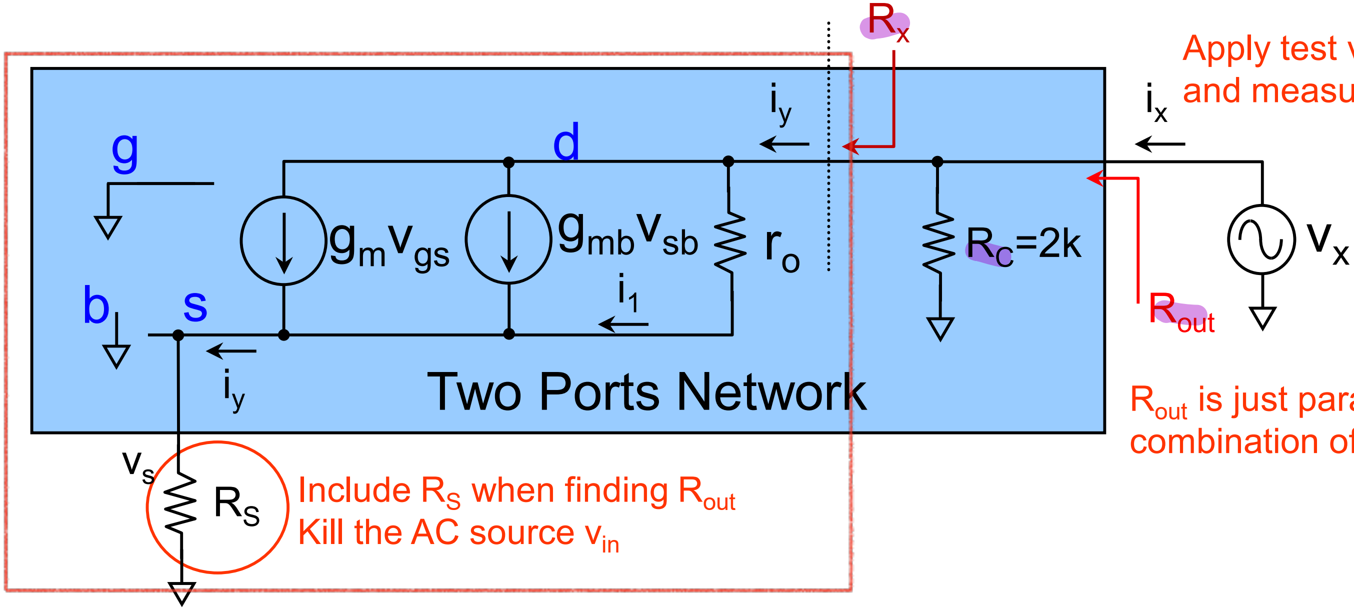
$$\Rightarrow R_T = (R_C // R_L) << r_o$$

$$\Rightarrow R_{in} = \frac{1}{g_m - g_{mb}} = 282$$

- If $R_C // R_L$ is negligible compared to r_o, the input resistance (R_{in}) is reduced to the inverse of the transconductance [$1/(g_m - g_{mb})$]

CG – Two-Port Network (R_{out})

Refer to slide SSA1-7 for the evaluation of R_{out}.



Apply test voltage v_x at output and measure test current i_x

$$v_{gs} = -v_s \quad v_{sb} = v_s$$

$$i_1 = i_y - g_m v_{gs} - g_{mb} v_{sb}$$
$$= i_y + (g_m - g_{mb}) v_s$$

R_{out} is just parallel combination of R_C and R_x

R_S<<$r_o[1+(g_m-g_{mb})R_S)]$

$$R_{out} = R_C // R_x$$

Eliminate v_s and keep v_x and i_y

$$R_x = \frac{v_x}{i_y} \begin{cases} v_s = i_y R_S \\ v_x = v_s + i_1 r_o \\ \quad = v_s + [i_y + (g_m - g_{mb}) v_s] r_o \end{cases}$$

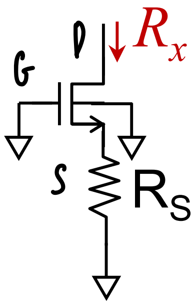
$$\Rightarrow v_x = i_y R_S + i_y r_o + i_y (g_m - g_{mb}) R_S r_o$$

$$\Rightarrow R_x = \frac{v_x}{i_y} = \cancel{R_S} + r_o [1 + (g_m - g_{mb}) R_S]$$

Here only can bcos R_C << R_x
($r_o \approx 100k$)

$$R_{out} = R_C // R_x$$
$$\approx R_C // \{ r_o [1 + (g_m - g_{mb}) R_S] \} \approx R_C$$

CG – Two-Port Network (R_{out})



$$R_x \approx r_o\left[1+\left(g_m - g_{mb}\right)R_S\right]$$

Important Result: *****
If you see a MOSFET connected in similar fashion, the equivalent resistance looking into the drain (R_x) is directly given by the formula. **No need to rederive.**

Example :

$$R_S = 50 \quad r_o = 100k$$

$$g_m = 2.83m \quad g_{mb} = -0.71m$$

$$\Rightarrow R_x \approx 118k >> R_C$$

$$\Rightarrow R_{out} \approx R_C = 2k$$

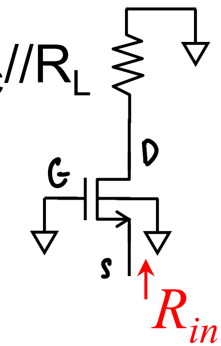
- Source side resistor helps boost up the output resistance of the transistor (R_x)

CG – Important Results ******

1

Important Result :
Transconductance (G_m) for **CG** is just $-(g_m - g_{mb})$

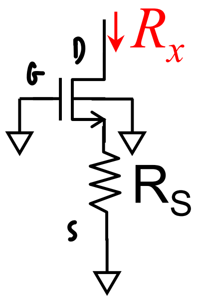
2



$$\boxed{R_{in} \approx \frac{1}{g_m - g_{mb}} \cdot \frac{r_o + R_T}{r_o}}$$

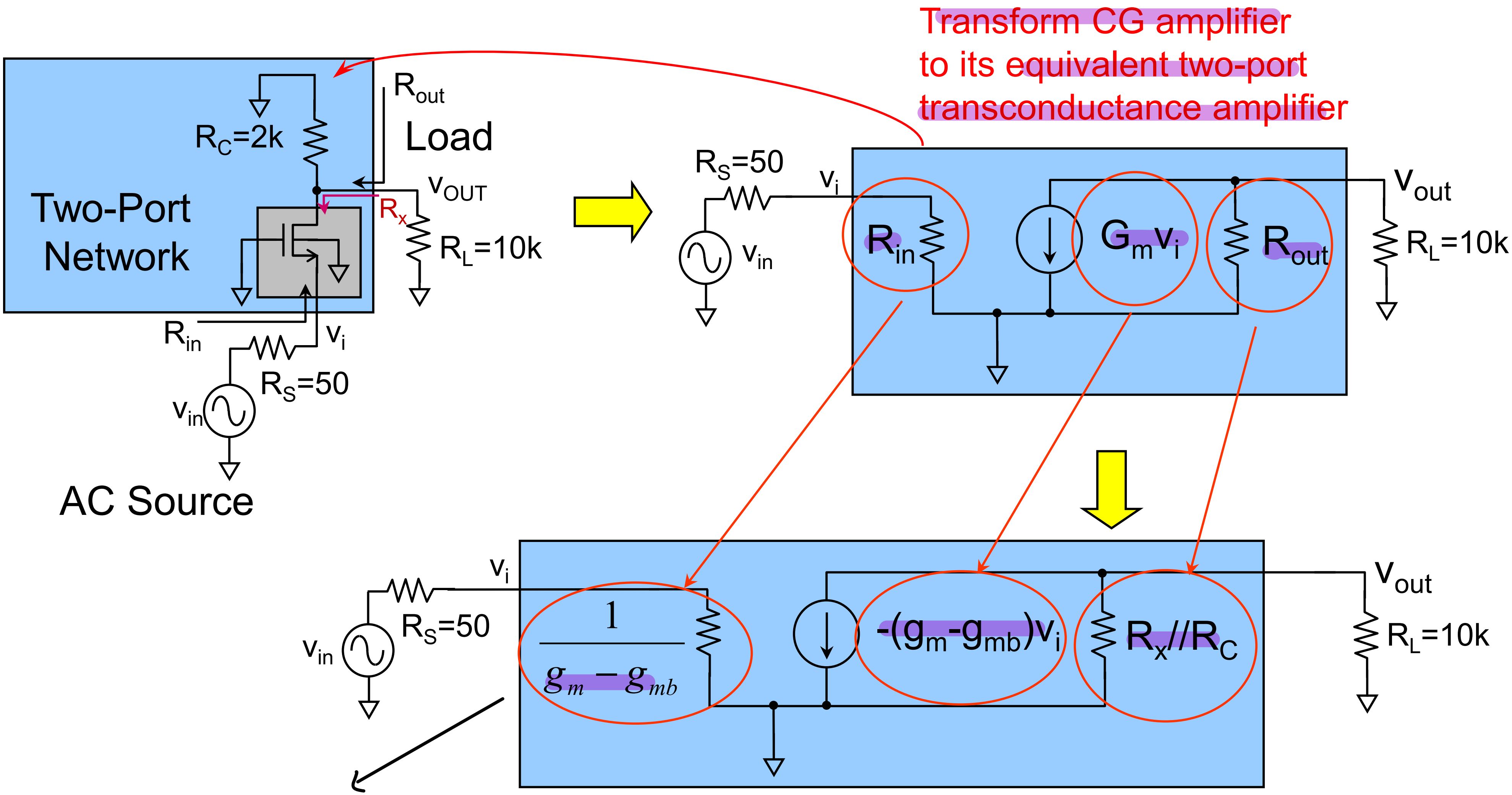
$$\approx \frac{1}{g_m - g_{mb}} \quad [If \quad R_T << r_o]$$

3



$$R_x \approx r_o[1 + (g_m - g_{mb})R_S]$$

CG – Two-Port Network

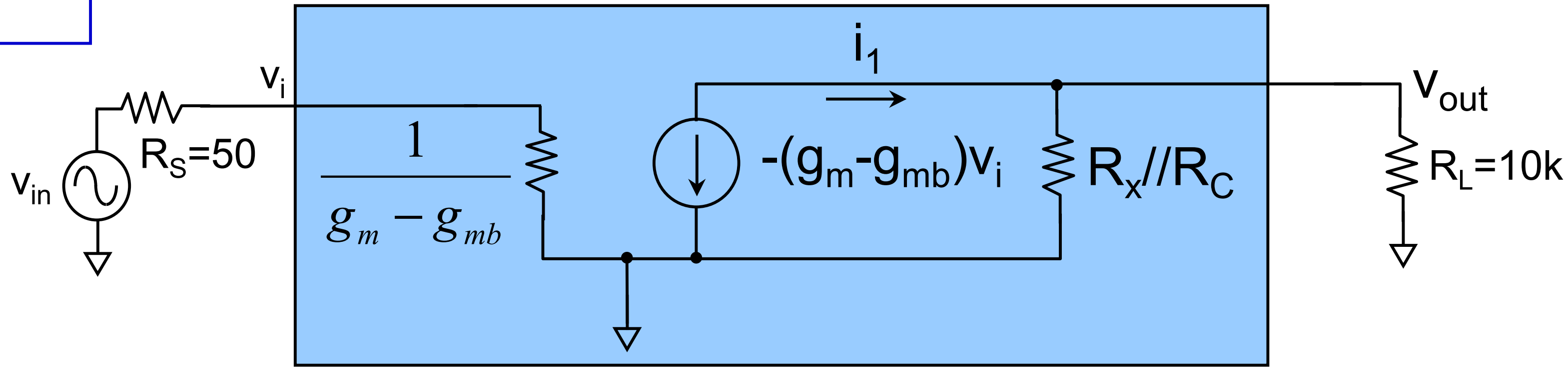


$$\frac{1}{g_m - g_{mb}} \cdot \frac{r_o + R_C//R_L}{r_o}$$ ⇒ since $r_o >> R_C//R_L$
100k 2k // 10k

$$\approx \frac{1}{g_m - g_{mb}}$$

CG – Two-Port Network (A_V)

g_m = 2.83 mA/V
g_{mb} = -0.71 mA/V
r_o = 100 kΩ



$$v_i = v_{in} \times \frac{R_{in}}{R_S + R_{in}}$$

$$R_{in} = \frac{1}{g_m - g_{mb}}$$

$$i_1 = -\left[-\left(g_m - g_{mb}\right)v_i\right]$$
$$= \left(g_m - g_{mb}\right)v_i$$

$$v_{out} = i_1 \times \left[\left(\cancel{R_x} // R_C\right) // R_L\right]$$

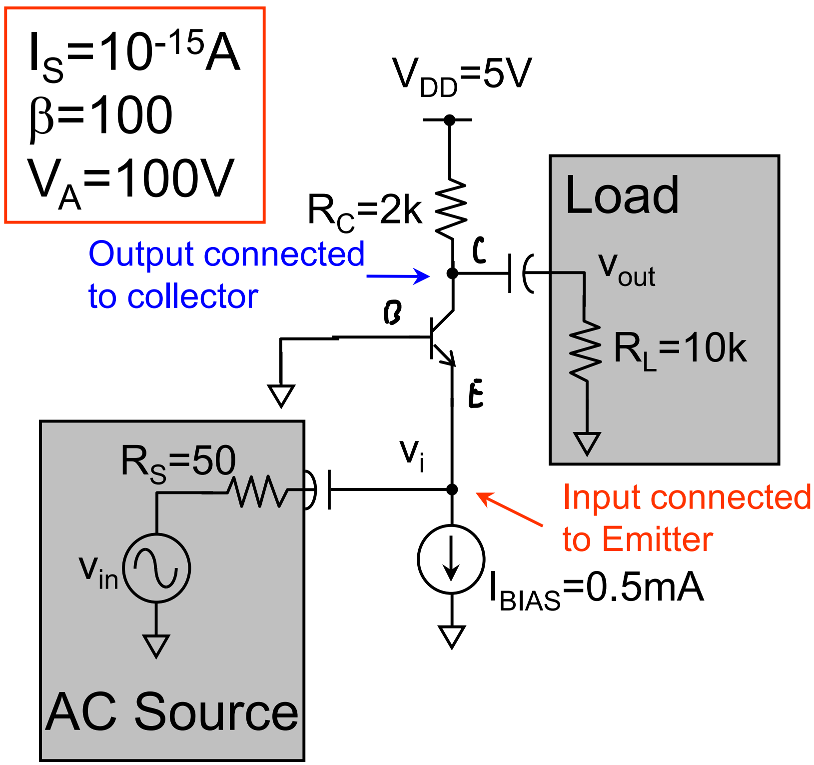
R_x is too big

$$\approx \left(g_m - g_{mb}\right) \times v_i \times \left(R_C // R_L\right)$$

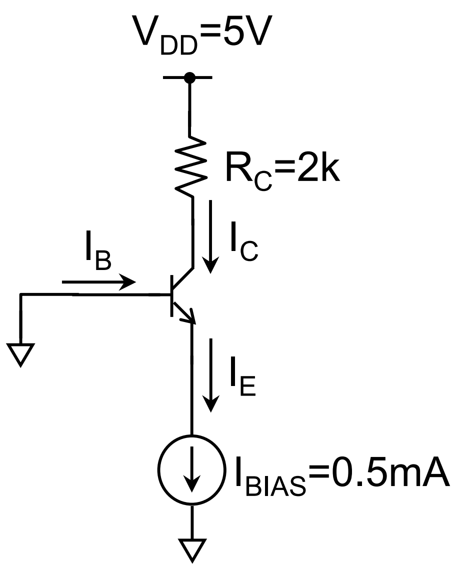
$$\approx \left(g_m - g_{mb}\right) \times \left(v_{in} \times \frac{R_{in}}{R_S + R_{in}}\right) \times \left(R_C // R_L\right)$$

$$\Rightarrow A_V = \frac{v_{out}}{v_{in}} = \frac{R_{in}}{R_S + R_{in}}\left(g_m - g_{mb}\right)\left(R_C // R_L\right)$$
$$= 5.01$$

Common Base (CB) Amplifier



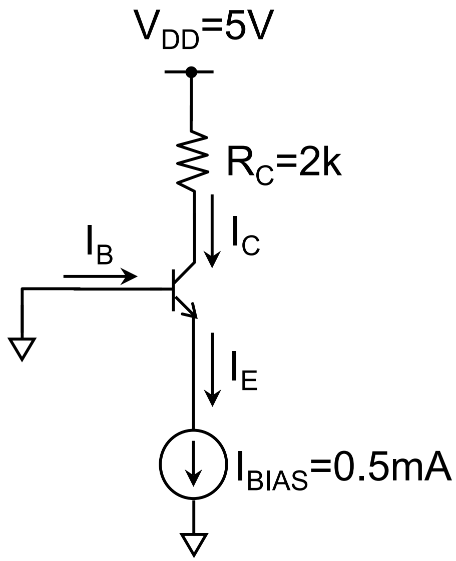
For DC analysis, remove AC source and open circuit capacitors



- Identify AC Source and Load
- To identify amplifier configuration, we need to consider AC equivalent circuit, i.e., short circuit capacitors
  - Input connected to Emitter, output connected to Collector, Base connected to neither input nor output ⇒ Common Base (CB)

DC Analysis for CB

- Remove AC source and open circuit capacitors when doing DC analysis



$I_S = 10^{-15}$A
$\beta = 100$
$V_A = 100$V

Determine DC biasing

$$V_T = \frac{kT}{q} = 26\,\text{mV}$$

$$I_E = I_{BIAS} = 0.5\,\text{mA}$$

$$I_C = \frac{\beta}{\beta+1} I_E = 0.495\,\text{mA}$$

$$I_B = \frac{I_C}{\beta} = 4.95\,\mu\text{A}$$

Determine AC small-signal parameters

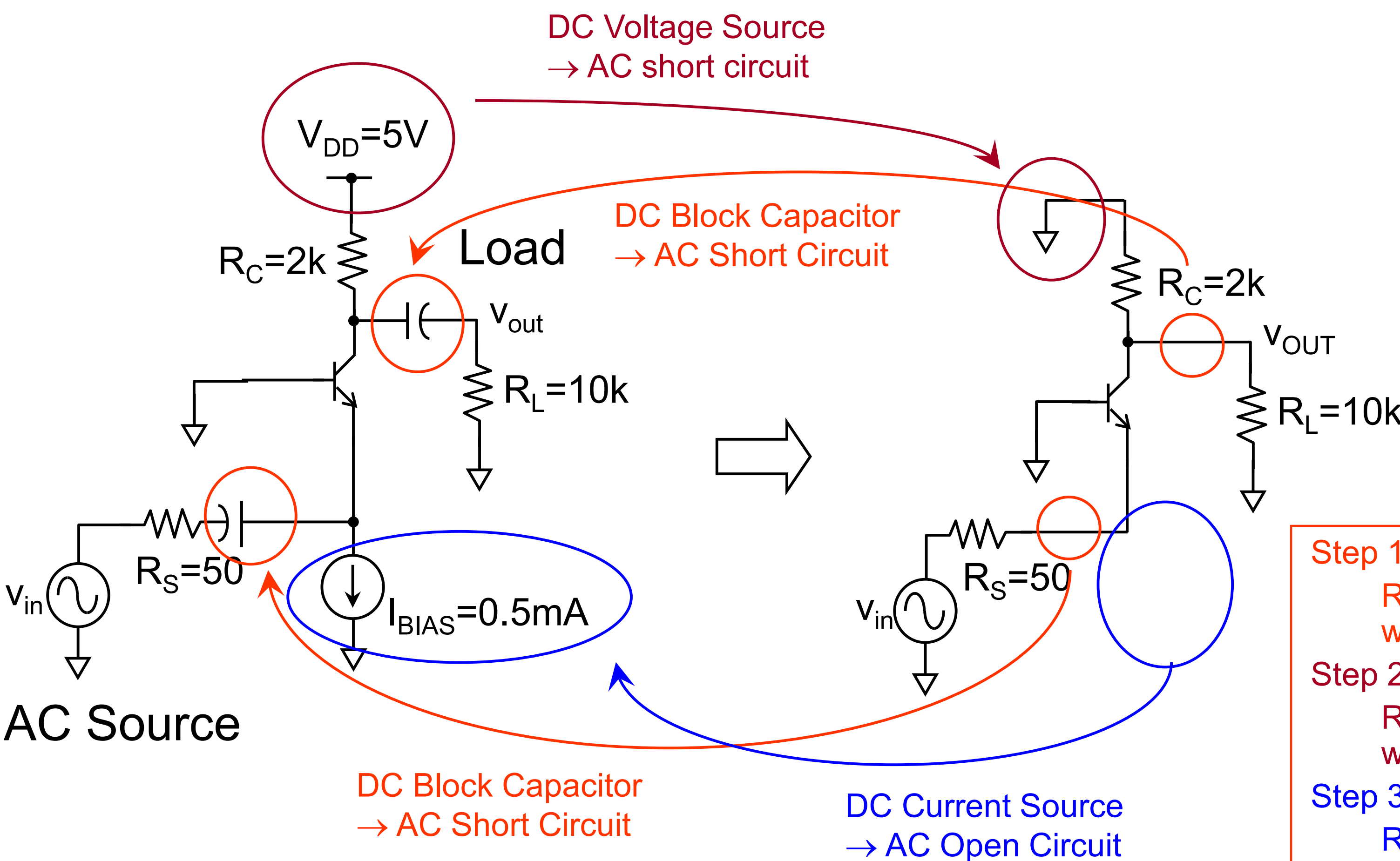
$$g_m = \frac{I_C}{V_T} = 19\,\text{mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = 5.26\,\text{k}\Omega$$

$$r_o = \frac{V_A}{I_C} = 202\,\text{k}\Omega$$

AC Analysis for CB (Self Reading)

For AC analysis, need to consider the AC equivalent circuit -



Step 1 :

Replace DC block capacitor with AC short circuit

Step 2 :

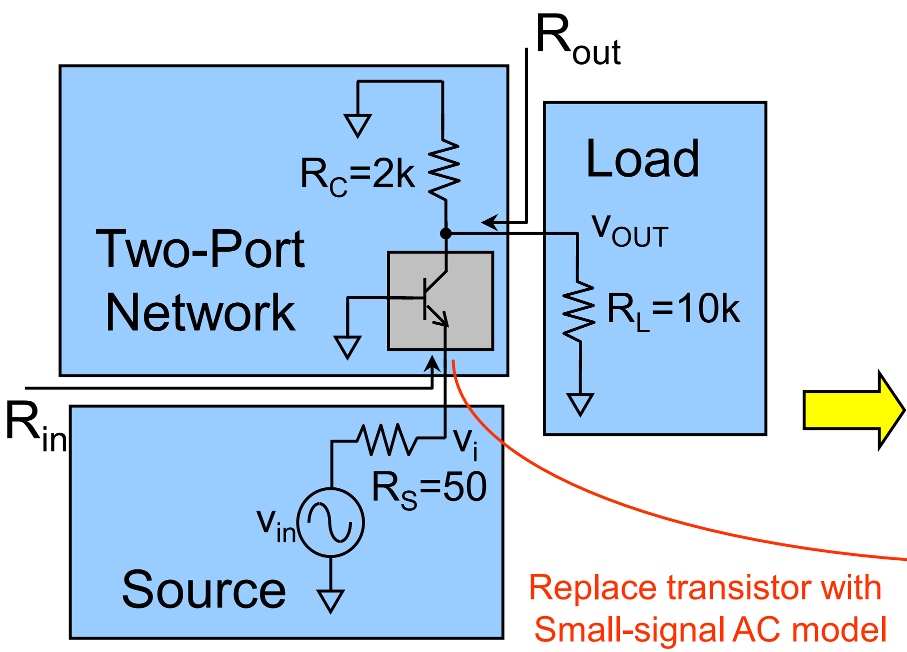
Replace DC voltage source with AC short circuit

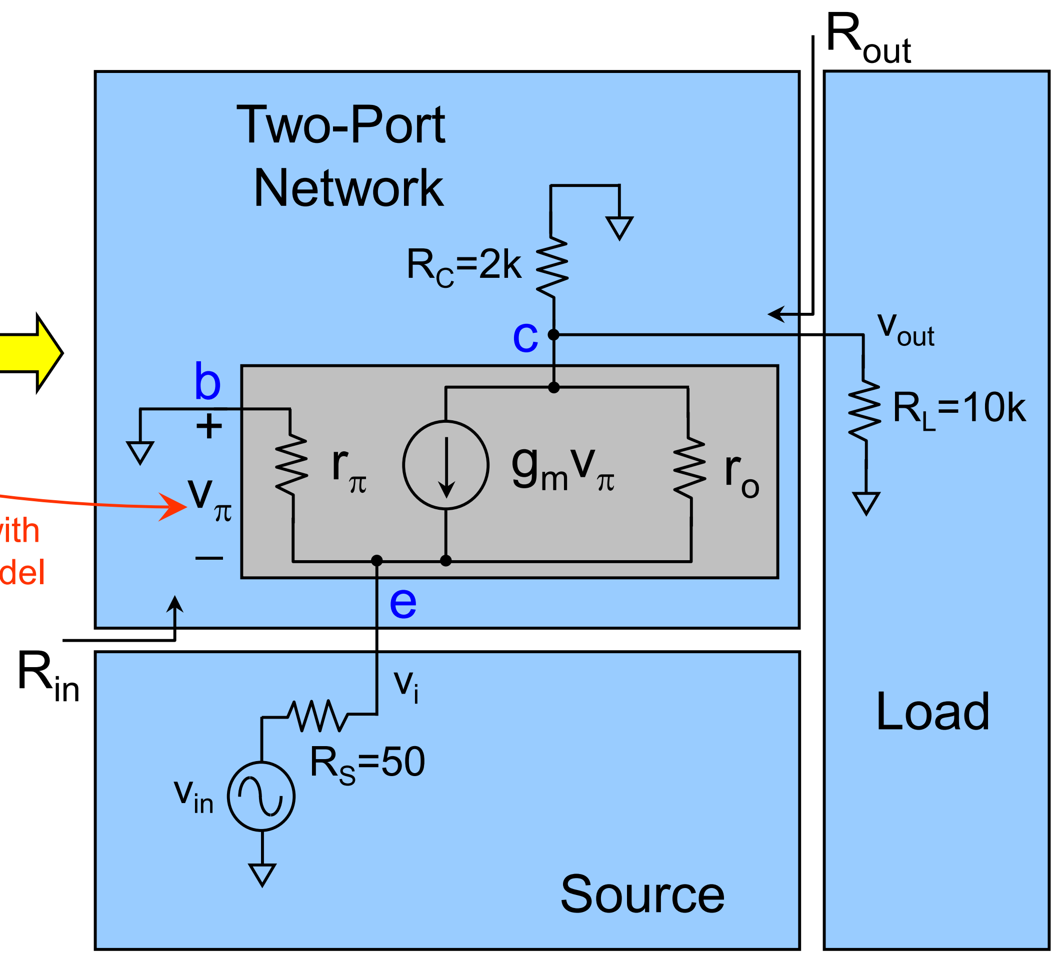
Step 3 :

Replace DC current source with AC open circuit

Continue the analysis with AC equivalent circuit on the right.

AC Analysis for CB (Self Reading)





Step 1 :

Identify source/load

Step 2 :

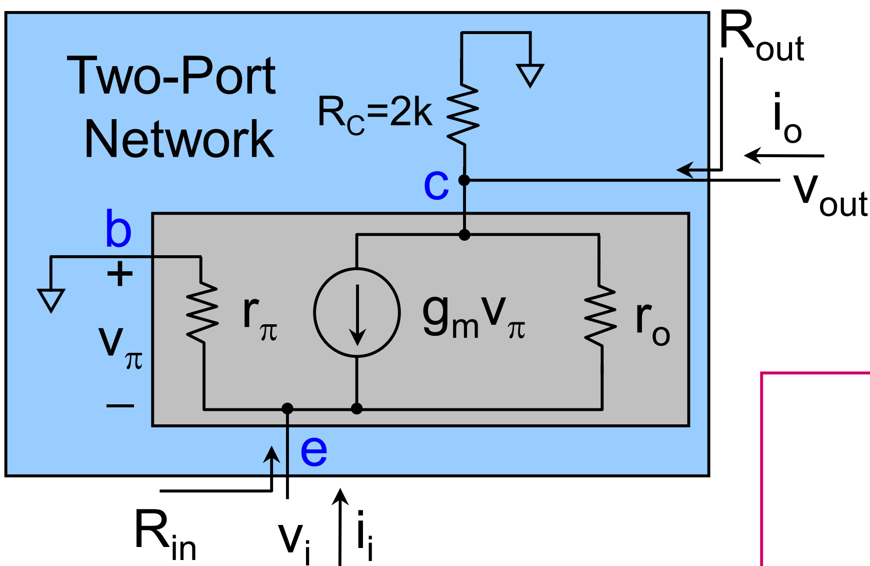
Group the remaining components (the amplifier circuit) into two-port network

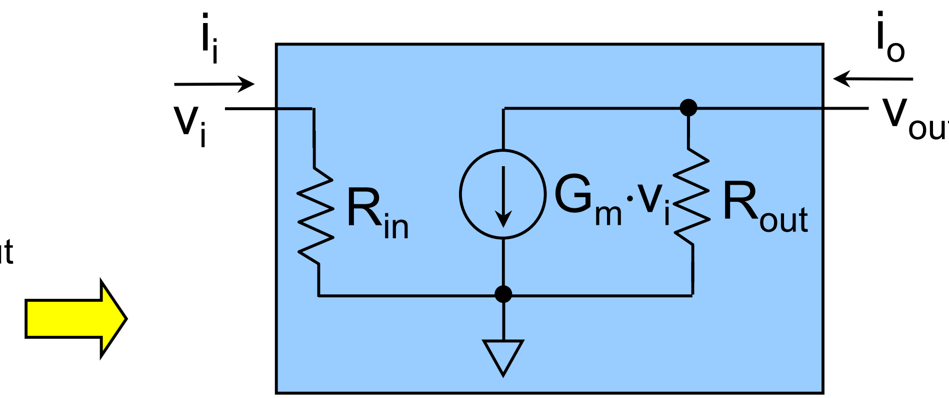
Step 3 :

Replace transistor with small signal AC model

For CB amplifier, transform to 2-port Transconductance Amplifier (based on experience of circuit designers). Need to evaluate G_m, R_{in} and R_{out}.

Mapping Two-Port Network for CB (Self Reading)



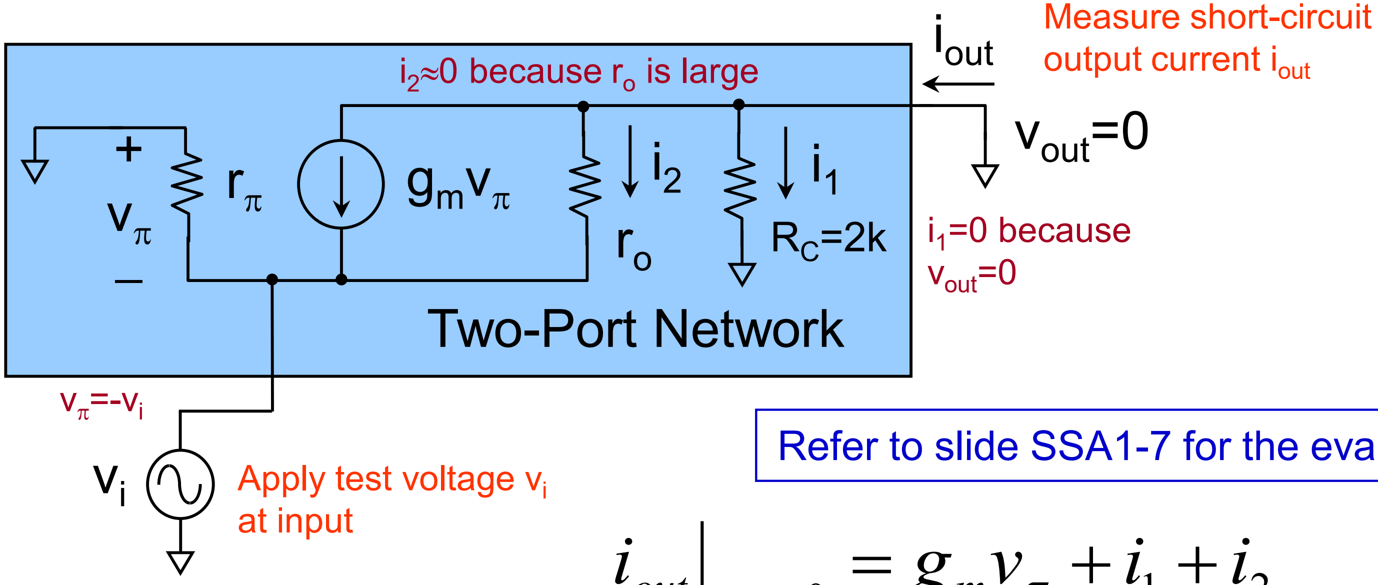


$$G_m = \left.\frac{i_o}{v_i}\right|_{v_{out}=0} \qquad R_{in} = \frac{v_i}{i_i} \qquad R_{out} = \frac{v_{out}}{i_o}$$

Refer to slide SSA1-7 for evaluation of 2-port parameters: G_m, R_{in}, R_{out}.

- Similar concept as Thevenin equivalent.
- Complicated two-port network can be represented by a two-port network characterized by only 3 parameters: R_{in}, R_{out} and G_m.

CB – Finding G_m (Self Reading)



Refer to slide SSA1-7 for the evaluation of G_m.

$$i_{out}\Big|_{v_{out}=0} = g_m v_\pi + i_1 + i_2$$

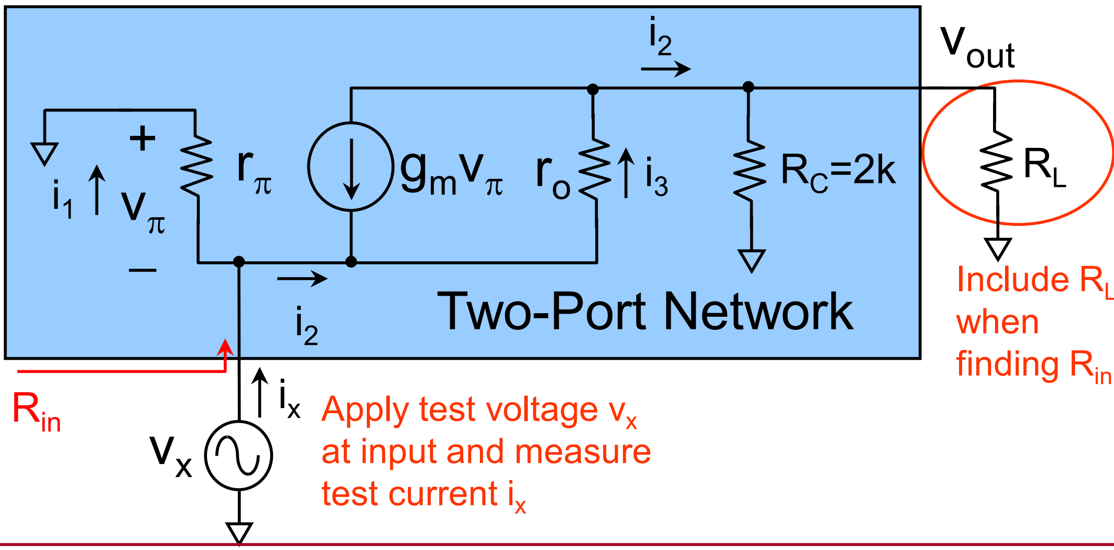
$$\approx -g_m v_i$$

$$G_m = \frac{i_{out}}{v_i}\Bigg|_{v_{out}=0} \approx -g_m = -19\,\mathrm{mA/V}$$

Important Result :
Transconductance (G_m) for CB is just $-g_m$

CB – Finding R_{in} (Self Reading)

Refer to slide SSA1-7 for the evaluation of R_{in}.



$$v_\pi = -v_x \quad i_1 = \frac{v_x}{r_\pi} \quad i_3 = \frac{v_x - v_{out}}{r_o}$$

$$i_2 = i_x - i_1 = i_x - \frac{v_x}{r_\pi}$$

KCL

Eliminate v_{out} and keep v_x and i_x

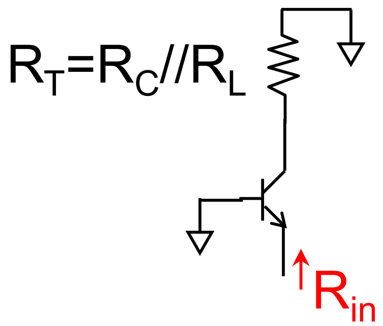
$$R_{in} = \frac{v_x}{i_x} \quad \begin{cases} i_x = i_1 - g_m v_\pi + i_3 = \dfrac{v_x}{r_\pi} + g_m v_x + \dfrac{v_x - v_{out}}{r_o} \\ v_{out} = i_2\left(R_C // R_L\right) = \left(i_x - \dfrac{v_x}{r_\pi}\right)\left(R_C // R_L\right) \end{cases}$$

$$\Rightarrow i_x = \frac{v_x}{r_\pi} + g_m v_x + \frac{v_x}{r_o} - i_x \frac{\left(R_C // R_L\right)}{r_o} + \frac{v_x\left(R_C // R_L\right)}{r_\pi r_o}$$

g_m>>$1/r_\pi$ and $1/r_o$

$$\Rightarrow R_{in} = \frac{1 + \dfrac{\left(R_C // R_L\right)}{r_o}}{g_m + \cancel{\dfrac{1}{r_\pi}} + \cancel{\dfrac{1}{r_o}} + \dfrac{\left(R_C // R_L\right)}{r_\pi r_o}} \approx \frac{1}{g_m} \cdot \frac{r_o + \left(R_C // R_L\right)}{r_o + \dfrac{\left(R_C // R_L\right)}{\beta}}$$

CB – Finding R_{in} (Self Reading)



$$R_{in} \approx \frac{1}{g_m} \cdot \frac{r_o + R_T}{r_o + \frac{R_T}{\beta}}$$

$$\approx \frac{1}{g_m} \quad \left[If \quad R_T << r_o \right]$$

Important Result:
If you see a BJT connected in similar fashion, the equivalent resistance looking into the emitter (R_{in}) is directly given by the formula. **No need to rederive.**

Example :

$$R_C = 2\text{k} \quad R_L = 10\text{k} \quad r_o = 202\text{k}$$

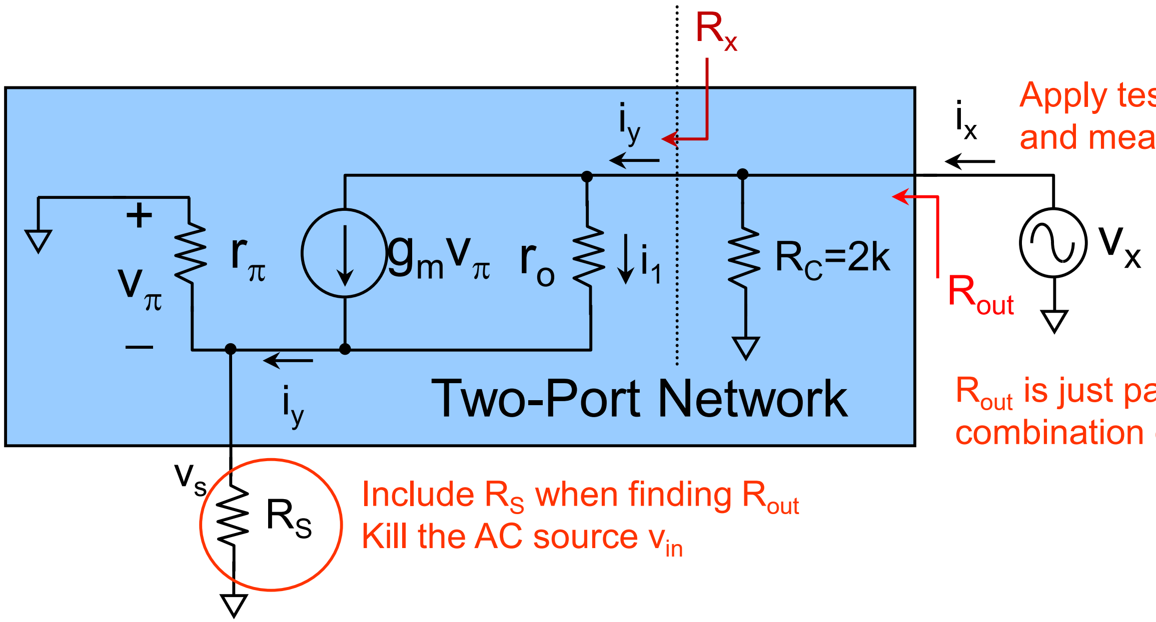
$$\Rightarrow R_T = (R_C // R_L) << r_o$$

$$\Rightarrow R_{in} = \frac{1}{g_m} = 53$$

- If $R_C//R_L$ is negligible compared to r_o, the input resistance (R_{in}) reduced to the inverse of the transconductance ($1/g_m$)

CB – Finding R_{out} (Self Reading)

Refer to slide SSA1-7 for the evaluation of R_{out}.



Apply test voltage v_x at output and measure test current i_x

$$v_\pi = -v_s$$
$$i_1 = i_y - g_m v_\pi = i_y + g_m v_s$$

R_{out} is just parallel combination of R_C and R_x

Include R_S when finding R_{out}

Kill the AC source v_{in}

$r_\pi // R_S << r_o[1+g_m(r_\pi // R_S)]$

$$R_{out} = R_C // R_x$$

$$R_x = \frac{v_x}{i_y}$$

Eliminate v_s and keep v_x and i_y

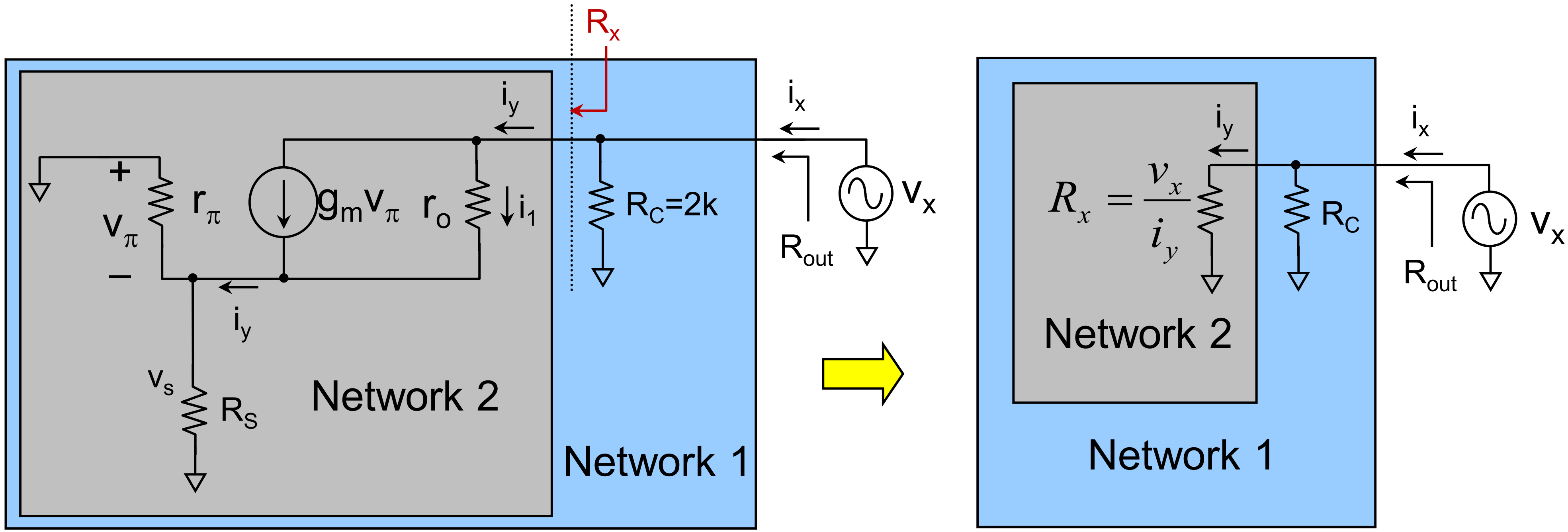
$$\begin{cases} v_s = i_y(r_\pi // R_S) \\ v_x = v_s + i_1 r_o = v_s + (i_y + g_m v_s) r_o \end{cases}$$

$$\Rightarrow v_x = i_y(r_\pi // R_S) + i_y r_o + i_y g_m (r_\pi // R_S) r_o$$

$$\Rightarrow R_x = \frac{v_x}{i_y} = \cancel{(r_\pi // R_S)} + r_o[1 + g_m(r_\pi // R_S)]$$

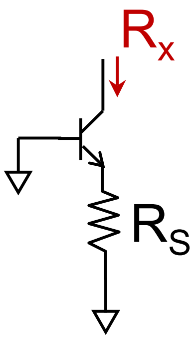
$$R_{out} = R_C // R_x$$
$$\approx R_C // \{r_o[1 + g_m(r_\pi // R_S)]\} \approx R_C$$

Equivalent Resistance (Self Reading)



- A complicated linear network (gray box) can be replaced by an equivalent resistance, R_x

CB – Finding R_{out} (Self Reading)



$$R_x \approx r_o\left[1+g_m\left(r_\pi // R_S\right)\right]$$

$$If \quad R_S << r_\pi \Rightarrow R_x = r_o\left(1+g_m R_S\right)$$

$$If \quad R_S >> r_\pi \Rightarrow R_x = r_o\left(1+g_m r_\pi\right)$$

$$= r_o\left(1+\beta\right)$$

Important Result:

If you see a BJT connected in similar fashion, the **equivalent resistance** looking into the **collector** (R_x) is directly given by the formula. **No need to rederive.**

Example :

$$R_S = 50 \quad r_\pi = 5.26\text{k}$$

$$r_o = 202\text{k} \quad g_m = 19\text{m}$$

$$\Rightarrow R_x \approx 394\text{k} >> R_C$$

$$\Rightarrow R_{out} \approx R_C = 2\text{k}$$

- Emitter side resistor helps boost up the output resistance of the transistor (R_x)
- For BJT, the maximum achievable boost up is $(1+\beta)r_o$

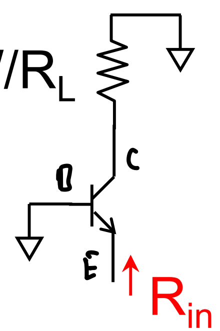
CB – Important Results

1

Important Result :
Transconductance (G_m) for CB is just $-g_m$

2

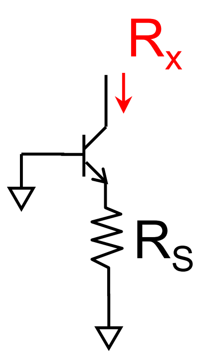
$R_T = R_C // R_L$



$$\boxed{R_{in} \approx \frac{1}{g_m} \cdot \frac{r_o + R_T}{r_o + \dfrac{R_T}{\beta}}}$$

$$\approx \frac{1}{g_m} \quad \left[If \quad R_T << r_o \right]$$

3



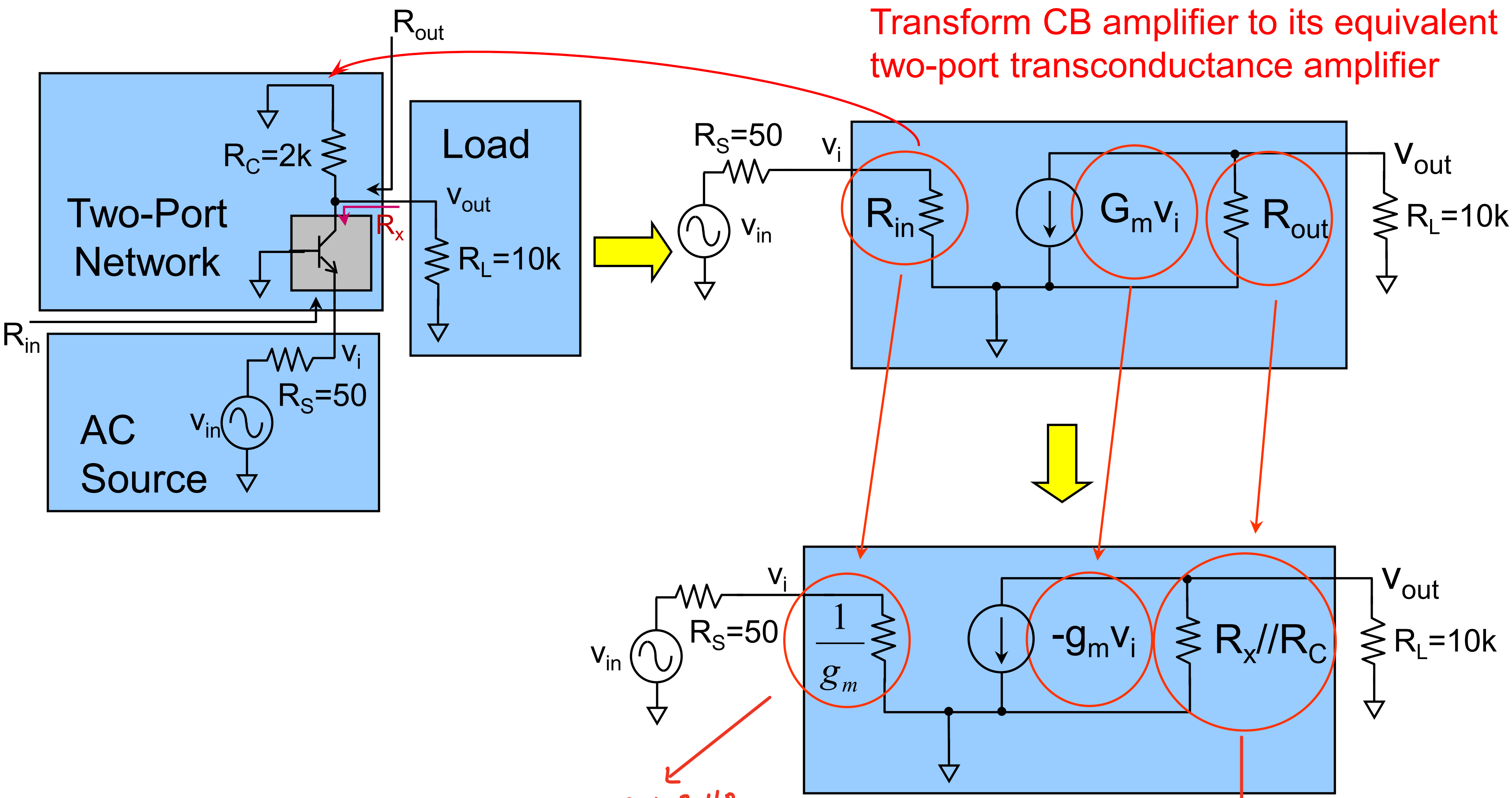
$$R_x \approx r_o \left[1 + g_m \left(r_\pi // R_S \right) \right]$$ use this

$$If \quad R_S << r_\pi \Rightarrow R_x = r_o \left(1 + g_m R_S \right)$$

$$If \quad R_S >> r_\pi \Rightarrow R_x = r_o \left(1 + g_m r_\pi \right)$$

$$= r_o \left(1 + \beta \right)$$

CB – Two-Port Network



$$\frac{1}{g_m}\cdot\frac{r_o + R_C//R_L}{r_o + \frac{R_C//R_L}{\beta}} \Rightarrow r_o >> R_C//R_L$$

$$\approx \frac{1}{g_m}$$

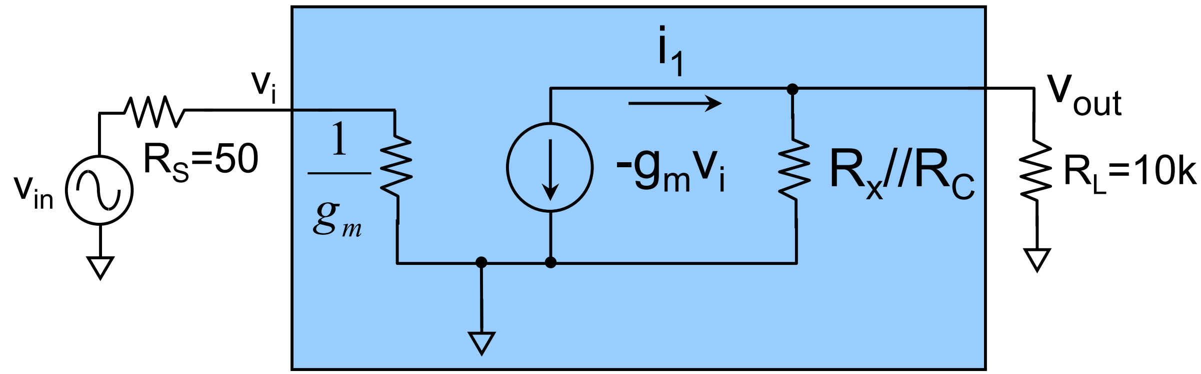
$$R_C // r_o\left[1+g_m(r_\pi // R_S)\right]$$

Since $r_o >> R_C$,

$$\approx R_C$$

CB – Two-Port Network (A_V)

g_m = 19 mA/V
r_π = 5.26 kΩ
r_o = 202 kΩ



$$v_i = v_{in} \times \frac{R_{in}}{R_S + R_{in}}$$

$$R_{in} = \frac{1}{g_m}$$

$$i_1 = -\left(-g_m v_i\right) = g_m v_i$$

$$v_{out} = i_1 \times \left[\left(R_x // R_C\right) // R_L\right]$$

$$= g_m \times v_i \times \left[\left(R_x // R_C\right) // R_L\right]$$

R_x is too big

$$\approx g_m \times \left(v_{in} \times \frac{R_{in}}{R_S + R_{in}}\right) \times \left(R_C // R_L\right)$$

$$\Rightarrow A_V = \frac{v_{out}}{v_{in}} \approx \frac{R_{in}}{R_S + R_{in}} g_m \left(R_C // R_L\right) = 16.3$$

Characteristics of CB/CG

- Low input resistance
- High output resistance
- Medium gain
- No polarity inversion
- The higher the G_m and the total output resistance, the higher the gain (A_V)
- BJT provides larger g_m than MOS (g_m-g_{mb})