NATIONAL UNIVERSITY OF SINGAPORE
Department of Electrical and Computer Engineering

EE2027 Electronic Circuits
Tutorial 4: Solution

- Unless otherwise stated, you may assume temperature, $T = 300$ K; thermal voltage, $V_T = 0.025$ V and make use of the equations given in the lecture notes directly, without having to derive them.
- All the symbols are as defined in the lecture notes.

Homework 4:
Homework 4 is Questions 5 of Tutorial 4. You will need to submit a softcopy of your handwritten homework to Canvas>Homework Submissions>HW4 half an hour after class (i.e., latest by 4:30 pm) on Tuesday, 16 April 2024. Failing to do that will mean zero mark for homework.

The softcopy submission of your homework must be in PDF format (in a single file) and named using the convention "<Your Name>_HW4".

Homework questions will not be discussed in class.

Q1. Figure Q1 shows a single-stage BJT amplifier circuit. The BJT Q_1 has $\beta = 100$.

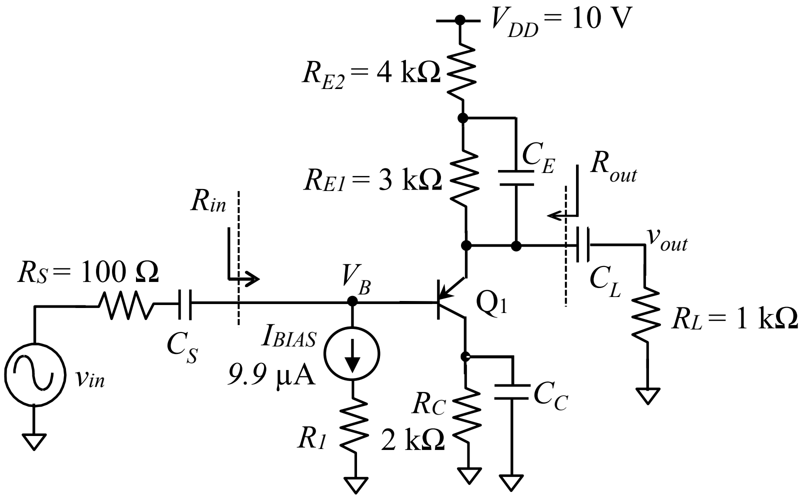


Fig. Q1

(a) Find the DC base voltage of Q_1 (V_B) and R_1. Verify any assumption you make in the calculation. (Note: The voltage drop across the DC current source, I_{BIAS}, has been designed to be approximately 0 V.) [Ans: $V_B = 2.3$ V, $R_1 = 232$ kΩ]

(b) Draw the small-signal equivalent of the circuit in Fig. Q1 and identify the amplifier configuration.

(c) Calculate the two-port network parameters of the amplifier, and draw the equivalent two-port network of the amplifier. Ans: [$R_{in} \approx 82.4$ kΩ, $R_{out} \approx 25.7$ Ω]

(d) Estimate the voltage gain, v_{out}/v_{in}. [Ans: $v_{out}/v_{in} \approx 0.97$]

Q1. Solution:

BJT Q_1 is of the pnp type.

(a) DC analysis:

Assume BJT Q_1 is working in ***forward active operation***, $V_{EB} \approx 0.7$ V.

$I_B = I_{BIAS} = 9.9$ μA; $I_E = (\beta + 1)I_B = 1$ mA.

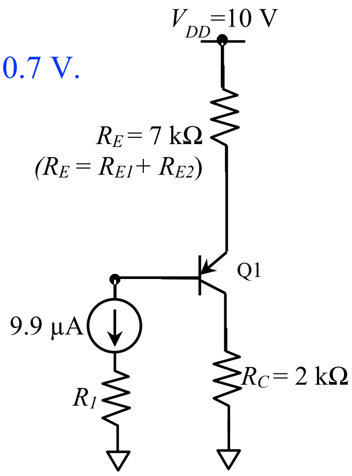
$V_E = V_{DD} - I_E R_E = 10 - 7 = 3$ V.

$V_B = V_E - 0.7 = 2.3$ V.

$R_1 = V_B / I_B = 232$ kΩ [Voltage drop across DC current source, I_{BIAS}, ≈ 0]

$V_C = I_C \times R_C = 990$ μA × 2 kΩ = 1.98 V.

$V_{BC} = V_B - V_C = 2.3 - 1.98 = 0.32$ V > 0.

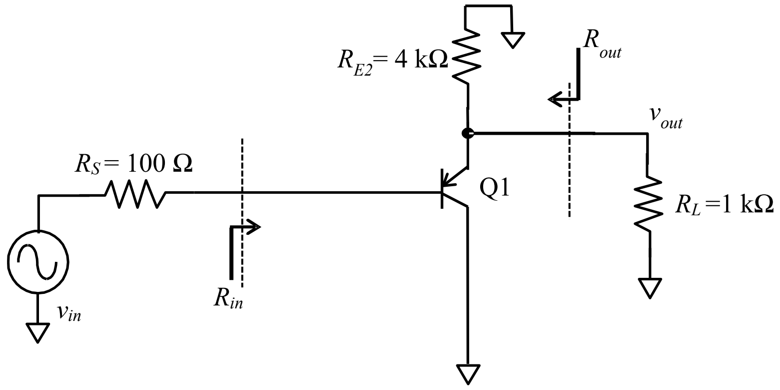


DC equivalent circuit

Check:
Base-emitter junction is in forward bias. Base-collector junction is in reverse bias. BJT Q1 is working in forward active operation.

(b) The amplifier configuration is Common Collector (CC).

The small-signal equivalent circuit is:



BJT Q_1 can also be replaced by its small-signal model in the above figure.

(c)

$$g_{m,Q1} = {}^{I_C}/_{V_T} = {}^{0.99\text{m}}/_{0.025} = 39.6\text{ mA/V} \text{ (or 40 mA/V assuming } I_C \approx I_E)$$

$$r_{\pi,Q1} = {}^{\beta}/_{g_{m,Q1}} = {}^{100}/_{39.6\text{m}} = 2.52\text{ k}\Omega \text{ (or 2.5 k}\Omega \text{ assuming } I_C \approx I_E)$$

$$r_{o,Q1} = {}^{V_A}/_{I_C} = \infty \text{ (}V_A \text{ is not given, by default, } V_A = \infty)$$

With the help of Table 1, Configuration A –

$$R_{in} = [r_{\pi,Q1}(1 + g_{m,Q1}\{R_{E2}//R_L\})] = [2.52\text{k} \times (1 + 39.6\text{m} \times \{4\text{k}//1\text{k}\})] = 82.4\text{ k}\Omega$$

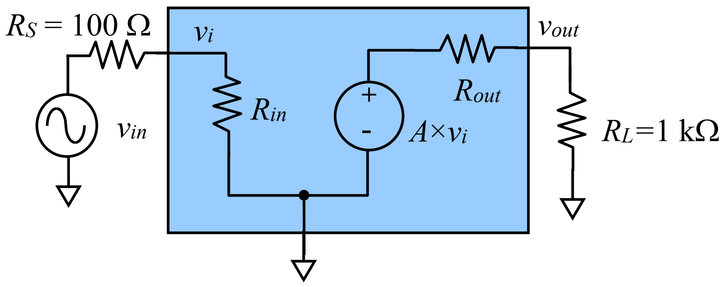
With the help of Table 1, Configuration C –

$$R_{out} = R_{E2}//\left[(R_S + r_{\pi,Q1})/(1 + \beta_{Q1})]//r_{o,Q1}\right] = 4\text{k}//[(0.1\text{k} + 2.52\text{k})/(1 + 100)]//\infty] = 25.7\ \Omega$$

With the help of Table 3, Configuration C –

$$A_v = \frac{v_{out}}{v_i} = \left[\frac{g_{m,Q1}(R_{E2}//R_L)}{1 + g_{m,Q1}(R_{E2}//R_L)}\right] = \left[\frac{39.6\text{m}(4\text{k}//1\text{k})}{1 + 39.6\text{m}(4\text{k}//1\text{k})}\right] = 0.97$$

The 2-port network of the amplifier (Voltage amplifier) is as follows -



Note: $A_v = \frac{v_{out}}{v_i}$ is not A in the 2-port network equivalent Voltage Amplifier (see figure above).

$$v_{out} = \frac{R_L}{R_L + R_{out}} \times Av_i$$

$$\Longrightarrow A = \frac{v_{out}}{v_i} \times \frac{R_L + R_{out}}{R_L} = A_v \times \frac{R_L + R_{out}}{R_L} = 1.067$$

(d) Estimate the voltage gain v_{out}/v_{in}.

With the help of Table 3, configuration C –

$$\frac{v_{out}}{v_{in}} = \frac{v_i}{v_{in}} \times \frac{v_{out}}{v_i} = \frac{R_{in}}{R_{in}+R_s}\left\{\frac{g_{m,Q1}(R_{E2}//R_L)}{1+g_{m,Q1}(R_{E2}//R_L)}\right\} \approx \frac{82.4\text{k}}{0.1+82.4\text{k}} \times \frac{39.6m\times(4\text{k}//1\text{k})}{1+39.6m\times(4\text{k}//1\text{k})} = 0.97$$

Q2. Consider the amplifier shown in Fig. Q2. The npn bipolar junction transistor has the following parameter values: $\beta = 100$, $V_A = 100$ V.

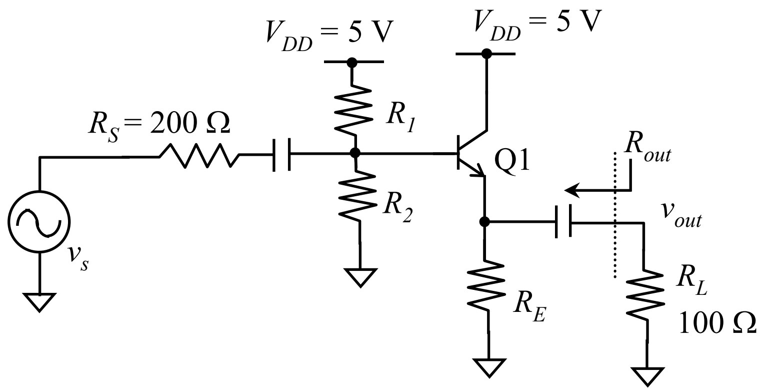


Fig. Q2

(a) For the maximum power transfer to load, R_L, what should R_{out} be?

[Ans: $R_{out} = 100\ \Omega$]

(b) To have the desired R_{out} of part (a), what should the specified $g_{m,Q1}$ be? (You may need to make some assumption for resistors R_1, R_2 and R_E)

[Ans: $g_{m,Q1} = 10$ mA/V]

(c) What is the amplifier configuration? Sketch the equivalent 2-port network of the amplifier circuit in Fig. Q2.

(d) Set $R_1 = 100$ kΩ and $R_E = 5$ kΩ, calculate R_2 to complete the design.

[Ans: $R_2 = 64$ kΩ]

(e) Calculate the input resistance of the amplifier, R_{in}.

[Ans: $R_{in} \approx 13.2$ kΩ]

(f) Calculate the voltage gain v_{out}/v_s.

[Ans: $v_{out}/v_s \approx 0.49$]

Q2. Solution:

(a) For the maximum power transfer to load, R_L, R_{out} must be designed to have the same value as R_L, so $R_{out} = 100\ \Omega$.

(b) $R_{out} = R_E \,//\, R_X$, where R_X is the equivalent resistance looking into the emitter of Q_1.

Resultant resistance connected to the base of Q_1 is $R_S//R_1//R_2 \approx R_S$ (since $R_S = 200\ \Omega$, which is small, we can assume $R_S << R_1, R_2$).

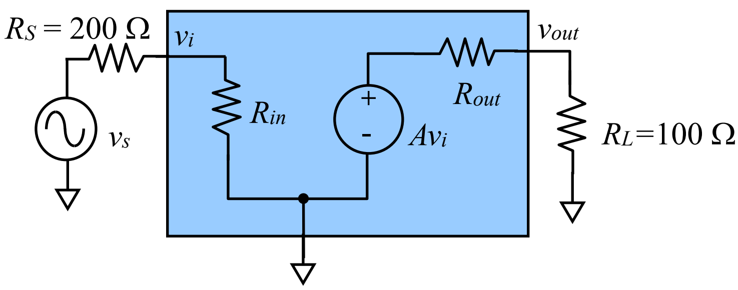
From Table 1, look into emitter, Configuration C –

$$R_X \approx \frac{R_S // R_1 // R_2}{\beta + 1} + \frac{1}{g_{m,Q1}} \approx \frac{1}{g_{m,Q1}} \quad [if \;\; R_S \;\; is \;\; small]$$

To have the desired $R_{out} = 100\ \Omega$, we can assume $R_E >> \dfrac{1}{g_{m,Q1}}$ $\Rightarrow R_{out} = R_E \,//\, R_X \approx$

$\dfrac{1}{g_{m,Q1}} = 100\ \Omega \;\Rightarrow g_{m,Q1} = 10$ mA/V

(c) The amplifier configuration is Common Collector (CC) and its equivalent 2-port network is as follows -



$$R_{out} = R_L \approx \frac{1}{g_{m,Q1}} = 100\ \Omega$$

(d) Need to find base voltage of BJT Q_1.

Since $R_E = 5$ kΩ $>> \dfrac{1}{g_{m,Q1}}$ [assumption made earlier in part (b) is valid], $R_{out} = 100\ \Omega$,

then $g_{m,Q1} = 10$ mA/V .

$$g_{m,Q1} = \frac{I_{C,Q1}}{V_T} \Rightarrow I_{C,Q1} = g_{m,Q1}V_T = 0.25 \text{ mA} \approx I_{E,Q1}$$

$$I_{E,Q1} = \frac{V_E}{R_E} \Rightarrow V_E = 1.25 \text{ V}$$

$$V_B = V_E + V_{BE} \approx V_E + 0.7 = 1.95 \text{ V}$$

Using the voltage divider method, base voltage of BJT Q1 -

$V_B = \dfrac{5R_2}{R_1 + R_2}$, Since $R_1 = 100$ kΩ, then $R_2 = 64$ kΩ. [Note: R_1, $R_2 >> R_S = 200$ Ω]

$I_1 \approx I_2 = \dfrac{5}{100k + 64k} = 30\,\mu\text{A} >> I_B = 2.5\,\mu\text{A} \Rightarrow$ Voltage divider method is valid.

(e)

$$g_{m,Q1} = 10 \text{ mA/V} \Rightarrow I_{C,Q1} = g_{m,Q1}V_T = 0.25 \text{ mA}$$

$$r_{o,Q1} = {V_{A,Q1}}/{I_{C,Q1}} = 400 \text{ k}\Omega$$

$$r_{\pi,Q1} = {\beta_{Q1}}/{g_{m,Q1}} = 10 \text{ k}\Omega$$

$$R_{out} \approx \frac{1}{g_{m,Q1}} = 100\ \Omega$$

With the help of Table 1, Configuration A –

$$R_{in} = R_1//R_2//[r_{\pi,Q1}(1 + g_{m,Q1}\{R_E//R_L\})]$$

$$=100\text{k}//64\text{k}//[10\text{k} \times (1 + 10\text{m} \times \{5\text{k}//0.1\text{k}\})] = 13.2 \text{ k}\Omega$$

(f) Calculate the voltage gain, v_{out} / v_S.

With the help of Table 3, configuration C –

$$A_v = \frac{v_{out}}{v_s} = \frac{v_i}{v_s} \times \frac{v_{out}}{v_i}$$

$$= \frac{R_{in}}{R_{in}+R_s}\left\{\frac{g_{m,Q1}(R_E//R_L)}{1+g_{m,Q1}(R_E//R_L)}\right\} \approx \frac{13.2}{0.2+13.2} \times \frac{10m\times0.098k}{1+10m\times0.098k} = 0.49$$

Q3. Figure Q3 shows a single-stage amplifier with current mirror biasing. For the two *p*-channel MOSFETs, $V_{THP1} = V_{THP2} = -1$ V, $\lambda_{P1} = \lambda_{P2} = 0$, $K_{p,M1} = K_{p,M2} = 2$ mAV^{-2}. For the pnp BJT, $\beta = 100$, $V_A = 100$ V.

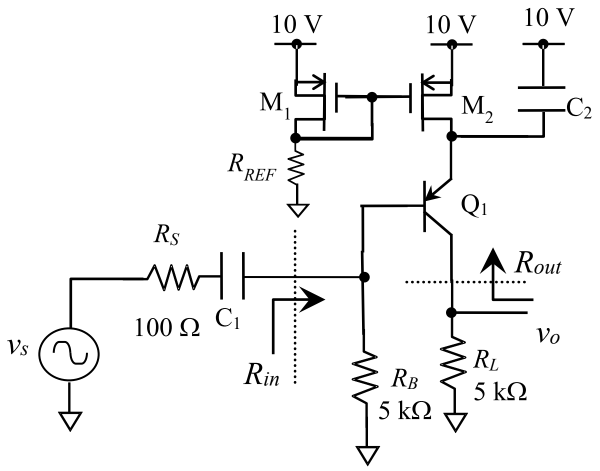


Fig. Q3

(a) Design R_{REF} such that $I_{D,M1} = 1$ mA. [Ans: $R_{REF} = 8.29$ kΩ]

(b) Calculate the small signal parameters of Q1, i.e., $g_{m,Q1}$, $r_{\pi,Q1}$ and $r_{o,Q1}$.

(c) Identify the configuration of the single-stage amplifier.

(d) Determine the parameters, G_m, R_{in} and R_{out} of the two-port network of the amplifier.

(e) Determine the gain v_o/v_s of the amplifier using the 2-port network parameters derived in part (d). [Ans: $v_o/v_s \approx -180$]

Q3. Solution:

(a) Note: M_1 and M_2 form a current mirror current source and they do not provide amplification function.

$V_G = V_D$ for $M_1 \Rightarrow M_1$ in saturation operation.

$$V_S = 10\,\text{V}; I_{D,M1} = K_p\left(|V_{GS}| - |V_{THP}|\right)^2$$

$$1\,\text{mA} = 2\,\text{mAV}^{-2}\left(10 - 1\,\text{mA} \times R_{REF} - 1\right)^2$$

$$R_{REF} = 9 \pm 0.707 = 9.707 \ \text{ or } 8.293\,\text{k}\Omega$$

$$R_{REF} = 9.707\,\text{k}\Omega\ , V_G = 9.707\,\text{V}, |V_{GS}| = 0.293\,\text{V} < |V_{THP}|, \text{ invalid.}$$
$$\therefore R_{REF} = 8.293\,\text{k}\Omega$$

(b)

$$I_C \approx I_E = I_{D,M2} = I_{D,M1} = 1\,\text{mA}$$
$$g_m = \frac{I_C}{V_T} = \frac{1\text{m}}{0.025} = 40\,\text{mA/V}$$
$$r_\pi = \frac{\beta}{g_m} = \frac{100}{40\text{m}} = 2.5\,\text{k}\Omega$$
$$r_o = \frac{V_A}{I_C} = \frac{100}{1\text{m}} = 100\,\text{k}\Omega$$

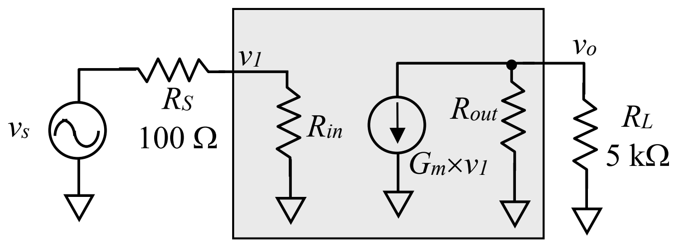
(c) The configuration of the single-stage amplifier is Common Emitter (CE).

(d) $G_m = g_m = 40$ mA/V (Table 3, CongfigurationA)

With the help of Table 1, Configuration A, with emitter shorted to gound
$R_{in} = R_B // r_\pi = 5\text{k}//2.5\text{k} = 1.67\ \text{k}\Omega$

With the help of Table 1, Configuration B, with emitter shorted to gound
$R_{out} = r_o = 100\ \text{k}\Omega$

(e) Determine the gain v_o/v_s of the amplifier using the 2-port network parameters derived in part (d).



$$A_V = \frac{v_o}{v_s} = \left(\frac{R_{in}}{R_S + R_{in}}\right) \times (-\,G_m) \times (R_{out} // R_L) = \frac{1.67\text{k}}{0.1\text{k}+1.67\text{k}} \times (-40\text{m}) \times \frac{100\text{k}\times 5\text{k}}{100\text{k}+5\text{k}} = -180$$

Q4. Consider the amplifier shown in Fig. Q4. The transistor M_1 has $K_p = 500\ \mu A/V^2$, $V_{THP} = -0.5$ V and $\lambda_p = 0.01\ V^{-1}$.

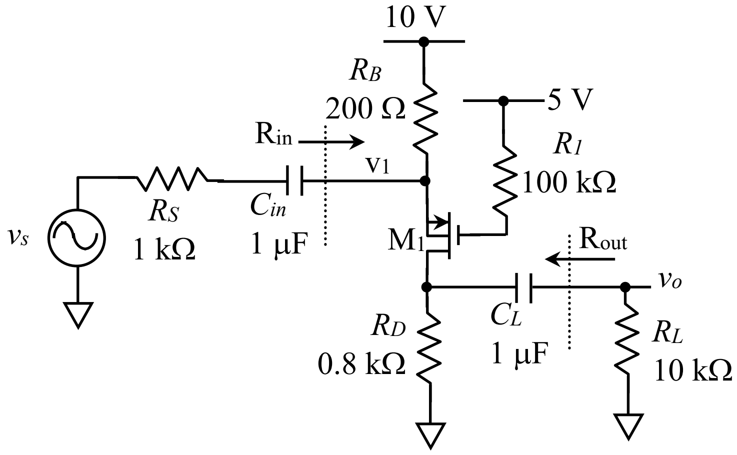


Fig. Q4

(a) What is the amplifier configuration?

(b) Determine the DC drain current of M_1, $I_{D,M1}$ and verify the assumption used.

[Ans: $I_{D,M1} = 5.65$ mA]

(c) What is the R_{in} for this amplifier? [Ans: $R_{in} \approx 120\ \Omega$]

(d) What is the R_{out} for this amplifier? [Ans: $R_{out} \approx 0.8\ \text{k}\ \Omega$]

(e) Use the concept of two-port network and the important results derived in the lecture notes, determine the voltage gain of this amplifier.

[Ans: gain ≈ 0.27]

Q4. Solution:

MOSFET M_1 is of the p-channel type.

(a) This amplifier configuration is **Common Gate (CG)**. Note that gate is effectively shorted to ground in AC operation as there is no current flowing through R_1 ($i_g = 0$)

(b) Determine DC drain current of M_1, $I_{D,M1}$.

Assume M_1 is operating in saturation region -

$$V_G = 5\text{ V (No Gate current} \Rightarrow V_G = 5\text{ V)}$$

$$I_{D,M1} = K_p\left(\left|V_{GS}\right| - \left|V_{THP}\right|\right)^2 = K_p\left(V_S - 5 - 0.5\right)^2 = \frac{10 - V_S}{R_B} \text{ [Square law and Ohm's law]}$$

$$\Rightarrow 0.1V_S^2 - 0.1V_S - 6.975 = 0$$

$$V_S = \frac{0.1 \pm \sqrt{0.01 + 2.79}}{0.2} = 8.87\text{ V } or\ -7.87\text{ V}$$

$$V_S \text{ cannot be negative} \Rightarrow V_S = 8.87\text{ V } and\ I_{D,M1} = 5.65\text{ mA}$$

Note: In the above calculation of DC drain current, $I_{D,M1}$, we can assume negligible Channel Length Modulation Effect as λ_p is small [See Tutorial 3, Q4(c)].

$$\left|V_{DS}\right| = \left|10 - I_{D,M1}R_B - I_{D,M1}R_D\right| = \left|10 - 5.65\text{m} \times 0.2\text{k} - 5.65\text{m} \times 0.8\text{k}\right| = 4.35\text{ V}$$

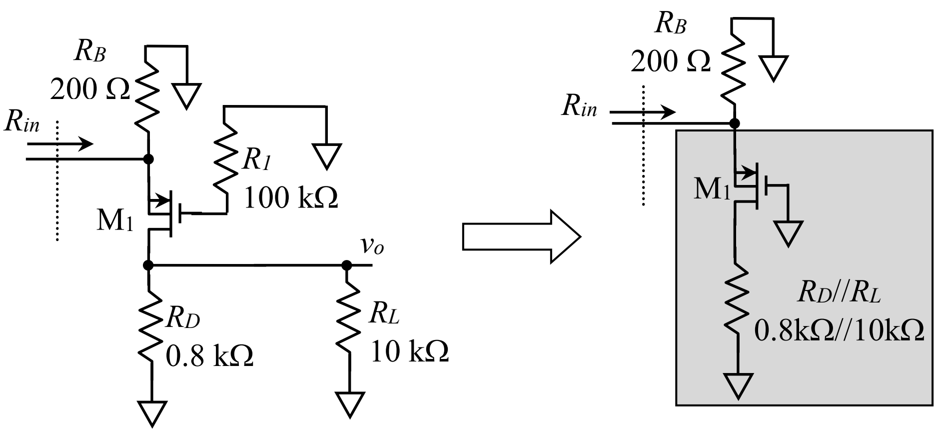
$$\left|V_{GS}\right| = \left|10 - I_{D,M1}R_B - 5\right| = \left|10 - 5.65\text{m} \times 0.2\text{k} - 5\right| = 3.87\text{ V}$$

Check whether the assumption of saturation operation is correct:

$$\left|V_{DSsat}\right| = \left|V_{GS}\right| - \left|V_{THP}\right| = 3.87 - 0.5 = 3.37\text{ V}$$

$$\Rightarrow \left|V_{DS}\right| > \left|V_{DSsat}\right|, \text{ hence } M_1 \text{ is in Saturation.}$$

(c)



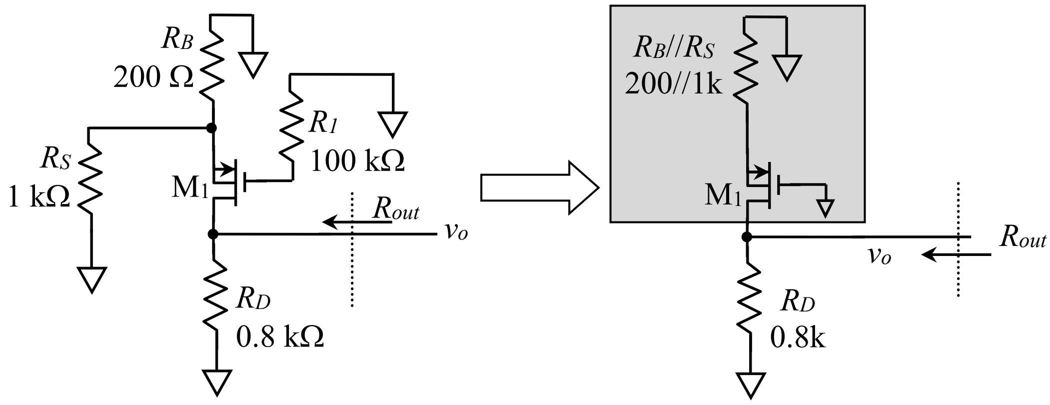
$$g_{m,M1} = 2\sqrt{K_P \left| I_{D,M1} \right|} = 2\sqrt{0.5\text{m} \times 5.65\text{m}} = 3.36\,\text{mA/V}$$

$$r_{o,M1} = \frac{1}{\lambda I_{D,M1}} = \frac{1}{0.01 \times 5.65\text{m}} = 17.7\,\text{k}\Omega$$

With the help of Table 2, Configuration D -

$$R_{in} = R_B // \left\{ \frac{1}{g_{m,M1}} \frac{r_{o,M1} + R_D//R_L}{r_{o,M1}} \right\} = 200//297 = 120\ \Omega$$

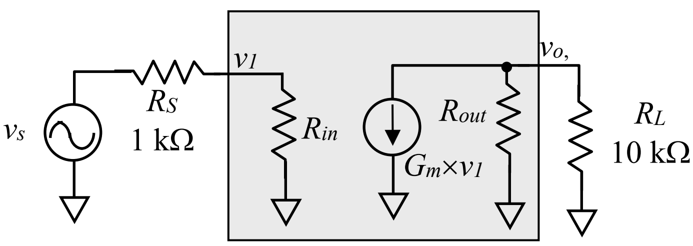
(d)



With the help of Table 2, Configuration B –

$$R_{out} = R_D // r_{o,M1}\{1 + g_{m,M1}(R_B//R_S)\} \approx 0.8\ \text{k}\Omega$$

(e) Common gate amplifier, $G_m = -g_{m,M1} = -3.36$ mA/V (Table 4, configuration B)



$$v_1 = \frac{R_{in}}{R_{in} + R_S} v_s = 0.107 v_s$$

$$v_o = -G_m v_1 (R_{out} // R_L) = 3.36\text{m} \times (0.8\text{k} // 10\text{k}) v_1 \approx 2.49 v_1 = 0.27 v_s$$

$$\frac{v_o}{v_s} = 0.27$$

Q5. In the amplifier circuit shown in Fig. Q5, the BJT Q_1 has $\beta = 100$ and $V_A = 100$ V. The MOSFETs, M_1 and M_2, are identical with $V_{TH} = -1.0$ V, $K_p = 2$ mA/V^2 and they do not experience Channel Length Modulation effect. The temperature is 300 K.

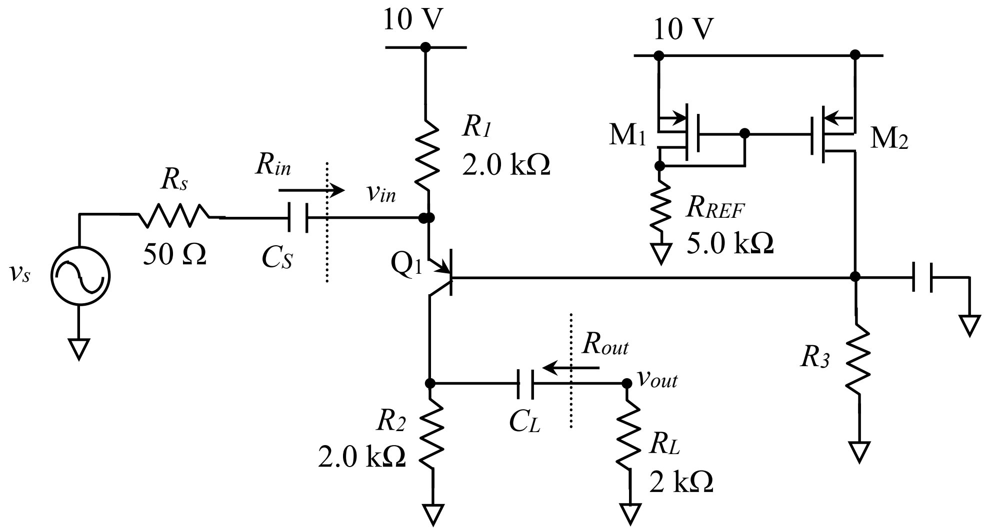


Fig. Q5

The BJT Q_1 operates in the forward active mode and its collector current, $I_{C,Q1} = 1.6$ mA. Ignore the Early effect of the BJT when performing DC analysis.

(a) Determine the drain current of the MOSFET M_2, $I_{D,M2}$.

(6 marks)

(b) Determine R_3 that will give rise to the desired $I_{C,Q1}$.

(4 marks)

(c) Determine the values of the 2-port network equivalent parameters of the amplifier: G_m (or A_v), R_{in} and R_{out}.

(8 marks)

(d) Determine the gain (v_{out}/v_s) of the amplifier.

(2 marks)

Q5. Solution:

M_1 and M_2 are identical p-MOSFETs and they form a current mirror -

- $V_{THP} = -1.0$ V
- $K_p = 2$ mA/V^2
- $\lambda_p = 0$ (no Channel Length Modulation effect)

BJT is of the pnp type, $\beta = 100$, $V_A = 100$ V

(a)

M_1 and M_2 form a current mirror $\Rightarrow I_{D,M2} = I_{D,M1}$. (1 mark)

$$I_{D,M1} = k_p\left(V_{SG,M1} - |V_{THP}|\right)^2$$
$$I_{D,M1} = k_p\left(10 - I_{D,M1} \times R_{REF} - |V_{THP}|\right)^2$$
$$I_{D,M1} = 2\text{m} \times \left(10 - I_{D,M1} \times 5.0\text{k} - 1.0\right)^2 \rightarrow 500 I_{D,M1} = \left(9 - 5.0\text{k} I_{D,M1}\right)^2$$
$$0 = 81 - 90.5\text{k} I_{D,M1} + 25.0\text{M} I_{D,M1}^2$$
$$I_{D,M1} = \frac{90.5\text{k} \pm \sqrt{90.25M}}{50.0\text{M}} = \frac{90.5\text{k} \pm 9.5\text{k}}{50.0\text{M}}$$
$$I_{D,M1} = \frac{81\text{k}}{50.0\text{M}} \text{ A or } \frac{100\text{k}}{50.0\text{M}} \text{ A}$$
$$I_{D,M1} = 1.62 \text{ mA or } 2.0 \text{ mA}$$

(4 marks)

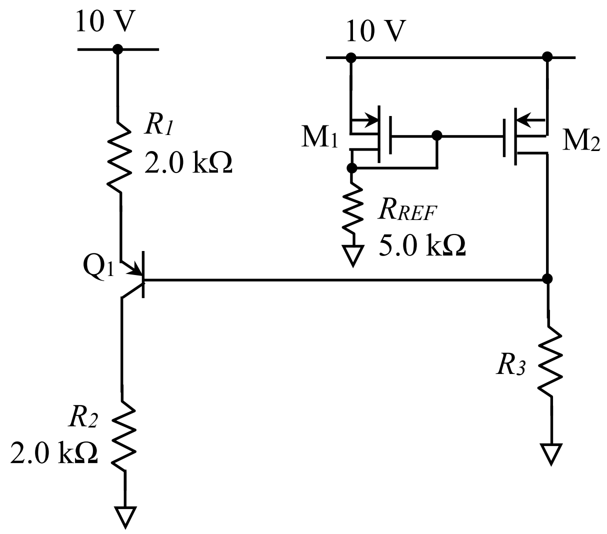
$$\boldsymbol{I_{D,M2} = I_{D,M1} = 1.62 \text{ mA}}$$

$I_{D,M1} = 2.0$ mA is not valid, as corresponding $V_{SG,M1} = 10 - I_{D,M1} \times R_{REF} = 0.0\,V < |V_{THP}| = 1.0$ V, i.e., no channel.

(1 mark)

(b)

The dc equivalent circuit –



- BJT operates in forward active mode $\Rightarrow V_{EB,Q1} \approx 0.7$ V
- $I_{C,Q1} = 1.6$ mA (Given) $\Rightarrow I_{B,Q1} = I_{C,Q1}/\beta = 0.016$ mA

By KVL,

$$10 = I_{E,Q1}R_1 + V_{EB,Q1} + \left(I_{B,Q1} + I_{D,M2}\right)R_3$$
$$= 1.01 \times 1.6\text{m} \times 2.0\text{k} + 0.7 + (0.016\text{m} + 1.62\text{m})R_3$$
$$\therefore\ \boldsymbol{R_3} = \mathbf{3.709\ k\Omega}\ (\text{or} = \mathbf{3.729\ k\Omega,}\ \text{assuming}\ I_{E,Q1} \approx I_{C,Q1})$$

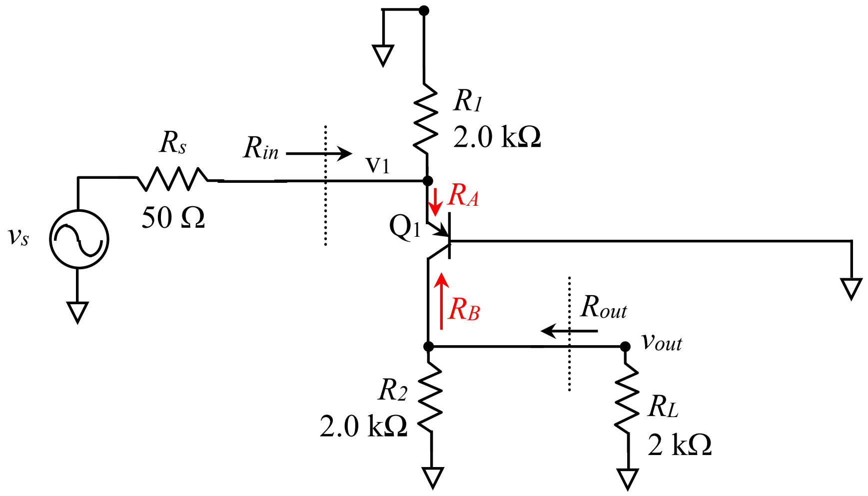
(4 marks)

(c)

- $g_{m,Q1} = I_{C,Q1}/\,V_T = 1.6\text{m}/0.025 = 64\ \text{mA/V}$
- $r_{\pi,Q1} = \beta_{Q1}/g_{m,Q1} = 100/64\text{m} = 1.563\ \text{k}\Omega$
- $r_{o,Q1} = V_{A,Q1}/I_{C,Q1} = 100/1.6\text{m} = 62.5\ \text{k}\Omega$

(3 marks)

The AC equivalent circuit –



$$\boldsymbol{G_m} = -g_{m,Q1} = \mathbf{-64\ mA/V}$$

(1 mark)

$$R_{in} = R_1//R_A$$

With the help of Table 1, Configuration D -

$$R_A = \left[\frac{1}{g_{m,Q1}} \times \frac{r_{o,Q1} + (R_2//R_L)}{r_{o,Q1} + (R_2//R_L)/\beta_{Q1}}\right] = 15.9\ \Omega\ \left(\text{or}\ \approx \frac{1}{g_{m,Q1}} = 15.60\ \Omega\right)$$
$$\therefore\ \boldsymbol{R_{in}} = R_1//R_A \approx \mathbf{15.77\ \Omega}$$

(2 marks)

$$R_{out} = R_2 // R_B$$

With the help of Table 1, Configuration B

$$R_B = r_{o,Q1}\{1 + g_{m,Q1}[r_{\pi,Q1}//(R_1//R_s)]\} = 251.71\ \mathrm{k\Omega}$$

$$\therefore \boldsymbol{R_{out}} = R_2//R_B = \mathbf{1.984\ k\Omega}$$

(2 marks)

(d)

$$v_{out}/v_s = \frac{R_{in}}{R_s + R_{in}} \times (-G_m) \times (R_{out}//R_L) = \frac{15.77}{50+15.77} \times (64\mathrm{m}) \times \frac{1.984\mathrm{k} \times 2\mathrm{k}}{1.984\mathrm{k} + 2\mathrm{k}} = 15.28$$

(2 marks)

Q6. For the amplifier circuit shown in Fig. Q6, the MOS Transistor M_1 has the following parameters: $K_n = 1$ mAV^{-2}, and $V_{THN} = 1$ V. MOS transistors M_2 and M_3 are identical and have the following parameters: $K_p = 1$ mAV^{-2}, and $V_{THP} = -1$ V.

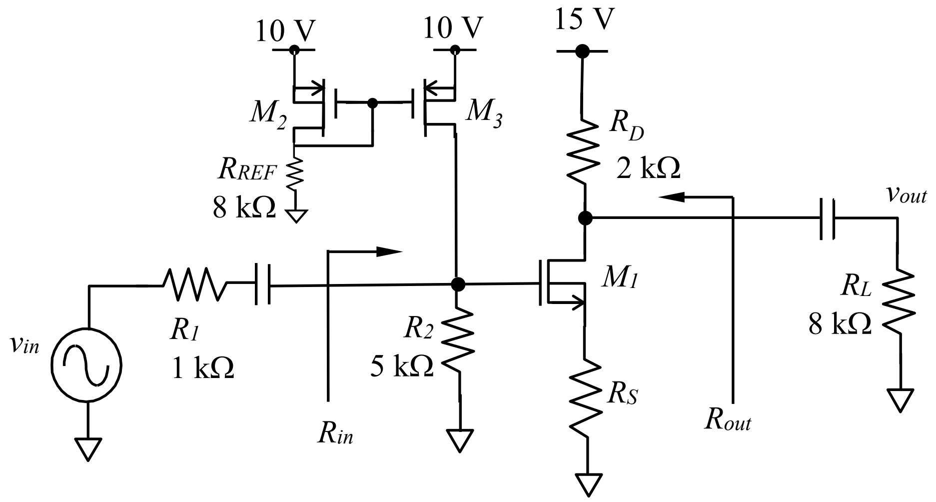


Fig. Q6

(a) Perform the DC analysis of the amplifier circuit shown in Fig. Q6, and show that the DC gate voltage of M_1 is 5 V. You need to explain briefly your answer.

(b) Calculate the value of R_S such that the drain current of M_1 is 4 mA. Verify that the M_1 is operating in the saturation region.

[Ans: $R_s = 0.5$ kΩ]

(c) Calculate the parameters, g_m and r_o, of the small-signal model of the transistor M_1.

[Ans: $g_{m,M1} = 4$ mA V^{-1}, $and\ r_{o,M1} = \infty$]

(d) What is the configuration of this amplifier?

(e) Calculate the values of R_{in}, and R_{out}.

[Ans: $R_{in} = 5$ kΩ and $R_{out} = 2$ kΩ]

(f) Estimate the voltage gain, A_v (= v_{out} / v_{in}).

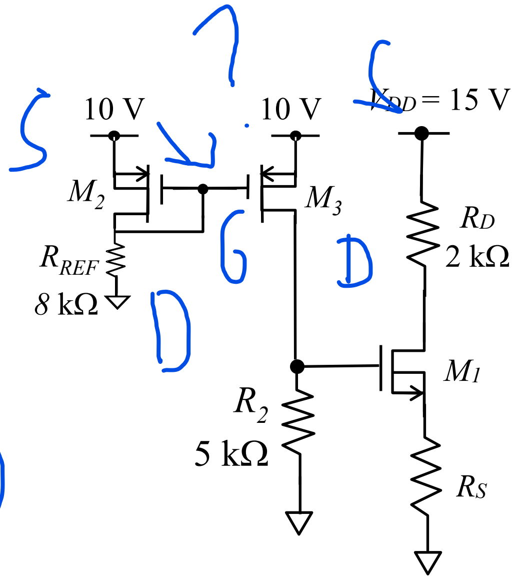
[Ans: $A_V = -1.78$]

Q6. Solution:

(a)

$V_{GS}=0$

$V_{DG}=0$



$V_{DS}=V_{GS}$

$V_D=V_G$

Equivalent circuit for DC analysis.

The two p-MOSFETs, M_2 and M_3, form a current mirror current source. $I_{D,M2} = I_{D,M3}$.

Gate and drain of M_2 shorted together, $V_{G,M2} = V_{D,M2} \Rightarrow$ M_2 is in saturation.

$$I_{D,M2} = K_{p,M2}\left(|V_{GS,M2}| - |V_{THP,M2}|\right)^2 = 1\text{m}\left([10 - V_{G,M2}] - 1\right)^2$$

$$I_{D,M2} = 1 \times \left(9 - I_{D,M2}\,R_{REF}\right)^2 = \left(9 - 8I_{D,M2}\right)^2 \qquad \text{(Note: } I_{D,M2} \text{ in mA)}$$

$$I_{D,M2} = 1 \text{ mA or } \frac{81}{64} \text{ mA}$$

$\boldsymbol{I_{D,M2} = 1\ \mathrm{mA}}$ $\left(I_{D,M2} = \frac{81}{64}\text{ mA not valid, as } V_{G,M2} = I_{D,M2}R_{REF} = 10.125\text{ V} > V_{S,M2} = 10\text{ V} \implies \text{no channel}\right)$

As $I_{G,M1} = 0$, $\boldsymbol{V_{G,M1}} = I_{D,M3} \times R_2 = 1\text{m} \times 5\text{k} = \mathbf{5\ V}$.

(b) Assume n-MOSFET M_1 is operating in saturation region -

$$I_{D,M1} = K_{n,M1}\left(V_{GS,M1} - V_{THN,M1}\right)^2$$

$$4\text{m} = 1\text{m}\left([V_{G,M1} - V_{S,M1}] - V_{THN,M1}\right)^2 = 1\text{m}(5 - 4R_S - 1)^2 = 1\text{m}(4 - 4R_S)^2$$

$$\therefore R_S = 0.5\text{ k}\Omega \text{ or } 1.5\text{ k}\Omega$$

$\boldsymbol{R_s = 0.5\ \mathrm{k\Omega}}$ ($R_S = 1.5$ kΩ not valid, as $V_{S,M1} = I_{D,M1} \times R_S = 6\text{ V} > V_{G,M1} = 5\text{ V} \implies$ no channel)

$V_{G,M1} = 5$ V, $V_{S,M1} = I_{D,M1} \times R_S = 2$ V, and

$V_{D,M1} = V_{DD} - I_{D,M1}R_D \;= 15 - 4 \times 2 = 7$ V

M1 is in saturation operation as,

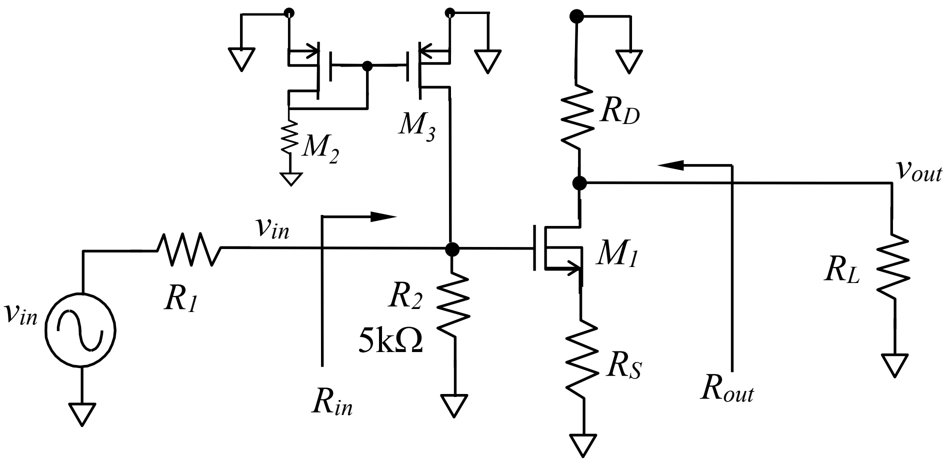
- $V_{GS,M1} = 5 - 2 = 3\;$ V $> \;V_{TH,M1} = 1$ V, and
- $V_{DS,M1} = 7 - 2 = 5\;$ V $> \;(V_{GS,M1} - V_{TH,M1}) = 2$ V .

(c) $g_{m,M1} = 2\sqrt{K_n I_D} = 2\sqrt{1\text{m} \;\times\; 4\text{m}} = 4 \text{ mA V}^{-1}$

$r_{o,M1} = \frac{1}{\lambda_n I_D} = \frac{1}{0 \times 4\text{m}} = \infty$

(d) The configuration of this amplifier is CS with degeneration.

(e)



Equivalent circuit for ac analysis.

$r_{o,M2} = r_{o,M3} = \infty \;\; (\lambda_p = 0)$

With the help of Table 2, Configuration A: $R_{in} = R_2 // r_{o,M3} //\infty \approx R_2 = 5$ kΩ

With the help of Table 2, Configuration B:

$R_{out} = R_D // \{r_{o,M1} [1 + g_{m,M1} R_S]\} = 2\text{k} // \infty = 2$ kΩ

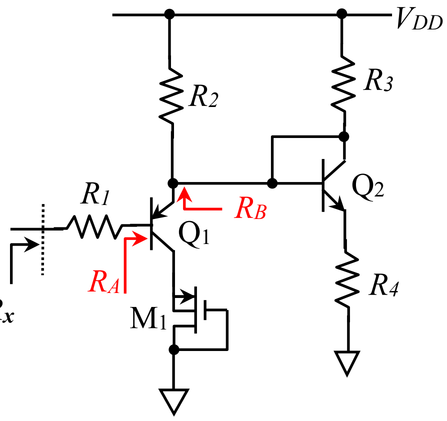
(f) With the help of Table 4, configuration D –

$$A_v = \frac{v_{out}}{v_{in}} = \frac{R_{in}}{R_1 + R_{in}} \times (-G_m) \times (\mathrm{R_{out}//R_L}) = -\frac{R_{in}}{R_1 + R_{in}} \left\{\frac{g_{m,M1}}{1 + g_{m,M1} R_S}\right\} (\mathrm{R_{out}//R_L})$$

$$\approx -\frac{5\text{k}}{1\text{k}+5\text{k}} \times \frac{4\text{m}}{1+4\text{m}\times0.5\text{k}} \times (2\text{k}//8\text{k}) = -1.78$$

Q7. Assume that the AC small-signal parameters of the BJT are $g_{m,Qi}$, $r_{\pi,Qi}$ and $r_{o,Qi}$, where i = 1, 2; and the AC small-signal parameters of the MOSFET are $g_{m,Mj}$, $g_{mb,Mj}$, $r_{i,Mj}$ and $r_{o,Mj}$, where j = 1, 2. Write down the expressions for the small-signal AC equivalent resistance (R_x) of the following configurations. V_{DD} and V_{BB} are DC voltage sources. I is a DC current source.

(a)



Solution:

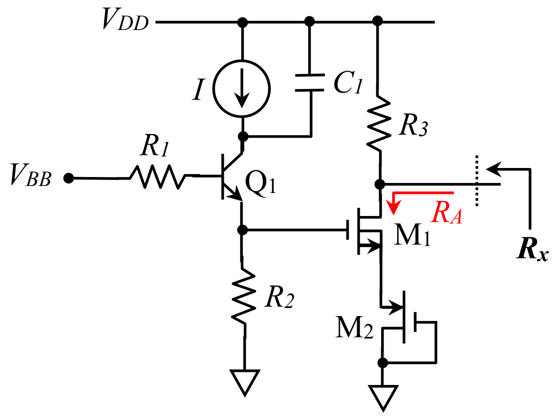
$R_x = R_1 + R_A$

- $R_A = r_{\pi,Q1} + \left(1+\beta_{Q1}\right) R_B$ (With the Help of Table 1, Configuration A)
- $R_B = \left[R_2//R_3//\left(\frac{1}{g_{m,Q2}} + R_4\right)\right]$ (With the Help of Table 1, Configuration E)

$$R_x = R_1 + R_A = R_1 + r_{\pi,Q1} + \left(1+\beta_{Q1}\right)\left[R_2//R_3//\left(\frac{1}{g_{m,Q2}} + R_4\right)\right]$$

$$= R_1 + r_{\pi,Q1}\left\{1 + g_{m,Q1}\left[R_2//R_3//\left(\frac{1}{g_{m,Q2}} + R_4\right)\right]\right\}$$

(b)



Solution: $R_x = R_3//R_A$

- $R_A = r_{o,M1}\left[1 + g_{m,M1}\left(\frac{1}{g_{m,M2}}\right)\right]$ (With the help of Table 1, Configuration B and Table 2, Configuration E)

$$R_x = R_3//R_A = R_3//\left\{r_{o,M1}\left[1 + g_{m,M1}\left(\frac{1}{g_{m,M2}}\right)\right]\right\}$$

Q8. A single-stage amplifier circuit is shown in Fig. Q8, where all the MOSFETs have identical Conductance Parameter of 0.5 mA/V^2 and Channel Length Modulation Factor of 0.01 V^{-1}. The Threshold Voltage of MOSFETs M_2 and M_3 are the same at -1 V, while that of MOSFET M_1 is -1.5 V with a body voltage of V_B = 10 V.

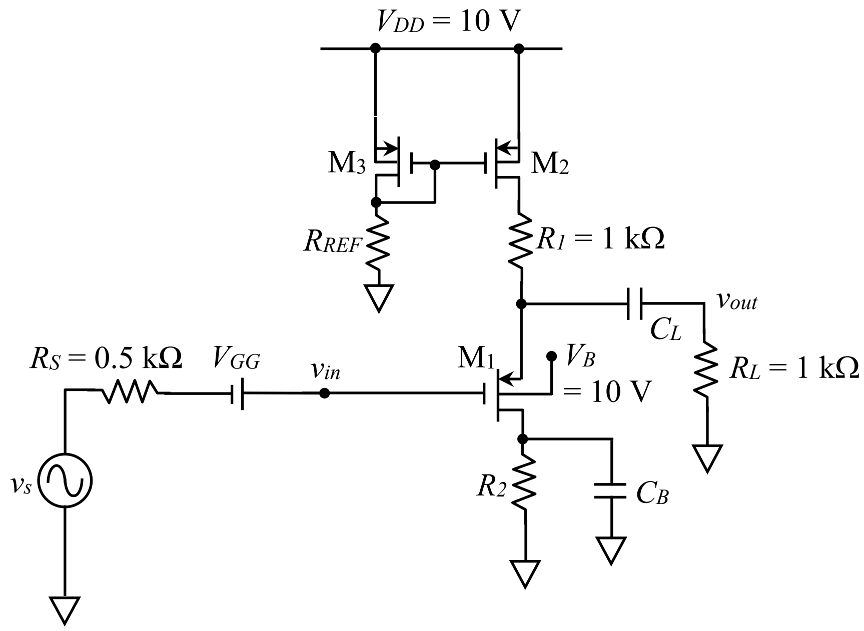


Fig. Q8

The amplifier circuit is designed to have a DC drain current of 1 mA for MOSFET M_1.

(a) Ignore Channel Length Modulation effect in the DC analysis, determine R_{REF}. You need to provide brief explanation to your calculation.

[Ans: $R_{REF} = 7.59$ kΩ]

(b) Identify the configuration of the amplifier circuit.

(c) Estimate the small-signal parameters of MOSFET M_1: $g_{m,M1}$, $r_{o,M1}$ and $g_{mb,M1}$.

[Ans: $g_{m,M1} = 21.41$ mA/V, $r_{o,M1} =$ 100 kΩ, $g_{mb,M1} = -0.35$ mA/V]

(d) Estimate the two-port network parameters of the amplifier circuit: R_{in}, and R_{out}.

[Ans: $R_{out} = \infty,\ R_{out} = 565\ \Omega$]

(e) Estimate the voltage gain v_{out}/v_s of the amplifier.

[Ans: $v_{out}/v_{in} \approx 0.51$]

(f) For the amplifier circuit to operate properly, what is the constraint on the value of R_2?

[Ans: $R_2 < 6.18$ kΩ]

Q8. Solution:

All MOSFETs are of the p-channel type with K_p = 0.5 mA/V^2, λ_p = 0.01 V^{-1}.
MOSFETs M_2 and M_3 have Threshold voltage, V_{THP} = -1 V,
MOSFET M_1 has Threshold voltage, V_{THP1} = -1.5 V (at V_B = 10 V)
$I_{D,M1} = 1$ mA

(a) ***Ignore Channel Length Modulation effect in the DC analysis***, determine R_{REF}.

$I_{D,M1} = 1\text{ mA} = I_{S,M1} = I_{D,M2} = I_{D,M3}$

($I_{D,M2} = I_{D,M3}$, as M_2 and M_3 form a **current mirror current source)**

Perform DC analysis –

M_3 in **saturation operation** (as **gate and drain** are **shorted together)** -

$$I_{D,M3} = K_p\left[|V_{GS,M3}| - |V_{THP}|\right]^2 = K_p\left[\left(V_{DD} - I_{D,M3}R_{REF}\right) - |V_{THP}|\right]^2$$

$$1m = 0.5\text{m}[(10 - 1\text{m} \times R_{REF}) - 1]^2$$

$$\pm\sqrt{1/0.5} = 9 - 1\text{m} \times R_{REF}$$

$$R_{REF} = 10.41 \text{ or } 7.59\text{ k}\Omega$$

$\boldsymbol{R_{REF} = 7.59\text{ k}\Omega}$ ($R_{REF} = 10.41\text{ k}\Omega$ is invalid, as $\left|V_{GS,M3}\right| = V_{DD} - I_{D,M3}R_{REF} = 10 - 1\text{m} \times 10.41\text{k} = -0.41\text{ V} < |V_{THP}| = 1\text{ V} \Rightarrow$ no channel)

OR,

$$I_{D,M3} = K_p\left[|V_{GS,M3}| - |V_{THP}|\right]^2$$

$$1m = 0.5\text{m}\left[V_{SG,M3} - 1\right]^2$$

$$\pm\sqrt{1/0.5} = V_{SG,M3} - 1$$

$$V_{SG,M3} = 2.41 \text{ or } -0.41\text{ V}$$

$\boldsymbol{V_{SG,M3} = 2.41\text{ V}}$ ($V_{SG,M3} = -0.41\text{ V} < |V_{THP}| = 1\text{ V} \Rightarrow$ no channel)

$\boldsymbol{R_{REF}} = \frac{(V_{DD} - V_{SG,M3})}{I_{REF}} = \boldsymbol{7.59\text{ k}\Omega}$

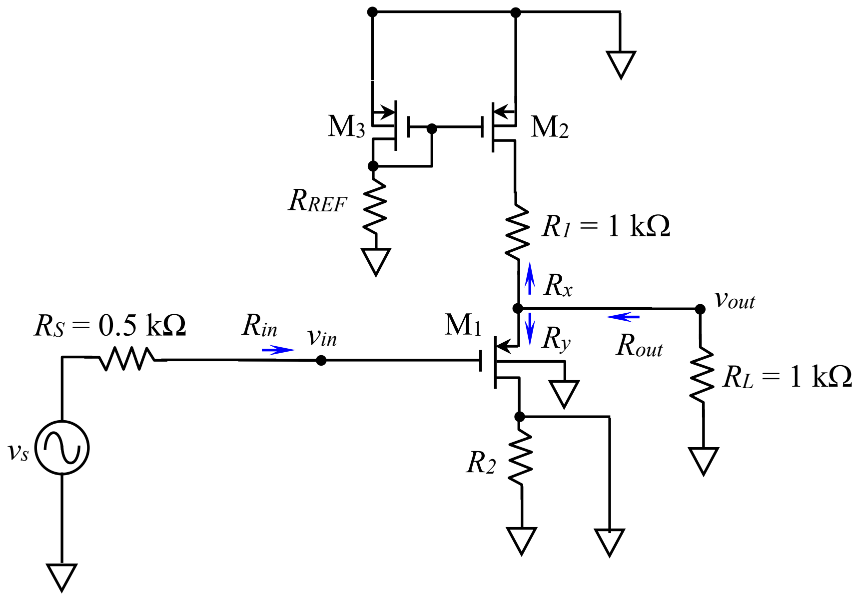
(b) Identify the configuration of the amplifier circuit.

Common Drain amplifier.

(c) Determine the small-signal parameters of MOSFET M1: $g_{m,M1}$, $r_{o,M1}$ and $g_{mb,M1}$.

- $g_{m,M1} = 2\sqrt{K_p I_D} = 2\sqrt{0.5\text{m} \times 1\text{m}} = 1.41 \text{ mA/V}$
- $r_{o,M1} = \frac{1}{\lambda_p I_D} = \frac{1}{0.01 \times 1\text{m}} = 100 \text{ k}\Omega$
- $g_{mb,M1} = -{g_{m,M1}}/{4} = -0.35 \text{ mA/V}$ (Body and source **not** shorted together)

(d) Estimate the two-port network parameters of the amplifier circuit: R_{in}, and R_{out}..



Equivalent circuit for small-signal ac analysis

With the help of Table 2, Configuration A: $\boldsymbol{R_{in}} = \infty$

$R_{out} = R_x // R_y$

- R_x is the equivalent resistance looking into the drain of M2 in series with R_1. With the help of Table 2, Configuration B: $R_x = r_{o,M2} + R_1 = 101 \text{ k}\Omega$
- R_y is the equivalent resistance looking into the source of M1. With the help of Table 2, Configuration C: $R_y = \frac{1}{g_{m,M1} - g_{mb,M1}} = \frac{1}{1.41\text{m} + 0.35\text{m}} = 568.2\ \Omega$

So, $\boldsymbol{R_{out}} = R_x // R_y = \boldsymbol{565\ \Omega}$

(e) Estimate the voltage gain v_{out}/v_s of the amplifier.

With the help of Table 4, Configuration C –

$$\frac{v_{out}}{v_s} = \frac{v_{out}}{v_{in}} = \left\{\frac{g_{m,M1}(R_x//R_L)}{1+(g_{m,M1}-g_{mb,M1})(R_x//R_L)}\right\} \approx \frac{1.41\text{m}\times(101\text{k}//1\text{k})}{1+(1.41\text{m}+0.35\text{m})\times(101\text{k}//1\text{k})} \approx 0.51$$

(f) For the amplifier circuit to operate properly, what is the constraint on the value of R_2?

For the amplifier circuit to operate properly, both M_1 and M_2 must operate in **saturation**, meaning

- $|V_{DS,M1}| = V_{SD,M1} > (|V_{GS,M1}| - |V_{THP1}|)$, and
- $|V_{DS,M2}| = V_{SD,M2} > (|V_{GS,M2}| - |V_{THP}|)$

$$|V_{GS,M2}| = |V_{GS,M3}| = V_{SG,M3} = V_{DD} - I_{D,M3} \times R_{REF} = 10 - 1\text{m} \times 7.59\text{k} = 2.41 \text{ V}$$

$$I_{D,M1} = K_p\left[|V_{GS,M1}| - |V_{THP1}|\right]^2$$
$$1\text{m} = 0.5\text{m}\left[|V_{GS,M1}| - 1.5|\right]^2$$
$$\pm\sqrt{1/0.5} = \left[|V_{GS,M1}| - 1.5\right]$$
$$\left|V_{GS,M1}\right| = 2.91 \text{ or } 0.086 \text{ V}$$
$$\left|V_{GS,M1}\right| = 2.91 \text{ V } (0.086 \text{ V} < |V_{THP1}| \text{ not valid})$$

By KVL,

$$V_{SD,M1} + V_{SD,M2} = V_{DD} - I_{D,M1}(R_1 + R_2)$$
$$(V_{SD,M1} + V_{SD,M2}) > \left[(|V_{GS,M1}| - |V_{THP1}|) + (|V_{GS,M2}| - |V_{THP}|)\right]$$
$$V_{DD} - I_{D,M1}(R_1 + R_2) > (2.91 - 1.5) + (2.41 - 1)$$
$$10 - 1\text{m} \times (1\text{k} + R_2) > (2.91 - 1.5) + (2.41 - 1)$$
$$\therefore \boldsymbol{R_2 < 6.18\ \mathrm{k\Omega}}$$