NATIONAL UNIVERSITY OF SINGAPORE
Department of Electrical and Computer Engineering

EE2027 Electronic Circuits
Tutorial 1: Solution

Homework 1:

Homework 1 consists of two questions, Q1 and Q7 of Tutorial 1. You will need to submit a softcopy of your handwritten homework to the Canvas folder: Files>Homework Submission>HW1 half an hour after class (i.e., latest by 12:30pm) on Thursday, 29 August 2024.

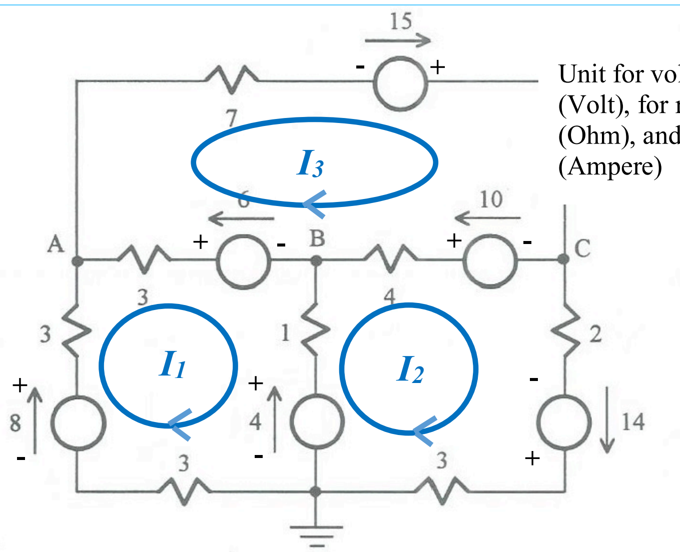
The softcopy submission of your homework must be in PDF format and in a single file. Name your file following the convention "Your_Name_HW1.pdf". Failing to do that will mean zero mark for homework.

Homework questions will not be discussed in class.

Q1. Formulate the equations for obtaining the loop currents, I_1, I_2 and I_3 of the circuit shown in Fig. Q1 by using the mesh analysis.

Derive also an equation each for the voltages of nodes A, B and C with respect to ground and in terms of the loop currents.

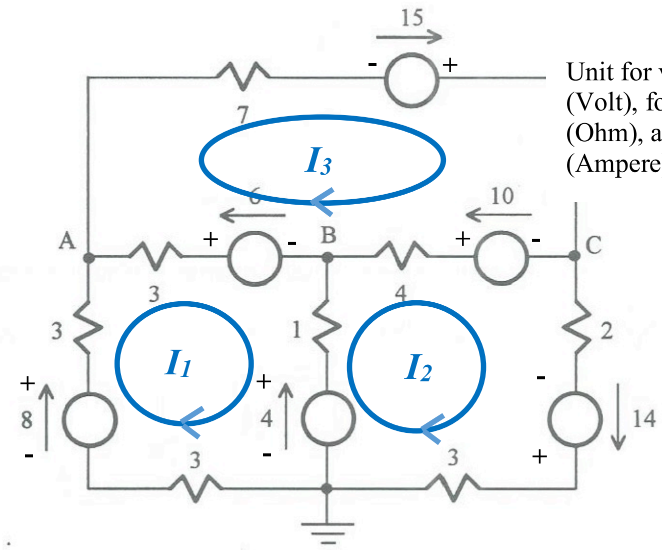
(10 marks)



Unit for voltage sources is V (Volt), for resistor is Ω (Ohm), and for current is A (Ampere)

Fig. Q1

Q1. **Solution:**



Unit for voltage sources is V (Volt), for resistor is Ω (Ohm), and for current is A (Ampere)

Applying KVL to the 3 independent loops in the above circuit:

- I_1 loop: $8 - 3I_1 - 3(I_1 - I_3) - 6 - (I_1 - I_2) - 4 - 3I_1 = 0$
- I_2 loop: $4 + (I_1 - I_2) - 4(I_2 - I_3) - 10 - 2I_2 + 14 - 3I_2 = 0$
- I_3 loop: $15 + 10 + 4(I_2 - I_3) + 6 + 3(I_1 - I_3) - 7I_3 = 0$

Simplifying the above equations leads to -

$10I_1 - I_2 - 3I_3 = -2$

$-I_1 + 10I_2 - 4I_3 = 8$

$-3I_1 - 4I_2 + 14I_3 = 31$

The above are the required equations for obtaining the loop currents I_1, I_2 and I_3. Actual solving of the equations is not required.

Note: Although the above 3 equations are likely the obvious choice, other equations are possible, e.g., another equation using the outer loop -

$$-3I_1 + 8 - 3I_1 - 7I_3 + 15 - 2I_2 + 14 - 3I_2 = 0 \;\Rightarrow\; 6I_1 + 5I_2 + 7I_3 = 37$$

(1 mark for showing all working, 2 marks per equation for total of 7 marks)

The voltages of nodes A, B and C with respect to ground are as follows:

$V_A = 8 - 6I_1$

$V_B = 4 + I_1 - I_2$

$V_C = -14 + 5I_2$

(1 mark per equation for 3 marks total)

Note: There are other possible equations for the node voltages V_A, V_B and V_C. For example, another possible equation for V_A is $V_A = 10 + 4I_1 - I_2 - 3I_3$. Although there are different possible equations for V_A, the numerical answer of V_A is the same.

Q2. (a) Using linear superposition, calculate the open-circuit voltage V_{OC} for the circuit shown in Fig. Q2.

[Ans: $V_{OC} = 5/6$ V]

(b) Calculate V_{OC} using the mesh analysis as well.

(c) Determine the Thevenin equivalent of the circuit to the left of node A (i.e., the circuit within the gray box) shown in Fig. Q2. What is the maximum power that can be delivered to a load R_L connected across node A and the ground?

[Ans: 208 μW]

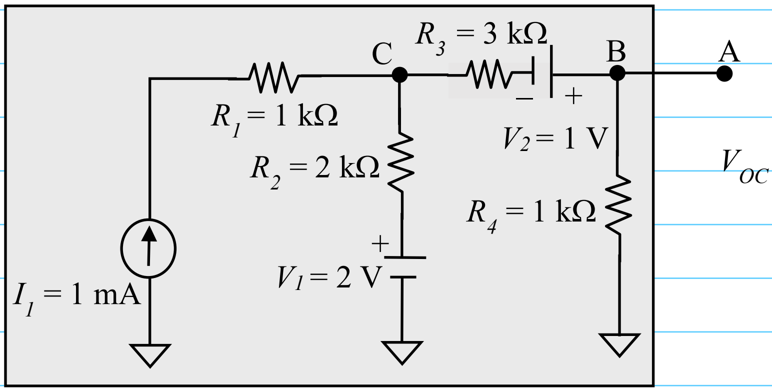
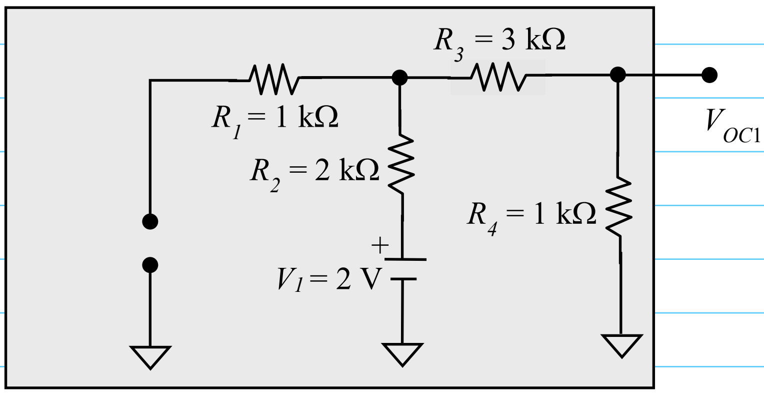


Fig. Q2

Q2. **Solution:**

(a)

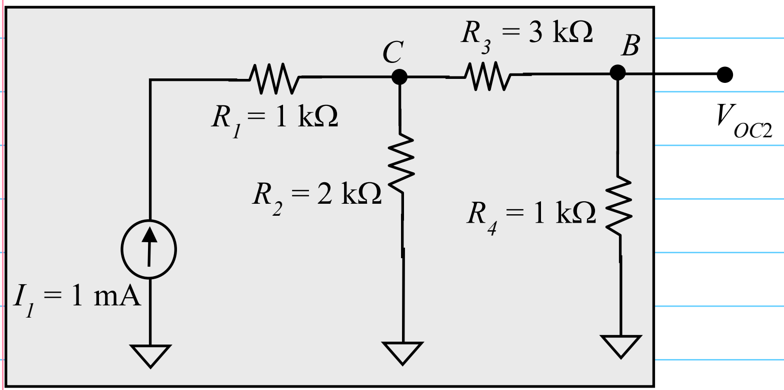
(i) Consider effect of only voltage source, V_1, first by killing current source, I_1, by open-circuit (O.C.); and voltage source, V_2, by short-circuit (S.C.):



By voltage divider method -

$$V_{OC1} = \frac{R_4}{R_2+R_3+R_4} \times 2\text{ V} = \frac{1}{(2+3+1)} \times 2 = \frac{2}{6}\text{ V} = \frac{1}{3}\text{ V}$$

(ii) Next consider only effect of current source, I_1, by killing voltage sources, V_1 and V_2, by S.C. Apply KCL to nodes C and B:



Node C: $\quad 1\, mA + \frac{0-V_C}{R_2} + \frac{V_{OC2}-V_C}{R_3} = 0$

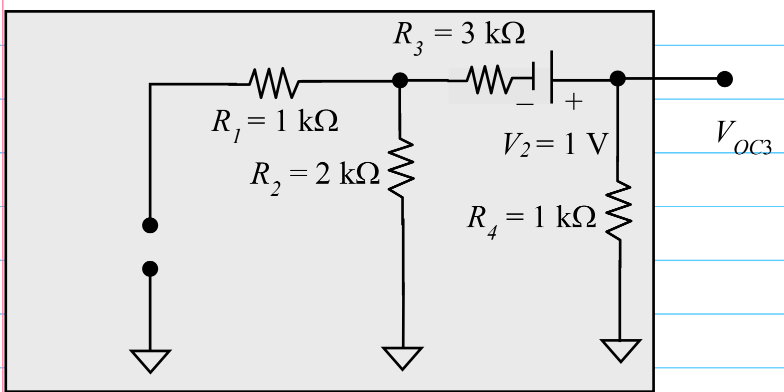
$$1\, mA + \frac{-V_C}{2\, k\Omega} + \frac{V_{OC2}-V_C}{3k\Omega} = 0 \;\Rightarrow 5V_C - 2\,V_{OC2} = 6 \;(1)$$

Node B: $\quad \frac{V_C - V_{OC2}}{R_3} + \frac{0-V_{OC2}}{R_4} = 0 \;\Rightarrow\; \frac{V_C - V_{OC2}}{3\, k\Omega} + \frac{-V_{OC2}}{1\, k\Omega} = 0$

$$\Rightarrow V_C = 4\,V_{OC2} \qquad (2)$$

Solving Eqs. (1) and (2), $V_{OC2} = \frac{1}{3}$ V

(iii) Finally, consider only effect of voltage source, V_2, by killing current source, I_1, by O.C.; and voltage source, V_1, by S.C.



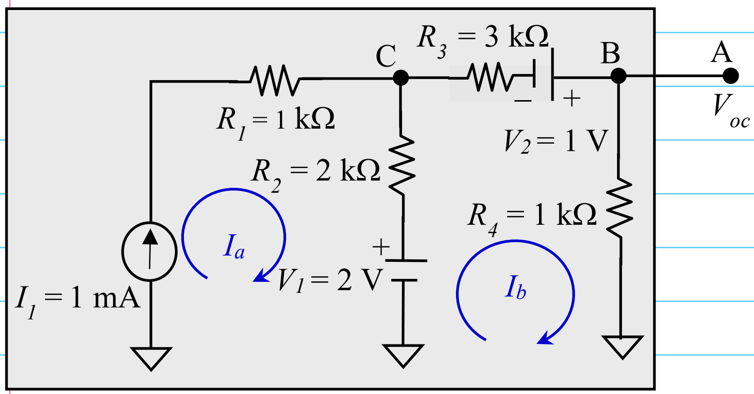
By voltage divider method –

$$V_{OC3} = \frac{R_4}{R_2+R_3+R_4} \times 1\text{ V} = \frac{1}{(2+3+1)} \times 1 = \frac{1}{6}\text{ V}$$

By linear superposition, the open-circuit voltage is given by:

$$V_{OC} = V_{OC1} + V_{OC2} + V_{OC3} = \frac{5}{6}\ \text{V}$$

(b)



From loop with current I_a: $I_a = 1$ mA

Apply KVL to the loop with current I_b:

$$2 + (I_a - I_b)R_2 - I_bR_3 + 1 - I_bR_4 = 0$$

$$2 + (1\text{mA} - I_b)R_2 - I_bR_3 + 1 - I_bR_4 = 0$$

$$\therefore I_b = \frac{5}{6}\ \text{mA}$$

$$V_{OC} = I_bR_4 = \frac{5}{6}\ \text{V}$$

Note: V_{OC} can be determined by different ways, as shown in this question, by means of linear superposition in part (a) or mesh analysis in part (b).

(c) Thevenin equivalent source voltage, $V_{THV} = V_{OC} = \frac{5}{6}$ V

To obtain the Thevenin equivalent resistance, R_{THV}, kill voltage sources, V_1 and V_2, by S.C. and kill current source, I_1, by O.C.:

$$R_{THV} = (R_2 + R_3)//R_4 = \frac{5\text{k}\times 1\text{k}}{1\text{k}+5\text{k}} = \frac{5}{6}\ \text{k}\Omega$$

For maximum power (P_{max}) transfer, $R_L = R_{THV} = \frac{5}{6}$ kΩ

$$P_{max} = \left(\frac{V_{THV}}{R_{THV}+R_L}\right)^2 \times R_L = \left(\frac{\frac{5}{6}}{2\times\frac{5}{6}\text{k}}\right)^2 \times \frac{5}{6}\text{k} = \frac{5}{24}\ \text{mW} = 208\ \mu\text{W}$$

Q3. (a) With the load resistor R_L removed, calculate the Thevenin equivalent voltage, V_{THV}, using the Node Aanalysis (KCL) method, and the Thevenin equivalent resistance, R_{THV}, across terminals A and B for the circuit in Fig. Q3.

(9 marks)

(b) If terminals A and B are short-circuited, what is the value of the short-circuit current?

(2 marks)

(c) What value should the load resistor R_L be for maximum power to be transferred to R_L? Calculate the maximum power delivered to R_L.

(4 marks)

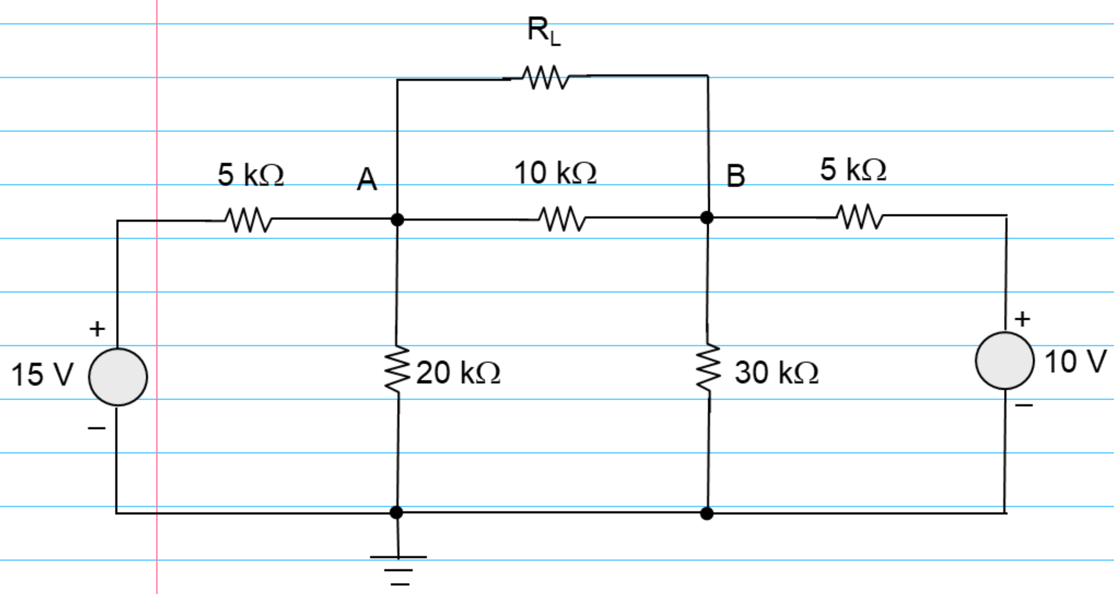
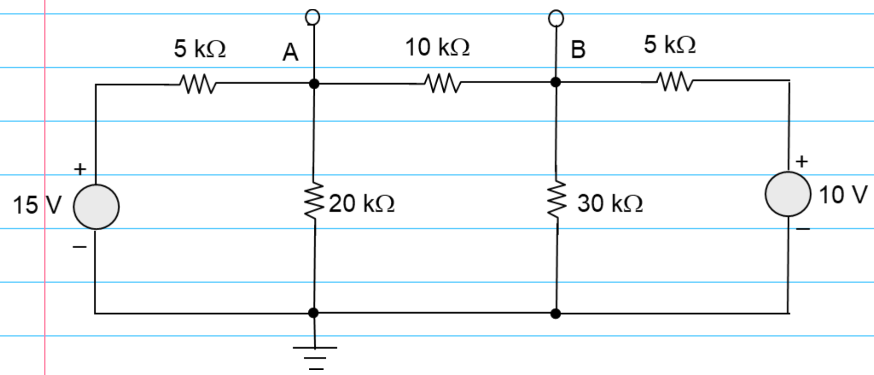


Fig. Q3

Q3. **Solution:**

(a) With the load resistor R_L removed, the resultant circuit with a single-port AB is shown below and can be replaced by its Thevenin equivalent circuit.



Applying KCL at Node A:

$$\frac{V_A}{20k} + \frac{V_A - V_B}{10k} = \frac{15 - V_A}{5k}$$

$$V_A + 2V_A - 2V_B = 60 - 4V_A$$

$$7V_A - 2V_B = 60 \qquad (1)$$

(1 mark)

Applying KCL at Node B:

$$\frac{V_B}{30k} + \frac{V_B - 10}{5k} = \frac{V_A - V_B}{10k}$$

$$V_B + 6V_B - 60 = 3V_A - 3V_B$$

$$-3V_A + 10V_B = 60 \qquad (2)$$

(1 mark)

Equating Eq. (1) and Eq. (2):

$$7V_A - 2V_B = -3V_A + 10V_B$$

Therefore, $V_A = 1.2\ V_B$ (3)

(1 mark)

Substituting Eq. (3) into Eq. (1)

$$8.4V_B - 2V_B = 60$$

Therefore,

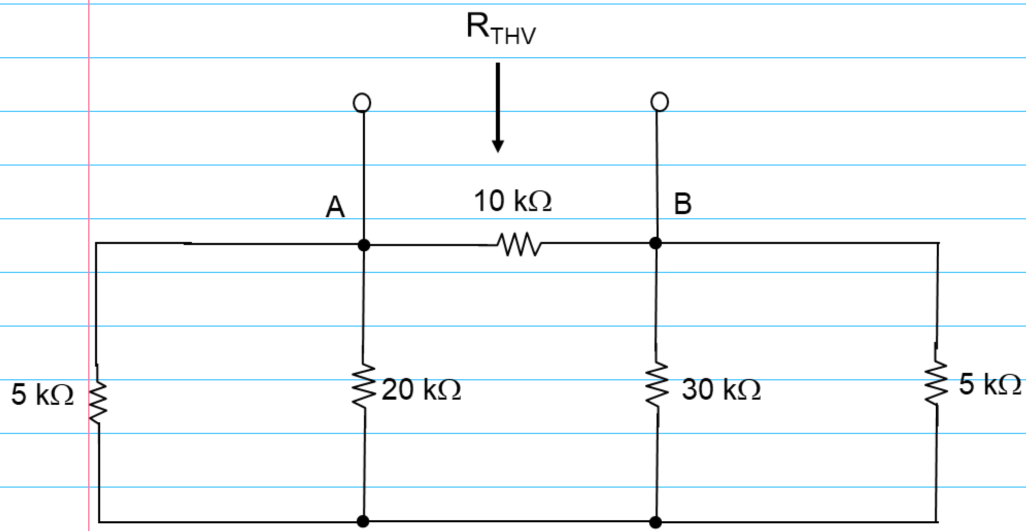
$$V_B = 9.375 \text{ V} \quad \text{and} \quad V_A = 11.25 \text{ V}$$

Hence, the Thevenin equivalent voltage is

$$V_{THV} = V_{AB} = V_A - V_B = 1.875 \text{ V}$$

(2 marks)

To find R_{THV}, kill the two independent voltage sources by replacing them with a short circuit. The resultant circuit is as follows:



$$\begin{aligned} R_{THV} &= 10 \text{ k}\Omega \,//\, [(5 \text{ k}\Omega \,//\, 20 \text{ k}\Omega) + (5 \text{ k}\Omega \,//\, 30 \text{ k}\Omega)] \\ &= 10 \text{ k}\Omega \,//\, [4 \text{ k}\Omega + 4.2857 \text{ k}\Omega] \\ &= 4.531 \text{ k}\Omega \end{aligned}$$

(4 marks)

(b) If terminals A and B are short-circuited, the short-circuit current can be calculated as:

$I_{SC} = V_{THV} / R_{THV} = 1.875 / 4.531k = 0.414$ mA

(2 marks)

(c) For maximum power transfer to R_L,

$R_L = R_{THV} = 4.531\ k\Omega$

(1 mark)

Maximum power delivered to R_L is

$$P_{RL(max)} = \left(\frac{V_{THV}}{R_{THV} + R_L}\right)^2 \text{ x } R_L = \left(\frac{1.875}{9.062\text{k}}\right)^2 \text{ x } 4.531\text{k} = 0.194 \text{ mW}$$

(3 marks)

Q4. The current (I) flowing through a semiconductor pn junction diode at various forward bias voltage (V) are measured at T = 300 K and tabulated as follows:

V (V)	I (A)
0.00	0.00
0.10	~0.00
0.20	~0.00
0.30	~0.00
0.40	~0.00
0.50	4.70×10^{-5}
0.55	3.70×10^{-4}
0.60	2.52×10^{-3}
0.62	5.91×10^{-3}
0.64	1.38×10^{-2}
0.66	2.73×10^{-2}
0.68	6.40×10^{-2}
0.70	1.51×10^{-1}

(a) Plot the I-V characteristic of the diode from V = 0 to 0.7 V, and estimate the cut-in voltage of the diode.

(b) The I-V characteristic of the semiconductor diode is given as follows:

$$I = I_S\left(e^{V/V_T} - 1\right), \text{ where } V_T = 0.025\text{V}.$$

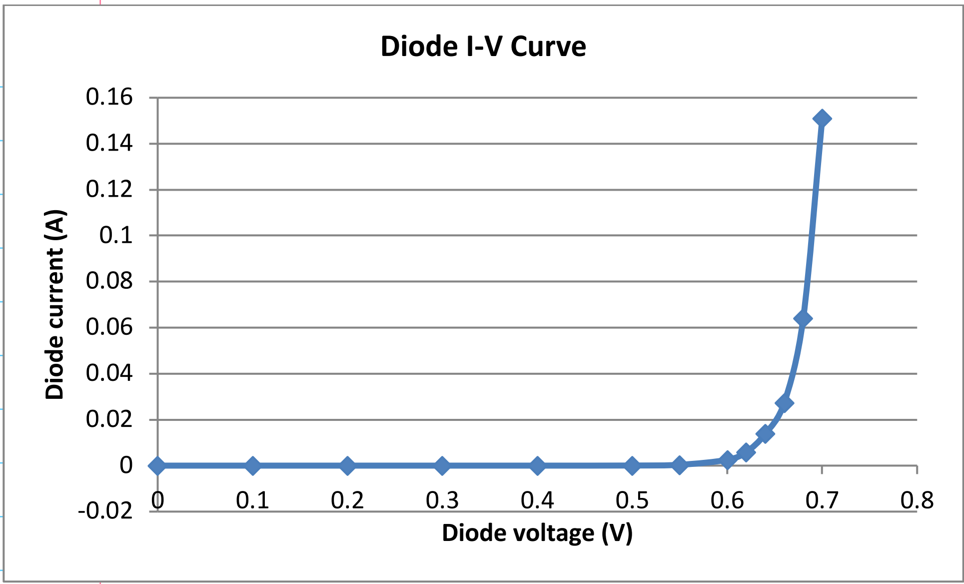
Estimate the value of the reverse saturation current (I_S) by devising a suitable plot between I and V in forward bias.*

Take note that the values of I in the above table are measured, and are therefore likely to have experimental errors. Hence, one will not estimate I_S by applying a single set of I versus V values from the table to the above equation, as that will not be accurate. Also, different sets of I versus V values are likely to give different I_S.

* Hint: The range of voltage (V) of the plot to be devised is an important consideration, and you need to transform the above equation into the form of $y = mx + C$, i.e., a linear relation between y and x, where m and C are constants.

Q4. **Solution:**

(a) Plot of the I-V characteristic of the diode from $V = 0$ to 0.7 V is shown below.



Cut-in voltage is ~0.6 V.

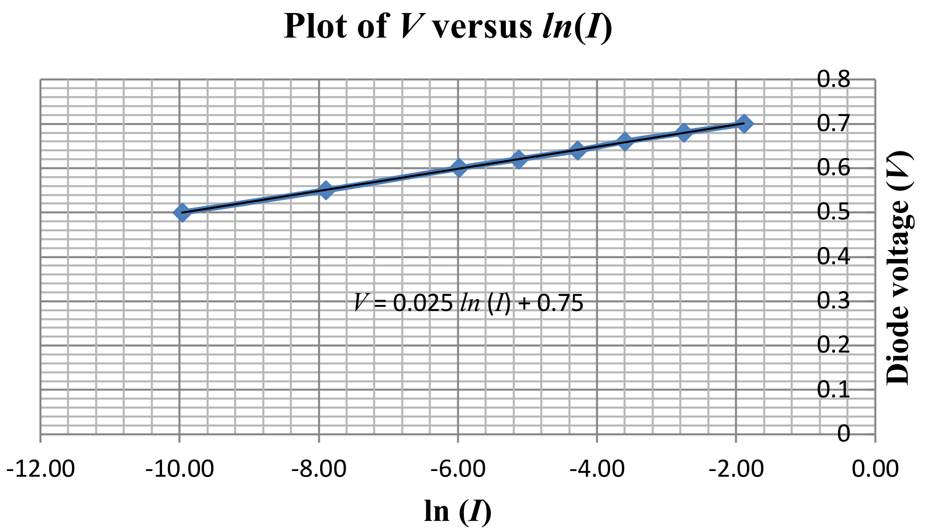
(b) Note that we only have measurements of I for $V > 0.5$ V, which is much higher than $V_T = 0.025$ V.

For $V >> V_T$, $e^{V/V_T} \gg 1,\ I = I_S\left(e^{V/V_T} - 1\right) \cong I_S e^{V/V_T}$

Hence, $I/I_S = e^{V/V_T} \ \Rightarrow\ V = V_T[ln(I) - ln(I_S)]$

So, V is seen as a ***linear*** function of $ln(I)$. By plotting V against $ln(I)$ for $V >> V_T$ (e.g., $V \geq 0.5$ V), a straight line is obtained and its intercept with the $ln(I)$-axis gives the value of $ln(I_S)$.

***V*(V)**	**ln (*I*)**
0.5	-9.97
0.55	-7.90
0.6	-5.98
0.62	-5.13
0.64	-4.28
0.66	-3.60
0.68	-2.75
0.7	-1.89



From the above plot of V versus $ln(I)$ curve, the best-fit straight line is found (using EXCEL) to be

$$V = 0.025\, ln\,(\mathrm{I}) + 0.75.$$

By means of a best-fit plot, the experimental errors are averaged out. It is also seen in the above plot that it is **not** practical to extrapolate the best-fit straight line to find the intercept with the $ln(I)$-axis so as to estimate I_S. Instead, we can compare the best-fit equation with $V = V_T[ln(I) - ln(I_S)]$, which yields

$$-V_T\, ln\,(I_S) = 0.75 \;\Rightarrow\; I_S = e^{-0.75/V_T} = e^{-30} = 9.36 \times 10^{-14}\ \text{A}.$$

Q5. The circuit shown in Fig. Q5 consists of a voltage source V_{DD}, two resistors R_1 and R_2, and a Zener diode. The breakdown voltage of the Zener diode, $V_Z = 7$ V. For reverse voltage less than V_Z, the reverse saturation current of the Zener diode is assumed to be negligible.

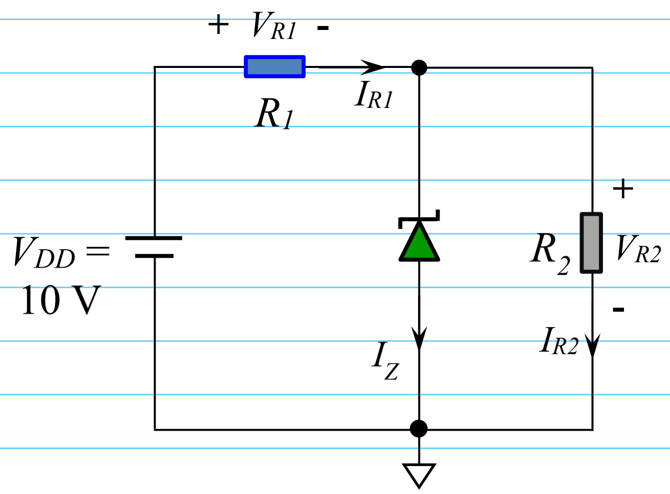


Fig. Q5

(a) The resistors are given as $R_1 = 1$ kΩ and $R_2 = 4$ kΩ. Assume first that the Zener diode is <u>NOT</u> operating in the breakdown region, calculate the voltage across the Zener diode, V_{R2}, and hence, verify whether the Zener diode is working in the breakdown region or not, and determine the actual voltage across the Zener diode.

Calculate also the voltages V_{R1} and V_{R2}, and the currents I_{R1}, I_{R2}, and I_Z.

[Ans: 3 V, 7 V, 3 mA, 1.75 mA, 1.25 mA]

(b) Repeat part (a) with R_1 changed to 6 kΩ , while keeping R_2 at 4 kΩ.

[Ans: 6 V, 4 V, 1 mA, 1 mA, 0 mA]

(c) The circuit in Fig. Q5 is to be used to provide a constant voltage of V_Z (the breakdown voltage of the Zener diode) to a given load R_2. How should a circuit designer choose the value of R_1?

Q5. **Solution:**

(a) By first assuming that the Zener diode is NOT operating in the breakdown region, then it behaves like an **open circuit** since its reverse saturation current is negligible. R_1 and R_2 are therefore in series, and the voltage across the Zener diode, which is also the voltage across R_2 is given by

$$V_{R2} = V_{DD} \times \frac{R_2}{R_1+R_2} = 10 \times \frac{4\text{k}}{1\text{k}+4\text{k}} = 8\,V$$

As V_{R2} (which is also the voltage of Zener diode) is **greater** than the breakdown voltage of the Zener diode, V_Z = 7 V, the assumption that the Zener diode is not operating in the breakdown region is not correct. Hence, the Zener diode is in fact operating in the breakdown region.

Since the Zener diode is operating in the breakdown region, the voltage across the Zener diode, V_{R2}, which is also the voltage across R_s, is clamped at the breakdown voltage, i.e.,

$$V_{R2} = V_Z = 7\,V$$

By KVL –

$$V_{R1} = V_{DD} - V_{R2} = 3\,V$$

$$I_{R2} = \frac{V_{R2}}{R_2} = \frac{7}{4\text{k}} = 1.75 \text{ mA}$$

$$I_{R1} = \frac{V_{R1}}{R_1} = \frac{37}{1\text{k}} = 3 \text{ mA}$$

$$I_Z = I_{R1} - I_{R2} = 1.25 \text{ mA}$$

(b) As in part (a), assume that the Zener diode is NOT operating in the breakdown region, then it behaves as an open circuit, and R_1 and R_2 are therefore in series.

$$V_{R2} = V_{DD} \times \frac{R_2}{R_1+R_2} = 10 \times \frac{4\text{k}}{6\text{k}+4\text{k}} = 4\,V \text{ (= voltage across Zener diode)}$$

As V_{R2}, also the voltage across the Zener diode, is **less than** the Zener diode breakdown voltage of V_Z = 7 V, the assumption that the Zener diode is not operating in the breakdown region is valid.

$$V_{R1} = V_{DD} - V_{R2} = 6\,V$$

Note: In this case, voltage across the Zener diode is not clamped at V_Z = 7 V, but determined by R_1 and R_2. This also means the circuit does not provide a constant (or regulated) voltage of V_Z to R_2.

As the Zener diode is <u>not</u> operating in the breakdown region, its current is negligible,

$$I_Z \approx 0$$

$$I_{R1} = I_{R2} = \frac{V_{DD}}{R_1+R_2} = \frac{10}{6\text{k}+4\text{k}} = 1 \text{ mA}$$

(b) For a given load R_2, an appropriate value for R_1 must be chosen to ensure that the Zener diode is indeed working in the breakdown region (meaning $I_Z \neq 0$) to provide a constant (or regulated) voltage of V_Z to R_2. From results of parts (a) and (b), the ratio of R_1/R_2 should be low enough.

Q6. The forward bias current of a *pn*-junction diode was measured at room temperature (V_T = 25 mV) as 1.066×10^{-7} A and 2.2×10^{-5} A at $V_D = 0.6$ V and 0.8 V, respectively.

(a) Determine the diode exponential factor and reverse saturation current, n and I_s, respectively.

(5 marks)

(b) The diode is used in the circuit shown in Fig. Q6, where $V_{DD} = 1$ V and $v_{dd}(t) = 0.01cos(\omega t)$ V. The diode operates at $V_D = 0.7$ V. Determine the diode small-signal resistance, r_d, and the maximum voltage across the diode v_D due to $v_{dd}(t)$ and V_{DD}.

(10 marks)

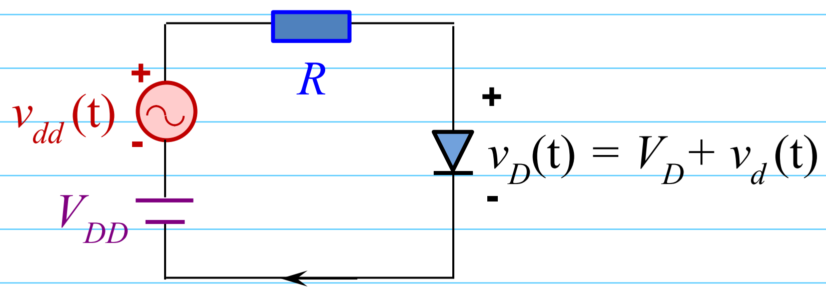


Fig. Q6

You are required to explain briefly your answers above.

Q6. **Solution:**

(a) $I = I_s(e^{\frac{V}{nV_T}} - 1) \approx I_s e^{\frac{V}{nV_T}}$

$I_1/I_2 = e^{\frac{V_1-V_2}{nV_T}} \quad \Rightarrow \quad \ln\left(\frac{I_1}{I_2}\right) = \frac{V_1-V_2}{nV_T}$

(1 mark)

$n = \frac{V_1-V_2}{V_T}/\ln\left(\frac{I_1}{I_2}\right) = 0.2/(25\times10^{-3})/\ln\left(\frac{2.2\times10^{-5}}{1.066\times10^{-7}}\right) = 1.5$

(2 marks)

$I = I_s e^{\frac{V}{1.5\times V_T}} \quad \Rightarrow \quad 1.066\times10^{-7} = I_s e^{\frac{0.6}{1.5\times25\times10^{-3}}}$

(1 mark)

Hence, $\quad I_s = 1.2\times10^{-14}$ A

(1 mark)

(b)

Perform the dc analysis on the circuit in Fig. Q6 with the ac source short-circuited:

$I_D = 1.2\times10^{-14} e^{\frac{0.7}{1.5\times25\times10^{-3}}} = 1.534\times10^{-6}$ A

(2 marks)

$R = \frac{1.0-0.7}{1.534\times10^{-6}} = 1.9557\times10^{5} \approx 196\ \text{k}\Omega$

(2 marks)

The diode small-signal resistance is obtained as follows:

$r_d = \frac{nV_T}{I_D} = \frac{1.5\times25\times10^{-3}}{1.534\times10^{-6}} = 24.4\ \text{k}\Omega.\ (\text{or}) \approx 25\ \text{k}\Omega$

(2 marks)

Perform the ac small-signal analysis on the circuit in Fig. Q6, with the dc source short-circuited and the diode replaced by r_d:

$v_{d,max} = \frac{r_d}{R+r_d} \times v_{dd,max} = \frac{25}{221} \times 0.01 \approx 0.0011\ \text{V} = 1.1\ \text{mV}$

(2 marks)

$v_{D,max} = V_D + v_{d,max} = 0.7 + 0.0011 = 0.7011\ \text{V}$

(2 marks)

Q7. For the circuit shown in Fig. Q7, $R_1 = R_2 = 1\ \text{k}\Omega$ and temperature, T = 300 K. The 2 pn-junction diodes, D_1 and D_2, are identical with a cut-in voltage of ~0.5 V, and a breakdown voltage of 6 V. When the diodes operate in substantial forward bias, they can be modelled by the constant-voltage-drop model with $V_{DO} = 0.72$ V.

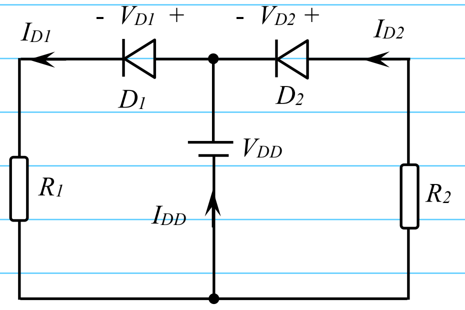


Fig. Q7

(a) Determine I_{D1} and I_{D2} for $V_{DD} = 0.3$ V.

(5 marks)

[Ans. $I_{D1} \approx 0$, $I_{D2} \approx 0$]

(b) Determine I_{D1}, I_{D2} and I_{DD} for $V_{DD} = 7$ V.

(5 marks)

[Ans. $I_{D1} = 6.28$ mA, $I_{D2} = -1$ mA, $I_{DD} = 7.28$ mA]

You need to provide explanation for your answers above.

Q7. **Solution:**

For $V_{DD} > 0$, D_1 is in forward bias and D_2 is in reverse bias.

(a) $V_{DD} = 0.3$ V

- $I_{D1} \approx 0$ [As $V_{DD} = 0.3$ V < cut-in voltage (~0.5 V), D_1 operates in forward bias and within the cut-in region]

  (2 marks for reasoning and 0.5 mark for numerical result for 2.5 total marks)

- $I_{D2} \approx 0$ [As $V_{DD} = 0.3$ V < $V_Z = 6$ V, D_2 operates in reverse bias, non-breakdown region]

  (2 marks for reasoning and 0.5 mark for numerical result for 2.5 total marks)

(b) $V_{DD} = 7$ V

- For $V_{DD} = 7$ V >> cut-in voltage (~0.5 V), D_1 is in substantial forward bias, $V_{D1} = V_{D0} = 0.72$ V.

  (1 mark)

  Apply KVL to the loop on the left,

$$I_{D1}R_1 + V_{D1} - V_{DD} = 0$$
$$I_{D1}R_1 + 0.72 - 7 = 0$$
$$I_{D1} = 6.28 \text{ mA}$$

  (1 mark)

- For $V_{DD} = 7$ V > $V_Z = 6$ V, D_2 is in reverse bias and in breakdown, $V_{D2} = -V_Z = -6$ V.

  (1 mark)

  Apply KVL to the loop on the right,

$$V_{DD} + V_{D2} + I_{D2}R_2 = 0$$
$$7 - 6 + I_{D2}R_2 = 0$$
$$I_{D2} = -1 \text{ mA}$$

  (1 mark)

- Apply KCL: $I_{DD} = I_{D1} - I_{D2} = 7.28$ mA.

  (1 mark)

Q8. The pn-junction diode used in the circuit of Fig. Q8(a) has the I-V characteristic shown in Fig. Q8(b) and its exponential factor, n = 1.2. In Fig. Q8(a), V_{DD} = 5 V, and the rms value of the small-signal ac source, v_{dd} (t), is 5 mV. The temperature, T = 300 K.

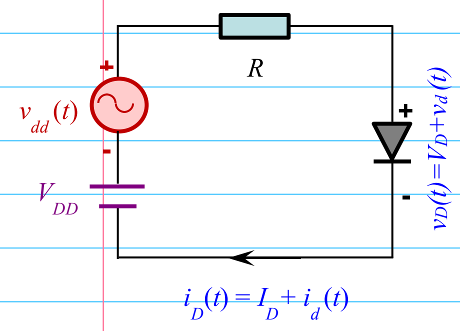


Fig. Q8(a)

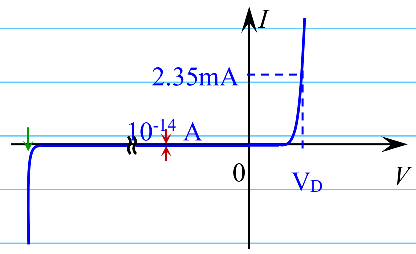


Fig. Q8(b)

(a) What is the value of the saturation current of the pn-junction diode?

[Ans: 10^{-14} A]

(b) The pn-junction diode in Fig. Q8(a) is operating with a DC current I_D of 2.35 mA. Determine the corresponding value of V_D.

[Ans: 0.785 V]

(c) What is the value of the small-signal resistance, r_d, of the pn-junction diode in Fig. Q8(a)?

[Ans: 12.8 Ω]

(c) What is the rms value of small-signal current $i_{d,rms}$ of the pn-junction diode in Fig. Q8(a)?

[Ans: 2.77 μA]

Q8. **Solution:**

(a)

$$I_S = 10^{-14}\text{A} \text{ (by observation in Fig. Q8(b))}$$

(b)

The pn-junction diode in Fig. Q8(a) operates in forward bias, as its p-type side is connected to the positive terminal of V_{DD}.

At a forward bias current, $I_D = 2.35$ mA, the diode voltage V_D can be calculated as follows:

$$I_D = I_S\left(e^{\frac{V_D}{nV_T}} - 1\right) \approx I_S e^{\frac{V_D}{nV_T}}$$

$$2.35m = 10^{-14} e^{\frac{V_D}{1.2\times0.025}}$$

$$V_D = 0.785\ V$$

(c)

$$r_d = \frac{nV_T}{I_D} = \frac{1.2\times0.025}{2.35\text{m}} = 12.8\ \Omega$$

(d) R needs to be determined first, which can be done by considering dc operation (i.e., in the absence of the ac small-signal voltage source, $v_{dd}(t)$). Apply KVL -

$$R = \frac{V_{DD}-V_D}{I_D} = \frac{5-0.785}{2.35\text{m}} = 1.79\ \text{k}\Omega$$

Consider ac operation (i.e., ac short the dc voltage, V_{DD}) and apply KVL –

$$i_{d,rms} = \frac{v_{dd,rms}}{R+r_d} = \frac{5\text{m}}{1.79\text{k}+12.8} = 2.77\ \mu\text{A}$$

Benedict Chin (A0284578J)

1)

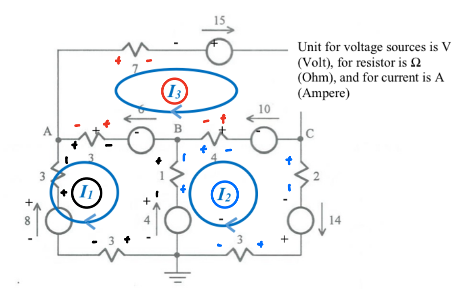


Fig. Q1

Using mesh analysis,

Loop 1: $-8 + 3I_1 + 3(I_1 - I_3) + 6 + (I_1 - I_2) + 4 + 3I_1 = 0$

$2 + 3I_1 + 3I_1 - 3I_3 + I_1 - I_2 + 3I_1 = 0$

$\therefore \quad 10I_1 - I_2 - 3I_3 = -2$ //

Loop 2: $-4 + (I_2 - I_1) + 4(I_2 - I_3) + 10 + 2I_2 - 14 + 3I_2 = 0$

$-4 + I_2 - I_1 + 4I_2 - 4I_3 + 10 + 2I_2 - 14 + 3I_2 = 0$

$\therefore \quad 10I_2 - I_1 - 4I_3 = 8$ //

Loop 3: $-15 - 10 + 4(I_3 - I_2) - 6 + 3(I_3 - I_1) + 7I_3 = 0$

$4I_3 - 4I_2 + 3I_3 - 3I_1 + 7I_3 = 31$

$\therefore \quad 14I_3 - 4I_2 - 3I_1 = 31$ //

Using node voltage analysis,

Node A: $V_a - 3(-I_1) - 8 - 3(-I_1) = 0$

$V_a = 8 - 6I_1$

Node B: $V_B - 1(I_1 - I_2) - 4 = 0$

$V_B = I_1 - I_2 + 4$

Node C: $V_C - 2(I_2) + 14 - 3(I_2) = 0$

$V_C = 2I_2 + 3I_2 - 14$

$= 5I_2 - 14$

2)

current is already 1mA no need to do KVL

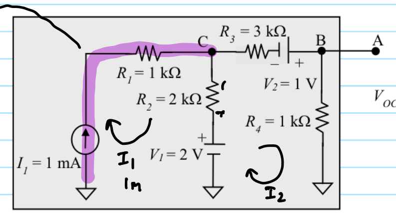
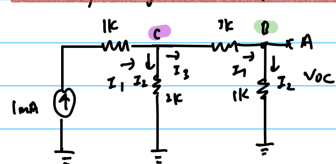


Fig. Q2

a) $\boxed{\text{Kill } V_1, V_2 \text{ by short circuit, find current source}}$

* RMB NEXT TIME SUPERPOSITION NEED TO DO ON INDIVIDUAL SOURCES (1 by 1)



KCL @ C : $\underset{(in)}{I_1} = \underset{(out)}{I_2 + I_3}$

$I_m = \frac{V_C - 0}{2k} + \frac{V_C - V_B}{3k}$

$I_m = \frac{V_C}{2k} + \frac{V_C - V_B}{3k}$ — ①

KCL @ B : $\underset{(in)}{I_1} = \underset{(out)}{I_2}$

$\frac{V_C - V_B}{3k} = \frac{V_B}{1k}$ — ②

$\frac{1k\,(V_C - V_B)}{3k} = V_B$

$V_B = \frac{1}{3}(V_C - V_B)$

$V_B + \frac{1}{3}V_B = \frac{1}{3}V_C$

$V_C = 4V_B$ — ③

Sub ③ into ①,

$I_m = \frac{4V_B}{2k} + \frac{4V_B - V_B}{3k}$

$I_m = \frac{2V_B}{1k} + \frac{V_B}{1k}$

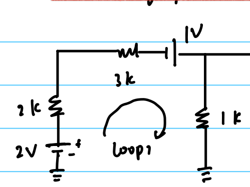
$I_m = \frac{3V_B}{1k}$

$3V_B = 1$

$V_B = \frac{1}{3}V$

$V_{B(V_1,V_2)} = \frac{1}{3}V$

$\boxed{\text{Kill } I_1 \text{ by open circuit}}$



By KVL,

$2 - 2k(I) - 3k(I) + 1 - 1k(I) = 0$

$3 = 6kI$

$I = 0.5mA$

$V_{BI_1} = 0.5m \times 1k$

$= \frac{1}{2}V$

$\therefore\ V_B = V_{OC} = V_{BI_1} + V_{B(V_1,V_2)}$

$= \frac{1}{2} + \frac{1}{3}$

$= \frac{5}{6}V$

b) Using mesh analysis, KVL (sum of P.D around closed-loop is zero)

Loop I_2: $2 - 2k(I_2 - I_1) - 3k(I_2) + 1 - 1k(I_2) = 0$

$2 - 2kI_2 + 2kI_1 - 3kI_2 + 1 - 1kI_2 = 0$

$3 = 6kI_2 - 2kI_1$ — (2)

Let $I_1 = 1mA$,

$3 = 6k\,I_2 - 2k(1m)$

$5 = 6kI_2$

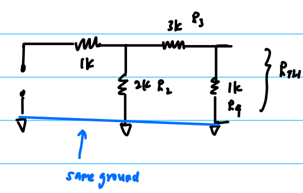
$I_2 = \frac{5}{6k}$

$V_B = V_{out} = I_2 \times 1k$

$= \frac{5}{6k} \times 1k$

$= \frac{5}{6} V$

Kill V_1 and V_2 by S.C and I_1 by O.C



c) $V_{OC} = V_{TH} = \frac{5}{6} V$

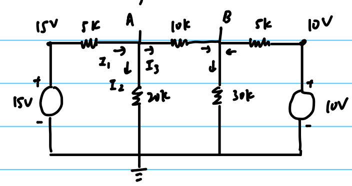
$R_{TH} = (R_2 + R_3) // R_4$

$= \left(\frac{5k \times 1k}{5k + 1k}\right) = \frac{5}{6} k\Omega$

For max power transfer, $R_{TH} = R_L = \frac{5}{6} k\Omega$

$P_{max} = \boxed{\left(\frac{V_{TH}}{R_{TH} + R_L}\right)^2 \times R_L} = \left(\frac{\frac{5}{6}}{\frac{5}{6}k \times 2}\right)^2 \times \frac{5}{6}k = \frac{5}{24} mW = 208 \mu W$

3) Vth R_L removed,



a) Using node analysis (KCL) method,

KCL at A: $\frac{15-A}{5k} = \frac{A}{20k} + \frac{A-B}{10k}$

$4(15-A) = A + 2(A-B)$

$60-4A-A = 2A-2B$

$60 = 7A-2B$

$2B = 7A-60$ —— (1)

KCL at B: $\frac{B}{30k} = \frac{A-B}{10k} + \frac{10-B}{5k}$

$B = 3(A-B) + 6(10-B)$

$B = 3A-3B+60-6B$

$60 = 10B-3A$ — (2)

Sub (1) into (2)

$60 = 5(7A-60) - 3A$

$60 = 35A - 300 - 3A$

$360 = 32A$

$A = 11.25V$ //

Sub A into (2)

$B = 9.375V$

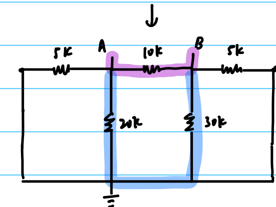
$\therefore V_{TH} = V_{ab} = V_A - V_B$

$= 1.875V$

To find R_{TH},

Kill V source by SC.

R_{THV}



$R_{THV} = (5k//20k + 5k//30k) // 10k$

$= (4k + 4.286kk) // 10k$

$= 4.53k\Omega$

b) $I_{sc} = \frac{V_{THV}}{R_{THV}} = \frac{1.875}{4.53k} = 0.414mA$ //

c) For max power xfer,

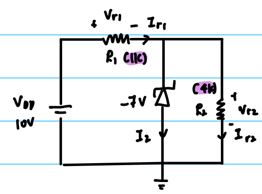
$R_L = R_{THV} = 4.53k\Omega$

Max power: $\left(\frac{V_{TH}}{R_{TH}+R_L}\right)^2 \times R_L$

$= \left(\frac{1.875}{4.53k\times2}\right)^2 \times 4.53k$

$= 0.194mW$

5)



Breakdown voltage $V_z = 7V$

a) Zener is NOT operating in breakdown region

Reverse saturation current ≈ 0

$I_{R1} = I_{R2}$

$I = \frac{10}{R_1 + R_2} = \frac{10}{1k + 4k} = 2mA$

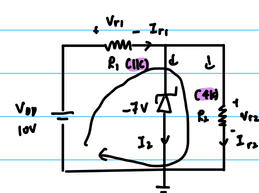
$V_{r2} = 8V$

Since $V_{r2} = 8V$ and it is in parallel with the zener diode, the zener diode should be in the breakdown region.

Hence, when zener diode is operating in breakdown region

$V_{r2} = 7V$

$I_{r2} = \frac{V_{r2}}{R_2} = \frac{7}{4k} = 1.75mA$



KVL : Loop1

$V_{DD} - V_{r1} - V_{r2} = 0$

$V_{r1} = V_{DD} - V_{r2}$

$= 3V$

$I_{r1} = \frac{3}{1k} = 3mA$

Using KCL,

$I_{r1} = I_z + I_{r2}$

$I_z = I_{r1} - I_{r2}$

$= 3mA - 1.75mA$

$= 1.25mA$

b) When R_1 changed to 6k,

$I_{r1} = \frac{V_{DD}}{R_1 + R_2} = \frac{10}{6k + 4k} = 1mA$

$V_{r1} = 1mA \times 6\Omega$

$= 6V$

Using KVL,

$V_{DD} - V_{r1} - V_{r2} = 0$

$V_{r2} = V_{DD} - V_{r1}$

$= 10 - 6$

$= 4V$

$I_{r2} = \frac{V_{r2}}{r_2} = \frac{4}{4k} = 1mA$

Since $V_{r2} = V_2 = 4V$ < $V_{zener\ breakdown}$ (7V), reverse saturation current $I_z = 0mA$

c) R_1 must be low enough (compare (a) and (b) results) such that the zener diode is able to work in the breakdown region to provide a constant voltage of V_z to R_2

6) Forward bias current of pn-junction at room temp : $V_T = 25mV$

$V_D = 0.6V$: $1.066 \times 10^{-7} A$

$V_D = 0.8V$: $2.2 \times 10^{-5} A$

a) $I = I_S (e^{\frac{V}{nV_T}} - 1) \approx I_S (e^{\frac{V}{nV_T}})$

$\frac{I_1}{I_2} = e^{\frac{V_1 - V_2}{nV_T}}$

$\ln\left(\frac{I_1}{I_2}\right) = \frac{V_1 - V_2}{nV_T}$

$n = \frac{V_1 - V_2}{V_T} \Big/ \ln\left(\frac{I_1}{I_2}\right)$

$= \frac{0.8 - 0.6}{25m} \Big/ \ln\left(\frac{2.2 \times 10^{-5}}{1.066 \times 10^{-7}}\right)$

$= 1.466$

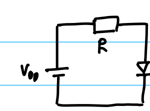
≈ 1.5 //

$I = I_S (e^{\frac{V}{nV_T}})$ (use either one to sub in)

$2.2 \times 10^{-5} = I_S \times (e^{\frac{0.8}{1.5(25m)}})$

$I_S \approx 1.2 \times 10^{-14}$ //

b) Perform DC analysis on the circuit, Short circuit ac signal



$v_D(t) = V_D + v_d(t)$
DC AC

$I_D = I_S e^{\frac{V}{nV_T}}$

$= (1.2 \times 10^{-14}) \times e^{\frac{0.7}{1.5(25m)}}$

$= 1.53 \mu A$

$R = \frac{V_{DD} - V_D}{I} = \frac{1 - 0.7}{1.53\mu}$

$\approx 195 k\Omega$

Diode small signal resistance,

$r_d = \frac{nV_T}{I_D} = \frac{1.5(25m)}{1.53\mu}$

$= 24.51 k\Omega$

Hence, perform ac analysis, short DC, diode replace with r_d

$v_d(t) = 0.01 \times \frac{r_d}{r_d + R}$

$= 0.01 \times \frac{24.511}{24.51 + 195}$

$= 1.11 mV$

Hence, $v_D(t) = V_D + v_d(t)$

$= 0.7V + 1.1mV$

$= 0.701 V$ //

7)

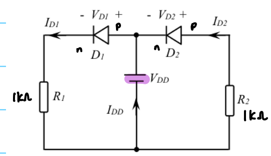


Fig. Q7

Cut in voltage: $V_{D1} = V_{D2} = 0.5V$

Breakdown voltage: $V_{D1} = V_{D2} = -6V$

In substantial forward bias, $V_{DD} = 0.72V$

a) When $V_{DD} = 0.3V$, D_1 is in forward bias and D_2 is in reverse bias. Since D_2 is in reverse bias, the reverse current is negligible, $I_{D2} \approx 0$ meaning that it can be treated like an open circuit. Hence, $I_{D2} = 0A$.

For diode D_1, since $V_{DD} = 0.3V < V_{cut\ in} (\sim 0.5V)$, forward current is negligible. Hence, $I_{D1} = 0A$.

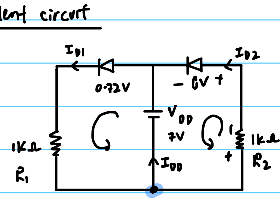
$\therefore I_{D1} : 0A$

$\therefore I_{D2} : 0A$

b) When $V_{DD} = 7V$, D_1 is forward bias and D_2 is reverse bias. Since $V_{DD} = 7V$ exceeds the diode breakdown voltage of 6V, reverse current passing through diode D_2 is no longer ≈ 0 but becomes very large. Furthermore, the voltage of diode D_2 remains fixed at $-V_{breakdown} = -6V$.

For diode D_1, since it is forward bias, and $V_{DD} = 7V > V_{cut\ in} = \sim 0.5V$, forward current flows through the diode. Since the diodes operate in substantial forward bias, voltage drop across D_1 can be modelled by constant-voltage drop model with $0.72V$.

Equivalent circuit



$I_{D1} = \frac{7 - 0.72}{1k} = 6.28mA$

$-I_{D2} R_2 - V_{D2} - V_{DD} = 0$

$7 + (-6) = -(1k)(R_2)$

$R_2 = \frac{1}{-1k}$

$= -1mA$

$I_{DD} + I_{D2} = I_{D1}$

$I_{DD} = 7.28m$

8)

a) Saturation current I_s = 10^{-14} A

b) PN junction diode operating at DC I_D : 2.35 mA (significant forward current)

$$I = I_s\left(e^{\frac{v}{nV_T}} - 1\right) \approx I_s e^{\frac{v}{nV_T}}$$

$$2.35m = 10^{-14}A\left(e^{\frac{v}{(1.2)(25m)}}\right)$$

$$v = \ln\left(\frac{2.35m}{10^{-14}}\right) \times (1.2)(25m)$$

$$= 0.785V$$

c) $\boxed{r_d = \frac{nV_T}{I_D}} = \frac{1.2(25m)}{2.35m} = 12.8\Omega$

d) $R = \frac{V_{DD} - V_D}{I_D} = \frac{5 - 0.785}{2.35m}$

$\approx 1.8k\Omega$

$i_{d,rms} = \frac{v_{dd}(t)}{R + r_d} = \frac{5m}{1.8k + 12.8} = 2.76\mu A$