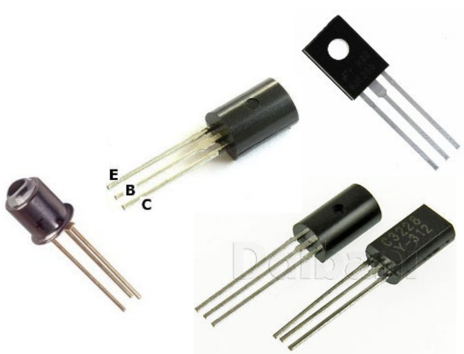
# *Bipolar Junction Transistor (BJT)*

## Bipolar Junction Transistor (BJT)

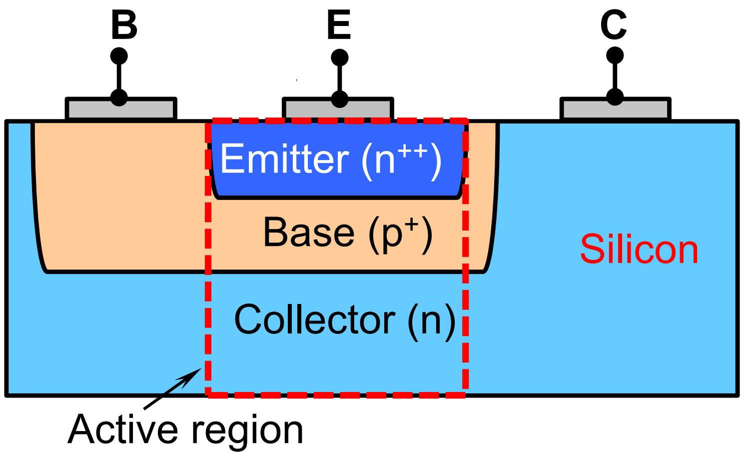
1. Introduction
2. Modes of Operation: Forward Active, Saturation, Cut-Off and Reverse Active
3. Forward Active Current-Voltage Characteristic
4. Large-Signal Model and dc Analysis
5. Amplifier Circuit
6. Small-Signal Model (Simplified Hybrid-$\pi$) and ac Analysis
7. Other Hybrid-$\pi$ Small-Signal Models

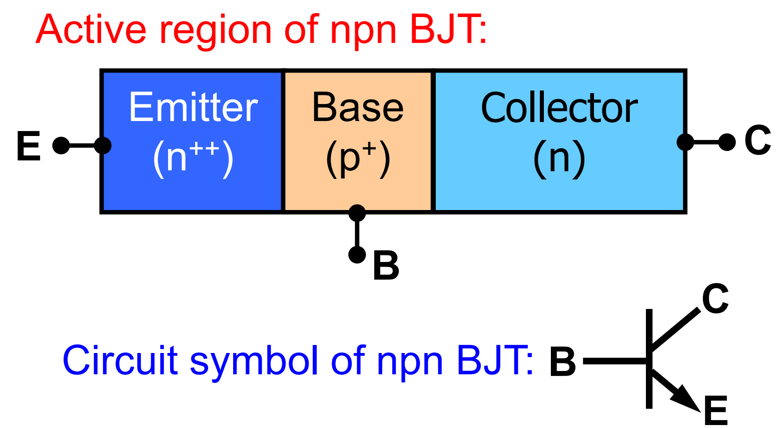



## Reference

- Sedra and Smith, Microelectronic Circuits, Fifth Edition, Oxford (2004), pp. 159 – 168, 173 - 183, 203 - 241.

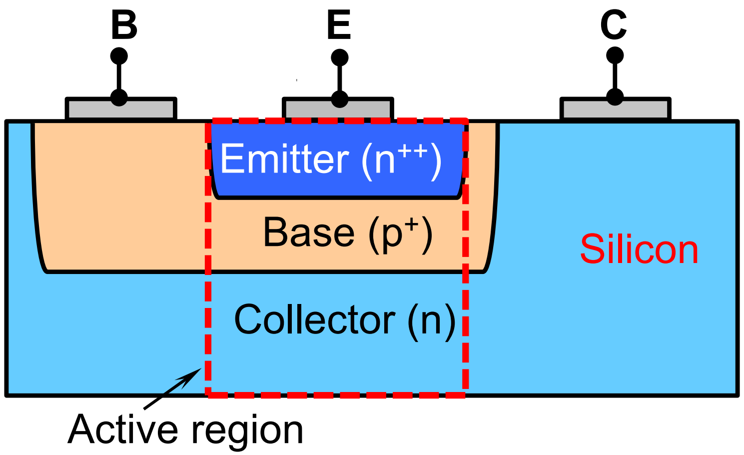
# *BJT* – Introduction (Structure)



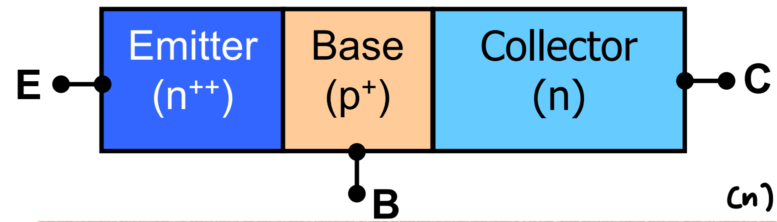




- Bipolar junction transistor (BJT) is a 3-terminal device made using a single crystal semiconductor (typically silicon), just like the *pn*-junction diode.
- BJT is made with 3 doped semiconductor regions, namely emitter, base and collector, corresponding to the 3 terminals.
- Left figure above shows a schematic cross-sectional view of an npn BJT, where the emitter is n-type, base is p-type and collector is n-type.
- The active region of the BJT is the region under (and including) the emitter. This is the part of the device that provides, for example, the amplifying function of the BJT. The rest of the structure is to facilitate the movement of the currents into or out of the transistor. We will focus on the active region of BJT.

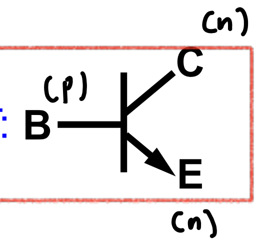
# *BJT* – Introduction (Structure)




Active region of npn BJT:




Circuit symbol of npn BJT:

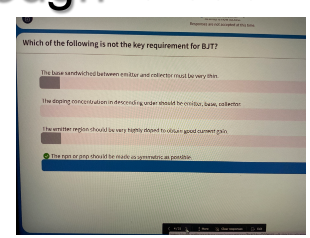



- BJT is not a symmetrical device, in particular, impurities added to the emitter is at a much higher concentration than that added to collector. Hence, emitter is indicated as $n^{++}$-type (very heavily doped and has many more electrons) and collector as n-type.

- Concentration of impurities (of a different type from that for emitter/collector) added to base is in between those of emitter and collector. Base is thus indicated as $p^{+}$-type.

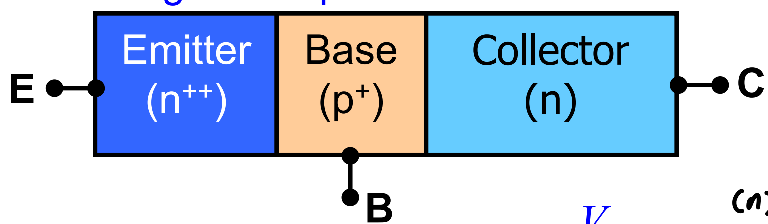
- BJT comprises two back-to-back *pn*-junctions*: emitter-base junction and base-collector junction. The two *pn*-junctions must be close enough to each other to interact, and this requires the base to be thin.

* BJT cannot be made by connecting two *pn*-junctions back-to-back.

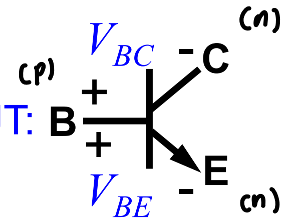


# *BJT* – Introduction (npn vs pnp BJT)

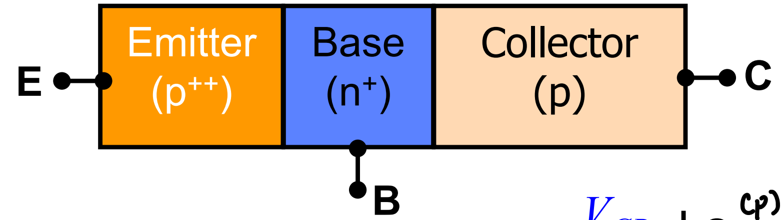
Active region of npn BJT:



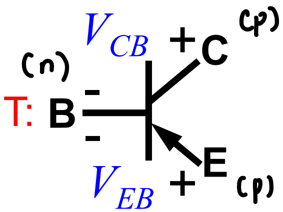

Circuit symbol of npn BJT:




Active region of pnp BJT:




Circuit symbol of pnp BJT:




- The emitter-base junction and base-collector junction can be either forward biased or reverse biased.
- There are two types of BJT: npn and pnp (as shown above). Note the difference between the 2 structures and circuit symbols.
- For pnp BJT, the emitter is $p^{++}$-type, base is $n^{+}$-type and collector is p-type.
- Both npn and pnp BJTs perform similar functions: amplification and switching. Their difference lies in the biasing polarity of the two junctions when operating. For a given mode of operation, the current directions and voltage polarities are exactly opposite between npn and pnp BJTs.
- Note that BJT is a nonlinear device, just like the *pn*-junction.

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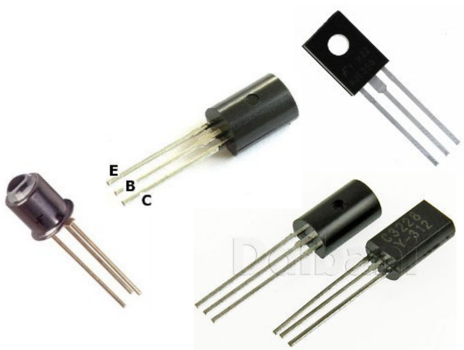


## Reference

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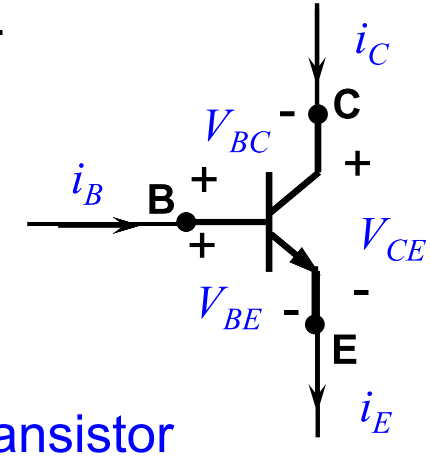
# *BJT* – Modes of Operation

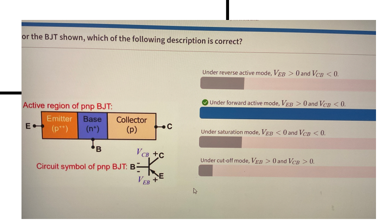
- BJT has 2 *pn*-junctions (emitter-base junction and collector-base junction), and each *pn*-junction can be either forward biased or reverse biased, hence there are 4 possible permutations of the biasing arrangement for a BJT. These correspond to different modes of operation of the BJT, as shown in the table below for an npn BJT*:




Modes of operation of the **npn** bipolar junction transistor

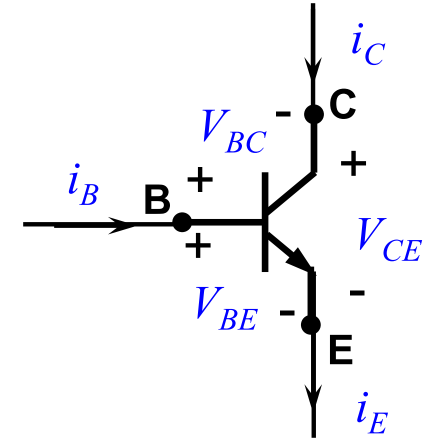
| Mode of Operation | Emitter-Base Junction | Collector-Base Junction | Application |
|---|---|---|---|
| Cut-off | Reverse biased ($V_{BE}$ < 0 for npn) | Reverse biased ($V_{BC}$ < 0 for npn) | Logic - OFF State |
| Forward Active | Forward biased ($V_{BE}$ > 0 for npn) | Reverse biased ($V_{BC}$ < 0 for npn) | Amplifier |
| Saturation | Forward biased ($V_{BE}$ > 0 for npn) | Forward biased ($V_{BC}$ >0 for npn) | Logic - ON State |
| Reverse Active | Reverse Biased ($V_{BE}$ < 0 for npn) | Forward Biased ($V_{BC}$ > 0 for npn) | Not used |

* A similar table for pnp bipolar junction transistor is shown in the Appendix A.



# *BJT* – Modes of Operation (for reference)

**Some definitions**:




- Voltage at the base with-respect-to the emitter (i.e., the base-emitter junction voltage) - $V_{BE}$
- Voltage at the base with-respect-to the collector (i.e., the base-collector junction voltage) - $V_{BC}$
- Voltage at the collector with-respect-to the emitter - $V_{CE}$ (= $V_{BE}$ - $V_{BC}$) or $V_{CE} = V_{CB} + V_{BE}$ or $V_{CE} = V_{CB} - V_{EB}$

❑ **Cut-off operation** (used as the OFF-state in logic circuit):

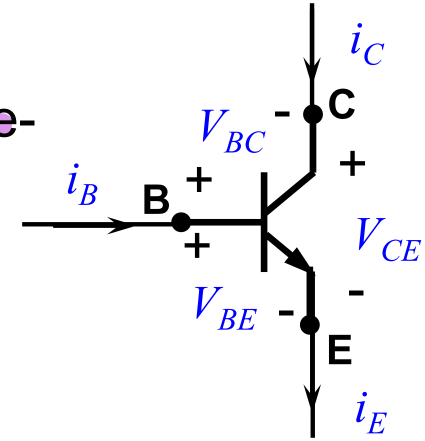
- Both emitter-base junction and base-collector junction are reverse biased.
- $V_{BE} < 0$ and $V_{BC} < 0$ for npn BJT.
- Practically no current flows in the BJT. Between collector and emitter is practically an open circuit.

❑ **Saturation operation** (Used as the ON-state in logic circuit):

- Both emitter-base junction and base-collector junction are forward biased.
- $V_{BE} > 0$ (~ 0.7 V) and $V_{BC} > 0$ (~0.7 V) for npn BJT.
- Large current flows in the BJT. Between collector and emitter is practically a short circuit, and $V_{CE}$ is a small voltage.

# *BJT* – Modes of Operation (for reference)

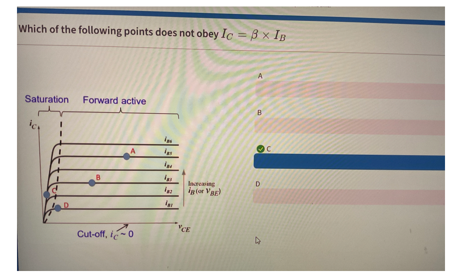
- **Forward active operation** (used in amplifier circuit):
  - Emitter-base ($n^{++}p^{+}$) junction is forward biased and base-collector ($p^{+}n$) junction is reverse biased.
  - $V_{BE} > 0$ and $V_{BC} < 0$ for npn BJT.
  - BJT works as a controlled current regulator. The main current through the BJT flows between the emitter and collector, $i_C$, and it is controlled by the base-emitter junction voltage, $v_{BE}$ (or equivalently, the base current, $i_B$). This is transistor action and is made possible by an $n^{++}$-type emitter and $p^{+}$-type base, and a thin base. See slides BJT-11 and -12.

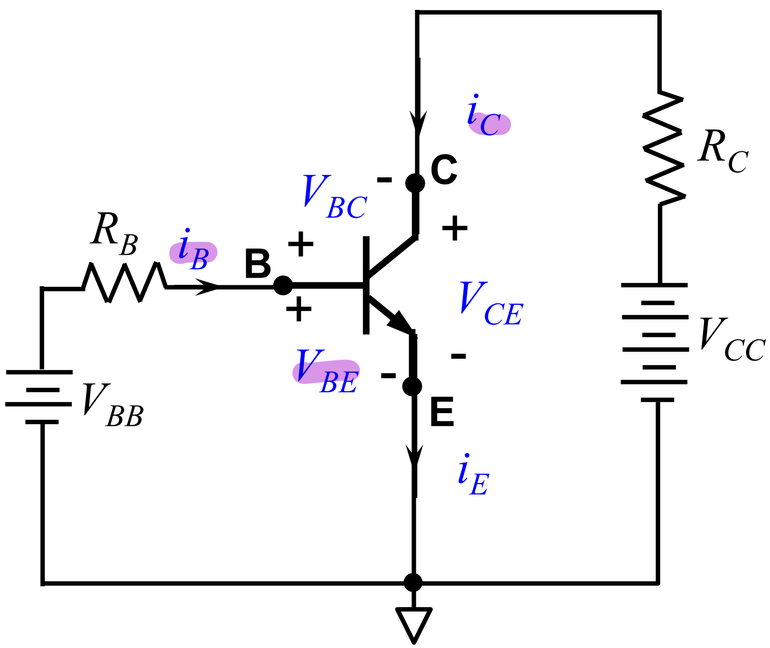


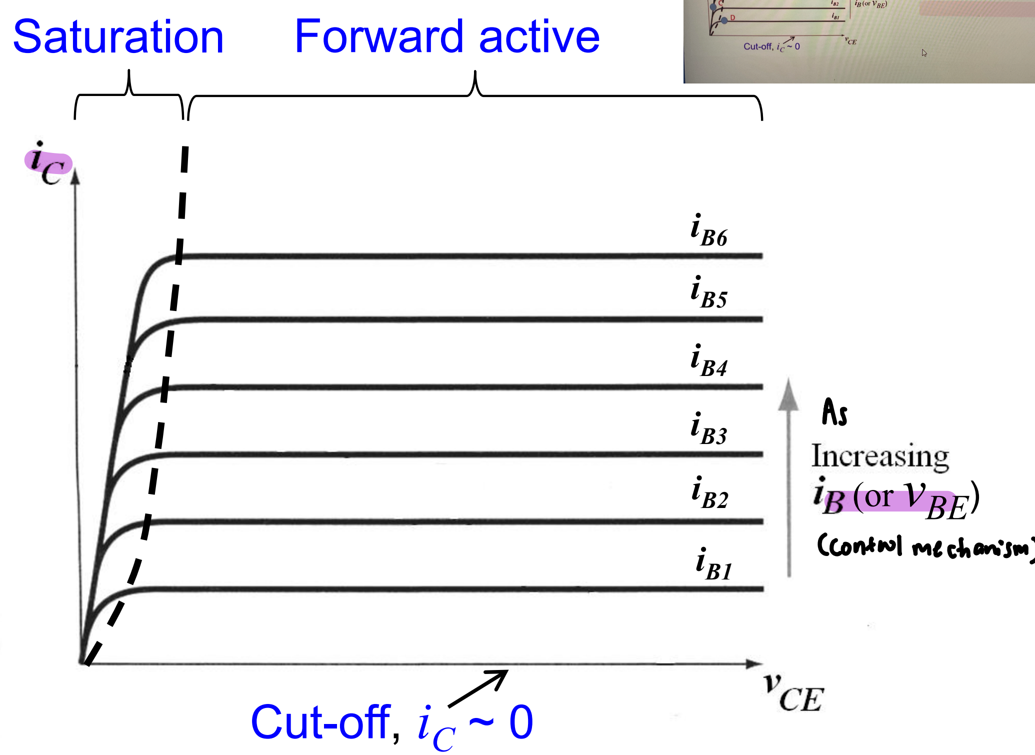

- **Reverse active operation** (not used):
  - Emitter-base ($n^{++}p^{+}$) junction is reverse biased and base-collector ($p^{+}n$) junction is forward biased.
  - $V_{BE} < 0$ and $V_{BC} > 0$ for npn BJT.
  - This mode of operation is not useful, as it does not provide amplification, unlike the forward active operation. This is owing to the non-symmetrical nature of BJT, specifically, collector being n-type while emitter is $n^{++}$-type.

# *BJT* – Modes of Operation

- The circuit below shows an npn BJT in common-emitter configuration. The common-emitter IV characteristics are also plotted, which shows the relationships between the collector current, $i_C$, and the collector-emitter voltage, $v_{CE}$, for different base currents, $i_B$, (or equivalently different $v_{BE}$).
- The regions corresponding to the forward active, saturation and cut-off modes of operation are as indicated in the plot below.
- We shall be focusing on BJT in forward active operation.



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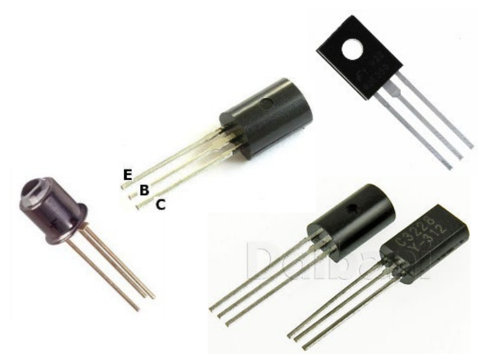


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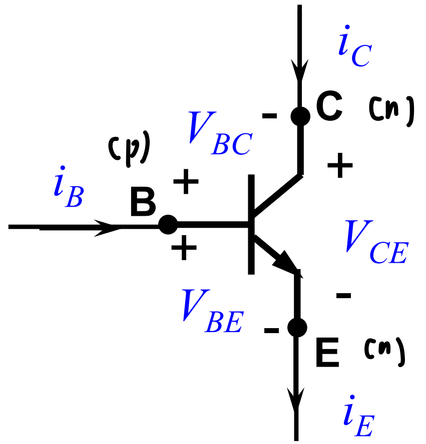
# *BJT* – Forward Active IV Characteristic

- Discussion will be based on an npn BJT, which applies to a pnp BJT as well, except for the bias voltage polarities and current directions (see Appendix B).
- In forward active operation, emitter-base junction is forward biased and base-collector junction is reverse biased: $v_{BE}$ > 0 and $v_{BC}$ < 0 for an npn BJT. The collector current, $i_C$, which flows into the collector and through the reverse-biased base-collector junction is controlled by $v_{BE}$ -

$$\boxed{i_C = I_S e^{v_{BE}/V_T}.} \qquad (3.1)$$




- The base current, $i_B$, which flows into the base and through the forward-biased base-emitter junction is

$$\boxed{i_B = \frac{I_S}{\beta} e^{v_{BE}/V_T}.} \qquad (3.2)$$

- In the above, $I_S$ is the saturation current; $V_T$ is the thermal voltage ($V_T = kT/q \approx$ 0.025 V at $T$ = 300 K); and $\beta$ is the common-emitter current gain ($\beta = i_C/i_B$).
- From equations (3.1) and (3.2) -

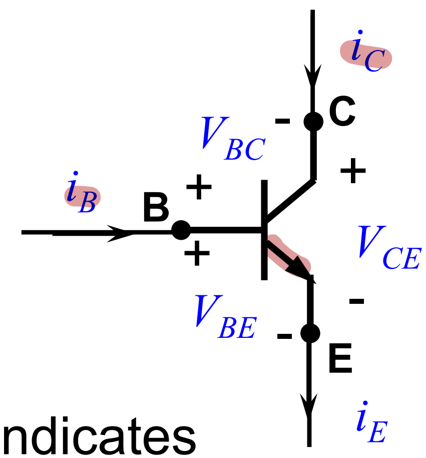
$$\boxed{i_C = \beta i_B} \qquad (3.3)$$

→ only true for forward active mode

# *BJT* – Forward Active IV Characteristic

- From equation (3.1): BJT works as a controlled current regulator, where the main current, $i_C$, that flows through the reverse-biased base-collector junction does not depend on $v_{BC}$, but is controlled by the voltage across a close by forward-biased base-emitter junction, $v_{BE}$. This is transistor action.
- Alternatively, $i_C$ is seen to be controlled by $i_B$ in equation (3.3). Typically, $i_C$ is much greater than $i_B$, e.g., 100 times.
- By Kirchhoff's Current Law (KCL), the emitter current is

$$\boxed{i_E = i_C + i_B} = I_B(\beta + 1) \qquad (3.4)$$




- $i_E$ flows out of emitter for npn BJT.
- The arrow head at the emitter in the BJT circuit symbol indicates the direction of current flow in forward active operation.

or

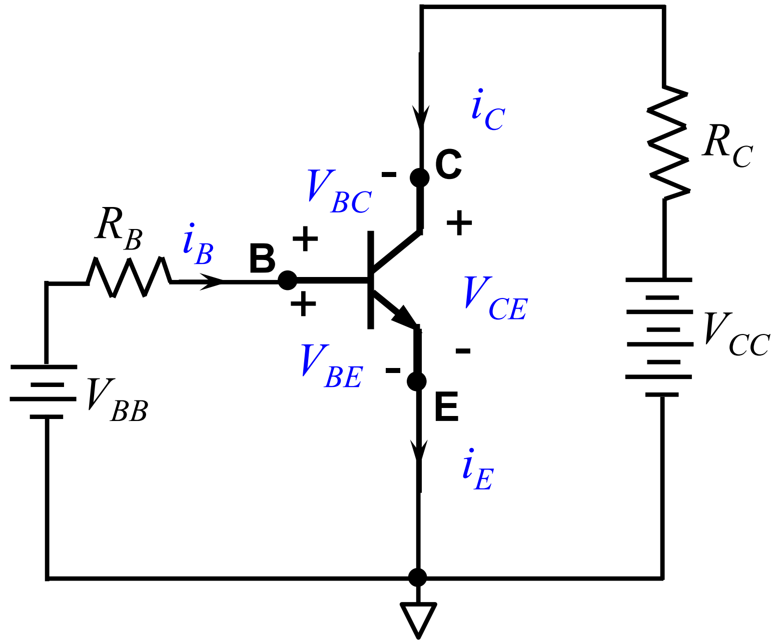
$$i_E = i_C + \frac{i_C}{\beta} = \left(\frac{\beta+1}{\beta}\right) i_C$$

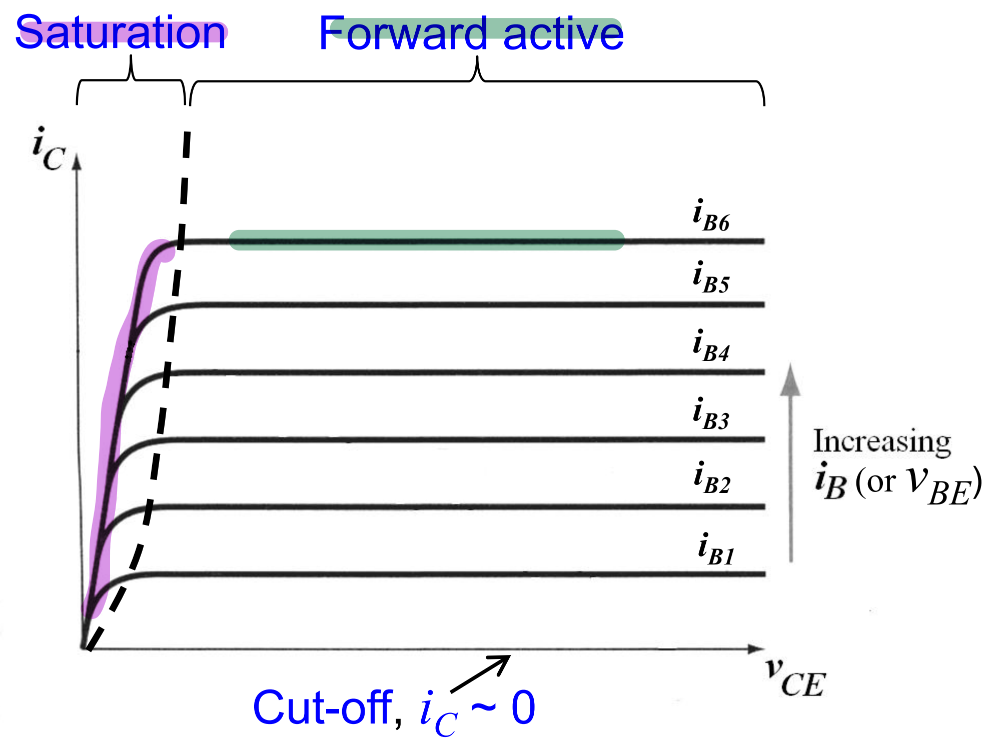
$$i_C = \frac{\overbrace{\beta}^{\approx\ 100-200}}{\beta+1}\, i_E$$

Hence approx, $i_C \simeq i_E$

# *BJT* – Forward Active IV Characteristic (Ideal)

Ideal common-emitter IV characteristics of an npn BJT -



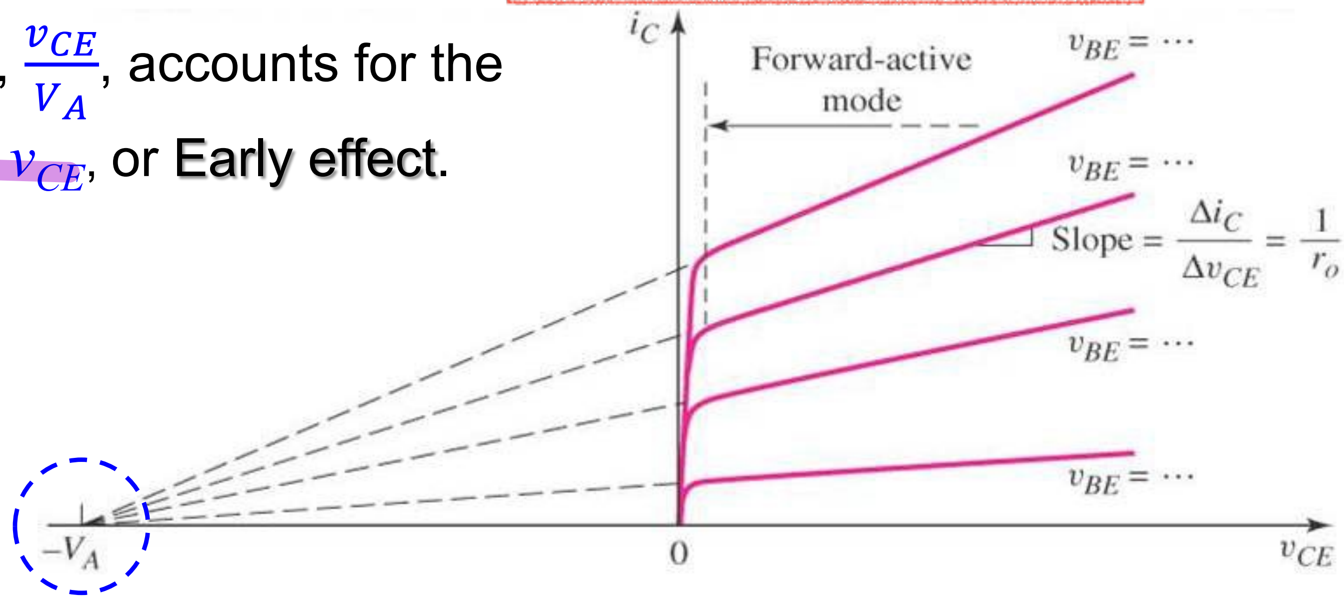




In forward active operation, emitter-base junction is forward biased, hence $v_{BE} \approx$ 0.7 V; while base-collector junction is reverse biased, $v_{BC} < 0$ for npn BJT.

- From eqns. (3.1) and (3.3), $i_C$, is a strong function of $v_{BE}$ (or $i_B$).
- $v_{CE} = v_{CB} + v_{BE} \approx v_{CB} = -v_{BC}$ for $v_{CE} \gg v_{BE} \approx 0.7$ V. much greater tha
- From eqn. (3.1), $i_C$, is not dependent on $v_{BC}$, hence not dependent on $v_{CE}$ in forward active operation, where $v_{CE} \gg v_{BE} \approx 0.7$ V.

# BJT – Forward Active IV Characteristic (Non-Ideal)

- In a real (non-ideal) BJT, $i_C$ is not independent on $v_{BC}$ (hence $v_{CE}$) in forward active operation. This is known as the Early Effect*.
- The $i_C$-$v_{CE}$ characteristics would then have a slight upward slope, as shown in the figure below.
- When all $i_C$-$v_{CE}$ characteristics are projected to the negative $v_{CE}$-axis, they meet approximately at the same voltage, -$V_A$.
  - $V_A$ is known as the Early voltage, and is typically large, e.g., ~100 V.
- Expression for $i_C$-$v_{CE}$ is then modified to: $\boxed{i_C = I_S e^{v_{BE}/V_T}\left(1 + \frac{v_{CE}}{V_A}\right)}$ (3.5)
- The additional factor, $\frac{v_{CE}}{V_A}$, accounts for the dependence of $i_C$ on $v_{CE}$, or Early effect.




*Early Effect is also known as the base-width modulation.

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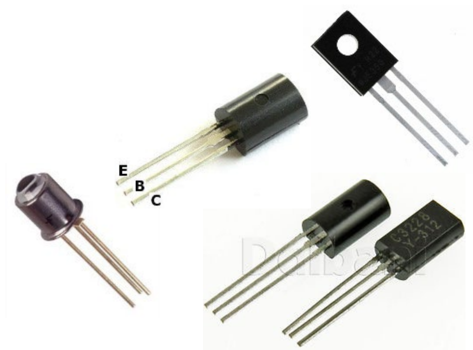


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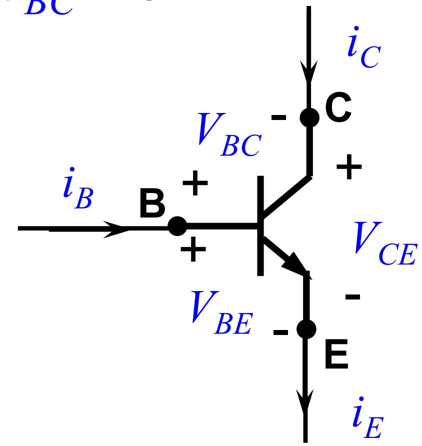
# *BJT* – Large-Signal Model (npn BJT)

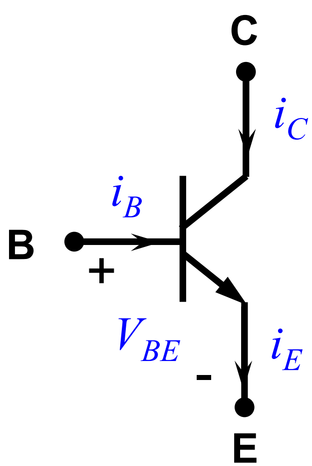
- For an npn BJT in forward active operation: $v_{BE} > 0$ and $v_{BC} < 0$.

  - $i_C = I_S e^{v_{BE}/V_T}$ (3.1)
  - $i_B = \frac{I_S}{\beta} e^{v_{BE}/V_T}$ (3.2)
  - $i_C = \beta i_B$ (3.3)

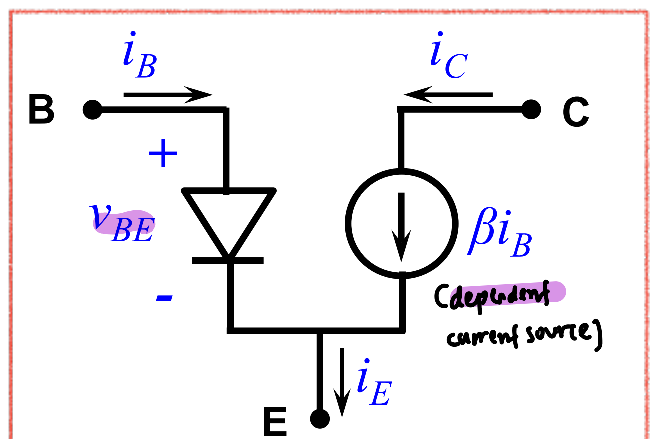



- We can model an npn BJT using

  - $i_B = \frac{I_S}{\beta} e^{v_{BE}/V_T}$ → a diode (between the emitter-base junction)
  - $i_C = \beta i_B$ → a dependent current source




**is equivalent to**

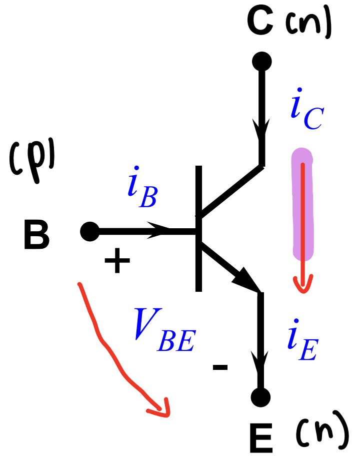



Under forward active, $V_{BE}$ (emitter-base) forward biased (diode), $V_{CE}$ (collector-emitter) is constant current (pg 12)

# *BJT* – Large-Signal Model (npn BJT)

- As the emitter-base junction is forward biased, $v_{BE} \approx 0.7$ V, it can be modeled by a constant voltage source (using the constant-voltage-drop model for a forward biased pn-junction).

npn BJT

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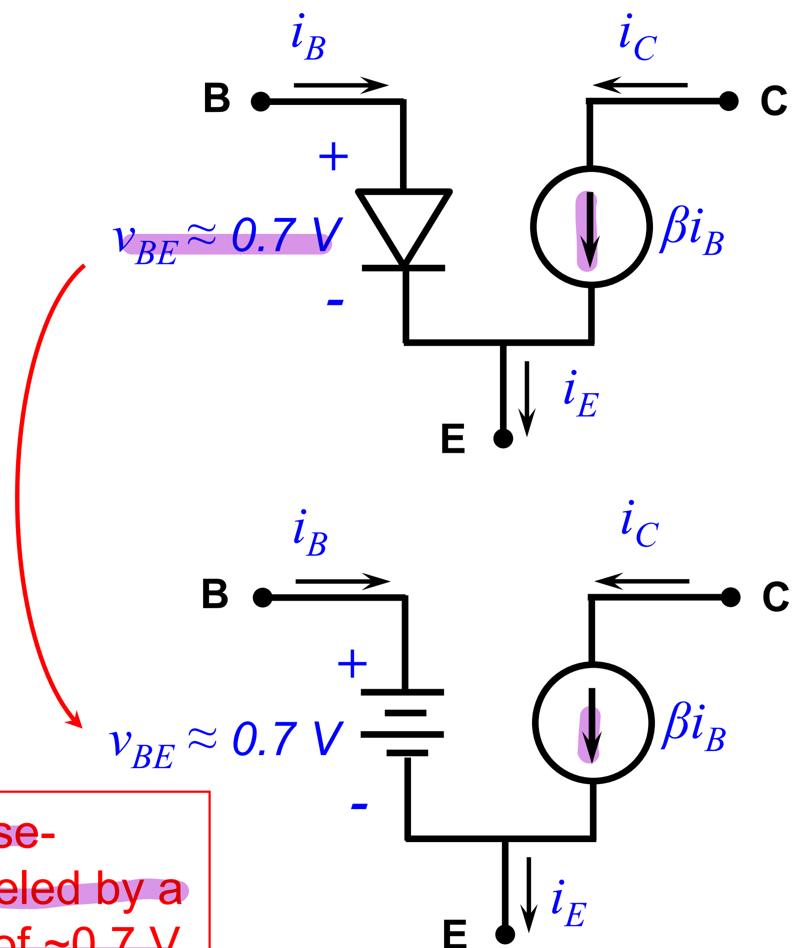
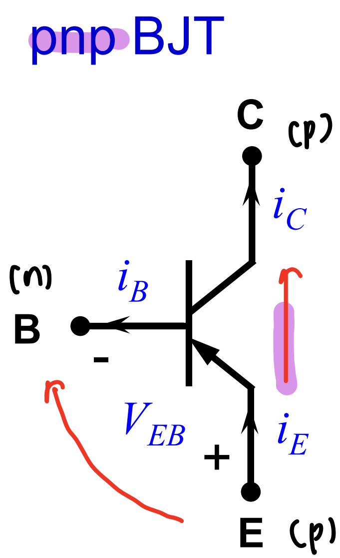



The forward biased base-emitter junction is modeled by a constant-voltage-drop of ~0.7 V

# *BJT* – Large-Signal Model (pnp BJT)

- Large-signal model of a pnp BJT: polarities of voltages and directions of currents are opposite to those of an npn BJT.

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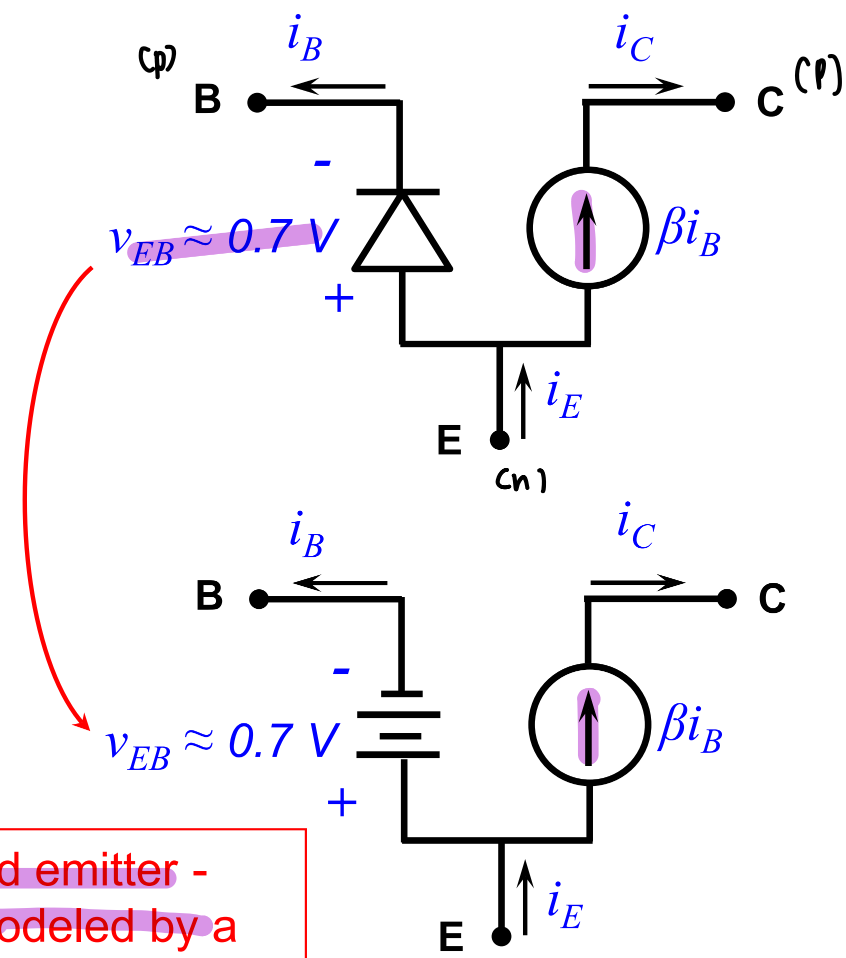
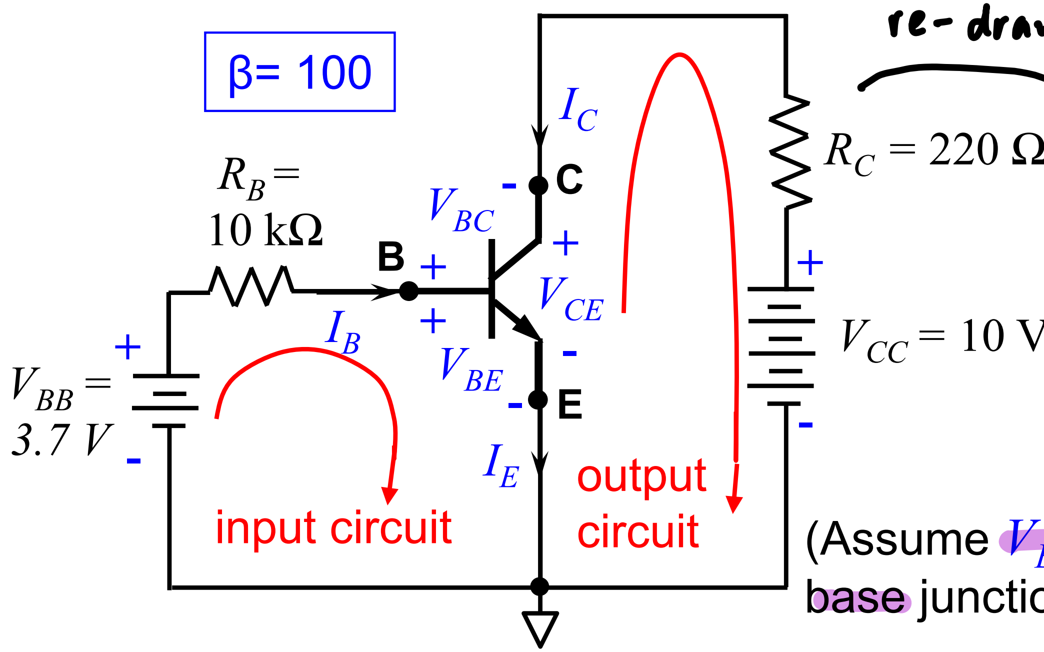
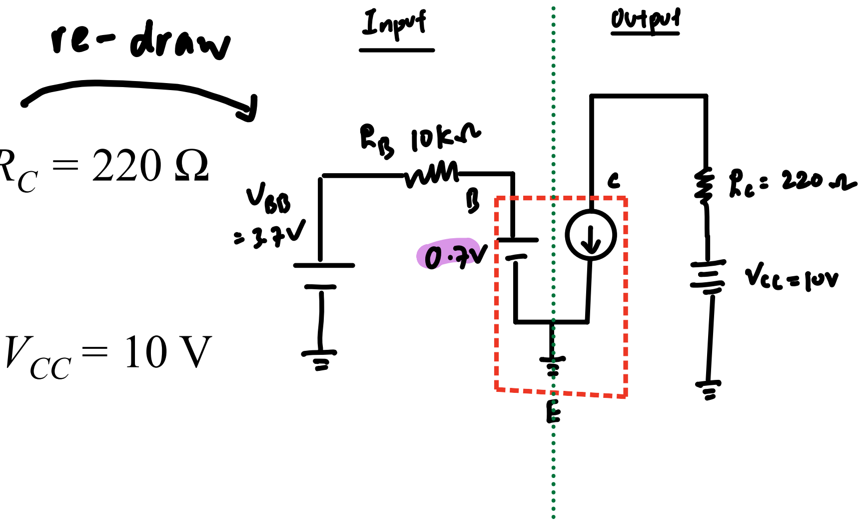



The forward biased emitter - base junction is modeled by a constant-voltage-drop of ~0.7 V

# *BJT* – Operating Point (Bias Point): DC Analysis



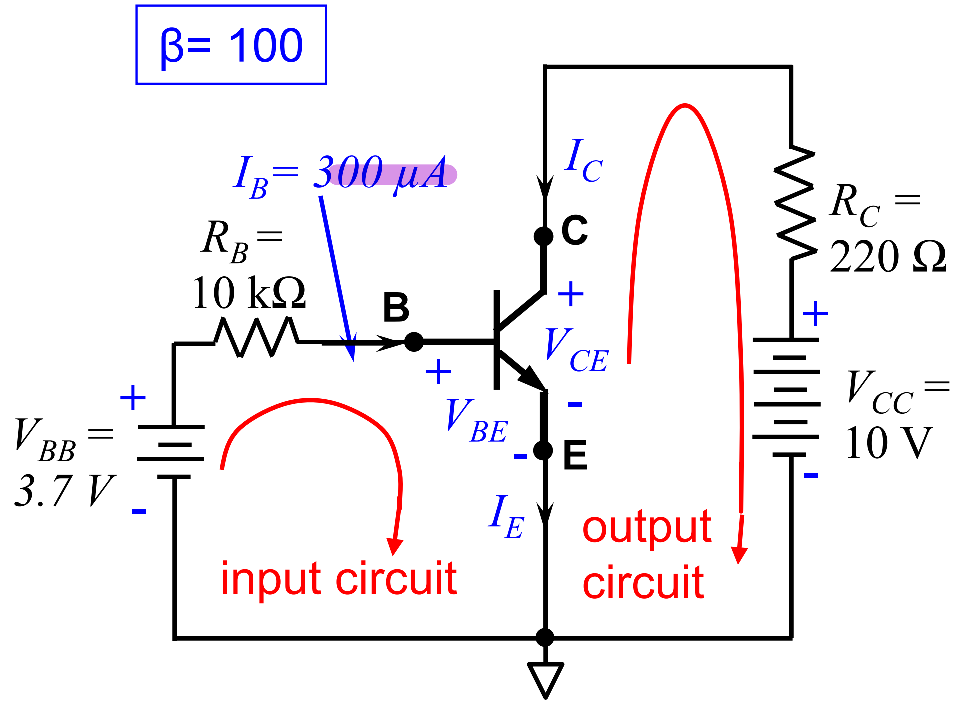

(Assume $V_{BE} \approx 0.7$ V, as emitter-base junction is forward biased)

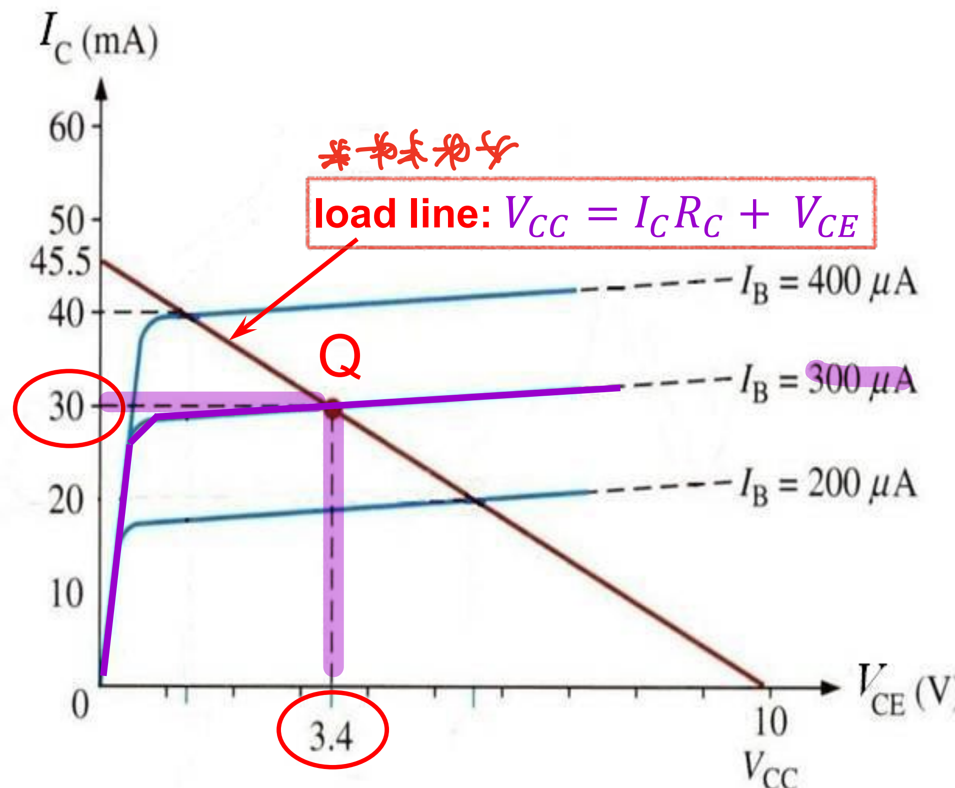
- We want to determine the dc operating point (also known as bias point) voltages and currents of the common-emitter amplifier circuit shown above.
- Note: all currents and voltages in above circuit are dc values and are denoted by capital letter symbol and capital subscript: for examples, $I_B$ for base current and $V_{BE}$ for base-emitter junction voltage.

$I_C = \beta I_B = 30\text{mA}$ ($\beta$ = 100)

- Input circuit, using KVL: $I_B = \frac{V_{BB} - V_{BE}}{R_B} \approx \frac{3.7 - 0.7}{10\text{k}} = 0.3 \text{ mA} = 300 \text{ µA}$
- Output circuit, using KVL: $V_{CC} = I_C R_C + V_{CE}$ (known as the load line)

# *BJT* – Operating Point (Bias Point): DC Analysis




- $I_C$ and $V_{CE}$ are also governed by the $I_C$-$V_{CE}$ characteristics of BJT shown above for different $I_B$).
- BJT is operating at $I_B$ = 300 μA, as determined earlier.
- Operating point of BJT (point Q) is the intersection between its $I_C$-$V_{CE}$ characteristic for $I_B$ = 300 μA and the output circuit load line:

$$V_{CC} = I_C R_C + V_{CE} \qquad \text{or} \qquad I_C = (V_{CC} - V_{CE})/R_C$$

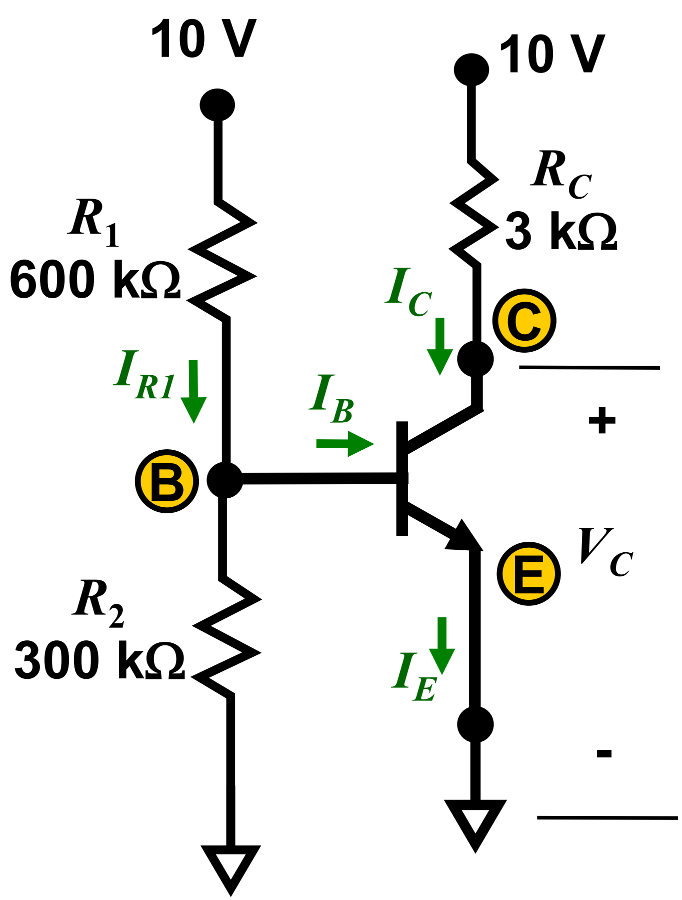
- At the operating point, Q: $I_C$ = 30 mA and $V_{CE}$ = 3.4 V (determined graphically).

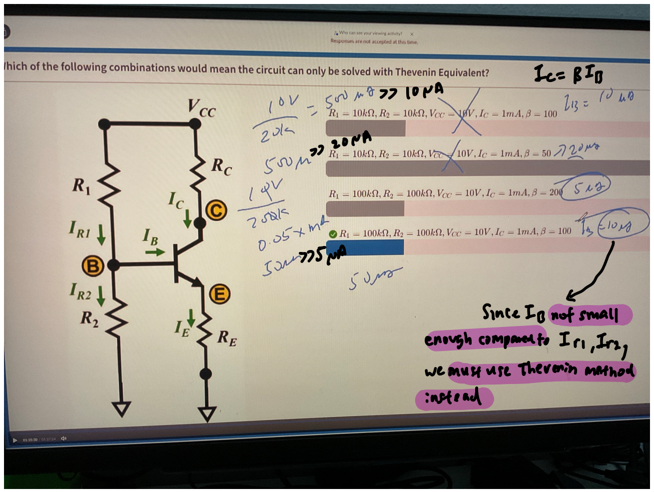
# *BJT* – Calculation of Operating Point

## Example 1 (Thevenin equivalent method)

→ Use this if method 2 assumption (pg 25) fails

Find the currents $I_C$, $I_B$, and $I_E$ , and the voltage $V_C$ using the large-signal model for the BJT, assuming that it is operating in the forward active region. The value of $\beta$ for the BJT is 100.

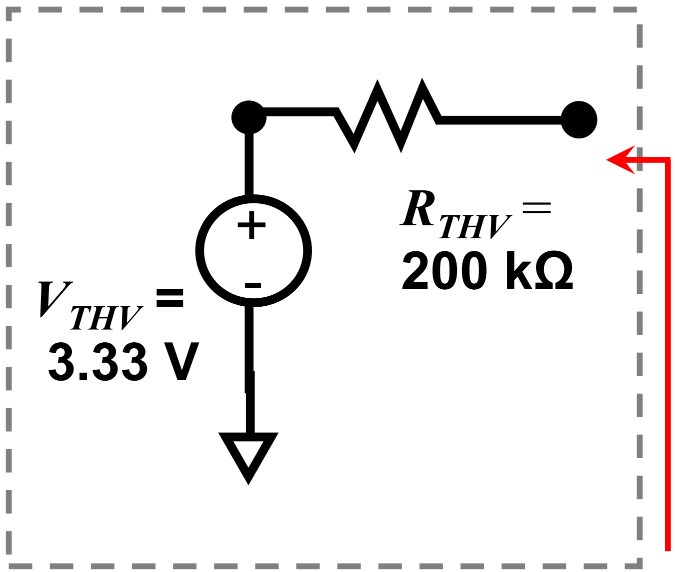


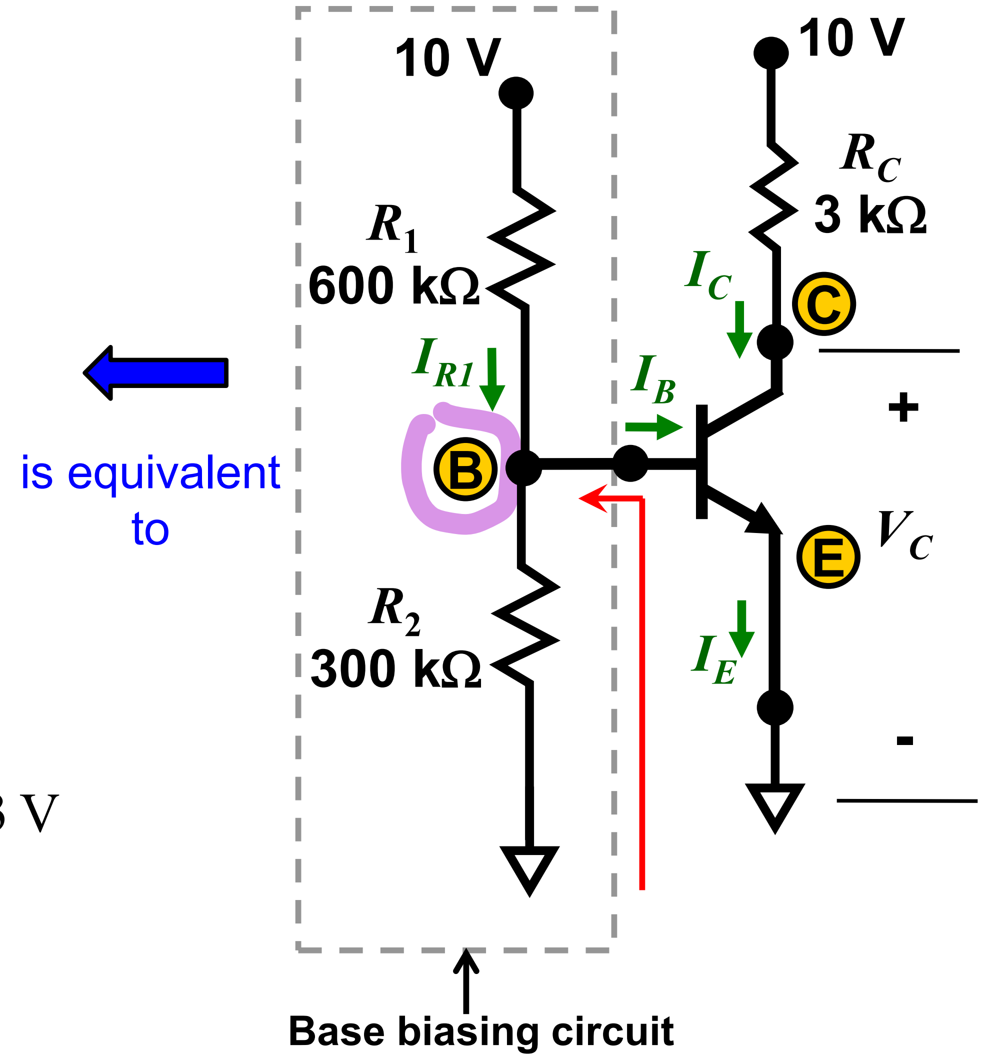



# *BJT* – Calculation of Operating Point

## Example 1 (Thevenin equivalent method)

- First, we obtain the Thevenin Equivalent Circuit of the base-biasing circuit, shown in the dashed grey box:




$$V_{THV} = \frac{R_2}{R_1 + R_2} \times 10 = 3.33 \text{ V}$$

$$R_{THV} = R_1 // R_2 = 200 \text{ k}\Omega.$$

Note: With the base biasing circuit replaced by its Thevenin's equivalent circuit, the above circuit has the same topography as the circuit of slide BJT-20.

# *BJT* – Calculation of Operating Point

## Example 1 (Thevenin equivalent method)

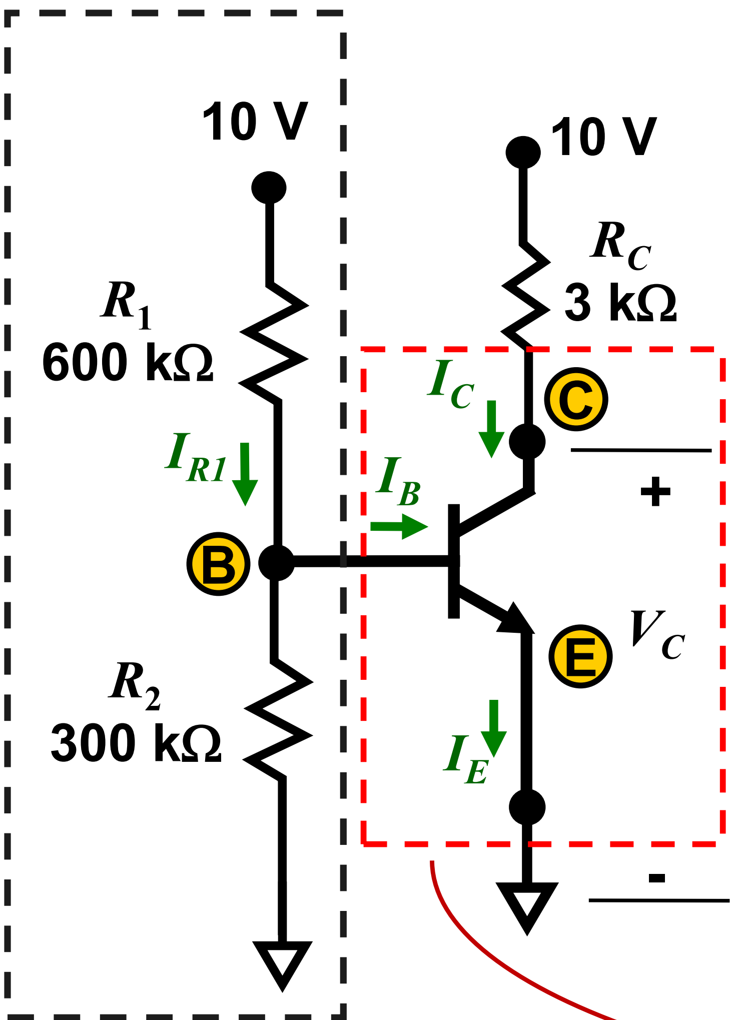
- Next, we replace the npn BJT by its large-signal model (Forward active).

********

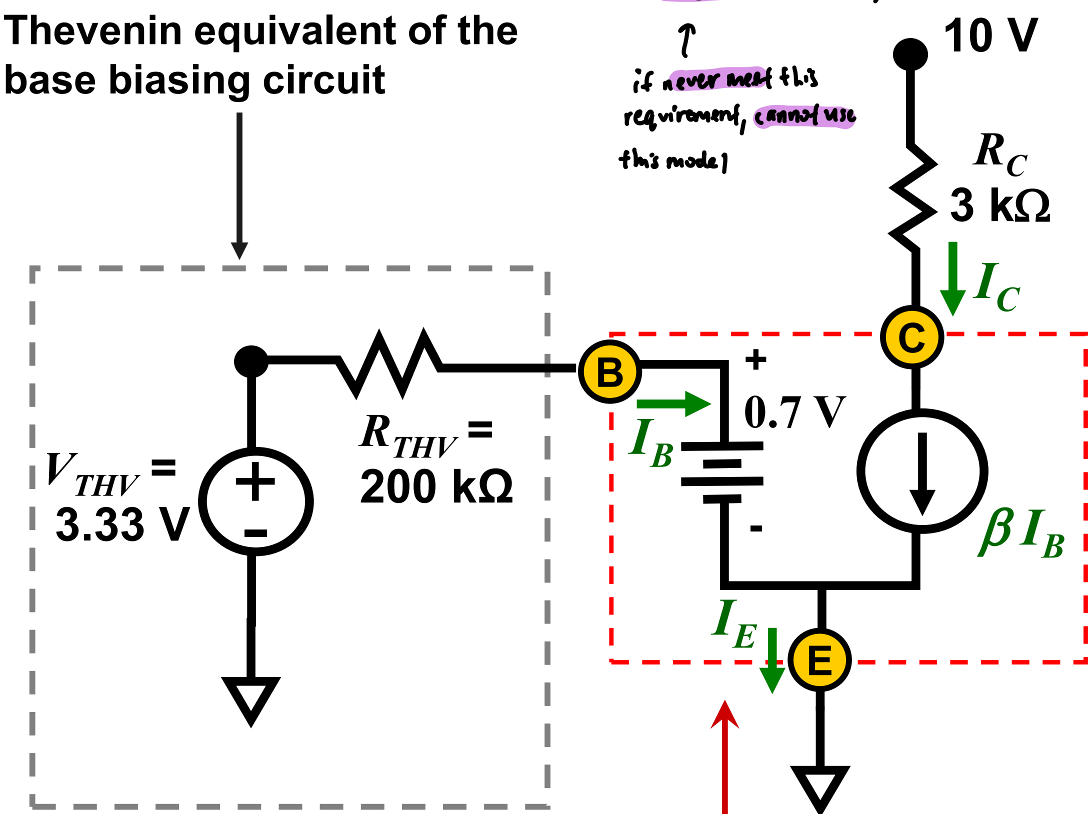
After finish finding values, need to check: $V_{BE} > 0$ (forward bias)

$V_{BC} < 0$ (reverse)

if never meet this requirement, cannot use this model



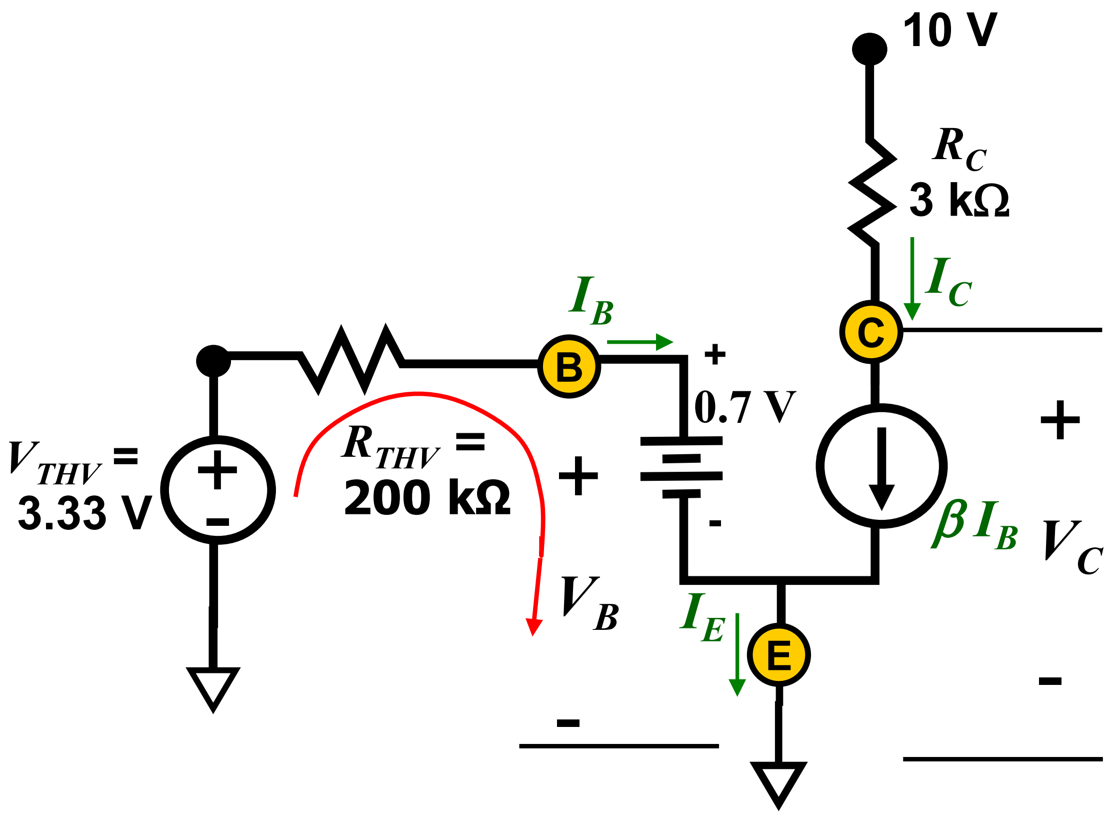

Thevenin equivalent of the base biasing circuit




Large-Signal Model for npn BJT

# *BJT* – Calculation of Operating Point

## Example 1 (Thevenin equivalent method)




$$I_B = \frac{3.33\ \text{V} - 0.7\ \text{V}}{200\ \text{k}\Omega} = \mathbf{0.0132\ mA.}$$

$$I_C = \beta I_B = 100 \times 0.0132\ \text{mA} = \mathbf{1.32\ mA}.$$

$$I_E = (\beta+1) I_B = 101 \times 0.0132\ \text{mA} = \mathbf{1.33\ mA}.$$

$$V_C = 10\ \text{V} - 3\ \text{k} \times (1.32\ \text{mA}) = \mathbf{6.04\ V.}$$

Check: Base voltage, $V_B = V_{BE} = 0.7$ V.

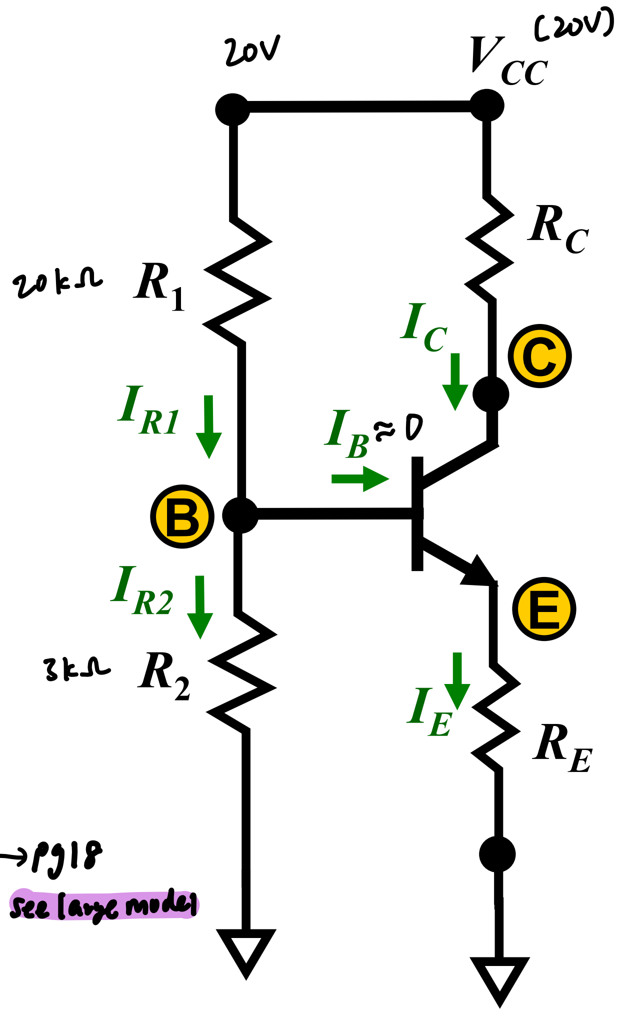
Base-collector voltage $V_{BC} = V_B - V_C = 0.7\ \text{V} - 6.04\ \text{V} = -5.34\ \text{V}$,

Since the base-emitter pn junction is forward biased, and the base-collector pn junction is reverse biased, the npn BJT is in operating in the forward active region.

# *BJT* – Calculation of Operating Point

## Example 2 (Voltage divider method)

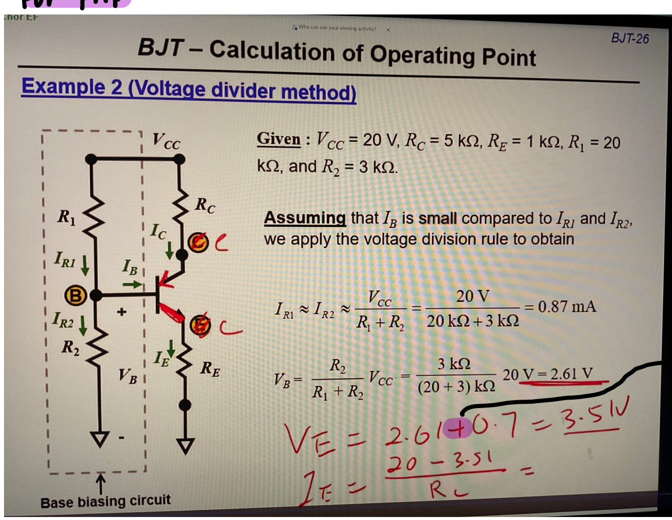
The circuit below has $V_{CC}$ = 20 V, $R_C$ = 5 kΩ, $R_E$ = 1 kΩ, $R_1$ = 20 kΩ, and $R_2$ = 3 kΩ. The value of $\beta$ for the BJT is 100. Assume BJT is operating in the forward active region, determine the values of $I_C$ and $I_B$ using the large-signal model for the BJT




One key assumption for this method

1) $I_B \approx 0$ (negligible) ie, $I_B << I_{R1}, I_{R2}$

For pnp


BJT-26

**BJT – Calculation of Operating Point**

Example 2 (Voltage divider method)

**Given** : $V_{CC}$ = 20 V, $R_C$ = 5 kΩ, $R_E$ = 1 kΩ, $R_1$ = 20 kΩ, and $R_2$ = 3 kΩ.

**Assuming** that $I_B$ is small compared to $I_{R1}$ and $I_{R2}$, we apply the voltage division rule to obtain

$$I_{R1} \approx I_{R2} \approx \frac{V_{CC}}{R_1 + R_2} = \frac{20\text{ V}}{20\text{ k}\Omega + 3\text{ k}\Omega} = 0.87\text{ mA}$$

$$V_B = \frac{R_2}{R_1 + R_2} V_{CC} = \frac{3\text{ k}\Omega}{(20+3)\text{ k}\Omega} 20\text{ V} = 2.61\text{ V}$$

$V_E = 2.61 + 0.7 = 3.51V$

$I_E = \frac{20 - 3.51}{R_C} =$

Base biasing circuit

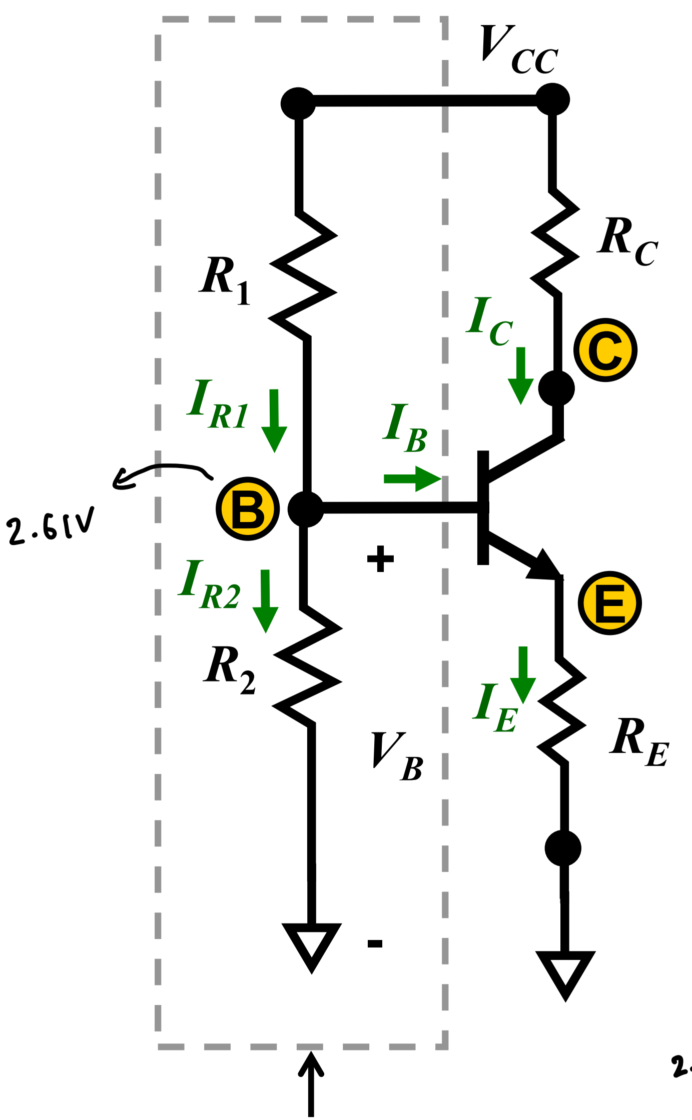


→ pg18

see large model

# *BJT* – Calculation of Operating Point

## Example 2 (Voltage divider method)




Base biasing circuit

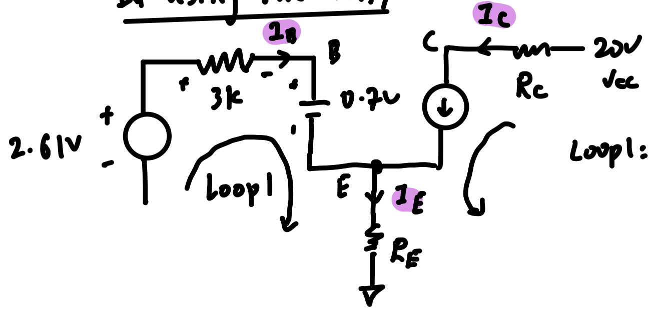
**Given** : $V_{CC}$ = 20 V, $R_C$ = 5 kΩ, $R_E$ = 1 kΩ, $R_1$ = 20 kΩ, and $R_2$ = 3 kΩ.

**Assuming** that $I_B$ is small compared to $I_{R1}$ and $I_{R2}$, we apply the voltage division rule to obtain

$$I_{R1} \approx I_{R2} \approx \frac{V_{CC}}{R_1 + R_2} = \frac{20\text{ V}}{20\text{ k}\Omega + 3\text{ k}\Omega} = 0.87\text{ mA}$$

$$V_B = \frac{R_2}{R_1 + R_2} V_{CC} = \frac{3\text{ k}\Omega}{(20+3)\text{ k}\Omega} 20\text{ V} = 2.61\text{ V}$$

If using thevenin,



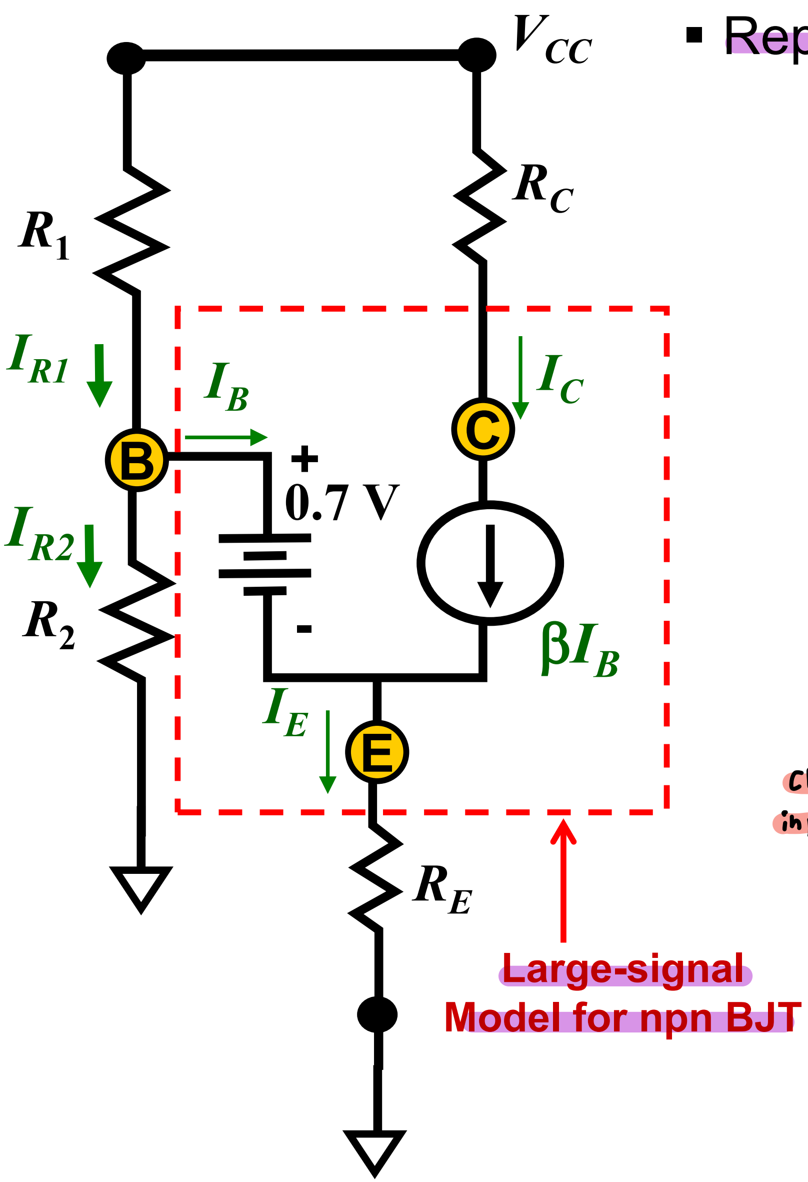

Loop1: $2.61 = I_B 3k + 0.7 + I_E R_E$

$I_B(1+\beta) = I_E$

solve simultaneous

# *BJT* – Calculation of Operating Point

## Example 2 (Voltage divider method)




- Replace the npn BJT by its large-signal model.

$$V_E = V_B - V_{BE} = 2.61 - 0.7 = 1.91 \text{ V.}$$

↳ constant voltage drop

Thus, $I_E = V_E / R_E = V_E / 1\text{ k}\Omega = 1.91$ mA.

100 constant

100 ~ 200

$I_C = \beta I_B$

$$I_C = [\beta / (\beta + 1)] I_E = [100/101] \times 1.91 \text{ mA}$$
$$= \mathbf{1.89 \text{ mA}}.$$

↳ $\boxed{I_C + \frac{I_C}{\beta} = I_E \rightarrow I_C \left(1 + \frac{1}{\beta}\right) = I_E}$

$$I_B = I_C / \beta = \mathbf{0.0189 \text{ mA} = 18.9\ \mu A}$$

Check: $I_B$ (0.0189 mA), is indeed much smaller than $I_{R1}$ and $I_{R2}$ (≈ 0.87 mA). Voltage divider assumption is valid.

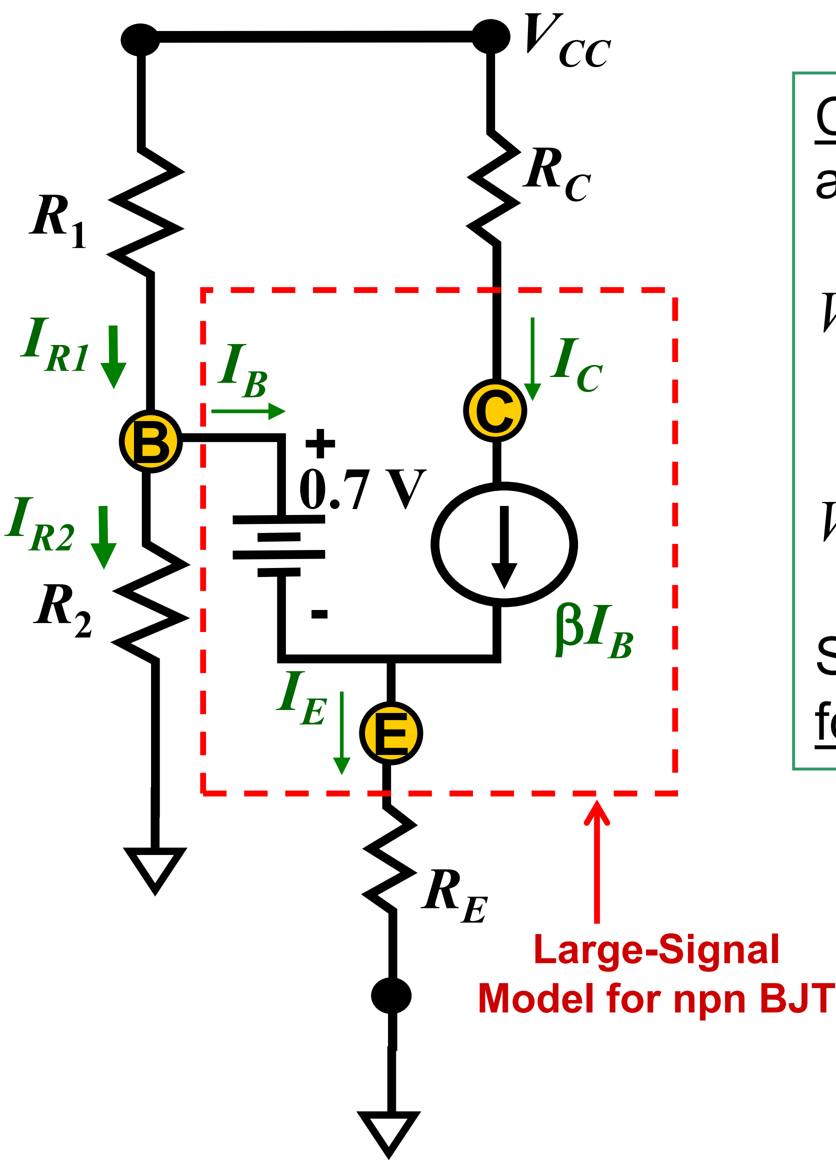
✓ Check assumption in pg 25

Note: If $I_B$ is not small compared to the currents flowing through $R_1$ and $R_2$, the voltage divider rule cannot be applied.

We would then need to use the Thevenin equivalent method.

# *BJT* – Calculation of Operating Point

## Example 2 (Voltage divider method)




Check that the BJT is indeed in the forward-active region of operation:

$V_C = V_{CC} - I_C R_C$ = 20 – 1.89 mA × 5 kΩ
= 10.55 V.

$V_{BC} = V_B - V_C$ = 2.61 – 10.55 = - 7.94 V.

Since $V_{BC} < 0$ and $V_{BE} > 0$, the npn BJT is in the forward active region of operation.

# Bipolar Junction Transistor (BJT)

## Bipolar Junction Transistor (BJT)

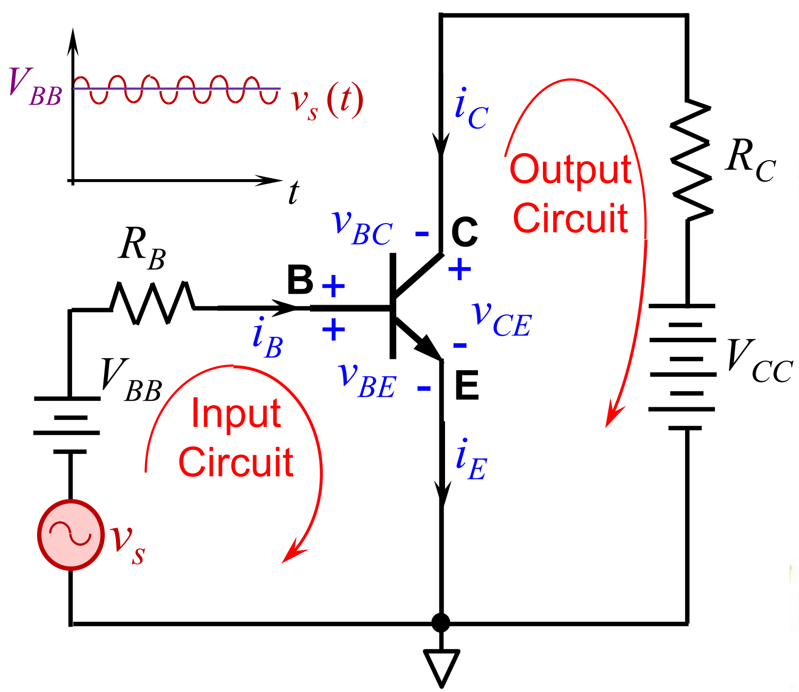
1. Introduction
2. Modes of Operation: Forward Active, Saturation, Cut-Off and Reverse Active
3. Forward Active Current-Voltage Characteristic
4. Large-Signal Model and dc Analysis
5. Amplifier Circuit
6. Small-Signal Model (Simplified Hybrid-$\pi$) and ac Analysis
7. Other Hybrid-$\pi$ Small-Signal Models

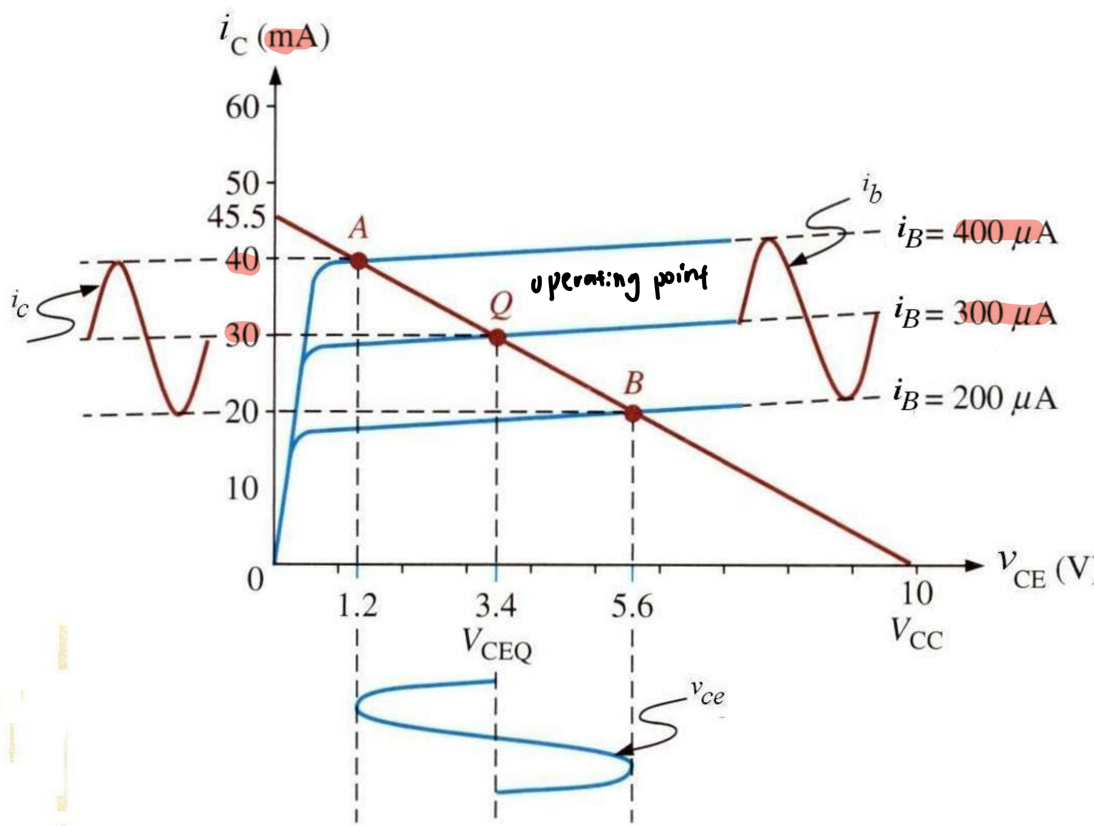

## Reference

- Sedra and Smith, Microelectronic Circuits, Fifth Edition, Oxford (2004), pp. 159 – 168, 173 - 183, 203 - 241.

# *BJT* – Amplifier Circuit

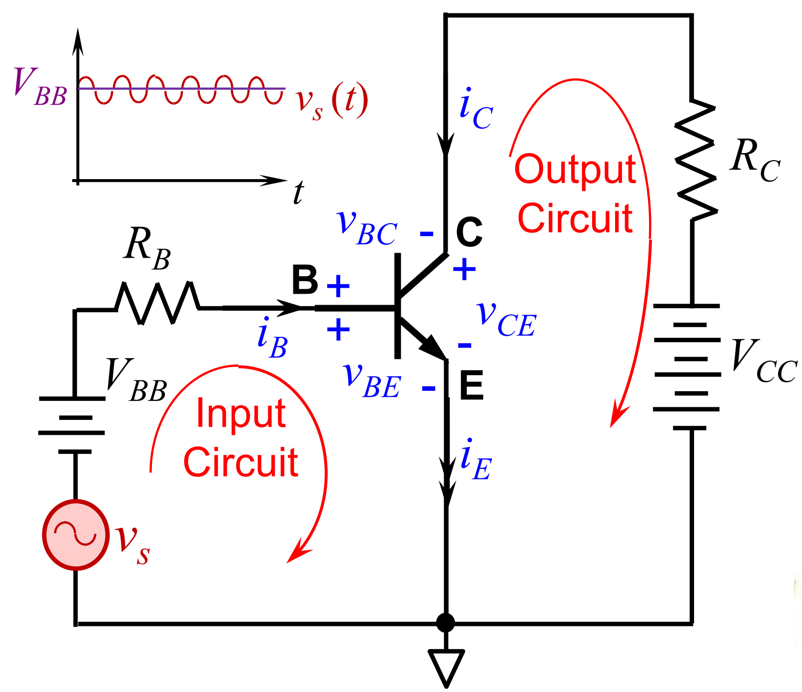
- In an amplifier circuit, BJT is biased in the forward active region. In addition to dc sources, $V_{BB}$ and $V_{CC}$, an **ac (small-signal) source**, $v_s$, is applied to the input circuit, as shown in the common-emitter amplifier circuit below. $v_s$ can be considered as a small add-on to $V_{BB}$.
- Two types of operation: dc and ac (small-signal) operations.
- The dc sources, $V_{BB}$ and $V_{CC}$ bias the BJT to operate in the **forward active region** and set the bias point (point Q) around which the small ac signal operates.

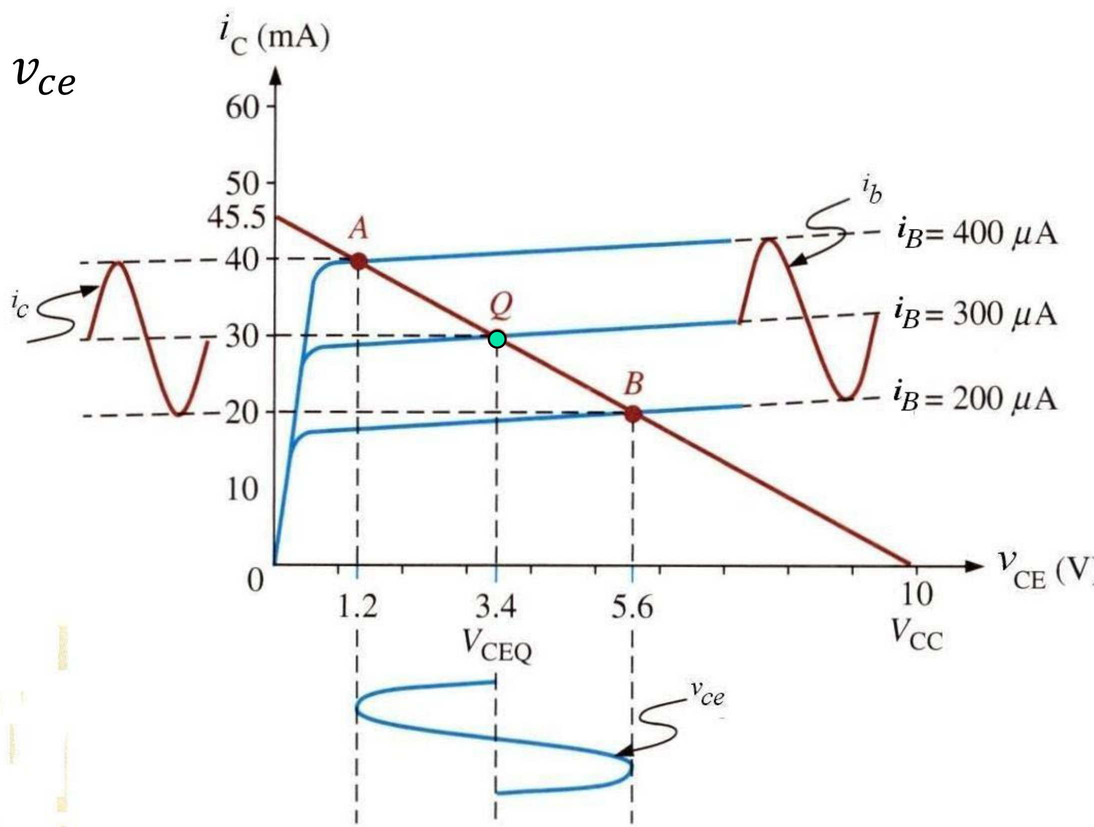


# *BJT* – Amplifier Circuit

- The dc bias point (Q) currents and voltages ($I_B$, $I_C$, $V_{CE}$) can be determined using the BJT <u>large-signal model</u>, in the absence of the ac (small-signal) source, $v_s$.
- In the presence of ac (small-signal) source, $v_s$, the total base current, $i_B$, in the input circuit has an ac (small-signal) value, $i_b$, in addition to the dc value, $I_B$:
  - $i_B = I_B + i_b$
- As a result, total collector current, $i_C$, and total collector-emitter voltage, $v_{CE}$, of the output circuit will be similarly changed, with the addition of a small-signal $i_c$ and $v_{ce}$, respectively:
  - $i_C = I_C + i_c$ and $v_{CE} = V_{CE} + v_{ce}$

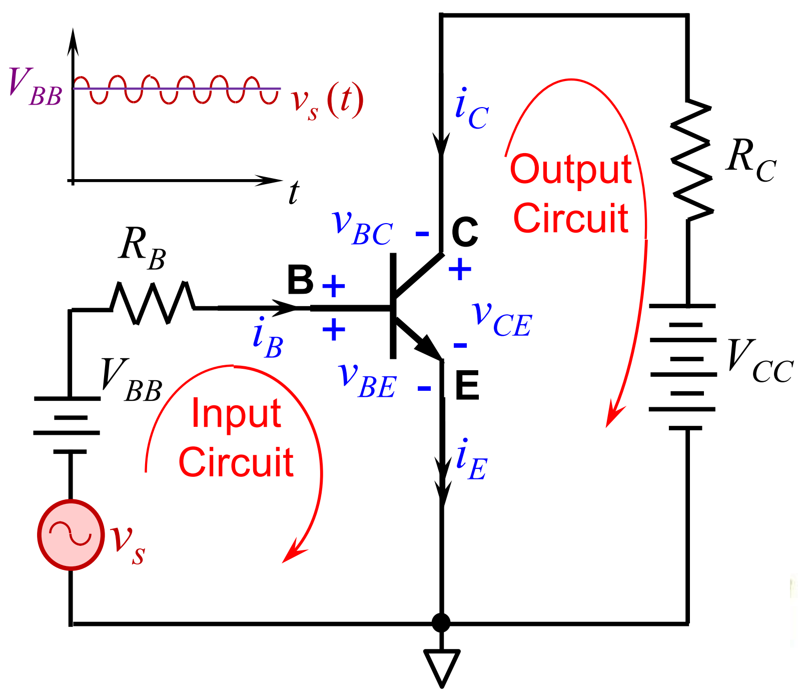


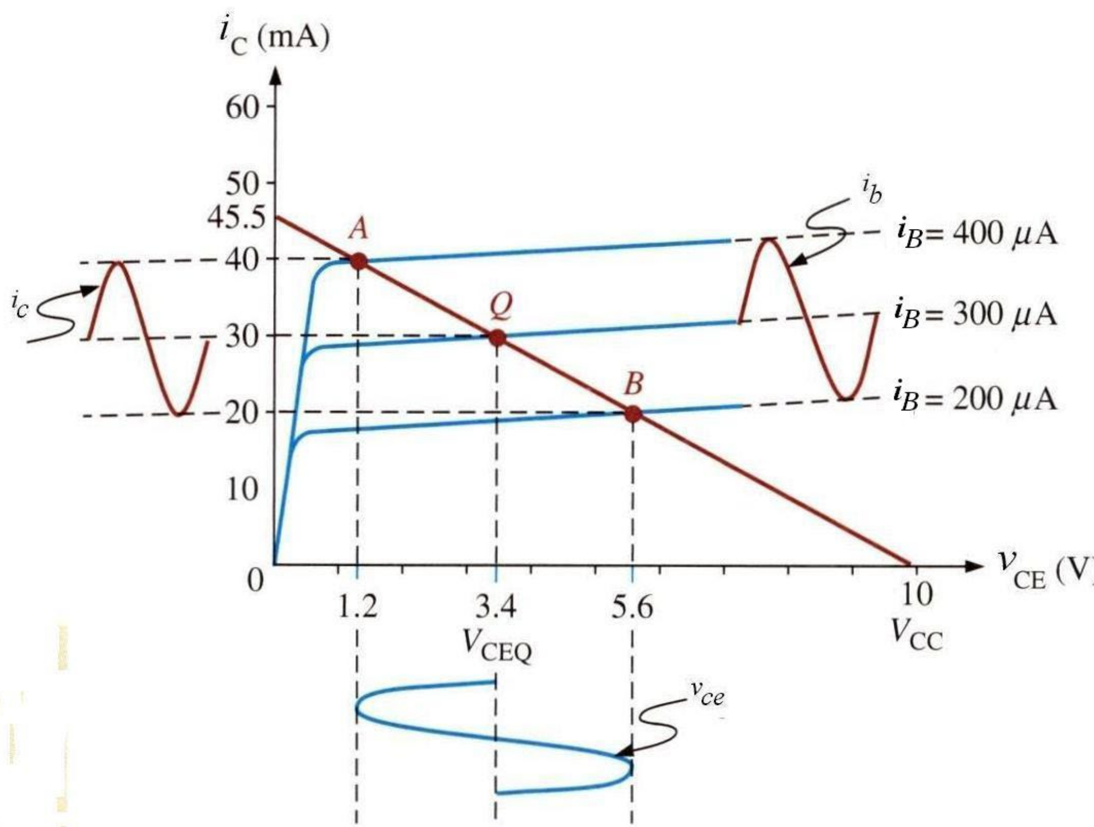



# *BJT* – Amplifier Circuit

Notations for dc and ac values (examples)

- dc current: $I_C$
  Capital symbol
  Capital subscript
- ac current: $i_c$
  Lower case symbol
  Lower case subscript
- Total current: $i_C = I_C + i_c$
  Lower case symbol
  Capital subscript



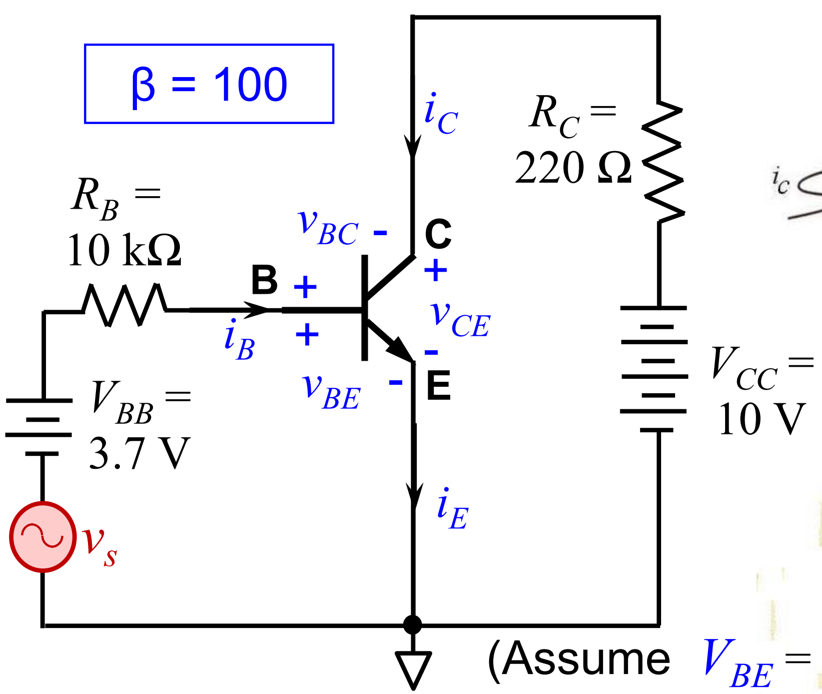



# *BJT* – Amplifier Circuit

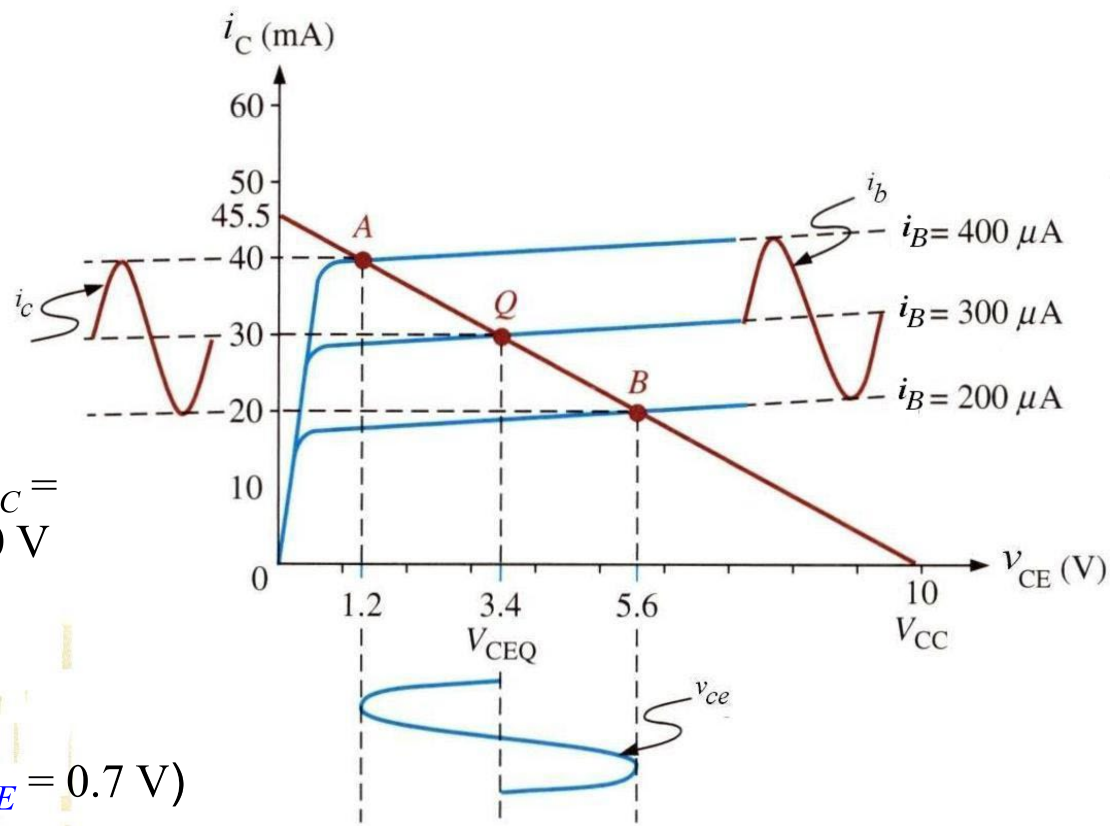
The common-emitter amplifier circuit below will demonstrate signal amplification, graphically with the help of the IV-characteristics.

❑ Without the ac (small-signal) source, i.e., $v_s$ = 0 -

- No ac (small-signal) values: $i_b$ = 0, $i_c$ = 0, and $v_{ce}$ = 0.
- $i_B = I_B$ = 300 μA, $i_C = I_C$ = 30 mA, and $v_{CE} = V_{CE}$ = 3.4 V (see slides BJT 19 & BJT-20 for dc operation analysis). These are dc biasing currents and voltages.
- Point Q, the dc bias point.



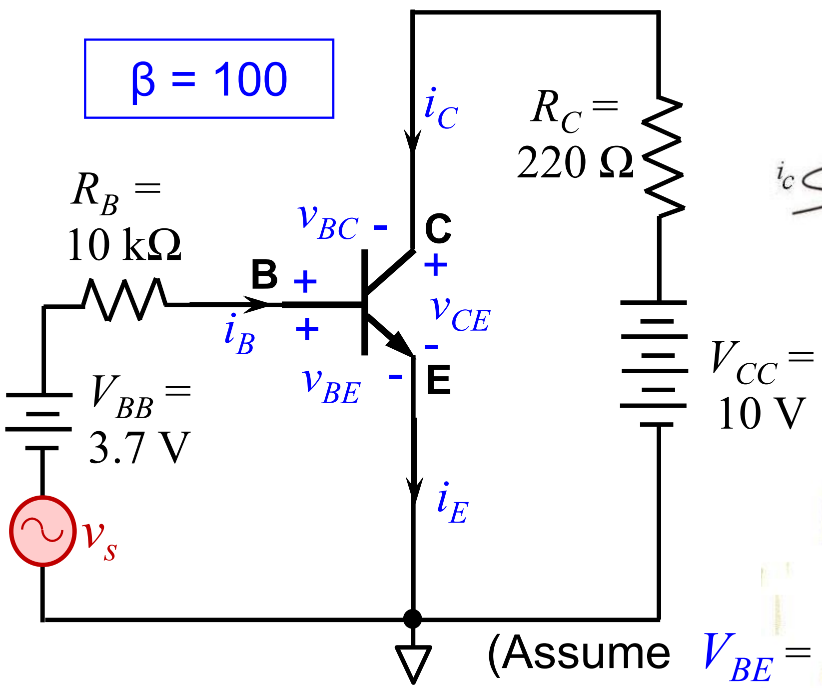

(Assume $V_{BE}$ = 0.7 V)



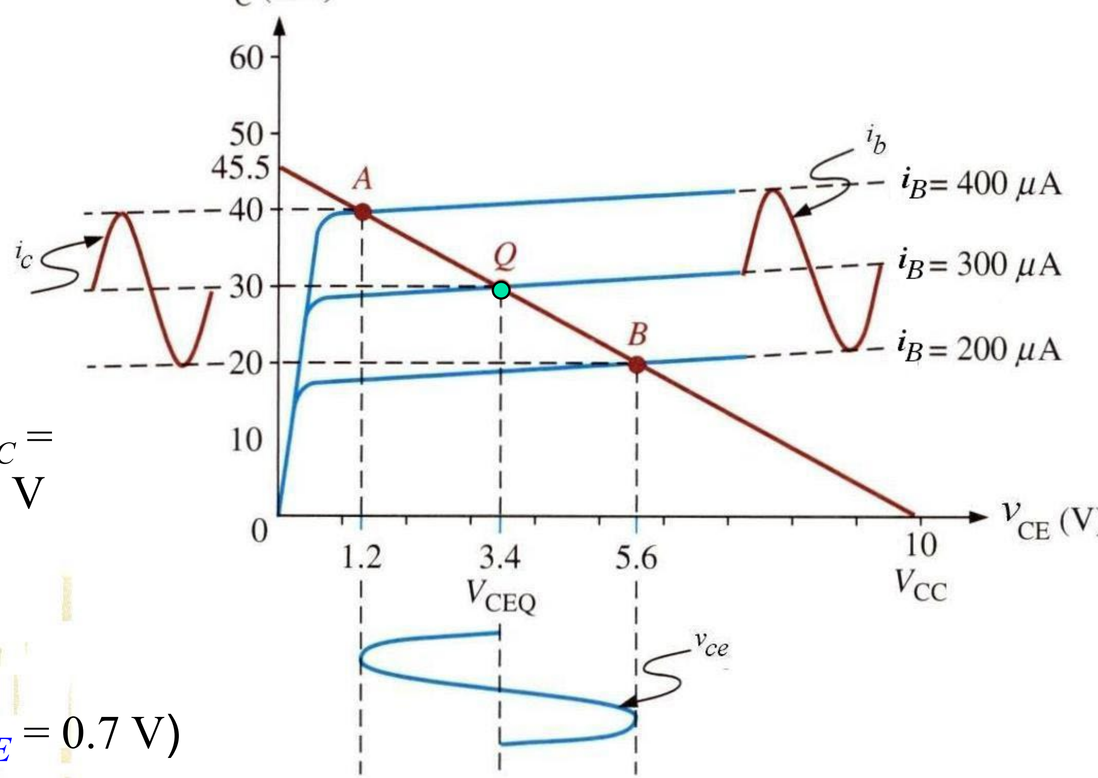
# *BJT* – Amplifier Circuit

With ac source $v_s$ -

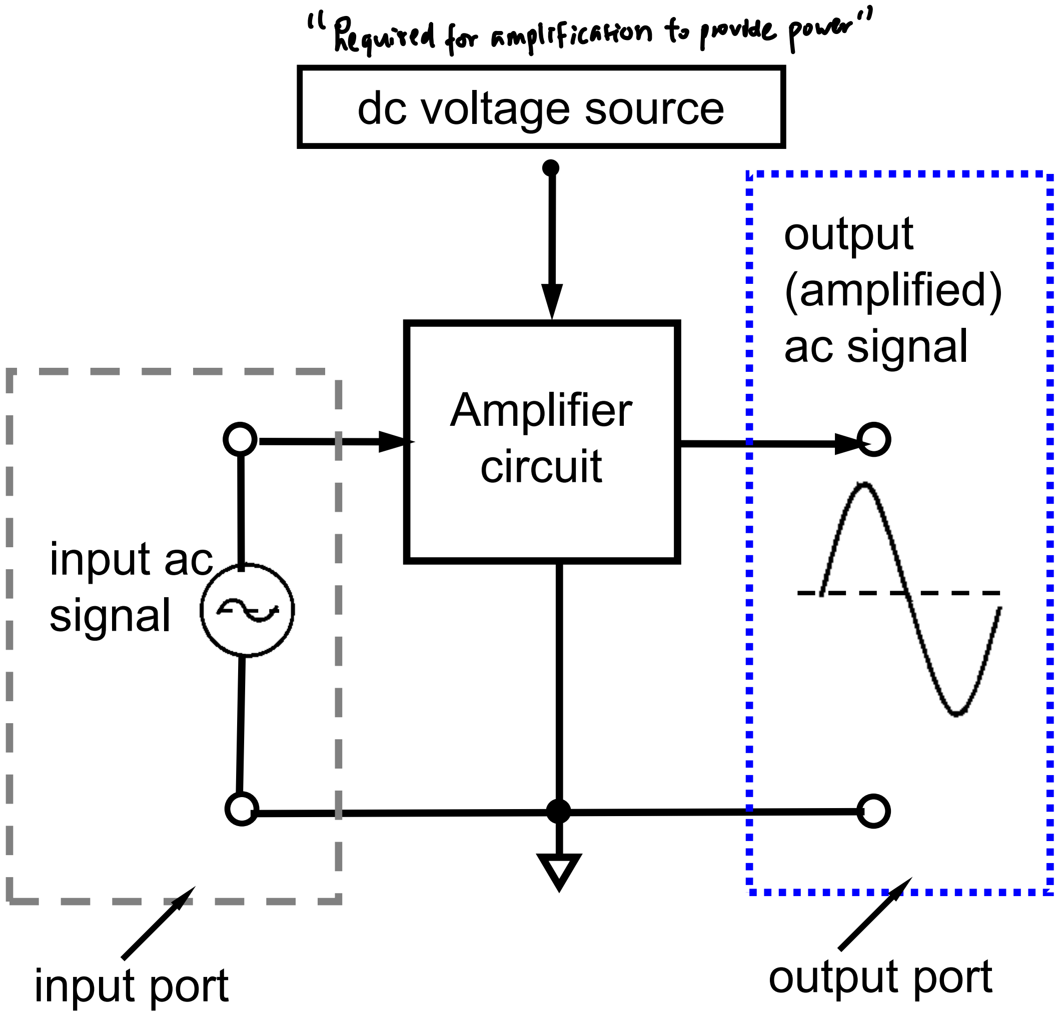
- When $i_b$ = 100 $\mu$A, $i_B = I_B + i_b$ = 300 $\mu$A +100 $\mu$A = 400 $\mu$A.
  - Point A: $i_C = I_C + i_c$ = 40 mA, and $v_{CE} = V_{CE} + v_{ce}$ = 1.2 V.
- When $i_b$ = -100 $\mu$A, $i_B = I_B + i_b$ = 300 $\mu$A -100 $\mu$A = 200 $\mu$A.
  - Point B: $i_C = I_C + i_c$ = 20 mA, and $v_{CE} = V_{CE} + v_{ce}$ = 5.6 V.
- Small-signal $i_b$ = 100 $\mu$A leads to small-signal $i_c$ = 10 mA, which is 100 ($\beta$) times bigger. This is signal amplification.




(Assume $V_{BE}$ = 0.7 V)



# *BJT* – Amplifier Circuit (The Principles)

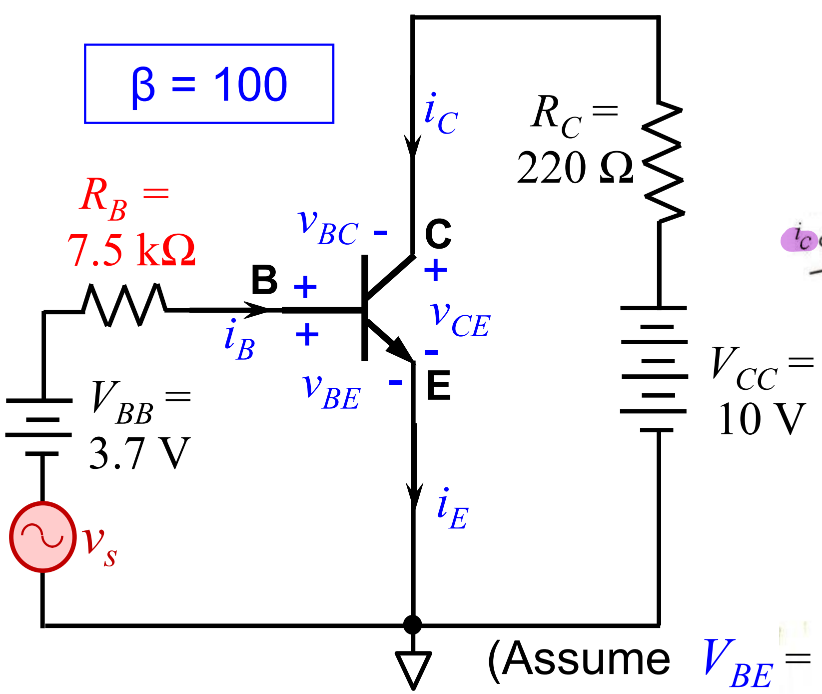



- Amplifier circuit is a 2-port network with the dc voltage source that supplies power to the entire circuit, while the ac signal is applied to the input port.
- The transistor in the amplifier circuit acts as a control device, which controls the flow of power from the dc source, to produce an enhanced (amplified) ac signal at the output port.
- The output ac signal is of the same waveform as the input ac signal, but is of larger amplitude (in current, voltage, or power).

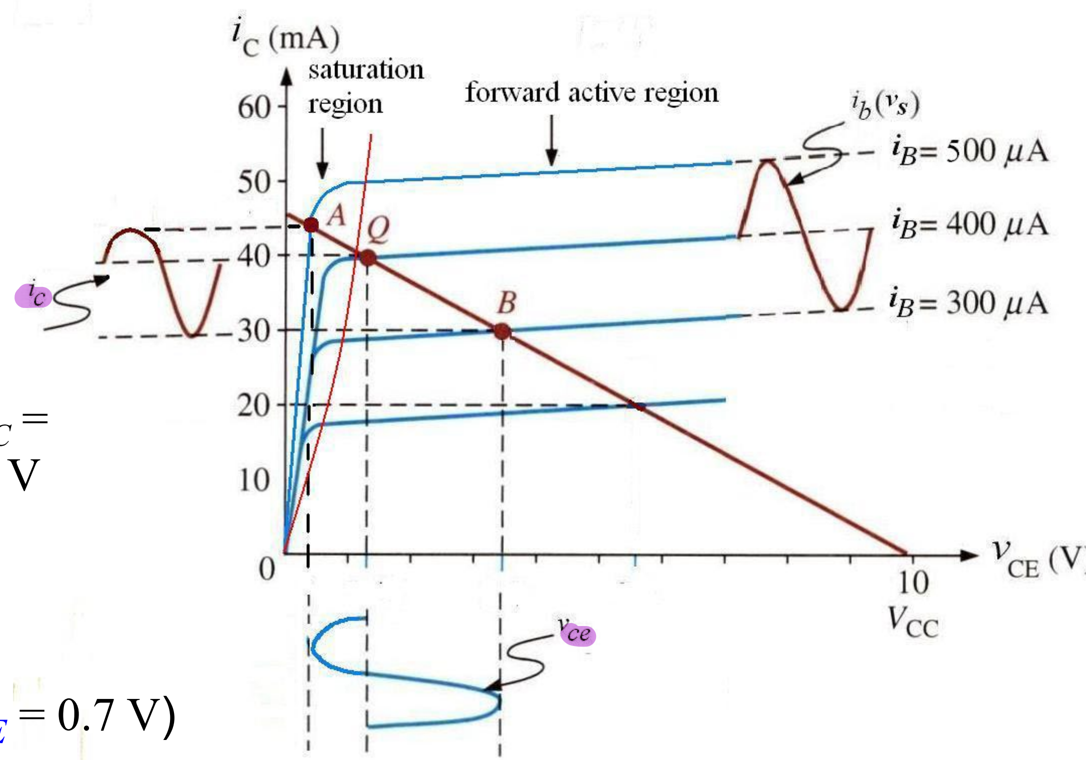
# *BJT* – Amplifier Circuit (Inappropriate Bias Point)

With $R_B$ reduced to 7.5 kΩ -

- dc base current, $I_B = (V_{BB} - V_{BE}) / R_B$ = (3.7 – 0.7)/7.5k = 400 μA. Bias point Q.
- With ac source $v_s$ - when $i_b$ = 100 μA, $i_B = I_B + i_b$ = 400 μA +100 μA = 500 μA.
  - Intersection between load line and BJT characteristics moves to point A, where the BJT no longer operates in the forward active region.
- The ac signal waveforms of $i_c$ and $v_{ce}$ are distorted.




(Assume $V_{BE}$ = 0.7 V)

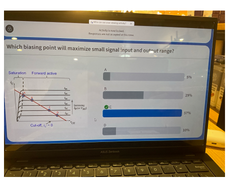


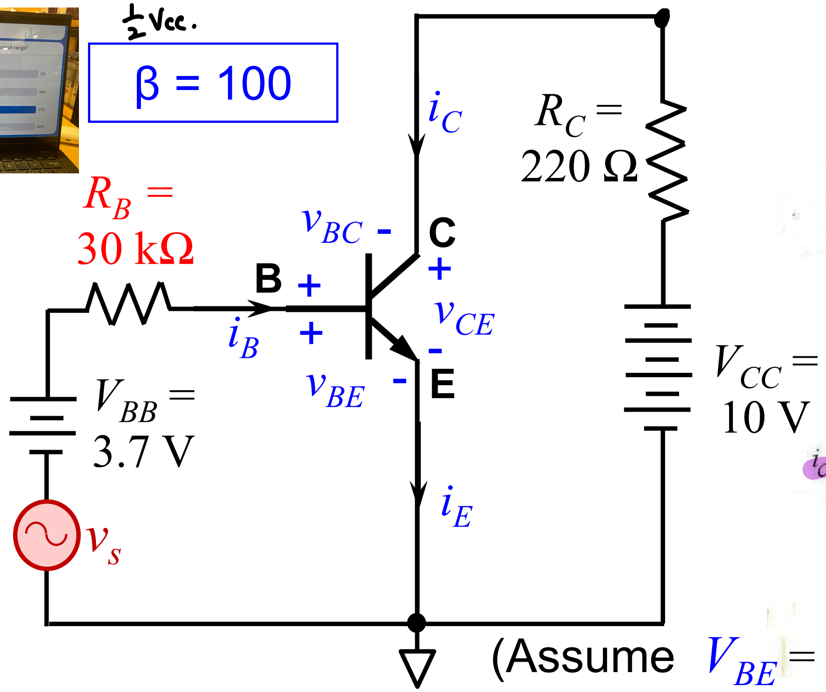
# *BJT* – Amplifier Circuit (Inappropriate Bias Point)

With $R_B$ increased to 30 kΩ -

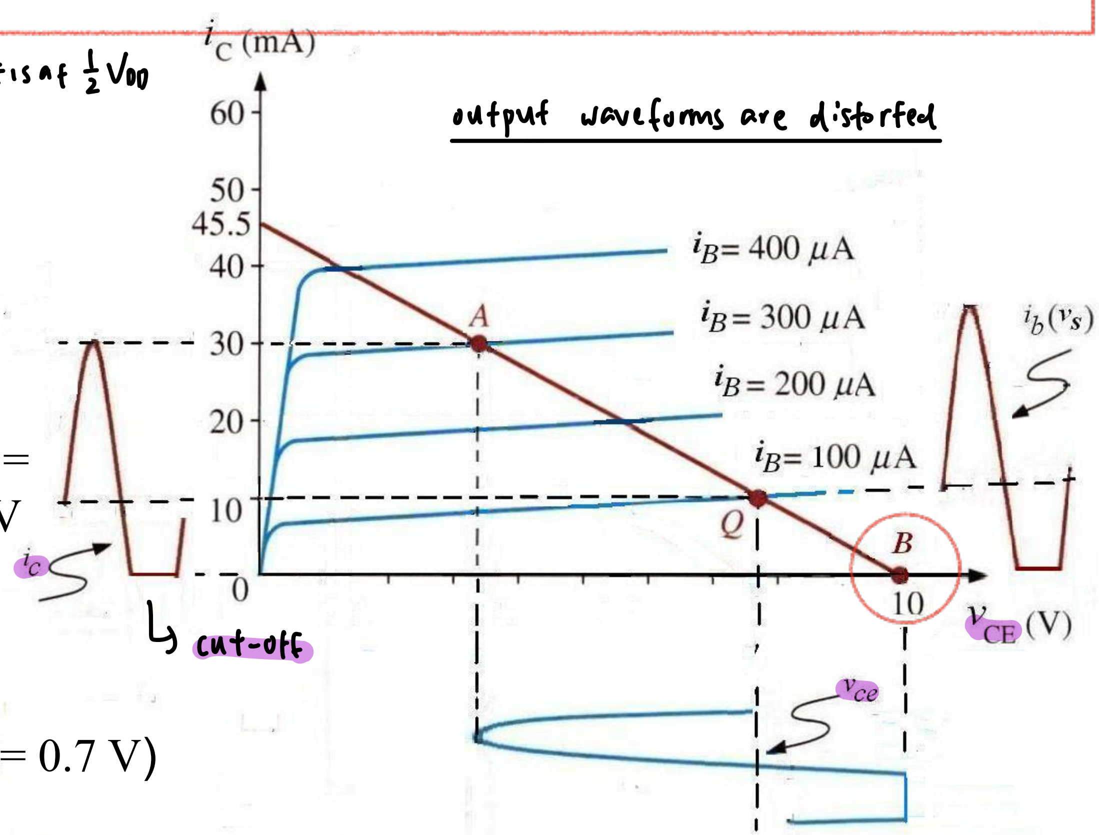
- dc base current, $I_B = (V_{BB} - V_{BE}) / R_B$ = (3.7 – 0.7)/30k = 100 μA. Bias point Q.
- With ac source $v_s$, assuming $i_b$ = 200 μA. At some point during the negative half-cycle of $i_b$ the intersection between load line and BJT characteristics is at point B.
  - ac signal $i_c$ is clipped at 0 A, as total $i_C$ cannot be negative.
  - ac signal $v_{ce}$ is clipped at 10 V, as total $v_{CE}$ cannot exceed the power supply voltage.

∴ Best way to pick Q-point is to select $I_C$ such that it is at $\frac{1}{2}V_{DD}$ $\frac{1}{2}V_{CC}$.






(Assume $V_{BE}$ = 0.7 V)



## Bipolar Junction Transistor (BJT)

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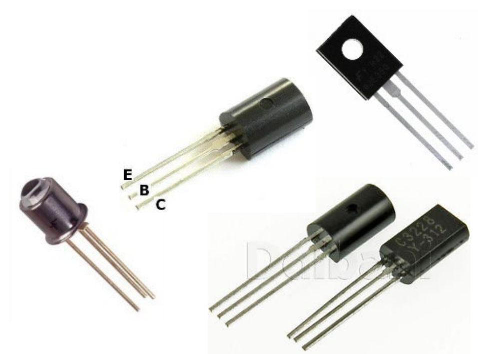


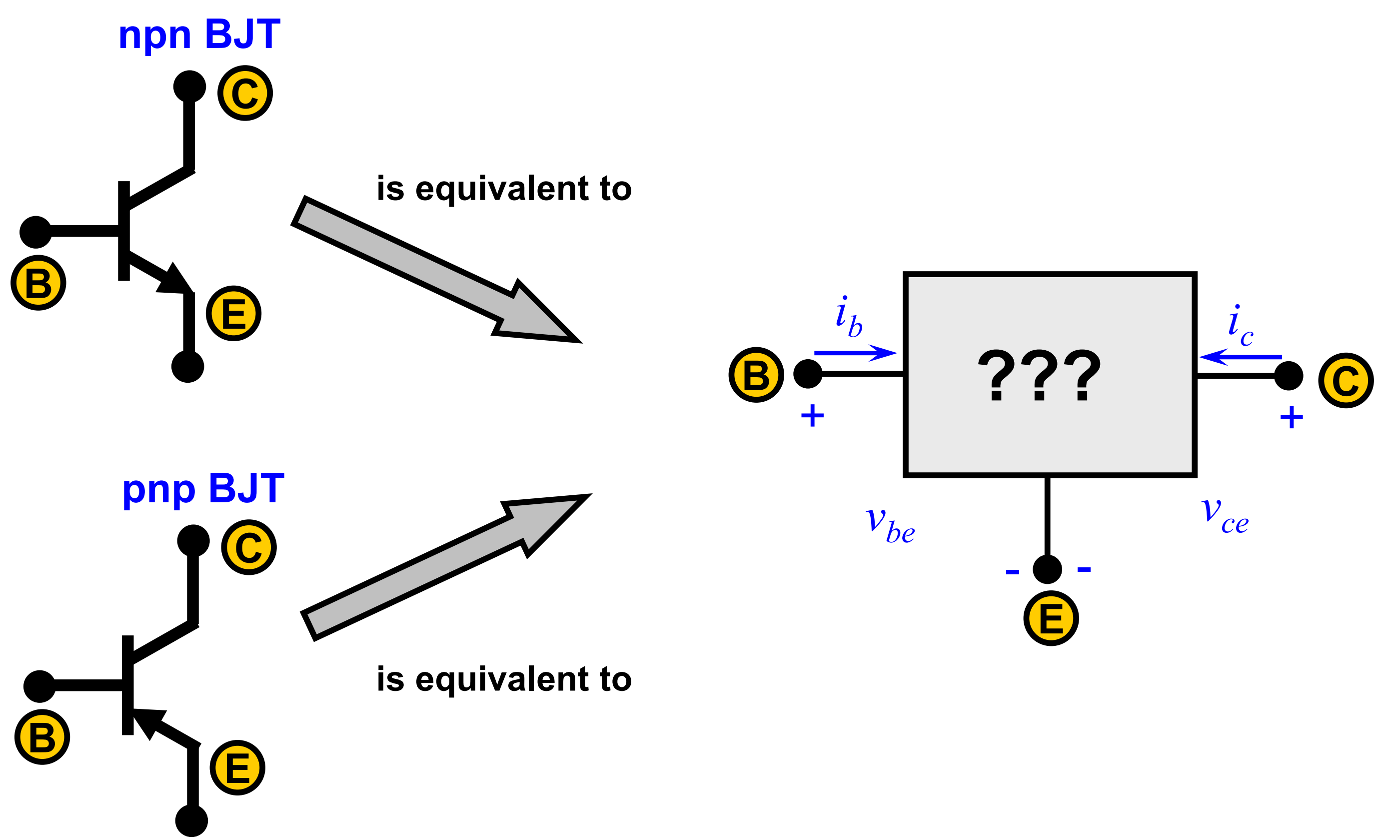

## Reference

- Sedra and Smith, Microelectronic Circuits, Fifth Edition, Oxford (2004), pp. 159 – 168, 173 - 183, 203 - 241.

# *BJT* – Small-Signal Model

To develop a small-signal model of BJT operating in the forward active region.

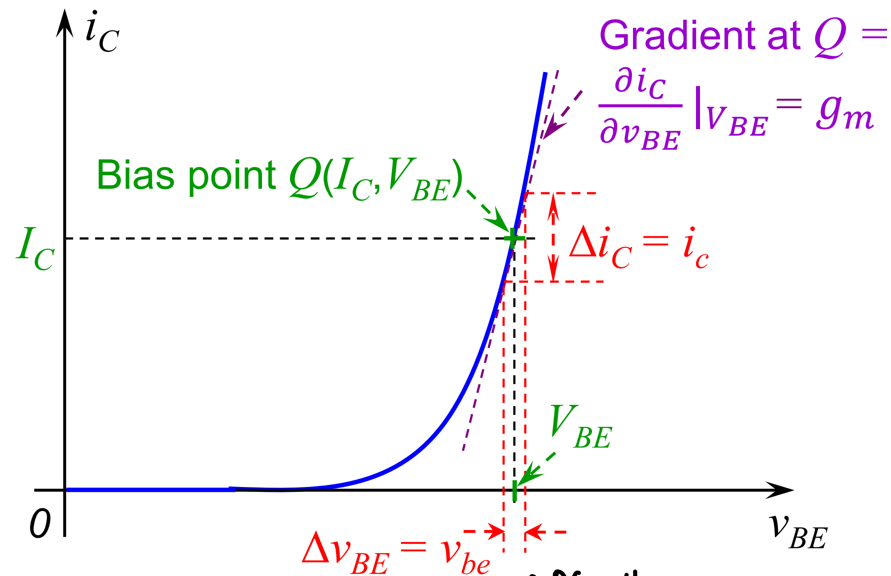
- We seek linear relationships* among the small-signal ac components of the base current, collector current, base-emitter voltage and collector-emitter voltage - $i_b$, $i_c$, $v_{be}$, and $v_{ce}$ (at a bias point).




* In the development of the small-signal model of a *pn*-junction diode, we obtain an approximate linear relationship between its small-signal current, $i_d$, and its small-signal voltage, $v_d$, at a bias point (see slides pn-30 to pn-35, in particular, equations (2.12) and (2.14)).

# BJT – Small-Signal Model Parameter - $g_m$

- From equation (3.1), collector current - $i_C = I_S e^{v_{BE}/V_T}$
- At the dc bias point (point Q), dc collector current - $I_C = I_S e^{V_{BE}/V_T}$
- A linear relationship between a small change in the collector current, $\Delta i_C$, and a small change in the base-emitter voltage, $\Delta v_{BE}$, can be found by linearizing the $i_C$-$v_{BE}$ characteristic.




- A small change in $i_C$ w.r.t a small change in $v_{BE}$, can be approximated by the derivative of $i_C$ w.r.t $v_{BE}$ –

0.7V

DC collector current

$$\frac{\Delta i_C}{\Delta v_{BE}}\Big|_{V_{BE}} = \frac{i_c}{v_{be}}\Big|_{V_{BE}} \approx \frac{\partial i_C}{\partial v_{BE}}\Big|_{V_{BE}} = \boxed{\frac{I_S e^{V_{BE}/V_T}}{V_T} = \frac{I_C}{V_T} = g_m} \qquad (3.6)$$

25mV

- **Transconductance** : $\boxed{g_m = \dfrac{i_c}{v_{be}} = \dfrac{I_C}{V_T}}$ (3.7)

- Transconductance, $g_m$, models a small change in the collector current, $i_c$, caused by a small change in base-emitter voltage, $v_{be}$.

# *BJT* – Small-Signal Model Parameter - $r_\pi$

- From equation (3.2), base current - $i_B = \frac{I_S}{\beta} e^{v_{BE}/V_T}$ $\therefore I_C = \beta I_B$

- At the dc bias point (point Q), dc base current - $I_B = \frac{I_S}{\beta} e^{V_{BE}/V_T}$

- A small change in $i_B$ w.r.t a small change in $v_{BE}$, can be approximated by the derivative of $i_B$ w.r.t $v_{BE}$ –

$$\frac{\Delta i_B}{\Delta v_{BE}}\Big|_{V_{BE}} = \frac{i_b}{v_{be}}\Big|_{V_{BE}} \approx \frac{\partial i_B}{\partial v_{BE}}\Big|_{V_{BE}} = \boxed{\frac{I_S e^{V_{BE}/V_T}}{\beta V_T} = \frac{I_C}{\beta V_T} = g_\pi} \quad (3.8)$$

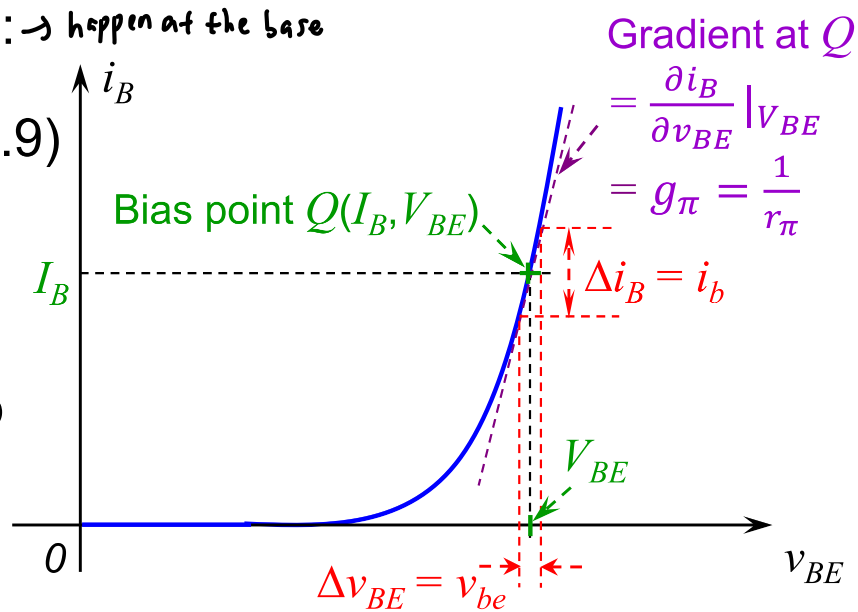
$\frac{g_m}{\beta}$ $\frac{1}{r_\pi}$

- Taking the reciprocal of $g_\pi$, input resistance: → happen at the base

$$r_\pi = \frac{1}{g_\pi} = \frac{v_{be}}{i_b} = \frac{\beta V_T}{I_C} = \frac{\beta}{g_m} = \frac{v_{be}}{i_b} \quad (3.9)$$

- Hence, $\beta = g_m r_\pi$ (3.10)

- Input resistance, $r_\pi$, models the relationship between a small change in the base current, $i_b$, and a small change in base-emitter voltage, $v_{be}$.



# *BJT* – Small-Signal Model Parameters (Summary)

- Equation (3.7) - Transconductance, $g_m$, models a small change in the collector current, $i_c$, caused by a small change in base-emitter voltage, $v_{be}$ –

AC $I_c$ current

$$i_c = g_m v_{be} \qquad \left[ g_m = \frac{I_C}{V_T} \right] \tag{3.7}$$

DC - bias point $I_c$

Small change in collector current

Small change in base-emitter voltage

- Equation (3.9) - Input resistance, $r_\pi$, models the relationship between a small change in the base current, $i_b$, and a small change in base-emitter voltage, $v_{be}$ –

$$v_{be} = r_\pi i_b \qquad \left[ r_\pi = \frac{\beta V_T}{I_C} \right] \tag{3.9}$$

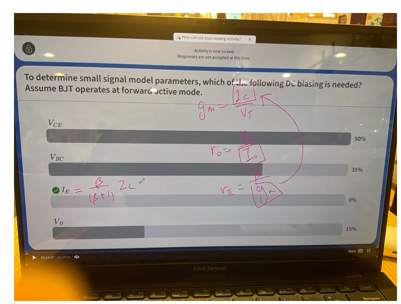
Small change in base-emitter voltage

Small change in base current

- Equation (3.10) - $\beta = g_m r_\pi$ (3.10)

- Combine equations (3.7), (3.9) and (3.10) -

$$i_c = g_m v_{be} = g_m r_\pi i_b = \beta i_b \tag{3.11}$$

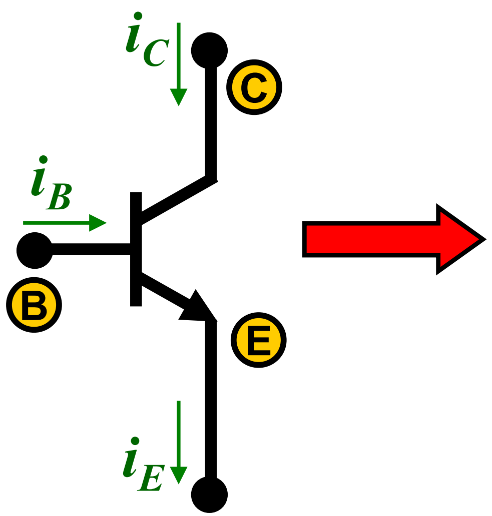


# *BJT* – Small-Signal Model (Simplified Hybrid-π)

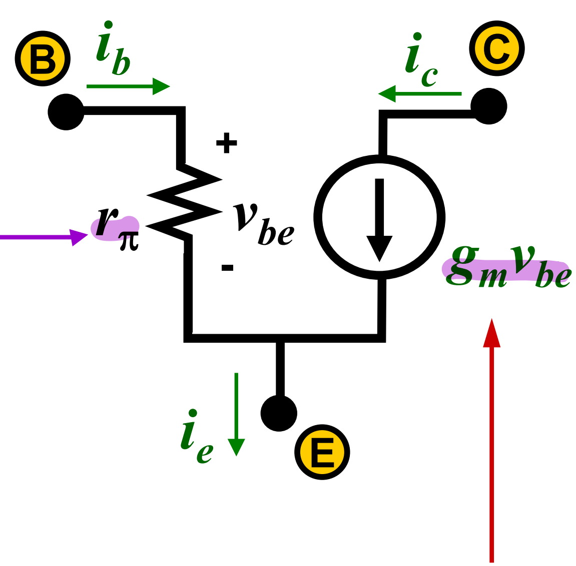
- Small-signal model of BJT is developed using equations (3.7), (3.9), (3.11) -

Eqn. (3.9): $v_{be} = r_\pi i_b$ - $v_{be}$ is linearly related to $i_b$ - base-emitter junction is modeled by a **resistor**, $r_\pi$

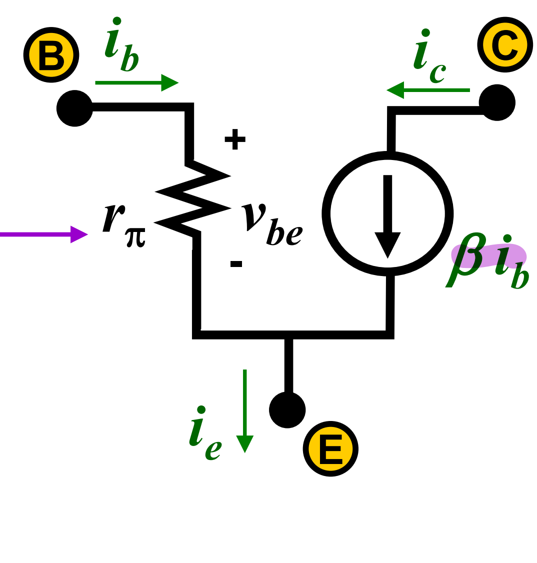
BJT




**Voltage-Controlled Version**




**Current-Controlled Version**

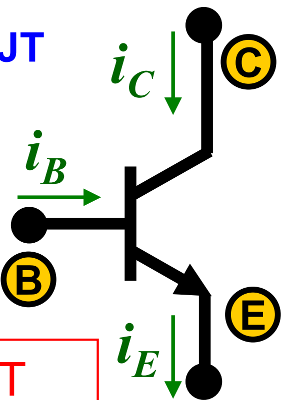



Eqn. (3.7): $i_c = g_m v_{be}$ - $i_c$ is dependent on $v_{be}$ – modeled by a **voltage-dependent** current source.

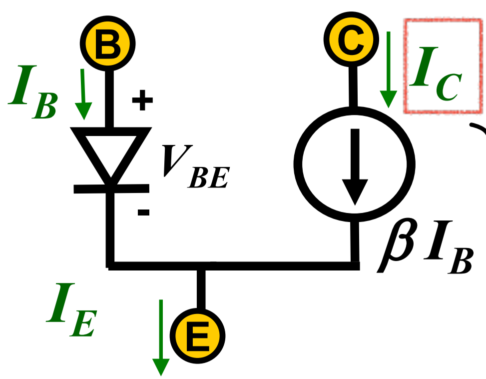
Eqn. (3.11): $i_c = \beta i_b$ – $i_c$ is dependent on $i_b$ - modeled by a **current-dependent** current source.

# *BJT* – Forward Active Mode Models (Summary)

Npn BJT




Replace BJT by large -signal model for dc analysis



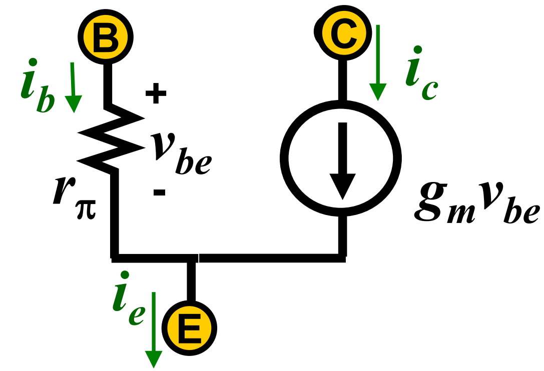

Large-Signal Model of the npn BJT

$g_m = \frac{I_C}{V_T}$

$r_\pi = \frac{\beta}{g_m}$

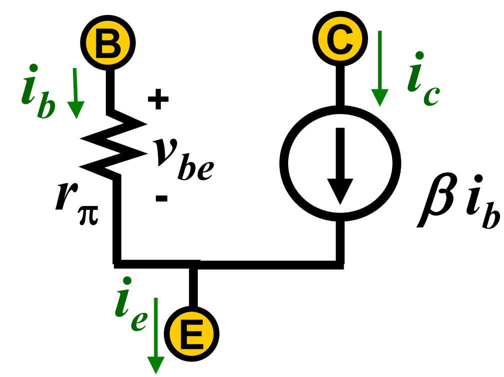
Replace BJT by hybrid-$\pi$ Model for small-signal analysis

Voltage-Controlled Version




or

Current-Controlled Version



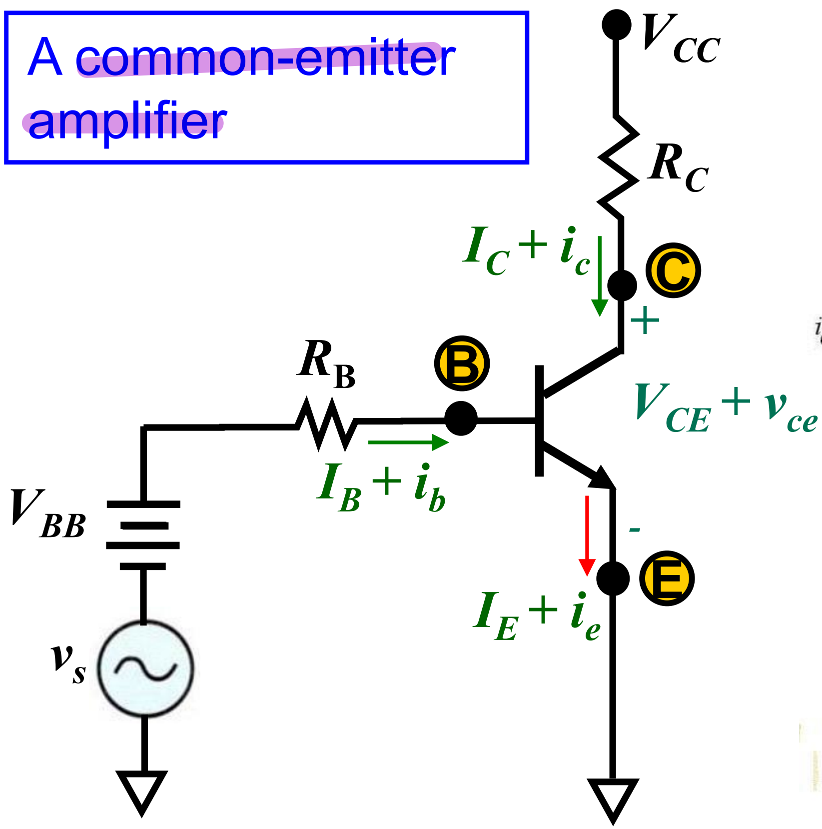

Small-Signal model (Simplified hybrid-$\pi$) of the npn BJT

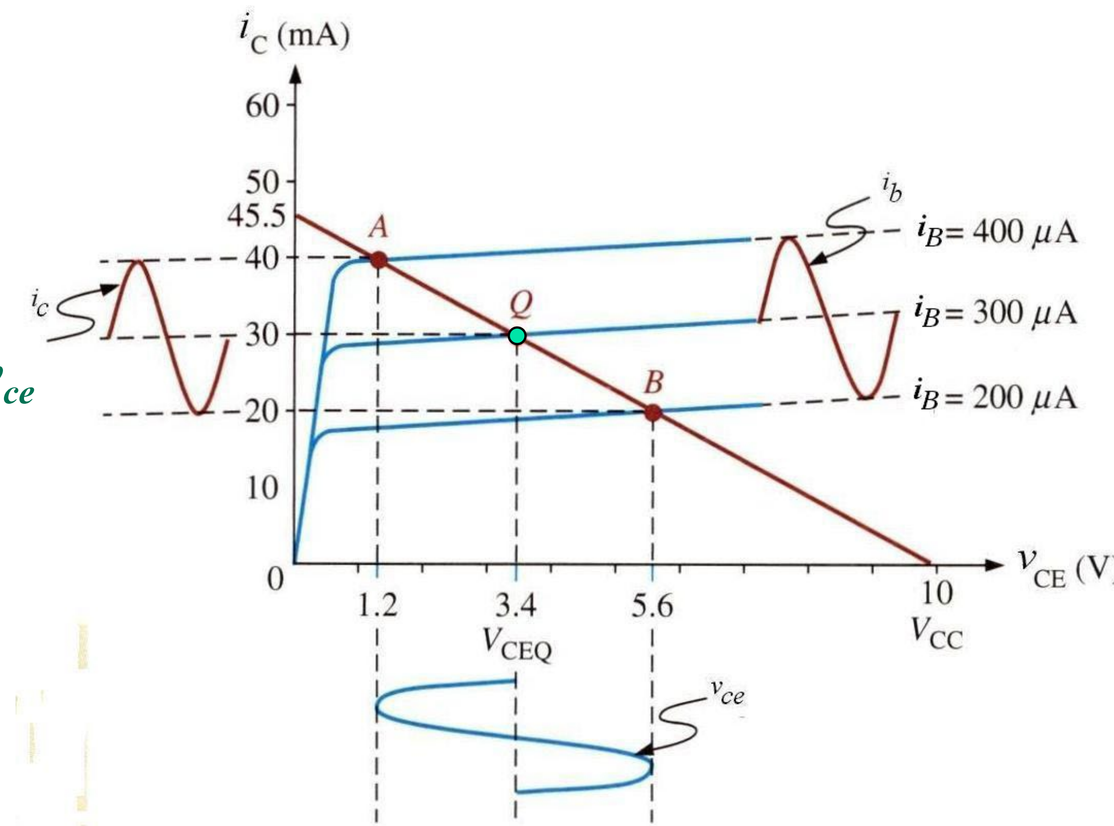
# *BJT* – Amplifier Circuit Operation & Analysis

- The operation of an amplifier circuit is shown in the $i_C$-$v_{CE}$ curve below, where the dc bias point (Q) and the small signals $i_b$, $i_c$ and $v_{ce}$ are indicated.
- In the presence of an ac (small-signal) source, $v_s$, -
  - $i_B = I_B + i_b$, $i_C = I_C + i_c$, and $v_{CE} = V_{CE} + v_{ce}$

  $i_B$, $i_C$ and $v_{CE}$ all have a small-signal component and they vary with time.

A common-emitter amplifier

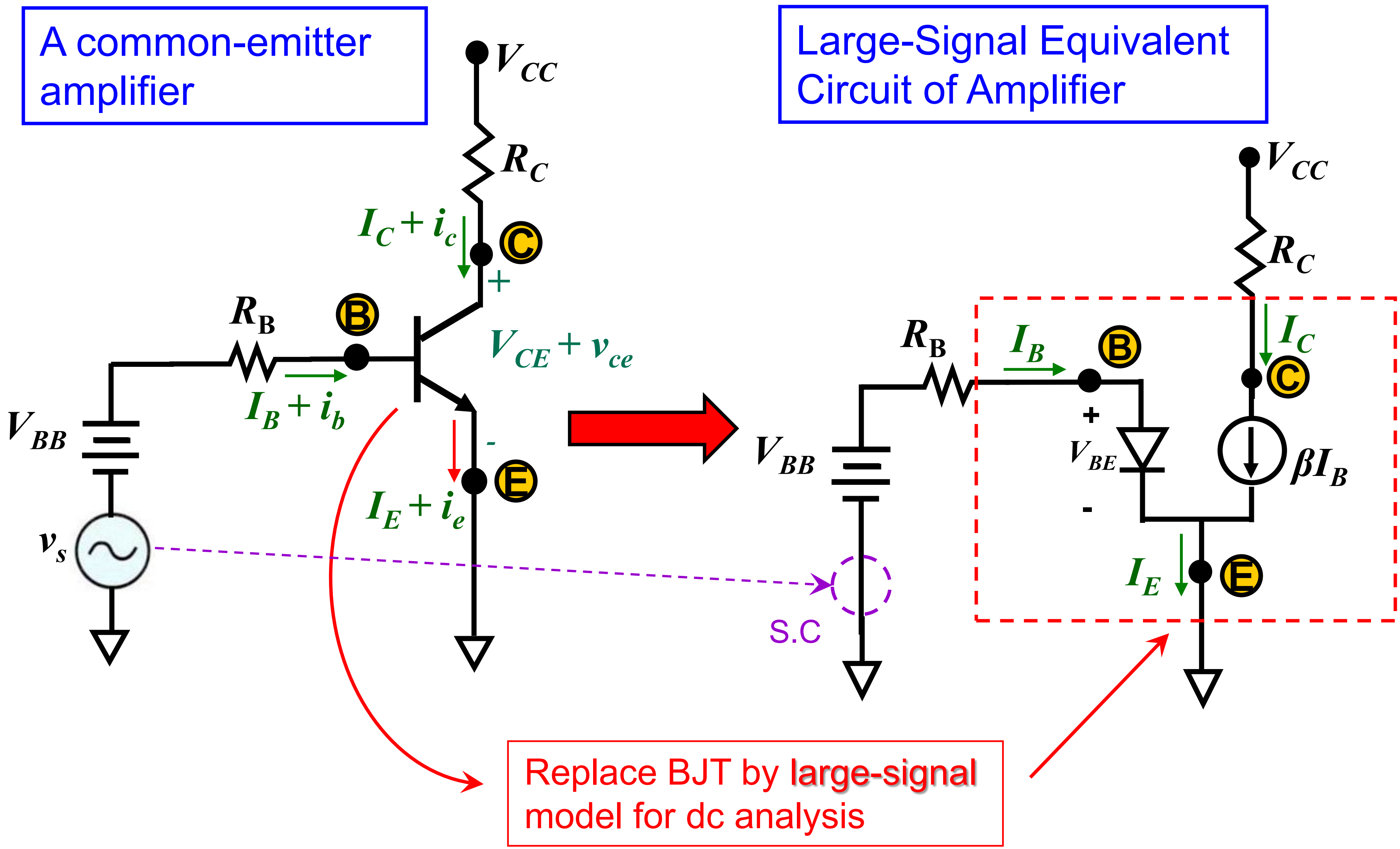


STEP 1

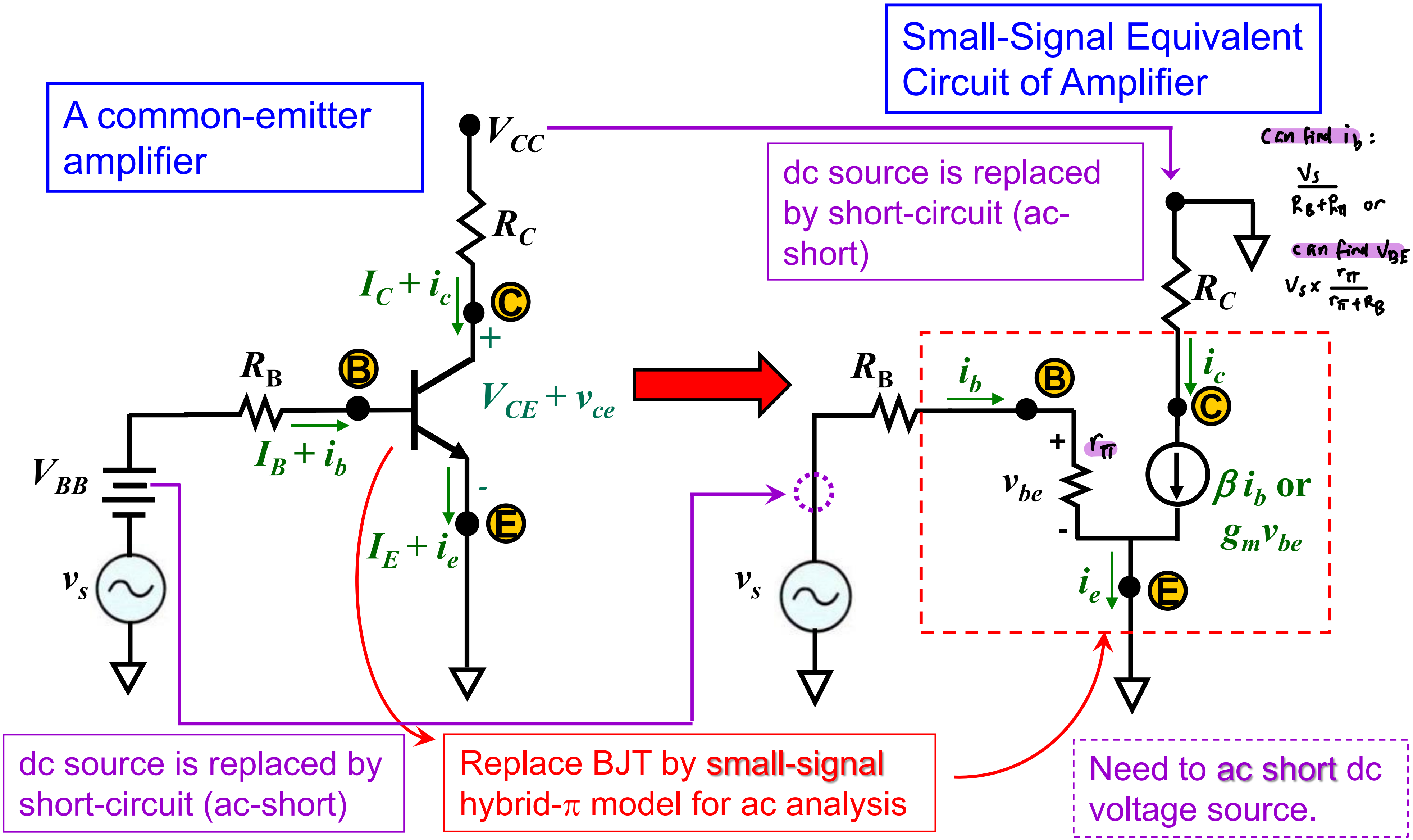
# *BJT* – Amplifier Circuit Operation (dc Analysis)

❑ The dc bias point of the amplifier can be determined using the **large-signal equivalent circuit**, in the **absence** of the ac (small signal) source, $v_s$.



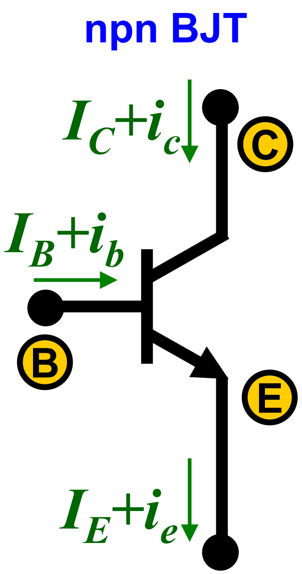
# *BJT* – Amplifier Circuit Operation (ac Analysis)

- The small-signal ac analysis of the amplifier is carried out using the **small-signal equivalent circuit,** and dc voltage sources need to be ac short.



# *BJT* – Small-Signal Model (pnp)

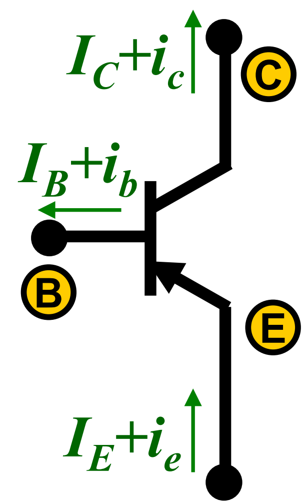
**npn BJT**




The change in collector current for the npn BJT is positive when there is an increase in the collector current flowing into the collector

$i_c = g_m v_{be} = \beta i_b$

**pnp BJT**



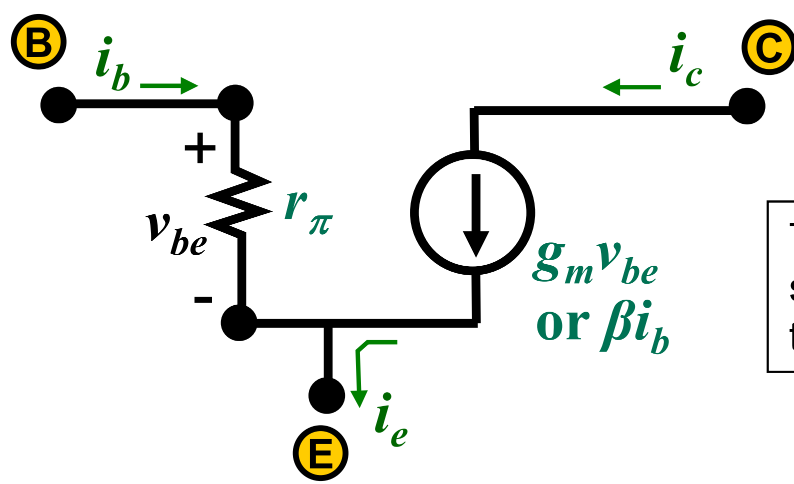

The change in collector current for the pnp BJT is positive when there is an increase in the collector current flowing out of the collector

$i_c = g_m v_{eb} = \beta i_b$

**Replace the BJT by the simplified hybrid-$\pi$ model for small-signal analysis**

***

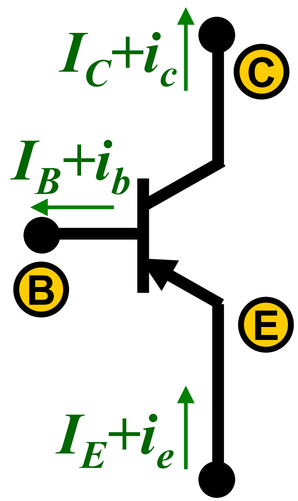
## Simplified Hybrid-$\pi$ Model ~~for npn BJT or pnp BJT~~

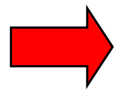


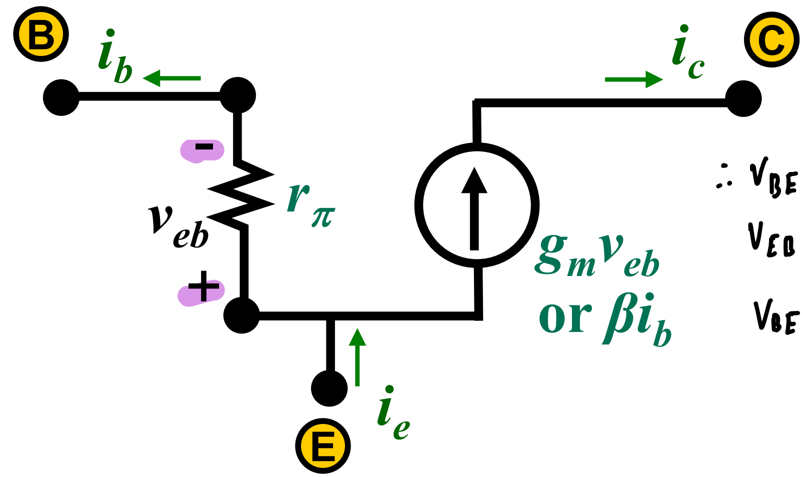

This Hybrid-$\pi$ Model is in its simplest form here. We will use this simplest form by default.

# *BJT* – Small-Signal Model (pnp vs npn)

**pnp BJT**



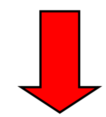



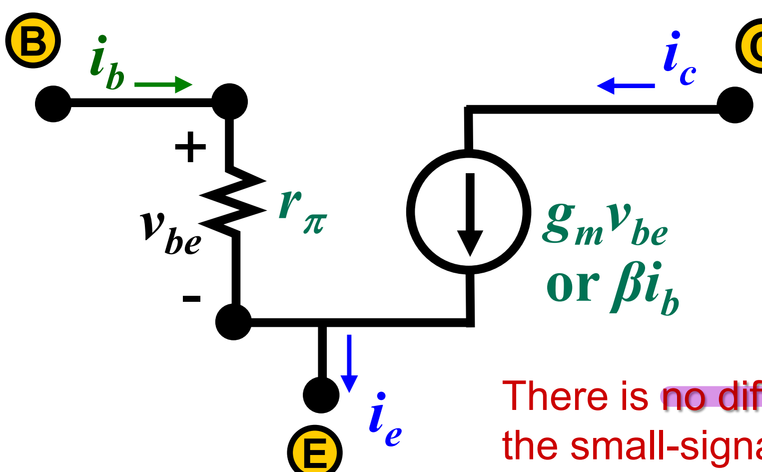



$\therefore V_{BE} = V_B - V_E$

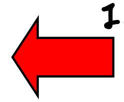
$V_{EB} = V_E - V_B$

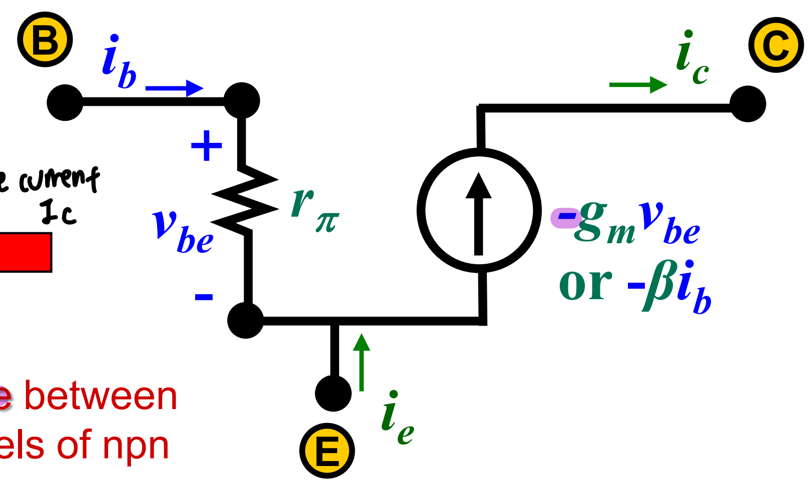
$V_{BE} = -V_{EB}$






reverse current $I_c$



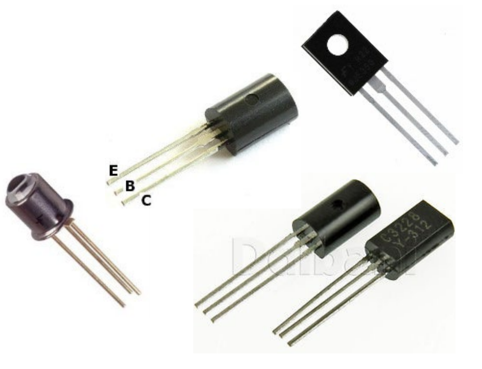



There is no difference between the small-signal models of npn BJT and pnp BJT.

# Bipolar Junction Transistor (BJT)

## Bipolar Junction Transistor (BJT)

1. Introduction
2. Modes of Operation: Forward Active, Saturation, Cut-Off and Reverse Active
3. Forward Active Current-Voltage Characteristic
4. Large-Signal Model and dc Analysis
5. Amplifier Circuit
6. Small-Signal Model (Simplified Hybrid-$\pi$) and ac Analysis
7. Other Hybrid-$\pi$ Small-Signal Models




## Reference

- Sedra and Smith, Microelectronic Circuits, Fifth Edition, Oxford (2004), pp. 159 – 168, 173 - 183, 203 - 241.

# *BJT* – Small-Signal Model (Simplified Hybrid-π)

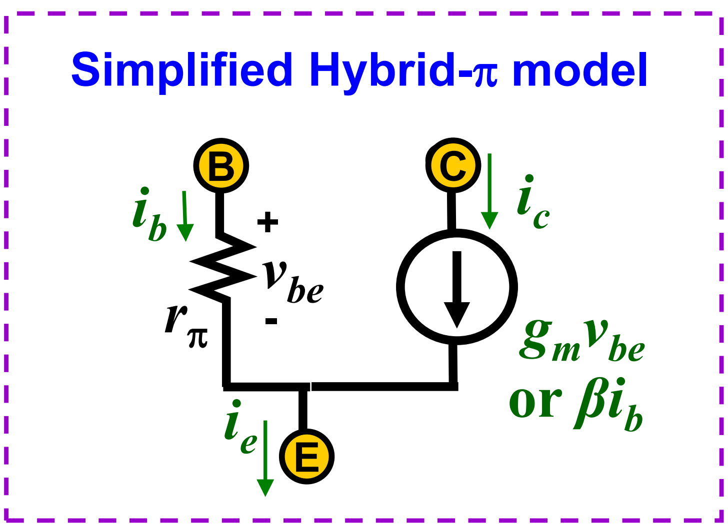
- In simplified hybrid-π model, Early effect was ignored and collector current,

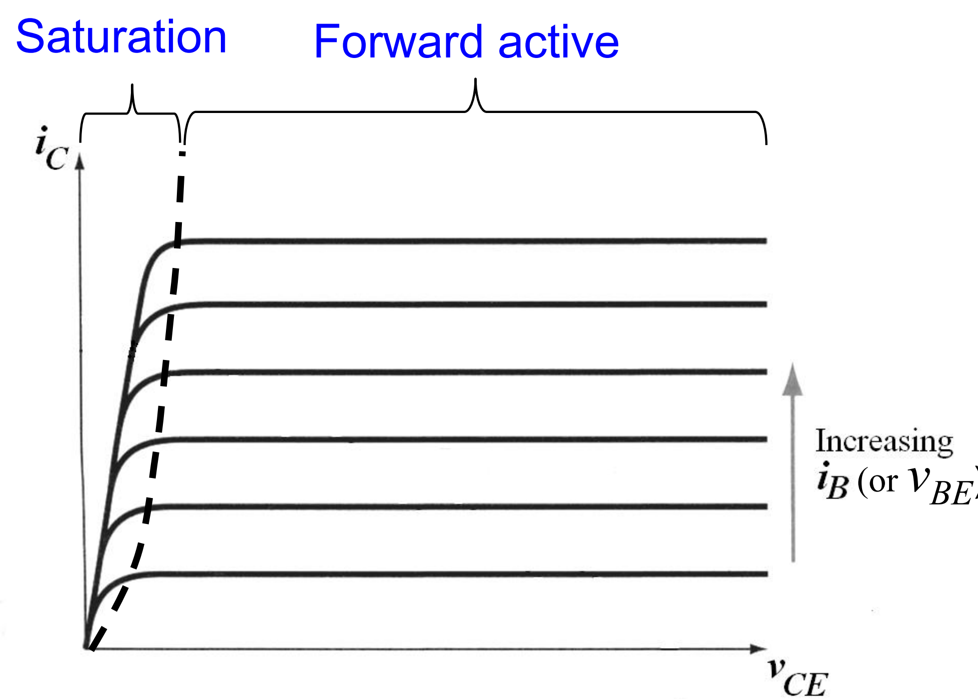
$$i_C = I_S e^{v_{BE}/V_T}, \qquad (3.1)$$

  is a function of only the base-emitter junction voltage, $v_{BE}$: $i_C = f(v_{BE})$.

- Small-signal collector current, $i_c$, is directly due to small-signal base-emitter junction voltage, $v_{be}$:

$$i_c = \frac{\partial i_C}{\partial v_{BE}}\Big|_{V_{BE}} \times v_{be} = g_m \times v_{be}$$ (see slide BJT-40, equation (3.6))



# *BJT* – Small-Signal Model (Hybrid-π)

- With Early effect (see slide BJT-14), collector current becomes

$$i_C = I_S e^{v_{BE}/V_T} \boxed{\left(1 + \frac{v_{CE}}{V_A}\right),} \tag{3.5}$$

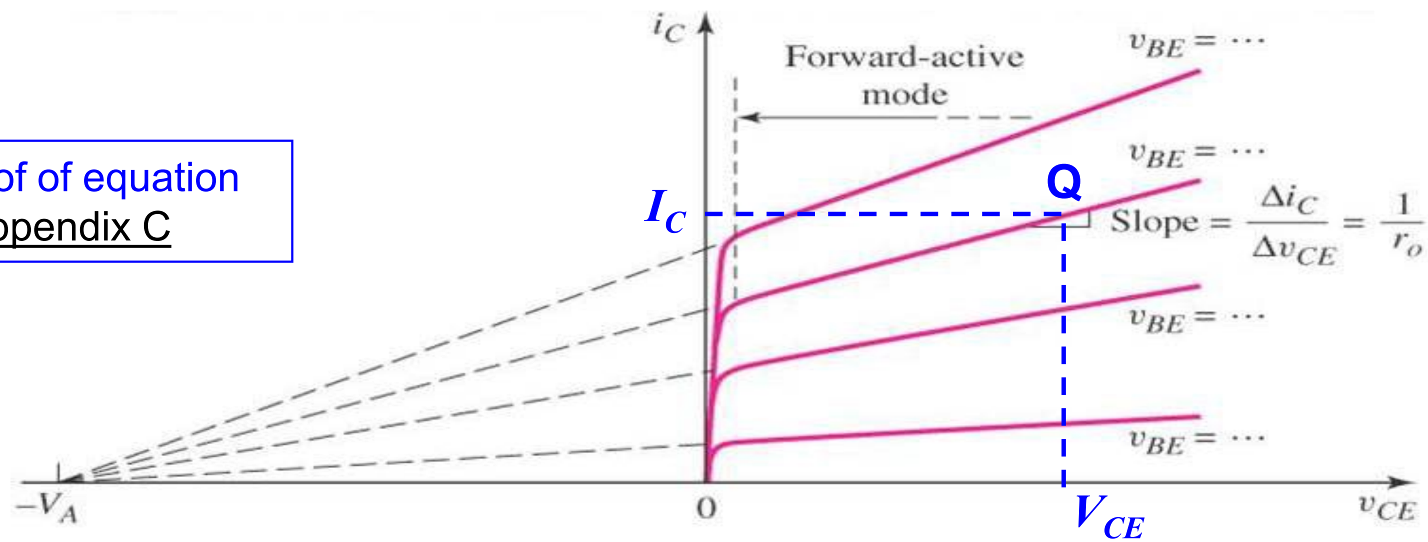
is also a function of collector-emitter voltage, $v_{CE}$, in addition to $v_{BE}$:

$$i_C = f(v_{BE}, v_{CE}).$$

- Small-signal collector current, $i_c$, will then be contributed by the small-signal collector-emitter voltage, $v_{ce}$, as well, in addition to that contributed by $v_{be}$ -

$$\Delta i_C = i_c \approx \underbrace{\frac{\partial i_C}{\partial v_{BE}}\Big|_{V_{BE}} \times v_{be}}_{\text{Contribution of } v_{be}} + \underbrace{\frac{\partial i_C}{\partial v_{CE}}\Big|_{V_{CE}} \times v_{ce}}_{\text{Contribution of } v_{ce}} \tag{3.12)*$$

* Mathematical proof of equation (3.12) is given in Appendix C



# *BJT* – Small-Signal Model (Hybrid-π)

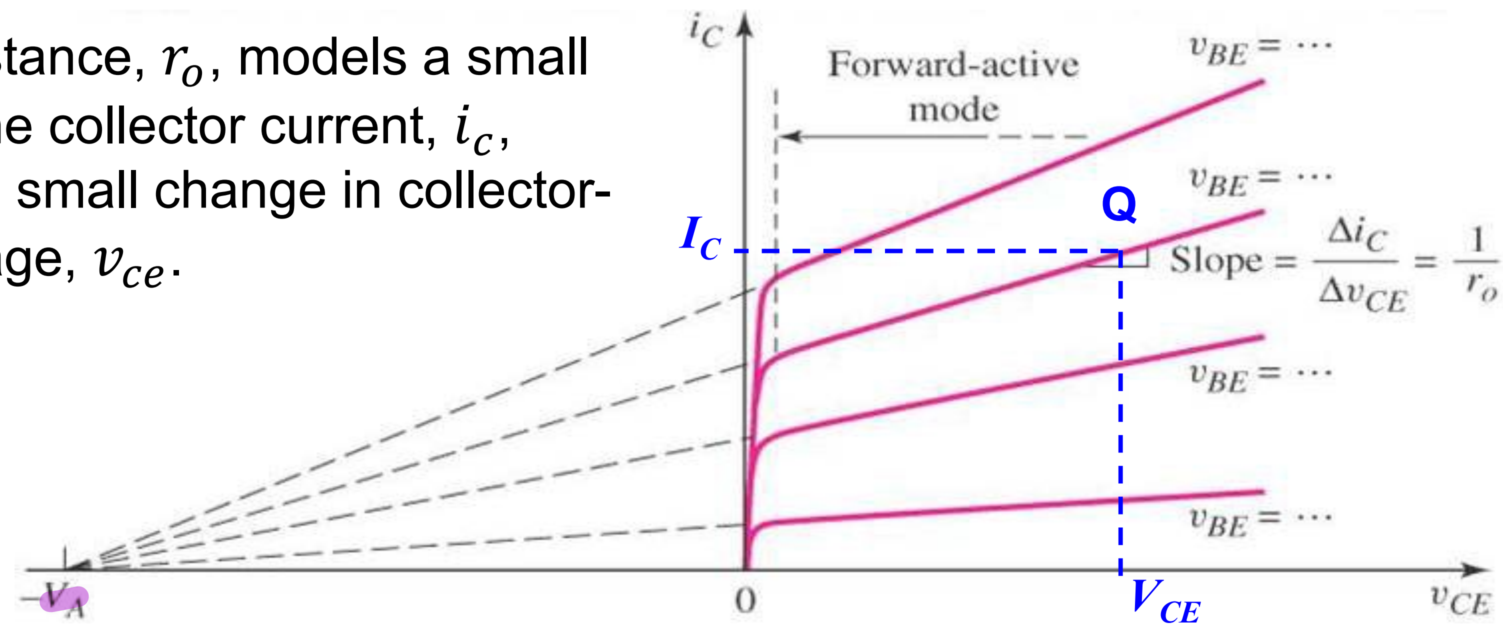
- Equation (3.5) - $$i_C = I_S e^{v_{BE}/V_T}\left(1 + \frac{v_{CE}}{V_A}\right)$$

- For given $V_{BE}$ and $V_{CE}$ (point Q), when $v_{CE}$ changes, $i_C$ will change. Change in $i_C$ owing to a small change in $v_{CE}$ is related to the slope (or derivative) of the corresponding $i_C$-$v_{CE}$ curve as follows –

$$\frac{\Delta i_C}{\Delta v_{CE}} = \frac{i_c}{v_{ce}} = \frac{\partial i_C}{\partial v_{CE}}\Big|_{V_{CE},V_{BE}} = \frac{I_S}{V_A} e^{V_{BE}/V_T} \approx \frac{I_C}{V_A} = \frac{1}{r_o} \qquad (3.13)$$

- Output resistance: $r_o = \frac{v_{ce}}{i_c} = \frac{V_A}{I_C}$ → early voltage if not given assume ∞ (3.14)

- Output resistance, $r_o$, models a small change in the collector current, $i_c$, caused by a small change in collector-emitter voltage, $v_{ce}$.



# *BJT* – Small-Signal Model (Hybrid-π)

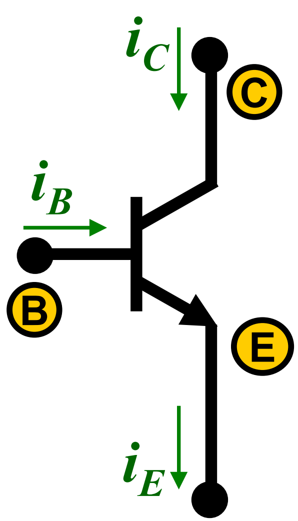
- From equation (3.12) –

$$i_c = \frac{\partial i_C}{\partial v_{BE}}|_{V_{BE}} \times v_{be} + \frac{\partial i_C}{\partial v_{CE}}|_{V_{CE}} \times v_{ce}$$

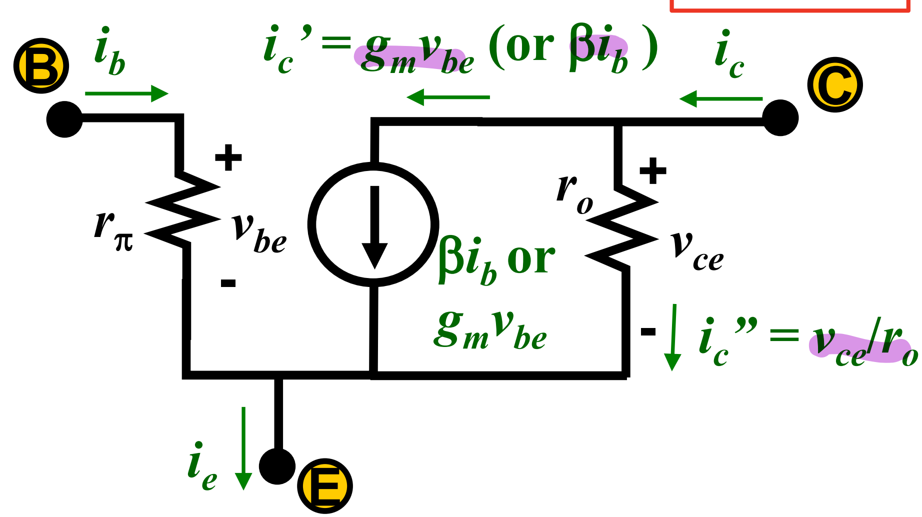
or, $$\boxed{i_c = g_m v_{be} + v_{ce}/r_o = \beta i_b + v_{ce}/r_o} \qquad (3.15)$$

- The term $v_{ce}/r_o$ accounts for the dependence of $i_C$ on $v_{CE}$ (or Early effect).
- To model the increase in $i_C$ owing to an increase in $v_{CE}$, a resistance $r_o$ is included between nodes C and E in the hybrid-π model.

Full Model




Replace BJT by this Hybrid-π Model for Small-Signal Analysis

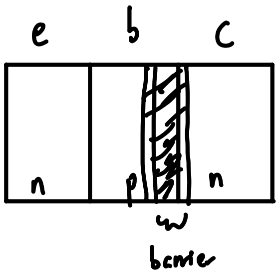



Note: In EE2027, you can assume that $r_o$ is infinite by default. However, if the Early voltage $V_A$ is given, then $r_o$ should be calculated and included in the small-signal analysis.

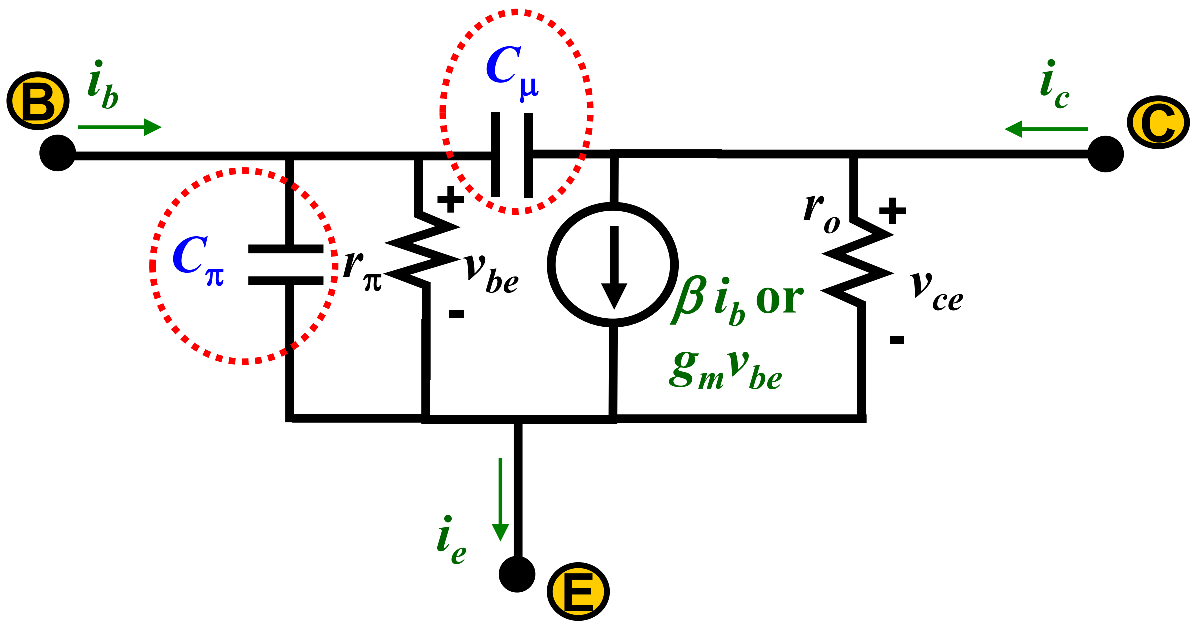
# *BJT* – Full Hybrid-π Model at High Frequency

- BJTs have capacitances associated with base-emitter and base-collector *pn*-junctions, which have charge storage.
- At high frequencies, two parasitic capacitors have to be included: $C_\pi$ and $C_\mu$, respectively across the base-emitter and base-collector junctions.

Early effect: also known as base width modulation




barier is being squeezed. So if barier is widened, it will cause electrons to flow into the collecter more easily. So by increasing collecter voltage, more electrons go into collecter side so collecter current increase slightly.



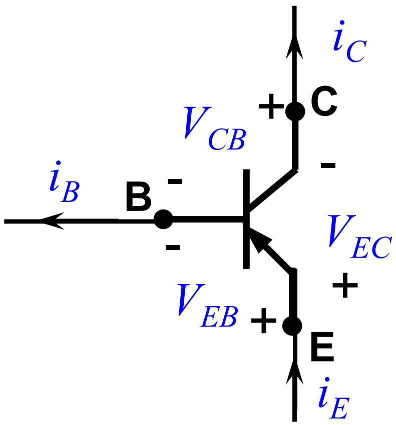

Note: We introduce the existence of these capacitances $C_\mu$ and $C_\pi$ here, and their inclusion in the Hybrid-π Model is only necessary when high frequency response is discussed. We will ignore these capacitances (in the pF range) for low frequency signals.

# BJT - Topics Discussed

- **Structure:** 3 regions/terminals - emitter, base and collector
- **Operation regions:** Forward active, saturation, cut-off
- **$i_C$-$v_{CE}$ curves**
- **Forward active IV characteristics (npn BJT):**
  - $i_C = I_S e^{v_{BE}/V_T},\ \ i_B = \frac{I_S}{\beta} e^{v_{BE}/V_T},\ \ i_C = \beta i_B$
- **Forward active $i_C$ with Early Effect (npn BJT):** $i_C = I_S e^{v_{BE}/V_T}\left(1 + \frac{v_{CE}}{V_A}\right)$
- **npn vs pnp BJT:** same functions, opposite bias voltage polarities and current directions
- **Forward active large-signal model & dc analysis**
  - Thevenin equivalent vs voltage divider method for base biasing circuit
- **Forward active small-signal model:** $g_m = \frac{I_C}{V_T},\ \ r_\pi = \frac{\beta V_T}{I_C},\ \ r_o = \frac{V_A}{I_C}$

# APPENDICES

# *APPENDIX A* – Operation of pnp BJT

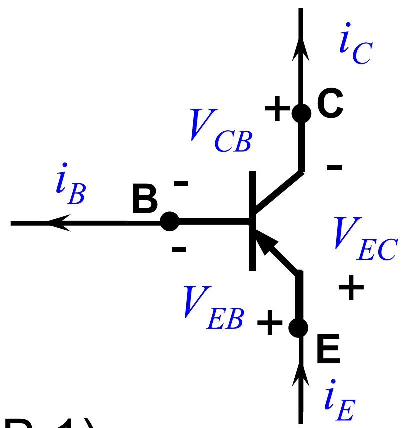



Modes of operation of the **pnp** bipolar junction transistor

| Mode of Operation | Emitter-Base Junction | Collector-Base Junction | Applications |
|---|---|---|---|
| Cut-off | Reverse biased ($V_{EB} < 0$ for pnp) | Reverse biased ($V_{CB} < 0$ for pnp) | Logic- OFF State |
| Forward Active | Forward biased ($V_{EB} > 0$ for pnp) | Reverse biased ($V_{CB} < 0$ for pnp) | Amplifier |
| Saturation | Forward biased ($V_{EB} > 0$ for pnp) | Forward biased ($V_{CB} >0$ for pnp) | Logic - ON State |
| Reverse Active | Reverse Biased ($V_{EB} < 0$ for pnp) | Forward Biased ($V_{CB} > 0$ for pnp) | Not used |

* For the same mode of operation, the current directions and voltage polarities of pnp BJT are exactly opposite from those of npn BJT.

# *APPENDIX B* – IV Characteristic of pnp BJT




- For pnp BJT in forward active operation –
  - Emitter-base junction is forward biased: $v_{EB} > 0$.
  - Collector-base junction is reverse biased: $v_{CB} < 0$.
  - Collector current, $i_C$, is controlled by $v_{EB}$ –

$$i_C = I_S e^{v_{EB}/V_T} \text{ (flows out of the collector),} \qquad \text{(B.1)}$$

  - Base current, $i_B$, is given by

$$i_B = \frac{I_S}{\beta} e^{v_{EB}/V_T} \text{ (flows out of the base),} \qquad \text{(B.2)}$$

where $I_S$ is the saturation current,

$V_T$ is the thermal voltage and $V_T = kT/q \approx 0.025$ at $T$ = 300 K, and

$\beta$ is the common-emitter current and $\beta = i_C/i_B$.

- From equations (B.1) and (B.2):

$$i_C = \beta i_B \qquad \text{(B.3)}$$

- Emitter current: $i_E = i_C + i_B$ (flows into the emitter). (B.4)

# *APPENDIX C* – Non-Ideal $i_C$-Expression

- With Early effect (see slide BJT-14), collector current in forward active region is given by

$$i_C = I_S e^{v_{BE}/V_T}\left(1+\frac{v_{CE}}{V_A}\right) = f(v_{BE}, v_{CE}) \qquad \text{(C.1)}$$

- We can express the total (instantaneous) currents and voltages in terms of the dc (operating point) and ac (signal) currents and voltages, that is
  - $i_C = I_C + i_c$
  - $v_{BE} = V_{BE} + v_{be}$
  - $v_{CE} = V_{CE} + v_{ce}$

- Hence, $i_C = I_C + i_c = f(v_{BE}, v_{CE}) = f(V_{BE}+v_{be}, V_{CE}+v_{ce})$ (C.2)

- Expand equation (C.2) as a Taylor series and neglecting higher order terms for small $v_{be}$and $v_{ce}$ -

$$i_C = I_C + i_c = f(V_{BE}+v_{be}, V_{CE}+v_{ce})$$
$$\approx f(V_{BE}, V_{CE}) + \frac{\partial i_C}{\partial v_{BE}}\Big|_{V_{BE}} \times v_{be} + \frac{\partial i_C}{\partial v_{CE}}\Big|_{V_{CE}} \times v_{ce}$$

- As $I_C = f(V_{BE}, V_{CE})$, small signal collector current

$$i_c \approx \frac{\partial i_C}{\partial v_{BE}}\Big|_{V_{BE}} \times v_{be} + \frac{\partial i_C}{\partial v_{CE}}\Big|_{V_{CE}} \times v_{ce}. \qquad \text{(C.3)}$$