EE2027
Electronic Circuits

Opamp and Opamp Based Circuits

Lecture Outline

- Overview of opamp and its analysis
- Opamp parameters
- Different types of opamp based amplifier
  - log amplifier, exponential amplifier, instrumentation amplifier
- Filter
  - Integrator, differentiator, 1^{st} order filter
  - 2^{nd} order SK filter and higher order filter
- Super diode and comparator
- Applications built using opamp
  - Triangular wave, multiplier, bandstop filter, full wave rectifier

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Opamp Schematic

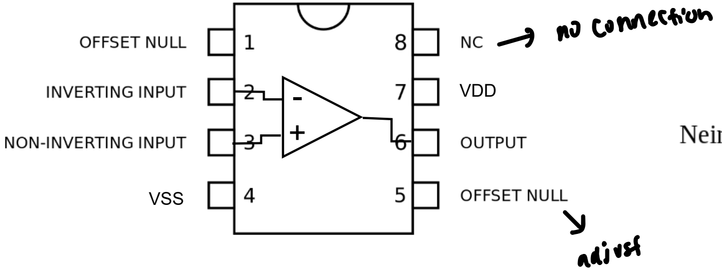
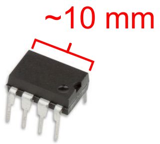


Diagram by Peter Halasz, CCASA 3.0



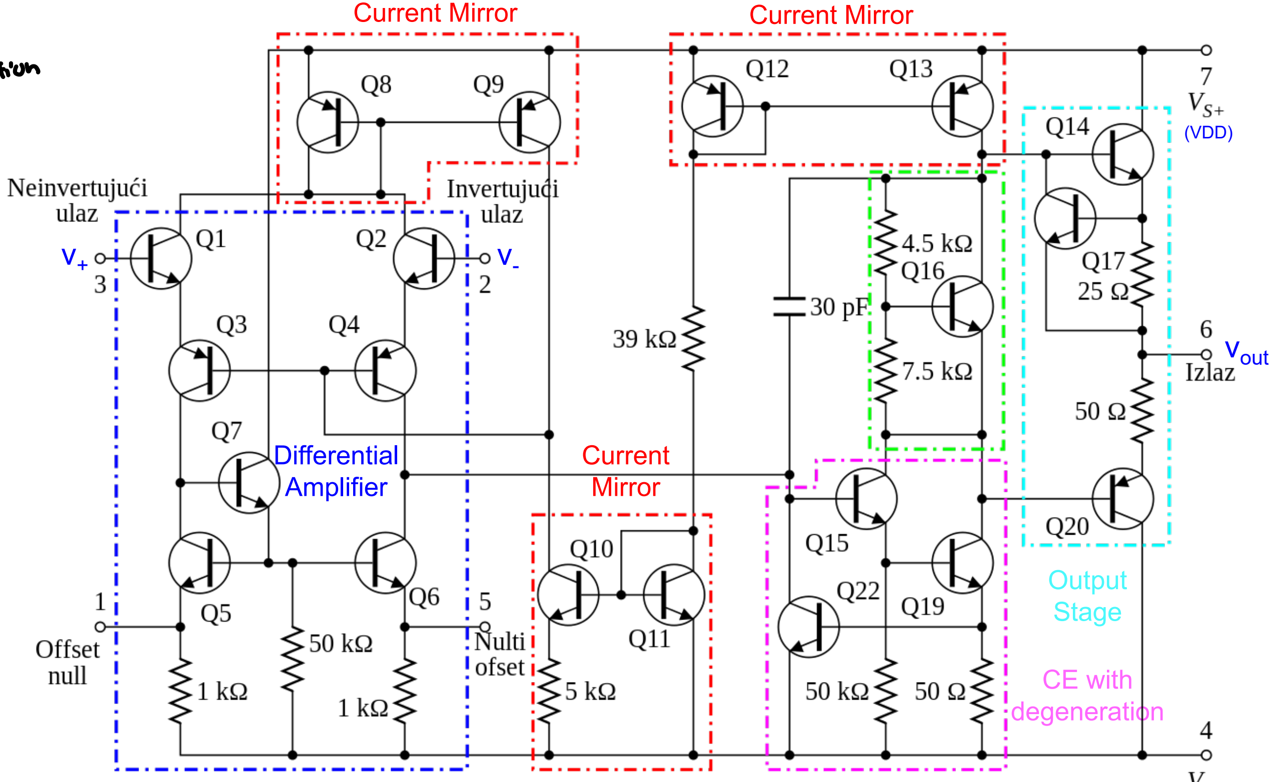
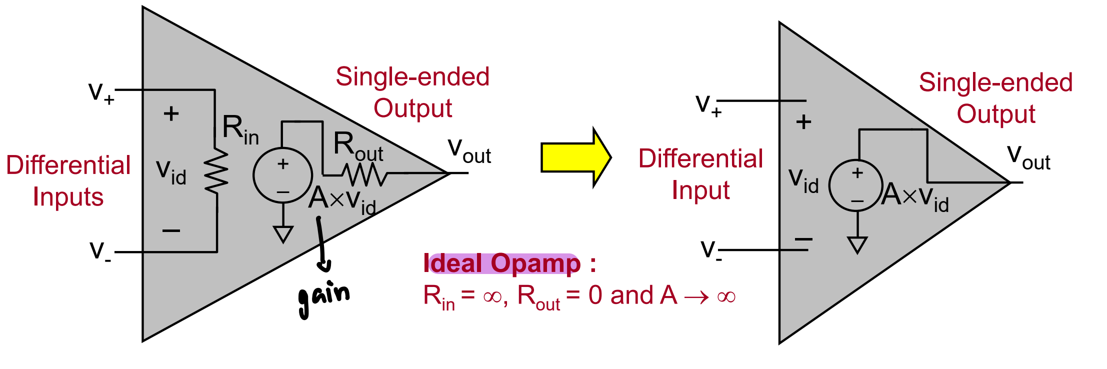


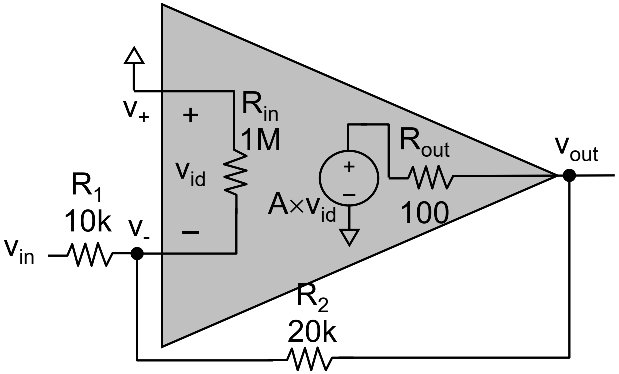
Diagram by Daniel Braun, CCASA 3.0

- Multistage Amplifier Analysis
- All the techniques learnt later in amplifier analysis will help you understand and design an opamp eventually

Opamp Applications



Model for Opamp **Model for Ideal Opamp**



$R_1, R_2 << R_{in} = 1M \Rightarrow R_{in} \rightarrow \infty$
$R_1, R_2 >> R_{out} = 100 \Rightarrow R_{out} \rightarrow 0$
$A = 10000 \rightarrow \infty$

For simplicity, we always analyze opamp based circuits with ideal opamp model

Possible Applications :

- Feedback Amplifier
- Integrator, Differentiator, Active Filter
- Simulated Inductor
- Switched-Capacitor Filter
- Oscillator
- A/D Converter

Connect some electronic components (Resistors, capacitors, transistors, diodes) surrounding the opamp will result in interesting applications

Lecture Summary

- What opamp is made up of inside
- Model used for opamp circuit analysis

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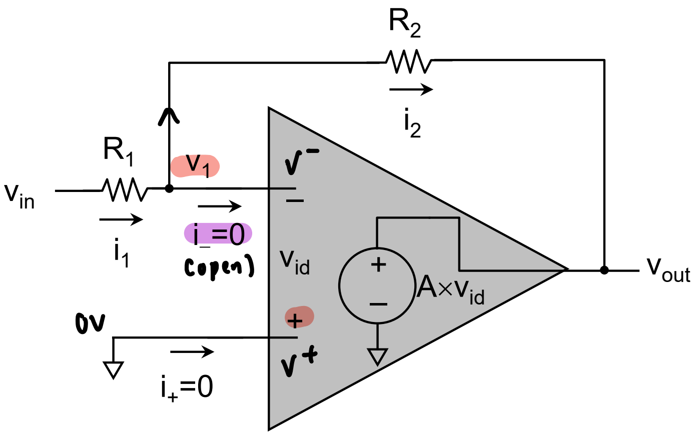
How to Derive Transfer Function for Opamp Circuit?



$*\!\!*$

Important Observations :

- No current flowing into the opamp, i.e., $i_- = i_+ = 0$
- If A is large and v_{out} is finite $\Rightarrow v_{id} = -v_1 \approx 0 \Rightarrow v+ \approx v-$
  $\Rightarrow$ Virtually short
- A real short, current can potentially flow through between two nodes. A virtual short, only the voltage appears to be zero, but there is no current flowing through between the two nodes.
- Since v+ is connected to ground, we can say v_1 is virtual ground

because V_{id} is virtually short

Transfer Function for Opamp Circuit

$$v_{id} = -v_1 \qquad V_{id} = 0 - V_1 \quad (V_{id} = V^+ - V^-)$$

$$v_{out} = Av_{id} = A(-v_1)$$

$$\Rightarrow v_1 = -\frac{v_{out}}{A}$$

$$\boxed{\frac{v_{in} - v_1}{R_1} = \frac{v_1 - v_{out}}{R_2}}$$

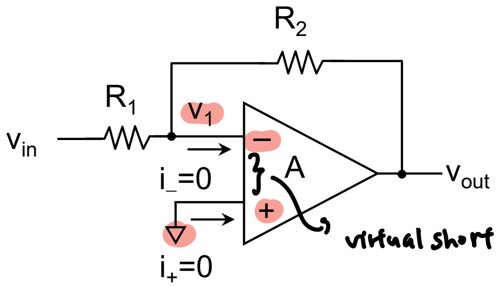
KCL:
$i_- = 0 \Rightarrow i_1 = i_2$

$$\Rightarrow \frac{v_{in} + \frac{v_{out}}{A}}{R_1} = \frac{-\frac{v_{out}}{A} - v_{out}}{R_2}$$

$$\Rightarrow \boxed{\frac{v_{out}}{v_{in}}} = -\frac{R_2}{R_1}\frac{1}{\boxed{\frac{R_1 + R_2}{AR_1}} + 1} \approx \boxed{-\frac{R_2}{R_1}} \quad if \quad A \to \infty$$

<<1 $\Rightarrow$ negligible even if A~10000
$\therefore A \to \infty$ is a good approximation

Inverting Amplifier



Transfer Function for Inverting Amplifier

$$v_1 = v_- \approx v_+ = 0 \quad [\because \textit{Virtually short to ground}]$$

$$\boxed{\frac{v_{in} - v_1}{R_1} = \frac{v_1 - v_{out}}{R_2}}$$

KCL:
$i\_ = 0 \Rightarrow i_1 = i_2$

$$\frac{v_{in}}{R_1} = \frac{-v_{out}}{R_2}$$

$$\boxed{\Rightarrow \frac{v_{out}}{v_{in}} = -\frac{R_2}{R_1}} \quad \cdots(1)$$

Using virtual short concept, the analysis is much simpler

$R_{out} << R_1, R_2 << R_{in}$

(10 times)

- The overall gain ($A_V = v_{out}/v_{in}$) is controlled by the ratio of resistors (R_2/R_1) ✓ because it can be accurately controlled
- The overall gain is independent on the opamp gain (A) ✓ because variation on A would not affect the gain
- It is a feedback amplifier ✓ because less distortion and more linear
- There is polarity inversion between v_{out} and v_{in} × Invert the output signal

output connect back to input

Lecture Summary

- **How to perform opamp circuit analysis using virtual short and no input current**

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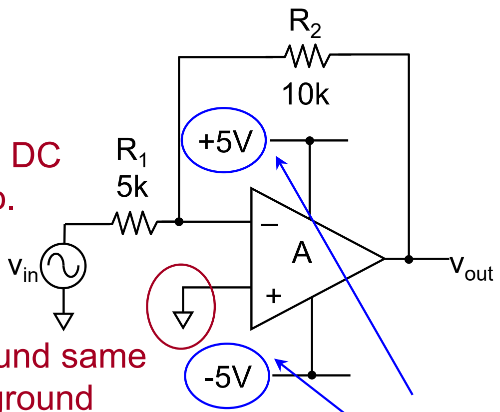
Opamp Biasing – Old Days

v_{in} has DC of zero.

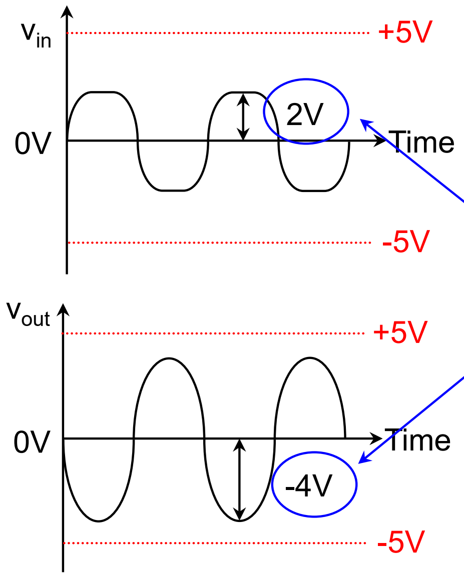


DC ground same as AC ground

Input (v_{in}) and output (v_{out}) must be within positive supply (+5V) and negative supply (-5V)

Supplies required to bias the opamp properly

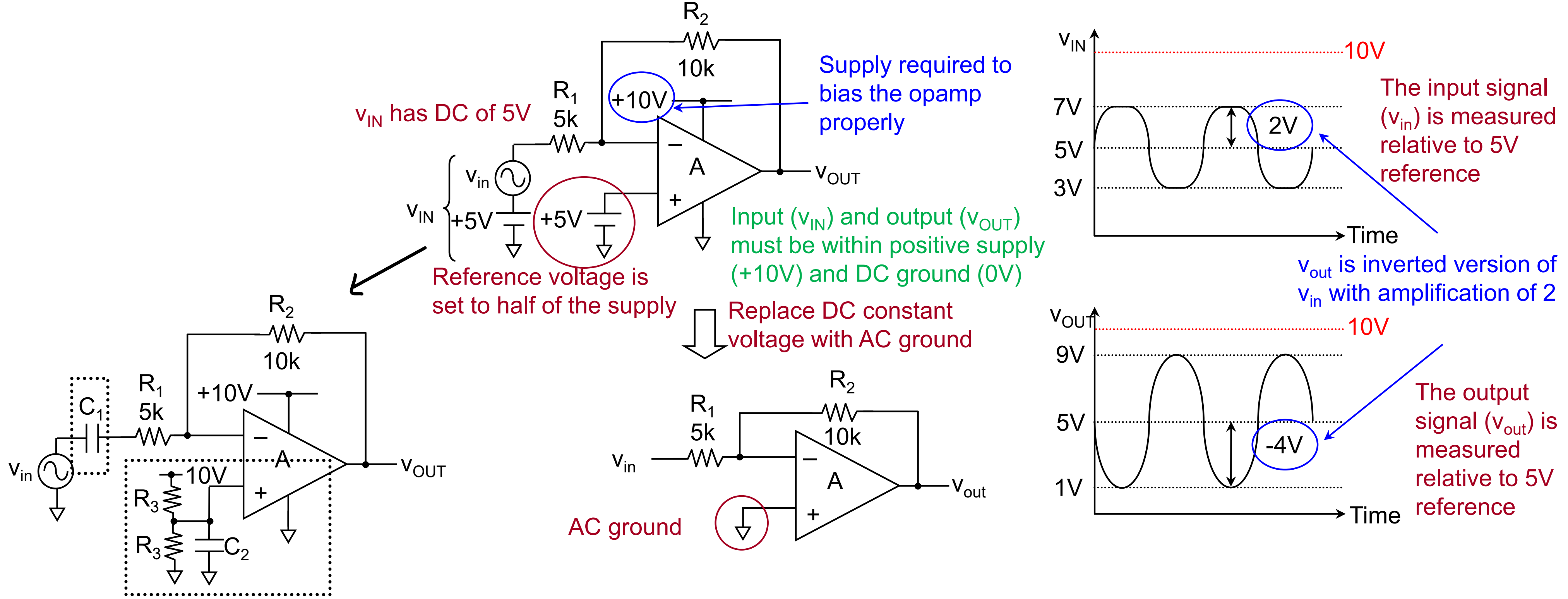
Inverting amp



v_{out} is inverted version of v_{in} with amplification of 2

- Luxury of dual supply (+5V and -5V)
- DC ground and AC ground are the same

Opamp Biasing – Nowadays



- Single supply only (+10V)
- DC ground and AC ground are not the same In this example, AC ground has DC voltage of 5V
- For analysis simplicity, we will always ignore the DC portion and perform the AC analysis
- The reference voltage is usually implemented using simple resistor divider, and stabilized with a capacitor

Lecture Summary

- **Different ways of biasing in old days and new days**

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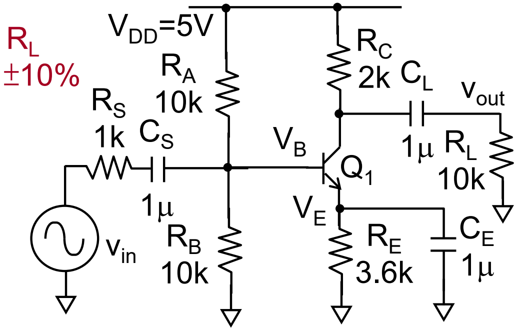
Why Feedback Amplifier?

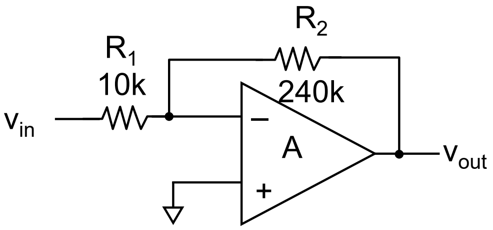
g_{m1}, R_C and R_L can vary by ±10%



$$A_V = \frac{v_{out}}{v_{in}} = -\frac{R_{in}}{R_{in}+R_S} g_{m1}\left(R_C // R_L\right) = -24$$

Open Loop Amplifier (CE)

VS



$$A_V = \frac{v_{out}}{v_{in}} = -\frac{R_2}{R_1} = -24$$

Feedback Amplifier

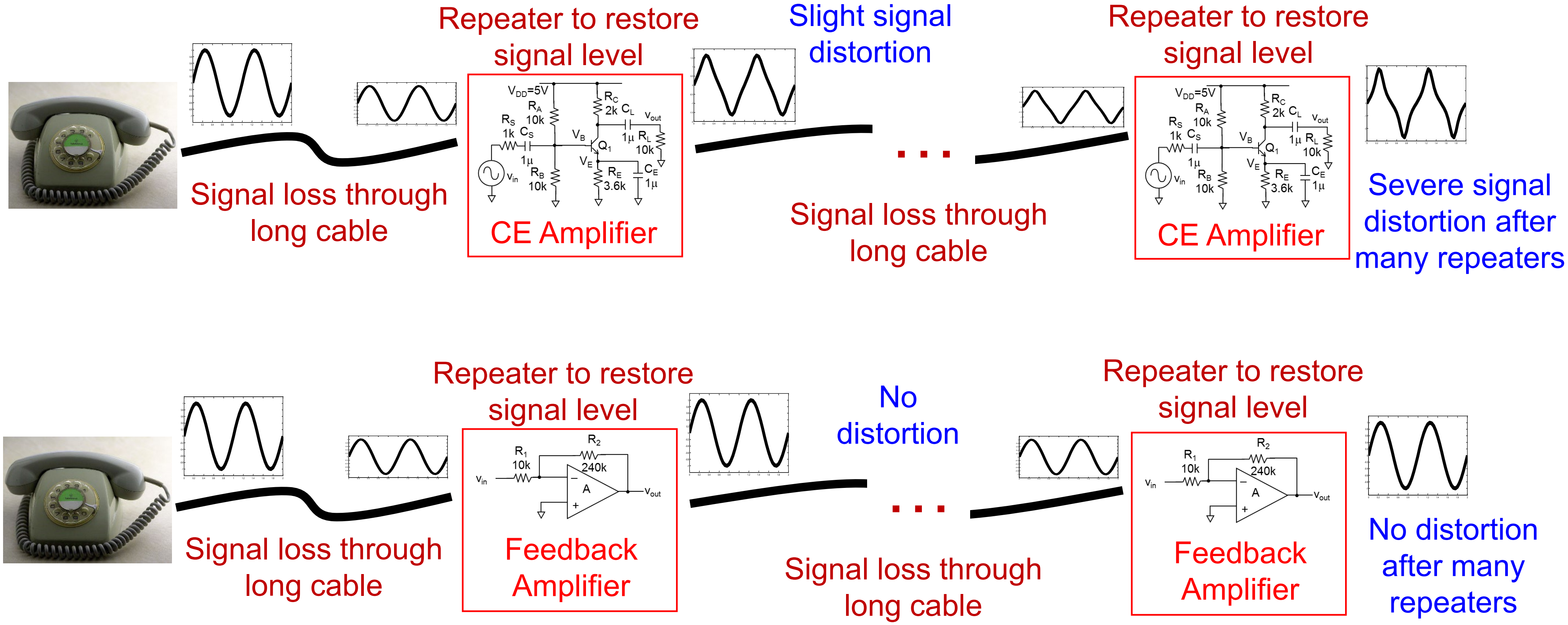
$R_2 = 24 \times R_1$

Resistors (R_1, R_2) coming from the same batch can match to each other accurately even its absolute value can vary

$$A_V = -\frac{R_2}{R_1} = -\frac{24 \times R_1}{R_1} = -24$$

	Open Loop Amplifier	Feedback Amplifier
Gain	Not accurate (Difficult to control transistor parameters) ×	Accurate gain (Gain only depends on the ratio of resistors, which can be accurately controlled) ✓
Complexity	Simple and consume less power ✓	Opamp is complicated and consume more power ×
Linearity	Poor Linearity (More distortion) ×	High Linearity (Less distortion) ✓

Linearity



Repeater to restore signal level
Slight signal distortion
Repeater to restore signal level
V_DD=5V
R_A 10k
R_S 1k
C_S 1μ
R_C 2k
C_L 1μ
v_out
R_L 10k
V_B
Q_1
V_E
R_E 3.6k
C_E 1μ
v_in
R_B 10k
Signal loss through long cable
CE Amplifier
Signal loss through long cable
CE Amplifier
Severe signal distortion after many repeaters
Repeater to restore signal level
No distortion
Repeater to restore signal level
R_1 10k
R_2 240k
A
v_in
v_out
Signal loss through long cable
Feedback Amplifier
Signal loss through long cable
Feedback Amplifier
No distortion after many repeaters

Lecture Summary

- **Why the need of using feedback amplifier**

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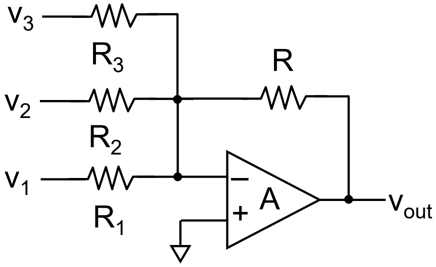
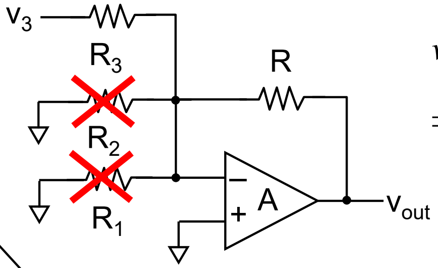
Summing Amplifier



1) Estimate output due to individual voltage source
2) Apply linear superposition to find the total contribution
3) While estimating contribution from one voltage source, kill all the other voltage source

Keep v_3 and kill v_1, v_2
Due to virtual ground, R_1 and R_2 are effectively removed

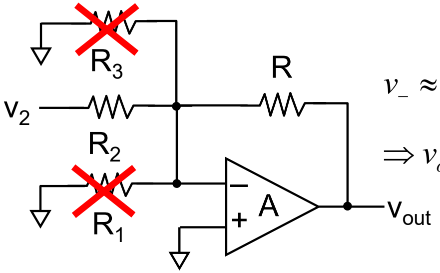


$$v_- \approx v_+ = 0$$

$$\Rightarrow v_{out,v3} = -\frac{R}{R_3} v_3$$

Equation (1) from slide 5-9

Keep v_2 and kill v_1, v_3
Due to virtual ground, R_1 and R_3 is effectively removed

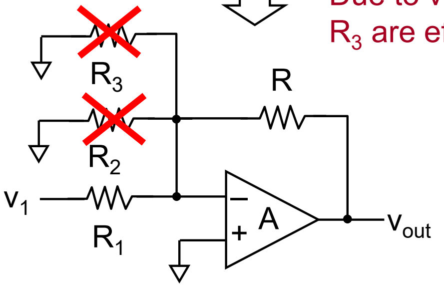


$$v_- \approx v_+ = 0$$

$$\Rightarrow v_{out,v2} = -\frac{R}{R_2} v_2$$

Equation (1) from slide 5-9

Keep v_1 and kill v_2, v_3
Due to virtual ground, R_2 and R_3 are effectively removed



$$v_- \approx v_+ = 0$$

$$\Rightarrow v_{out,v1} = -\frac{R}{R_1} v_1$$

Equation (1) from slide 5-9

Transfer Function for Summing Amplifier

$$\boxed{v_{out} = v_{out,v_1} + v_{out,v_2} + v_{out,v_3}}$$ Superposition

$$= -\left(\frac{R}{R_1} v_1 + \frac{R}{R_2} v_2 + \frac{R}{R_3} v_3\right)$$

$$= -(v_1 + v_2 + v_3) \quad if \quad R_1 = R_2 = R_3 = R$$

Steps for Opamp Circuit Analysis

Opamp Circuit Analysis

⇩

Assume virtual short
$v_+ \approx v_-$

⇩

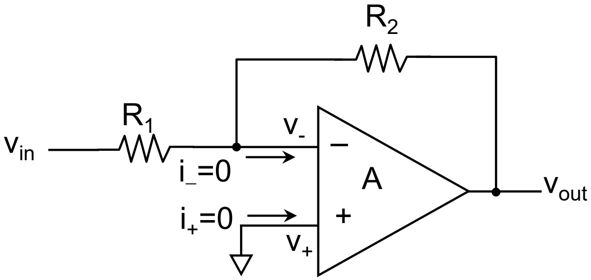
Assume no input current
$i_+ = i_- = 0$

⇩

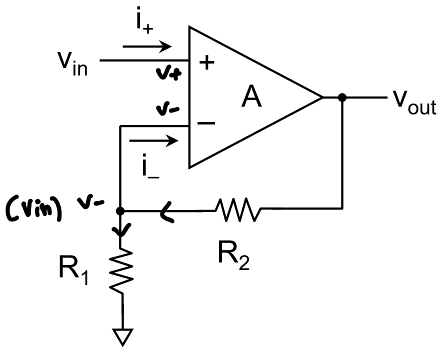
Apply linear superposition for multiple input voltage sources

⇩

Derive $A_V = v_{out}/v_{in}$ using KCL and KVL



Non-Inverting Amplifier



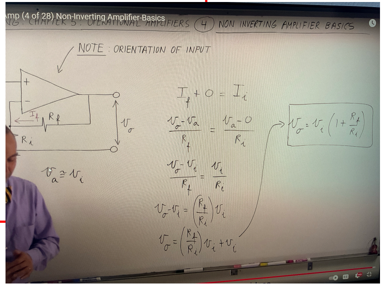
close to equal

Transfer Function for Non-Inverting Amplifier

$$\boxed{v_- \approx v_+} = v_{in} \quad [\because Virtual\ \ Short]$$

$$v_- = v_{out} \times \frac{R_1}{R_1 + R_2} = v_{in} \qquad [\because i_+ = i_- = 0]$$

$$\Rightarrow \frac{v_{out}}{v_{in}} = \left(1 + \frac{R_2}{R_1}\right) \qquad (2)$$

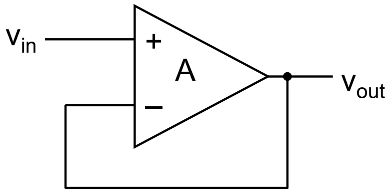


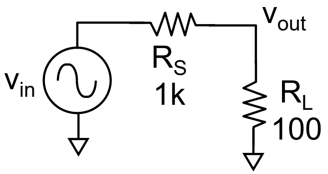
What is the limitation on the gain of non-inverting amplifier?

→ cannot have gain less than 1

- The overall gain ($A_V = v_{out}/v_{in}$) is controlled by the ratio of resistors ($1+R_2/R_1$)
- The overall gain is independent on the opamp gain (A)
- It is a feedback amplifier
- There is no polarity inversion between v_{out} and v_{in}

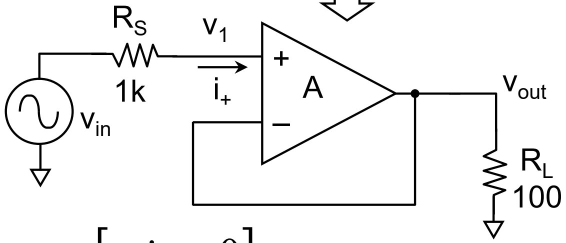
Unity Gain Buffer (Source Follower) → power gain





Due to loading effect ($R_L < R_S$)
$\Rightarrow A_V = v_{out}/v_{in} < 1$

$$\frac{v_{out}}{v_{in}} = \frac{R_L}{R_L + R_S} = 0.09$$



Transfer Function for
Unity Gain Buffer

$$v_{out} = v_- \approx v_+ = v_{in} \quad [\because \mathit{Virtual\ \ Short}]$$

$$\Rightarrow \frac{v_{out}}{v_{in}} = 1$$

$$R_{in} = \infty$$

$$v_1 = v_{in} [\because i_+ = 0]$$

$$\Rightarrow v_{out} = v_- \approx v_+ = v_1 = v_{in}$$

$$\Rightarrow \frac{v_{out}}{v_{in}} = 1$$

The unity gain buffer presents high R_{in} to the source and is able to drive low impedance load

- Ideal for driving low impedance load

Lecture Summary

- The three important assumptions to perform opamp circuit analysis, i.e., virtual short, no input current and superposition
- Summing amplifier, non-inverting amplifier, buffer

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Key Opamp Parameters

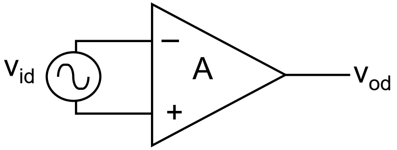
Parameter	ABBV	Unit	Definition
Common-mode rejection ratio	CMRR	dB	The ratio of differential voltage amplification to common-mode voltage amplification.
Gain-bandwidth product	GBW	MHz	The product of the open-loop voltage gain and the frequency at which it is measured.
Input noise voltage spectral density	V_n	$nV/\sqrt{Hz}$	The internal noise voltage reflected back to an ideal voltage source in parallel with the input pins.
Input offset voltage	V_{OS}	mV	The DC voltage that must be applied between the input terminals to cancel the DC offsets within the opamp. (cause output voltage to be zero)
Input resistance	r_i	MΩ	The DC resistance between the input terminals with either input grounded. This is R_{in} in the opamp model in slide 5-5.
Input voltage range	V_I	V	The range of input voltages that may be applied to either IN+ or IN- inputs.
Maximum peak-to-peak output voltage swing	$V_{O(PP)}$	V	The maximum peak-to-peak output voltage that can be obtained without waveform clipping.

Key Opamp Parameters

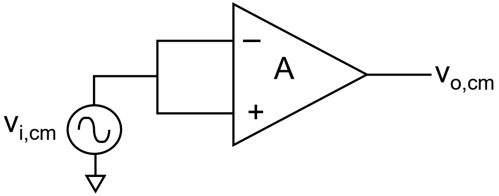
Parameter	ABBV	Unit	Definition
Open-loop gain	A_{OL}	dB	The ratio of the change in output voltage to the change in voltage across the input terminals. This is the differential voltage amplification, i.e., A in the opamp model in slide 5-5.
Output resistance	r_o	Ω	DC resistance that is placed in series with ideal amplifier and the output terminal. This is R_{out} in the opamp model in slide 5-5.
Phase margin	Φ_m	°	The absolute value of the open-loop phase shift at the frequency where the open-loop amplification first equals one. (Key for stability consideration)
Power supply rejection ratio	PSRR	dB	The ratio of differential voltage amplification to supply-to-output voltage amplification
Slew rate	SR	V/μs	The rate of change in the output voltage with respect to time for a step change at the input.

Note: The list is by no means exhaustive, we just show key ones that are simple enough to be understood at this level.

A_{OL} and CMRR (Common Mode Rejection Ratio)



A_{OL}: open loop gain $\quad \underbrace{A_{OL} = \frac{v_{od}}{v_{id}}}_{\text{ideal} = \infty}$

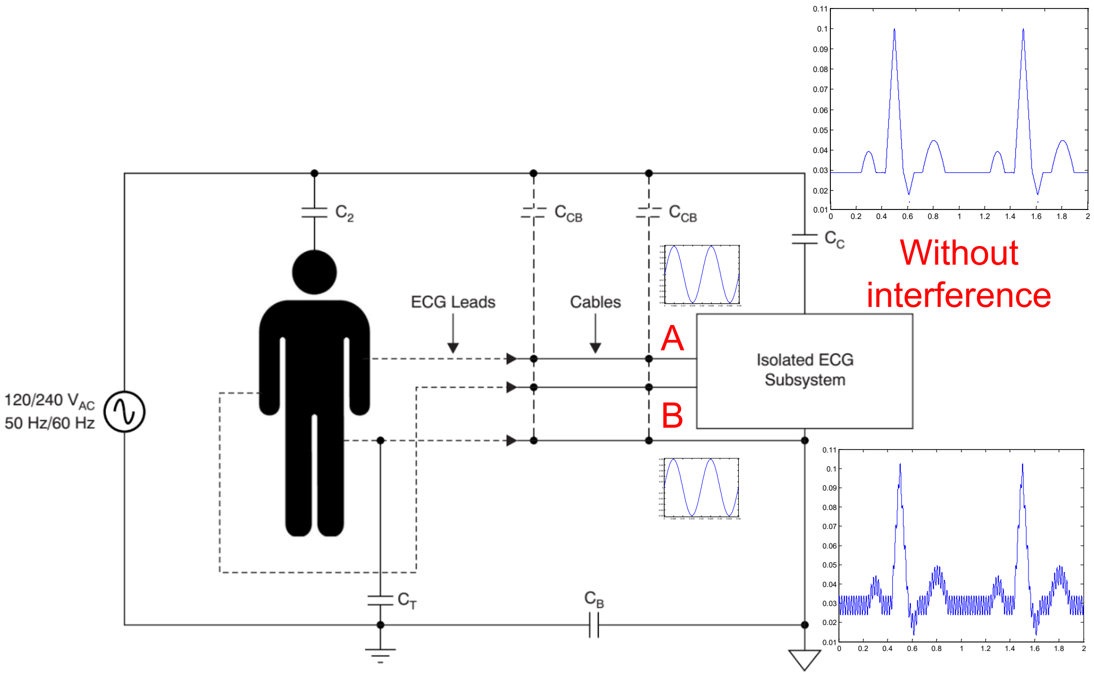


A_{CM}: common-mode gain $\quad \underbrace{A_{CM} = \frac{v_{o,cm}}{v_{i,cm}}}_{\text{ideal} = 0}$

CMRR: $\underbrace{CMRR}_{\text{ideal: large number}} = 20log\left(\frac{A_{OL}}{A_{CM}}\right)$

- For ideal opamp, A_{CM} (common-mode voltage amplification) should be zero, which leads to infinite CMRR.
- $v_{i,cm}$ (common-mode input voltage) includes interference and common-mode noise that readily appear at both input terminals.
- Opamp with good CMRR should reject common-mode input voltage without impacting the output.

CMRR and its Implication



Example:
Assuming the coupled power line interference signal at points A and B are 50 mV and 50 Hz, estimate the power line interference amplitude at the output. You may assume the ECG system is built using opamp with A_{OL} of 60 dB and CMRR of 80 dB. Also the ECG system has a gain of 40 dB.

$$20log(A_{CM}) = 20log(A_{OL}) - CMRR$$
$$= -20dB$$
$$A_{CM} = 0.1$$

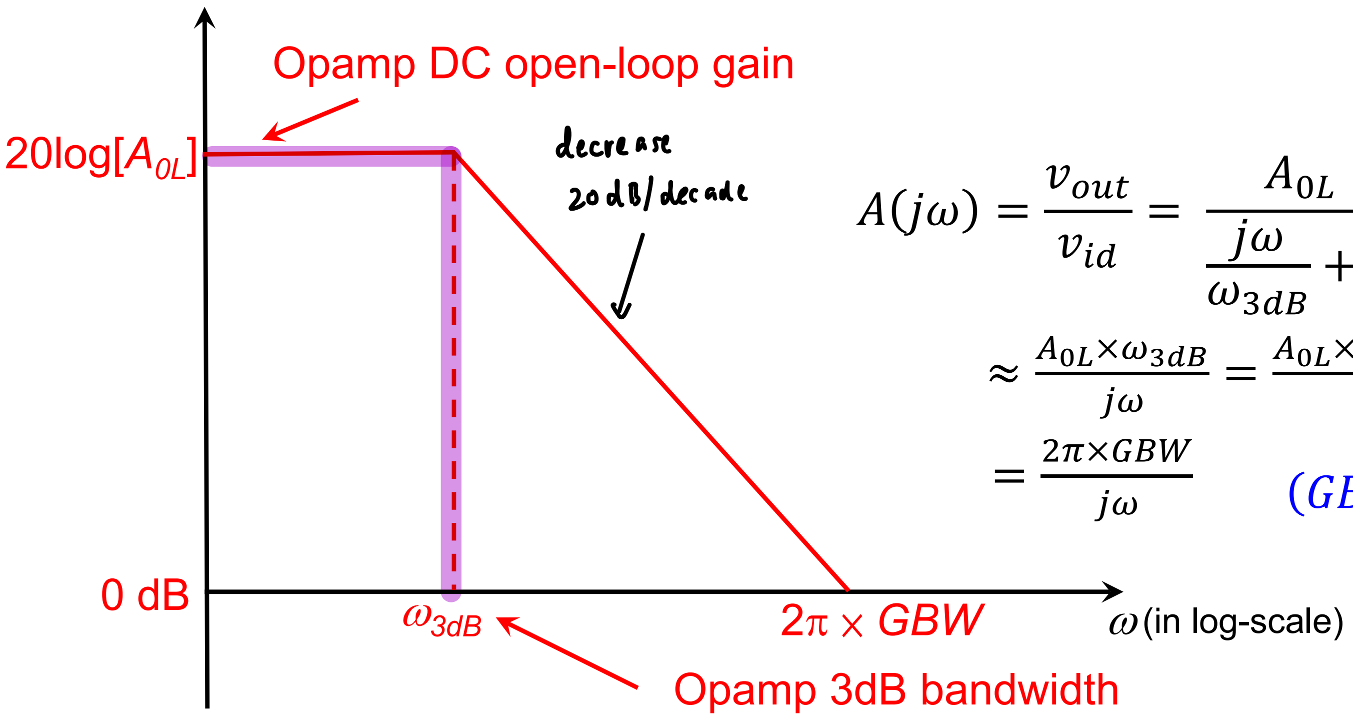
The output will exhibit 5 mV power line interference of 50 Hz.

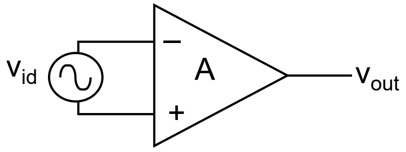
Note: ECG signal can be quite small, and power line interference of 5 mV at the output may not be acceptable.

↓
up to user to determine if output is small enough

GBW (Gain-BandWidth product)

Opamp Transfer Function, $A(j\omega)$





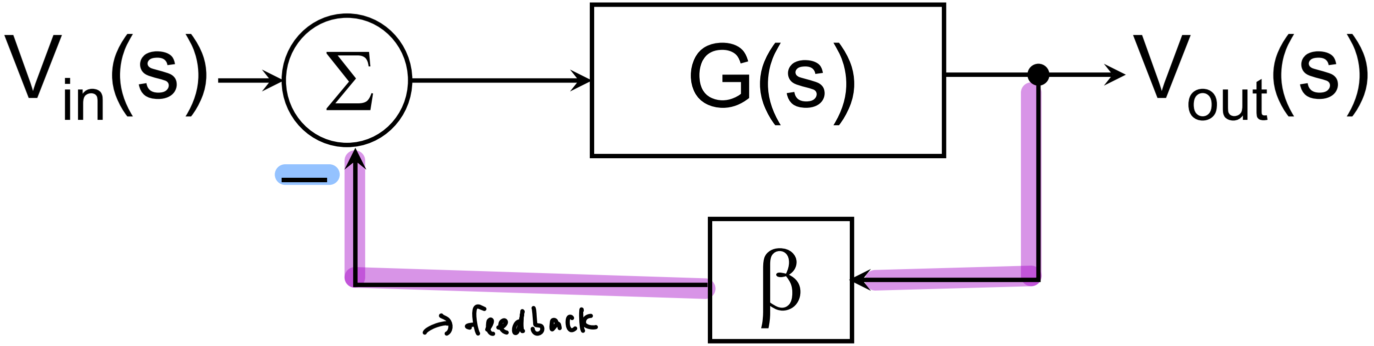
$$A(j\omega) = \frac{v_{out}}{v_{id}} = \frac{A_{0L}}{\frac{j\omega}{\omega_{3dB}} + 1}$$

$$\approx \frac{A_{0L} \times \omega_{3dB}}{j\omega} = \frac{A_{0L} \times 2\pi \times f_{3dB}}{j\omega} \quad [\text{If } \omega >> \omega_{3dB}]$$

$$= \frac{2\pi \times GBW}{j\omega} \qquad (GBW = A_{0L} \times f_{3dB})$$

- GBW impacts the frequency range of opamp based circuit.
- To understand how GBW determines the frequency range, we need to review feedback theory.

Feedback Theory



→ feedback

$$V_{out}(s) = G(s)[V_{in}(s) - \beta \times V_{out}(s)]$$

→ much bigger

$$\Rightarrow H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{G(s)}{1 + \beta \cdot G(s)} \approx \frac{1}{\beta} \leftarrow \beta \cdot G(s) >> 1$$ → ignore +1

$$20log[|H(s)|] = 20\log\left[\left|\frac{G(s)}{1 + \beta \cdot G(s)}\right|\right]$$ Loop Gain

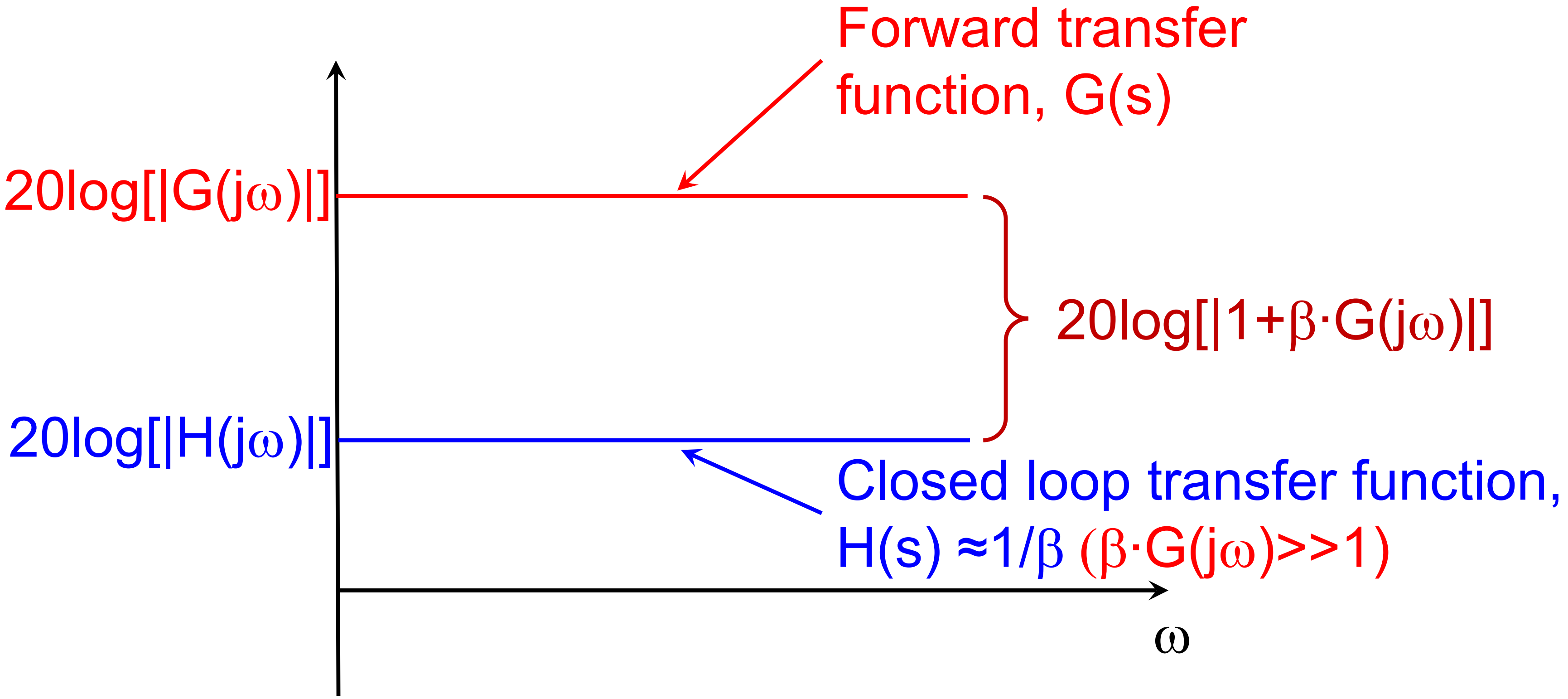
$$= 20\log[|G(s)|] - 20\log[|1 + \overbrace{\beta \cdot G(s)}^{\text{Loop Gain}}|]$$

$$\approx 20\log[|G(s)|] - 20\log[|\beta \cdot G(s)|] \leftarrow \beta \cdot G(s) >> 1$$

$$\approx -20log(\beta) = 20log(1/\beta)$$

Note: Log function transforms the operation to simple addition or subtraction

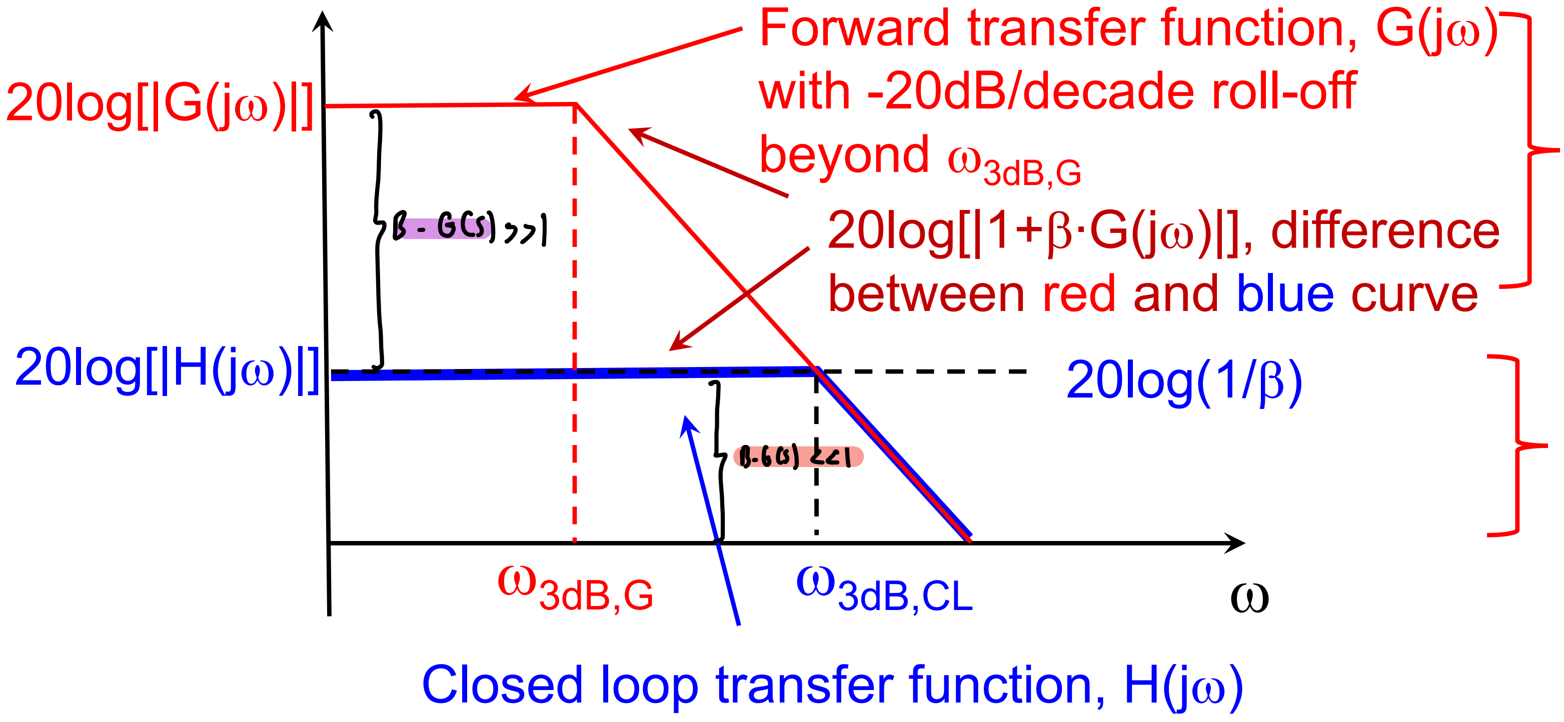
Bode Plot



Bode plot obtained by replacing s with jω

Bode Plot

- In reality, G(jω) is a function of frequency.



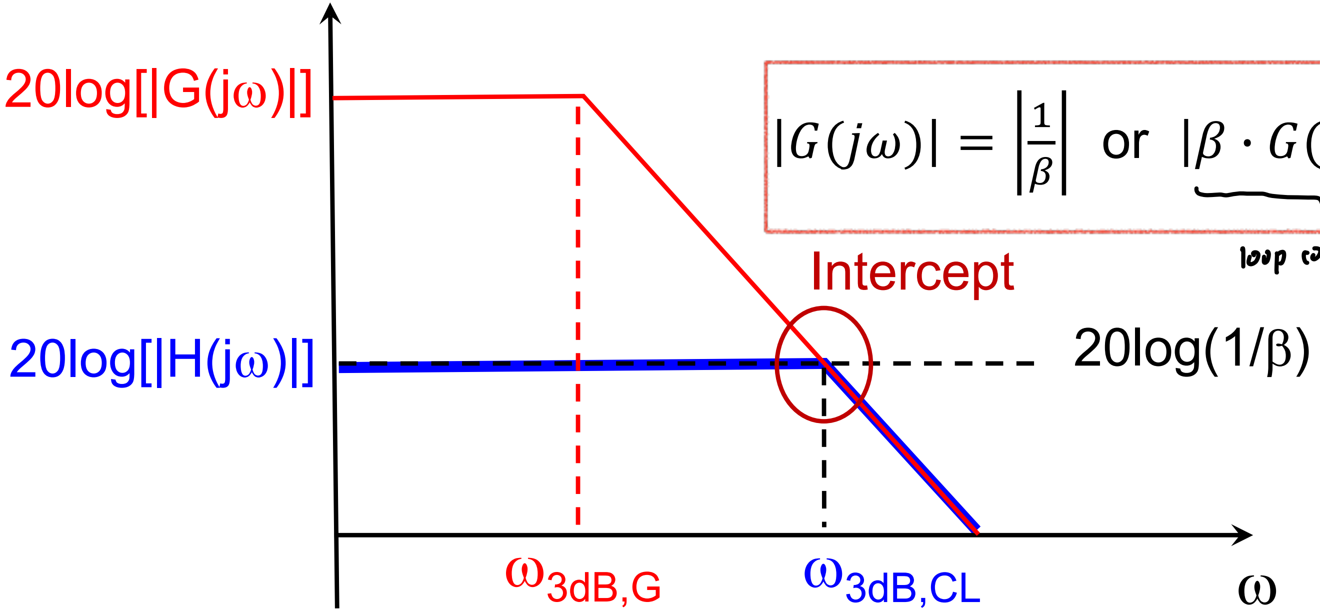
$$H(s) = \frac{G(s)}{1+\beta \cdot G(s)} \approx \frac{1}{\beta}$$

$\beta \cdot G(s) >> 1$

$$H(s) = \frac{G(s)}{1+\beta \cdot G(s)} \approx G(s)$$

$\beta \cdot G(s) << 1$

Bode Plot



$$|G(j\omega)| = \left|\frac{1}{\beta}\right| \text{ or } \underbrace{|\beta \cdot G(j\omega)|}_{\text{loop control}} = 1 \text{ at } \omega = \omega_{3dB,CL}$$

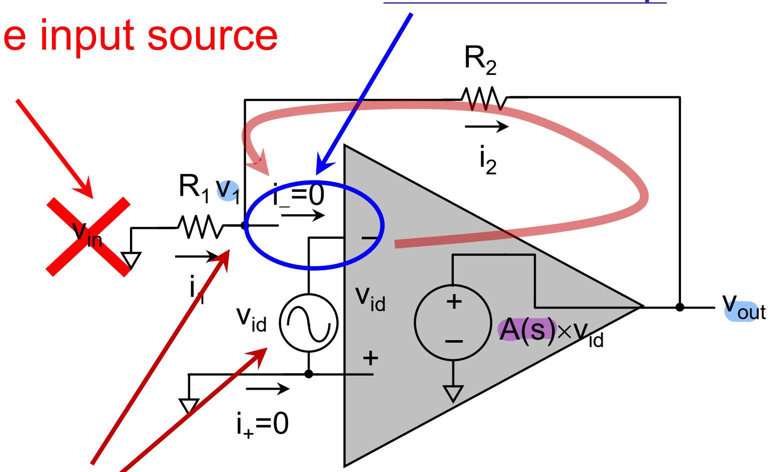
- Closed loop bandwidth ($\omega_{3dB,CL}$) can be found by finding intercept of 20log(1/β) and 20log[|G(jω)|]
- Intercept of 20log(1/β) and 20log[|G(jω)|] ⇒ |1/β| = |G(jω)| or |β·G(jω)| = 1
- |β·G(jω)| = 1 ⇒ 20log[|β·G(jω)|] = 0 dB
- In any system with negative feedback loop, once G(jω) and β are given, we can determine $\omega_{3dB,CL}$ easily.

Implication and Reality

In opamp circuit, finding β and G(s) individually is not straightforward. But finding loop gain, β·G(s), is readily easier.

1. Kill the input source

2. Break the loop



3. Apply test signal (v_{id}) at the input of the loop, and find the voltage at the output (v_1) of the loop

Obtain the loop gain by tracing the path from v_{id} to v_1:

$$\beta \cdot G(s) = \frac{v_1}{v_{id}} = \boxed{\frac{v_{out}}{v_{id}}} \times \frac{v_1}{v_{out}} = A(s) \times \frac{R_1}{R_1 + R_2}$$

As A(s) is given for certain opamp, R_1 and R_2 are known, closed loop bandwidth can be easily deduced by solving:

$$\left| A(j\omega_{3dB,CL}) \times \frac{R_1}{R_1 + R_2} \right| = 1$$

N.B. $A(j\omega)$ is the transfer function (open-loop gain) of the opamp and its expression is given in slide 5-30

Example (Main Concept)

Example:

Inverting amplifier with R_2 = 400 kΩ and R_1 = 100 kΩ. Opamp has GBW of 1MHz.

$-\left(\frac{R_2}{R_1}\right)$

$j = \sqrt{-1}$

$|j| = 1$

$$\left| A(j\omega_{3dB,CL}) \times \frac{R_1}{R_1 + R_2} \right| = 1$$

$$\Rightarrow \left| \frac{2\pi \times GBW}{j\omega_{3dB,CL}} \times \frac{R_1}{R_1 + R_2} \right| = 1$$

$$\Rightarrow \boxed{\omega_{3dB,CL} = 2\pi \times GBW \times \frac{R_1}{R_1 + R_2}} = 2\pi \times 200\text{kHz}$$

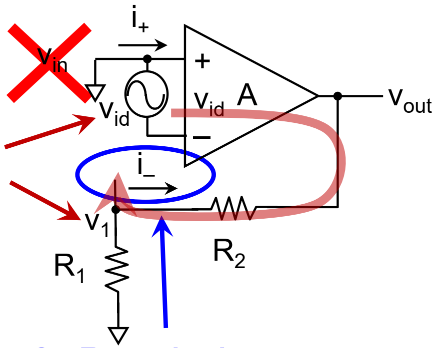
$\downarrow$

$$\boxed{f_{3dB,CL} = GBW \times \frac{R_1}{R_1 + R_2}} \rightarrow \text{in hz}$$

Another Example

1. Kill the input source

3. Apply test signal (v_{id}) at the input of the loop, and find the voltage at the output (v_1) of the loop



2. Break the loop

Obtain the loop gain by tracing the path from v_{id} to v_1:

$$\beta \cdot G(s) = \frac{v_1}{v_{id}} = \frac{v_{out}}{v_{id}} \times \frac{v_1}{v_{out}}$$
$$= A(s) \times \frac{R_1}{R_1+R_2}$$

Example: $\left(1+\frac{R_2}{R_1}\right)$

Non-inverting amplifier with R_2 = 300 kΩ and R_1 = 100 kΩ. Opamp has GBW of 1MHz.

$$\left|A(j\omega_{3dB,CL}) \times \frac{R_1}{R_1+R_2}\right| = 1$$

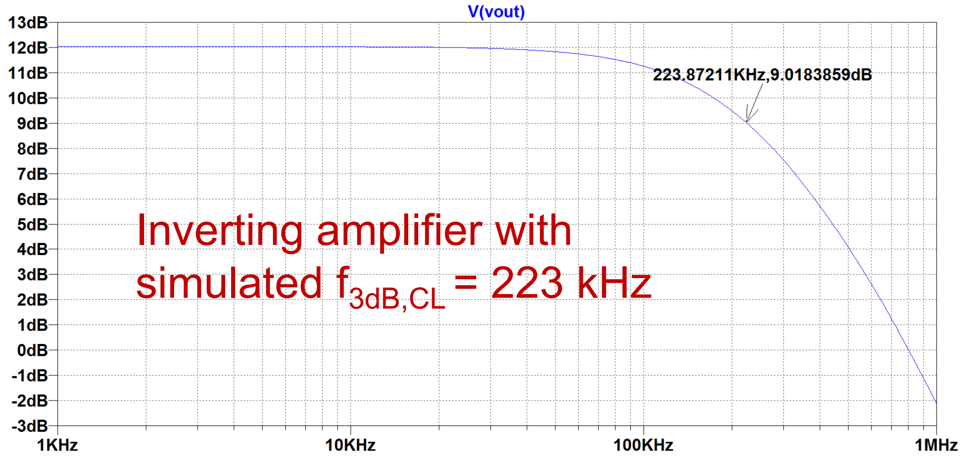
$$\Rightarrow \left|\frac{2\pi \times GBW}{j\omega_{3dB,CL}} \times \frac{R_1}{R_1+R_2}\right| = 1$$

$$\Rightarrow \omega_{3dB,CL} = 2\pi \times GBW \times \frac{R_1}{R_1+R_2} = 2\pi \times 250\text{kHz}$$

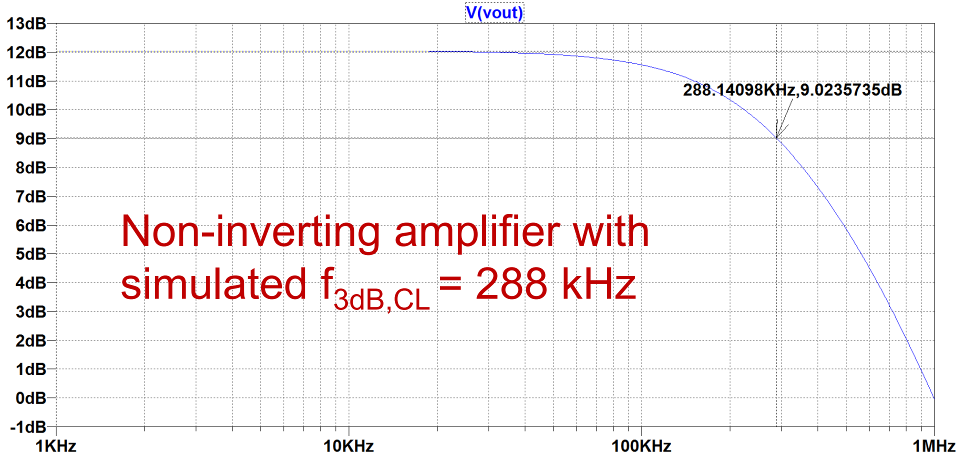
Note: Both non-inverting amplifier here and inverting amplifier in previous slide have the same absolute gain value of 4, but non-inverting amplifier has relatively better closed-loop bandwidth performance.

UA-741 Simulation

Example:
Inverting amplifier with R_2 = 400 kΩ and R_1 = 100 kΩ and non-inverting amplifier with R_2=300 kΩ and R_1=100 kΩ. UA-741 has GBW of 1MHz.



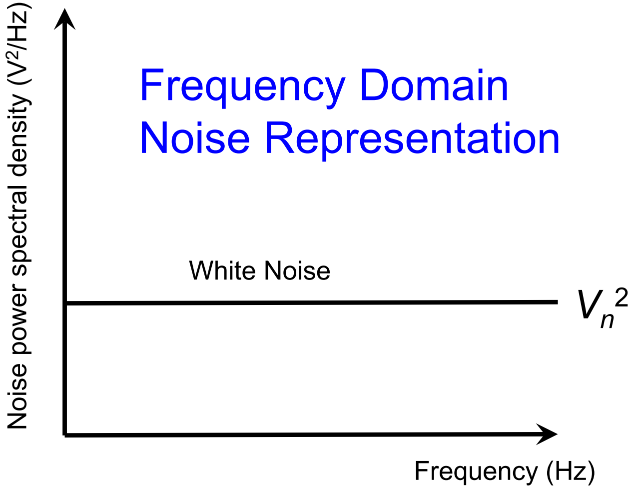
Inverting amplifier with simulated $f_{3dB,CL}$ = 223 kHz

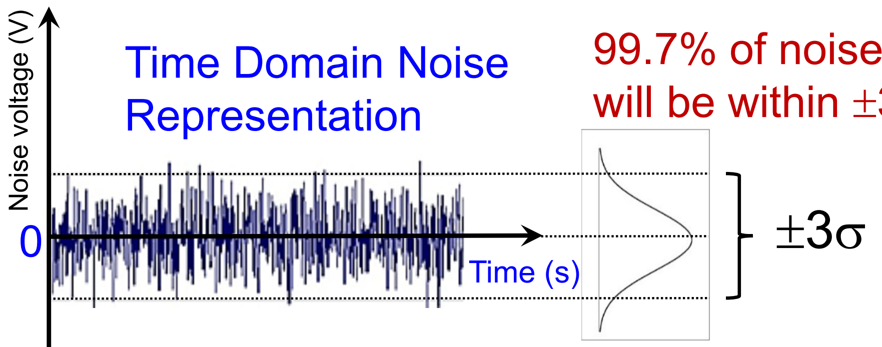


Non-inverting amplifier with simulated $f_{3dB,CL}$ = 288 kHz

Note: The opamp r_i and r_o could impact the final $\beta \cdot G(s)$, which cause the slight difference in $\omega_{3dB,CL}$.

V_n (Input Noise Voltage Spectral Density)





99.7% of noise voltage will be within $\pm 3\sigma$

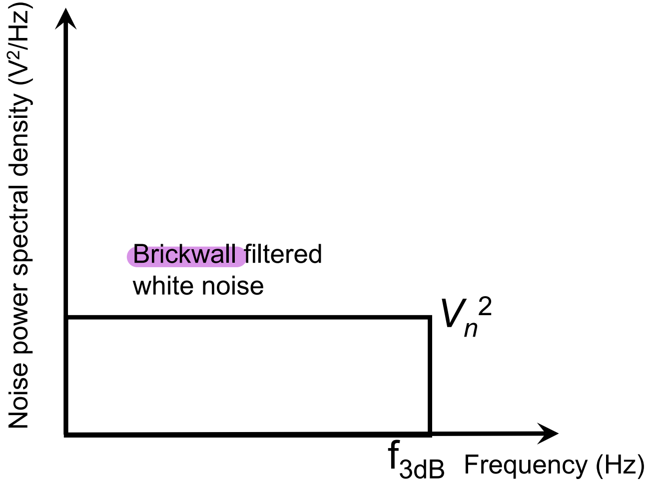
White noise can be considered as Gaussian random number with mean (μ) of zero, and standard deviation (σ) of $v_{noise,rms}$, i.e. $\sigma = v_{noise,rms}$

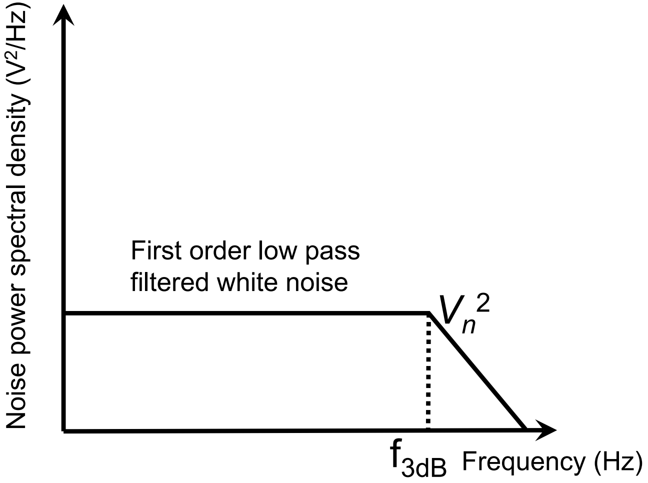
$$\sqrt{\int_0^\infty V_n^2 df} = v_{noise,rms}$$

$v_{noise,rms}$ is finite as bandwidth is limited

Relationship between frequency domain noise representation and time domain noise representation

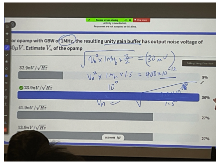
Equivalent Noise Bandwidth



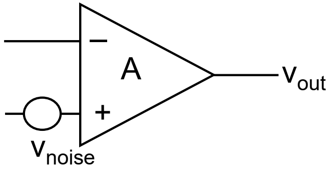


$$\sqrt{\int_0^\infty V_n^2 df} = \boxed{\sqrt{V_n^2 \times f_{3dB}} = v_{noise,rms}}$$

$$\sqrt{\int_0^\infty \left|\frac{1}{\frac{jf}{f_{3dB}}+1}\right|^2 V_n^2 df} = \boxed{\sqrt{V_n^2 \times \underbrace{f_{3dB} \times \frac{\pi}{2}}_{\text{Equivalent Noise Bandwidth}}} = v_{noise,rms}}$$



v_n and its Implication



Opamp consists of transistors and resistors, which generate noise. We model all the noise contribution with an input referred voltage noise source, v_{noise}.

Analysis (Using superposition):

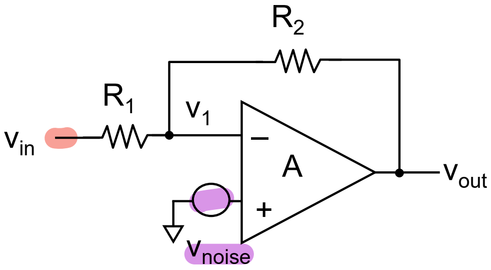
Kill V noise

$$v_{out,in} = -\frac{R_2}{R_1} \times v_{in}$$

Kill Vin

$$v_{out,n} = \overbrace{\left(1 + \frac{R_2}{R_1}\right)}^{\text{non-inverting}} \times v_{noise}$$

(look at output terminal)



Actual inverting amplifier with noise consideration

Example:

Inverting amplifier with R_2 = 400 kΩ and R_1 = 100 kΩ. Opamp has GBW of 1 MHz and V_n of 10nV/√Hz

$$\omega_{3dB,CL} = 2\pi \times GBW \times \frac{R_1}{R_1 + R_2} = 2\pi \times 200 \text{ kHz}$$

(slide 35)

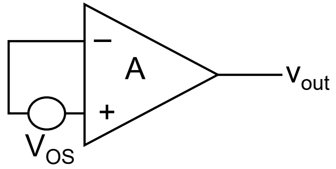
$$v_{out,n} = \left(1 + \frac{R_2}{R_1}\right) \times \sqrt{V_n^2 \times \underbrace{200kHz \times \frac{\pi}{2}}_{\text{Equivalent Noise Bandwidth}}} = 28\ \mu V \rightarrow \text{RMS}$$

Equivalent Noise Bandwidth

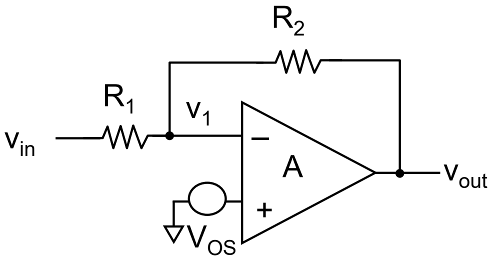
This inverting amplifier is only good for input signal, v_{in}, larger than 28μV/4 = 7 μV. → inverting configuration (look at input terminal)

Note: Noise contribution from R_1 and R_2 are ignored here. In reality, this will lead to even higher minimum input signal amplitude.

V_{OS} (Input Offset Voltage)



- Ideally, v_{out} should be zero when input terminals are shorted.
- Due to device mismatches within opamp, $-V_{OS}$ needs to be applied across the two input terminals to ensure $v_{out} = 0$.



Actual inverting amplifier with offset

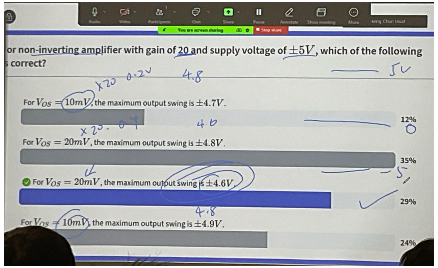
Analysis (Using superposition):

Kill Vos

$$v_{out,in} = -\frac{R_2}{R_1} \times v_{in}$$

Kill Vin (non-inverting configuration)

$$v_{out,OS} = \left(1 + \frac{R_2}{R_1}\right) \times V_{OS}$$



V_{OS} and its Implication

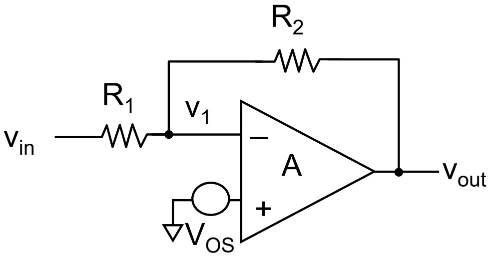
Example:

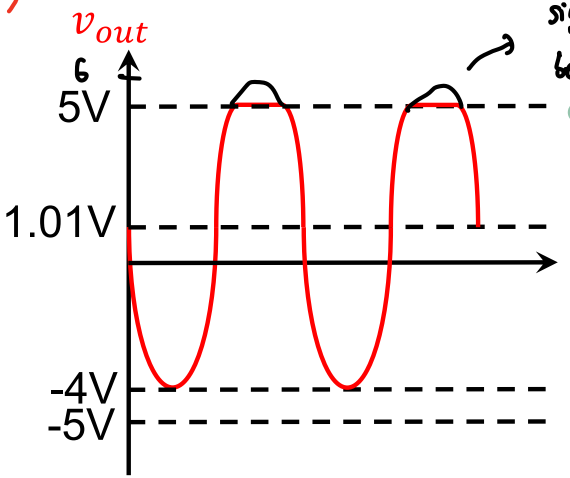
Inverting amplifier with R_2 = 1000 kΩ and R_1 = 10 kΩ. Opamp has supply voltage of ±5 V, V_{OS} of 10 mV, v_{in} is a 1 kHz sinusoidal input with 50 mV amplitude, sketch the output.

→ inverting

non-inverting (ie gain +1)

$$v_{out} = -100 \times 0.05sin(2\pi \times 1kt) + 101 \times 0.01$$
$$= -5sin(2\pi \times 1kt) + 1.01$$





signal gets clipped because cannot exceed 5V.

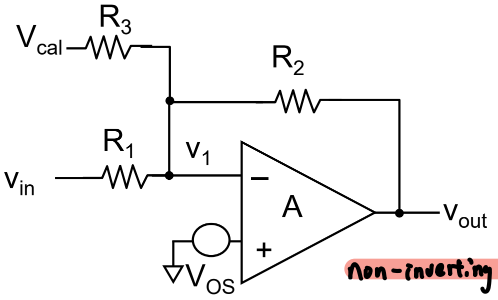
Constant DC output shift due to V_{OS} reduces the applicable input voltage amplitude. We can only apply v_{in} with an amplitude of slightly less than 40 mV to avoid output clipping.

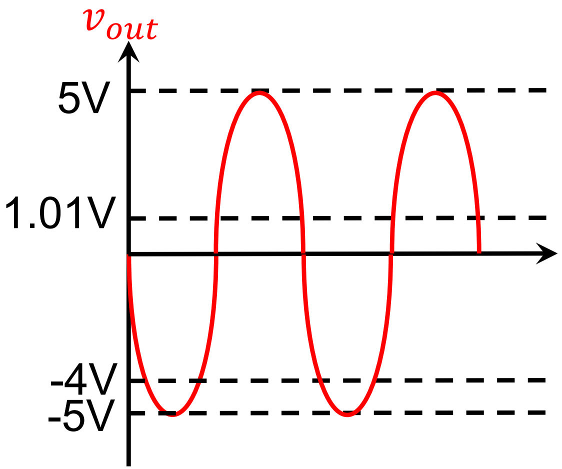
V_{OS} and its Compensation

Example:

Inverting amplifier with R_2 = 1000 kΩ and R_1 = 10 kΩ. Opamp has supply voltage of ±5 V, V_{OS} of 10 mV, v_{in} is a 1 kHz sinusoidal input with 50 mV amplitude, sketch the output.

To compensate the effect of V_{OS}, summing circuit can be applied (see figure below, where R_3 branch has been added).





$$v_{out} = \underbrace{-5sin(2\pi \times 1kt)}_{\text{inverting config.}} + \overbrace{1.01}^{\text{non-inverting config}} - \underbrace{\frac{R_2}{R_3} \times V_{cal}}_{\text{remove offset?}}$$

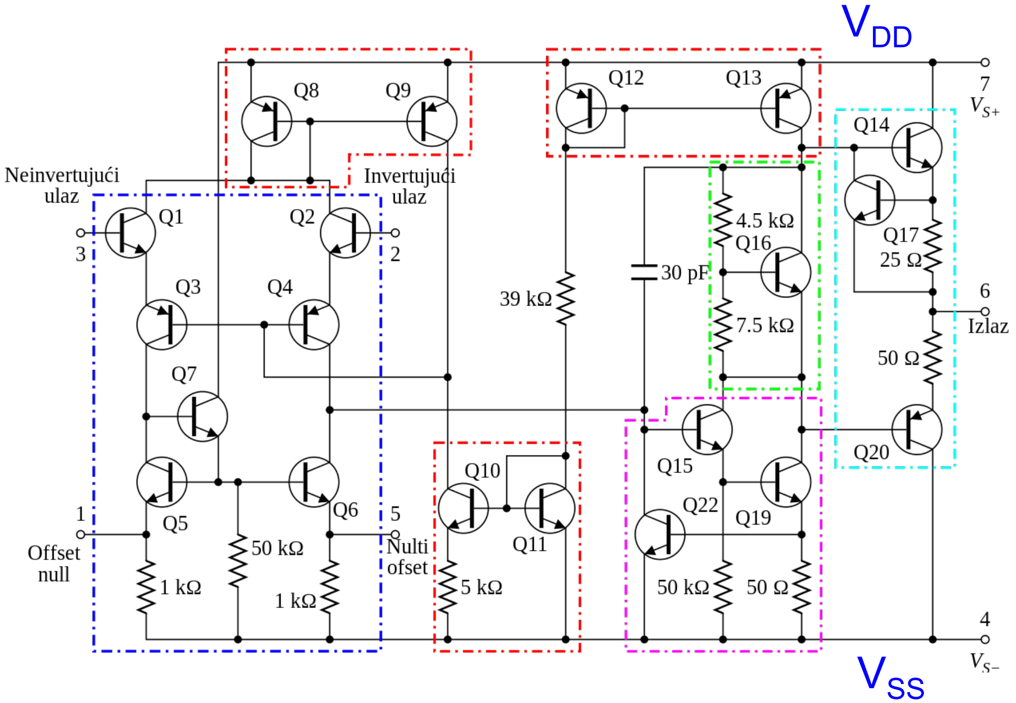
R_3 and V_{cal} are chosen to cancel DC output shift at the output. (How to choose R_2/R_3?)

↳ $R_2 < R_3$ (easier to adjust voltage)

With constant DC output shift due to V_{OS} compensated, there is no output clipping

V_I and $V_{O(PP)}$

For Q1 and Q2 to operate properly (forward active mode), $V_{B,Q1}$ and $V_{B,Q2}$ need to be within $V_I \in (V_{in,min}, V_{in,max}) < V_{DD} - V_{SS}$



For Q14 and Q20 to operate properly (forward active mode), $V_{E,Q14}$ and $V_{E,Q20}$ need to be within $V_{O(PP)} \in (V_{out,min}, V_{out,max}) < V_{DD} - V_{SS}$

If the opamp is built using MOSFETs, the consideration is for the input and output transistors to operate at saturation region.

Φ_m (Feedback Theory Revisit)

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{G(s)}{1 + \beta \cdot G(s)}$$

If $\beta \cdot G(j\omega) = -1 \Rightarrow H(j\omega) \to \infty$

infinite gain

Feedback circuit is unstable and oscillates

$a + jb$ $\quad \sqrt{a^2+b^2} \quad \tan^{-1}\frac{b}{a}$

$$\beta \cdot G(j\omega) = |M(\omega)| \cdot e^{j\theta(\omega)}$$

Complex number can be represented as magnitude and phase

$e^{j180} = \cos 180° + j \underbrace{\sin 180}_{0}$

$= -1$

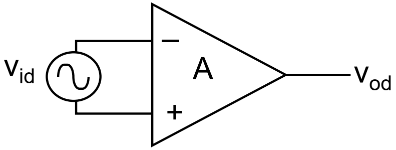
$\beta \cdot G(j\omega) =$ -1 occurs when |M(ω)| = 1 and θ(ω) = 180°

$$\Phi_m = 180° - \theta(\omega)$$

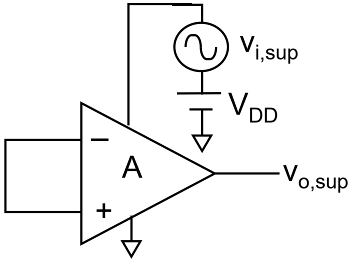
at ω when $|M(\omega)| = 1$

Φ_m is thus a measure of feedback circuit stability performance, $\Phi_m > 60°$ is desirable for good stability performance for feedback circuit.

PSRR (Power Supply Rejection Ratio)



A_{OL}: $A_{OL} = \dfrac{v_{od}}{v_{id}}$



A_{SUP}: $A_{SUP} = \dfrac{v_{o,sup}}{v_{i,sup}}$

PSRR: $PSRR = 20log\left(\dfrac{A_{OL}}{A_{SUP}}\right)$

ideal: big

ideal : small

- PSRR is a measure of how good the opamp rejects the supply line ripples or interference ($v_{i,sup}$).
- For ideal opamp, A_{SUP} should be zero, which leads to infinite PSRR.

PSRR Example

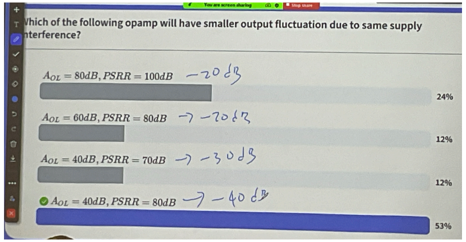
Example:

An AC-to-DC supply is used to power up the opamp circuit. The supply has an output ripple voltage of 10 mV. Assume the opamp has A_{OL} of 60 dB and PSRR of 70 dB, estimate the output voltage amplitude due to the supply ripple.

$$20log(A_{SUP}) = 20log(A_{OL}) - PSRR$$
$$= -10dB$$
$$A_{SUP} = 0.32$$

The output voltage amplitude due to the supply ripple would be 3.2 mV.

Besides ripple, depending on how complex the system is, the supply voltage could also be corrupted by the switching of digital circuit or other noise source.

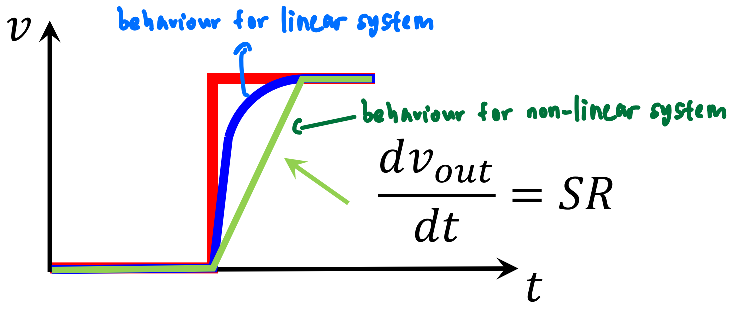


SR (Slew Rate)

$$\boxed{\frac{dv_{out}}{dt} < SR}$$

SR is a measure of how fast the rate of change of the output (v_{out}) can be. This is often limited by the current driving capability within the opamp.

For step response, SR limits the rise time of the output.



— Input

— Non-slew rate limited output

— Slew rate limited output

For sinusoidal output [$v_{out} = v_{out,pk}cos(2\pi f t)$], SR limits the maximum signal frequency and $v_{out,pk}$ that the circuit can handle without distortion

must be less than if not op-amp cannot handle

$$\frac{dv_{out}}{dt} = \boxed{2\pi f \times v_{out,pk} sin(2\pi f t) < SR}$$

(for all time, t)

Maximum slope of v_{out} occurs when $sin(2\pi ft) = 1$

SR Example

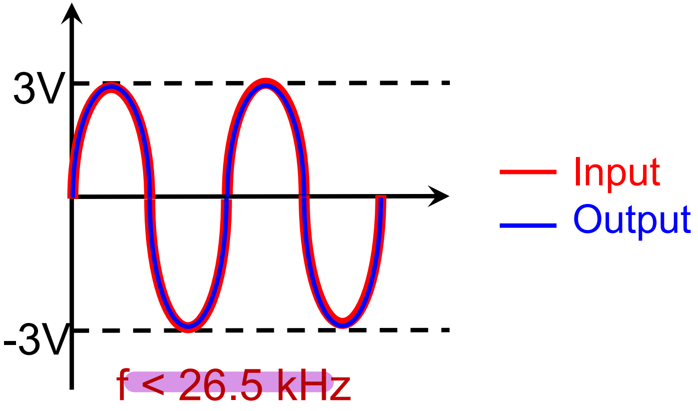
Example:

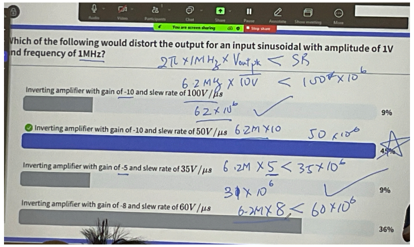
An unity gain buffer is used to drive the output. The sinusoidal input has amplitude of 3 V. Assuming the opamp has SR of 0.5 V/μs, estimate the maximum sinusoidal input frequency that the unity gain buffer can handle without distortion. → follow the "weakest link", look at either GBW and slew rate. Lower value is the "weakest link" and use that as max input that op-amp can handle.

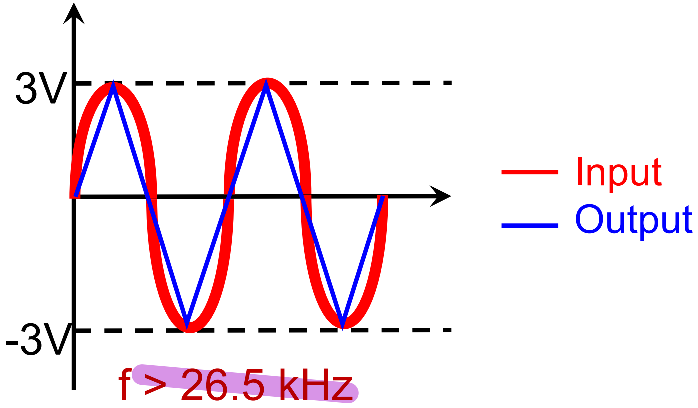
$$\frac{dv_{out}}{dt} = 2\pi f \times v_{out,pk} sin(2\pi f t) < SR$$

$$2\pi f \times 3 < 0.5 \text{ V/μs}$$

$$\Rightarrow f < 26.5 \text{ kHz}$$

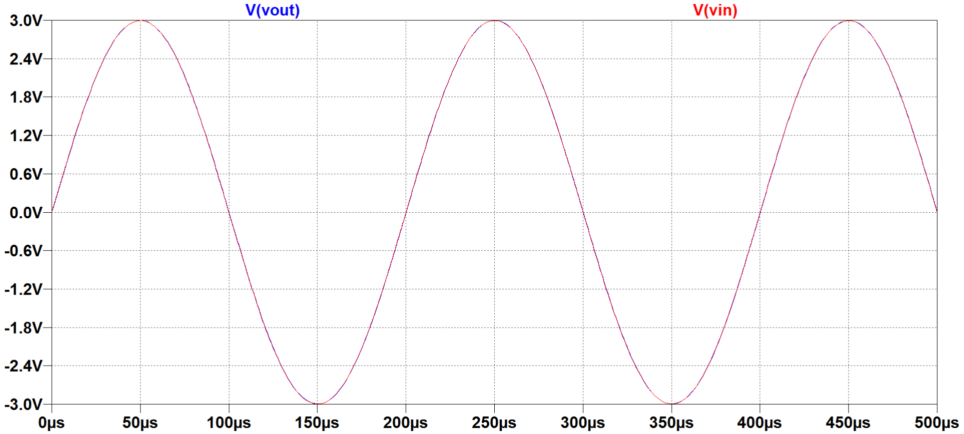




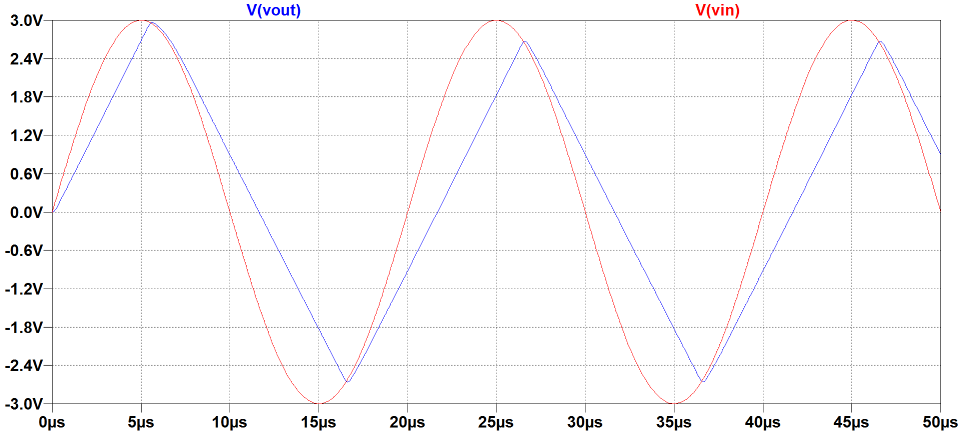


SR Simulation

Example:
An unity gain buffer is used to drive the output. The sinusoidal input has amplitude of 3 V. Assuming the opamp has SR of 0.5 V/μs and GBW of 1 MHz. Simulate signal with input frequency of 5 kHz and 50 kHz.



Buffer with input frequency of 5 kHz



Buffer with input frequency of 50 kHz

Note: If we analyze based on β·G(s) and GBW, unity gain buffer should be able to handle signal up to 1 MHz ($f_{3dB,CL}$ = 1 MHz). Here, SR becomes the limiting factor on signal frequency.

Typical UA741 Parameter Value

Parameter	ABBV	Units	Value
Common-mode rejection ratio	CMRR	dB	90
Gain-bandwidth product	GBW	MHz	1
Input noise voltage spectral density	V_n	$nV/\sqrt{Hz}$	23
Input offset voltage	V_{OS}	mV	1
Input resistance	r_i	MΩ	2
Input voltage range	V_I	V	±13
Maximum peak-to-peak ouput voltage swing	$V_{O(PP)}$	V	±13

Typical UA741 Parameter Value

Parameter	ABBV	Units	Value
Open-loop gain	A_{OL}	dB	106
Output resistance	r_o	Ω	75
Phase margin	Φ_m	$\circ$	50
Power supply rejection ratio	PSRR	dB	90
Slew rate	SR	V/μs	0.5

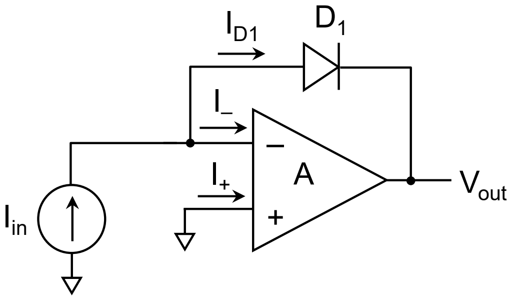
Lecture Summary

- Overview of key opamp parameters
- Examples on how such parameters impact the design

Lecture Outline

- Overview of opamp and its analysis
- Opamp parameters
- **Different types of opamp based amplifier**
  - **log amplifier, exponential amplifier, instrumentation amplifier**
- Filter
  - Integrator, differentiator, 1st order filter
  - 2nd order SK filter and higher order filter
- Super diode and comparator
- Applications built using opamp
  - Triangular wave, multiplier, bandstop filter, full wave rectifier

Logarithmic Amplifier



Because the signals we are dealing with consist of DC and AC components, we use upper case (V_{out}, I_{in}) rather than lower case (v_{out}, i_{in})

1	$\log_{10}(1) = 0$
10	$\log_{10}(10) = 1$
100	$\log_{10}(100) = 2$
1000	$\log_{10}(1000) = 3$
Wide range	⇨ Narrow range

Transfer Function for Logarithmic Amplifier

$$V_- \approx V_+ = 0 \quad [\because Virtual\ \ Short]$$

$$V_{D1} = V_- - V_{out} \approx -V_{out}$$

$$\boxed{I_{D1} = I_S\left(e^{V_{D1}/V_T} - 1\right) \approx I_S e^{-V_{out}/V_T}}$$

Diode characteristic equation

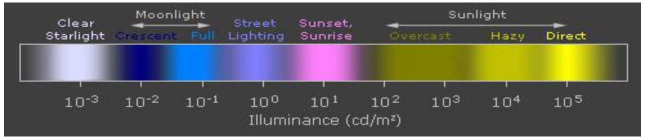
$$I_{D1} = I_{in} \approx I_S e^{-V_{out}/V_T} \quad [\because i_+ = i_- = 0]$$

$$\boxed{\Rightarrow V_{out} = -V_T \ln\left(\frac{I_{in}}{I_S}\right)}$$

Output is natural log of input

- Suitable for applications where the input has very wide dynamic range

Logarithmic Amplifier (Camera)

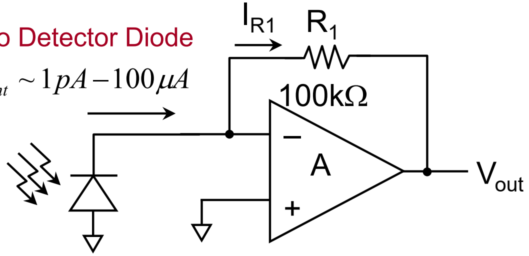


I_{light} 1pA 10pA 100pA 1nA 10nA 100n 1µA 10µA 100µA

I_{light} changes with light intensity
Light intensity changes by many orders of magnitude $\Rightarrow$ I_{light} also changes by many orders of magnitude

Photo Detector Diode

$I_{light} \sim 1pA - 100\mu A$



$$V_{out} = -I_{light} \times R_1$$
$$\sim \underbrace{-0.1\mu V}_{\text{Clear Starlight}} \ \ to \ \ \underbrace{-10V}_{\text{Direct Sunlight}}$$

Output changes linearly with input $\Rightarrow$ Difficult to detect and differentiate the input signal

V_{out} < 1 mV cannot be easily measured
$\Rightarrow$ Can only detect sunset/sunrise and above

$I_S = 10^{-15} A, \ \ V_T = 25mV$

$I_{light} \sim 1pA - 100\mu A$

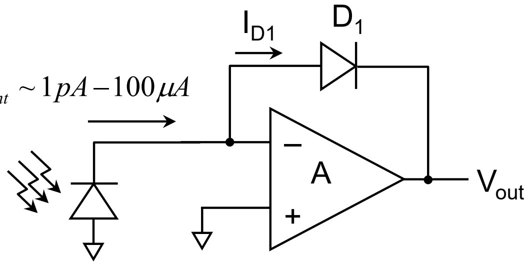


Photo Detector Diode

$$\boxed{I_{light} \sim 1pA - 100\mu A}$$

Input changes by eight orders of magnitude ($10^{-12} – 10^{-4}$)

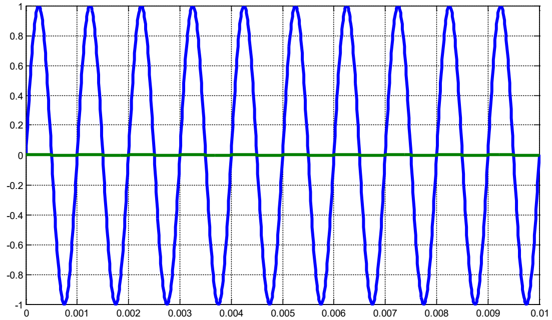
$$V_{out} = -V_T \ln\left(\frac{I_{light}}{I_S}\right) \sim \underbrace{-0.17V}_{\text{Clear Starlight}} \ \ to \ \ \underbrace{-6.3V}_{\text{Direct Sunlight}}$$

Output changes in log scale with input $\Rightarrow$ Easy to detect and differentiate the input signal

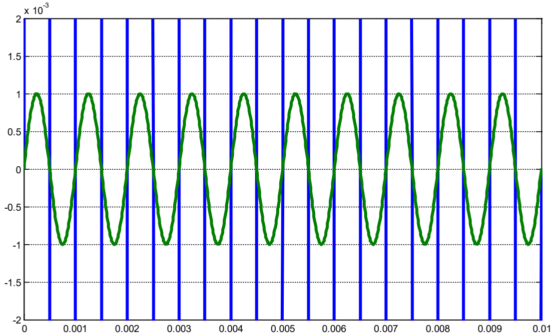
-0.17 V and -0.63 V can be easily measured
$\Rightarrow$ All light conditions can be detected

Issue with signals of wide dynamic range:

sin(2π1000t)
0.001×sin(2π1000t)

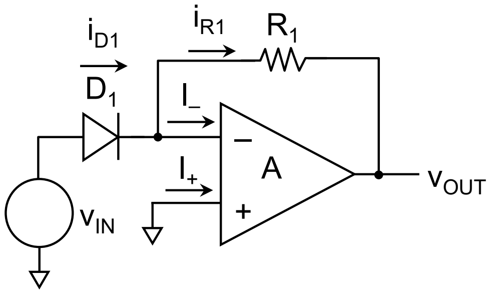


Zoom out for large signal and lose the details on small signal



Zoom in for small signal and lose the overview on large signal

Exponential Amplifier



Because the signals we are dealing with consist of DC and AC components, we use lower case variable with upper case subscript (v_{OUT}, v_{IN})

Transfer Function for Exponential Amplifier

$$V_- \approx V_+ = 0 \quad [\because \textit{Virtual Short}]$$

$$v_{D1} = v_{IN} - V_- \approx v_{IN}$$

$$\boxed{i_{D1} = I_S\left(e^{v_{IN}/V_T} - 1\right) \approx I_S e^{v_{IN}/V_T}}$$

Diode characteristic equation

$$i_{D1} = i_{R1} = \frac{V_- - v_{OUT}}{R_1} \approx \boxed{\frac{-v_{OUT}}{R_1}} \quad [\because i_+ = i_- = 0]$$

$$\boxed{\Rightarrow v_{OUT} = -R_1 I_S e^{v_{IN}/V_T}}$$

Output is exponential of input

- Suitable for applications where the input has very narrow dynamic range and you want to expand it for easier differentiation

Lecture Summary

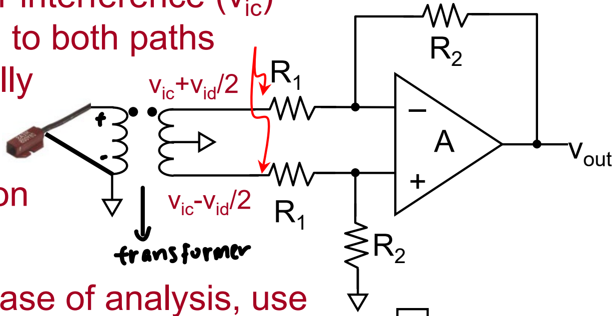
- **Exponential and log amplifiers**

Lecture Outline

- Overview of opamp and its analysis
- Opamp parameters
- **Different types of opamp based amplifier**
  - **log amplifier, exponential amplifier, instrumentation amplifier**
- Filter
  - Integrator, differentiator, 1^{st} order filter
  - 2^{nd} order SK filter and higher order filter
- Super diode and comparator
- Applications built using opamp
  - Triangular wave, multiplier, bandstop filter, full wave rectifier

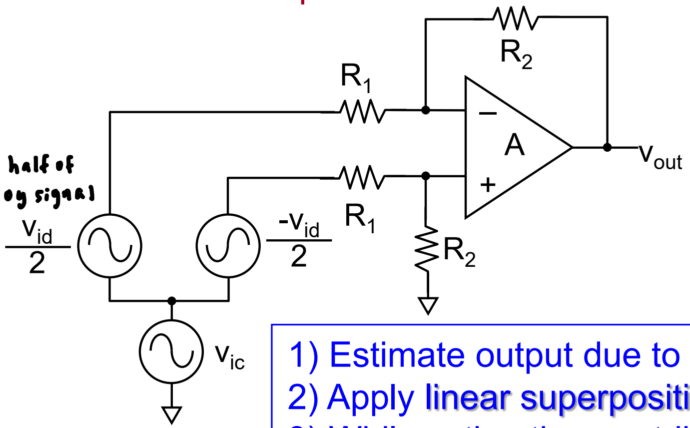
Instrumentation Amplifier

Noise or interference (v_{ic}) coupled to both paths identically



Vibration sensor

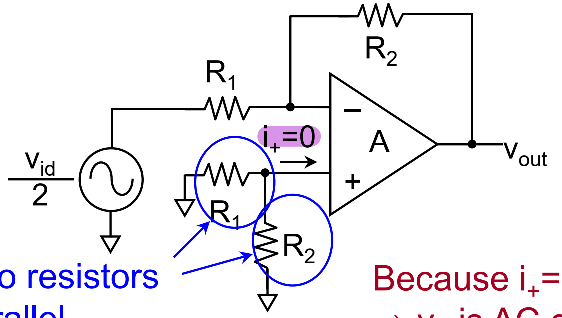
For ease of analysis, use three separate sources to model the input



1) Estimate output due to individual voltage sources ($v_{id}/2$, $-v_{id}/2$, v_{ic})
2) Apply linear superposition to find the total contribution
3) While estimating contribution from one voltage source, kill all the other voltage sources

Keep $v_{id}/2$

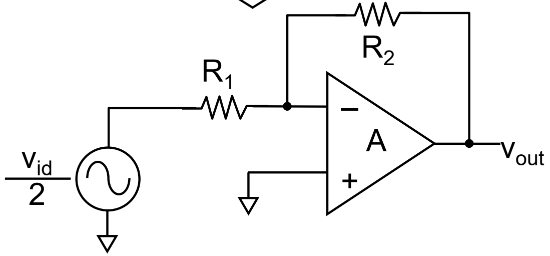
Kill $-v_{id}/2$ and v_{ic} by short circuit them



These two resistors are in parallel

(voltage =0)

Because $i_+=0$
$\Rightarrow$ v_+ is AC ground

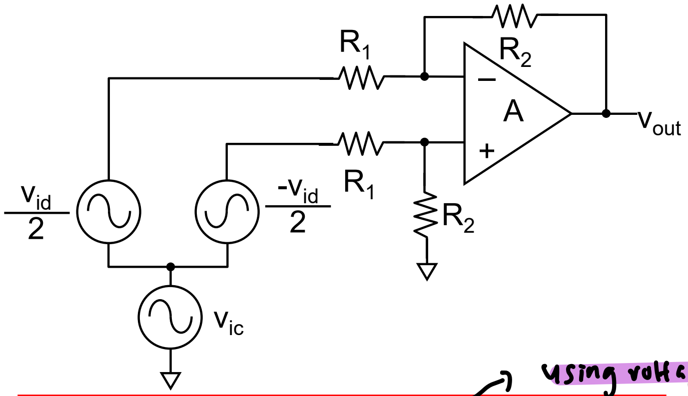


$$\underbrace{v_{out,\frac{v_{id}}{2}}}_{} = -\frac{R_2}{R_1}\left(\frac{v_{id}}{2}\right) \qquad \cdots(1)$$

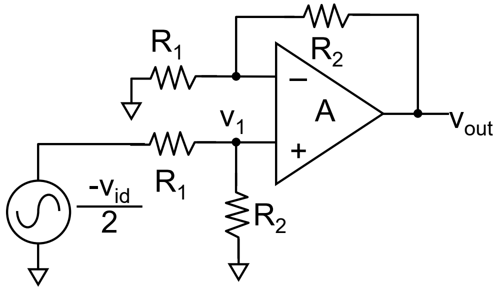
v_{out} due to $v_{id}/2$

Equation (1) from slide 5-9

Instrumentation Amplifier



Keep $-v_{id}/2$
Kill $v_{id}/2$ and v_{ic} by short circuit them



using voltage divider

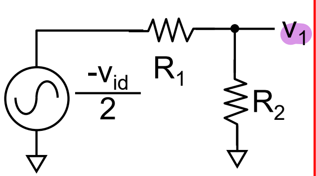
$$v_{1,-\frac{v_{id}}{2}} = \frac{R_2}{R_1+R_2}\left(\frac{-v_{id}}{2}\right)$$

$$\boxed{v_{out,-\frac{v_{id}}{2}} = \left(1+\frac{R_2}{R_1}\right)v_{1,-\frac{v_{id}}{2}}}$$

Equation (2) from slide 8-22

$$= \left(1+\frac{R_2}{R_1}\right)\frac{R_2}{R_1+R_2}\left(\frac{-v_{id}}{2}\right)$$

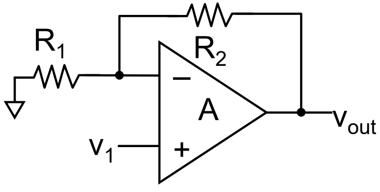
$$\Rightarrow \underbrace{v_{out,-\frac{v_{id}}{2}}}_{v_{out}\text{ due to }-v_{id}/2} = \frac{R_2}{R_1}\left(\frac{-v_{id}}{2}\right) \qquad (2)$$



Configuration A

Simple resistor divider

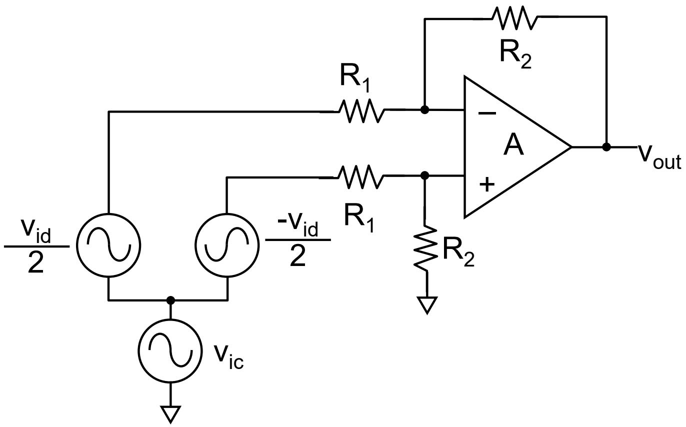
$\times$



Configuration B

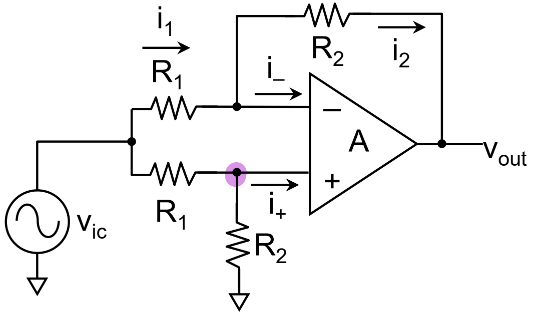
Non-inverting Amplifier
Equation (2) from slide 5-22

Instrumentation Amplifier



Keep v_{ic}

Kill $v_{id}/2$ and $-v_{id}/2$ by short circuit them



$$v_+ = v_{ic} \times \frac{R_2}{R_1 + R_2} \approx v_- \left[\because Virtual \quad short \right]$$

$$i_1 = i_2 \left[\because i_+ = i_- = 0 \right]$$

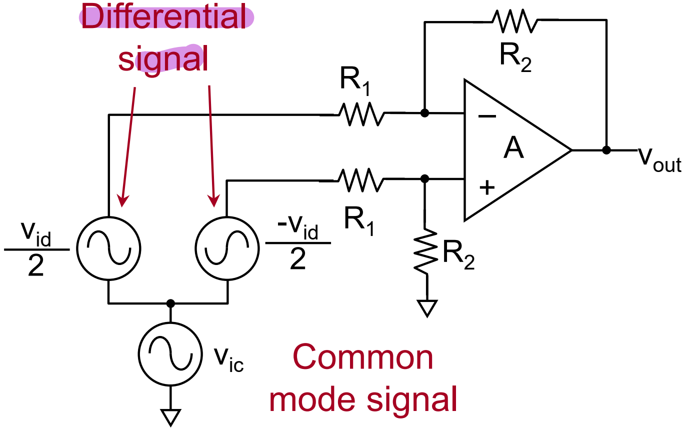
$$\Rightarrow \frac{v_{ic} - v_-}{R_1} = \frac{v_- - v_{out,v_{ic}}}{R_2}$$

$$R_2 v_{ic} - v_{ic} \times \frac{R_2^2}{R_1 + R_2} = v_{ic} \times \frac{R_1 R_2}{R_1 + R_2} - R_1 v_{out,v_{ic}}$$

$$\Rightarrow \underbrace{v_{out,v_{ic}}}_{v_{out} \text{ due to } v_{ic}} = 0 \qquad \cdot (3)$$

Instrumentation Amplifier

Differential signal



Common mode signal

Transfer Function for Instrumentation Amplifier

$$\boxed{v_{out} = v_{out,\frac{v_{id}}{2}} + v_{out,-\frac{v_{id}}{2}} + v_{out,v_{ic}}}$$

Superposition : Combine eqn (1), (2) and (3) from previous slides

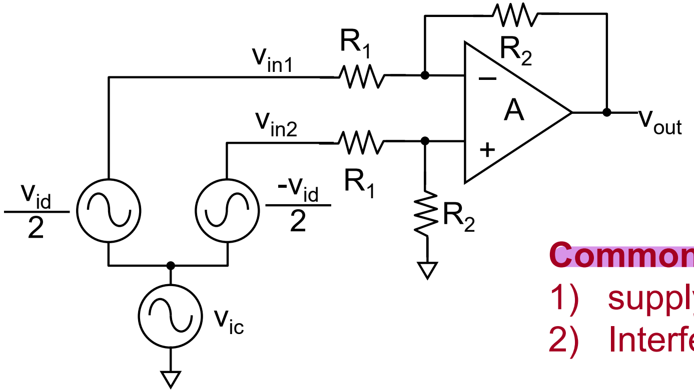
$$= -\frac{R_2}{R_1}\left(\frac{v_{id}}{2}\right) + \frac{R_2}{R_1}\left(-\frac{v_{id}}{2}\right) + 0$$

$$= -\frac{R_2}{R_1}(v_{id})$$

→ only depend on differential signal

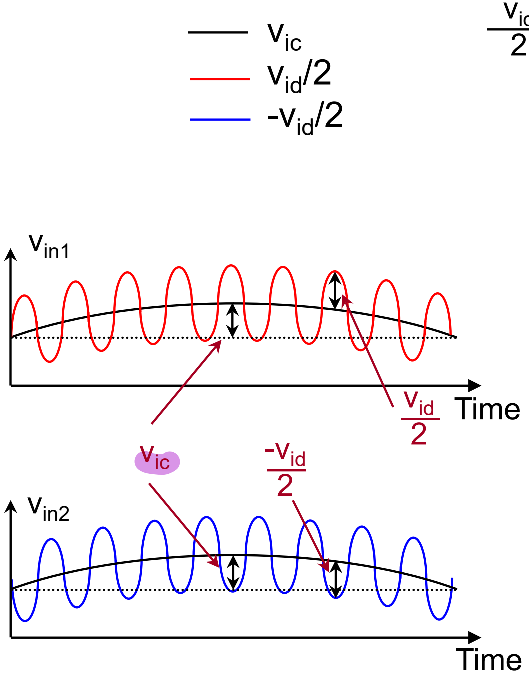
- It rejects common mode signal and only amplifies differential signal
- Good rejection for common mode interference and noise
- Required for measurement instrument
- Limited by the matching properties of the resistors

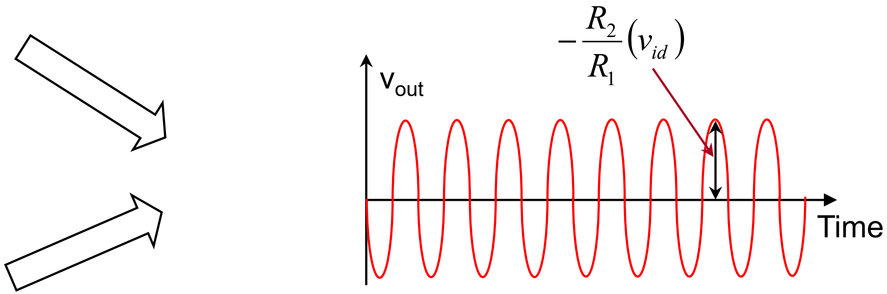
Common Mode Interference Rejection



Common mode interference (v_{ic}) :

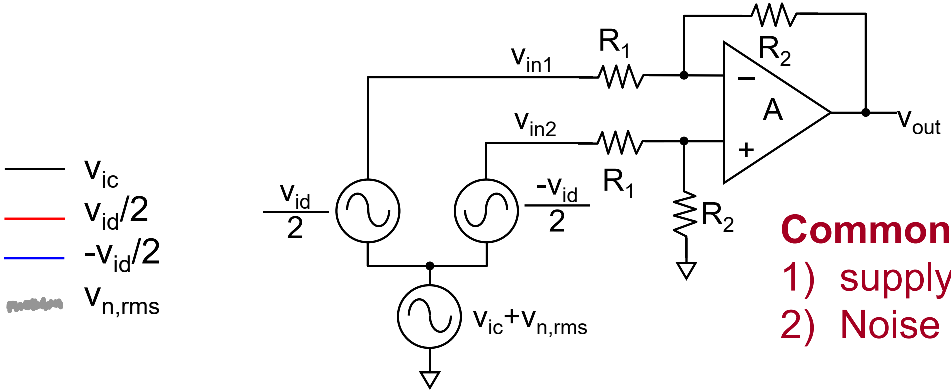
1) supply fluctuation (50~60 Hz from power line)
2) Interference from other signal source





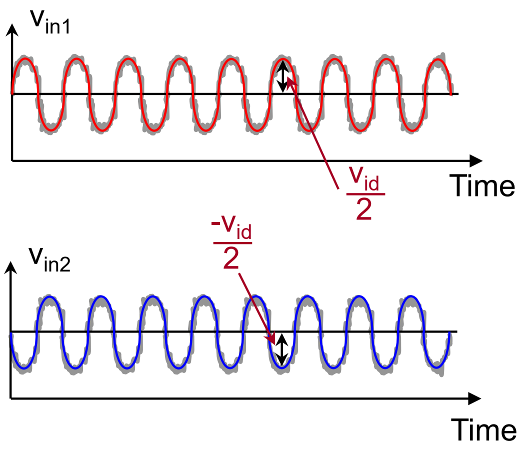
Free of common mode interference

Common Mode Noise Rejection

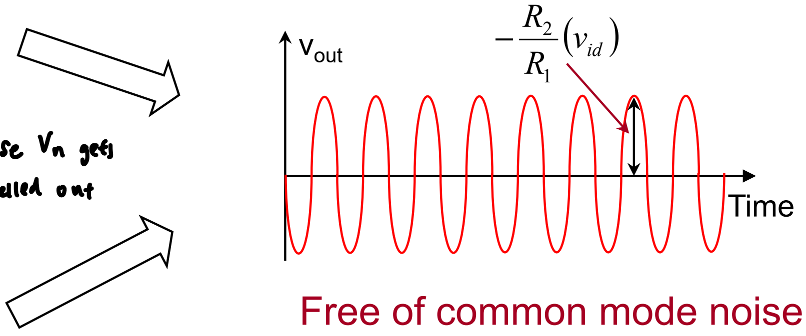


Common mode noise ($v_{n,rms}$) :

1) supply noise
2) Noise coupling from other noise source



noise V_n gets cancelled out



Free of common mode noise

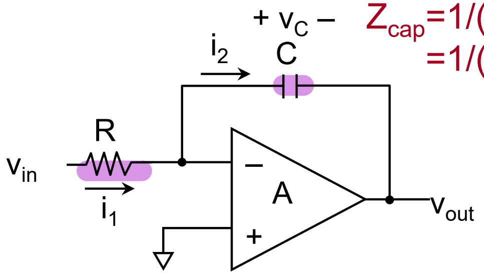
Lecture Summary

- **Instrumentation amplifier and its advantages**

Lecture Outline

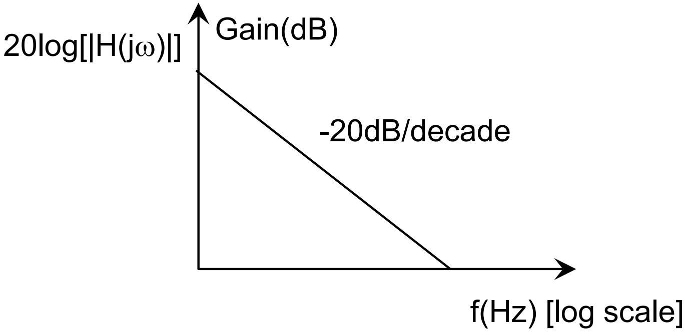
- Overview of opamp and its analysis
- Opamp parameters
- Different types of opamp based amplifier
  - log amplifier, exponential amplifier, instrumentation amplifier
- **Filter**
  - **Integrator, differentiator, 1st order filter**
  - 2nd order SK filter and higher order filter
- Super diode and comparator
- Applications built using opamp
  - Triangular wave, multiplier, bandstop filter, full wave rectifier

Integrator



$Z_{cap}=1/(j\omega C)$
$=1/(sC)$

The circuit performs analog integration



- $H(j\omega)$ reduces with increasing ω ($\omega = 2\pi f$)
- $dB(H(j\omega)) = 20 \times \log_{10}(|H(j\omega)|)$
- -20dB/decade means $H(j\omega)$ decreases by 20 dB when the frequency (f or ω) increases by 10 times.

Transfer Function for Integrator

$$v_- \approx v_+ = 0 \qquad [\because Virtual\ \ Short]$$

$$v_C(t) = v_- - v_{out}(t) \approx -v_{out}(t)$$

$$i_1(t) = \frac{v_{in}(t)}{R} \qquad i_2(t) = C\frac{dv_C(t)}{dt} \approx -C\frac{dv_{out}(t)}{dt}$$

$$i_1(t) = i_2(t) \qquad [\because i_+ = i_- = 0]$$

$$\Rightarrow -C\frac{dv_{out}(t)}{dt} = \frac{v_{in}(t)}{R}$$

$$\Rightarrow v_{out}(t) = -\frac{1}{RC}\int_0^t v_{in}(t)dt$$ (Output is integration of input)

Alternatively based on inverting amplifier (for ac signal)

$$\boxed{H(j\omega) = \frac{v_{out}}{v_{in}} = -\frac{Z_{cap}}{R} = -\frac{1}{j\omega CR} = -\frac{1}{sCR} = H(s)}$$

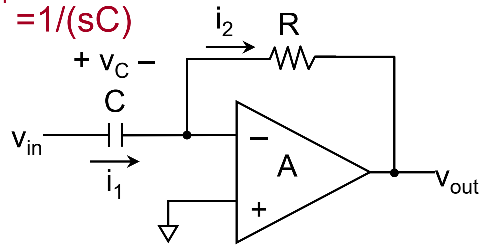
Slide 5-9 equation 1

$$\boxed{\frac{1}{j\omega} = \frac{1}{s} \rightarrow \int(\cdot)dt}$$

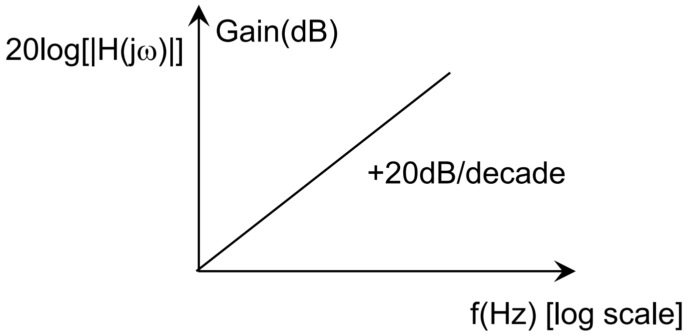
S-Transform (Learnt in EE2023, Signals and Systems)

Differentiator

$Z_{cap} = 1/(j\omega C)$
$= 1/(sC)$

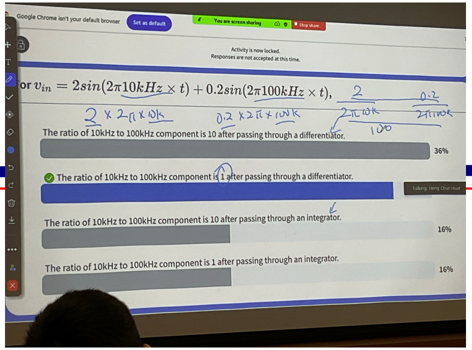


The circuit performs analog differentiation



- $H(j\omega)$ increases with increasing ω ($\omega = 2\pi f$)
- $dB(H(j\omega)) = 20 \times \log_{10}(|H(j\omega)|)$
- +20dB/decade means $H(j\omega)$ increases by 20 dB when the frequency (f or ω) increases by 10 times.

Transfer Function for Differentiator



$$v_- \approx v_+ = 0 \qquad [\because Virtual\ \ Short]$$

$$v_C(t) = v_{in}(t) - v_- \approx v_{in}(t)$$

$$i_1(t) = C\frac{dv_C(t)}{dt} \approx C\frac{dv_{in}(t)}{dt} \qquad i_2(t) = \frac{v_- - v_{out}(t)}{R} \approx -\frac{v_{out}(t)}{R}$$

$$i_1(t) = i_2(t) \qquad [\because i_+ = i_- = 0]$$

$$\Rightarrow -\frac{v_{out}(t)}{R} = C\frac{dv_{in}(t)}{dt}$$

$$\Rightarrow v_{out}(t) = -RC\frac{dv_{in}(t)}{dt}$$

(Output is differentiation of input)

Alternatively based on inverting amplifier (for ac signal)

$$\boxed{H(j\omega) = \frac{v_{out}}{v_{in}} = -\frac{R}{Z_{cap}} = -j\omega CR = -sCR = H(s)}$$

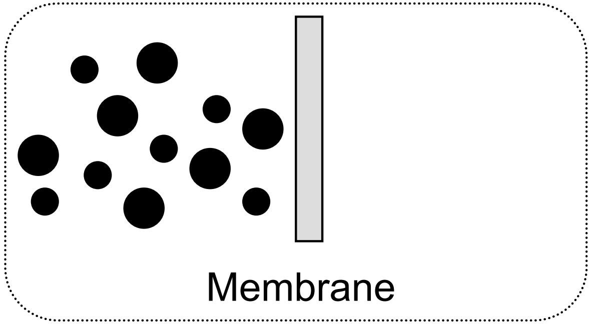
Slide 5-9 equation 1

$$\boxed{\Rightarrow \quad j\omega = s \rightarrow \frac{d(\cdot)}{dt}}$$

S-Transform (Learnt in EE2023, Signals and Systems)

What is Filter?

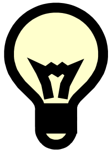
Filtering consists of the following factors:
1) Filtering criteria
2) Filtering action



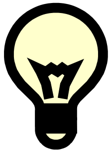
The membrane behaves like a filter which allows small particles to get through and block big particles

Selection Criteria

Action

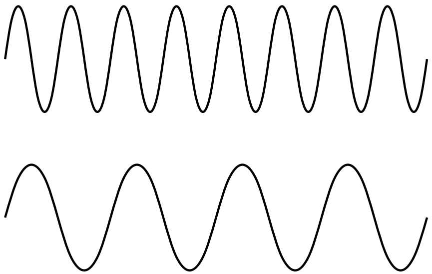


Size

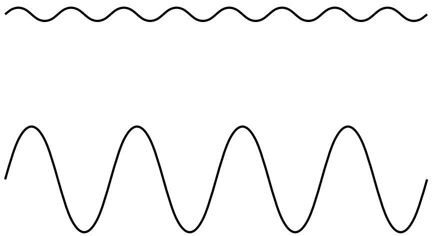


Small size goes through

What is Electronic Filter?



Electronic Lowpass Filter



The electronic lowpass filter only allows low frequency signal to go through and suppresses the high frequency signal

Selection Criteria

Action

Frequency

Pass low frequency
Reject high frequency

Lecture Summary

- Integrator and differentiator as a special form of filter
- What is filter

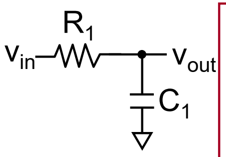
Lecture Outline

- Overview of opamp and its analysis
- Opamp parameters
- Different types of opamp based amplifier
  - log amplifier, exponential amplifier, instrumentation amplifier
- **Filter**
  - **Integrator, differentiator, 1st order filter**
  - 2nd order SK filter and higher order filter
- Super diode and comparator
- Applications built using opamp
  - Triangular wave, multiplier, bandstop filter, full wave rectifier

Simple Lowpass Filter

Passive 1st Order Lowpass Filter

Simple resistor divider



$$H(j\omega)=\frac{v_{out}}{v_{in}}=\frac{\frac{1}{j\omega C_1}}{\frac{1}{j\omega C_1}+R_1}=\boxed{\frac{1}{j\omega C_1 R_1+1}}$$

- At low frequency ($\omega \to 0$), ignore 1st term of denominator ($j\omega C_1R_1$) $\Rightarrow$ $H(j\omega) \approx 1$ $\Rightarrow$ Unity gain
- At high frequency ($\omega \to \infty$), ignore 2nd term of denominator (1) $\Rightarrow$ $H(j\omega) \approx 1/(j\omega C_1R_1)$ $\Rightarrow$ Gain reduces with increasing frequency

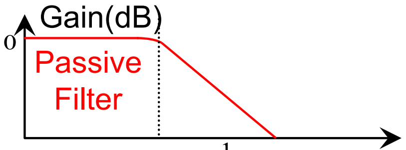
$$\boxed{\left|H(j\omega_{3dB})\right|=\frac{1}{\sqrt{2}}}$$

Interested in finding frequency (ω_{3dB}) where the power drops by half (Power is square of voltage)

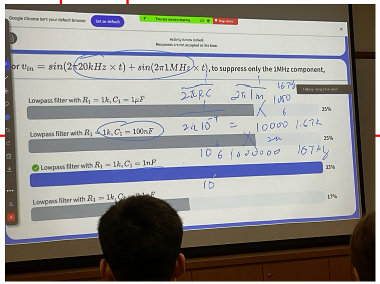
$$\Rightarrow \frac{1}{\sqrt{(\omega_{3dB}R_1C_1)^2+1}}=\frac{1}{\sqrt{2}}$$

$$\Rightarrow \omega_{3dB}=\frac{1}{R_1C_1} \quad or \quad f_{3dB}=\frac{1}{2\pi R_1C_1}$$

Gain reduced by √2, is equivalent to 3 dB reduction

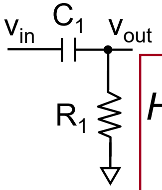


$$f_{3dB}=\frac{1}{2\pi\times(R_1C_1)}$$



Simple Highpass Filter

Passive 1st Order Highpass Filter



Simple resistor divider

$$\boxed{H(j\omega)=\frac{v_{out}}{v_{in}}=\frac{R_1}{\frac{1}{j\omega C_1}+R_1}=\frac{j\omega C_1R_1}{j\omega C_1R_1+1}=\boxed{\frac{j\omega}{j\omega+\frac{1}{C_1R_1}}}}$$

- At low frequency ($\omega \to 0$), ignore 1st term of denominator ($j\omega$) $\Rightarrow H(j\omega) \approx j\omega C_1R_1 \Rightarrow$ Gain increases with increasing frequency
- At high frequency ($\omega \to \infty$), ignore 2nd term of denominator ($1/(C_1R_1)$) $\Rightarrow H(j\omega) \approx 1 \Rightarrow$ Unity gain

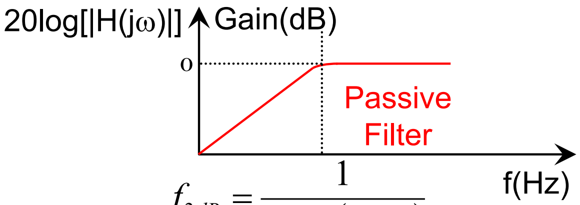
$$\boxed{\left|H(j\omega_{3dB})\right|=\frac{1}{\sqrt{2}}}$$

Interested in finding frequency (ω_{3dB}) where the power drop by half (Power is square of voltage)

$$\Rightarrow \frac{1}{\sqrt{\left(\frac{1}{\omega_{3dB}R_1C_1}\right)^2+1}}=\frac{1}{\sqrt{2}}$$

$$\Rightarrow \omega_{3dB}=\frac{1}{R_1C_1} \quad or \quad f_{3dB}=\frac{1}{2\pi R_1C_1}$$

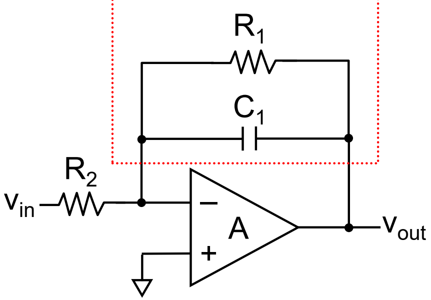
Gain reduced by √2, is equivalent to 3dB reduction



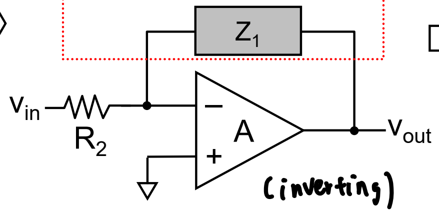
$$f_{3dB}=\frac{1}{2\pi\times(R_1C_1)}$$

First Order Lowpass Filter

Active 1st order Lowpass Filter



$$Z_1 = \left(\frac{1}{j\omega C_1} // R_1\right)$$



(inverting)

Equation (1) from slide 5-9

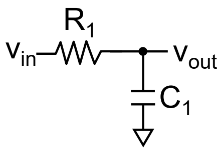
$$\boxed{H(j\omega) = \frac{v_{out}}{v_{in}} = -\frac{Z_1}{R_2} = -\frac{\left(\frac{1}{j\omega C_1} // R_1\right)}{R_2}}$$

Gain term

$$= -\frac{\frac{1}{j\omega C_1 + 1/R_1}}{R_2} = -\overbrace{\frac{R_1}{R_2}} \times \frac{1}{j\omega C_1 R_1 + 1}$$

Passive 1st Order Lowpass Filter

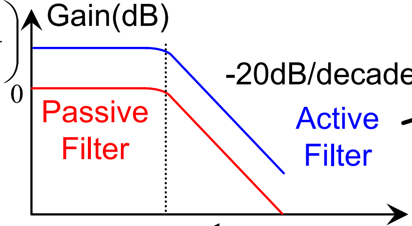
Simple resistor divider



$$\boxed{H(j\omega) = \frac{v_{out}}{v_{in}} = \frac{1}{j\omega C_1 R_1 + 1}}$$

Unity gain

$20 \times \log_{10}\left(\frac{R_1}{R_2}\right)$



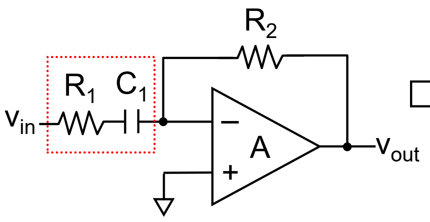
does amplification and filtering at the same time

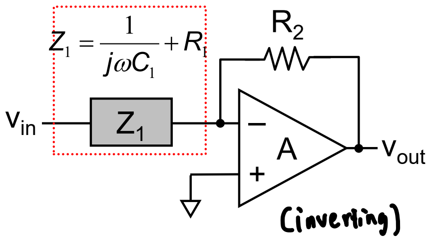
$$f_{3dB} = \frac{1}{2\pi \times (R_1 C_1)}$$

- Active 1st order lowpass has gain of R_1/R_2 versus gain of 1 for passive 1st order lowpass

First Order Highpass Filter

Active 1^{st} order Highpass Filter



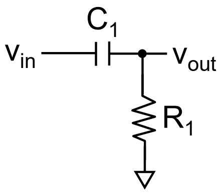


Equation (1) from slide 5-9

$$\boxed{H(j\omega) = \frac{v_{out}}{v_{in}} = -\frac{R_2}{Z_1} = -\frac{R_2}{\frac{1}{j\omega C_1} + R_1}}$$

$$= -\frac{j\omega C_1 R_2}{j\omega R_1 C_1 + 1} = -\underbrace{\frac{R_2}{R_1}}_{\text{Gain term}} \times \frac{j\omega}{j\omega + \frac{1}{R_1 C_1}}$$

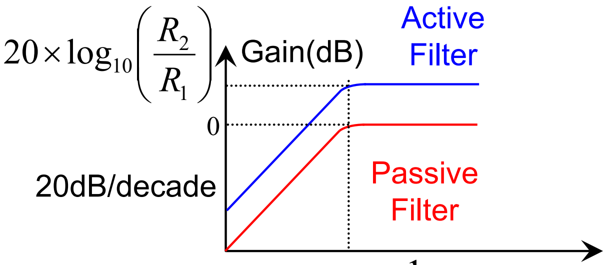
Passive 1^{st} Order Highpass Filter



Simple resistor divider

$$\boxed{H(j\omega) = \frac{v_{out}}{v_{in}} = \frac{j\omega}{j\omega + \frac{1}{R_1 C_1}}}$$

Unity gain



$$f_{3dB} = \frac{1}{2\pi \times (R_1 C_1)}$$

- Active 1^{st} order highpass has gain of R_2/R_1 versus gain of 1 for passive 1^{st} order highpass

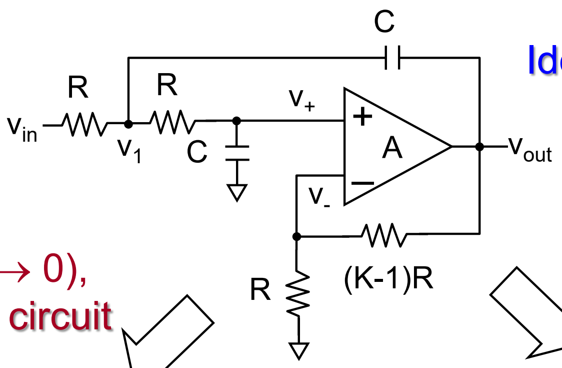
Lecture Summary

- **First order active filter**

Lecture Outline

- Overview of opamp and its analysis
- Opamp parameters
- Different types of opamp based amplifier
  - log amplifier, exponential amplifier, instrumentation amplifier
- **Filter**
  - Integrator, differentiator, 1st order filter
  - **2nd order SK filter and higher order filter**
- Super diode and comparator
- Applications built using opamp
  - Triangular wave, multiplier, bandstop filter, full wave rectifier

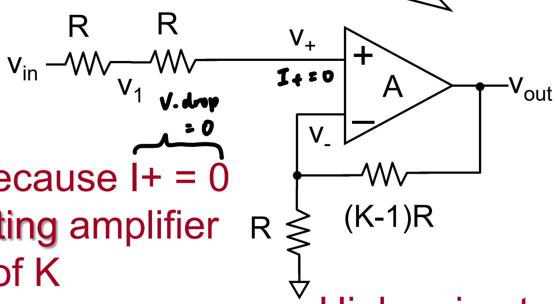
Simplified Sallen-Key Lowpass Filter



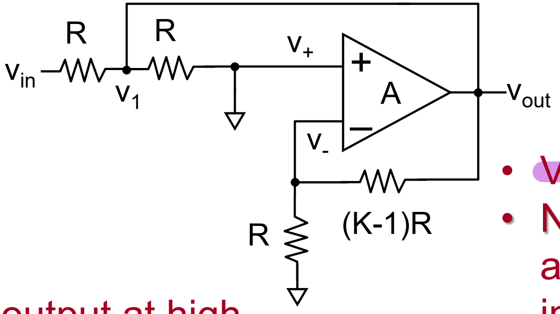
Identical R and C except for (K-1)R

At low frequency ($\omega \to 0$),
C behaves like open circuit

At high frequency ($\omega \to \infty$),
C behaves like short circuit



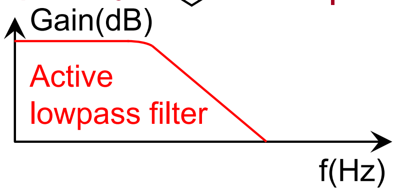
- $V+ = v_{in}$ because $I+ = 0$
- Non-inverting amplifier with gain of K



- $V+ = 0$
- Non-inverting amplifier with zero input will have zero output

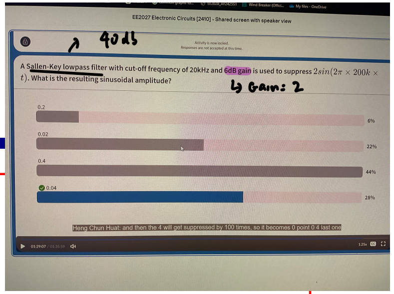
High gain at low frequency

Zero output at high frequency

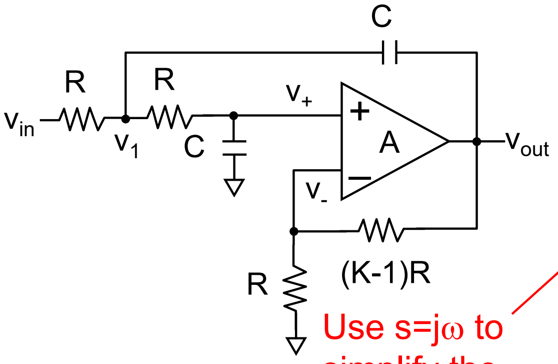


Sallen-Key Lowpass Filter

Signal: 2x2 = 4
sallen-key: -40db,
gain: 0.01
resulting: 0.01x4 = 0.04



Simplified Design Equation



$$H(j\omega)=\frac{v_{out}}{v_{in}}=\frac{\frac{K}{(RC)^2}}{(j\omega)^2+(j\omega)\left(\frac{3-K}{RC}\right)+\frac{1}{(RC)^2}}=\boxed{\frac{H_o\omega_o^2}{(j\omega)^2+\frac{\omega_o}{Q}(j\omega)+\omega_o^2}}$$

Filter function

Use s=jω to simplify the notation

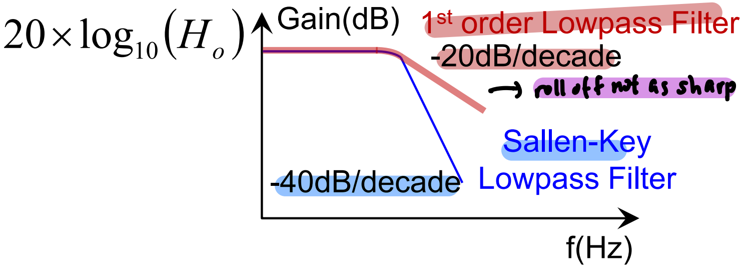
$$\boxed{=\frac{\frac{K}{(RC)^2}}{s^2+s\left(\frac{3-K}{RC}\right)+\frac{1}{(RC)^2}}=\frac{H_o\omega_o^2}{s^2+\frac{\omega_o}{Q}s+\omega_o^2}}$$

Filter function is usually given by system designer, circuit designer only concerns about how to implement the desired filter function through opamp.

$$\boxed{\Rightarrow \omega_o=\frac{1}{RC} \qquad \frac{1}{Q}=3-K \qquad H_o=K}$$

Important design equations

Why using 2nd order lowpass filter?



- At low frequency, ignore the 1st and 2nd terms of denominator of H(jω) $\Rightarrow H(j\omega)\approx H_o$
- At high frequency, ignore the 2nd and 3rd terms of denominator of H(jω) $\Rightarrow H(j\omega)\approx H_o\omega_o^2/(j\omega)^2$
- The gain reduces with $\omega^2 \Rightarrow$ sharper roll-off of -40dB/decade compared to 1st order lowpass filter
- It can suppress unwanted high frequency component more compared to 1st order lowpass filter

Lowpass Design Example

Design a second order lowpass filter that has Q of 0.7071 and cut-off frequency (f_o) at 1 kHz [$\omega_o = 2\pi f_o$]

$$\frac{v_{out}}{v_{in}} = \frac{H_o \omega_o^2}{(j\omega)^2 + \frac{\omega_o}{Q}(j\omega) + \omega_o^2} = \frac{H_o (2\pi \times 1k)^2}{(j\omega)^2 + \frac{2\pi \times 1k}{0.7071}(j\omega) + (2\pi \times 1k)^2}$$

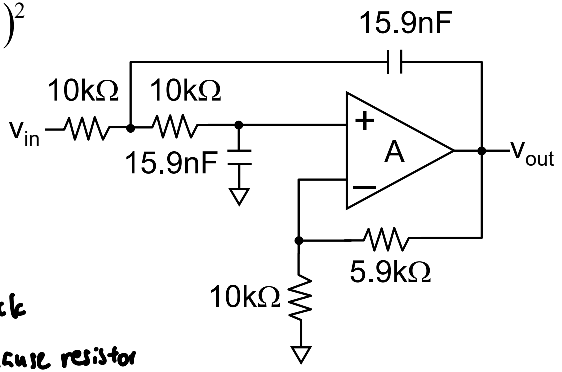
Using simplified design equations

$$\frac{1}{Q} = 3 - K = \frac{1}{0.7071} \Rightarrow K = 1.59 \qquad H_o = K = 1.59$$

$$\omega_o = \frac{1}{RC} = 2\pi \times 1k$$

Choose $R = 10k\Omega \Rightarrow C = 15.9nF$

For design, better to pick capacitor value first because resistor can use trimmer / potentiometer to adjust



Important Notes on Sallen-Key Lowpass Filter :

1) Be able to identify the circuit topology as Sallen-Key Lowpass Filter
2) Be able to locate the design equations for Sallen-Key Lowpass Filter
3) Be able to use the design equations to design the desired filter

Lecture Summary

- 2^{nd} order SK lowpass filter

Lecture Outline

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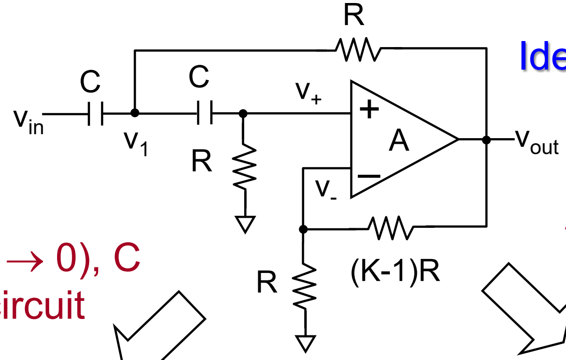
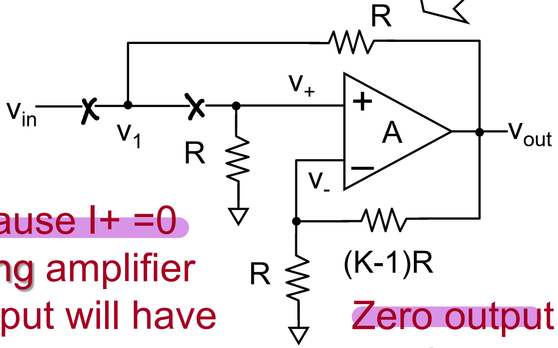
- Applications built using opamp
  - Triangular wave, multiplier, bandstop filter, full wave rectifier

Simplified Sallen-Key Highpass Filter



Identical R and C except for (K-1)R

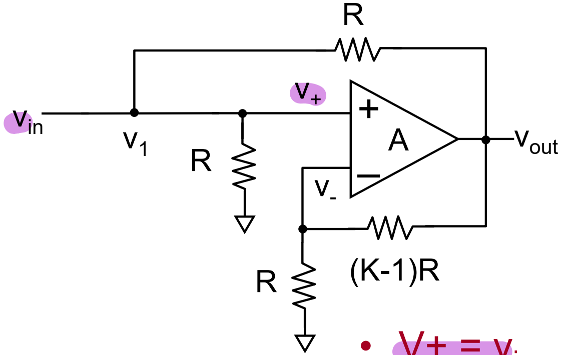
At low frequency ($\omega \rightarrow 0$), C behaves like open circuit



- V+ = 0 because I+ =0
- Non-inverting amplifier with zero input will have zero output

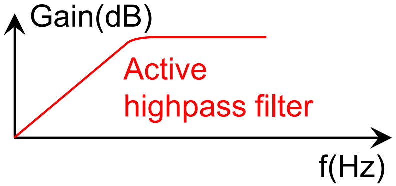
Zero output at low frequency

At high frequency ($\omega \rightarrow \infty$), C behaves like short circuit



- V+ = v_{in}
- Non-inverting amplifier with gain of K

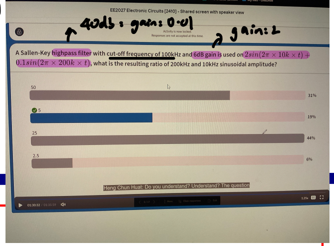
High gain at high frequency



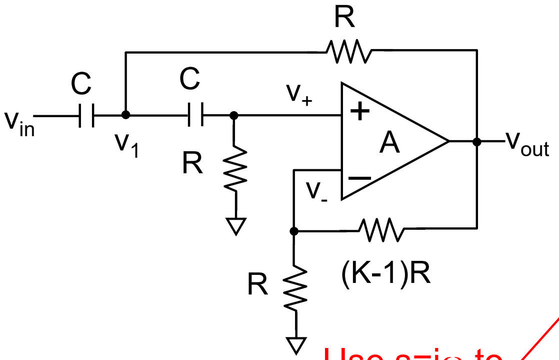
Sallen-Key Highpass Filter

200KHz: 0.1×2
= 0.2
10KHz: 2×2
= 4
10kHz < 100kHz cutoff
4×0.01 = 0.04

ratio: $\frac{0.2}{0.04}$ (200kHz) (10kHz)
= 5



Simplified Design Equation



$$H(j\omega)=\frac{v_{out}}{v_{in}}=\frac{K(j\omega)^2}{(j\omega)^2+(j\omega)\left(\frac{3-K}{RC}\right)+\frac{1}{(RC)^2}}=\boxed{\frac{H_o(j\omega)^2}{(j\omega)^2+\frac{\omega_o}{Q}(j\omega)+\omega_o^2}}$$

Filter function

$$\boxed{=\frac{Ks^2}{s^2+s\left(\frac{3-K}{RC}\right)+\frac{1}{(RC)^2}}=\frac{H_os^2}{s^2+\frac{\omega_o}{Q}s+\omega_o^2}}$$

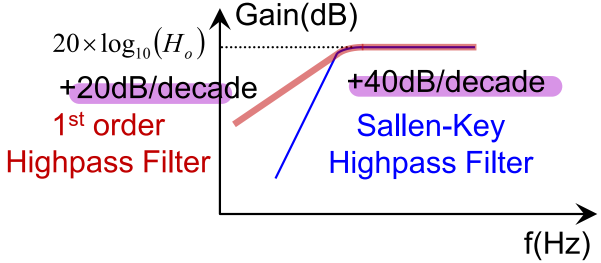
Use s=jω to simplify the notation

$$\boxed{\Rightarrow \omega_o=\frac{1}{RC} \qquad \frac{1}{Q}=3-K \qquad H_o=K}$$

Important design equations

Filter function is usually given by system designer, circuit designer only concerns about how to implement the desired filter function through opamp.

Why using 2nd order highpass filter?



- At low frequency, ignore the 1st and 2nd terms of denominator of H(jω) $\Rightarrow H(j\omega) \approx H_o(j\omega)^2/\omega_o^2$
- At high frequency, ignore the 2nd and 3rd terms of denominator of H(jω) $\Rightarrow H(j\omega) \approx H_o$
- The gain increases with $\omega^2 \Rightarrow$ sharper roll-off of +40dB/decade compared to 1st order highpass filter
- It can suppress unwanted low frequency component more compared to 1st order highpass filter

Highpass Design Example

Design a **second order** highpass filter that has Q of 0.7071 and cut-off frequency (f_o) at 5 kHz

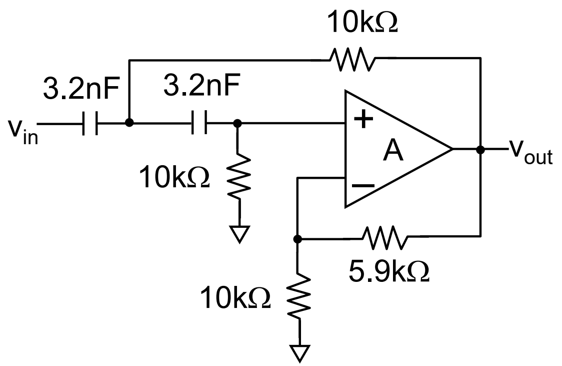
$$\frac{v_{out}}{v_{in}} = \frac{H_o s^2}{s^2 + \frac{\omega_o}{Q} s + \omega_o^2} = \frac{H_o s^2}{s^2 + \frac{2\pi \times 5k}{0.7071} s + (2\pi \times 5k)^2}$$

Using simplified design equations

$$\frac{1}{Q} = 3 - K = \frac{1}{0.7071} \Rightarrow K = 1.59 \qquad H_o = K = 1.59$$

$$\omega_o = \frac{1}{RC} = 2\pi \times 5k$$

$$\text{Choose} \quad R = 10k\Omega \Rightarrow C = 3.2nF \longrightarrow$$ make smart choice even if there is infinite combos for soln



Important Notes on Sallen-Key Highpass Filter :

1) Be able to identify the circuit topology as Sallen-Key Highpass Filter
2) Be able to locate the design equations for Sallen-Key Highpass Filter
3) Be able to use the design equations to design the desired filter

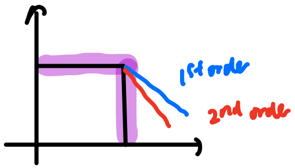
How to Build Higher Order Filter?

7th Order Lowpass Filter:

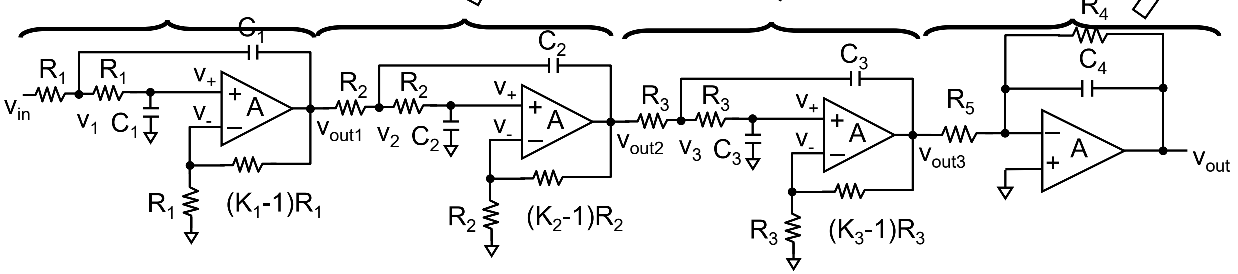
$$H(j\omega)=\frac{V_{out}}{V_{in}}=\frac{H_o}{a_7(j\omega)^7+a_6(j\omega)^6+a_5(j\omega)^5+a_4(j\omega)^4+a_3(j\omega)^3+a_2(j\omega)^2+a_1(j\omega)^1+1}$$

$$=\underbrace{\frac{H_{o1}\left(\omega_{o1}^2\right)}{(j\omega)^2+\frac{\omega_{o1}}{Q_1}(j\omega)+\omega_{o1}^2}}_{1^{st}\ \text{Sallen Key}}\times\underbrace{\frac{H_{o2}\left(\omega_{o2}^2\right)}{(j\omega)^2+\frac{\omega_{o2}}{Q_2}(j\omega)+\omega_{o2}^2}}_{2^{nd}\ \text{Sallen Key}}\times\underbrace{\frac{H_{o3}\left(\omega_{o3}^2\right)}{(j\omega)^2+\frac{\omega_{o3}}{Q_3}(j\omega)+\omega_{o3}^2}}_{3^{rd}\ \text{Sallen Key}}\times\underbrace{\frac{H_{o4}\left(\omega_{o4}\right)}{j\omega+\omega_{o4}}}_{4^{th}\ \text{First Order}}$$

Higher order filters attempt to approximate brickwall filter



(2nd order)   (2nd order)   (2nd order)



Lecture Summary

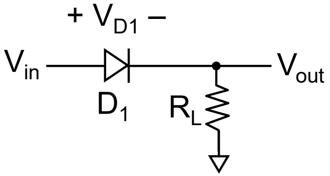
- 2^{nd} order SK highpass filter
- Higher order filter design

Lecture Outline

- Overview of opamp and its analysis
- Opamp parameters
- Different types of opamp based amplifier
  - log amplifier, exponential amplifier, instrumentation amplifier
- Filter
  - Integrator, differentiator, 1^{st} order filter
  - 2^{nd} order SK filter and higher order filter
- **Super diode and comparator**
- Applications built using opamp
  - Triangular wave, multiplier, bandstop filter, full wave rectifier

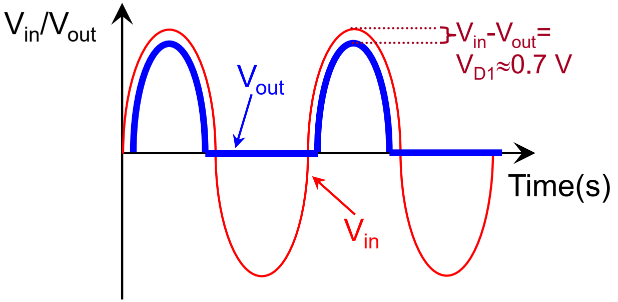
Superdiode

Conventional Rectifier

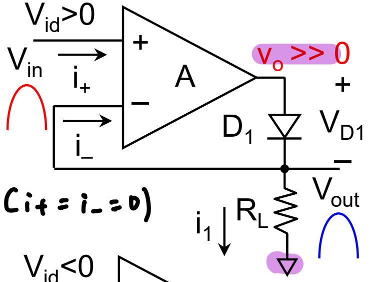


The rectifier circuit only rectifies input when the input (V_{in}) is bigger than the diode voltage (V_{D1}) by 0.7 V

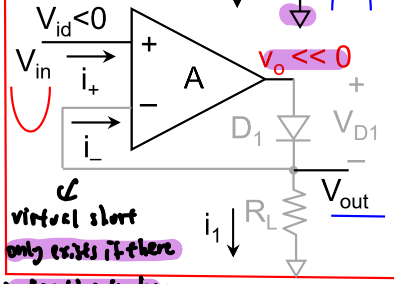
Because the signals we are dealing with consist of DC and AC components, we use upper case (V_{out}, V_{in}, V_{D1}) rather than lower case (v_{out}, v_{in}, v_{D1})



Superdiode Rectifier



$(i_+ = i_- = 0)$



virtual short only exists if there is feedback loop

Because the signals we are dealing with consist of DC and AC components, we use upper case (V_{out}, V_{in}, V_{D1}) rather than lower case (v_{out}, v_{in}, v_{D1})

$V_{in} > 0$:

$V_o >> 0 \Rightarrow$ diode starts conducting

$\Rightarrow V_{D1} \sim 0.7$ V

$V_+ - V_- = (V_{out}+V_{D1})/A \approx 0$ if $A \rightarrow \infty$

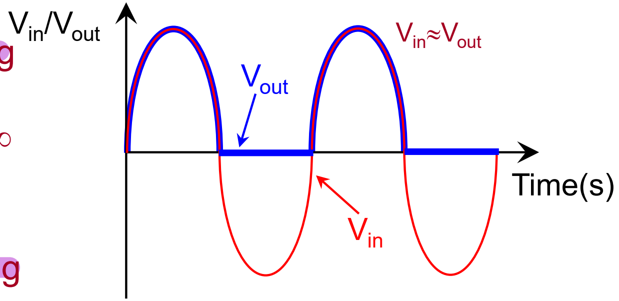
$\Rightarrow V_{out} \approx V_{in}$

$V_{in} < 0$:

$V_o << 0 \Rightarrow$ diode stops conducting

$i_- = 0$ and $i_{D1} \approx 0 \Rightarrow i_1 = 0$

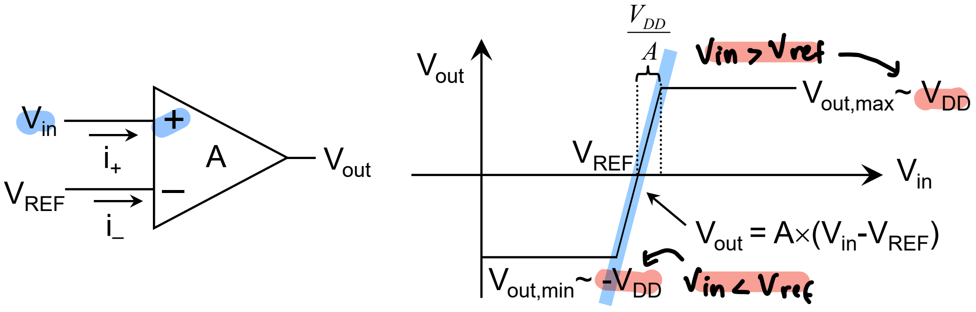
$\Rightarrow V_{out} \approx 0$



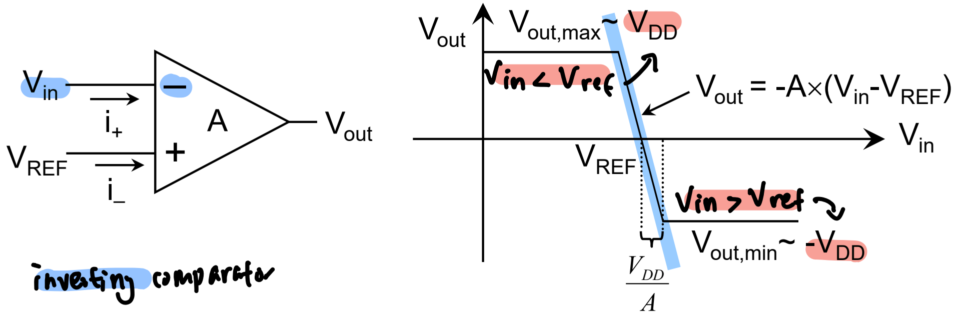
- Ideal rectifier circuit should rectify input when the input (V_{in}) is bigger than zero
- Useful for AM demodulator when signal is small

Opamp Used as Comparator

→ op-amp without feedback loop
→ good for logic circuit bcos it goes high and low



$V_{out} = A\times(V_{in}-V_{REF})$, but owing to the huge gain and limited supply voltage, the output grows rapidly and saturates at positive and negative supply voltage. If $A \rightarrow \infty$, the transfer characteristic behaves like a step function centered at V_{REF}.



inverting comparator

$V_{out} = -A\times(V_{in}-V_{REF})$, but owing to the huge gain and limited supply voltage, the output grows rapidly and saturates at positive and negative supply voltage. If $A \rightarrow \infty$, the transfer characteristic behaves like an inverted step function centered at V_{REF}.

As an example, opamp with gain A = 10000 and supply voltage of ±10 V, when $|V_{in}-V_{ref}| > 1$ mV, V_{out} would saturate to ±10 V.

Important Notes :

When opamp operates in open loop, → no feedback loop
virtual short ($v_+ \approx v_-$) no longer applies.

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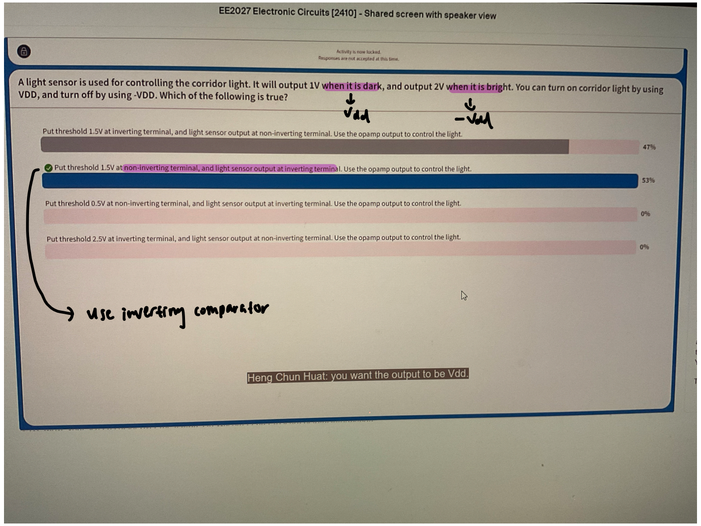
Lecture Summary

- Super diode operation
- Comparator – opamp analysis technique cannot be applied due to open loop nature of the circuit



Lecture Outline

- Overview of opamp and its analysis
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  - log amplifier, exponential amplifier, instrumentation amplifier
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  - 2nd order SK filter and higher order filter
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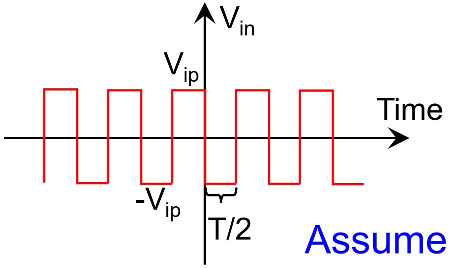
- **Applications built using opamp**
  - **Triangular wave, multiplier, bandstop filter, full wave rectifier**

Applications Built using Opamp

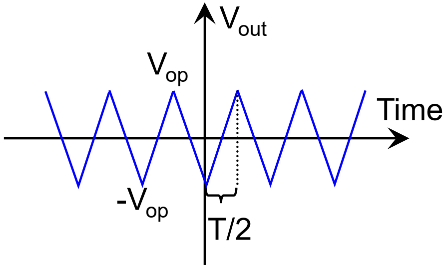
- Now we have a library of opamp-based circuits
- We can realize more complicated functions by combining a number of opamp circuits
- This will form sub-system
- The concept will be illustrated in the following few examples

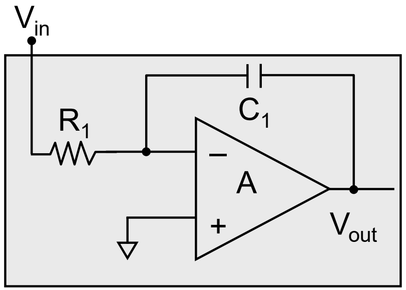
Triangular Wave



Integration of square wave results in triangle wave. Intuitively, integration is finding the area under square wave.

Assume $V_{ip} = V_{DD}/2$





$$V_{out1}\left(\frac{T}{2}\right) - V_{out1}(0) = -\frac{1}{R_1C_1}\int_0^{\frac{T}{2}}\left(-V_{ip}\right)dt$$

$$= -\frac{1}{R_1C_1}\int_0^{\frac{T}{2}}\left(-\frac{V_{DD}}{2}\right)dt$$

$$= -\frac{1}{R_1C_1}\left[-\frac{V_{DD}}{2}t\right]_{t=0}^{t=\frac{T}{2}}$$

$$= \frac{V_{DD}T}{4R_1C_1} = 2V_{op}$$

If want sine wave, can use low-pass filte

If want sawtooth, can use

Multiplier

In exam, follow the hints, hints will be given.

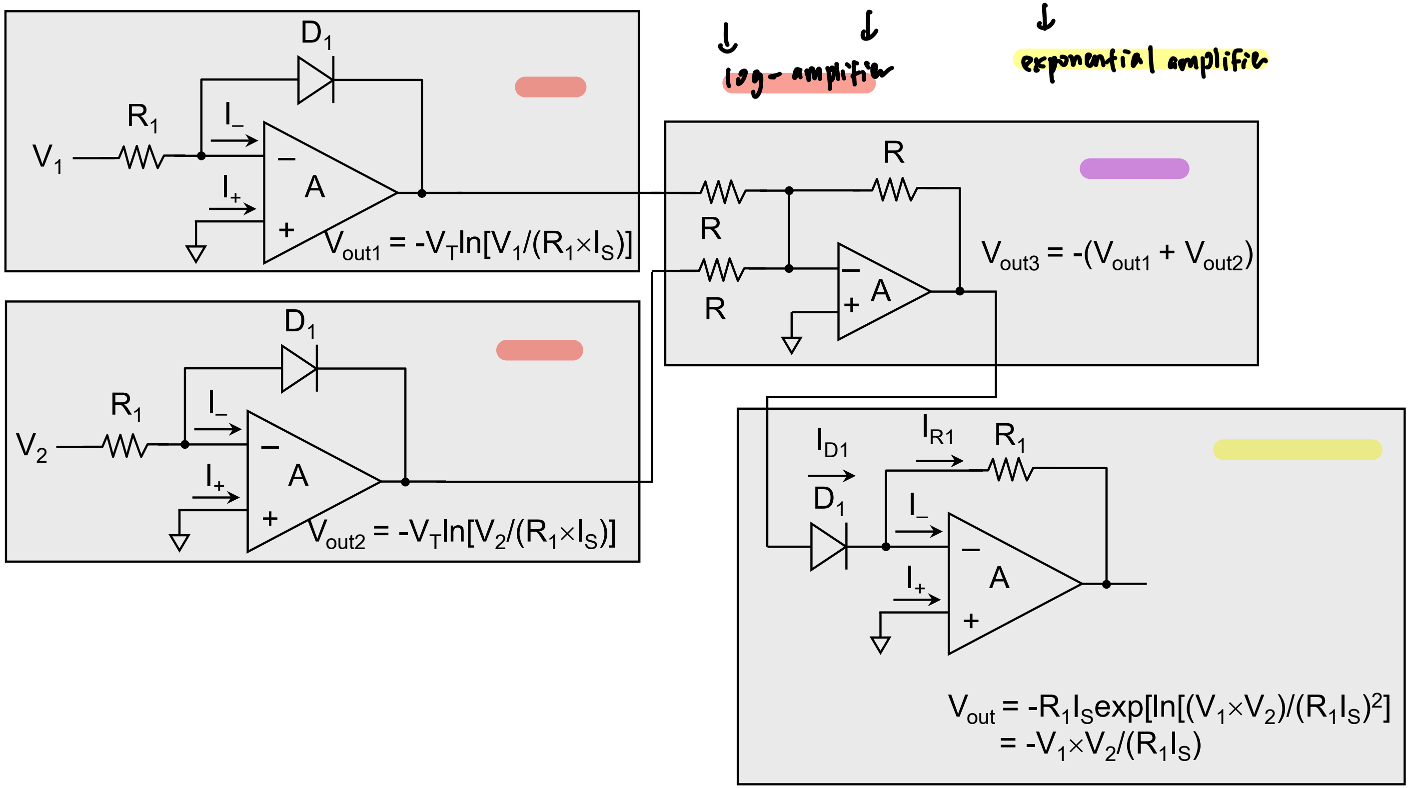
- How to create multiplier using existing opamp circuits covered so far, i.e. $V_{out} = kV_1 \times V_2$?

(Hints: Pre-university mathematics, $\log(A\times B) = \log(A) + \log(B)$ and $e^{\ln(X)} = X$)

summing amplifier

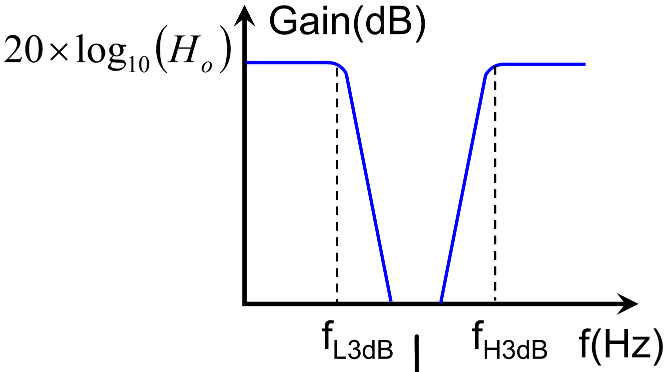
log-amplifier

exponential amplifier

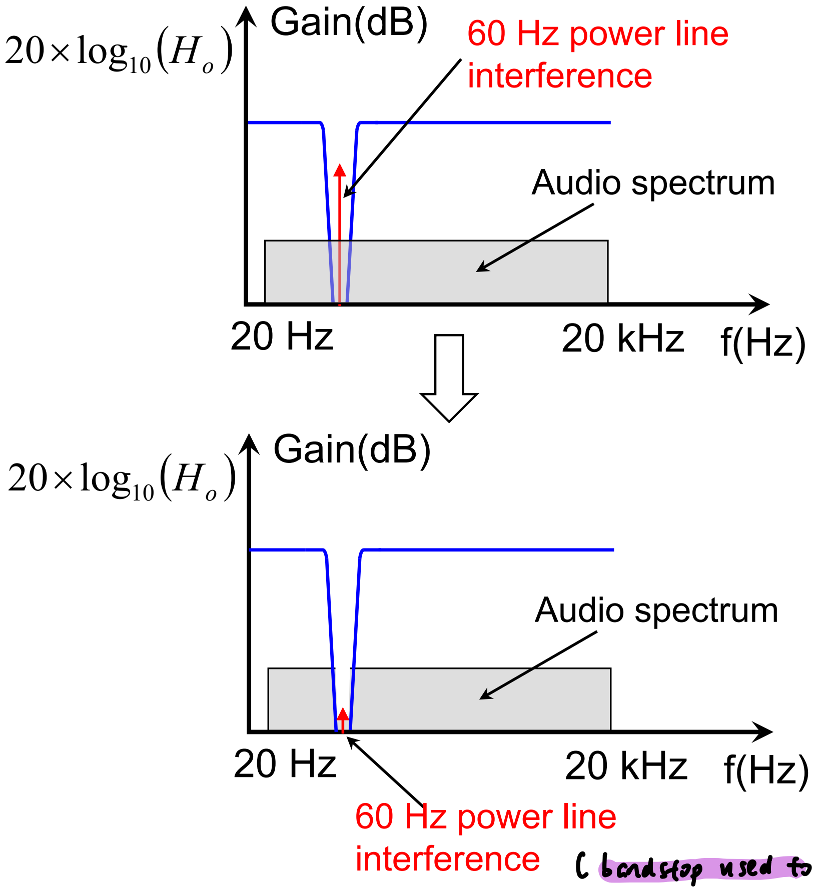


Bandstop Filter → low pass + high pass, how to combine?

- How to create bandstop filter that has the following characteristic?
- What is the use of bandstop?



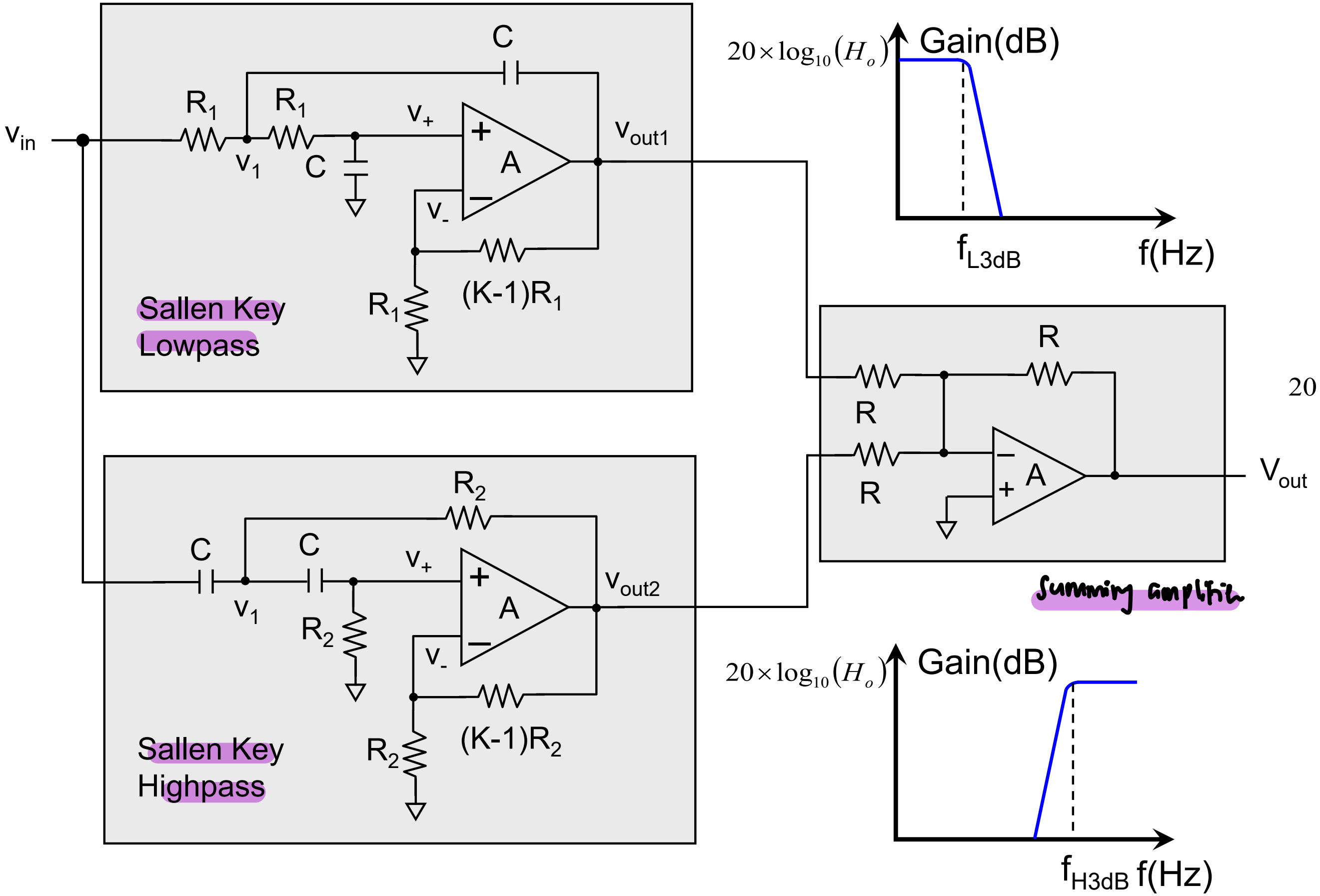
stop at middle band



(bandstop used to

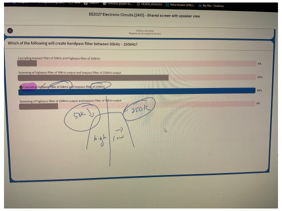
remove this (60Hz)

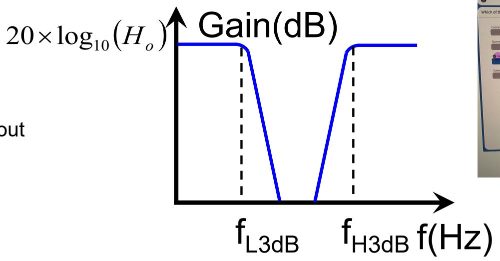
Bandstop Filter



What about band-pass?

Signal must past through both low-pass and high pass and still survive, so can put lowpass, then high pass, side by side



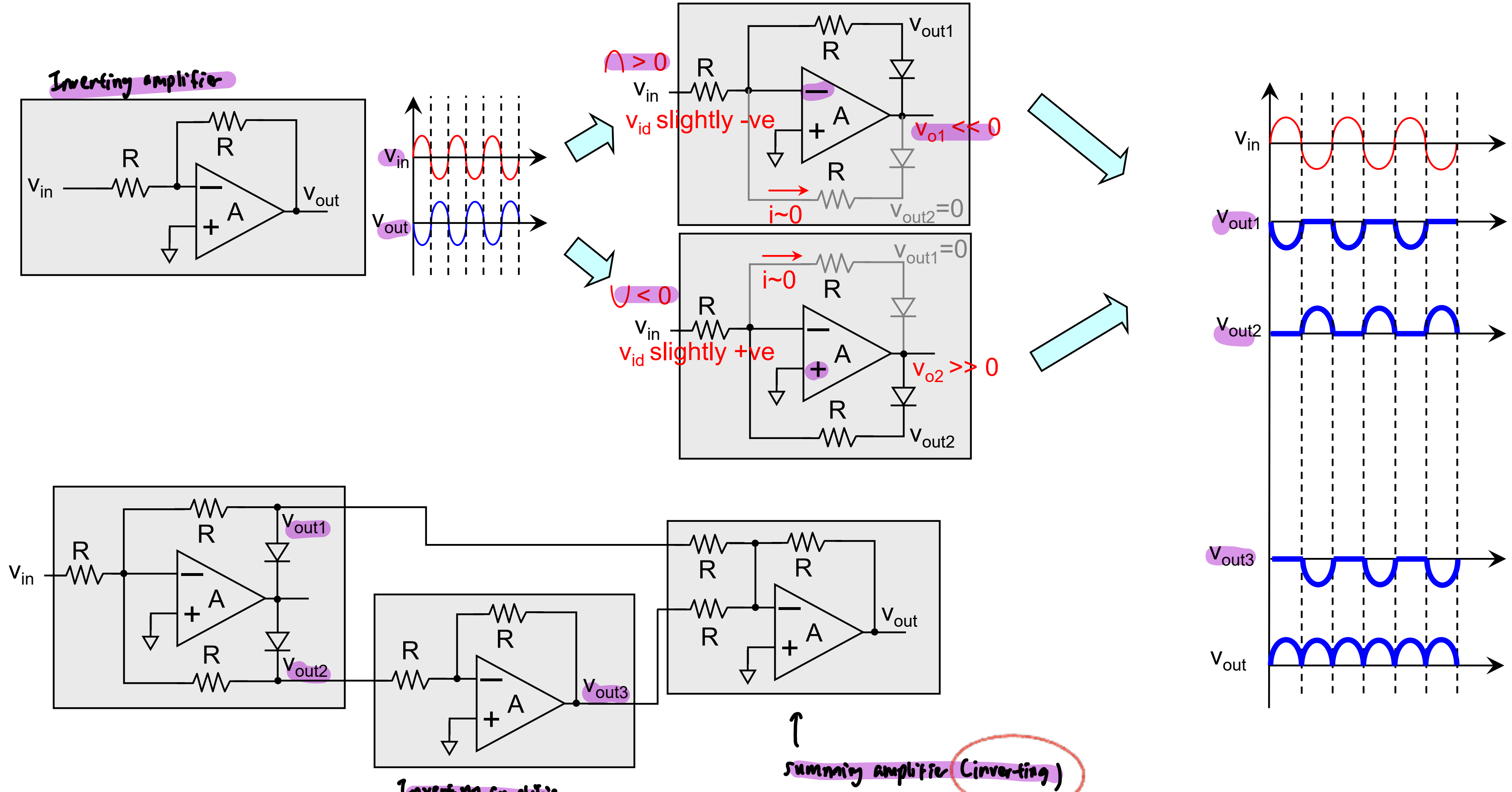


bandstop requires $f_{H3dB} > f_{L3dB}$

Full Wave Rectifier

negative : output = 0

(super diode → half wave rectifier)



Lecture Summary

- Overview of opamp and its analysis
- Different opamp based amplifier
- Opamp based filter design
- Super diode and comparator
- Various functions built using combination of opamp circuits

Opamp – Topics Discussed

- **Opamp model: 2-port network with 3 parameters (R_{in}, R_{out}, A)**
  - **Ideal Opamp model:** $R_{in} = \infty$, $R_{out} = 0$, $A \to \infty$
- **Opamp circuit analysis (with negative feedback):**
  - **Virtual short:** $v_+ \approx v_-$
  - **No input current:** $i_+ = i_- = 0$
- **Opamp biasing (dual vs single voltage supply)**
- **Opamp key parameters & their effects -**
  - **CMRR:** $CMRR = 20log\left(\frac{A_{OL}}{A_{CM}}\right)$, $A_{OL} = \frac{v_{od}}{v_{id}}$, $A_{CM} = \frac{v_{o,cm}}{v_{i,cm}}$
  - **GBW affects operating frequency range of opamp circuit:**
    - $A(j\omega) = \frac{A_{0L}}{\frac{j\omega}{\omega_{3dB}}+1} \approx \frac{A_{0L}\times\omega_{3dB}}{j\omega} = \frac{2\pi\times GBW}{j\omega}$, $\left|\beta \cdot G(j\omega_{3dB,CL})\right| = 1$

Opamp – Topics Discussed (Cont.)

- **Opamp key parameters & their effects (Cont.) -**
  - **v_n limits the minimum v_{in} amplitude**
    - $v_{noise,rms} = \sqrt{V_n^2 \times f_{3dB} \times \frac{\pi}{2}}$
  - **V_{OS} limits the maximum v_{in} amplitude**
  - **PSRR:** $PSRR = 20log\left(\frac{A_{OL}}{A_{sup}}\right)$, $A_{OL} = \frac{v_{od}}{v_{id}}$, $A_{CM} = \frac{v_{o,sup}}{v_{i,sup}}$
  - **SR:** $SR > \frac{dv_{out}}{dt} = 2\pi f \times v_{out,pk} sin(2\pi f t)$
- **Library of basic Opamp based circuits: amplifiers, integrator, differentiator, filters (LPF & HPF), super diode, comparator**
- **More complex Opamp based circuits (e.g., multiplier, bandstop filter, full-wave rectifier)**

Reading Assignment

- **Reading: Reference Book (Sedra & Smith)**
  Chapter 5, pp. 473 – 543. (Opamp)