NATIONAL UNIVERSITY OF SINGAPORE

Department of Electrical and Computer Engineering

EE2027 Electronic Circuits

Supplementary Questions 2: Solution

These are supplementary questions to Tutorial 2, and they aim to provide more work examples. They will not be discussed in class, but solutions will be provided after Tutorial discussion.

<u>Supplementary Questions</u> (Will not be discussed in class)

S1. (a) Draw an opamp circuit that can convert a square wave to a triangular wave.

(b) Figure S1 shows a comparator circuit on the right. For the input waveform, $V_{in}(t)$, shown on the left, sketch the comparator output waveform, $V_{out}(t)$. Label your sketch clearly with time and voltage information. The comparator is powered by ±5 V voltage source.

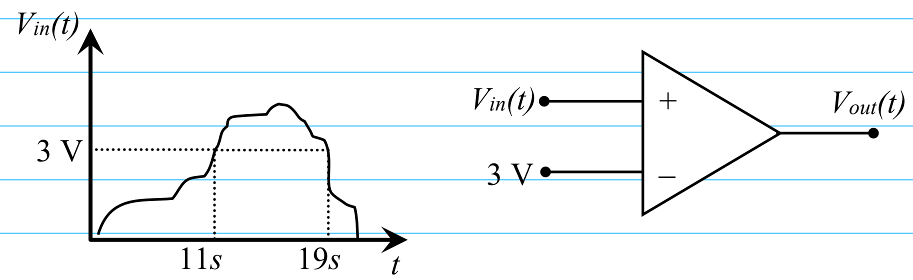


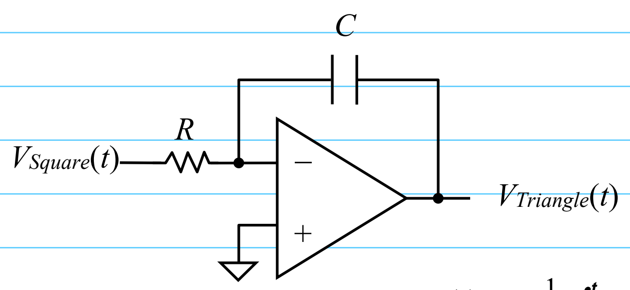
Fig. S1

(c) Propose an opamp-based circuit that can produce a square wave with an arbitrary duty cycle from a square wave with 50% duty cycle. Duty cycle is defined as the ratio of the duration that the waveform is high to the period (T_{period}). For example, a 40% duty cycle means that the square wave is high for $0.4 \times T_{period}$ and low for $0.6 \times T_{period}$.

S1. <u>Solution:</u>

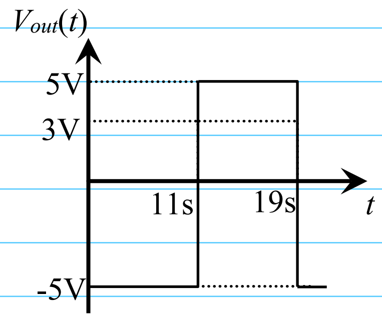
(a) Draw an opamp circuit that can convert a square wave to a triangular wave.

An integrator will convert a square wave to a triangular wave -

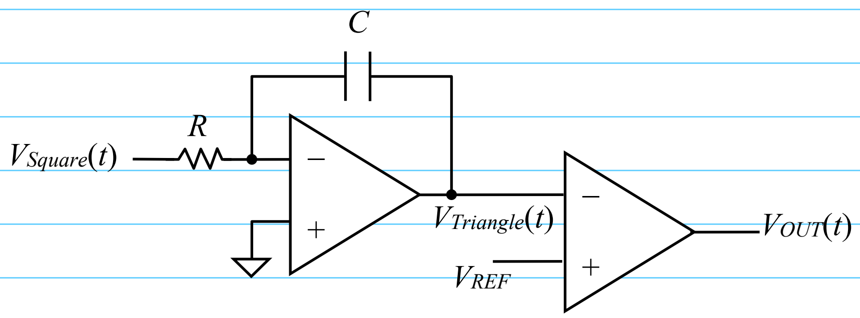


$$V_{Triangle}(t) = -\frac{1}{RC}\int_0^t V_{Square}(\tau)d\tau$$

(b) Figure S1 shows a comparator circuit on the right. For the input waveform, $V_{in}(t)$, shown in the left, sketch the comparator output waveform, $V_{out}(t)$. Label your sketch clearly with time and voltage information. The comparator is powered by ±5 V voltage source.



(c) Propose an opamp-based circuit that can produce a square wave with an arbitrary duty cycle from a square wave with 50% duty cycle. Duty cycle is defined as the ratio of the duration that the waveform is high to the period (T_{period}). For example, a 40% duty cycle means that the square wave is high for $0.4\times T_{period}$ and low for $0.6\times T_{period}$.



The duty cycle of $V_{OUT}(t)$ can be varied by V_{REF}.

S2. You may need to use the following trigonometry identity:

$$\cos A \cos B = \frac{\cos(A+B)}{2} + \frac{\cos(A-B)}{2}$$

(a) Expand $cos(\omega t) \times cos(100\omega t)$ using the identity shown above.

(b) Given an input of $cos(\omega t) \times cos^2(100\omega t)$, express the input in terms of the sum of cosine terms. Explain how you can extract the $cos(\omega t)$ term from the given input.

(c) Show an opamp circuit that can perform the task in part (b).

S2. <u>**Solution**</u>**:**

(a) Expand $cos(\omega t) \times cos(100\omega t)$ using the identity shown above.

$$cos(\omega t) \times cos(100\omega t) = \frac{cos(101\omega t)}{2} + \frac{cos(99\omega t)}{2}$$

(b) Given an input of $cos(\omega t) \times cos^2(100\omega t)$, express the input in terms of the sum of cosine terms. Explain how you can extract the $cos(\omega t)$ term from the given input.

$$\begin{aligned} &cos(\omega t) \times cos^2(100\omega t) \\ &= \left[\frac{cos(101\omega t)}{2} + \frac{cos(99\omega t)}{2}\right] \times cos(100\omega t) \\ &= \frac{cos(\omega t)}{4} + \frac{cos(201\omega t)}{4} + \frac{cos(\omega t)}{4} + \frac{cos(199\omega t)}{4} \\ &= \frac{cos(\omega t)}{2} + \frac{cos(201\omega t)}{4} + \frac{cos(199\omega t)}{4} \end{aligned}$$

Input signal is a sum of 3 sinusoidal waves of different angular frequencies of ω, 199ω, and 201ω.

A low pass filter (e.g., with ω_{3dB} lower than 100ω) will be able to extract the $cos(\omega t)$ term.

(c) Show an opamp circuit that can perform the task in part (b).

Either sallen key low pass filter or first order low pass filter (e.g., with ω_{3dB} lower than 100ω).

S3. One engineer has decided to design an inverting amplifier, as shown in Fig. S3. The opamp has the following parameters, and is powered by ±5 V voltage sources.

Opamp	
GBW	1 MHz
V_{OS}	20 mV

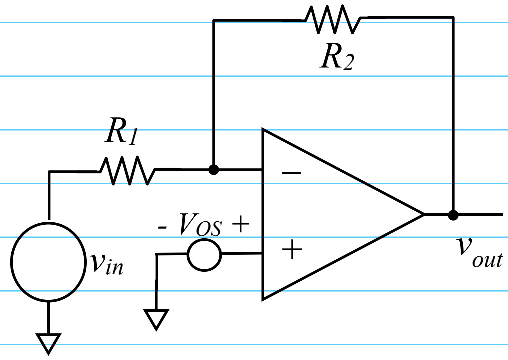


Fig. S3 Inverting amplifier

(a) For the inverting amplifier with a gain of -200, determine the maximum allowable peak v_{in} such that v_{out} is not distorted or clipped. v_{in} is a sinusoidal voltage waveform with zero dc offset.

[Ans. $v_{in,pk} \approx 5$ mV]

(b) Specify R_1 and R_2 values to achieve the gain in part (a). Assume that the opamp is ideal.

(c) Estimate the closed-loop 3dB cut-off frequency ($f_{3dB,CL}$) of the inverting amplifier.

[Ans. $f_{3dB,CL} \approx 5$ kHz]

(d) Assume the opamp has an input noise voltage spectral density, $v_n = 100\ nV/\sqrt{Hz}$. Estimate the minimum allowable input voltage, v_{in}, that the inverting amplifier can handle. (Hint: You may assume that v_n appears in a similar position as V_{OS} and you may ignore the noise contribution from R_1 and R_2.)

[Ans. $v_{in,min} = 8.9$ µV]

(e) Comment on whether the minimum allowable input voltage, v_{in}, is larger or smaller if the noise contribution from R_1 and R_2 are considered. Explain your answer.

S3. Solution:

(a) For the inverting amplifier with a gain of -200, determine the maximum allowable peak v_{in} such that v_{out} is not distorted or clipped.

$$v_{out} = v_{in} \times \left(-\frac{R_2}{R_1}\right) + V_{OS} \times \left(1 + \frac{R_2}{R_1}\right) = -200 \times v_{in} + 4.02\text{V}$$

$$-5\text{V} < v_{out} < 5\text{V}$$

$$\Rightarrow -9.02\text{V} < -200 \times v_{in} < 0.98\text{V}$$

$$\Rightarrow 45.1\text{ mV} > v_{in} > -4.9\text{ mV}$$

$$v_{in} = v_{in,pk} \times sin(\omega t)$$

To avoid distortion, $v_{in,pk}$ cannot be more than 4.9 mV ≈ 5 mV.

Alternatively, as the DC offset for v_{out} is 4.02 V ≈ 4 V, that leaves only (5V – 4.02 V) ≈ 1 V difference for the positive sinusoidal output. Hence, $v_{in,pk}$ = 1/200 ≈ 5 mV.

(b) Specify R_1 and R_2 values to achieve the gain in part (a).

We can choose R_2 = 200 kΩ, and R_1 = 1 kΩ.

(c) Estimate the closed-loop 3dB cut-off frequency ($f_{3dB,CL}$) for the inverting amplifier.

$$f_{3dB,CL} = GBW \times \frac{R_1}{R_2 + R_1} = 1\text{MHz} \times \frac{1}{201} \approx 5\text{ kHz}$$

(d) Assume the opamp has an input noise voltage spectral density, $v_n = 100\, nV/\sqrt{Hz}$. Estimate the minimum allowable input voltage, v_{in}, that the inverting amplifier can handle. (Hint: You may assume v_n appear in similar position as V_{OS} and you may ignore the noise contribution from R_1 and R_2.)

All the quantities are rms value.

$$v_{out,n} = \left(1 + \frac{R_2}{R_1}\right) \times \sqrt{V_n^2 \times f_{3dB,CL} \times \frac{\pi}{2}} = 1.78\text{ mV}$$

$$v_{in,min} > \frac{v_{out,n}}{200} = 8.9\text{ µV}$$

(e) Comment on whether the minimum allowable input voltage, v_{in}, is larger or smaller if the noise contribution from R_1 and R_2 are considered. Explain your answer.

With R_1 and R_2 noise contribution, the total output noise will be larger, and thus when the noise is referred back to the input, it will also be larger. Hence, the $v_{in,min}$ will need to be larger.

S4. All the opamps used in Fig. S4 have GBW of 1 MHz, slew rate of 0.5 V/μs, and offset voltage of 10 mV.

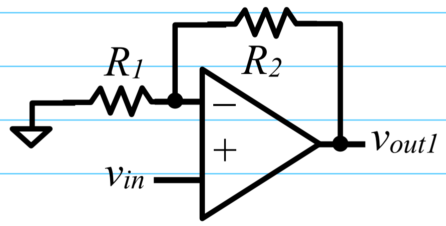


Fig. S4(a)

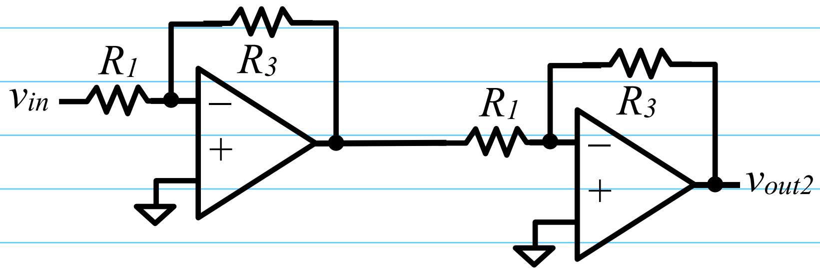


Fig. S4(b)

(a) Derive the voltage gains, v_{out1}/v_{in} and v_{out2}/v_{in}.

(b) If both of the gains in part (a) are identical, derive the relationship among R_1, R_2 and R_3.

If a gain of 100 is desired for both amplifiers in Figs. S4(a) and S4(b), calculate the values for R_2 and R_3 if R_1 is 1kΩ.

(c) If a gain of 100 is desired for both amplifiers in Figs. S4(a) and S4(b), estimate their $f_{3dB,CL}$.

[Ans. 10 kHz, 90.9 kHz]

(d) In order to operate with an input signal up to $f_{3dB,CL}$ in part (c), what is the maximum v_{in} that each amplifier can handle?

[Ans. 79.6 mV, 8.75 mV]

S4. **Solution:**

(a) Derive the voltage gains, v_{out1}/v_{in} and v_{out2}/v_{in}:

Circuit (a) is a non-inverting amplifier: $\frac{v_{out1}}{v_{in}} = \frac{R_2}{R_1} + 1$

Circuit (b) is a cascade of two inverting amplifiers: $\frac{v_{out2}}{v_{in}} = \left(\frac{R_3}{R_1}\right)^2$

(b) Derive the relationship among R_1, R_2 and R_3:

$$\left(\frac{R_3}{R_1}\right)^2 = \frac{R_2}{R_1} + 1$$

$$\left(\frac{R_3}{1k}\right)^2 = \frac{R_2}{1k} + 1 = 100$$

Therefore, $R_2 = 99\ \text{k}\Omega$ and $R_3 = 10\ \text{k}\Omega$

(c) If gain of 100 is desired, estimate the maximum operating frequency for both amplifiers:

Non-inverting amplifier:

$$\frac{R_2}{R_1} + 1 = 100 \Rightarrow \frac{R_2}{R_1} = 99 \Rightarrow f_{3dB,CL1} = GBW \times \frac{1}{1 + \frac{R_2}{R_1}} = 10\ \text{kHz}$$

Solution 1 (2 cascaded inverting amplifiers):

$$\left(\frac{R_3}{R_1}\right)^2 = 100 \Rightarrow \frac{R_3}{R_1} = 10 \Rightarrow f_{3dB,CL2} = GBW \times \frac{1}{1 + \frac{R_3}{R_1}} = 90.9\ \text{kHz}$$

This solution for $f_{3dB,CL2}$ is for a single stage and NOT for two cascaded amplifiers. At $f_{3dB,CL2}$, the resulting gain is in fact at -6dB instead of -3dB. However, this frequency is accepted as it is more straight forward and easier to obtain.

Solution 2 (2 cascaded inverting amplifiers):

$$\left(\frac{R_3}{R_1}\right)^2 = 100 \Rightarrow \frac{R_3}{R_1} = 10 \Rightarrow f_{6dB,CL2} = GBW \times \frac{1}{1 + \frac{R_3}{R_1}} = 90.9kHz$$

(Above 3dB frequency is only for a single stage of inverting amplifier)

For two cascaded inverting amplifiers -

$$\frac{v_{out2}}{v_{in}} = \left[\frac{R_3/R_1}{1+\dfrac{j\omega}{\omega_{6dB,CL2}}}\right] \times \left[\frac{R_3/R_1}{1+\dfrac{j\omega}{\omega_{6dB,CL2}}}\right] = \left[\frac{100}{\left[1+\dfrac{j\omega}{\omega_{6dB,CL2}}\right] \times \left[1+\dfrac{j\omega}{\omega_{6dB,CL2}}\right]}\right]$$

At $\omega_{3dB,CL2}$,

$$\left|\frac{100}{\left(\dfrac{j\omega_{3dB,CL2}}{\omega_{6dB,CL2}}+1\right)\left(\dfrac{j\omega_{3dB,CL2}}{\omega_{6dB,CL2}}+1\right)}\right| = \frac{100}{\sqrt{2}} \Rightarrow \left(\frac{\omega_{3dB,CL2}}{\omega_{6dB,CL2}}\right)^2 + 1^2 = \sqrt{2}$$

$$\Rightarrow \omega_{3dB,CL2} = \omega_{6dB,CL2} \times \sqrt{\sqrt{2}-1}$$

$$\Rightarrow f_{3dB,CL2} = f_{6dB,CL2} \times \sqrt{\sqrt{2}-1} = 58.5k$$

This solution takes the two cascaded amplifiers into account and estimate the $f_{3dB,CL2}$ accurately. It is more complicated to estimate, but is the exact solution.

Either solution of $f_{3dB,CL2}$ is acceptable.

(d) The maximum v_{in} that each amplifier can handle in order to operate at the maximum operating frequency in (c):

Non-inverting amplifiers:

$$\frac{dv_{out1}}{dt} < SR \Rightarrow 2\pi \times 10\text{kHz} \times v_{out1,pk} < \frac{0.5\text{V}}{\mu\text{s}} \Rightarrow v_{out1,pk} < 7.96\text{ V}$$

$$\Rightarrow v_{in} = v_{out1,pk}/100 < 79.6\text{ mV}$$

Solution 1 (2 cascaded inverting amplifiers):

$$\frac{dv_{out2}}{dt} < SR \Rightarrow 2\pi \times 90.9\text{kHz} \times v_{out2,pk} < \frac{0.5\text{V}}{\mu\text{s}} \Rightarrow v_{out2,pk} < 0.875\text{ V}$$

$$\Rightarrow \quad v_{in} = v_{out2,pk}/100 < 8.75\text{ mV}$$

Solution 2 (2 cascaded inverting amplifiers):

$$\frac{dv_{out2}}{dt} < SR \Rightarrow 2\pi \times 58.5\text{kHz} \times v_{out2,pk} < \frac{0.5\text{V}}{\mu\text{s}} \Rightarrow v_{out2,pk} < 1.36\text{ V}$$

$$\Rightarrow \quad v_{in} = v_{out2,pk}/100 < 13.6\text{ mV}$$

Due to the different $f_{3dB,CL2}$ in part (c), different maximum v_{in} can be obtained. Both solutions are acceptable.

S5. Figure S5 shows a circuit with two opamps, A and B, that are identical.

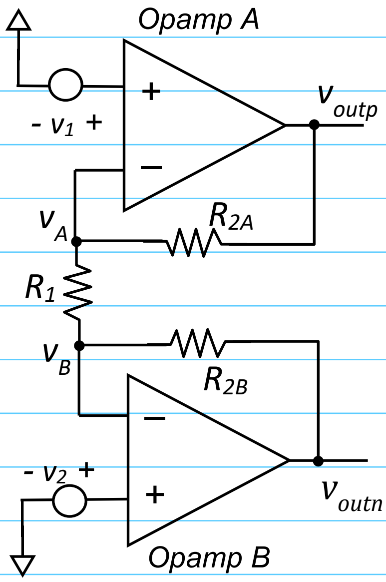


Fig. S5

(a) Determine v_A and v_B in terms of v_1 and v_2.

(b) Derive (v_{outp} - v_{outn}) in terms of (v_1 - v_2) for the circuit shown in Fig. S5. You may assume R_{2A} = R_{2B} = R_2. (**Hint: Consider R_{2A}, R_1 and R_{2B} as a voltage divider network.**)

(c) If $v_1 = v_{ic} + \frac{v_{id}}{2}$ and $v_2 = v_{ic} - \frac{v_{id}}{2}$, derive ($v_{outp}$ - v_{outn}) in terms of v_{ic} and v_{id}.

(d) What is (v_{outp} - v_{outn}) in terms of v_{ic} and v_{id}, if R_{2A} is 10% larger than R_{2B}, i.e., $R_{2A} = 1.1 \times R_{2B}$?

(e) The circuit shown in Fig. S5 functions similarly to an opamp circuit covered in lectures. Which is this circuit? (Hint: Specify whether the circuit is similar to inverting amplifier, buffer, filter, etc.)

Based on (c) and (d), comment on the desirable characteristics of the circuit shown in Fig. S5.

S5. **Solution:**

(a) Determine v_A and v_B in terms of v_1 and v_2.

$v_A = v_1$ and $v_B = v_2$ (applying virtual short to opamp A and opamp B)

(b) Derive $(v_{outp} - v_{outn})$ in terms of $(v_1 - v_2)$ for the circuit shown in Fig. S5. You may assume $R_{2A} = R_{2B} = R_2$. (Hints: Consider R_{2A}, R_1 and R_{2B} as a voltage divider network.)

Because there is no input current to the opamp, we can derive voltage across R_1 using the voltage divider network comprising R_{2A}, R_1 and R_{2B}.

$$
\begin{aligned}
v_A - v_B &= v_1 - v_2 \\
&= (v_{outp} - v_{outn}) \times \frac{R_1}{R_1 + R_{2A} + R_{2B}} \\
&= (v_{outp} - v_{outn}) \times \frac{R_1}{R_1 + 2R_2} \\
\Rightarrow v_{outp} - v_{outn} &= \left(1 + \frac{2R_2}{R_1}\right) \times (v_1 - v_2)
\end{aligned}
$$

(c) If $v_1 = v_{ic} + \frac{v_{id}}{2}$ and $v_2 = v_{ic} - \frac{v_{id}}{2}$, derive $(v_{outp} - v_{outn})$ in terms of v_{ic} and v_{id}.

$$v_{outp} - v_{outn} = \left(1 + \frac{2R_2}{R_1}\right) \times \left(v_{ic} + \frac{v_{id}}{2} - v_{ic} + \frac{v_{id}}{2}\right) = \left(1 + \frac{2R_2}{R_1}\right) \times v_{id}$$

Or

$$
\begin{aligned}
v_{outp} - v_{outn} &= \left(1 + \frac{R_{2A} + R_{2B}}{R_1}\right) \times \left(v_{ic} + \frac{v_{id}}{2} - v_{ic} + \frac{v_{id}}{2}\right) \\
&= \left(1 + \frac{R_{2A} + R_{2B}}{R_1}\right) \times v_{id}
\end{aligned}
$$

(d) What is $(v_{outp} - v_{outn})$ in terms of v_{ic} and v_{id} if R_{2A} is 10% larger than R_{2B}, i.e., $R_{2A} = 1.1 \times R_{2B}$?

$$v_{outp} - v_{outn} = \left(1 + \frac{2.1 \times R_{2B}}{R_1}\right) \times \left(v_{ic} + \frac{v_{id}}{2} - v_{ic} + \frac{v_{id}}{2}\right) = \left(1 + \frac{2.1 \times R_{2B}}{R_1}\right) \times v_{id}$$

(e) Based on (c) and (d), comment on the desirable characteristics of this circuit, and its similarity with which opamp circuit function that we have covered in the lecture. (Hint: Specify whether it is similar to inverting amplifier, buffer, filter or etc.)

It is similar to the instrumentation amplifier.

From (c), it rejects the common mode signal (v_{ic}) and only amplify the differential signal (v_{id}). This is good for rejecting common mode noise and interference.

From (d), the rejection of common mode signal (v_{ic}) is insensitive to component mismatch in R_2, making it better than the instrumentation amplifier covered in the lecture which depends on the matching of R_2.

S6. Figure S6 shows a first-order band-pass (BP) filter, where both opamps have a GBW of 10 MHz and slew rate of 10 V/µs. The BP filter is obtained by connecting a first-order high-pass (HP) filter and a first-order low-pass (LP) filter in series. The transfer function (based on ideal opamp analysis) and 3-dB frequency of the HP and LP filters are given as follows:

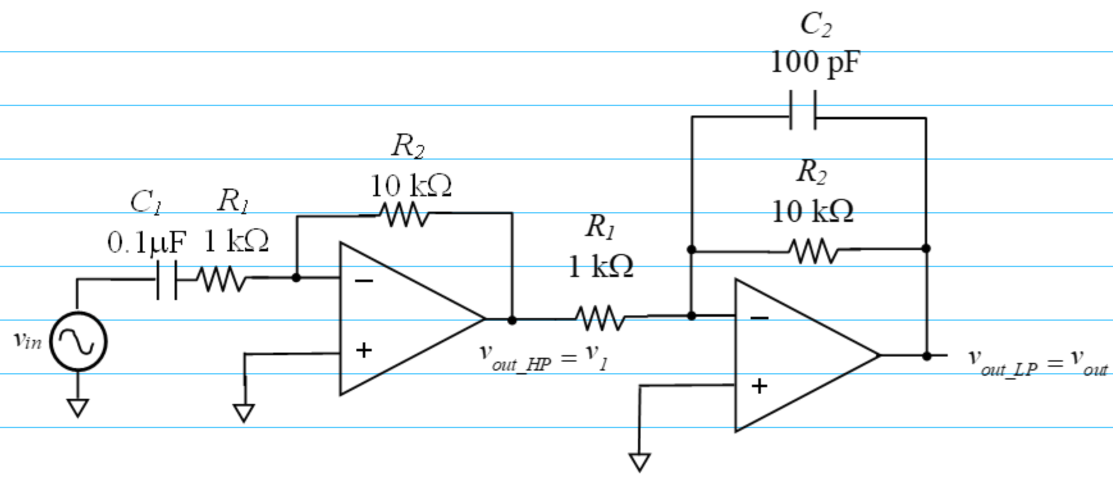


Fig. S6

$$\frac{v_{out_{HP}}}{v_{in}} = -\frac{R_2}{R_1} \times \frac{j\omega R_1 C_1}{j\omega R_1 C_1 + 1}$$

$$\frac{v_{out\_LP}}{v_1} = -\frac{R_2}{R_1} \times \frac{1}{j\omega R_2 C_2 + 1}$$

$$f_{3dB\_LP} = \frac{1}{2\pi R_2 C_2} \quad \text{and} \quad f_{3dB\_HP} = \frac{1}{2\pi R_1 C_1}$$

(a) Based on ideal opamp analysis, determine the lower 3-dB cutoff frequency (f_{low_BP}) and upper (higher) 3-dB cutoff frequency (f_{high_BP}) of the BP filter.

(b) At mid-band frequencies f (i.e., $f_{low\_BP} << f << f_{high\_BP}$), the capacitor C_1 can be considered as a short circuit while the capacitor C_2 can be considered as an open circuit and both LP and HP filters reduce to an inverting amplifier configuration with a gain of $-R_2/R_1$. Owing to limited GBW of the opamps, the gain of the inverting amplifier will decrease at high frequencies. Determine the $f_{3dB,CL}$ (i.e., closed loop bandwidth) of the inverting amplifier.

Will the BP filter work properly (i.e., according to the 3-dB cutoff frequencies determined in part (a)), based on GBW consideration only? Explain the reason for this.

(c) If $v_{in} = 0.03sin(2\pi f_{in}t)$ V, determine the maximum allowable f_{in} without distortion in v_{out}, based on slew rate consideration.

If the frequency of the input signal is equal to the upper 3-dB cutoff frequency (f_{high_BP}) of the BP filter, will the output signal be distorted, based on slew rate consideration only? Explain the reason for this.

(d) Both opamps in the BP filter are replaced by another type of opamp with a reduced GBW of 1 MHz, but with the slew rate unchanged. Will the upper 3-dB cutoff frequency of the BP filter, determined in part (a), be affected? If so, what is the upper 3-dB cutoff frequency of the BP filter for this situation? Explain.
Assume a single-stage amplifier (and not two cascaded amplifiers) in your analysis for simplification.

S6. Solution:

(a) $f_{low\_BP} = f_{3dB\_HP} = \frac{1}{2\pi R_1 C_1} = \frac{1}{2\pi \times 1k \times 0.1\mu} = 1.59 \text{ kHz}$

$f_{high\_BP} = f_{3dB\_LP} = \frac{1}{2\pi R_2 C_2} = \frac{1}{2\pi \times 10k \times 100p} = 159 \text{ kHz}$

(b) $f_{3dB,CL} = GBW \times \frac{R_1}{R_1+R_2} = 10\text{MHz} \times \frac{1}{11} = 909 \text{ kHz}$

Since the passband of the BP filter is **between 15.9 Hz and 159 kHz**, which is less than $f_{3dB,CL}$, the **BP filter will work properly**.

(c) Amplified by 100 times (i.e., $(R_2/R_1)^2$), the output signal of the BP filter, v_{out}, will have a peak amplitude of 3 V.

$$\frac{dv_{out}}{dt} = 3 \times 2\pi f_{in} cos(2\pi f_{in} t) \leq SR = 10 \text{ V/µs}$$
$$\Rightarrow 3 \times 2\pi f_{in} \leq 10 \text{ V/µs} \Rightarrow f_{in} \leq 530 \text{ kHz}$$

Hence, the maximum input frequency $f_{in(max)}$ is 530 kHz based on slew rate consideration.

Since the input signal frequency is equal to the upper 3-dB cutoff (f_{high_BP}) of 159 kHz, which is less than $f_{in(max)}$, the output **will not be distorted**.

(d) With a GBW of 1 MHz, the closed loop bandwidth of the inverting amplifiers will change to:

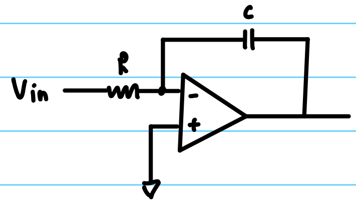
$$f_{3dB,CL} = GBW \times \frac{R_1}{R_1+R_2} = 1\text{MHz} \times \frac{1}{11} = 90.9 \text{ kHz}$$

Since $f_{3dB,CL} < f_{high\_BP}$, the upper (higher) 3dB cutoff frequency will no longer be at 159 kHz (i.e., f_{high_BP} is no longer determined by R_2 and C_2).

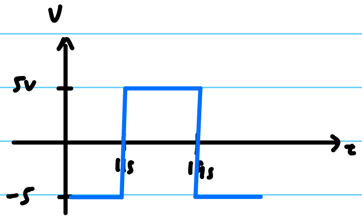
Instead, f_{high_BP} of the BP filter will be determined by the GBW and will be at 90.9 kHz.

New f_{high_BP} = 90.9 kHz

1a) Integrator → area under curve of input signal

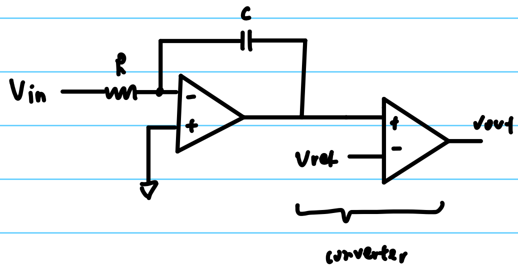


b)



powered by ±5V voltage source

c)



Vref will determine the duty cycle. output will either be high or low depending on Vref

a) $\cos A \cos B = \dfrac{\cos(A+B)}{2} + \dfrac{\cos(A-B)}{2}$

$$\cos(wt) \times \cos(100wt) = \frac{\cos(wt+100wt)}{2} + \frac{\cos(wt-100wt)}{2}$$

$$= \frac{1}{2}\cos(101wt) + \frac{1}{2}\cos(-99wt)$$

$$= \frac{1}{2}\cos(101wt) + \frac{1}{2}\cos(99wt)$$

b) $\cos(wt) \times \cos^2(100wt)$

$$= \cos(wt) \times (\cos(100wt)\cos(100wt))$$

$$= \cos(wt) \times \left(\frac{1}{2}\cos(200wt) + \frac{1}{2}\cos(0)\right)$$

$$= \cos(wt) \times \left(\frac{1}{2}\cos(200wt) + \frac{1}{2}\right)$$

$$= \cos(wt) \times \frac{1}{2}\cos(200wt) + \frac{1}{2}\cos(wt)$$

$$= \frac{1}{2}\left(\cos(201wt) + \cos(199wt)\right) + \frac{1}{2}\cos(wt)$$

$w = 2\pi f$

$$= \frac{1}{2}\left(\frac{\cos(201wt)}{2} + \frac{\cos(199wt)}{2}\right) + \frac{\cos(wt)}{2}$$

$$= \frac{\cos(201wt)}{4} + \frac{\cos(199wt)}{4} + \frac{\cos(wt)}{2}$$

$\therefore$ To extract $\cos(wt)$, we can use low-pass filter with w_{3dB} lower than $100w$

c) first-order low pass filter design / sallen-key low pass

3a) inverting amplifier gain: -200, $-\frac{R_2}{R_1} : \frac{200}{1}$

$V_{out(os)} = (1 + \frac{200}{1}) \times 20m$ $\quad$ $V_{in} = -200 \times V_{in}$

$= 4.02V$ $\quad$ $= -200V_{in}$

Maximum allowable

$-5 < V_{out} < 5$

$-5 < 4.02V - 200V_{in} < 5$

$-9.02 < -200\ V_{in} < 0.98$

$-4.9mV < V_{in} < 45mV$

$\therefore V_{in,pk} \simeq 5mA$

b) For gain of -200, R_2 can be $200k\Omega$ and R_1 can be $1k\Omega$

c) $f_{3db,CL} = GBW \times \frac{R_1}{R_1 + R_2}$

$= 1M \times \frac{1k}{200k + 1k}$

$= 4975.1$

$\simeq 5kHz$

d) $V_{out} = (1 + \frac{200}{1}) \times \sqrt{(100n)^2 \times 5kHz \times \frac{\pi}{2}}$

$= 1.78mV$

$V_{in} = V_{out} \div 200$

$= 8.9nV$

e) If noise contribution from R_1 and R_2 is considered, total noise output will be larger, hence the minimum allowable input voltage will have to be large.

4a) $V_{out1}/v_{in} = \left(1+\frac{R_2}{R_1}\right)$ → non-inverting

$V_{out2}/v_{in} = \left(-\frac{R_3}{R_1}\right)\times\left(-\frac{R_3}{R_1}\right)$ → two-inverting

$= \left(\frac{R_3}{R_1}\right)^2$

b) If identical gain,

$\left(\frac{R_3}{R_1}\right)^2 = \left(1+\frac{R_2}{R_1}\right)$

$\left(\frac{R_3}{R_1}\right)^2 = \left(1+\frac{R_2}{R_1}\right) = 100$

$\left(\frac{R_3}{1k}\right) = \sqrt{100}$ $\quad\quad$ $1+\frac{R_2}{1k} = 100$

$R_2 = 99 \times 1k$

$= 99k\Omega$ //

$R_3 = 10 \times 1k$

$= 10k\Omega$ //

c) Amplifier for $S4_a$: $GBW \times \frac{R_1}{R_1+R_2}$

$= 1M \times \frac{1k}{1k+99k}$

$= 10kHz$ //

Amplifier for $S4_b$: $GBW \times \frac{R_1}{R_1+R_3}$

$= 1M \times \frac{1k}{1k+10k}$

$= 90.9kHz$ //

d) $\frac{dV_{out1}}{d_{in}} = 2\pi f \times V_{out,pk} < 0.5V/\mu s$

$V_{out,pk} < 7.96V$

$\text{max } V_{in} = \frac{V_{out,pk}}{100}$

$= 79.6mV$ //

$\frac{dV_{out2}}{d_{in}} = 2\pi f \times V_{out,pk} < 0.5V/\mu s$

$V_{out,pk} < 0.875V$

$\text{max } V_{in} = \frac{V_{out,pk}}{100}$

$= 8.75mV$ //

5a) Based on virtual short,

$V_a = V_1 \qquad V_B = V_2$

* Question states, consider as voltage divider network

b) $V_1 = V_a = \dfrac{R_1}{R_1 + R_{2A} + R_{2B}} \qquad V_2 = V_B = \dfrac{R_1}{R_1 + R_{2A} + R_{2B}}$

$$V_a - V_B = V_1 - V_2 = \frac{R_1}{R_1 + R_{2A} + R_{2B}} \times V_{outp} - \frac{R_1}{R_1 + R_{2A} + R_{2B}} \times V_{outn}$$

$$V_1 - V_2 = \frac{R_1}{R_1 + 2R_2} (V_{outp} - V_{outn})$$

$$(V_{outp} - V_{outn}) = (V_1 - V_2) \times \left(1 + \frac{2R_2}{R_1}\right) \quad //$$

c) $V_1 = V_{ic} + \dfrac{V_{id}}{2} \qquad V_2 = V_{ic} - \dfrac{V_{id}}{2}$

$$(V_{outp} - V_{outn}) = \left(\left(V_{ic} + \frac{V_{id}}{2}\right) - \left(V_{ic} - \frac{V_{id}}{2}\right)\right) \times \left(1 + \frac{2R_2}{R_1}\right)$$

$$= V_{id}\left(1 + \frac{2R_2}{R_1}\right)$$

d) $$V_{id}\left(1 + \frac{2R_2}{R_1}\right) = V_{id}\left(1 + \frac{R_{2A} + R_{2B}}{R_1}\right)$$

<u>If $R_{2A} = 1.1 R_{2B}$,</u>

$$V_{id}\left(1 + \frac{1.1R_{2B} + R_{2B}}{R_1}\right)$$

$$= V_{id}\left(1 + \frac{2.1R_{2B}}{R_1}\right) \quad //$$

e) Instrumentation amplifier. Since it removes V_{ic}.

- removes common mode signal
- only amplifies differential signal

6a) $f_{low-BP} = \dfrac{1}{2\pi R_2 C_2}$

$= \dfrac{1}{2\pi (10k)(100p)}$

$= 159kHz$

$f_{3dB-HP} = \dfrac{1}{2\pi R_1 C_1}$

$= \dfrac{1}{2\pi (1k)(0.1\mu)}$

$= 1.59kHz$

b) GBW: 10MHz

$f_{3dB,CL} = 10MHz \times \dfrac{1k}{1k+10k}$

$\approx 909kHz$

Yes. Since the passband is between 1.59kHz and 159kHz which is lesser than the $f_{3dB,CL}$, it will work properly

c) $V_{in} = 0.03\sin(2\pi f_{in} t)$

$V_{out} = 0.03\sin(2\pi f_{in} t) \times (10 \times 10)$

$= 3\sin(2\pi f_{in} t)$

$\dfrac{dV_{out}}{dt} = 2\pi f_{in} \times V_{out} < 10V/\mu s$

$= 2\pi f_{in} \times 3 < 10000000$

$f_{in} < 531kHz$

No. If the frequency of the input signal is equal to the upper 3db cut-off (159kHz), the output will not be distorted as the input frequency is lower than $f_{in(max)} = 531kHz$

d) <u>If GBW changed to 1MHz</u>

$$f_{3dB,CL} = 1M \times \frac{R_1}{R_1+R_2}$$
$$= 1M \times \frac{1k}{1k+10k}$$
$$= 90.9kHz$$

Yes, the upper 3db cut-off frequency will be affected as it is higher than the $f_{3db,CL}$ of 90.9kHz.
Upper cut off-frequency will now be 90.9kHz.