NATIONAL UNIVERSITY OF SINGAPORE
Department of Electrical and Computer Engineering

EE2027 Electronic Circuits
Supplementary Questions 1: Solution

These are supplementary questions to Tutorial 1, and they aim to provide more work examples. They will not be discussed in class, but solutions will be provided after Tutorial discussion.

S1. By applying Thevenin equivalent or Norton equivalent to the circuit shown in Fig. S1, transform it in a **sequence** (meaning in multiple transformations) to a single loop equivalent circuit to determine the current flowing through R_5. Show the sequence of transformation in your answer.

Hint: You may start the transformation with V_A in series with R_S.

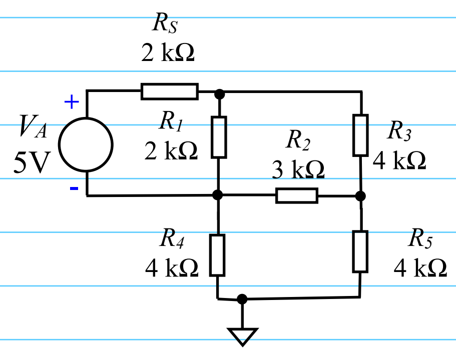
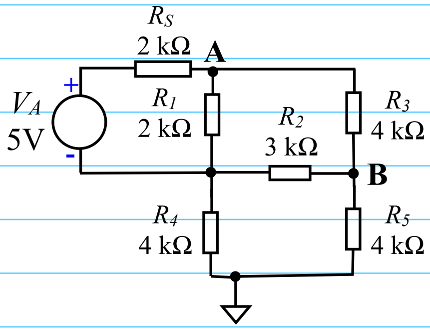


Fig. S1

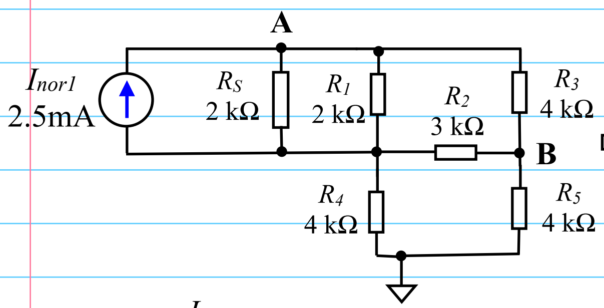
[Ans: 0.095 mA]

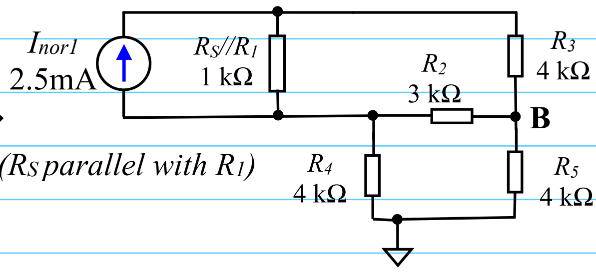
S1. **Solution:**

Note: As the question specifies that multiple transformations must be used, we cannot transform the entire circuit excluding R_5 in a single step using either Thevenin or Norton equivalent.



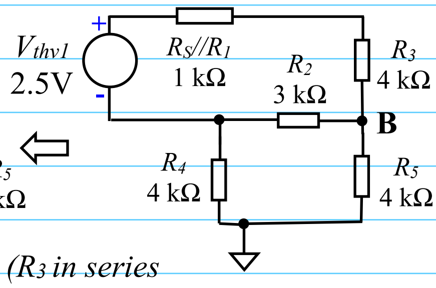
(Convert V_A in series with R_s into the Norton equivalent)



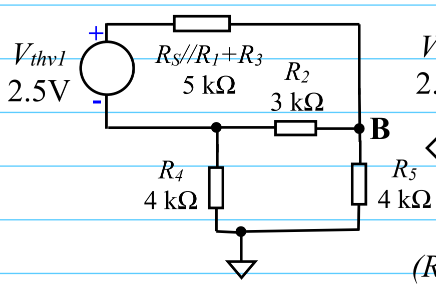


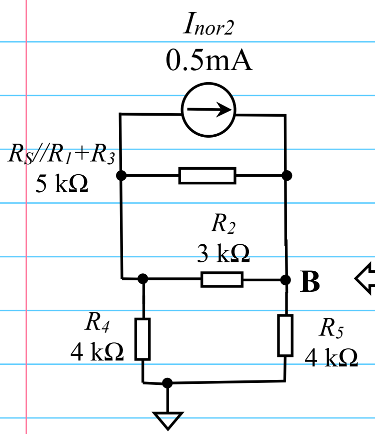
(R_S parallel with R_1)

(Convert I_{nor1} in parallel with $R_S//R_1$ into the Thevenin equivalent)



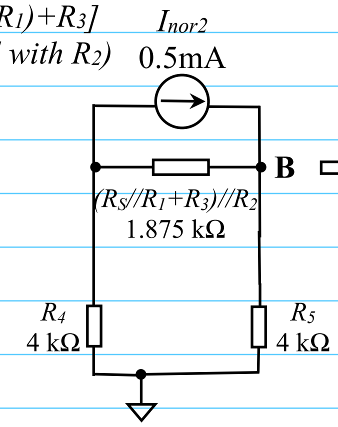
(R_3 in series with $R_S//R_1$)

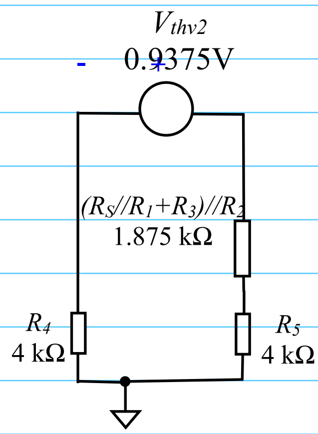




(Convert V_{thv1} in series with $(R_s//R_1 + R_s)$ into the Norton equivalent)

($[(R_S//R_1)+R_3]$ parallel with R_2)





$$I_{R5} = \frac{0.9375}{4k + 4k + 1.875k}\ \mathrm{A}$$
$$= 0.095\ \mathrm{mA}$$

(Convert I_{nor2} in parallel with $(R_s//R_1+R_s)//R_1$ into the Thevenin equivalent)

S2. (a) Using mesh analysis with Phasor notation, calculate the current $i(t)$ for the circuit in Fig. S2.

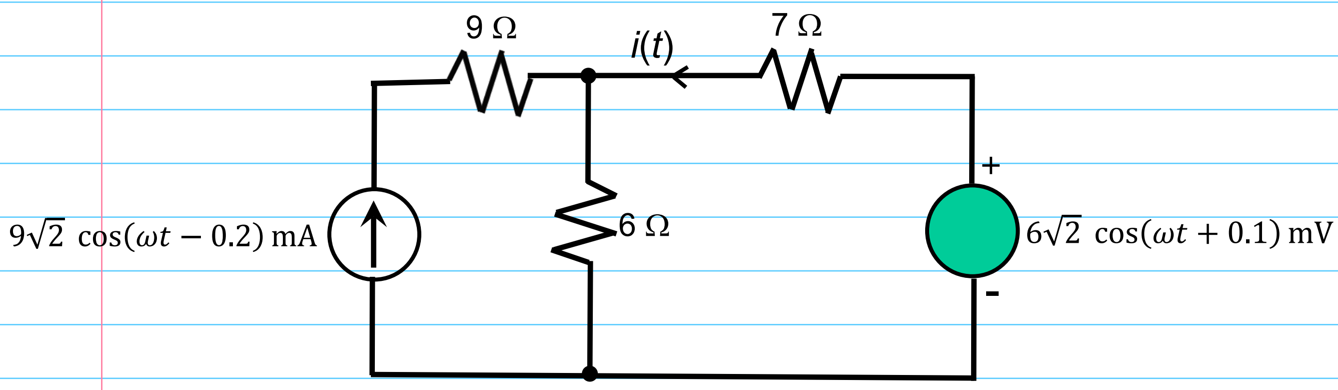


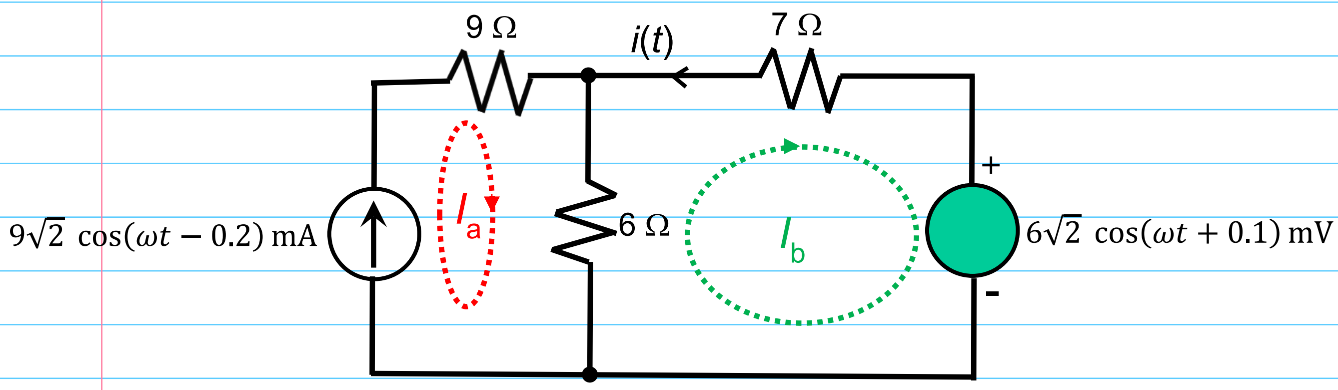
Fig. S2

(b) Calculate the average power dissipated in the 6 Ω resistor in Fig. Q5. Refer to the "self-reading" slides 43 to 45 in the lecture notes on "Review of Basic Concepts" for the calculation of average power in ac circuits.

[Ans: 168 μW]

S2. **Solution:**

(a)



Assign phasor loop currents, I_a and I_b, where

$$I_a = 9e^{-j0.2} \qquad (1)$$

Note: I_a and I_b are in mA.

Write the KVL equation for the right-hand mesh/loop:

$$6(I_a - I_b) - 7I_b - 6e^{j0.1} = 0$$

$$6I_a - 13I_b = 6e^{j0.1} \qquad (2)$$

Substituting Eq. (1) into Eq. (2):

$$13I_b = 54e^{-j0.2} - 6e^{j0.1}$$

$$13I_b = 54[\cos(-0.2) + j\sin(-0.2)] - 6[\cos(0.1) + j\sin(0.1)]$$

$$13I_b = (52.9236 - j10.7281) - (5.9700 + j0.5990) = 46.9536 - j11.3271$$

$$I_b = 3.6118 - j\,0.8713 \text{ mA}$$

$$I_b = \sqrt{(3.6118)^2 + (0.8713)^2}\ e^{j\tan^{-1}\left(\frac{-0.8713}{3.6118}\right)} = 3.7154e^{-j0.241} \text{ mA}$$

Note: The phase angle should be in the fourth quadrant of the complex plane as the real part is positive and the imaginary part is negative.

Since $i(t) = -i_b(t)$, ∴ $i(t) = -3.72\sqrt{2}\cos(\omega t - 0.241)$ mA

Alternatively,

$$i(t) = 3.72\sqrt{2}\cos(\omega t - 0.241 \pm \pi)\text{ mA}$$

$\therefore \quad i(t) = 3.72\sqrt{2}\cos(\omega t + 2.901)\text{ mA}$

(or) $\quad i(t) = 3.72\sqrt{2}\cos(\omega t - 3.383)\text{ mA}$

<u>Note</u>: All the above answers for $i(t)$ are acceptable, with a minus sign or remove the minus sign by adding or subtracting a phase change of π radians.

(b) Net current flowing downwards through the 6 Ω resistor:

$$I_a - I_b = 9e^{-j0.2} - (3.6118 - j\,0.8713)$$
$$= 8.8206 - j1.7880 - 3.6118 + j0.8713 = (5.2088 - j0.9167)\text{ mA}$$

Average power $= [(5.2088\times10^{-3})^2 + (-0.9167\times10^{-3})^2]\times 6$ W

$= [27.13+0.84]\times 6\ \mu\text{W} = 168\ \mu\text{W}$

S3. For the circuit shown in Fig. S3, assume diodes D1 and D2 have the following I-V characteristics:

$$I_{D1} = I_S e^{V_{D1}/V_T} \text{ and } I_{D2} = 10 \times I_S e^{V_{D2}/V_T}$$

(a) Under what biasing conditions are the above equations valid?

(b) Estimate V_{OUT}.

(c) Discuss the potential application of this circuit.

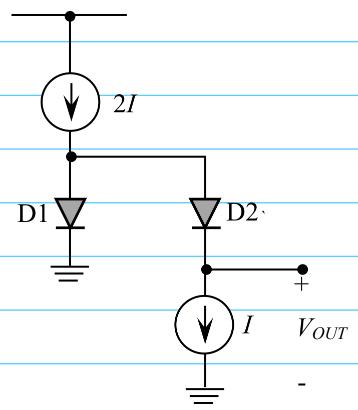


Fig. S3

S3. **Solution:**

(a) For $I_{D1} = I_S e^{V_{D1}/V_T}$ and $I_{D2} = 10 \times I_S e^{V_{D2}/V_T}$ to be valid, diodes D1 and D2 must be in forward biased with V_{D1} and $V_{D2} >> V_T$.

(b) Current flowing through D2 is I, and through D1 is $2I - I = I$ (by KCL). Both currents are in forward biased direction.

- For Diode D2: $I_{D2} = 10 \times I_S e^{V_{D2}/V_T} = I \quad \Rightarrow \quad V_{D2} = V_T \ln\left(\frac{I}{10 I_S}\right)$
- For Diode D1: $I_{D1} = I_S e^{V_{D1}/V_T} = I \quad \Rightarrow \quad V_{D1} = V_T \ln\left(\frac{I}{I_S}\right)$

Hence, by means of Kirchhoff Voltage Law (KVL):

$$V_{OUT} = V_{D1} - V_{D2} = V_T \ln\left(\frac{I}{I_S}\right) - V_T \ln\left(\frac{I}{10 I_S}\right) = \frac{kT}{q} \ln(10)$$

(c) V_{OUT} is proportional to temperature (T), hence it can be used as a temperature sensor.

S4. Figure S4 shows a pn-junction diode circuit where S1 is a switch. In the circuit, the current source I_1 provides a current of 2 mA, the voltage source V_2 provides a voltage of 10 V, $R_1 = 1$ kΩ, $R_2 = 1.5$ kΩ, and $R_3 = 2$ kΩ. The diode has a reverse saturation current, $I_S = 1.24\times10^{-15}$ A; an exponential factor, $n = 1$; and a breakdown voltage, $V_Z = 5$ V. The temperature is 300 K.

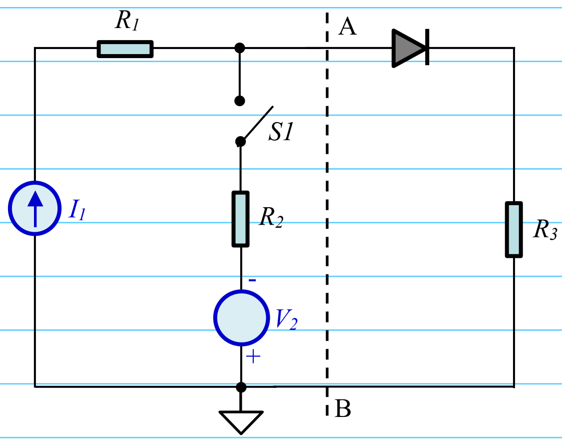


Fig. S4

(a) With S1 opened, determine the diode voltage (including the polarity). Briefly explain your calculation.

[Ans: 0.703 V]

(b) With S1 closed, determine the diode voltage (including the polarity) and current (including the direction). Briefly explain your calculations.

Estimate also the corresponding small-signal resistance. Briefly explain your answer.

Hint: You may want to first determine the Thevenin equivalent of the sub-circuit to the left of the line A-B.

[Ans: 5 V, 0.57 mA]

S4. **Solution:**

(a)

With S1 opened, the current flowing through the diode is I_I = 2 mA, which is substantial (>> $I_S = 1.24 \times 10^{-15}$ A), from the p-type side to n-type side. Diode operates in forward bias, beyond cut-in voltage.

$$I_D \approx I_S e^{V_D/(nV_T)} = 1.24 \text{ x } 10^{-15} e^{V_D/0.025} = 2 \text{ x } 10^{-3}$$

$$\therefore \; V_D = 0.703 \text{ V}$$

(Higher at the p-type side w.r.t. the n-type side)

(b)

Determine first the Thevenin equivalent of the sub-circuit to the left of the line A-B -

$$V_{THV} = V_{OC,AB} = -V_2 + I_1 \times R_2 = -10 + 2\text{m} \times 1.5\text{k} = -7 \text{ V}$$

$$R_{THV} = R_2 = 1.5 \text{ k}\Omega$$

V_{THV} reverse biases the diode, as its negative terminal is connected to the p-type side of the diode. Since $|V_{THV}| > V_Z$ (= 5 V) $\Rightarrow$ diode is operating in the breakdown region.

$|V_D| = V_Z = 5$ V (higher at the n-type side w.r.t. the p-type side)

$$|I_D| = \frac{|V_{THV}| - V_Z}{R_{THV} + R_3} = \frac{7-5}{1.5\text{k}+2\text{k}} = 0.57 \text{ mA}$$

(flowing from n-type side to p-type side)

Small-signal resistance = ratio of small change in voltage to small change in current ≈ 0 (As the magnitude of I_D increases rapidly with marginal increase in the magnitude of V_D in breakdown operation)

N.B.: Small-signal resistance of *pn*-junction diode, $r_d = \frac{nV_T}{I_D}$, is applicable only in forward bias and cannot be used here.

S5. A *pn*-junction diode circuit is shown in Fig. S5(a) and the forward biased large-signal model of the diode is shown in Fig. S5(b). In the circuit,

$$v_s(t) = 5\text{Cos}(\omega t + \theta) \text{ mV},$$

is a small-signal ac voltage source. The *pn*-junction diode has an exponential factor, $n = 1.2$, and the temperature is 300 K.

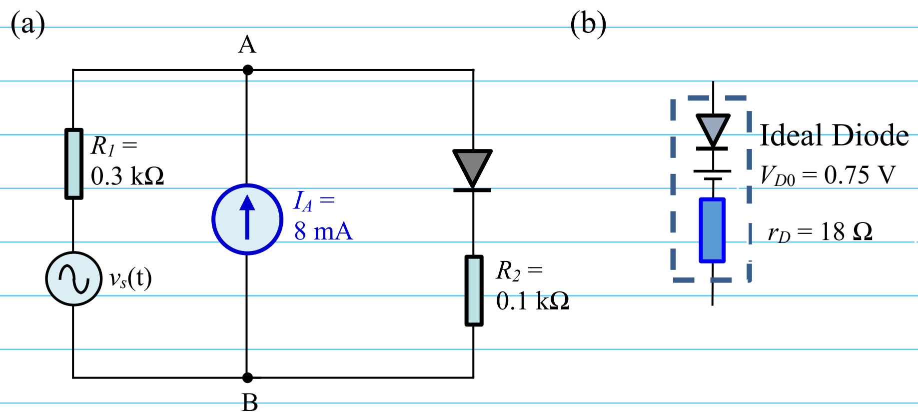


Fig. S5

(a) Perform the dc analysis of the circuit of Fig. S5(a) and show that the diode is in forward bias.

Hint: You may want to convert a current source in parallel with a resistance into its Thevenin equivalent.

(b) Determine the current and voltage of the diode at the dc operating point (i.e., I_D and V_D, respectively) in the circuit of Fig. S5(a).

[Ans: 3.95 mA, 0.82 V]

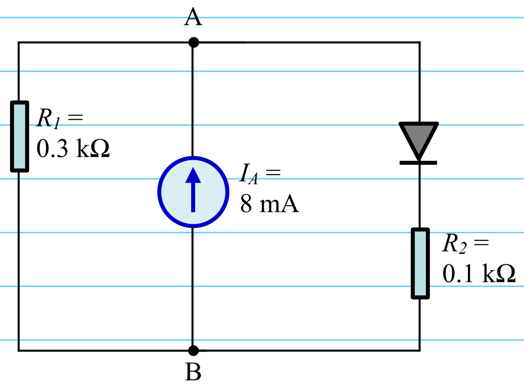
(c) Draw the ac equivalent circuit of Fig. S5(a) and perform the ac analysis. Hence, determine the small-signal current of the diode, $i_d(t)$, and its rms (root mean square) value.

[Ans: $i_{d,rms} = 8.7\ \mu$A]

S5. **Solution:**

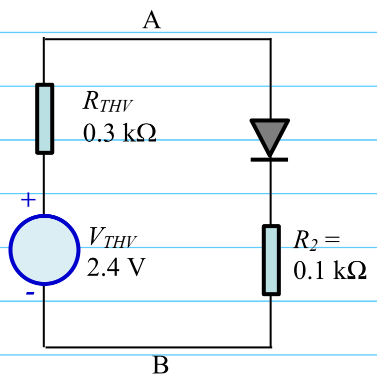
(a)

In dc operation, consider the circuit in the absence of the small-signal ac voltage source (i.e., ‘kill’ the ac voltage source by short-circuiting it), and the dc equivalent circuit is as follows -



Convert the current source, I_A, in parallel with R_1 to the Thevenin equivalent -

- $V_{THV} = I_A \times R_1 = 2.4$ V
- $R_{THV} = R_1 = 0.3$ kΩ



It can be seen that polarity of V_{THV} is such that it forward biases the diode, with its positive terminal connected to the p-type side of diode.

Alternatively, using the DC equivalent circuit, students may argue that since the current source I_A is the only source in the circuit, by applying KCL at node A, it can be concluded that the current flowing through the diode is from p-type to n-type side, i.e., in the forward biased direction. Hence, diode is in forward bias.

(b)

We need to find the dc current and voltage of the diode, i.e., I_D and V_D.

Since the diode is in forward bias and $V_{THV} >> V_{DO} = 0.75$ V, we can replace the diode by its forward biased large-signal model.

By means of KVL,

$$I_D \approx \frac{V_{THV} - V_{D0}}{R_{THV} + R_2 + r_D} = \frac{2.4 - 0.75}{300 + 100 + 18} = 3.95 \text{ mA}$$

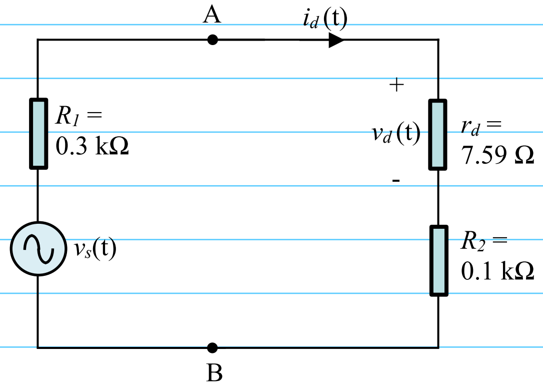
$$V_D \approx V_{D0} + I_D \times r_D = 0.75 + 3.95\text{m} \times 18 = 0.82 \text{ V}$$

(c)

In small-signal ac operation, dc <u>current source</u> is replaced by an <u>open-circuit</u> and the diode can be modelled by a resistor of value -

$$r_d = \frac{nV_T}{I_D} = \frac{1.2 \times 0.025}{3.95\text{m}} = 7.59\ \Omega$$

The ac equivalent circuit -



By means of KVL, small-signal current of diode -

$$i_d(t) = \frac{v_s(t)}{R_1 + R_2 + r_d} = \frac{5\text{Cos}(\omega t + \theta)\text{ mV}}{300 + 100 + 7.59} = 12.3\text{Cos}(\omega t + \theta)\ \mu\text{A}$$

The rms value of $i_d(t) = \frac{12.3}{\sqrt{2}} = 8.7\ \mu\text{A}$

S6. The sub-circuit within the dashed box in Fig. S6 is used to power a diode, D_1. The diode has a cut-in voltage of ~0.5 V, and a breakdown voltage of 6 V. When the diode operates in substantial forward bias, it can be modeled using the constant-voltage-drop model with $V_{DO} = 0.72$ V.

(a) Convert the sub-circuit within the dashed box into its Thevenin equivalent, i.e., to determine its Thevenin voltage, V_{THV}, and Thevenin resistance, R_{THV}.

[Ans: 10.5 V, 6.5 kΩ]

(b) With the help of part (a) or otherwise, determine the current flowing through the diode (including the magnitude and direction). Briefly explain your calculation.

[Ans: 1.505 mA]

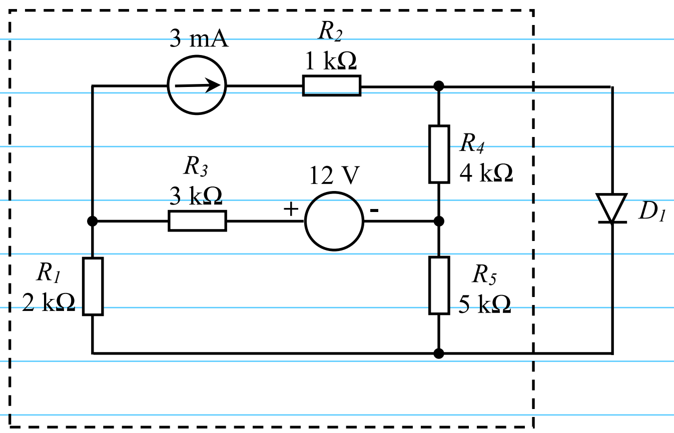
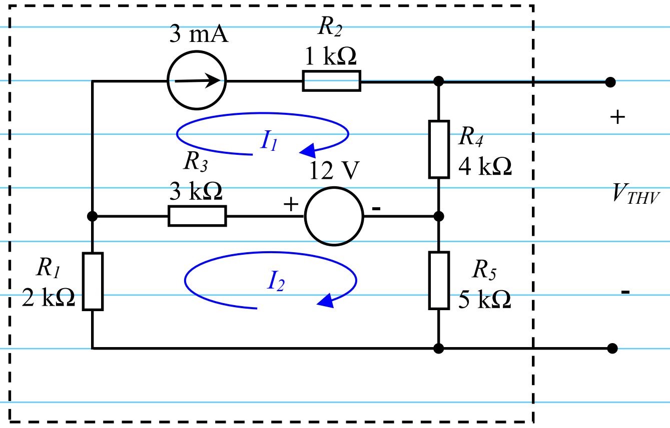


Fig. S6

S6. **Solution:**

(a) The sub-circuit within the dashed box is a single-port linear network. Hence, it can be replaced by its Thevenin equivalent.



From the top loop, $I_1 = 3$ mA

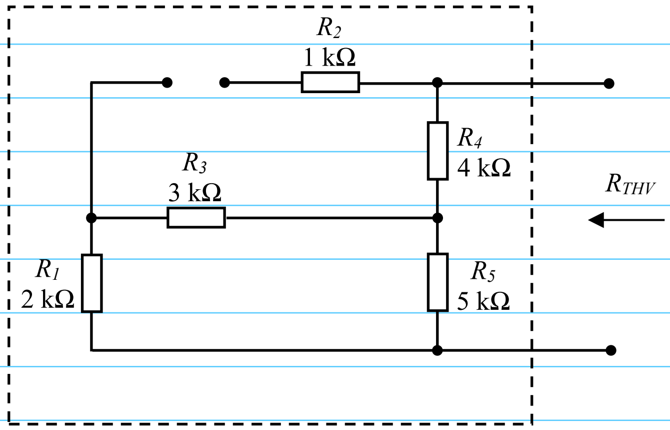
Apply KVL to the bottom loop,

$$12 + I_2R_5 + I_2R_1 + (I_2 - I_1)R_3 = 0$$

$$I_2 = -0.3 \text{ mA}$$

$$V_{THV} = I_1R_4 + I_2R_5 = 10.5 \text{ V}$$

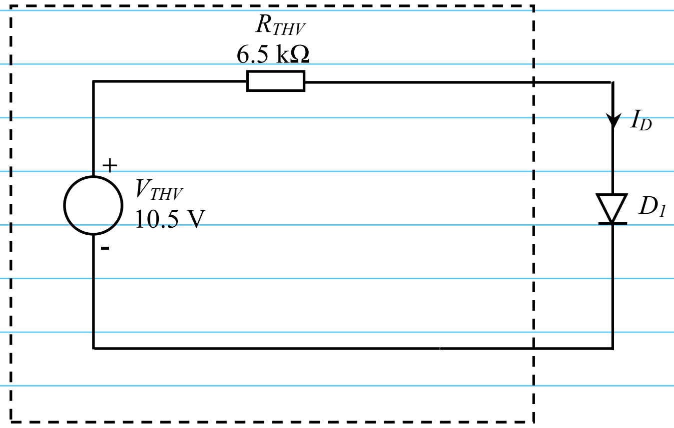
To find R_{THV}, kill current source by open circuiting it, and kill voltage source by short circuiting it –



$$R_{THV} = R_4 + R_5//(R_1 + R_3)$$

$$R_{THV} = 6.5 \text{ k}\Omega$$

(b) Replace the sub-circuit within the dashed box by its Thevenin equivalent –



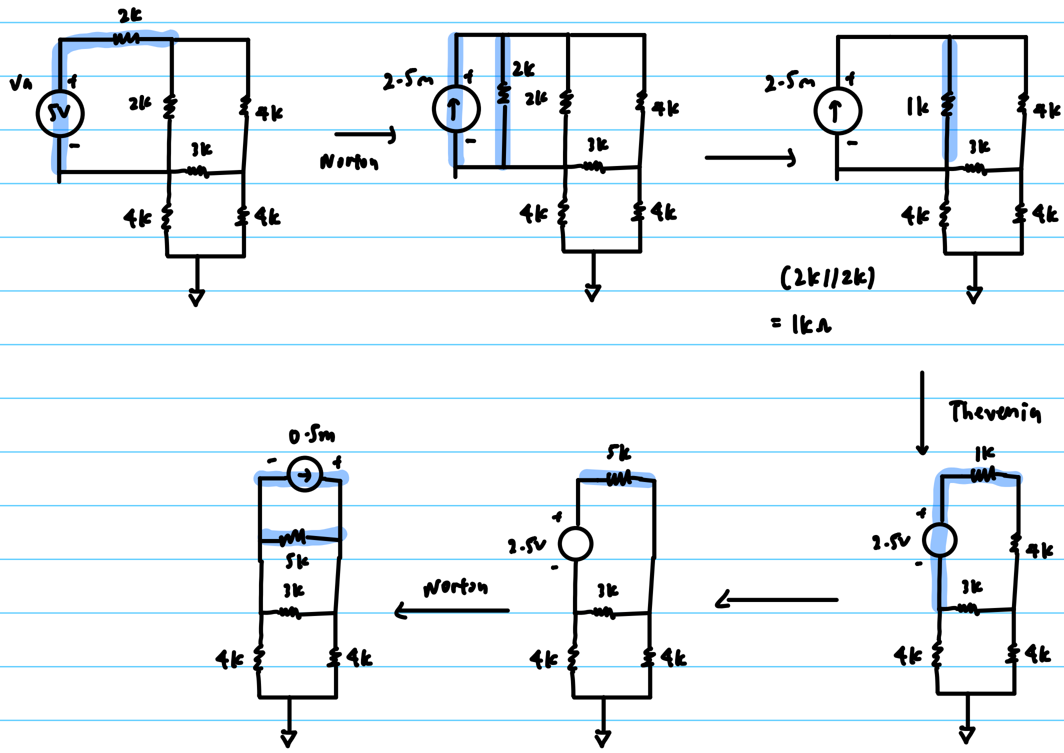
Diode is in **forward bias** as the positive terminal of V_{THV} is connected to the p-type side of diode.
As V_{THV} >> cut-in voltage of ~0.5 V, diode is in **substantial forward bias**.

Hence, diode can be modelled by the constant-voltage-drop model with V_{DO} = 0.72 V. Apply KVL,

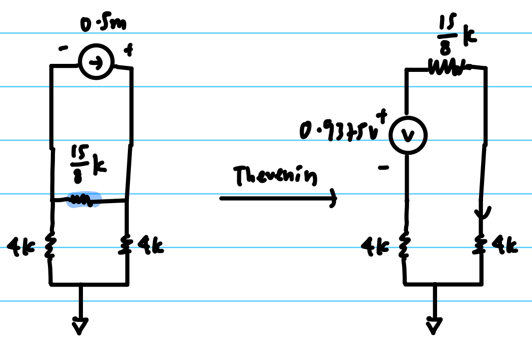
$$I_D = \frac{V_{THV} - V_{DO}}{R_{THV}} = \frac{10.5 - 0.72}{6.5\text{k}} = 1.505 \text{ mA}$$

I_D flows from the p-type to n-type side through the diode, as indicated in the above figure.

1)



(2k//2k)
= 1kΩ

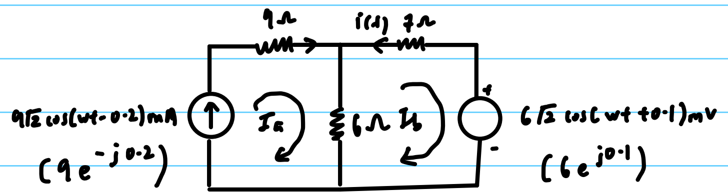


(3k//5k)
$= \frac{15}{8}$ kΩ

→ current flowing through $R_{4k} = \dfrac{0.9375}{\frac{15}{8}k + 4k + 4k}$

$= 0.095\,mA$

2a) Using mesh analysis,



Loop I_a : $I_a = 9e^{-j0.2}$

Loop I_b : $I_b: -6e^{j0.1} - 7I_b - 6(I_b - I_a) = 0$

$$-6\angle 0.1 - 7I_b - 6I_b + 6I_a = 0$$

$$-6\angle 0.1 + 6(9\angle -0.2) = 13I_b$$

$$I_b = 3.611 - 0.871j$$

$$I_b = 3.715\angle -0.237$$

$I(t) = -I_b(t) = -3.715\sqrt{2}\cos(wt - 0.237)$ mA

(opposite direction)

b) $\boxed{\text{Average power} = V \times I}$ → For DC element, like resistor, AC will use $\frac{1}{2}VI\cos\theta$

$= (I \times R) \times I$

$= I^2 R$

Current flowing in toward 6Ω resistor, using KCL

$I_a = I_{6\Omega} + I_B$

$I_{6\Omega} = I_a - I_B$

$= 9e^{-j0.2} - (3.715e^{-j0.237})$

$= (9\angle -0.2) - (3.715\angle -0.237)$

$= 5.209 - 0.9157j$ mA

$((5.209m)^2 + (-0.9157m)^2) \times 6\Omega$

$= 167.83\,\mu W$

$\approx 168\,\mu W$

3) $I_{D1} = I_s e^{\frac{V_{D1}}{V_T}}$ $\qquad$ $I_{D2} = 10 \times I_s e^{\frac{V_{D2}}{V_T}}$

a) Forward bias with V_{D1} and $V_{D2} \gg V_T$

b) Using KCL at node A,

$2I = I_{D1} + I_{D2}$

$2I = I_{D1} + I$

$I_{D1} = 2I - I$

$= I$

∴ You want to estimate the value of Vout, hence, you need to find the estimated values of V_{D1} and V_{D2} using the eqn above.

$\therefore\ I_{D2} = 10 \times I_s e^{V_{D2}/V_T}$

$I = 10 \times I_s e^{V_{D2}/V_T}$

$\frac{I}{10 I_s} = e^{V_{D2}/V_T}$

$\ln\left(\frac{I}{10 I_s}\right) = \frac{V_{D2}}{V_T}$

$V_{D2} = V_T \ln\left(\frac{I}{10 I_s}\right)$

$I_{D1} = I_s e^{V_{D1}/V_T}$

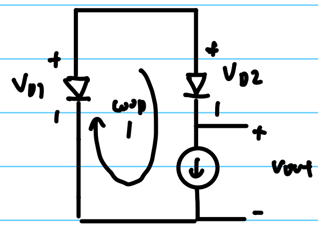
$I = I_s e^{V_{D1}/V_T}$

$\frac{I}{I_s} = e^{V_{D1}/V_T}$

$\ln\left(\frac{I}{I_s}\right) = \frac{V_{D1}}{V_T}$

$V_{D1} = V_T \ln\left(\frac{I}{I_s}\right)$

Using KVL



equivalent circuit

$V_{D1} - V_{D2} - V_{out} = 0$

$V_{out} = V_{D1} - V_{D2}$

$= V_T \ln\left(\frac{I}{I_s}\right) - V_T \ln\left(\frac{I}{10 I_s}\right)$

$= V_T \left(\ln\left(\frac{I}{I_s}\right) - \ln\left(\frac{I}{10 I_s}\right)\right)$

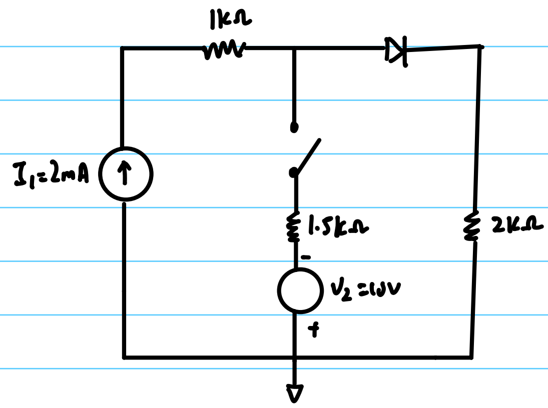
$= V_T \left(\ln\left(\frac{\frac{I}{I_s}}{\frac{I}{10 I_s}}\right)\right)$

$= V_T \ln(10)$

$\therefore\ V_{out} = \frac{kT}{q} \ln(10)$ //

c) Since Vout is proportional with temperature, it can be a temperature sensor.

4a) With S1 opened, R_2 and V_2 is open circuit.
Hence, since the diode is forward-biased, the current will pass through the diode.



$$I_D \approx I_S\left(e^{\frac{V_D}{nV_T}}\right)$$

$$2m = \left(1.24\times10^{-15}\, e^{\frac{V_D}{1(25m)}}\right)$$

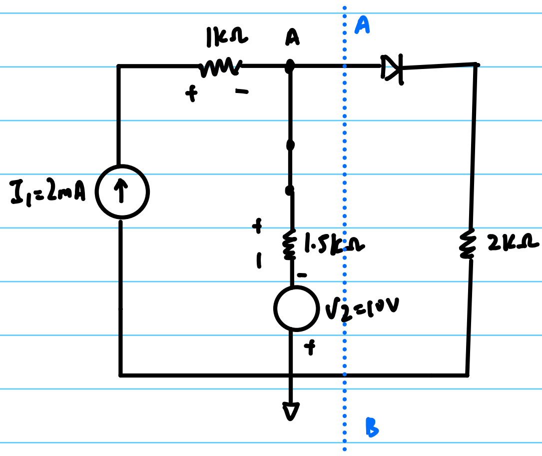
$$\ln\left(\frac{2m}{1.24\times10^{-15}}\right) = \frac{V_D}{25m}$$

$$V_D = \ln\left(\frac{2m}{1.24\times10^{-15}}\right)\times 25m$$

$$= 0.7027$$

$$\simeq 0.703V$$

b) Using thevenin, $V_{oc} = V_{ab}$



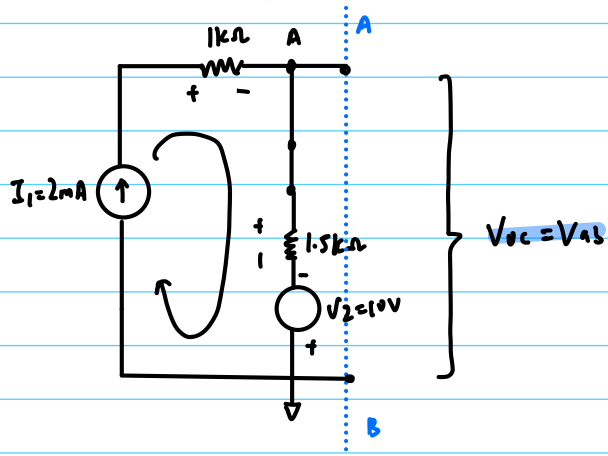
$$V_{oc} = V_{ab} = V_a = -10V + 1.5k(2m)$$

$$= -10 + 3$$

$$= -7V$$

Short voltage source, open current source,

$R_{TH} = 1.5k\Omega$

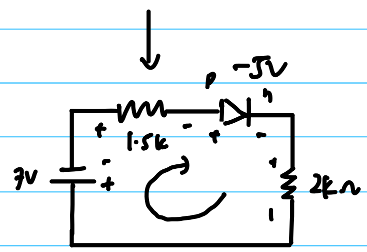


Since $V_{oc} = -7V$, $|V| = 7V > V_{breakdown}\ 5V$,
hence diode is in breakdown region and diode $V_Z = -5V$

$$\therefore\ -7 - 1.5kI - (-5) - 2kI = 0$$

$$I = \frac{-2}{1.5k+2k}$$

$$= -0.571mA$$



$\therefore$ current is 0.571mA from n-type to p-type

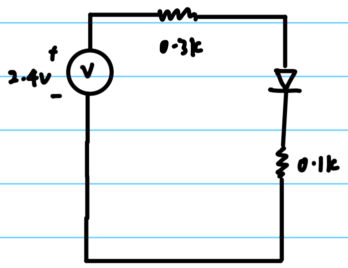
$V = IR$

small signal resistance: ratio of small change in voltage over small change in current. Since in break down region, a small change in voltage can result in a very large increase in current, $\frac{V \cdot \text{small}}{V \cdot \text{large}} \approx 0$.
Hence, small signal resistance ≈ 0.

5) $V_s(t) = 5\cos(\omega t - \theta)$ mV

a) In DC analysis, 'kill' ac source by short circuit, hence following circuit

Converting from Norton to Thevenin,



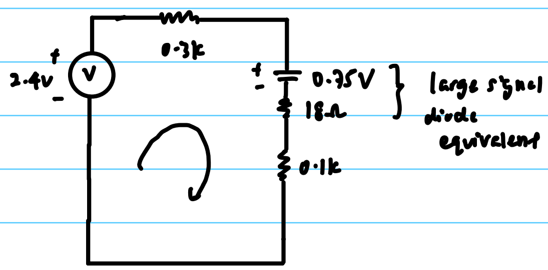
$V_{TH} = R_1 \times I_A$

$= 2.4V$

$R_{TH} = R_1 = 0.3k\Omega$

Based on the polarity of V_{TH}V, it forward biases the diode as the positive terminal is connected to p-type side of the battery

b) Since $V_{TH} >> V_{D0} = 0.75V$ and diode is in forward bias, we can replace diode with large signal model



$$I_D = \frac{2.4 - (0.75)}{0.3k + 0.1k + 18\Omega} = 3.95mA$$

$V = I_D \times r_D = (3.95m) \times (18\Omega)$

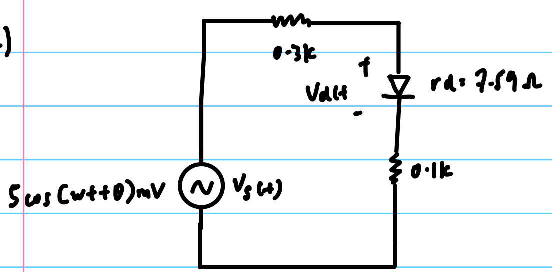
$= 0.0711V$

$V_D = V_{D0} + V$

$= 0.75V + 0.0711V$

$= 0.821V$

c)

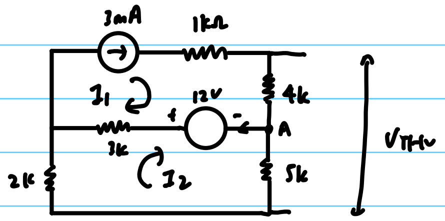


$$r_d = \frac{nV_T}{I_d} = \frac{(1.2)(25m)}{3.95m} = 7.59\Omega$$

$$i_d(t) = \frac{5\cos(\omega t + \theta)m}{0.3k + 0.1k + 7.59} = 12.3\cos(\omega t + \theta)\ \mu A$$

$$i_d(t)_{rms} = \frac{12.3\mu A}{\sqrt{2}} = 8.7\mu A$$

6)



<u>Short voltage, open current source</u>

$R_{TH} = ((2+3) \parallel 5) + 4$

$= 2.5k\Omega + 4k$

$= 6.5k\Omega$

Using KVL,

$I_1 = 3mA$

$-2kI_2 - 3k(I_2 - I_1) - 12 - 5kI_2 = 0$

$-2kI_2 - 3kI_2 + 3k(3m) - 12 - 5kI_2 = 0$

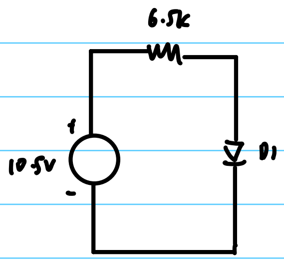
$-3 = 10k$

$I_2 = -0.3mA$ (opp)

$V_{TH} = (3 \times 4) + (-0.3m \times 5)$

$= 10.5V$

5)



Since $V_{DD} >> V_{cutin}$ (0.5V), diode is in forward bias.

Using constant voltage drop model, $V_{DO} = 0.72V$

$$I = \frac{10.5 - 0.72}{6.5k}$$

$= 1.505mA$

Current is 1.505mA from p-type to n-type