NATIONAL UNIVERSITY OF SINGAPORE
Department of Electrical and Computer Engineering

EE2027 Electronic Circuits
Supplementary Questions 4: Solution

- Unless otherwise stated, you may assume temperature, T = 300 K, thermal voltage, $V_T \approx$ 0.025 V.

These are supplementary questions to Tutorial 4, and they aim to provide more work examples. They will not be discussed in class, but solutions will be provided after Tutorial discussion.

Supplementary Questions (Will not be discussed in class)

S1. For the single-stage amplifier circuit shown in Fig. S1, the BJT Q$_1$ has the following parameters: $I_S = 10^{-14}$ A, $\beta = 100$ and $V_A = 50$ V.

(a) What is the configuration of the amplifier?

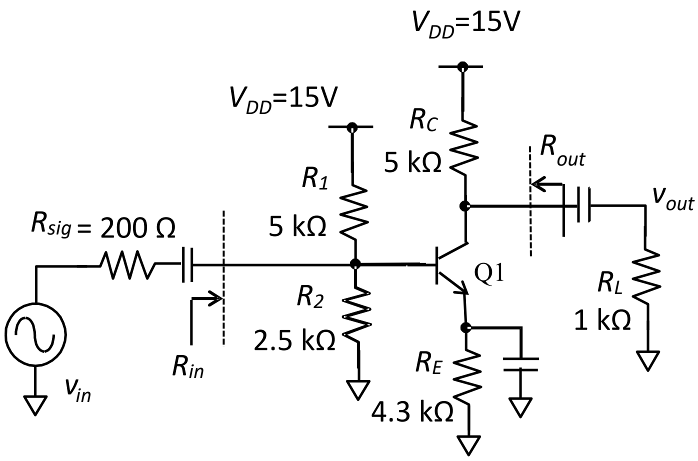
(b) Calculate the dc emitter, base and collector current of the BJT assuming the voltage divider method is applicable. Verify any assumption(s) made in your calculation.
[Ans: $I_C \approx I_E = 1$ mA; $I_B = 10$ µA]

(c) Calculate the parameters of the ac small-signal model of the transistor Q$_1$.
[Ans: $g_{m,Q1}$ = 40 mA/V; $r_{\pi,Q1}$ = 2.5 kΩ; $r_{o,Q1}$= 50 kΩ]

(d) Calculate the input resistance, R_{in} and output resistance, R_{out}.
[Ans: $R_{in} \approx$ 1 kΩ; $R_{out} \approx$ 4.55 kΩ]

(e) Draw the 2-port network equivalent for the amplifier shown in Fig. S1. Estimate the voltage gain v_{out} / v_{in}.
[Ans: $A_v \approx$ -27.32]

(f) Which two-port parameter(s) will change if R_C is changed to 9 kΩ? Calculate the new value(s). Is there any adverse impact on the amplifier due to this change? Explain with quantitative reasoning.



V_{DD}=15V
V_{DD}=15V
R_C
5 kΩ
R_{out}
v_{out}
R_1
5 kΩ
R_{sig} = 200 Ω
Q1
R_2
2.5 kΩ
R_L
1 kΩ
R_{in}
R_E
4.3 kΩ
v_{in}

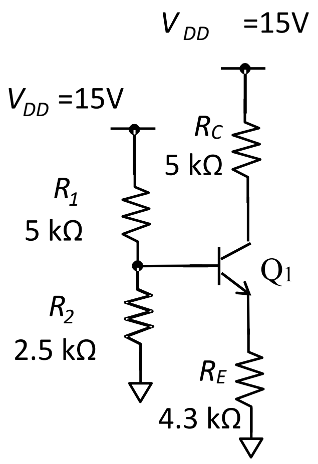
Fig. S1

S1. <u>Solution</u>:

BJT Q_1 is of the npn type.

(a) Common Emitter configuration.

(b) dc analysis:



Assume Q_1 in forward active mode, $V_{BE,Q1} \approx 0.7$ V.

$V_{B,Q1} = 15\times2.5/(5+2.5) = 5$ V

$V_{E,Q1} = V_{B,Q1} - V_{BE,Q1} \approx 5\text{ V} - 0.7\text{ V} = 4.3$ V

$I_{E,Q1} = V_E / R_E = 4.3 / 4.3\text{k} = 1$ mA

$I_{C,Q1} = [\beta / (\beta+1)]\, I_{E,Q1} = 100/101 \times 1\text{m} \approx 1$ mA

$I_{B,Q1} = I_{C,Q1} / \beta = 10\ \mu$A.

$V_{C,Q1} = 15 - 5\text{k}\times1\text{m} = 10$ V

Since $V_{C,Q1} > V_{B,Q1}$ and $V_{BE,Q1} \approx 0.7$ V, BJT Q_1 is indeed in forward active mode.
The current flowing through the voltage divider resistors, R_1 and R_2, ≈ 2 mA >> 10 μA.
Hence the assumptions are valid.

Note: Never use $I_C = I_S e^{V_{BE}/V_T} = I_S e^{0.7/V_T}$ to evaluate I_C as $V_{BE} \approx 0.7\,V$ is only an approximation, and the exponential relation will blow it up, giving a I_C that is not acceptable.

(c) Parameters of the ac small-signal model:

$g_{m,Q1} = I_C / V_T = 1\text{m} / 0.025 = 40$ mA/V
$r_{\pi,Q1} = \beta / g_{m,Q1} = 100 / 40\text{m} = 2.5\ \text{k}\Omega$
$r_{o,Q1} = V_A / I_C = 50\text{ V} / 1\text{m} = 50\ \text{k}\Omega$

(d) ac small signal analysis:

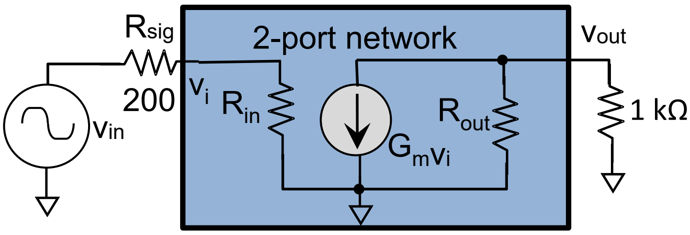
With the help of Table 1, Configuration A -

$R_{in} = R_1 \,//\, R_2 \,//\, r_{\pi,Q1} = 5 \,//\, 2.5 \,//\, 2.5\ \text{k}\Omega = 1\ \text{k}\Omega$

With the help of Table 1, Configuration B -

$R_{out} = R_C \,//\, r_{o,Q1} = 5\text{k} \,//\, 50\text{k} = 4.55\ \text{k}\Omega$

(e) 2-port network equivalent of amplifier:



$R_{in} = 1\ \text{k}\Omega$, $R_{out} = 4.55\ \text{k}\Omega$, $G_m = g_{m,Q1} = 40$ mA/V

Voltage gain $v_{out} / v_{in} = -\,(R_L \,//\, R_{out}) \times G_m \times [R_{in}/(R_{in} + R_{sig})]$

$= -\,(1 \,//\, 4.55)\text{k} \times 40\text{m} \times [1/\,(0.2 + 1\,)] = -0.82 \times 40 \times 0.833 = -27.32$

(f) As the increase in R_C does not affect the BJT DC currents in this case, small-signal parameters: $g_{m,Q1}$, $r_{\pi,Q1}$ and $r_{o,Q1}$ do not change. Hence, G_m and R_{in} do not change, and only R_{out} will increase -

$R_{out} = R_C \,//\, r_{o,Q1} = 9\text{k} \,//\, 50\text{k} = 7.63\ \text{k}\Omega$, which leads to an increase in gain.

However, with the increase in R_C to 9 kΩ, the dc collector voltage now becomes 6 V (= 15 - 1m x 9k) while the dc biasing collector-emitter voltage (V_{CE}) reduces to 1.7 V (= 15 - 1m x 9k – 1m x 4.3k). This will result in the negative output small-signal voltage (v_{ce}) swing being limited to less than 1.7 V (otherwise, the BJT will enter into the saturation region), when there is a positive ac voltage swing at the input (refer to BJT course notes, slide BJT-36). Hence, the positive ac voltage range at the input will be very limited. In fact, this will limit severely the overall input ac voltage range, if a sinusoidal ac signal is applied at the input, and if the output is not to be distorted.

S2. For the amplifier circuit shown in Fig. S2, the NMOS transistor M_1 has the following parameters: $K_n = 0.75$ mA/V^2, $\lambda = 0.01$ V^{-1} and $V_{TH} = 1$ V.

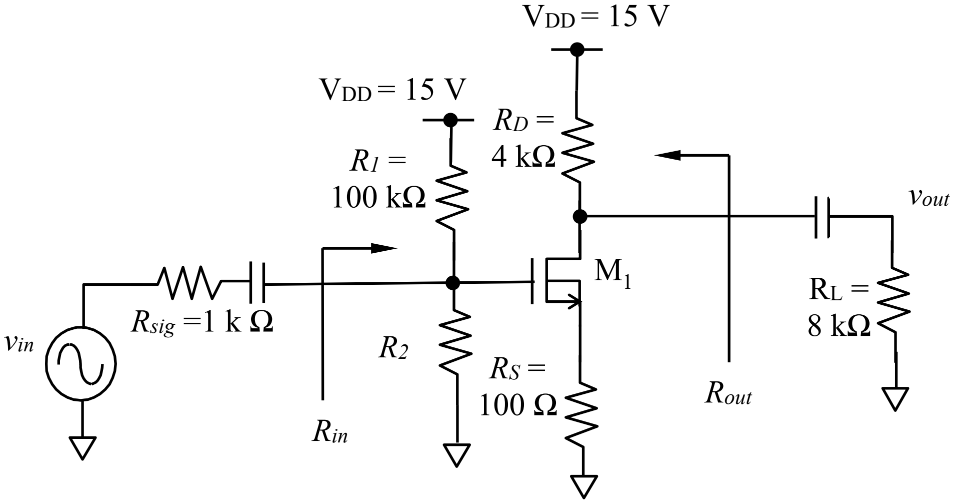


Fig. S2

(a) Calculate R_2 such that the DC drain current through M_1 is 1.5 mA. Verify that the MOSFET is operating in the saturation region.

[Ans: $R_2 = 20.6$ kΩ]

(b) Calculate the parameters, g_m and r_o, of the AC small signal model of the transistor M_1.

(c) What is the configuration of this amplifier?

(d) Calculate the input resistance, R_{in}. [Ans: $R_{in} \approx 17.1$ kΩ]

(e) Calculate the output resistance, R_{out}. [Ans: $R_{out} \approx 3.8$ kΩ]

(f) Estimate the voltage gain v_{out} / v_{in}. [Ans: $v_{out} / v_{in} \approx -4.26$]

S2. <u>Solution</u>:

(a)

$I_{D,M1} = 1.5\,\text{mA}$, assume working in saturation region.

$I_{D,M1} = K_n (V_{GS} - V_{THN})^2$; $1.5\text{mA} = 0.75\text{mAV}^{-2}(V_{GS} - 1)^2$

$\Rightarrow V_{GS} = 1 \pm 1.414 = -0.414$ (invalid as $V_{GS} < V_{TH}$) or $2.414V$

$$V_S = I_S R_s = 1.5\text{m} \times 0.1\text{k} = 0.15\,\text{V} \Rightarrow V_G = 2.564\,\text{V}$$

$$V_G = \frac{15R_2}{R_1 + R_2} \Rightarrow R_2 = 20.6\,\text{k}\Omega$$

$$V_{DS} = 15 - I_{D,M1}R_D - 0.15 = 15 - 1.5\text{m} \times 4\text{k} - 0.15 = 8.85\text{ V}$$

$$V_{DSsat} = V_{GS} - V_{TH} = 2.414 - 1 = 1.414\text{ V}$$

$V_{DS} > V_{DSsat}$, hence in saturation.

(b) $$g_m = 2\sqrt{K_n I_{D,M1}} = 2\sqrt{0.75\text{m} \times 1.5\text{m}} = 2.12\text{ mA/V}$$

$$r_o = \frac{1}{\lambda I_{D,M1}} = \frac{1}{0.01 \times 1.5\text{m}} = 66.7\,\text{k}\Omega$$

(c) The configuration of this amplifier is CS with degeneration.

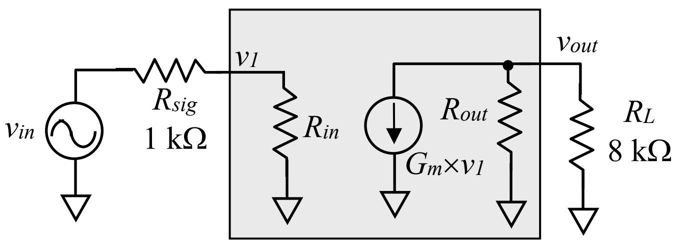
(d) With the help of Table 2, Configuration A –

$$R_{in} = R_1 // R_2 = 100k // 20.6k = 17.1k\Omega$$

(e) With the help of Table 2, Configuration B –

$$R_{out} = R_D // r_{o,M1}\{1 + g_{m,M1}R_S\} = 4\text{k} // 66.7\text{k}(1 + 2.12\text{m} \times 0.1\text{k}) = 3.8\,\text{k}\Omega$$

(a) Estimate the voltage gain v_{out} / v_{in}.



With the help of Table 4, configuration D –

$$A_v = \frac{v_{out}}{v_{in}} = \frac{R_{in}}{R_{sig}+R_{in}} \times (-G_m) \times (\mathrm{R_{out}//R_L}) = -\frac{R_{in}}{R_{sig}+R_{in}}\left\{\frac{g_m}{1+g_m R_S}\right\}(\mathrm{R_{out}//R_L})$$

$$\approx -\frac{17.1k}{1k+17.1k}\frac{2.12m}{1+2.12m\times 0.1k}(3.8k//8k) = -4.26$$

S3. For the single-stage npn BJT amplifier circuit shown in Fig. S3, the BJT Q1 has the following parameters: $I_S = 10^{-15}$ A, $\beta = 100$ and $V_A = 100$ V.

(a) What is the amplifier configuration?

(b) Calculate the biasing DC collector current of BJT Q1, $I_{C,Q1}$.

[Ans: $I_{C,Q1} \approx 0.4$ mA]

(c) Calculate the AC small-signal model parameters of BJT Q1.

(d) Calculate the two-port network parameters, G_m, R_{in} and R_{out} of the amplifier.

[Ans: $G_m = -16$ mA/V, $R_{in} \approx 62.5\ \Omega$, $R_{out} \approx 5$ kΩ]

(e) Estimate the voltage gain (v_{out}/v_{in}) of the amplifier.

[Ans: $v_{out}/v_{in} \approx 29.9$]

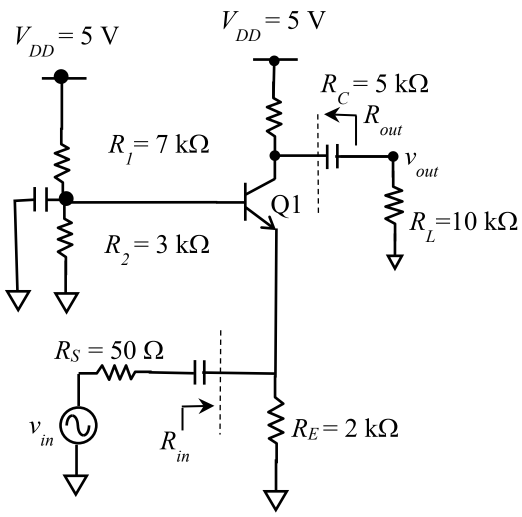


Fig. S3

S3. <u>Solution</u>:

(a) Common Base configuration

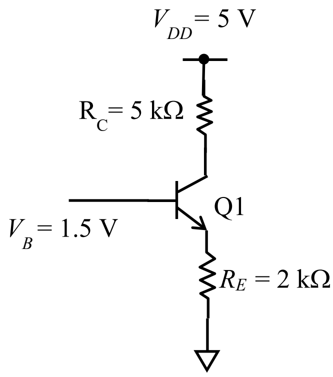
(b) Assume BJT Q1 is in forward active mode, $V_{BE,Q1} \approx 0.7$ V.

Assume voltage divider applies -

$V_{B,Q1} = (3/(3+7)) \times 5 = 1.5\text{V}$
$V_{E,Q1} = 1.5\text{ V} - 0.7\text{ V} = 0.8\text{ V}$

$I_{E,Q1} = V_{E,Q1} / R_E = 0.8 / 2\text{k} = 0.4\text{ mA}$

$I_{C,Q1} = \beta_{Q1} / (\beta_{Q1} + 1)\, I_{E,Q1} = 0.4\text{m} \times 100/101 \approx 0.4\text{ mA}$



Check voltage divider assumption –
$I_{B,Q1} = I_{C,Q1}/\beta_{Q1} = 4\ \mu\text{A}$; $I_{R1}, I_{R2} \approx V_{DD}/(R_1 + R_2) = 0.5\text{ mA} >> I_{B,Q1} \Rightarrow$ valid

Check forward active operation –
$V_{C,Q1} = V_{DD} - I_{C,Q1} \times R_C \approx 3\text{ V}$
$V_{BC,Q1} = V_{B,Q1} - V_{C,Q1} = -1.5\text{ V} < 0$ and $V_{BE,Q1} \approx 0.7\text{ V} > 0 \Rightarrow$ valid

(c) $g_{m,Q1} = I_C / V_T = 0.4\text{m} / 0.025 = 16\text{ mA/V}$

$r_{\pi,Q1} = \beta / g_m = 100 / 16\text{m} = 6.25\text{ k}\Omega$

$r_{o,Q1} = V_A / I_C = 100\text{ V} / 0.4\text{m} = 250\text{ k}\Omega$

(d) With the help of Table 3, Configuration B -
$G_m = -g_{m,Q1} = -16\text{ mA/V}$.

With the help of Table 1, Configuration D -

$$R_{in} = \frac{1}{g_m}\left[\frac{r_{o,Q1} + R_C // R_L}{r_{o,Q1} + \dfrac{R_C // R_L}{\beta}}\right] // R_E = \frac{1}{16m}\left[\frac{250k + 5k//10k}{250k + \dfrac{5k//10k}{100}}\right] // 2k = 62.5\Omega$$

With the help of Table 1, Configuration B –

$$R_{out} = r_{o,Q1}\{1 + g_{m,Q1}[(r_{\pi,Q1})//(R_E//R_s)]\}// R_C$$
$$\approx r_{o,Q1}\{1 + g_{m,Q1}R_s\}// R_C \approx R_C = 5\text{ k}\Omega$$

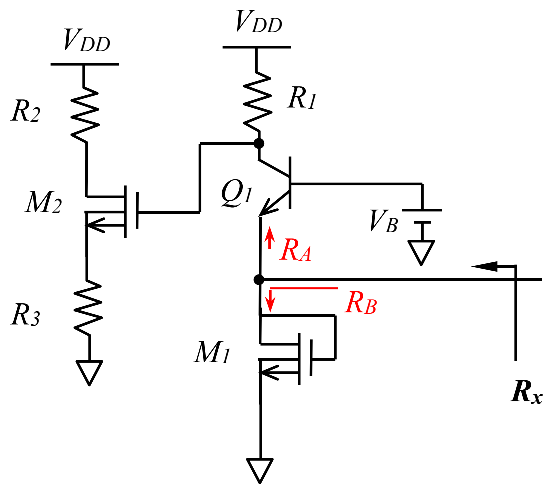
(e) $$v_i = \left(\frac{R_{in}}{R_s + R_{in}}\right) v_{in} = \frac{62.5}{50+62.5} v_{in} = 0.56 v_{in}$$

$$v_{out} = -G_m \times (R_{out}//R_L) \times v_i = g_{m,Q1} \times (R_{out}//R_L) \times v_i$$

$$\frac{v_{out}}{v_{in}} = 16\text{m} \times \frac{10\text{k}\times 5\text{k}}{10\text{k}+5\text{k}} \times 0.56 = 29.9$$

S4. Assume that the AC small signal parameters of the BJT are $g_{m,Q1}$, $r_{\pi,Q1}$, $r_{o,Q1}$, β_{Q1}, and the AC small signal parameters of the MOSFETs are $g_{m,Mi}$, $g_{mb,Mi}$, $r_{o,Mi}$, where $i = 1, 2$. Write down the expression for the small signal AC equivalent resistance (R_x) of each of the following circuit configurations. V_{DD}, V_B and V_G are DC voltage sources. I is a DC current source.

(a)



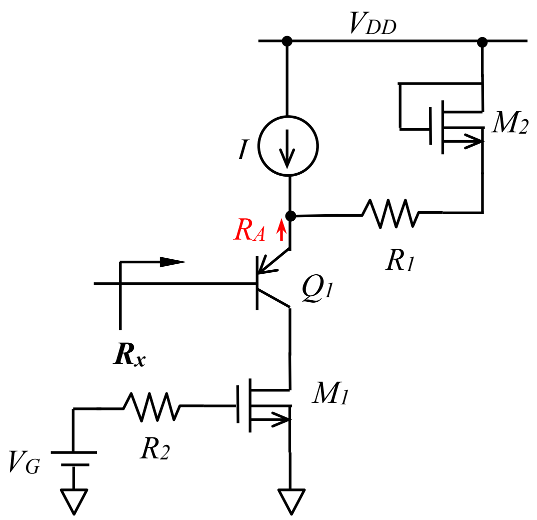
Solution:

$R_x = R_A // R_B$

- $R_A = \frac{1}{g_{m,Q1}} \times \frac{r_{o,Q1}+R_1}{r_{o,Q1}+R_1/\beta_{Q1}}$ (With the help of Table 1, configuration D)–
- $R_B = \frac{1}{g_{m,M1}}$ (With the help of Table 2, configuration E)

$$R_x = R_A // R_B = \left(\frac{1}{g_{m,Q1}} \times \frac{r_{o,Q1}+R_1}{r_{o,Q1}+R_1//[g_{m,Q1} r_{\pi,Q1}]}\right) // \left(\frac{1}{g_{m,M1}}\right)$$

(b)



Solution:

$R_x = r_{\pi,Q1} + (1+\beta_{Q1}) R_A$ (With With the help of Table 1, Configuration A)

- $R_A = R_1 + \frac{1}{g_{m,M2}}$ (With the help of Table 2, Configuration E)

$$R_x = r_{\pi,Q1} + (1+\beta_{Q1}) \times \left(R_1 + \frac{1}{g_{m,M2}}\right) = r_{\pi,Q1} + (1 + r_{\pi,Q1} g_{m,Q1}) \times \left(R_1 + \frac{1}{g_{m,M2}}\right)$$

S5. An amplifier circuit using an n-MOSFET is shown in Fig. S5. The n-MOSFET has the following parameters: K_n = 1 mA/V^2 and λ = 0.02 V^{-1}. v_{in} is a small signal AC voltage source.

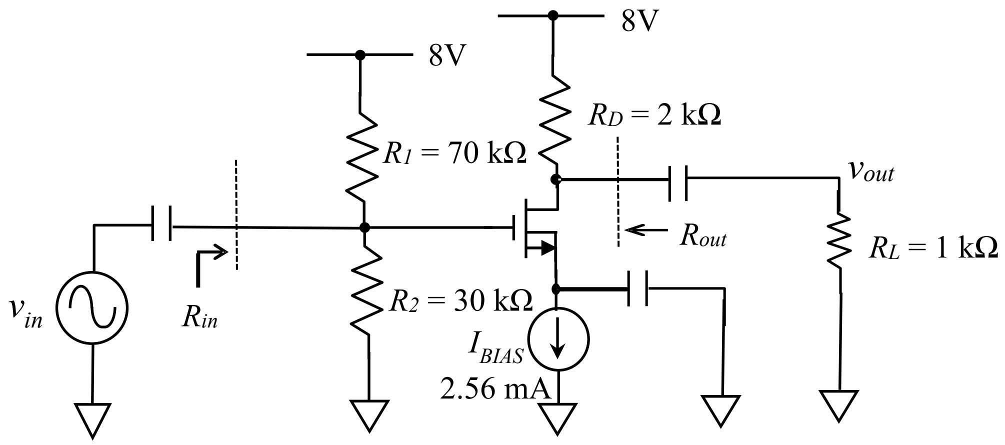


Fig. S5

(a) Calculate the DC gate voltage, V_G. [Ans: V_G = 2.4 V]

(b) Assuming that the n-MOSFET is operating in the saturation region and neglecting Channel Length Modulation, calculate the threshold voltage, V_{THN}, given that the voltage drop across the dc current source, I_{BIAS}, has been designed to be approximately 0 V. Verify that the n-MOSFET is operating in the saturation region. [Ans: V_{THN} = 0.8 V]

(c) Find the values of small-signal parameters g_m and r_o of the n-MOSFET.

(d) What is the configuration of the amplifier circuit?

(e) Draw the small-signal ac equivalent circuit of Fig. Q6. You may assume that in the small signal ac equivalent circuit, all the coupling capacitors behave like short circuits.

(f) Calculate the input resistance, R_{in}, and output resistance, R_{out}.
[Ans: $R_{in} \approx$ 21 kΩ, $R_{out} \approx$ 1.81 kΩ]

(g) Calculate the voltage gain, v_{out} / v_{in}.
[Ans: $v_{out} / v_{in} \approx$ -2.06]

S5. <u>Solution</u>:

(a)

$V_G = R_2 / (R_1 + R_2) \times 8 \text{ V} = 2.4 \text{ V}$

This is because the gate current $I_G = 0$.

(b)

$V_{GS} = V_G = 2.4 \text{ V}$ (since voltage drop across the dc current source is 0 V)

$I_D = K_n(V_{GS} - V_{THN})^2 = 1\text{m} \times (2.4 - V_{THN})^2 = 2.56 \text{ mA}$

$\Rightarrow (2.4 - V_{THN}) = \pm\sqrt{2.56}$

$\therefore \; V_{THN} = 0.8 \text{ V, or } 4 \text{ V}$

$V_{THN} = 4$ V is not valid as the n-MOSFET will be operating in cut-off since $V_{GS} < V_{THN}$
Thus, $V_{THN} = 0.8$ V.

$V_{GS} = 2.4 \text{ V} > V_{THN}$

$V_{DSsat} = (V_{GS} - V_{THN}) = 2.4 - 0.8 = 1.6 \text{ V}$

$V_{DS} = 8 - I_D R_D = 8 - (2.56\text{m} \times 2\text{k}) = 2.88 \text{ V} > V_{DSsat}$

Hence, the nMOSFET is operating in the saturation region.

(c)

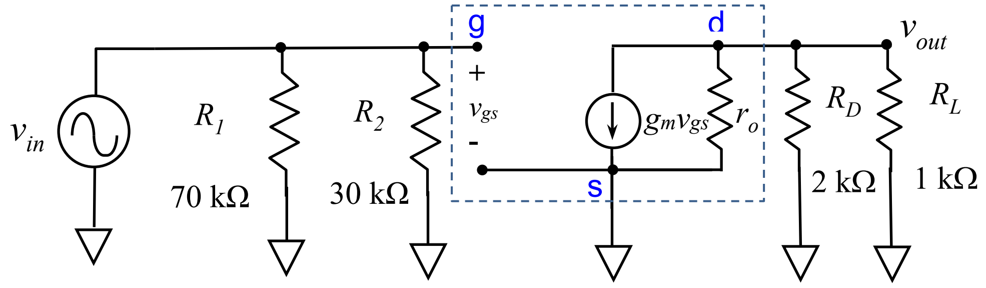
$g_m = 2\sqrt{K_n I_D} = 2\sqrt{1\text{m x } 2.56\text{m}} = 3.2 \text{ mA V}^{-1}$

$r_o = \frac{1}{\lambda I_D} = \frac{1}{0.02 \times 2.56\text{m}} = 19.5 \text{ k}\Omega$

(d)

Common source

(e)



(f)

With the help of Table 2, Configuration A -

$R_{in} = R_1 \,//\, R_2 = 70 \,//\, 30\ \text{k}\Omega = 21\ \text{k}\Omega$

With the help of Table 2, Configuration B -

$R_{out} = R_D \,//\, r_o = 2 \,//\, 19.5\ \text{k}\Omega = 1.81\ \text{k}\Omega$

(g)

$v_{in} = v_{gs}$

$v_{out} = -g_m v_{gs} \times (r_o \,//\, R_D \,//\, R_L)$

$v_{out} / v_{in} = -g_m (r_o \,//\, R_D \,//\, R_L) = -3.2\text{m} \times (19.5\text{k}//2\text{k}//1\text{k}) = -2.06$

S6. Figure S6 shows a single-stage amplifier circuit, where V_{DD} = 5 V, R_S = 100 Ω, R_{B1} = R_{S1} = 2 kΩ, R_{D1} = 2.2 kΩ and R_L = 5 kΩ. The MOSFET M_1 has the following parameters: K_n = 2 mA/V^2, V_{THN} = 1 V, λ_n = 0.

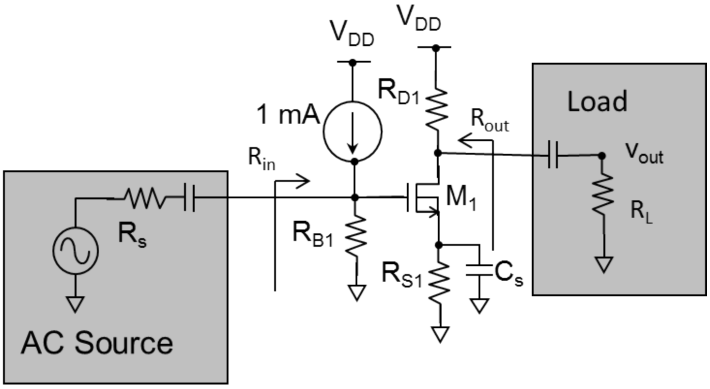


Fig. S6

(a) Find I_D, V_{GS} and V_{DS} of MOSFET M_1. Verify any assumption you make in the calculations.

[Ans: I_D = 0.305 mA; V_{GS} = 1.39 V; V_{DS} = 3.72 V]

(b) Identify the amplifier configuration.

(c) Calculate the two-port network parameters of the amplifier, i.e., G_m, R_{in}, R_{out}, and draw the two-port network of the amplifier.

[Ans: G_m = 1.56 mA/V; $R_{in} \approx$ 2 kΩ; $R_{out} \approx$ 2.2 kΩ]

(d) Estimate the voltage gain v_{out}/v_{in} of the amplifier. [Ans: $v_{out}/v_{in} \approx$ - 2.27]

S6. <u>Solution</u>:

(a) For DC analysis, remove the AC source. Load is also removed because it is isolated from the rest of the circuit by the capacitor (which behaves like an open circuit in DC).

$V_{G,M1}$ = 1 mA × R_{B1} = 1m × 2k = 2 V.

Assume that the MOSFET is operating in saturation.

$$I_{D,M1} = K_n\left(V_{GS} - V_{THN}\right)^2 = K_n\left(V_{G,M1} - V_{S,M1} - V_{THN}\right)^2$$
$$= 2\text{ m} \quad \times\left(1 - V_{S,M1}\right)^2$$

$$I_{D,M1} = I_{S,M1} = \frac{V_{S,M1}}{R_{S1}} = \frac{V_{S,M1}}{2\text{ k}}$$

$$\frac{V_{S,M1}}{2\text{ k}} = 2\text{ m} \times \left(1 - V_{S,M1}\right)^2 \quad \Rightarrow \quad 4V_{S,M1}^2 - 9V_{S,M1} + 4 = 0$$

$$\Rightarrow V_{S,M1} = 1.64\text{ V} \quad \text{or} \quad 0.61\text{ V}.$$

For $V_{S,M1} = 1.64\text{ V}, V_{GS,M1} = 0.36\text{ V} < V_{THN} \Rightarrow$ invalid. Hence, $V_{S,M1} = 0.61\text{ V}$.

$$I_{D,M1} = \frac{0.61}{2} = 0.305\text{ mA}$$

$$V_{DS} = V_{DD} - I_{D,M1}R_{D1} - V_{S,M1} = 5 - 0.305 \times 2.2 - 0.61 = 3.72\text{ V}$$

$$V_{GS} = 2 - 0.61 = 1.39\text{ V}$$

Check whether the assumption of saturation operation is correct:

$$V_{GS} - V_{TH} = 0.39\text{ V}$$

$\Rightarrow V_{DS} > V_{GS} - V_{TH}$, hence in Saturation.

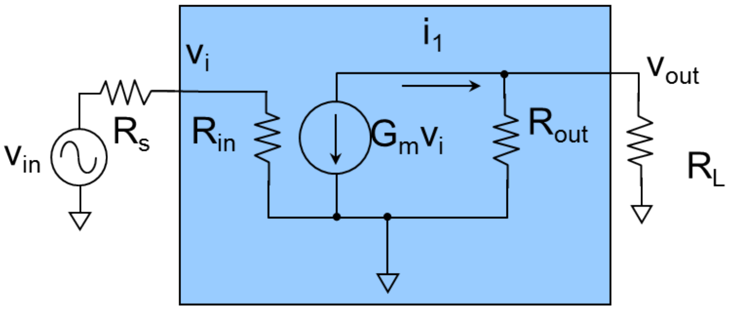
(b) Common source amplifier.

(c) $G_m = g_m = 2\sqrt{K_n I_{D,M1}} = 2\sqrt{2 \times 0.305} = 1.56\text{ mA/V}$

$R_{in} = R_{B1} = 2\text{ k}\Omega$

$$r_o = \frac{1}{\lambda I_{D,M1}} = \infty$$

$R_{out} = r_o \,//\, R_{D1} = 2.2\text{ k}\Omega$



(d)

$$\frac{v_{out}}{v_{in}} = \frac{R_{in}}{R_S + R_{in}} \times (-G_m) \times (R_{out}//R_l) = -\frac{2\text{k}}{2\text{k} + 0.1\text{k}} \times 1.56\text{m} \times \frac{2.2\text{k} \times 5\text{k}}{2.2\text{k} + 5\text{k}} = -2.27$$

S7. Figure S7 shows a single-stage amplifier circuit. The device parameters are as follows -

- n-channel MOSFETs: $K_n = 1$ mA/V^2, $V_{THN} = 1$ V and $\lambda_n = 0.01$ V^{-1}.
- p-channel MOSFETs: $K_p = 2.5$ mA/V^2, $V_{THP} = -1$ V, and $\lambda_p = 0.01$ V^{-1}.

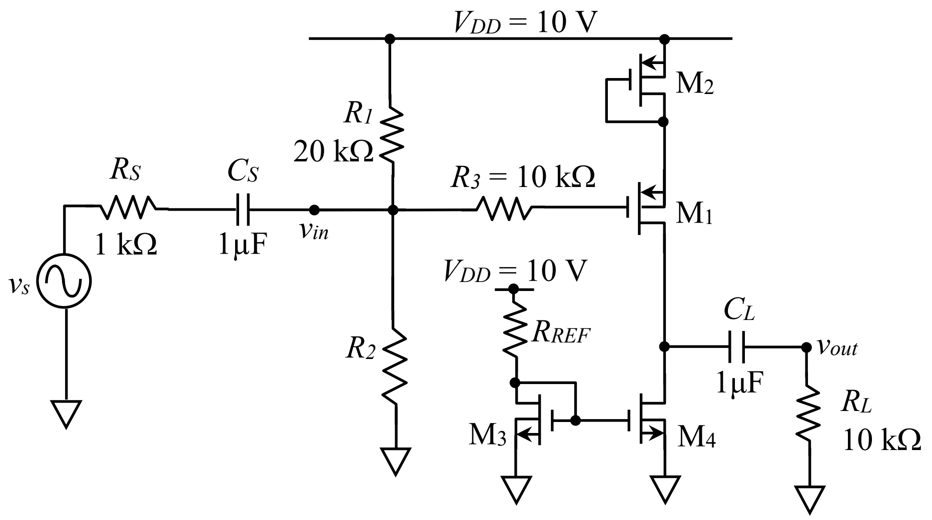


Fig. S7

MOSFET M1 is operating in saturation region and its DC drain current is 1 mA. Ignore Channel Length Modulation effect in DC analysis.

(a) Determine the DC drain current of M3 and R_{REF}.

[Ans: $I_{D,M3} = 1$ mA, $R_{REF} = 8$ kΩ]

(b) Determine R_2.

[Ans: $R_2 = 41.35$ kΩ]

(c) Identify the amplifier configuration.

(d) Determine the small-signal parameters of MOSFET M1: $g_{m,M1}$, $r_{o,M1}$ and $g_{mb,M1}$.

[Ans: $g_{m,M1} = 3.162$ mA/V, $r_{o,M1} = 100$ kΩ, $g_{mb,M1} = 0$]

(e) Calculate the two-port network parameters of the amplifier: R_{in}, R_{out} and G_m (or A_v).

[Ans: $R_{in} = 13.48$ kΩ, $R_{out} = 66.67$ kΩ, $G_m = 1.581$ mA/V]

(f) Estimate the voltage gain v_{out}/v_s of the amplifier.

[Ans: $v_{out}/v_s = -12.8$]

S7. <u>Solution</u>:

(a) Determine the DC drain current of M_3 and R_{REF}.

MOSFET M_3 and M_4 are of the n-channel type.

Perform DC analysis -

- $I_{D,M1}$ = 1 mA (Given)
- $I_{D,M4} = I_{D,M1}$ = 1 mA
- $I_{D,M3} = I_{D,M4}$ = 1 mA (M3 and M4 form a **current mirror current source)**

M3 in **saturation operation** (as **gate and drain** are **shorted together)** -

$$I_{D,M3} = K_n\left[V_{GS,M3} - V_{THN}\right]^2 = K_n\left[\left(V_{DD} - I_{D,M3}R_{REF}\right) - V_{THN}\right]^2$$

$$1\text{m} = 1\text{m}[(10 - 1\text{m} \times R_{REF}) - 1]^2$$

$$\pm 1 = (10 - 1\text{m} \times R_{REF}) - 1$$

$$R_{REF} = 10\ \text{k}\Omega \ \text{ or } \ 8\ \text{k}\Omega$$

$$R_{REF} = 8\ \text{k}\Omega$$

($R_{REF} = 10$ kΩ is invalid as $V_{GS,M3} = V_{DD} - I_{D,M3}R_{REF} = 0 < V_{THN} = 1$ V)

(b) Determine R_2.

To determine R_2, we need to find $V_{G,M1}$ first.

Voltage between R_1 and R_2 = $V_{G,M1}$ (as $I_{G,M1} = 0$)

Hence, $V_{G,M1} = \frac{R_2}{R_1+R_2} \times V_{DD}$

Apply KVL, $V_{G,M1} = V_{DD} - V_{SG,M1} - V_{SG,M2} = \frac{R_2}{R_1+R_2} \times V_{DD}$ (1)

- M1 (p-MOSFET) is in saturation operation,

$$I_{D,M1} = K_p\left[V_{SG,M1} - |V_{THP}|\right]^2 = 2.5\text{m}\left[V_{SG,M1} - 1\right]^2 = 1\text{m}$$

$$V_{SG,M1} = 1.63 \text{ V or } 0.37 \text{ V}$$

$V_{SG,M1} = 0.37$ V (< $|V_{THP}| = 1$ V, is not valid as M1 is turned off)

$$\therefore\ V_{SG,M1} = 1.63 \text{ V}$$

- M2 (p-MOSFET) is in saturation (as gate and drain are shorted together), and is identical to M1

$$I_{D,M2} = K_p\left[V_{SG,M2} - |V_{THP}|\right]^2 = 2.5\text{m}\left[V_{SG,M2} - 1\right]^2 = 1\text{m}$$

$\therefore\ V_{SG,M2} = 1.63$ V (Same as M1)

Substituting $V_{SG,M1}$ and $V_{SG,M2}$ values into eqn. (1) above -

$$V_{G,M1} = V_{DD} - V_{SG,M1} - V_{SG,M2} = 10 - 1.63 - 1.63 = 6.74\text{ V} = \frac{R_2}{R_1+R_2} \times V_{DD}$$

$$\therefore\ R_2 = 41.35\text{ k}\Omega$$

(c) Identify the amplifier configuration.

CS with degeneration.

(d) Determine the small-signal parameters of MOSFET M1: $g_{m,M1}$, $r_{o,M1}$ and $g_{mb,M1}$.

M1 is p-channel MOSFET -

- $g_{m,M1} = 2\sqrt{I_{D,M1}K_p} = 2\sqrt{1\text{m} \times 2.5\text{m}} = 3.162\text{ mA/V}$
- $r_{o,M1} = \frac{1}{I_{D,M1}\lambda_p} = \frac{1}{1\text{m}\times 0.01} = 100\text{ k}\Omega$
- $g_{mb,M1} = 0$ (source and body are shorted together, no body effect)

(e) Calculate the two-port network parameters of the amplifier: R_{in}, R_{out} and G_m (or A_v).

- With the help of Table 2, Configuration A –

  $R_{in} = R_1//R_2 = 13.48\text{ k}\Omega$

- With the help of Table 2, Configuration B –

  $$R_{out} = \left\{r_{o,M1}\left[1 + g_{m,M1} \times \frac{1}{g_{m,M2}}\right]\right\} //r_{o,M4} = 66.67\text{ k}\Omega$$

- With the help of Table 4, Configuration D –

  $$G_m = \frac{g_{m,M1}}{1+g_{m,M1}\left[\frac{1}{g_{m,M2}}\right]} = 1.581\text{ mA/V}$$

(f) Estimate the voltage gain v_{out}/v_s of the amplifier.

$$A_V = -\left(\frac{R_{in}}{R_S + R_{in}}\right) \times G_m \times (R_{out}//R_L)$$

$$= -\left(\frac{13.48}{1+13.48}\right) \times 1.581\text{m} \times (66.67\text{k}//10\text{k}) = -12.80$$