# EE2027 Electronic Circuits

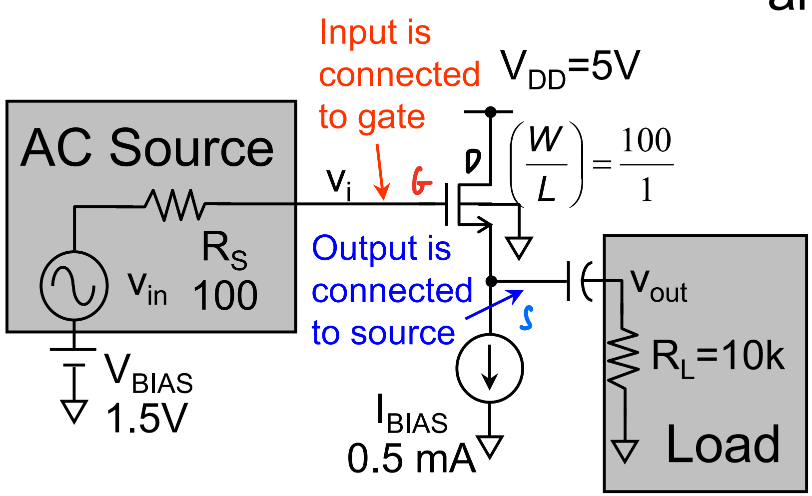
## Single-Stage Amplifier

### Lecture Outline

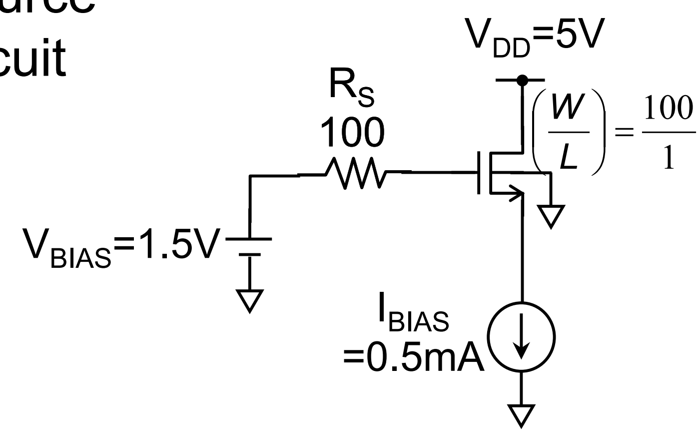
- **Single-Stage Amplifiers**
  CC/CD, CE/CS with degeneration
- **Effect of Capacitors**
  Frequency response
- **Current Source**
  Current mirror
- **Multi-Stage Amplifier Analysis Concept**

# Common Drain (CD)

$\mu_n C_{ox}$=80$\mu$A/V$^2$
$V_{THN}$=0.7V
$\lambda_n$=0.02V$^{-1}$



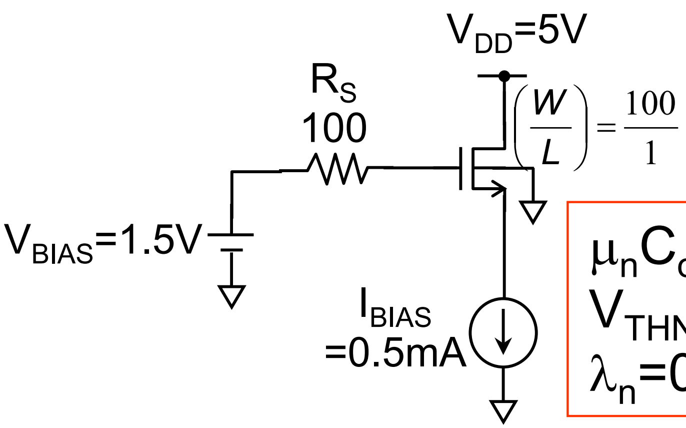

For DC analysis, remove AC source and open circuit capacitor




- Identify AC Source and Load
- To identify amplifier configuration, we need to consider AC equivalent circuit, i.e., short circuit capacitors
  - Input connected to Gate, output connected to Source, Drain connected to neither input nor output ⇒ Common Drain (CD)

# DC Analysis - CD

- Remove AC source and open circuit capacitor when doing DC analysis




$\mu_n C_{ox}$=80μA/V$^2$
$V_{THN}$=0.7V
$\lambda_n$=0.02V$^{-1}$

**Determine DC biasing**

$$I_D = I_S = I_{BIAS} = 0.5\,\text{mA}$$

**Determine AC small-signal parameters**

$$\left(\frac{W}{L}\right) = \frac{100}{1}$$

$2\sqrt{k_n I_D}$

$$g_m = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right) I_D} = 2.83\,\text{mA/V}$$

MOSFET has body effect

(body not shorted to source)

$$\boxed{g_{mb} \approx -\frac{g_m}{4} = -0.71\,\text{mA/V}}$$

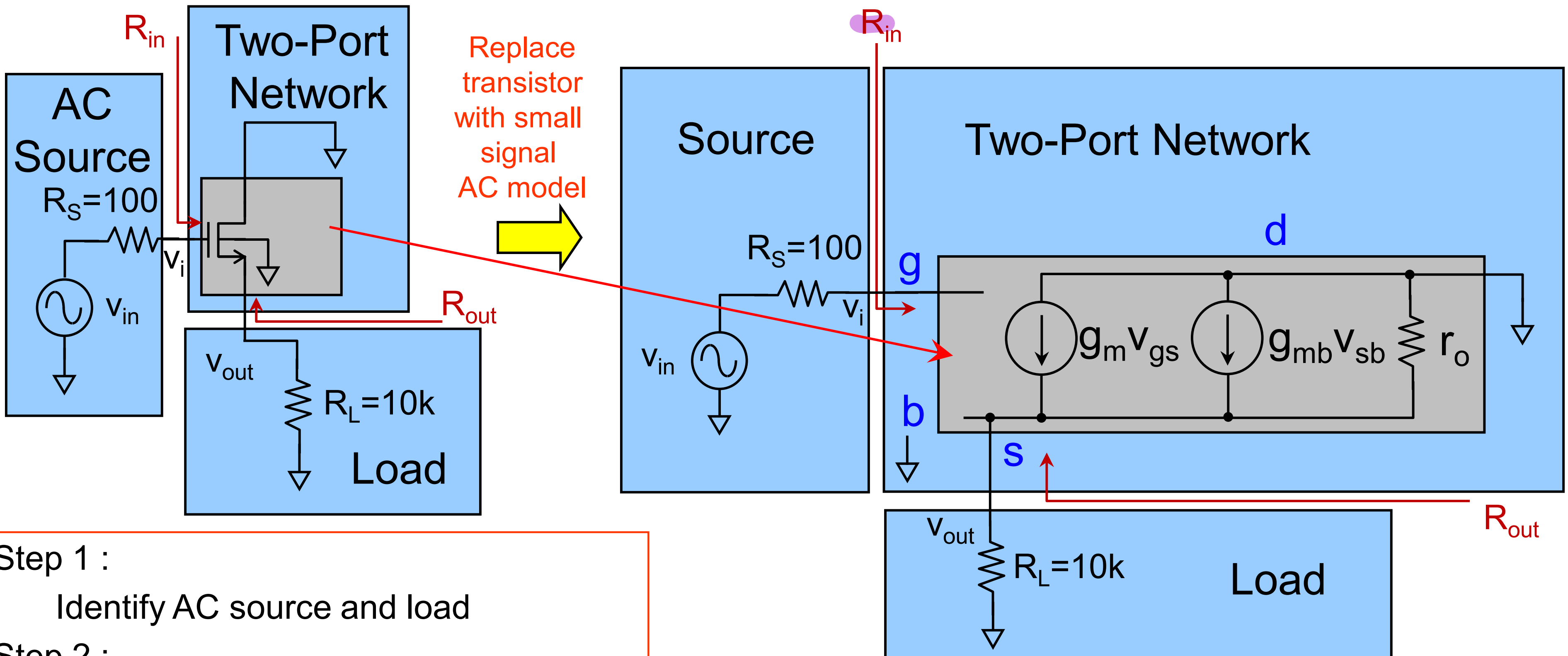
Good approximation, no need to go through detailed calculations

$$r_i = \infty$$

$$r_o = \frac{1}{\lambda_n I_D} = 100\,\text{k}\Omega$$

# AC Analysis - CD

For AC analysis, need to consider the AC equivalent circuit (i.e., short circuit capacitor, replace DC voltage source by short circuit and replace DC current source by open circuit).




Step 1 :

Identify AC source and load

Step 2 :

Group the remaining components (the amplifier circuit) into two-port network

Step 3 :

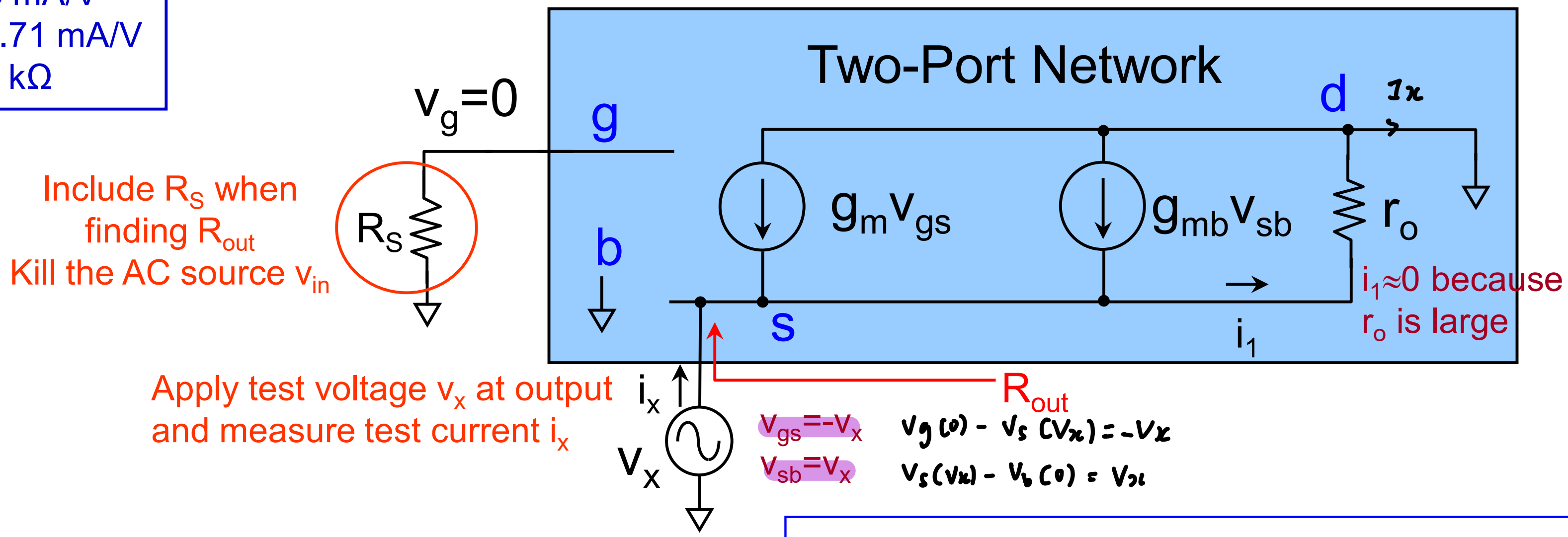
Replace transistor with small-signal model

For CD amplifier, transform to 2-port Voltage Amplifier (based on experience of circuit designers). Need to evaluate $R_{in}$, $R_{out}$ and A.

$$R_{in} = \infty$$

# CD – Two-Port Network ($R_{out}$)

$g_m$ = 2.8 mA/V
$g_{mb}$ = -0.71 mA/V
$r_o$ = 100 kΩ

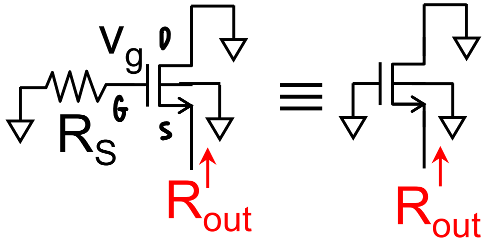



Refer to slide SSA1-10 for the evaluation of $R_{out}$.

$$R_{out} = \frac{v_x}{i_x} \left\{ \begin{array}{l} i_x \approx -g_m v_{gs} - g_{mb} v_{sb} = (g_m - g_{mb}) v_x \\ \Rightarrow R_{out} \approx \dfrac{1}{g_m - g_{mb}} \end{array} \right.$$

# CD – Two-Port Network ($R_{out}$)

Since no current flow through $R_S$, $v_g$ is same as AC ground




$$R_{out} \approx \frac{1}{g_m - g_{mb}}$$

**Important Result:**

If you see a MOSFET connected in similar fashion, the equivalent resistance looking into the source ($R_{out}$) is directly given by the formula. **No need to rederive.**

Example :

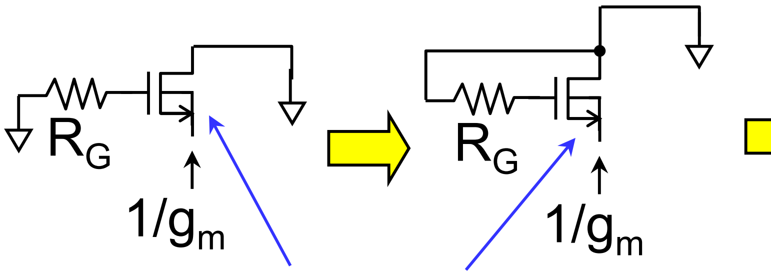
$$R_S = 100 \quad r_o = 100\text{k}$$

$$g_m = 2.83\text{m} \quad g_{mb} = -0.71\text{m}$$

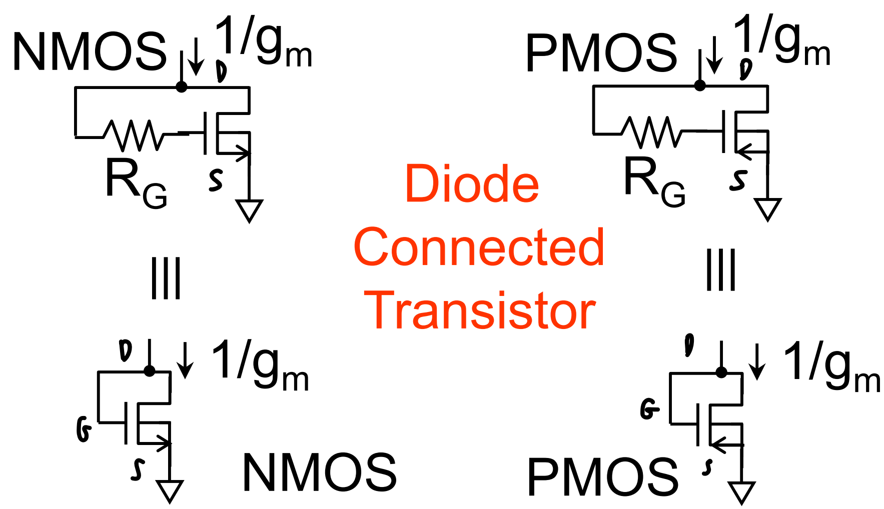
$$\Rightarrow R_{out} \approx 283$$

- The output resistance does not depend on $R_S$

# MOS – Equivalent Resistance

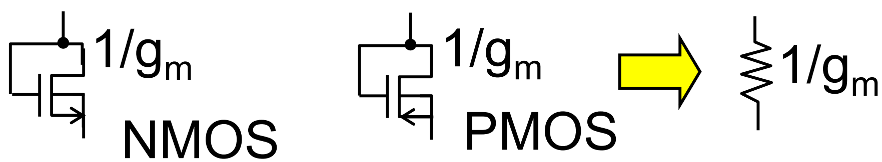


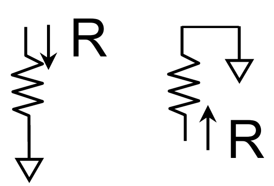

When body and source connected together, i.e., $v_{sb}=0$
$\Rightarrow$ there is no body effect
$\Rightarrow$ $g_{mb}v_{sb}=0$, eliminate $g_{mb}$




Diode Connected Transistor

Since no current flows through $R_G$,
there is no voltage drop across $R_G$,
$R_G$ is effectively short circuited



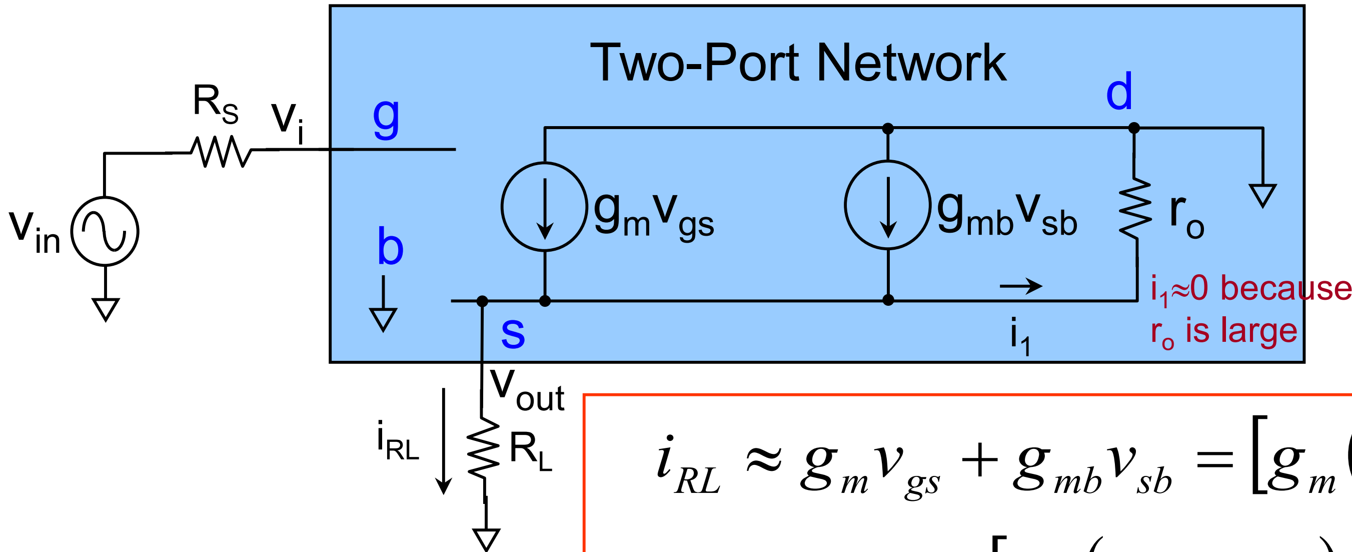




A resistor, whether you look from top or bottom, is always a resistor with resistance R

Only APPLICABLE for AC analysis

- When the drain and gate are connected together, the transistor is called diode connected transistor
- Diode connected transistor has an equivalent resistance of $1/g_m$

# CD – Two-Port Network ($A_V$)




$$v_{gs} = v_{in} - v_{out}$$
$$v_{sb} = v_{out}$$

$$i_{RL} \approx g_m v_{gs} + g_{mb} v_{sb} = \left[g_m\left(v_{in} - v_{out}\right) + g_{mb} v_{out}\right]$$
$$v_{out} = i_{RL} R_L \approx \left[g_m\left(v_{in} - v_{out}\right) + g_{mb} v_{out}\right] R_L$$
$$\Rightarrow \left[1 + \left(g_m - g_{mb}\right) R_L\right] v_{out} = g_m R_L v_{in}$$
$$\Rightarrow A_V = \frac{v_{out}}{v_{in}} = \frac{g_m R_L}{1 + \left(g_m - g_{mb}\right) R_L} \approx \frac{g_m}{g_m - g_{mb}} \approx 0.8$$

If $(g_m - g_{mb}) R_L >> 1$

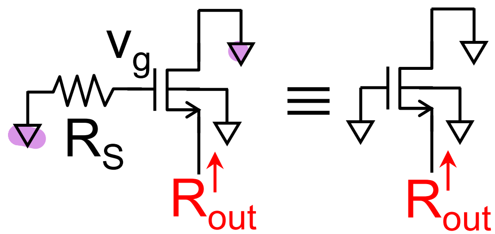
No body effect means source and body are connected together
$\Rightarrow g_{mb}$ can be eliminated in the equation

- Without body effect, $g_{mb} v_{sb} = 0 \Rightarrow A_V \approx 1$
- For CD amplifier, we calculate $v_{out}/v_{in}$ directly.

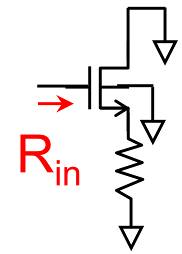
# CD – Important Results

1

Since no current flows through $R_S$, $v_g$ is same as AC ground



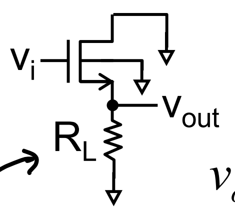

$$R_{out} \approx \frac{1}{g_m - g_{mb}}$$




$$R_{in} = \infty$$

2

Relationship between $v_i$ and $v_{out}$ is defined as follows:

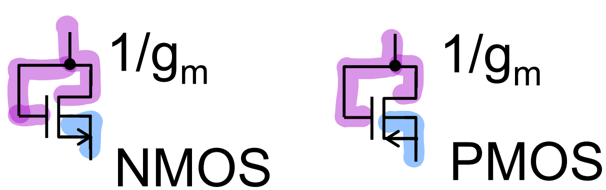


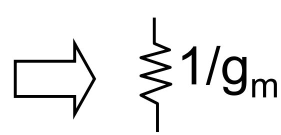

one resistor only

$$v_{out} = \frac{g_m R_L}{1 + (g_m - g_{mb}) R_L} v_i$$

3

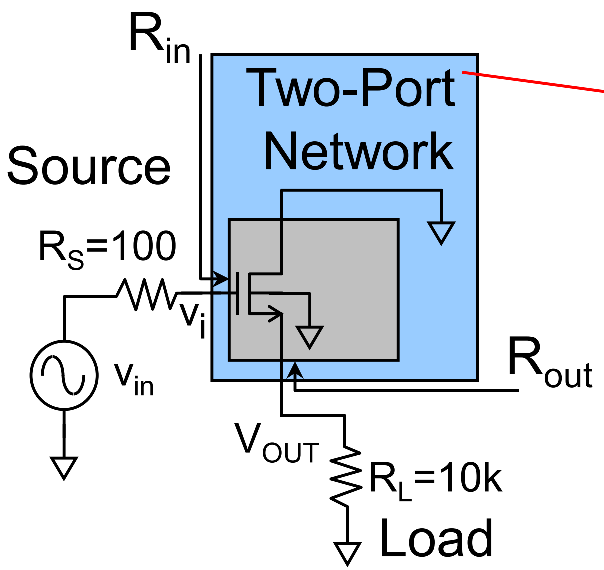
Diode Connected Transistor (Gate and drain shorted) (No body effect)

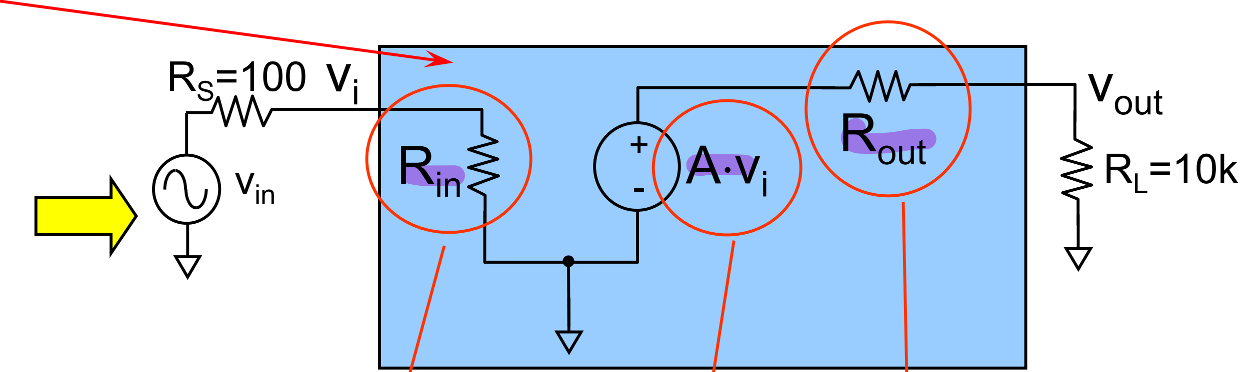


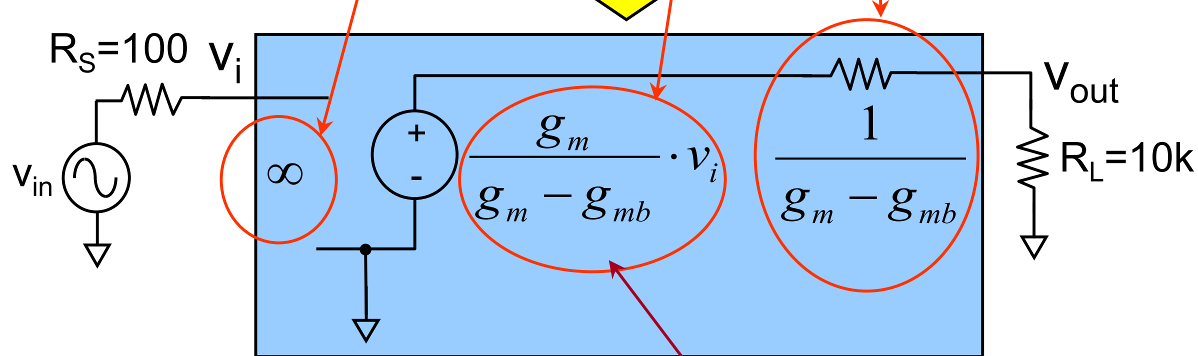



# CD – Two-Port Network

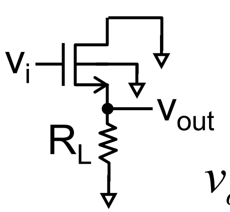
Transform the CD amplifier to 2-port voltage amplifier




Relationship between $v_i$ and $v_{out}$ is defined as follows:




$$v_{out} = \frac{g_m R_L}{1+(g_m - g_{mb})R_L} v_i$$

It is simpler to remember $A_v$ (=$v_{out}/v_i$) rather than A

A can be found based on the $A_v$.

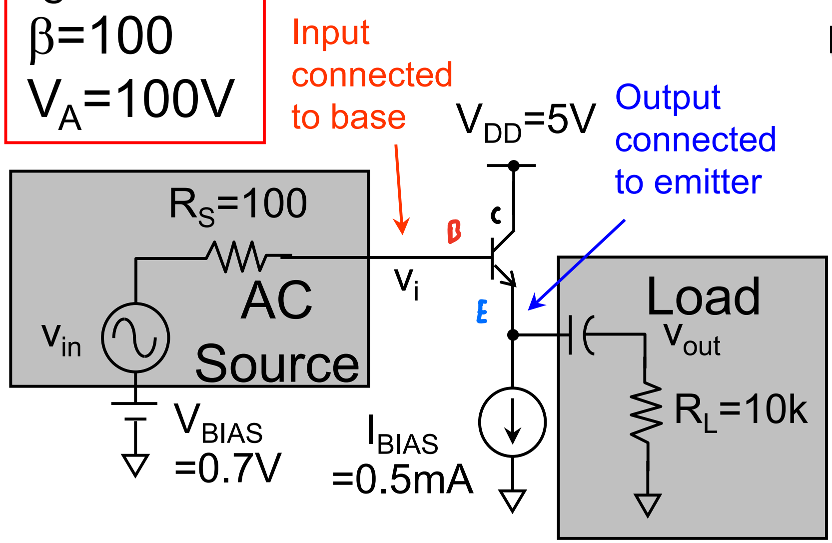
$$\frac{v_{out}}{v_{in}} = \frac{v_i}{v_{in}} \times \frac{v_{out}}{v_i} = \frac{g_m R_L}{1+(g_m - g_{mb})R_L}$$

$\Rightarrow$ $v_i = \frac{R_{in}}{R_{in}+R_s} v_{in}$

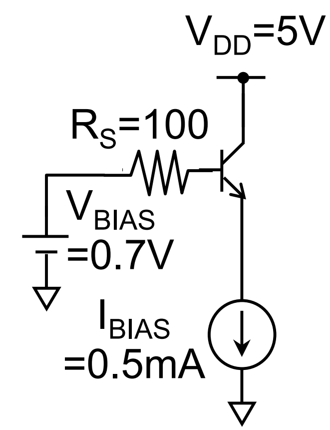
$$\frac{v_i}{v_{in}} \times \frac{v_{out}}{v_i} = \frac{R_{in} \to \infty}{R_{in} + R_s} \times \frac{g_m R_L}{1+(g_m-g_{mb})R_L} \approx \frac{\cancel{R_{in}}}{\cancel{R_{in}}} \times \frac{g_m R_L}{1+(g_m-g_{mb})R_L}$$

# Common Collector (CC)

$I_S$=$10^{-15}$A
$\beta$=100
$V_A$=100V



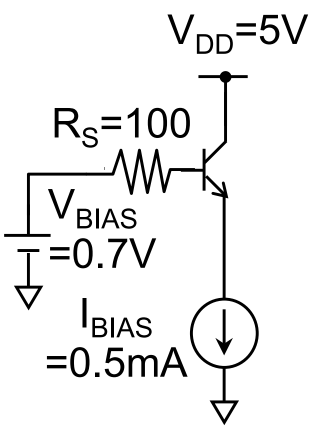

For DC analysis,
remove AC source
and open circuit
capacitor




- Identify AC Source and Load
- To identify amplifier configuration, we need to consider AC equivalent circuit, i.e., short circuit capacitors
  - Input connected to Base, output connected to Emitter, Collector connected to neither input nor output ⇒ Common Collector (CC)

# DC Analysis - CC

- Remove AC source and open circuit capacitors when doing DC analysis




$I_S=10^{-15}$A
β=100
$V_A$=100V

**Determine DC biasing**

$$V_T = 26\,\text{mV}$$

$$I_E = I_{BIAS} = 0.5\,\text{mA}$$

$$I_C = \frac{\beta}{\beta+1} I_E = 0.495\,\text{mA}$$

$$I_B = 4.95\,\mu\text{A}$$

$$I_B = \frac{I_C}{\beta}$$

$I_C \approx I_E$

**Determine AC small-signal parameters**

$$g_m = \frac{I_C}{V_T} = 19\,\text{mA/V}$$

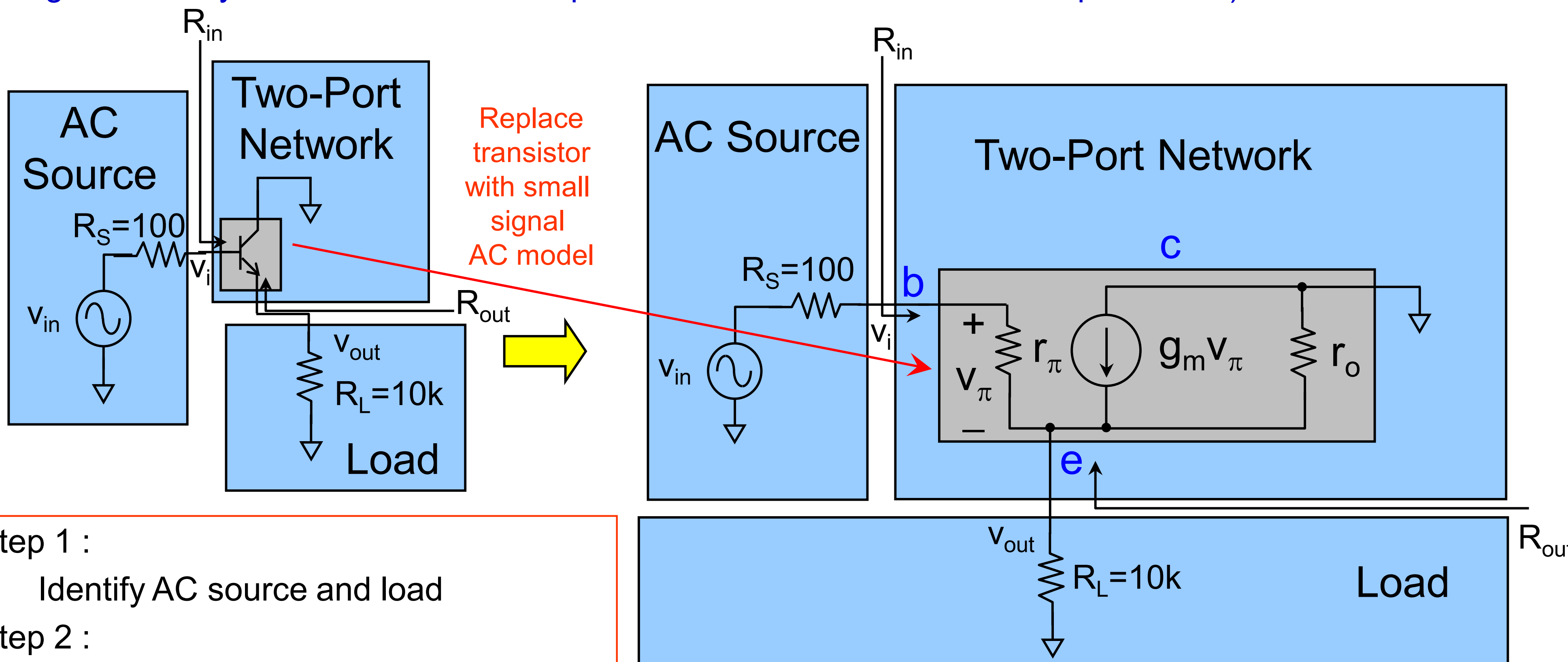
$$r_\pi = \frac{\beta}{g_m} = 5.26\,\text{k}\Omega$$

$$r_o = \frac{V_A}{I_C} = 202\,\text{k}\Omega$$

Finding the current $I_C$ is key

# CC - AC Analysis (Self Reading)

For AC analysis, need to consider the AC equivalent circuit (i.e., short circuit capacitor, replace DC voltage source by AC short circuit and replace DC current source with AC open circuit).




Step 1 :

Identify AC source and load

Step 2 :

Group the remaining components into two-port network

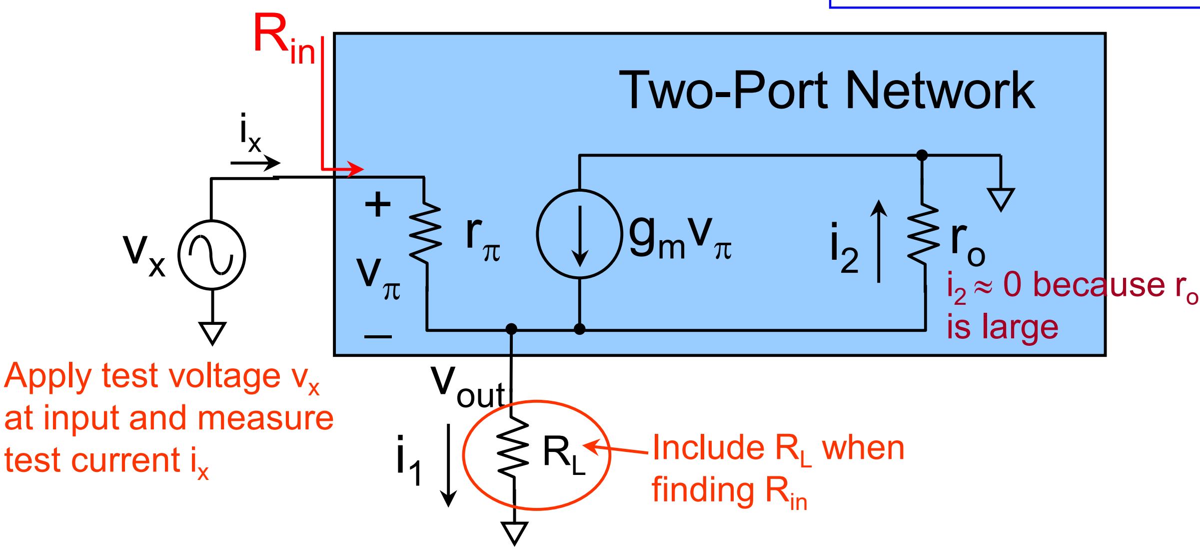
Step 3 :

Replace transistor with small-signal model

For CC amplifier, transform to 2-port Voltage Amplifier (based on experience of circuit designers). Need to evaluate $R_{in}$, $R_{out}$ and A.

# CC – Two-Port Network ($R_{in}$) (Self Reading)

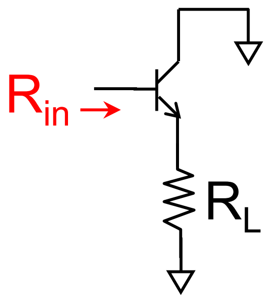
Refer to slide SSA1-10 for the evaluation of $R_{in}$.




$$R_{in} = \frac{v_x}{i_x}\;\left\{\begin{array}{l} v_x = v_\pi + v_{out} = i_x r_\pi + v_{out} \quad \text{Eliminate } v_{out} \text{ and keep } v_x \text{ and } i_x \\ \bullet \quad v_{out} = i_1 R_L \\ \bullet \quad i_1 = i_x + g_m v_\pi - \cancel{i_2} \approx i_x + g_m v_\pi = i_x + i_x g_m r_\pi \\ \therefore v_x = i_x r_\pi + v_{out} \approx i_x r_\pi + (i_x + i_x g_m r_\pi) R_L \\ \Rightarrow R_{in} = \dfrac{v_x}{i_x} \approx r_\pi + (1 + g_m r_\pi) R_L = r_\pi + (1+\beta) R_L \approx r_\pi (1 + g_m R_L) \end{array}\right.$$

$$(\mathrm{N.B.}\ g_m r_\pi = \beta \gg 1)$$

# CC – Two-Port Network ($R_{in}$) (Self Reading)




$$R_{in} \approx r_\pi + (1+\beta)R_L$$
$$\approx r_\pi + \beta R_L$$
$$\approx r_\pi(1 + g_m R_L)$$

**Important Result:**
If you see a BJT connected in similar fashion, the **equivalent resistance** looking into the **base** ($R_{in}$) is directly given by the formula. **No need to rederive.**

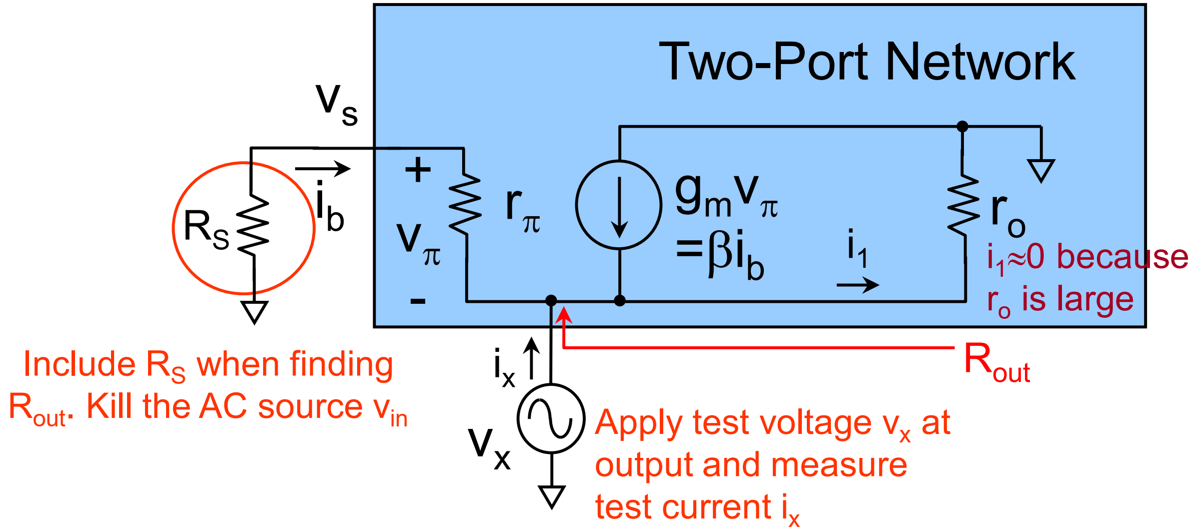
Example :

$$R_L = 10\text{k} \quad r_o = 202\text{k}$$
$$\Rightarrow R_{in} = r_\pi + (\beta + 1)R_L$$
$$= 1.01\text{M}$$

- Emitter side resistor helps boost up the input resistance looking into the base

# CC – Two-Port Network ($R_{out}$) (Self Reading)

Refer to slide SSA1-10 for the evaluation of $R_{out}$.



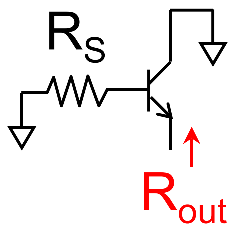

Eliminate $i_b$ and keep $v_x$ and $i_x$

$$R_{out} = \frac{v_x}{i_x} \quad \Rightarrow \begin{cases} i_x \approx -(\beta+1)i_b \\ i_b = -\dfrac{v_x}{R_S + r_\pi} \end{cases}$$

$$\Rightarrow R_{out} = \frac{R_S + r_\pi}{\beta + 1}$$

$$\approx \frac{R_S}{\beta+1} + \frac{1}{g_m}$$

# CC – Two-Port Network ($R_{out}$) (Self Reading)




$$R_{out} \approx \frac{R_S}{\beta+1} + \frac{1}{g_m}$$

$$\approx \frac{1}{g_m} \quad [\text{if} \quad R_S \quad \text{is} \quad \text{small}]$$

**Important Result:**
If you see a BJT connected in similar fashion, the equivalent resistance looking into the emitter ($R_{out}$) is directly given by the formula. **No need to rederive.**

Example :

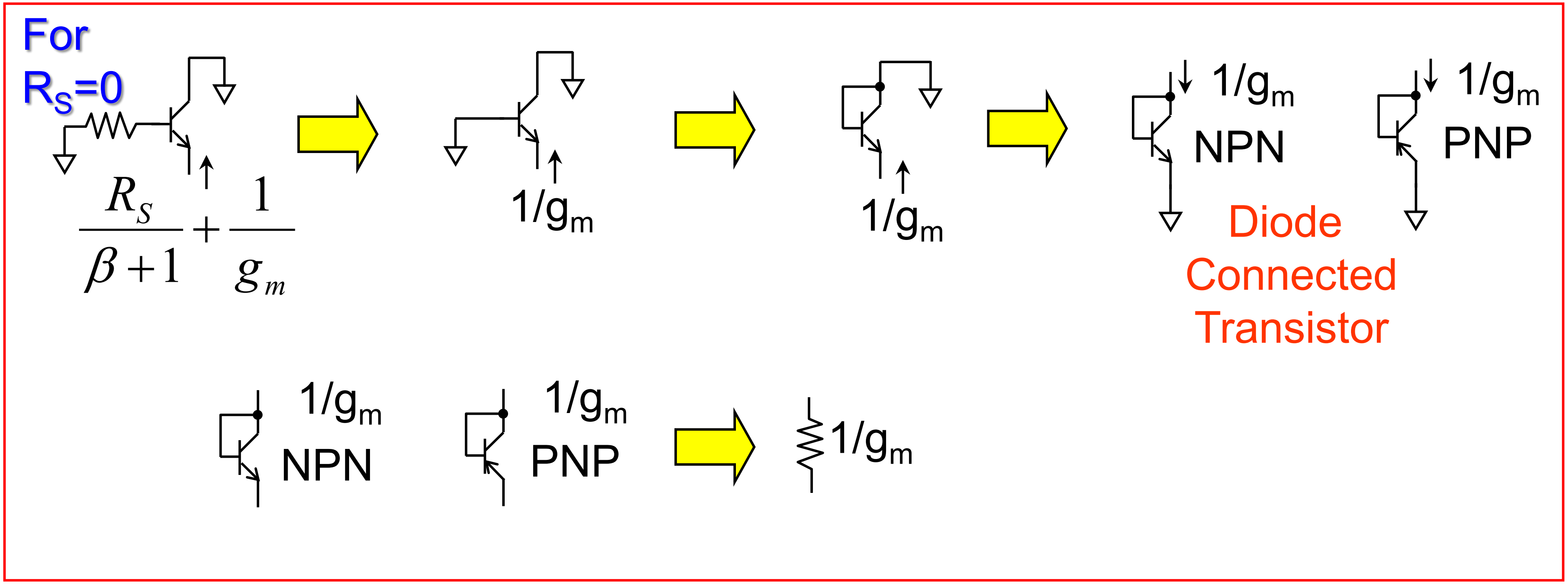
$R_S = 100 \quad r_\pi = 5.26\,\text{k}$

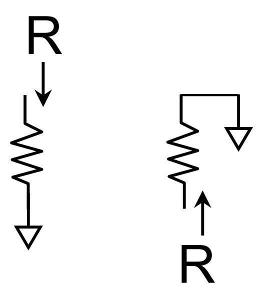
$r_o = 202\,\text{k} \quad g_m = 19\,\text{m}$

$\Rightarrow R_{out} \approx 54$

- If $R_S$ is small, the output impedance is just the inverse of the transconductance of BJT

# BJT – Diode Connected Transistor Equivalent Resistance (Self Reading)



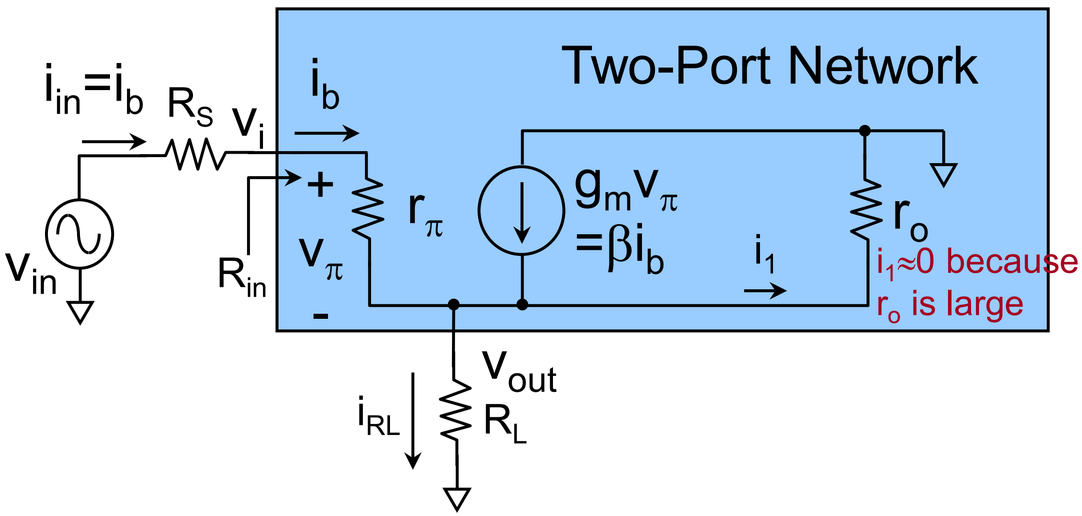




A resistor, whether you look from top or bottom, always is a resistor with resistance R

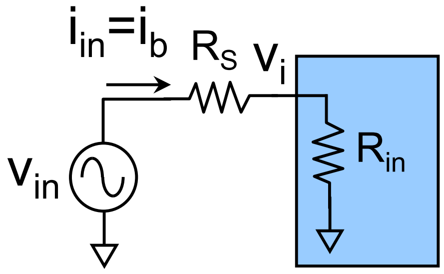
Only APPLICABLE for AC analysis

- When the collector and base are connected together, the transistor is called diode connected transistor
- Diode connected transistor has an equivalent resistance of $1/g_m$

# CC – Two-Port Network ($A_V$) (Self Reading)




To determine $i_b$, we can replace the two-port network by an equivalent input resistance, $R_{in}$




$$i_b = \frac{v_{in}}{R_{in}+R_S}$$

$$R_{in} = r_\pi + (\beta+1)R_L$$

$$i_{RL} = (\beta+1)i_b - i_1$$

$$v_{out} = i_{RL} \times R_L$$

$$= (\beta+1)i_b \times R_L$$

Eliminate $i_b$ and keep $v_{out}$ and $v_{in}$

$$\Rightarrow A_V = \frac{v_{out}}{v_{in}} = \frac{(\beta+1)i_b \times R_L}{[R_S + r_\pi + (\beta+1)R_L]i_b}$$

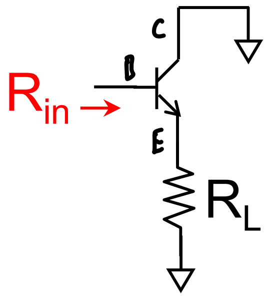
$$= \frac{(\beta+1)R_L}{\cancel{R_S} + r_\pi + (\beta+1)R_L}$$

$$= \frac{g_m R_L}{1+g_m R_L} \approx 1 \quad [\text{If} \quad g_m R_L >> 1]$$

- For CC amplifier, we calculate $v_{out}/v_{in}$ directly.
- $v_{out}/v_{in} \approx v_{out}/v_i$ as both $i_b$ and $R_s$ are small.

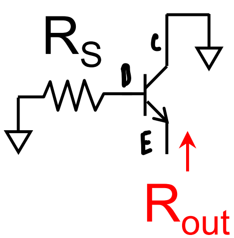
# CC – Important Results

1



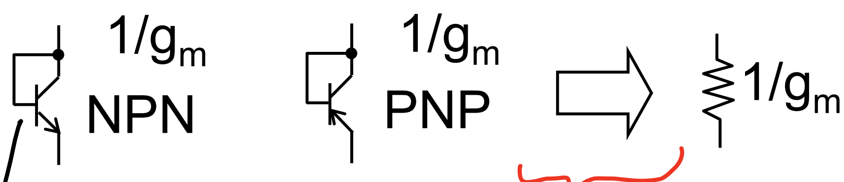

$$R_{in} \approx r_\pi + (1+\beta)R_L$$
$$\approx r_\pi + \beta R_L$$
$$\approx r_\pi(1 + g_m R_L) \quad **$$

2




$$R_{out} \approx \frac{R_S}{\beta+1} + \frac{1}{g_m} \quad **$$
$$\approx \frac{1}{g_m} \quad [if \quad R_S \quad is \quad small]$$

3

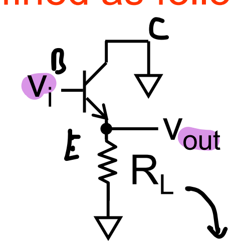



Base and collector shorted

During AC* analysis only

4

Relationship between $v_i$ and $v_{out}$ is defined as follows:

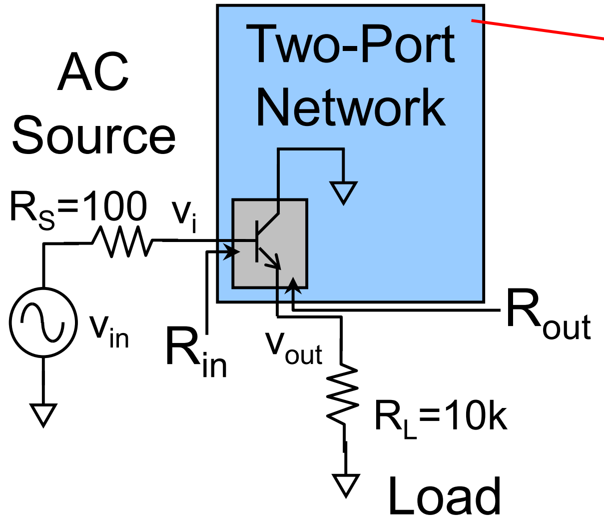


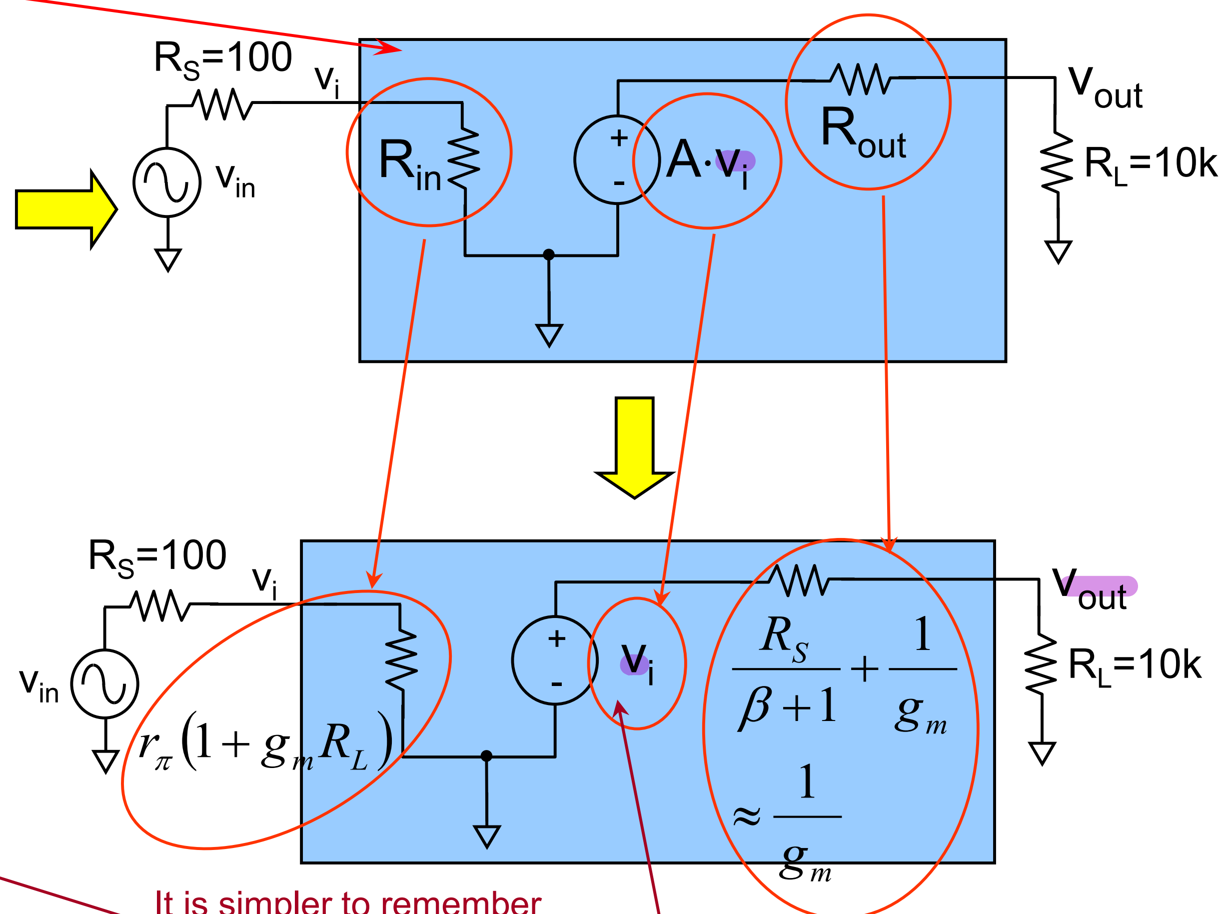

$$v_{out} = \frac{g_m R_L}{1 + g_m R_L} v_i$$

only 1 resistor, if more than one need to simplify to 1

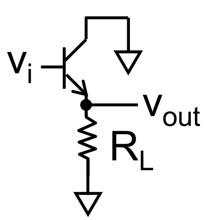
# CC – Two-Port Network

Transform the CC amplifier
to 2-port voltage amplifier




Relationship between $v_i$ and $v_{out}$ is defined as follows:




$$v_{out} = \frac{g_m R_L}{1+g_m R_L} v_i$$

It is simpler to remember $A_v$ (=$v_{out}/v_i$) rather than A

A can be found based on $A_v$

$$\frac{v_{out}}{v_{in}} = \frac{v_i}{v_{in}} \times \frac{v_{out}}{v_i} = \frac{R_{in}}{R_S+R_{in}} \times \frac{g_m R_L}{1+g_m R_L}$$

# Characteristics of CC/CD

- High input resistance
- Low output resistance ***
- Close to unity gain
- No polarity inversion
- Ideal buffer (Emitter/Source follower)
- ~~BJT provides larger $g_m$ than MOS ($g_m$-$g_{mb}$~~)

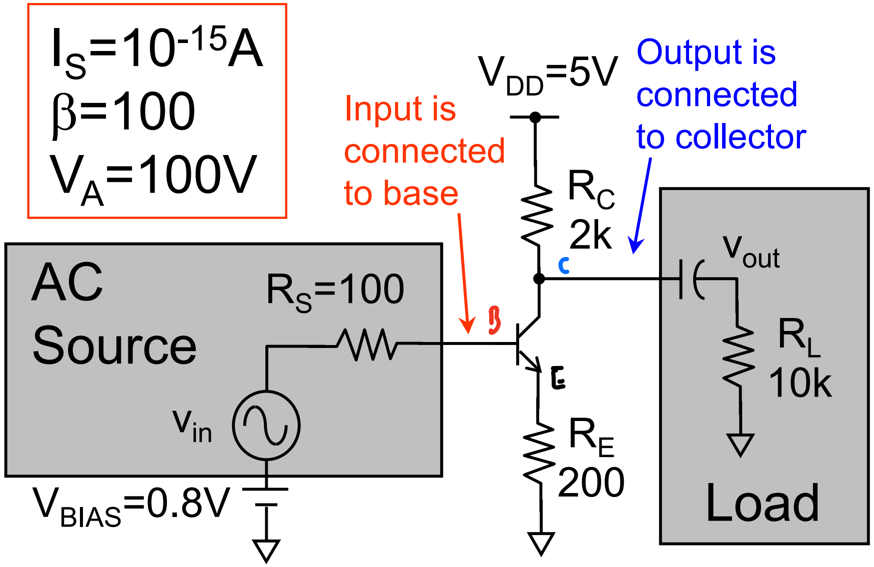
  $\Rightarrow$ Lower output resistance

  $\Rightarrow$ Better buffer

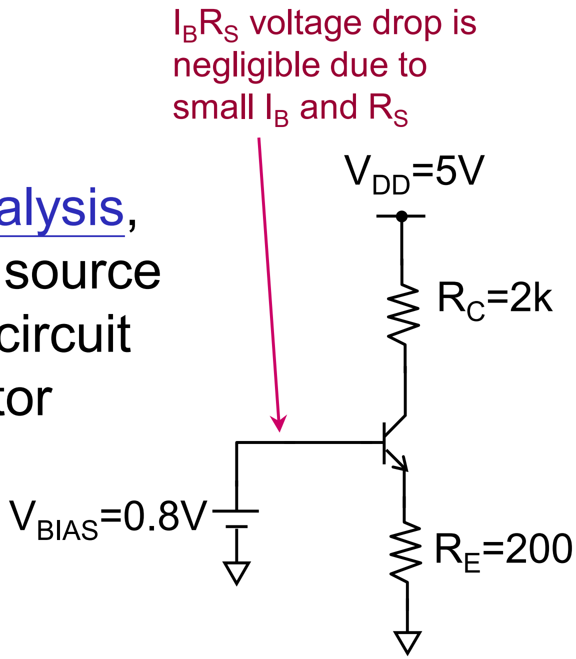
$$\frac{g_m R_L}{1+g_m R_L}$$

# CE with Emitter Degeneration

Emitter connected to some resistor



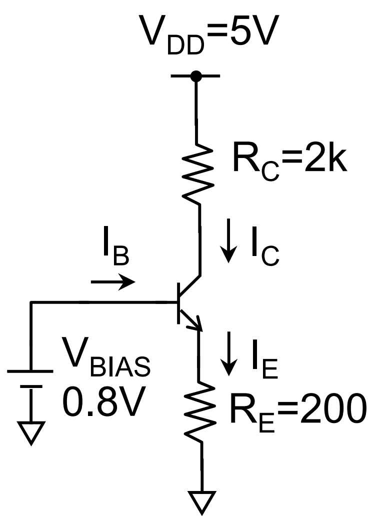

For DC analysis, remove AC source and open circuit capacitor




- Identify AC Source and Load
- To identify amplifier configuration, we need to consider AC equivalent circuit, i.e., short circuit capacitors
    - Input connected to Base, output connected to Collector, Emitter connected to neither input nor output and is connected to a resistor ⇒ CE with Emitter Degeneration

# DC Analysis

- Remove AC source and open circuit capacitor when doing DC analysis




$I_S = 10^{-15}$ A
$\beta = 100$
$V_A = 100$ V

**Determine DC biasing**

$$V_T = 26\,\text{mV}$$

Assume $V_{BE} \approx 0.7\,\text{V}$

$$I_E = \frac{V_{BIAS} - V_{BE}}{R_E} = 0.5\,\text{mA}$$

$$I_C = \frac{\beta}{\beta+1} I_E = 0.495\,\text{mA}$$

$$V_{BE} = V_T \ln\left(\frac{I_C}{I_S}\right) \approx 0.7001$$

$\therefore$ Valid Assumption

$$I_B = 4.95\,\mu\text{A}$$

$V_B$ is small so need to check

**Determine AC Small-signal parameters**

$$g_m = \frac{I_C}{V_T} = 19\,\text{mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = 5.26\,\text{k}\Omega$$

$$r_o = \frac{V_A}{I_C} = 202\,\text{k}\Omega$$

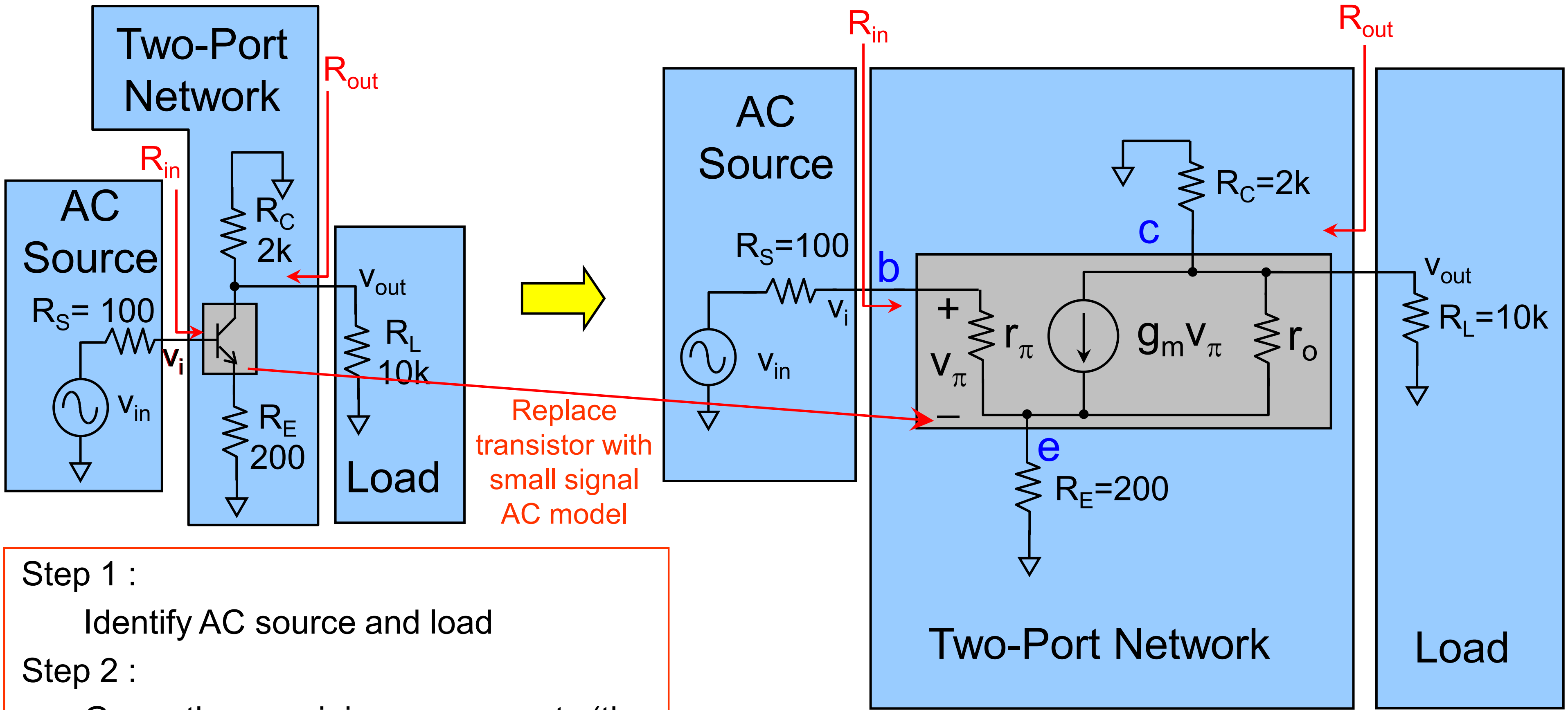
If the assumption is invalid, use the updated $V_{BE}$, re-estimate $I_C$, a few iterations might be needed.

* Never estimate $I_C$ using the exponential transistor equation using $V_{BE} \approx 0.7$ V.

$$i_C = I_S e^{v_{BE}/V_T},$$

# AC Analysis

For AC analysis, need to consider the AC equivalent circuit (i.e., short circuit capacitor, replace DC voltage source by AC short circuit and replace DC current source with AC open circuit).




Step 1 :

Identify AC source and load

Step 2 :

Group the remaining components (the amplifier circuit) into two-port network

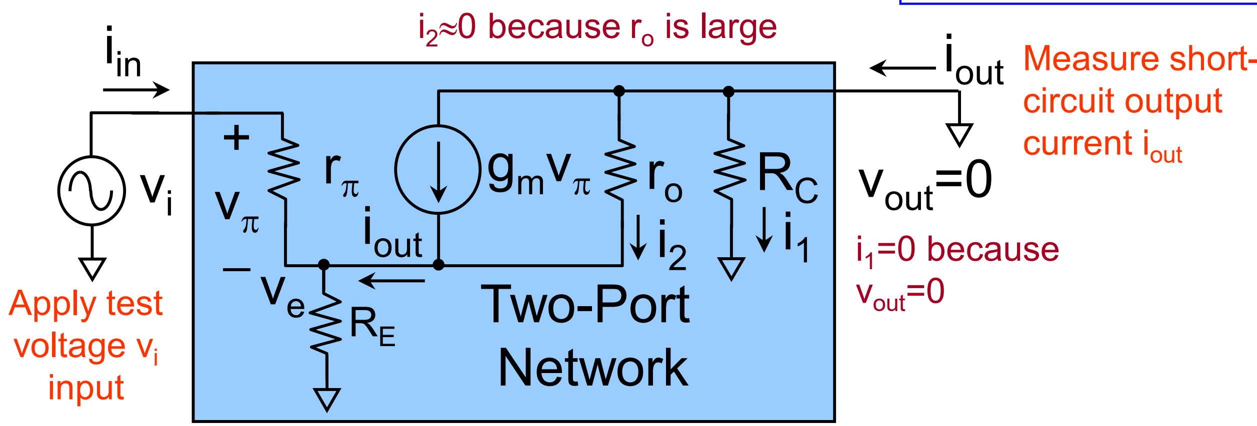
Step 3 :

Replace transistor with small signal AC model

For CE with emitter degeneration amplifier, transform to 2-port Transconductance Amplifier (based on experience of circuit designers). Need to evaluate $G_m$, $R_{in}$ and $R_{out}$.

# CE with Emitter Degeneration – Two-Port Network ($G_m$)

Refer to slide SSA1-7 for the evaluation of $G_m$.




**Important Result :** Transconductance ($G_m$) is just $g_m/(1+g_mR_E)$

In normal CE, ($G_m$) is $g_m$, hence CE with degen → $g_m$ smaller

$$G_m = \frac{i_{out}}{v_i}\Big|_{v_{out}=0}$$

$$v_i = v_\pi + v_e = i_{in}r_\pi + (i_{in}+i_{out})R_E = i_{in}(r_\pi + R_E) + i_{out}R_E$$

Eliminate $i_{in}$ and keep $v_i$ and $i_{out}$

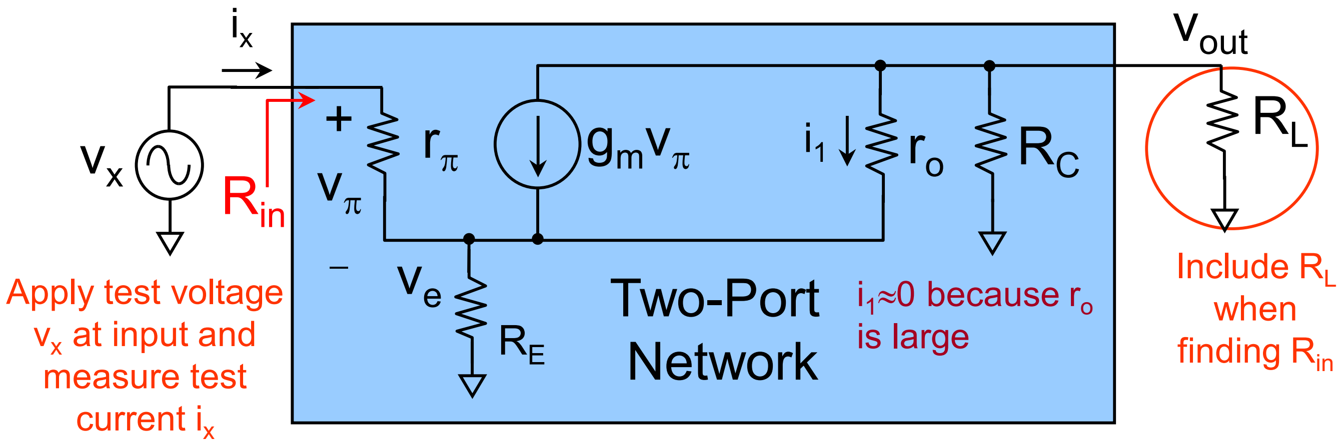
$$i_{out} = g_m v_\pi + i_2 \approx g_m v_\pi = g_m i_{in} r_\pi \;\Rightarrow\; i_{in} = \frac{i_{out}}{g_m r_\pi}$$

$$\Rightarrow v_i = \left[\frac{i_{out}}{g_m r_\pi}\right] \times (r_\pi + R_E) + i_{out}R_E = i_{out} \times \frac{r_\pi + R_E(1+g_m r_\pi)}{g_m r_\pi}$$

$$\Rightarrow G_m = \boxed{\frac{i_{out}}{v_i} = \frac{g_m r_\pi}{r_\pi + R_E(1+g_m r_\pi)} \approx \frac{g_m}{1+g_m R_E}} = 4\ \mathrm{mA/V}\ \ (\mathrm{N.B.}\ g_m r_\pi = \beta \gg 1)$$

# CE with Emitter Degeneration – Two-Port Network ($R_{in}$)

Refer to slide SSA1-7 for the evaluation of $R_{in}$.

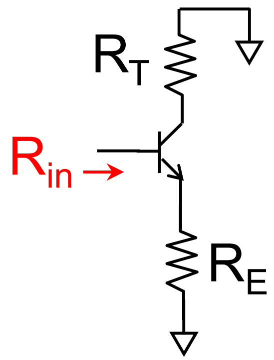



$$R_{in} = \frac{v_x}{i_x} \quad \begin{cases} i_x \approx \dfrac{v_e}{R_E} - g_m v_\pi = \dfrac{v_e}{R_E} - g_m i_x r_\pi \\ v_e = v_x - v_\pi = v_x - i_x r_\pi \\ \Rightarrow i_x \approx \dfrac{(v_x - i_x r_\pi)}{R_E} - g_m i_x r_\pi \end{cases}$$

Eliminate $v_e$ and keep $v_x$ and $i_x$

$$\Rightarrow R_{in} = \frac{v_x}{i_x} = r_\pi + g_m r_\pi R_E + R_E$$
$$= \boxed{r_\pi + (\beta + 1) R_E}$$
$$\approx \boxed{r_\pi [1 + g_m R_E]}$$

# CE with Emitter Degeneration – Two-Port Network ($R_{in}$)




$$R_{in} = r_\pi + (\beta + 1)R_E$$
$$\approx r_\pi[1 + g_m R_E]$$

**Important Result:**
If you see a BJT connected in similar fashion, the equivalent resistance looking into the base ($R_{in}$) is directly given by the formula. **No need to rederive.** The resistance in the collector branch has no effect on $R_{in}$.

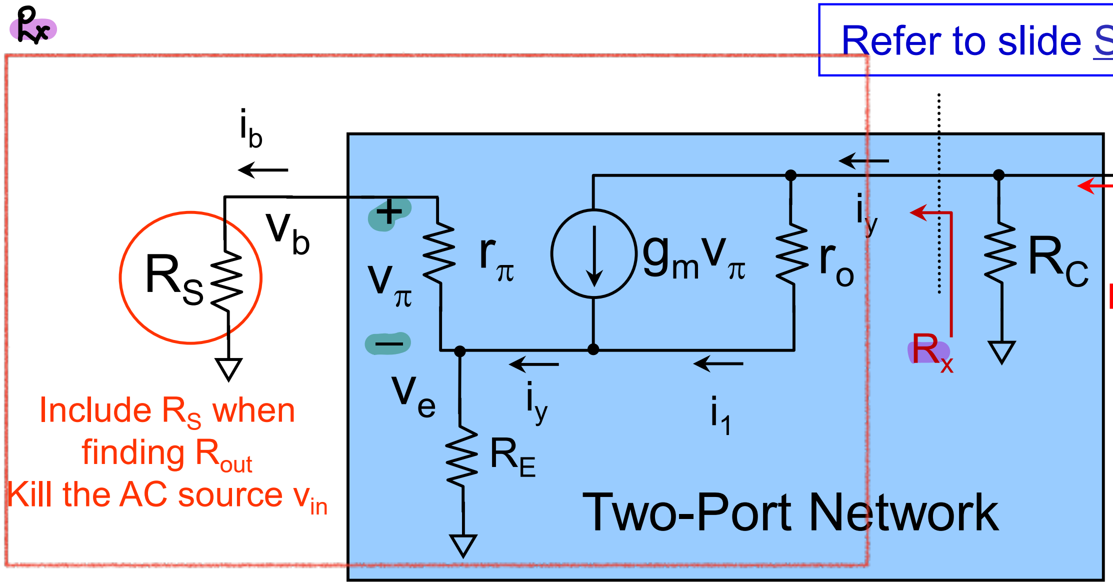
Example :

$$R_E = 200 \quad r_o = 202\text{k}$$
$$\Rightarrow R_{in} = r_\pi + (\beta + 1)R_E$$
$$= 25.5\text{k}$$

- Emitter side resistor helps boost up the input resistance looking into the base, same formula as CC

# CE with Emitter Degeneration – Two-Port Network ($R_{out}$)

Refer to slide SSA1-7 for the evaluation of $R_{out}$.




Apply test voltage $v_x$ at output and measure test current $i_x$

$R_{out}$ is just parallel combination of $R_C$ and $R_x$

$$i_1 = i_y - g_m v_\pi = i_y + g_m i_b r_\pi$$

$i_y - g_m(-i_b r_\pi)$

$$R_{out} = R_x // R_C$$

$$R_x = \frac{v_x}{i_y}$$

$$v_x = v_e + i_1 r_o = v_e + (i_y - g_m v_\pi) r_o = v_e + (i_y + g_m i_b r_\pi) r_o$$

Eliminate $v_e$, $i_b$ and keep $v_x$ and $i_y$

$$v_e = i_y[(R_S + r_\pi)//R_E]$$

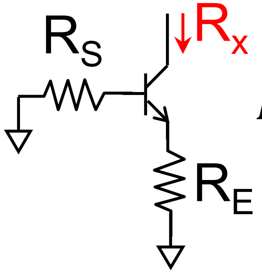
$$i_b = \frac{v_e}{R_S + r_\pi} = \frac{i_y}{R_S + r_\pi}[(R_S + r_\pi)//R_E]$$

$$\Rightarrow v_x = \cancel{i_y[(R_S + r_\pi)//R_E]} + i_y r_o + i_y g_m \left(\frac{r_\pi}{R_S + r_\pi}\right)[(R_S + r_\pi)//R_E]\, r_o$$

$$\Rightarrow R_x = \frac{v_x}{i_y} = r_o \left\{1 + g_m[(R_S + r_\pi)//R_E]\left(\frac{r_\pi}{R_S + r_\pi}\right)\right\}$$

$$\Rightarrow R_{out} = R_x // R_C \approx R_C$$

# CE with Emitter Degeneration – Two-Port Network ($R_{out}$)




$$R_x = r_o\left\{1 + g_m[(R_S + r_\pi)//R_E]\left(\frac{r_\pi}{R_S + r_\pi}\right)\right\}$$

**Important Result:**

If you see a BJT connected in similar fashion, the equivalent resistance looking into the collector ($R_x$) is directly given by the formula.

**No need to rederive.**

Example :

$R_S = 100 \quad r_\pi = 5.26\text{k}$

$R_E = 200 \quad r_o = 202\text{k}$

$g_m = 19\text{m}$

$\Rightarrow R_x \approx 928\text{k}$

$\Rightarrow R_{out} \approx R_C = 2\text{k}$

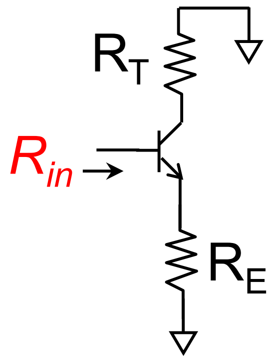
- Emitter side resistor helps boost up the transistor output resistance ($R_x$)
- Very similar to CB configuration. Same formula as CB if $R_S = 0$.

# CE with Degeneration – Important Results

1

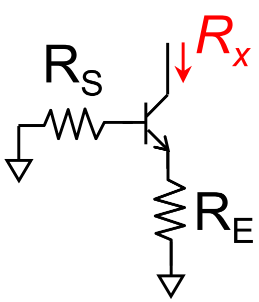
**Important Result :**
Transconductance ($G_m$) is just $g_m/(1+g_mR_E)$

2




$$R_{in} = r_\pi + (\beta + 1)R_E$$
$$\approx r_\pi[1 + g_mR_E]$$

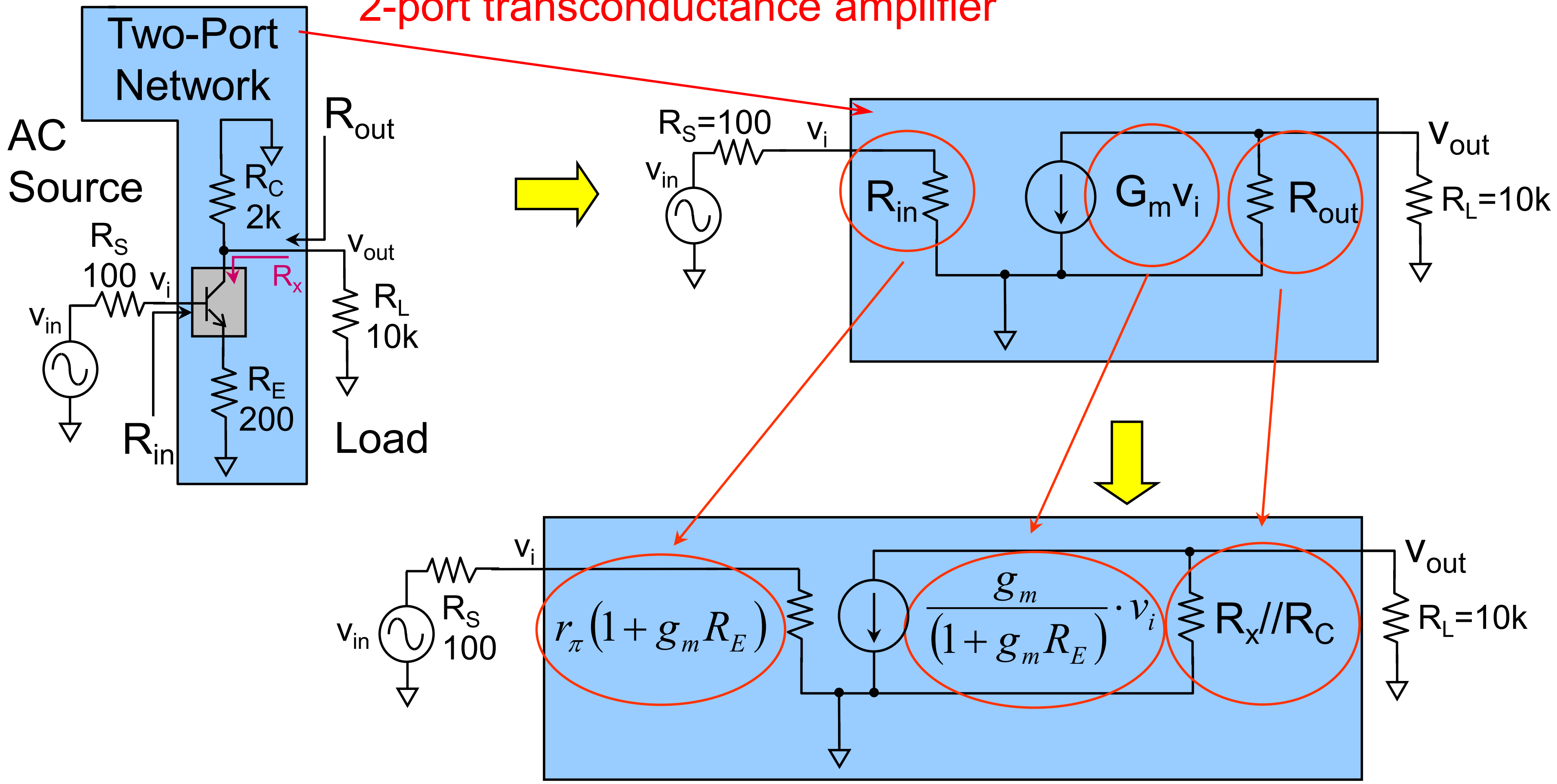
3



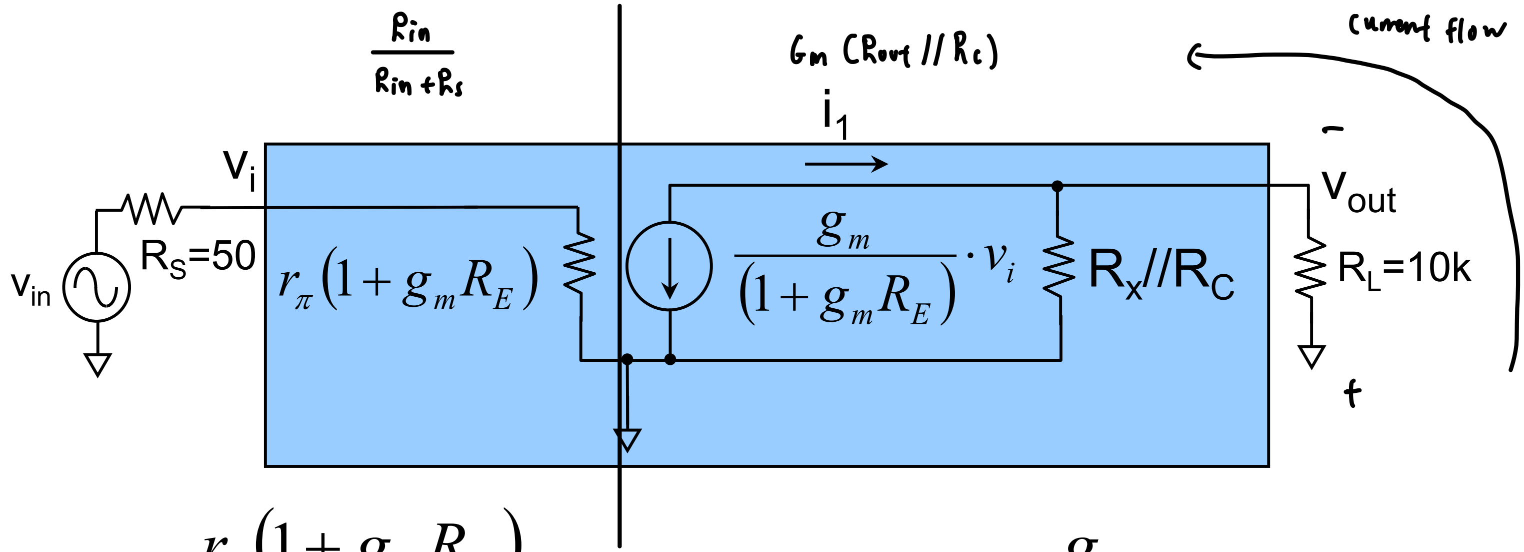

$$R_x = r_o\left\{1 + g_m[(R_S + r_\pi)//R_E]\left(\frac{r_\pi}{R_S + r_\pi}\right)\right\}$$

# CE with Emitter Degeneration – Two-Port Network

Transform the CE with degeneration amplifier to 2-port transconductance amplifier



# CE with Emitter Degeneration – Two-Port Network ($A_V$)




$$v_i = \frac{r_\pi(1+g_m R_E)}{R_S + r_\pi(1+g_m R_E)} v_{in} \qquad i_1 = -\frac{g_m}{1+g_m R_E} v_i$$

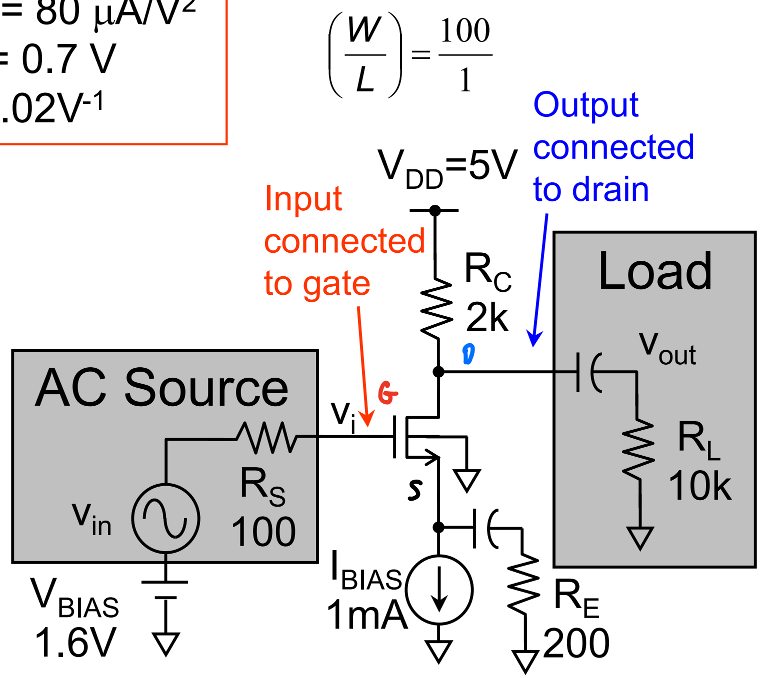
$$v_{out} = i_1 \times \left[(R_x // R_C) // R_L\right]$$

$$= -\frac{g_m}{1+g_m R_E} v_i \times \left[(R_x // R_C) // R_L\right]$$

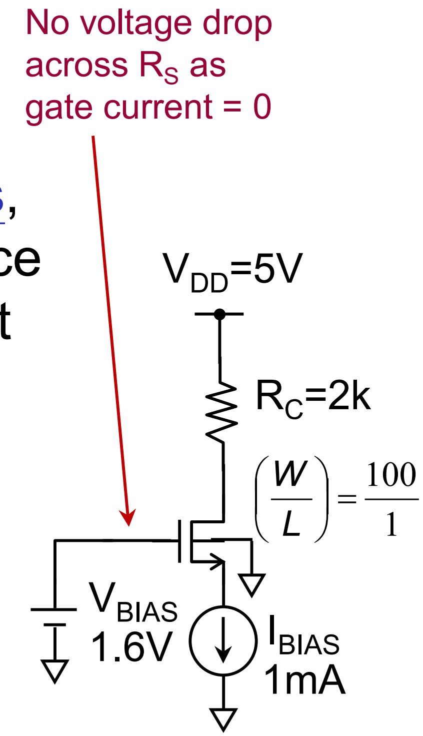
$$A_V = \frac{v_{out}}{v_{in}} = -\frac{r_\pi(1+g_m R_E)}{R_S + r_\pi(1+g_m R_E)} \times \frac{g_m}{1+g_m R_E} \times \left[(R_x // R_C) // R_L\right]$$

# CS with Source Degeneration

$\mu_n C_{ox} = 80\ \mu A/V^2$
$V_{THN} = 0.7$ V
$\mu_n = 0.02V^{-1}$



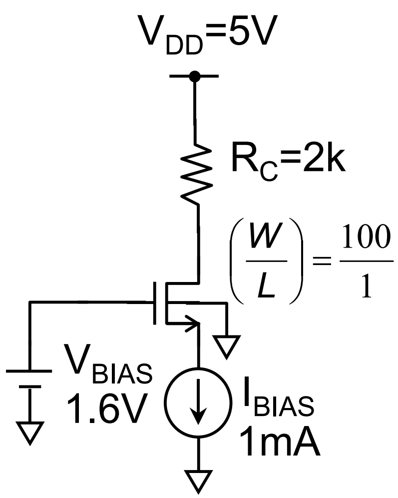

For DC analysis, remove AC source and open circuit capacitors




- Identify Source and Load
- To identify amplifier configuration, we need to consider AC equivalent circuit, i.e., short circuit capacitors
  - Input connected to Gate, output connected to Drain, Source connected to neither input nor output and is connected to a resistor $\Rightarrow$ CS with Source Degeneration

# DC Analysis

- Remove AC source and open circuit capacitors when doing DC analysis




$\mu_n C_{ox}$ = 80 μA/V²
$V_{THN}$ = 0.7 V
$\mu_n = 0.02V^{-1}$

**Determine DC biasing**

$$I_D = I_S = I_{BIAS} = 1mA$$

$$\left(\frac{W}{L}\right) = \frac{100}{1}$$

**Determine AC small-signal parameters**

$$g_m = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right) I_D}$$

$$= 4\,\text{mA/V}$$

MOSFET has body effect (body not shorted to source)

Good approximation, no need to go through detailed calculations

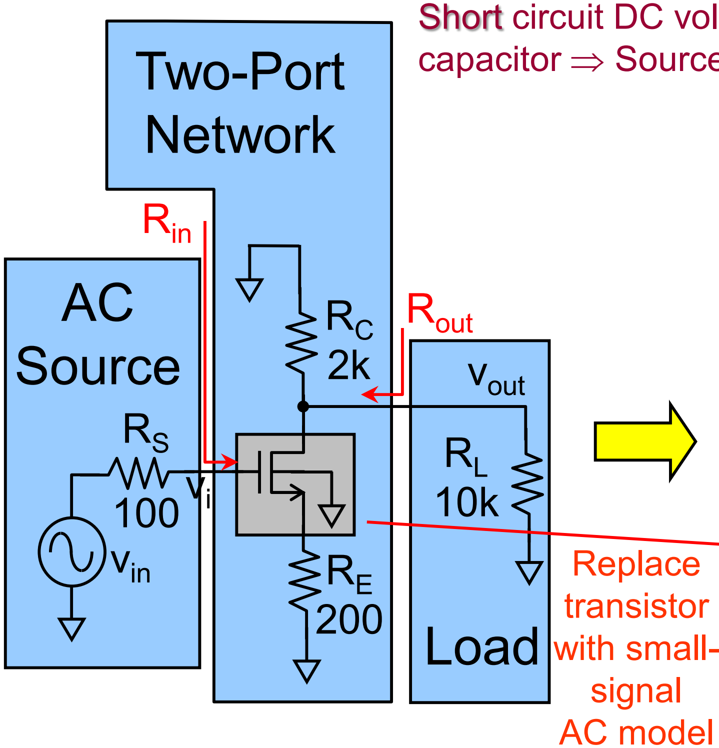
$$\boxed{g_{mb} \approx -\frac{g_m}{4} = -1\,\text{m}A/V}$$

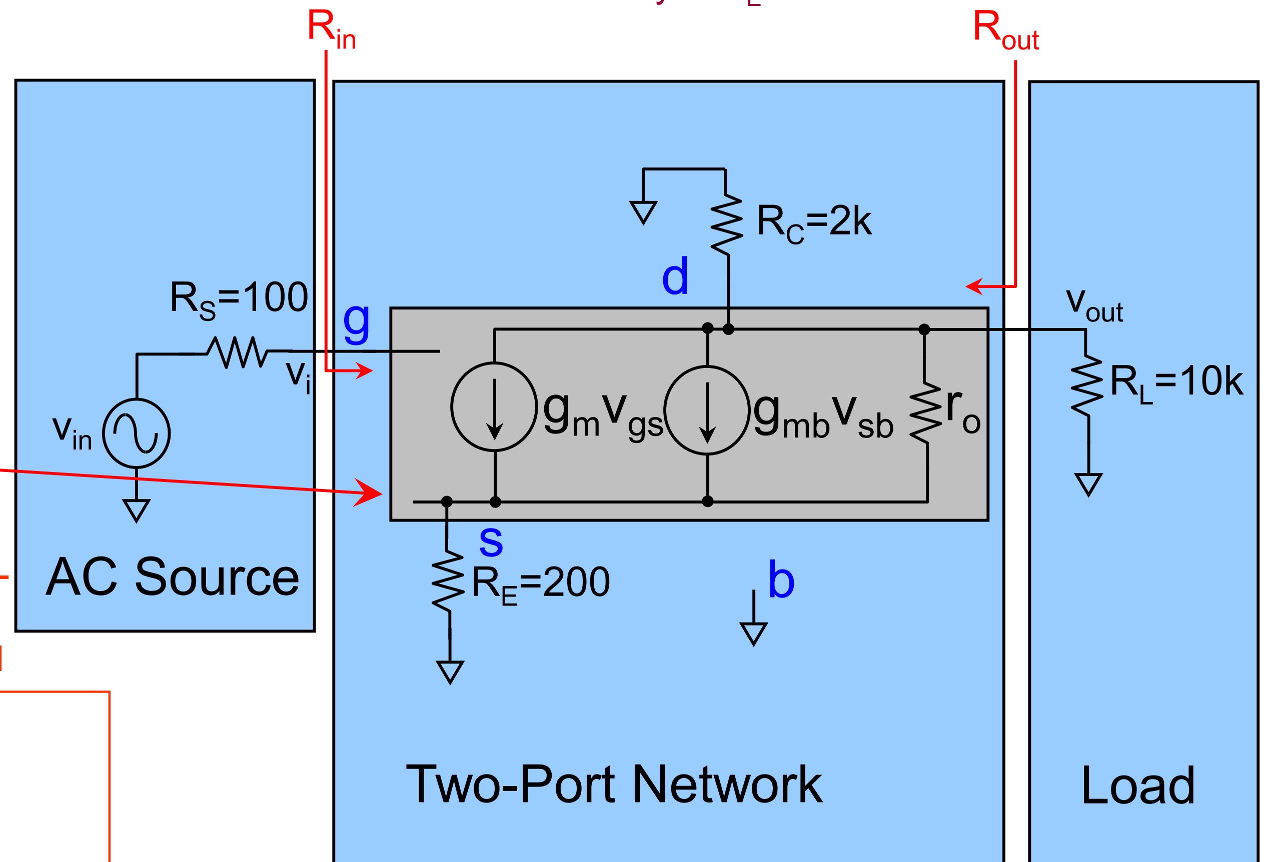
$$r_i = \infty$$

$$r_o = \frac{1}{\lambda_n I_D} = 50\,\text{k}\Omega$$

# AC Analysis (Self Reading)

**Short** circuit DC voltage source, **open** circuit DC current source, **short** circuit capacitor ⇒ Source terminal of transistor is connected directly to $R_E$




Step 1 :

Identify AC source and load

Step 2 :

Group the remaining component into two-port network

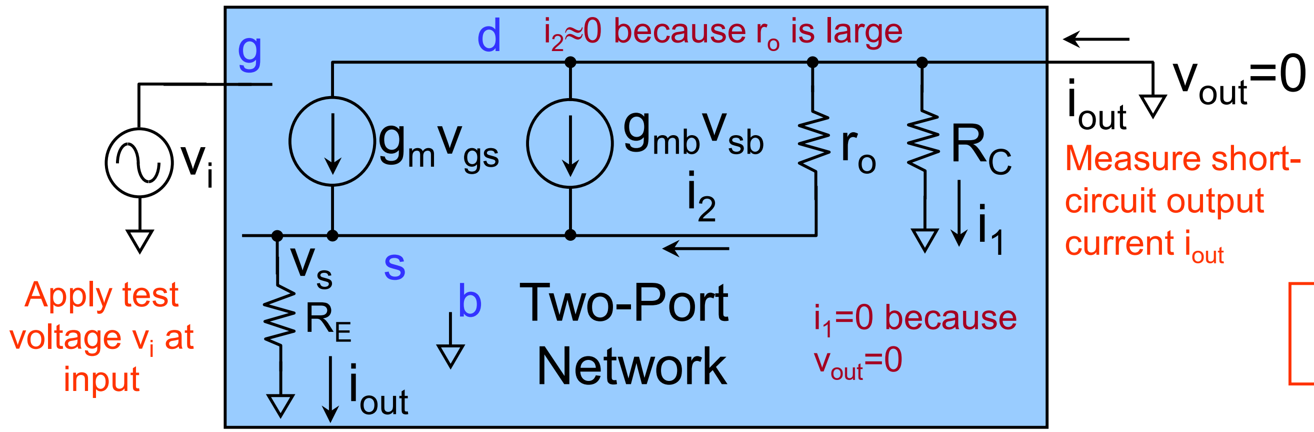
Step 3 :

Replace transistor with small-signal AC model

For CS with emitter degeneration amplifier, transform to 2-port <u>Transconductance Amplifier</u> (based on experience of circuit designers). Need to evaluate $G_m$, $R_{in}$ and $R_{out}$.

$$R_{in} = \infty$$

# CS with Source Degeneration – Two-Port Network ($G_m$) (Self Reading)




**Important Result :**
Transconductance ($G_m$) is just $g_m/[1+(g_m-g_{mb})R_E]$

$$v_{gs} = v_i - v_s \qquad v_{sb} = v_s$$

Eliminate $v_s$ and keep $v_i$ and $i_{out}$

$$G_m = \left.\frac{i_{out}}{v_i}\right|_{v_{out}=0}$$

$$\begin{cases} v_s \approx i_{out}\big|_{v_{out}=0} \times R_E \\ i_{out}\big|_{v_{out}=0} \approx g_m v_{gs} + g_{mb} v_{sb} \\ \qquad \approx g_m (v_i - v_s) + g_{mb} v_s \end{cases}$$

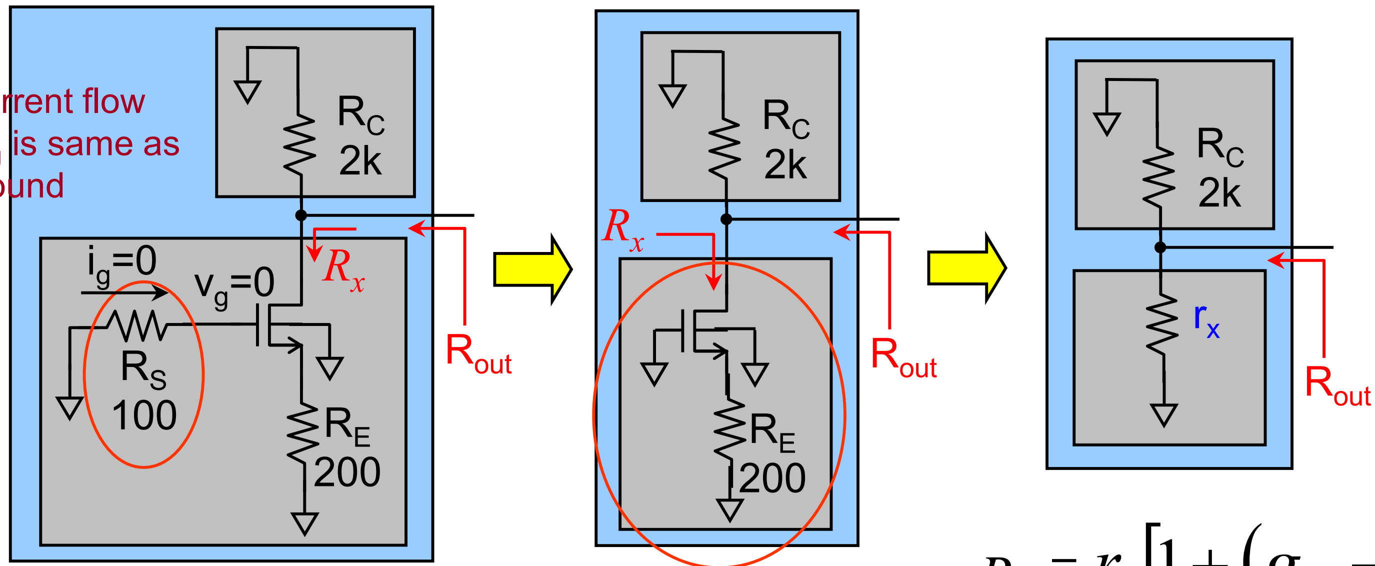
$$G_m = \frac{g_m}{1+(g_m - g_{mb})R_E} = 2\,\mathrm{mA/V}$$

$$\Rightarrow i_{out}\big|_{v_{out}=0} \approx g_m v_i - (g_m - g_{mb}) i_{out}\big|_{v_{out}=0} \times R_E$$

# CS with Source Degeneration – Two-Port Network ($R_{out}$) (Self Reading)

Since no current flow through $R_S$, $v_g$ is same as AC ground

Include $R_S$ when finding $R_{out}$
Kill the AC source $v_{in}$




Any complicated network can be reduced to an equivalent resistance (AC analysis only)

Slide SSA1-47
CG

$$R_x = r_o\left[1+\left(g_m - g_{mb}\right)R_E\right]$$
$$\approx 100\text{k}$$
$$R_{out} = R_C \,/\, R_x \approx R_C = 2\text{k}$$

- Source side resistance helps boost up the transistor output resistance ($R_x$)
- Same as CG (slide SSA1-47)

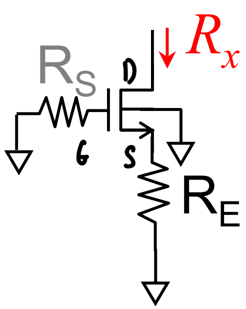
# CS with Degeneration – Important

1

**Important Result :**
Transconductance ($G_m$) is just $g_m/[1+(g_m-g_{mb})R_E]$

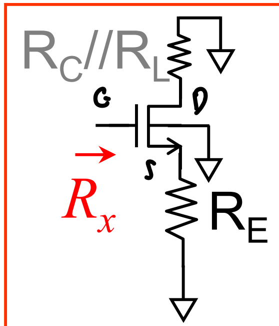
In normal CS,
$G_m = g_m$

2




$$R_x \approx r_o\left[1+\left(g_m - g_{mb}\right)R_E\right]$$

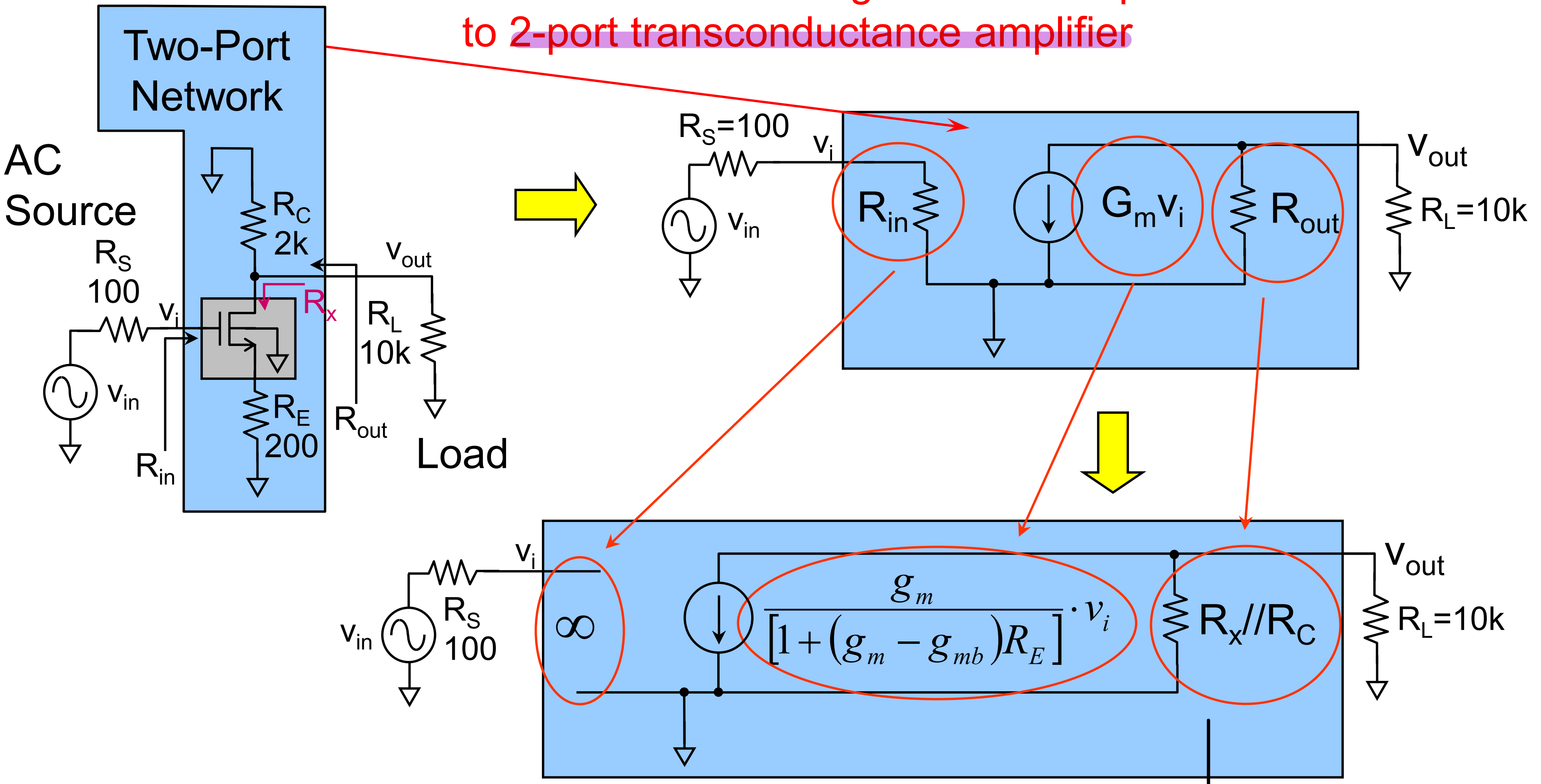
3




$$R_x = \infty$$

# CS with Source Degeneration – Two-Port Network

Transform the CS with degeneration amplifier to 2-port transconductance amplifier

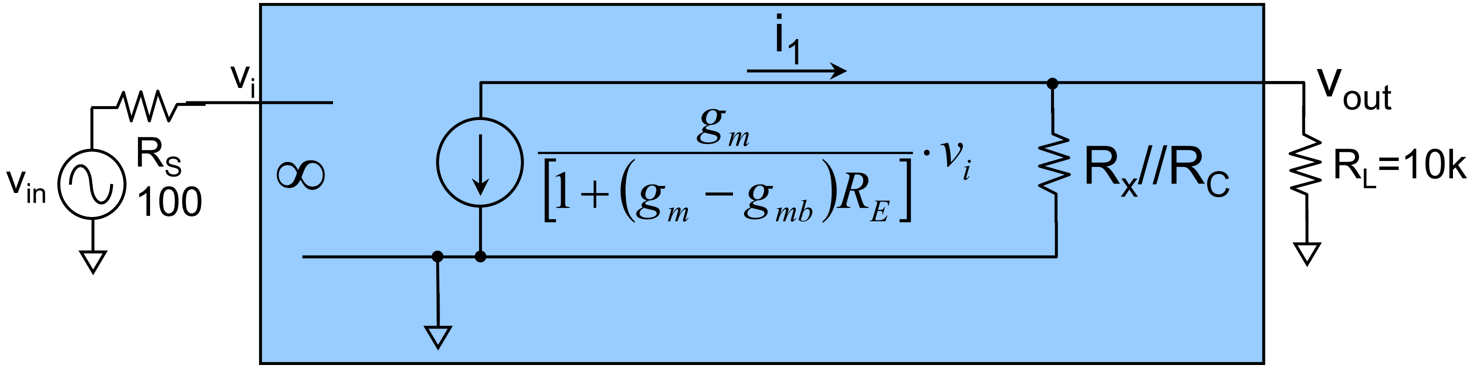



$R_C \, // \, r_o(1+?)$

(choose which to ignore, ie if $r_o(1+?) >> r_c$, can approx. it to $\approx R_c$

Only do this if you know the values

# CS with Source Degeneration – Two-Port Network ($A_V$)




$$v_i = v_{in} \qquad i_1 = -\frac{g_m}{1+(g_m-g_{mb})R_E} \times v_i$$

$$v_{out} = i_1 \times (R_{out} // R_L) = -\frac{g_m}{1+(g_m-g_{mb})R_E} \times v_i \times (R_{out} // R_L)$$

$$\approx -v_{in} \times \frac{g_m}{1+(g_m-g_{mb})R_E} \times [(R_C // R_x) // R_L]$$

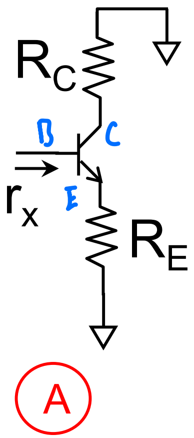
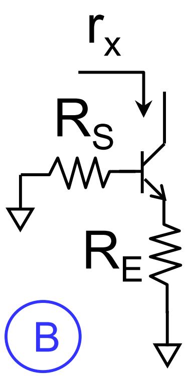
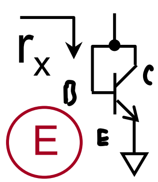
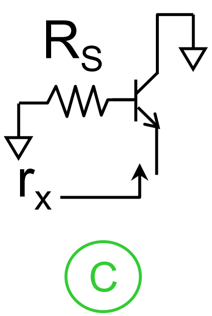
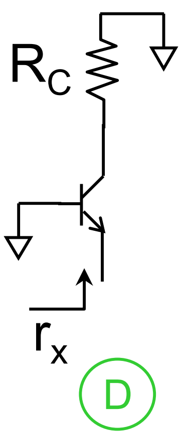
$$\Rightarrow A_V \approx -\frac{g_m}{1+(g_m-g_{mb})R_E} \times (R_C // R_L)$$

# Characteristic of CE/CS with Emitter/Source Degeneration

- High input resistance
- High output resistance
- Lower Gain than CE/CS → to get better linearity
- Polarity inversion

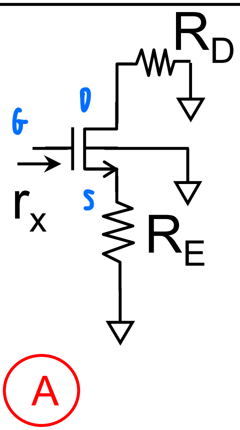
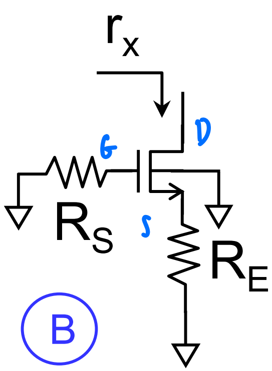
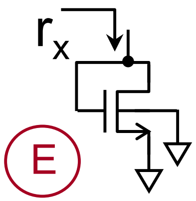
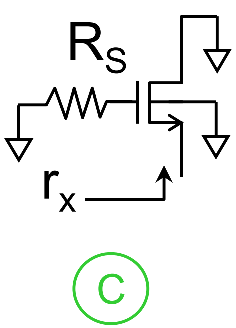
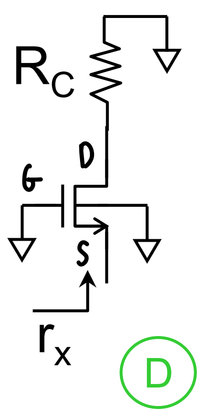
# BJT Equivalent Resistance Summary (Table 1)

Blue: look into collector terminal Red: look into base terminal Green: look into emitter terminal

| Conf | $r_x$ | Conf | $r_x$ | Conf | $r_x$ |
|---|---|---|---|---|---|
|  | $r_\pi + (1+\beta)R_E$ <br> $\approx r_\pi(1+g_m R_E)$ <br> If $R_E = 0$ <br> $r_x = r_\pi$ |  | $r_o\left\{1+g_m\left[(r_\pi+R_S)//R_E\right]\left(\frac{r_\pi}{r_\pi+R_S}\right)\right\}$ <br> If $R_S = 0$ and $r_\pi << R_E$ <br> $\Rightarrow r_{x,\max} = r_o(\beta+1)$ <br> If $R_E = 0$, $r_x = r_o$ |  | $\frac{1}{g_m}$ |
|  | $\frac{R_S+r_\pi}{1+\beta}//r_o$ <br> $\approx \frac{R_S}{1+\beta}+\frac{1}{g_m}$ <br> If $R_S = 0$ <br> $r_x \approx \frac{1}{g_m}$ |  | $\frac{1}{g_m}\times\frac{r_o+R_C}{r_o+R_C/\beta}$ <br> If $R_C = 0, r_x \approx \frac{1}{g_m}$ | | |

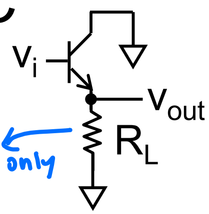
# MOS Equivalent Resistance Summary (Table 2) ********

Blue: look into drain terminal  Red: look into gate terminal  Green: look into source terminal

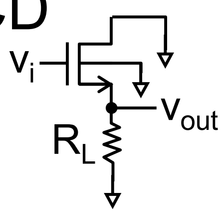
| Conf | $r_x$ | Conf | $r_x$ | Conf | $r_x$ |
|---|---|---|---|---|---|
|  A | $\infty$ |  B | If source and body shorted, $g_{mb}=0$, no body effect<br>$r_o\left[1+\left(g_m-g_{mb}\right)R_E\right]$<br>$If \quad R_E=0, r_x=r_0$ |  E | $\frac{1}{g_m}$ |
|  C | $\frac{1}{g_m-g_{mb}}$ |  D | $\frac{1}{g_m-g_{mb}}\times\frac{r_o+R_C}{r_o}$<br>$If\ R_C=0, r_x\approx\frac{1}{g_m-g_{mb}}$ | | |

Return

# BJT Amplifier Configurations (Table 3)

| BJT | $G_m$ | $A_V$ (gain) |
|---|---|---|
| CE<br>A | $g_m$ | Derive Based on 2-port Network<br>-ve<br>transconductance |
| CB<br>B | $-g_m$ | Derive Based on 2-port Network<br>+ve<br>transconductance |
| CC<br>C  <br>simplify to only 1 resistor | Too Complex To Be Useful<br>Two port voltage | $\dfrac{g_m R_L}{1+g_m R_L}$<br>+ve<br>voltage amplifier |
| CE with Degeneration ($R_E$)<br>D | $\dfrac{g_m}{1+g_m R_E}$ | Derive Based on 2-port Network<br>-ve<br>transconductance |

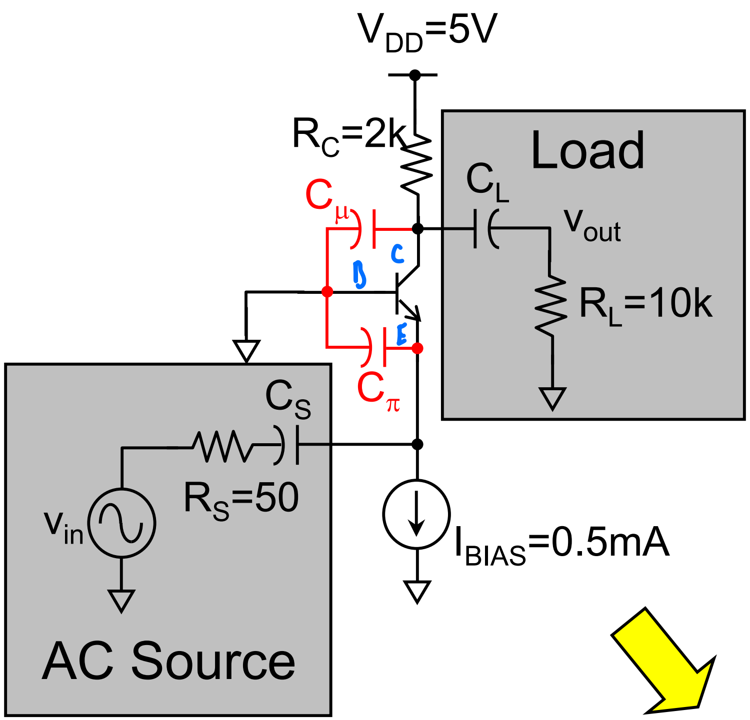
# MOS Amplifier Configurations (Table 4)

| MOS | $G_m$ | $A_V$ |
|---|---|---|
| CS<br>A | $g_m$ | Derive Based on 2-port Network<br>transconductance |
| CG<br>B | $-(g_m - g_{mb})$<br>*Drop $g_{mb}$ if no body effect* | Derive Based on 2-port Network<br>transconductance |
| CD<br>C<br><br> | Too Complex To Be Useful | $\frac{g_m R_L}{1+(g_m - g_{mb})R_L} \approx \frac{g_m}{g_m - g_{mb}}$<br>*Drop $g_{mb}$ if no body effect*<br>voltage amplifier |
| CS with degeneration ($R_E$)<br>D | $\frac{g_m}{1+(g_m - g_{mb})R_E}$<br>*Drop $g_{mb}$ if no body effect* | Derive Based on 2-port Network<br>transconductance |

# Effect of Capacitors

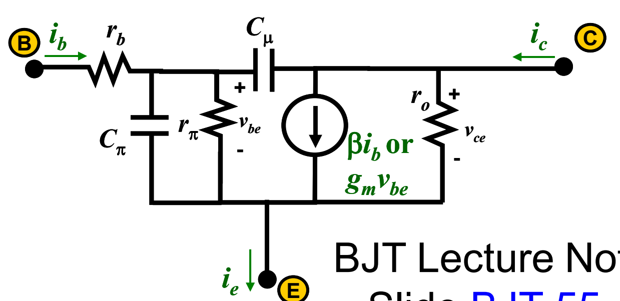
Consider CB amplifier example (Slide SSA1-50):

- $C_L$ and $C_S$ are external capacitors added to the circuit, they are usually large (~µF range)
- $C_\mu$ and $C_\pi$ are parasitic capacitors inherent to BJT, they are usually small (~pF range)

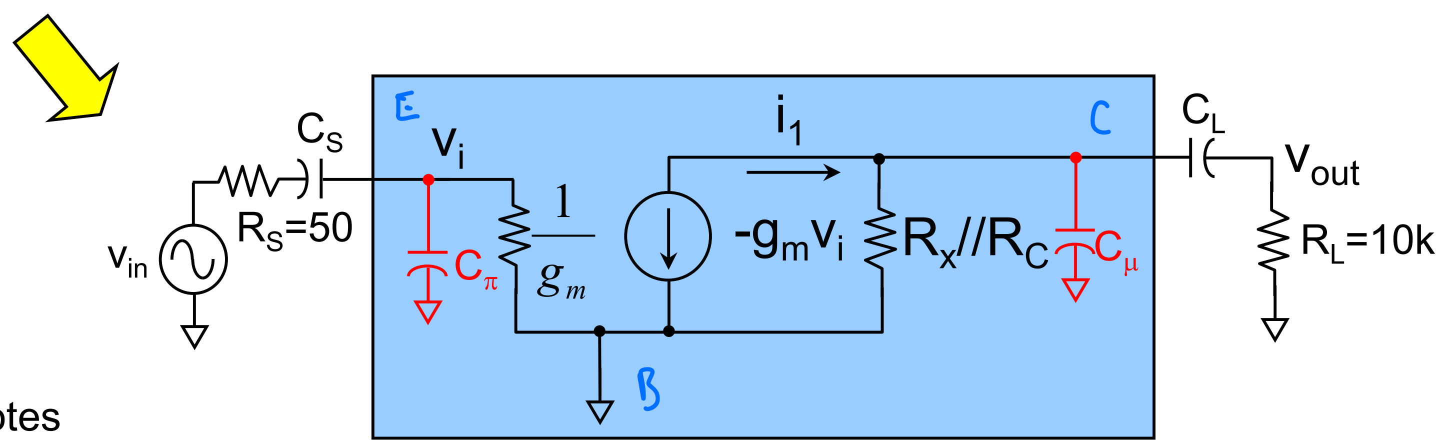



| | Low freq (→DC) | Mid freq | High freq |
|---|---|---|---|
| $C_S$, $C_L$ | Open | Short | Short |
| $C_\mu$, $C_\pi$ | Open | Open | Short |

Audio : <20kHz
Video : <6MHz
RF : ~GHz

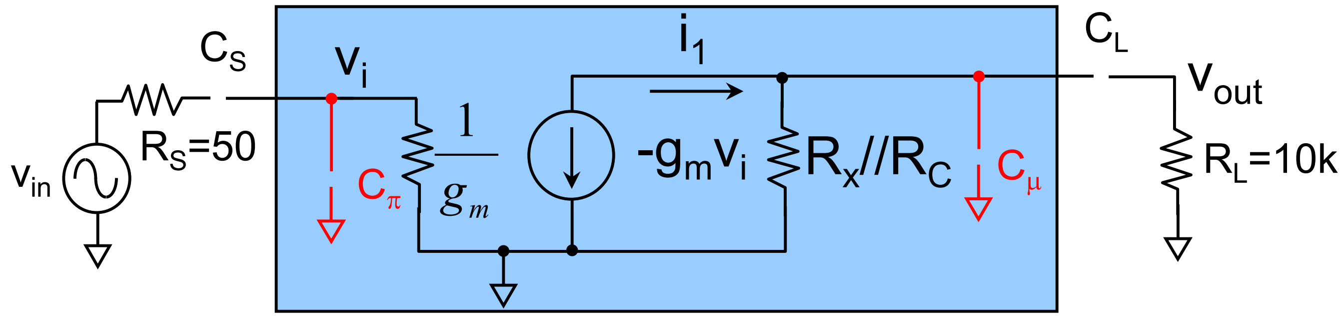



BJT Lecture Notes – Slide BJT-55

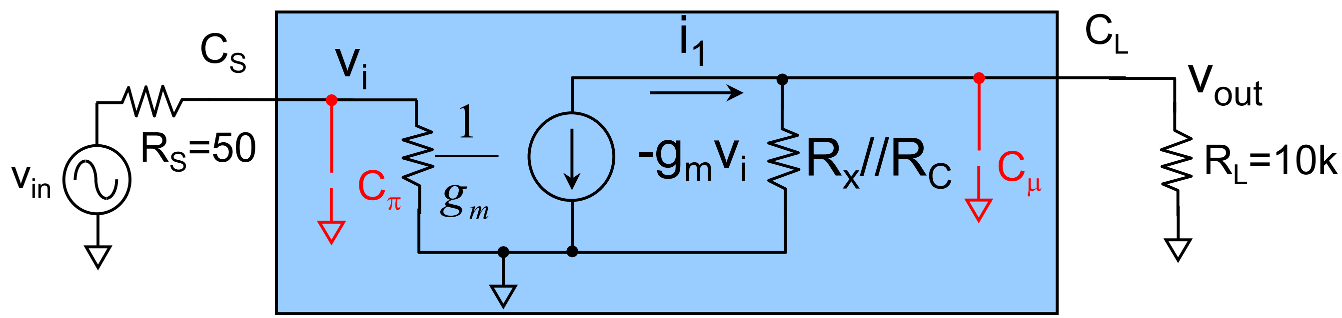


# Effect of Capacitors

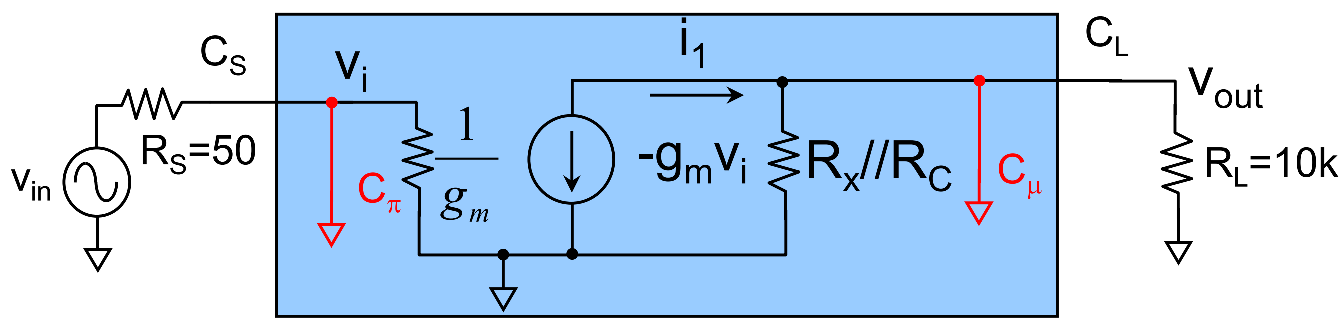
At low frequency,
$C_S$, $C_L$, $C_\mu$, $C_\pi \rightarrow$ open circuit
$v_{out}$ isolated from $v_{in} \Rightarrow v_{out} = 0$



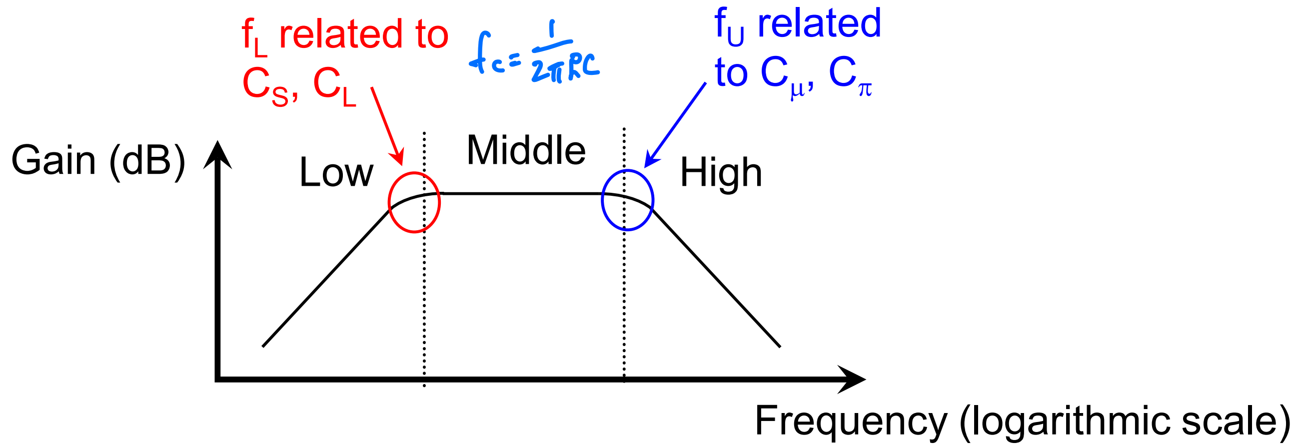

At mid frequency,
$C_S$, $C_L \rightarrow$ short circuit
$C_\mu$, $C_\pi \rightarrow$ open circuit
2-port network $\Rightarrow v_{out} = A_V \times v_{in}$




At high frequency,
$C_S$, $C_L \rightarrow$ short circuit
$C_\mu$, $C_\pi \rightarrow$ short circuit
$v_{in}$ and $v_{out}$ short to ground
$\Rightarrow v_{out} = 0$



# Frequency Response




- $C_S$ and $C_L$ cause the gain to drop at low frequency due to open circuit at low frequency.
- $C_\mu$ and $C_\pi$ cause the gain to drop at high frequency due to short circuit at high frequency.
- At the mid frequency, $C_S$ and $C_L$ are short circuit whereas $C_\mu$ and $C_\pi$ are open circuit. The amplifier behaves like 2-port network with gain.
- The lower cut-off point ($f_L$), i.e., the frequency below which the gain starts to drop is related to RC time constant related to $C_S$ and $C_L$.
- The upper cut-off point ($f_U$), i.e. the frequency above which the gain starts to drop is related to RC time constant related to $C_\mu$ and $C_\pi$.

# Definition of Decibel

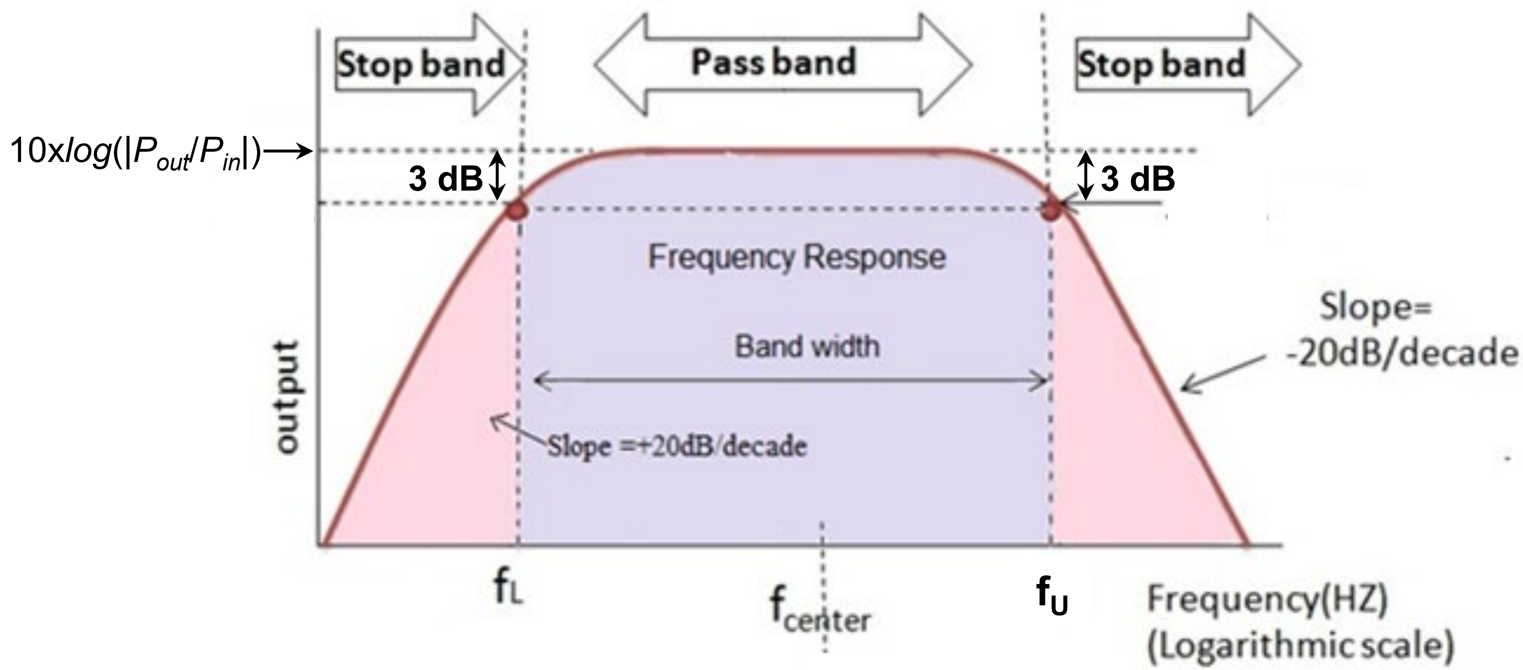
$$\textit{Decibel (dB)}[\textit{Power Ratio}]$$

$$Y(dB) = 10 \times \log\left(\left|\frac{P_{out}}{P_{in}}\right|\right) = 10 \times \log(|\textit{Power Gain}|)$$

$$\textit{Decibel (dB)}[\textit{Voltage Ratio}]$$

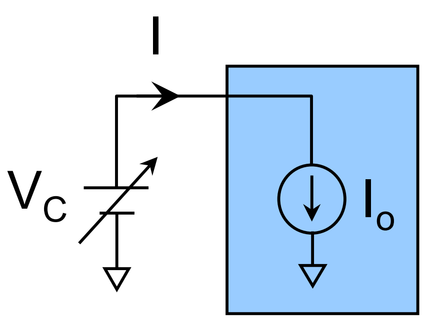
$$Y(dB) = 20 \times \log\left(\left|\frac{V_{out}}{V_{in}}\right|\right) = 20 \times \log(|\textit{Gain}|)$$

Half-power point of an electronic amplifier is the frequency at which the output power has dropped to half of its mid-band value (equivalent to a drop of 3 dB). The half-power point is a commonly used definition of cut-off frequency (see slides 1-30 & 1-31)

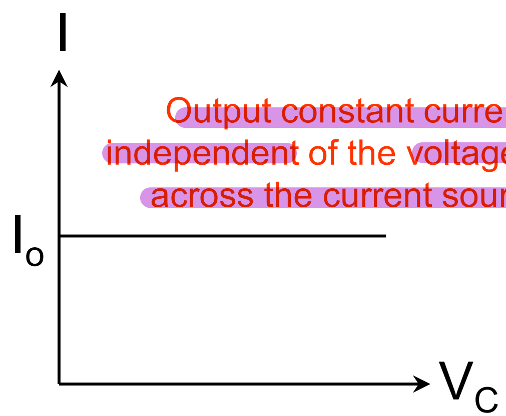


# Ideal versus Non-ideal Current Source

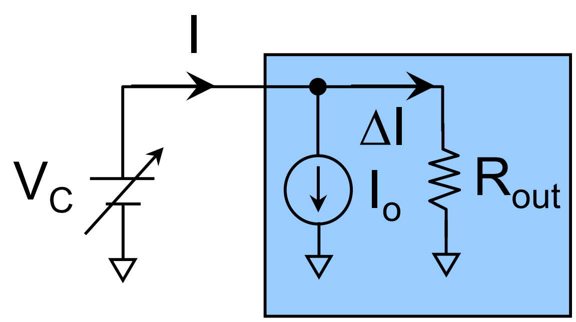
## Ideal Current Source



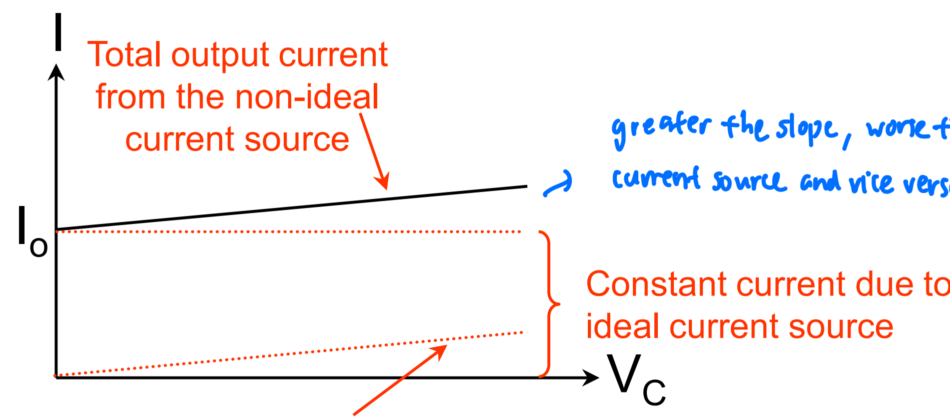

Ideal current source is simply a parallel combination of ideal current source ($I_o$) with infinite output resistance ($\infty$)




## Non-ideal Current Source




Non-ideal current source is simply a parallel combination of ideal current source ($I_o$) with finite output resistance ($R_{out}$)

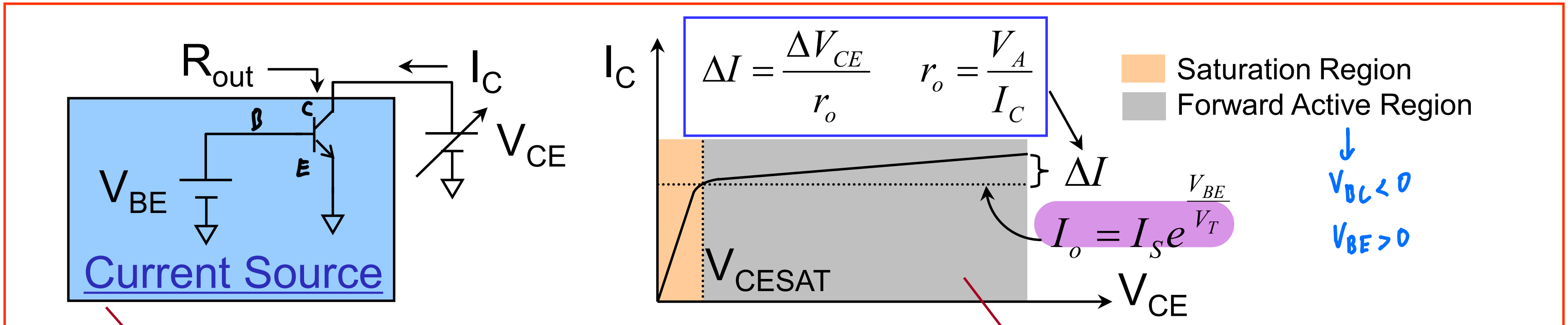


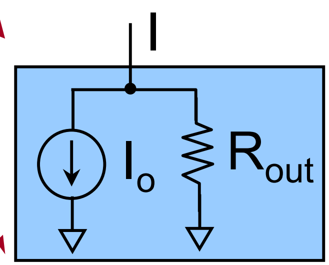

greater the slope, worse the current source and vice versa

Small variable current ($\Delta I$) due to finite output resistance ($R_{out}$)

$$\Delta I = \frac{V_C}{R_{out}}$$

# Transistor as Current Source

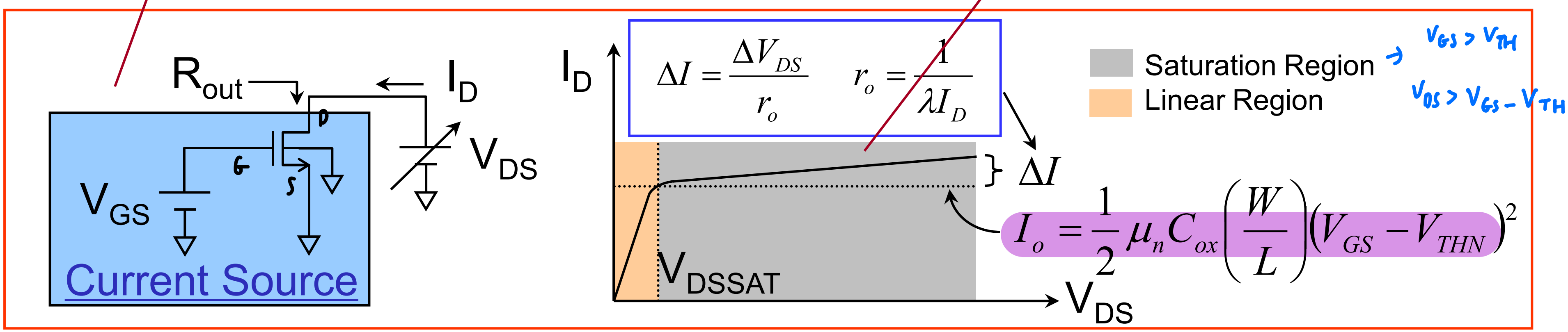


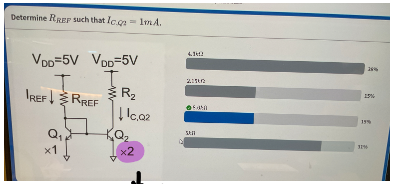

$R_{out}=r_o=V_A/I_C$ for BJT transistor current source

$R_{out}=r_o=1/(\lambda I_D)$ for MOS transistor current source

Transistor with voltage bias behaves very similarly to non-ideal current source as long as it operates in **GRAY** region

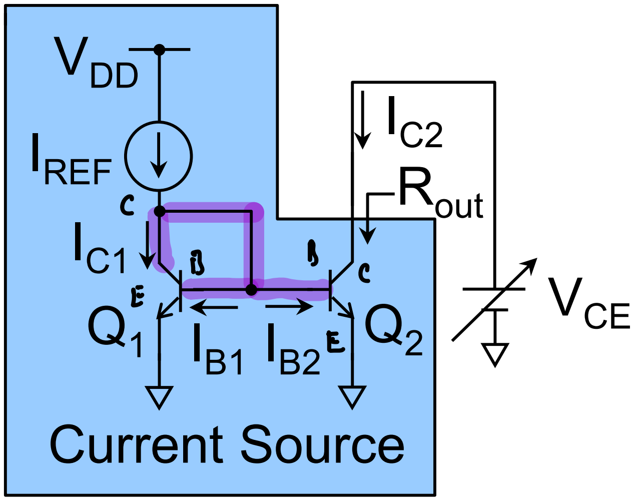


# BJT Current Mirror




↓ $I_{C,Q2} = 1mA$, since 2 times of $Q_1$, $I_{C,Q1} = 0.5mA$

$I_{ref}(0.5m) = \frac{5 - V_{BE}}{R_{ref}} = \frac{5-0.7}{0.5m} = 8.6k$




## Determine DC Biasing

$$I_{REF} = I_{C1} + I_{B1} + I_{B2}$$

$$\because V_{BE1} = V_{BE2} \rightarrow I_C = I_S e^{\frac{V_{BE}}{V_T}}$$

$$\Rightarrow I_{B1} = I_{B2} = \frac{I_{C1}}{\beta}$$

$$\Rightarrow \boxed{I_{C2} \approx I_{C1} \approx \frac{\beta}{\beta+2} I_{REF}}$$

$I_{C2}$ not exactly equal to $I_{REF}$ due to finite current gain $\beta$

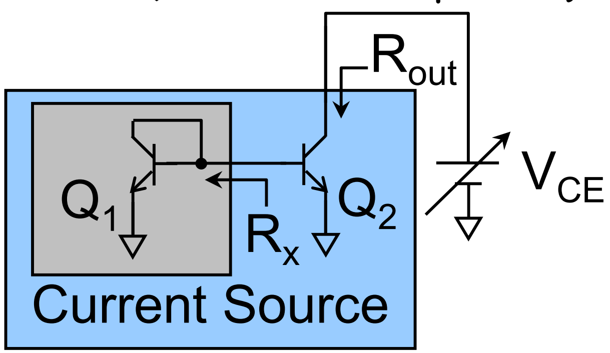
## Determine AC small signal parameters

$$g_{m1} = g_{m2} = \frac{I_C}{V_T}$$

$$R_{out} = r_o = \frac{V_A}{I_C}$$

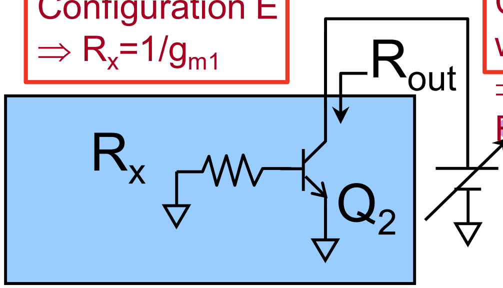
## AC Analysis

(Current source open circuit)



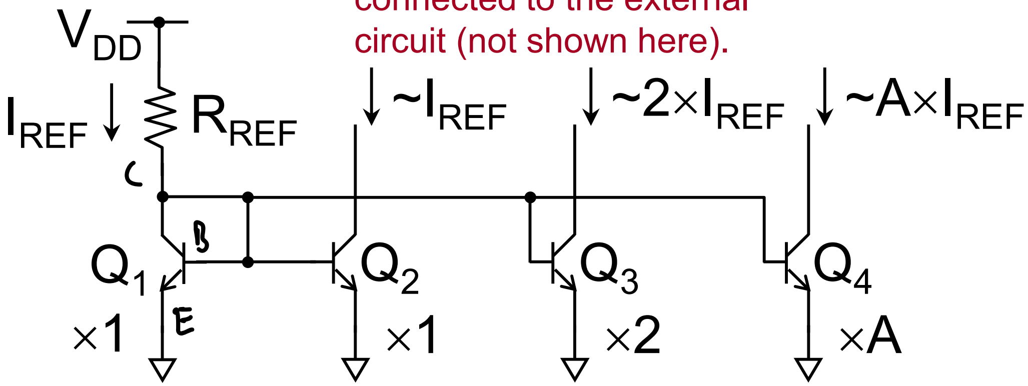

Look-up Table 1

Configuration E
$\Rightarrow R_x = 1/g_{m1}$




Configuration B with $R_E = 0$
$\Rightarrow$
$R_{out} = r_o = V_A/I_{C2}$

# BJT Current Mirror




$$I_{REF} = \frac{V_{DD} - V_{BE}}{R_{REF}}$$

$I_{REF}$ can be obtained easily through a resistor

$$I_{C1} \approx I_{REF} = J_S e^{\frac{V_{BE}}{V_T}} \times Area^{*}$$

$$I_{C2} \approx I_{C1} \quad I_{C3} \approx 2 \times I_{C1}$$

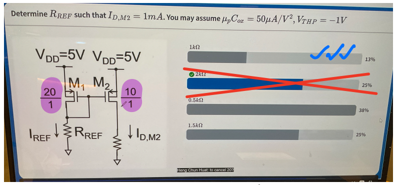
$$I_{C4} \approx A \times I_{C1}$$

- Using same $V_{BE}$ created from the reference branch, different current sources can be created by laying out different area of BJT
- The current source current is not identical to $I_{REF}$ due to:
  - Finite current gain $\beta$
  - Finite output resistance $r_o$ and different $V_{CE}$ from the reference branch

*Area refers to the cross-sectional area of the BJT active region (i.e., base-emitter junction), see BJT notes, slide BJT-2.

*$J_S$ is the Saturation current density of BJT = $I_S$ / Area

# MOS Current Mirror




$I_{D,M1}$ have to be 2times bigger,

$2mA = \frac{1}{2} \mu_p C_{ox} \frac{W}{L} (|V_{GS}| - |V_{TH}|)^2$

$4 = (|V_{GS}| - |V_{TH}|)^2$

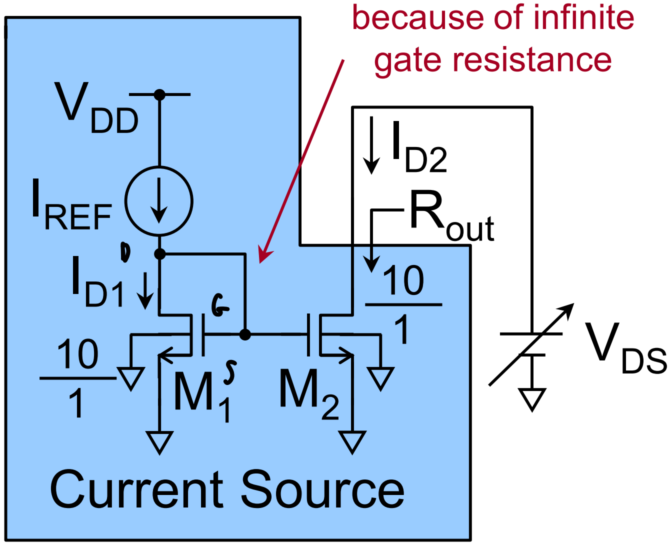
$2 = |V_{GS}| - 1$

$|V_{GS}| = 3$

$R_{ref} = \frac{V_{DD} - |V_{GS}|}{2m}$

$= 1k\Omega$ //

No gate current because of infinite gate resistance




## Determine DC Biasing

$$I_{REF} = I_{D1}$$

$$\because V_{GS1} = V_{GS2}$$

$$\Rightarrow \boxed{I_{D1} = I_{D2} = I_{REF}}$$

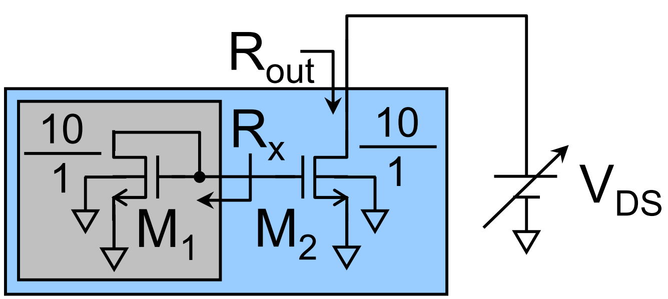
## Determine AC small signal parameters

$$g_{m1} = g_{m2}$$

$$= \sqrt{2\mu_n C_{ox}\left(\frac{10}{1}\right) I_{D1}}$$

$$R_{out} = r_o = \frac{1}{\lambda I_{D2}}$$

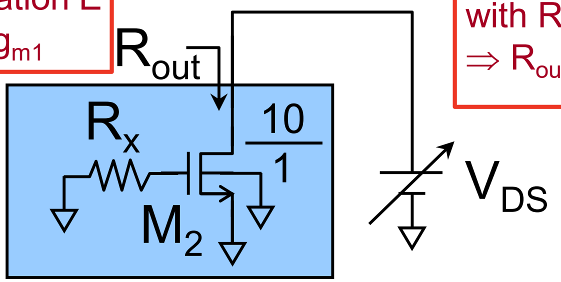
AC Analysis




Look-up Table 2

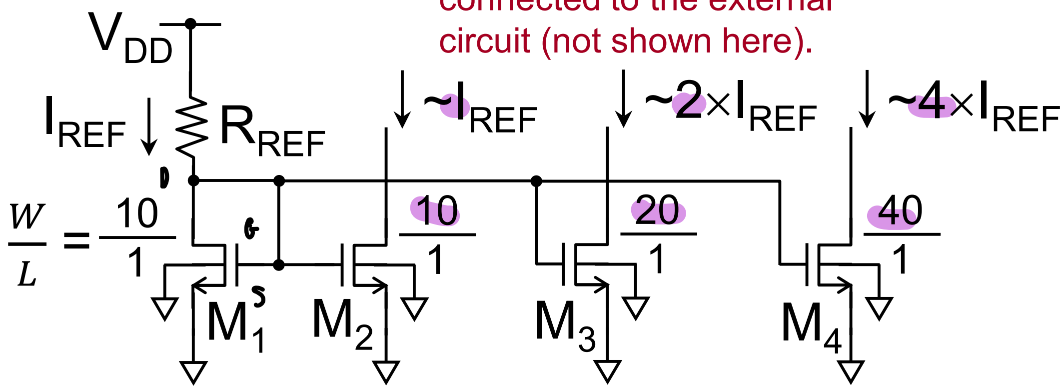
Configuration E
$\Rightarrow R_x = 1/g_{m1}$

Configuration B with $R_E=0$
$\Rightarrow R_{out} = r_o = 1/\lambda I_{D2}$



# MOS Current Mirror

The drain should be connected to the external circuit (not shown here).




Because all MOSFETs share the same $V_{GS}$, the transistor currents are proportional to their corresponding sizing ratio

$$I_{REF} = \frac{V_{DD} - V_{GS}}{R_{REF}}$$

$I_{REF}$ can be obtained easily through a resistor

$$I_{D1} = I_{REF}$$

$$= \frac{\mu_n C_{ox}}{2}\left(\frac{W}{L}\right)_{M1}(V_{GS} - V_{THN})^2$$

$$I_{D2} = I_{D1} \qquad I_{D3} = 2 \times I_{D1} \qquad I_{D4} = 4 \times I_{D1}$$

- Using same $V_{GS}$ created from the reference branch, different current sources can be created by laying out different sizing (e.g., *w*/*L*) of MOS transistors
- The current source current is not identical to $I_{REF}$ due to finite output resistance $r_o$ and different $V_{DS}$ from the reference branch

# EE2027 Electronic Circuits

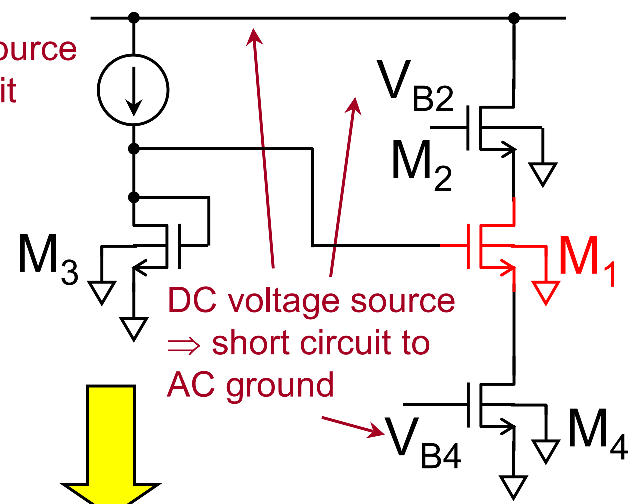
## Multi-Stage Amplifier

### Lecture Outline

- Multi-Stage Amplifier Analysis Concept

# Old Ways of Analyzing Circuits

DC current source
⇒ open circuit

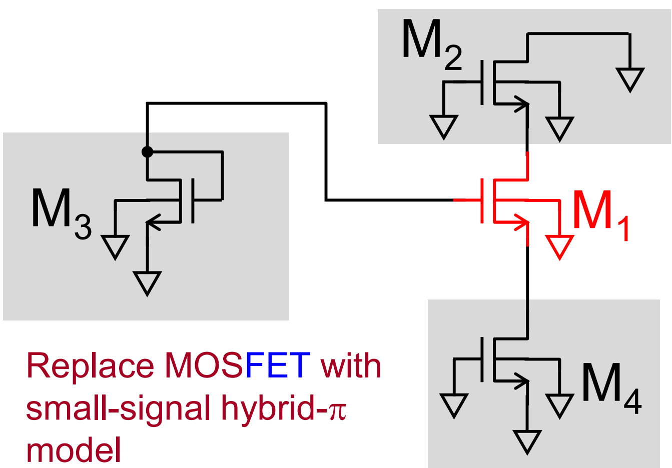



AC
Circuit

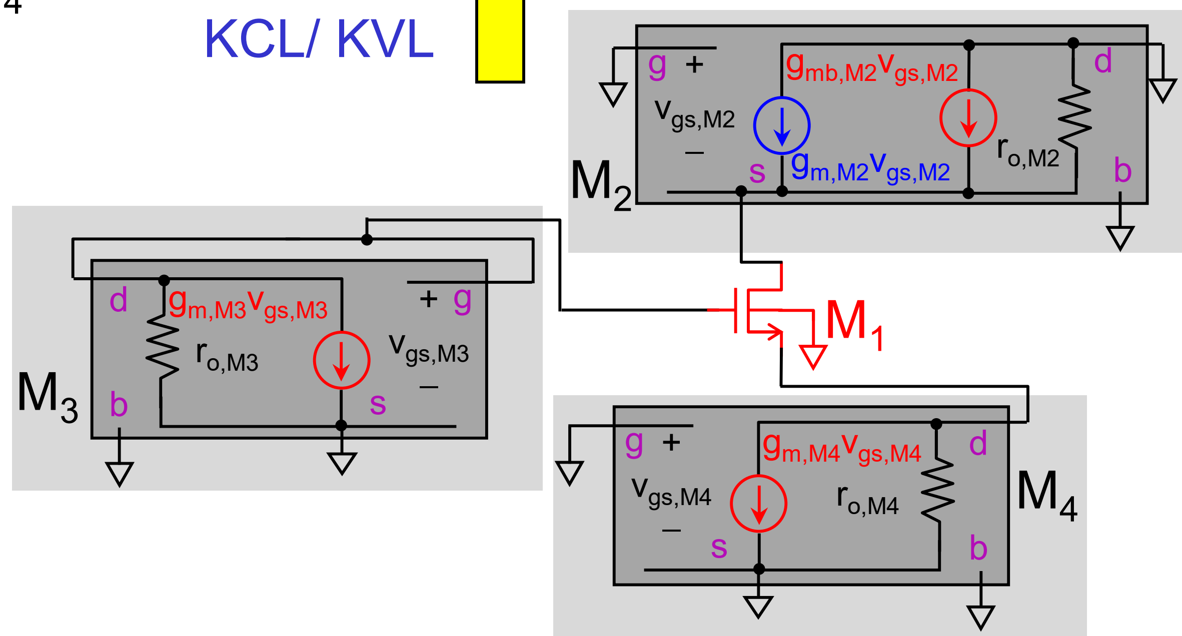
$$v_{g,M1} = v_{gs,M3} = -g_{m,M3}v_{gs,M3} \times r_{o,M3} \cdots (1)$$
$$v_{s,M1} = v_{ds,M4} = \left(i_{d,M1} - g_{m,M4}v_{gs,M4}\right) \times r_{o,M4} \cdots (2)$$
$$\vdots$$

Tedious simultaneous algebra and difficult to solve with number of transistors >2

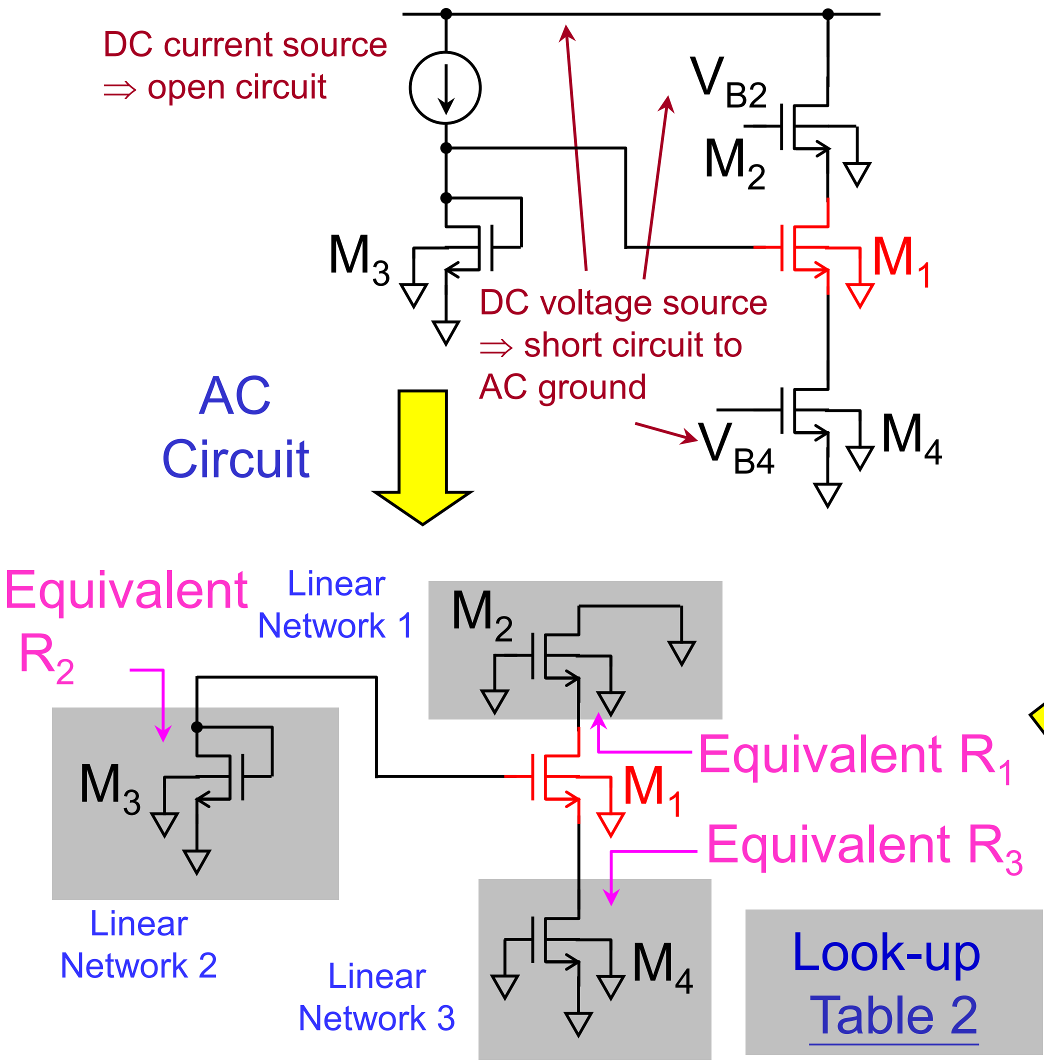
KCL/ KVL

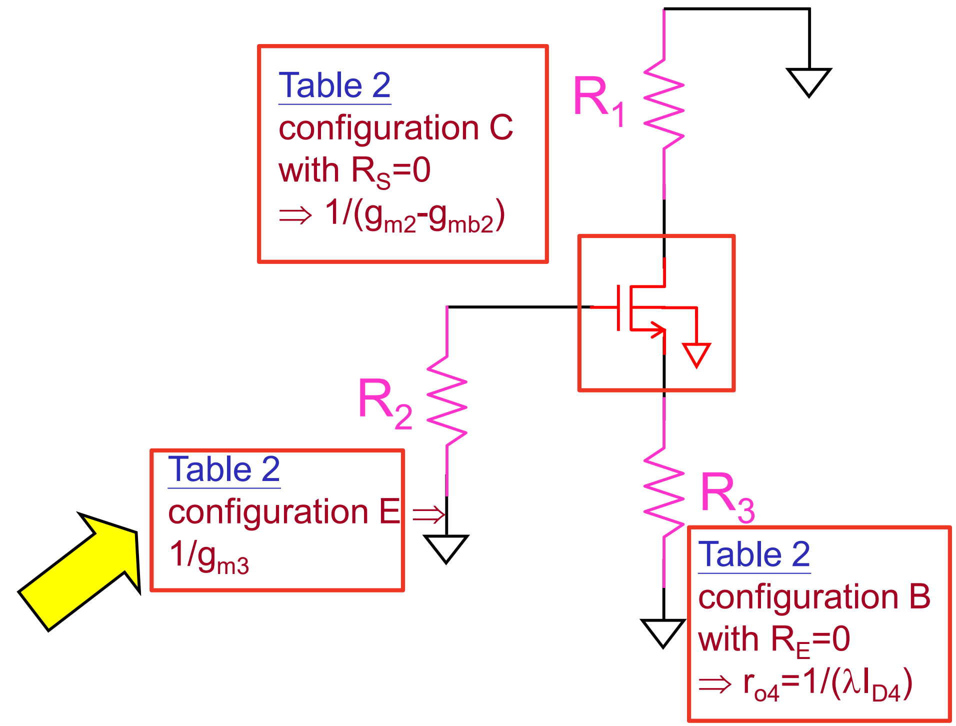



Replace MOSFET with small-signal hybrid-π model



# AC Analysis of Complicated Circuit

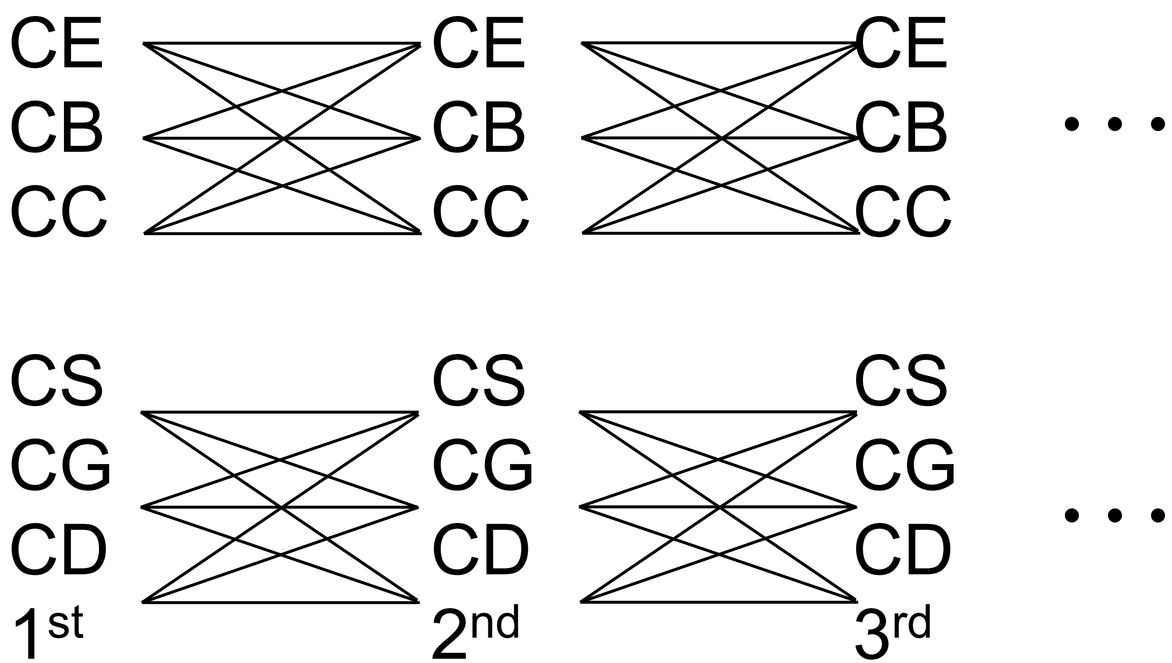



Identify familiar transistor configurations and apply formula directly for the equivalent resistance

# Multi-Stage Amplifier

Multi-stage amplifier is cascade of single-stage amplifiers-




1. We can mix the usage of BJT or MOS amplifier
2. We can extend by more than 3 stages
3. CE/CS with degeneration can also be used.

How many combinations can we have?

Why multi-stage?

- To modify $R_{in}$, $R_{out}$ or the overall gain

# Steps for Multi-Stage Amplifier Analysis

**DC Analysis**

Remove AC source

Replace capacitor with open circuit

Obtain DC biasing of transistor

→ Find $I_D$, $I_C$

Obtain AC small-signal parameters of transistor (e.g., $g_m$, $r_\pi$, $r_o$)

**AC Analysis**

DC voltage source ⇒ short circuit
DC current source ⇒ open circuit

Replace capacitor with short circuit

Identify AC source, load and **different single-stage amplifiers** within the multi-stage amplifier

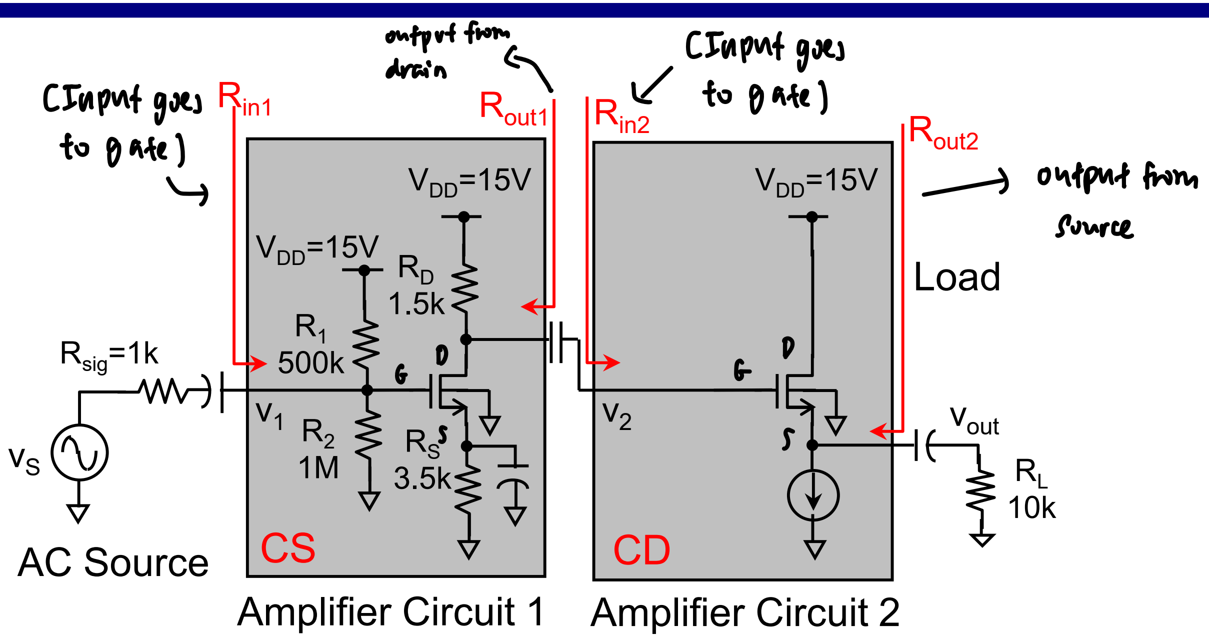
Finding the two-port equivalent parameters associated with each single-stage amplifier, i.e., $R_{in}$, $R_{out}$, $G_m$/A

Replace each single-stage amplifier with the corresponding two-port network and find the $A_V$

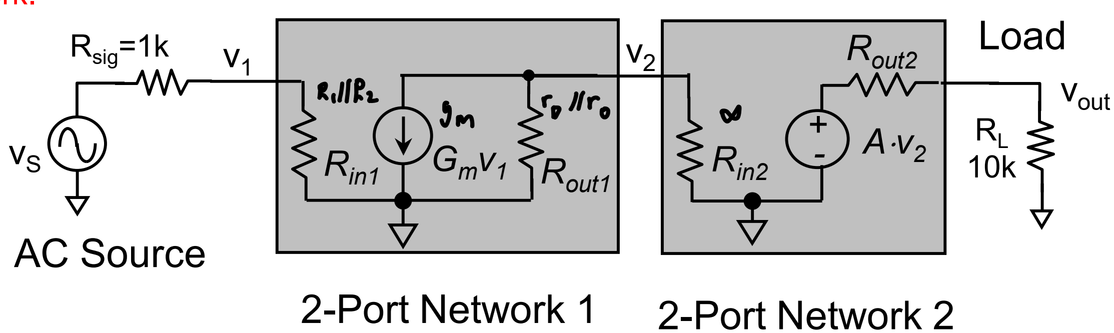
Different from single-stage

Compare above with slide SSA1-13 for steps for single-stage amplifier analysis

# Steps for Two-Stage Amplifier Analysis




Easier to analyze complicated circuit (e.g., multi-stage amplifier) as each amplifier is simplified to its equivalent 2-port network.

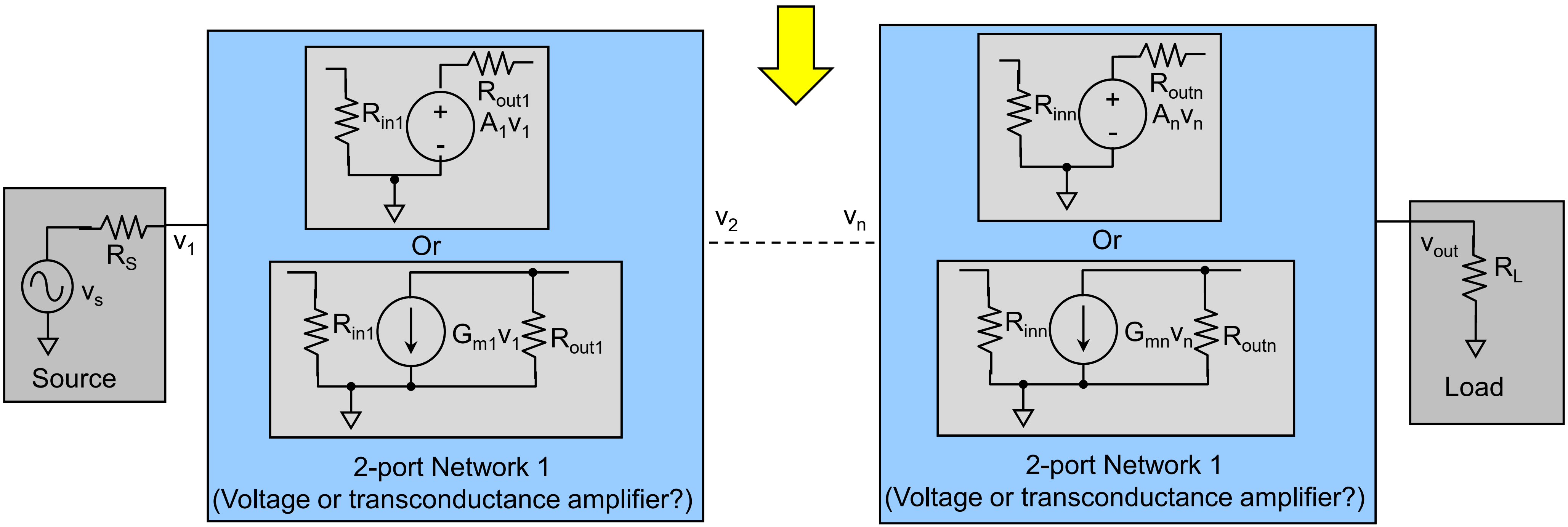



$$A_V = \frac{v_{out}}{v_s} = \frac{v_1}{v_s} \times \frac{v_2}{v_1} \times \frac{v_{out}}{v_2}$$

N.B. $R_{in1}$ includes the contribution of Amplifier Circuit 2, and $R_{out2}$ includes the contribution of Amplifier Circuit 1.

# How about >2 Stage?

1. Transform into AC equivalent circuit
2. Identify signal path (Source to Load)
3. Identify transistors that get cut across by signal path
4. Determine the number of single-stages
5. Identify the amplifier configuration
6. Replace each stage by its 2-port network equivalent




$$\frac{v_{out}}{v_s} = \frac{v_1}{v_s} \times \frac{v_2}{v_1} \times \cdots \times \frac{v_{out}}{v_n}$$

# How about Amplifier Design?

**Amplifier Analysis**

1. Resistors, capacitors and transistors are all given
2. Perform DC Analysis using Large Signal Model
3. Determine transistor small signal AC parameters, i.e. $g_m$, $g_{mb}$, $r_o$, $r_\pi$
4. Determine amplifier configuration and 2-port network parameters, i.e. $R_{in}$, $R_{out}$, $G_m$, $A_V$
5. Determine the overall gain of amplifier

**Amplifier Design**

- Choose the resistors, capacitors, transistors and proper voltage or current source to give the desired DC biasing
- Determine the DC biasing required to give the desired $g_m$, $g_{mb}$, $r_o$, $r_\pi$.
- Determine transistor small signal AC parameters, i.e. $g_m$, $g_{mb}$, $r_o$, $r_\pi$, to give the desired 2-port network parameters
- Determine suitable amplifier configuration and corresponding 2-port network parameters, i.e. $R_{in}$, $R_{out}$, $G_m$, $A_V$ to give the desired gain
- The overall gain of amplifier is specified