EE2027

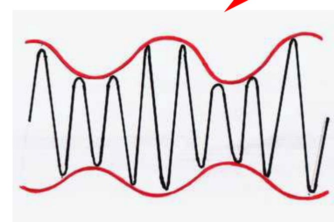
Electronic Circuits

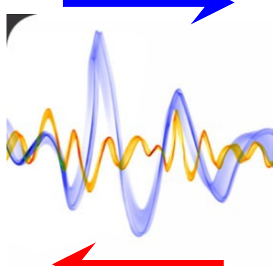
Review of Basic Concepts

World of Signals

Radio: 850, 900, 1800, 1900, 2100MHz
Transmit: +33dBm (10V)
Receive: -104dBm (1.4μV_{rms})

Audio:20Hz~20kHz
Mic: 5~50mV
Earphone: 0.316V_{rms} (nominal)



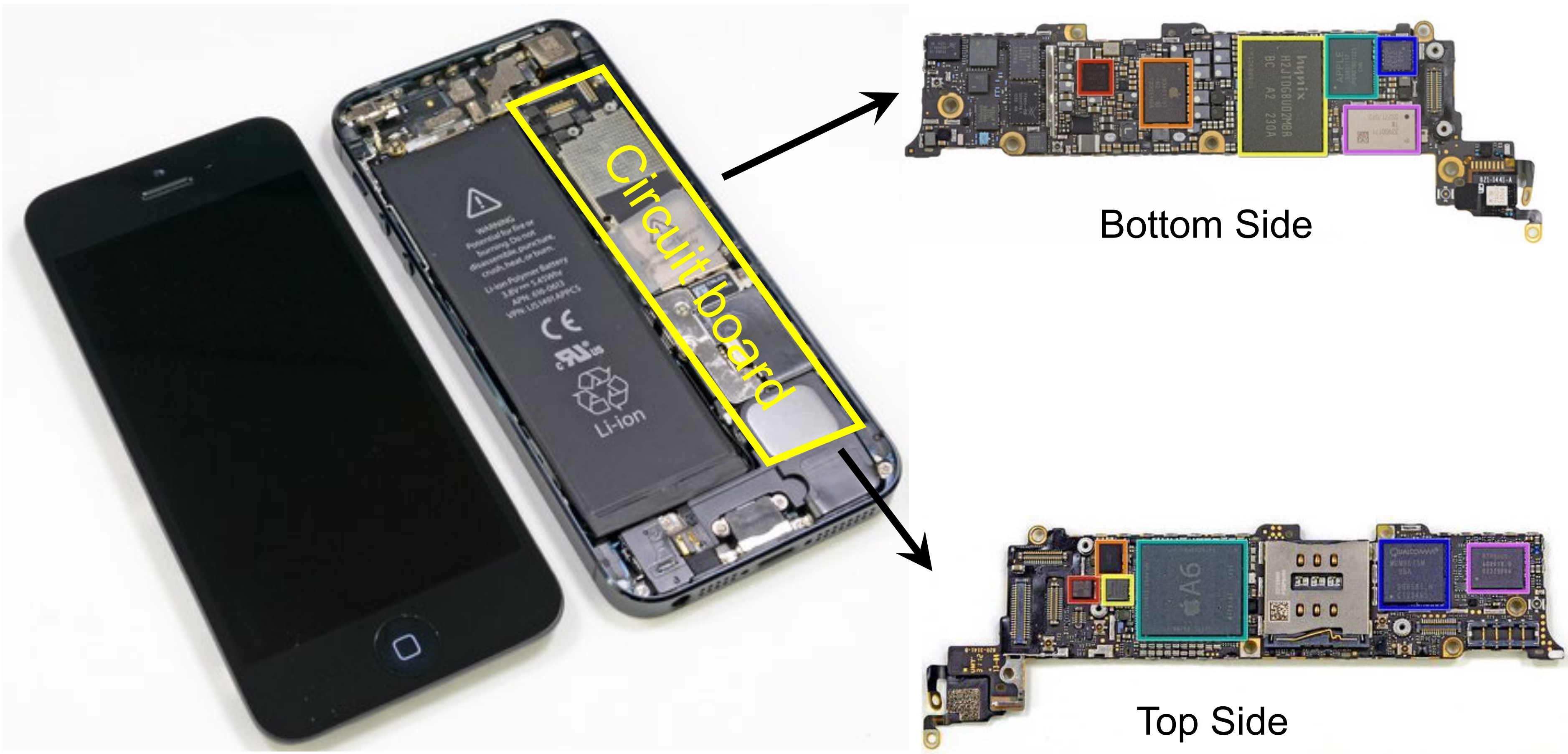


Motion:
Vibration: 50Hz@5V
Movement: ±8g, 12-bit

Camera:
Picture: 8 Megapixel
Video: HD, 30fps

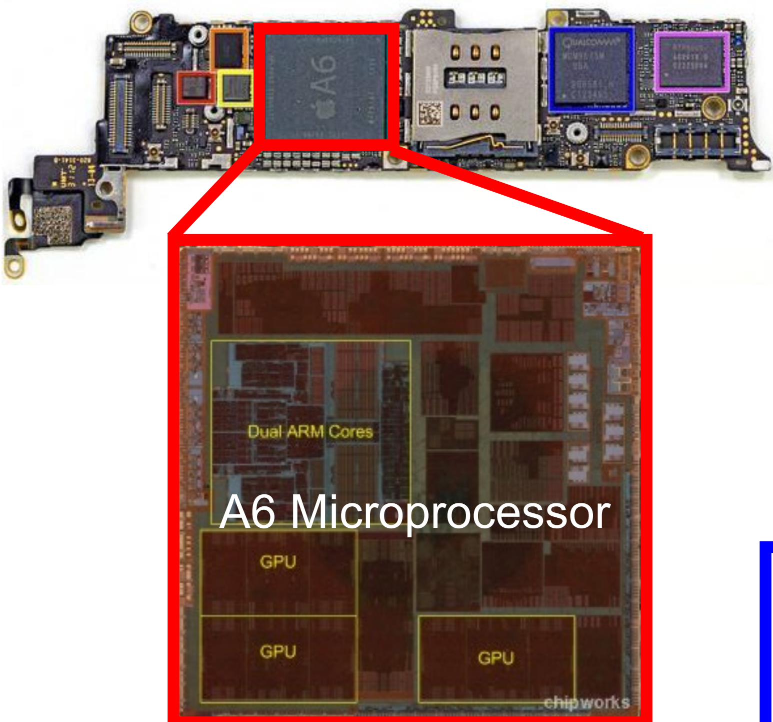
Important things:
1. Signal level
2. Signal frequency

Teardown

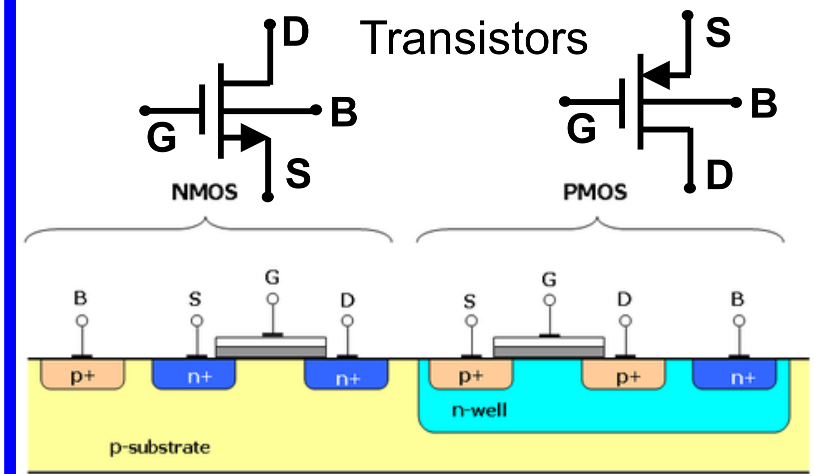


http://www.ifixit.com/Teardown/iPhone+5+Teardown/10525/1

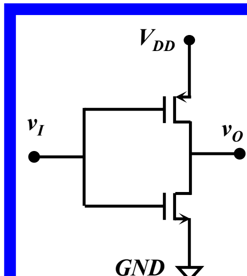
Microprocessor



Transistors

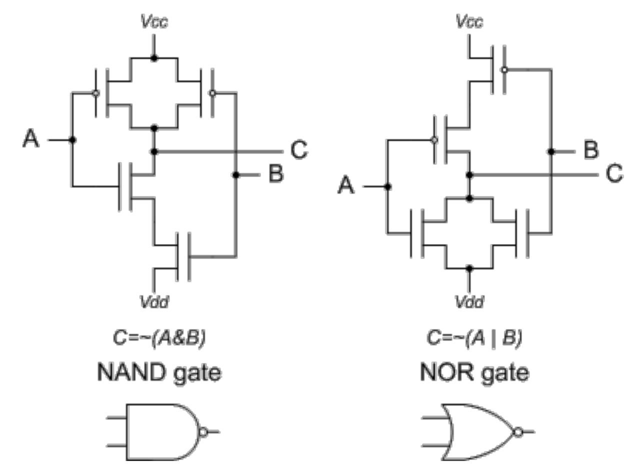


Lecture on MOSFET and its characteristics



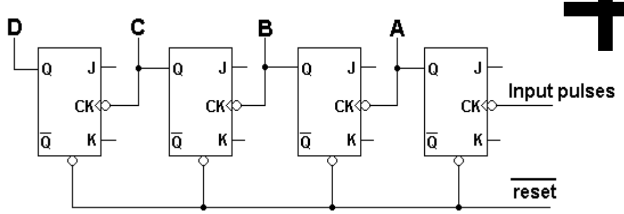
Lecture on CMOS Inverter

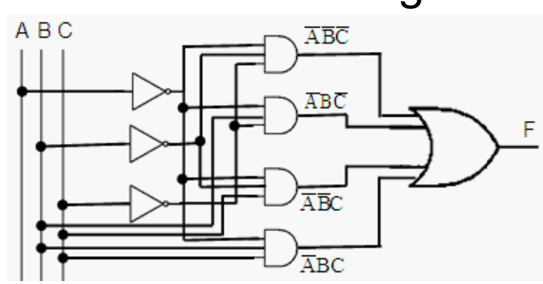
Logic Gates



EE2xxx Digital Fundamentals

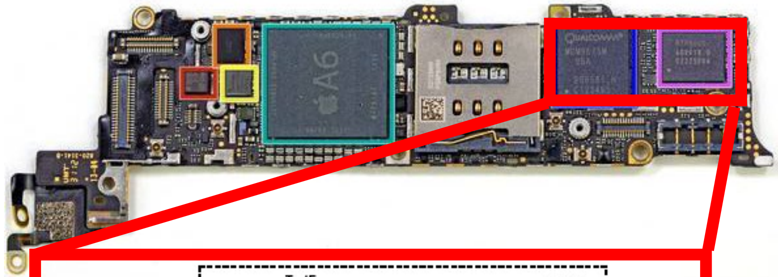
Sequential Logic + Combinational Logic

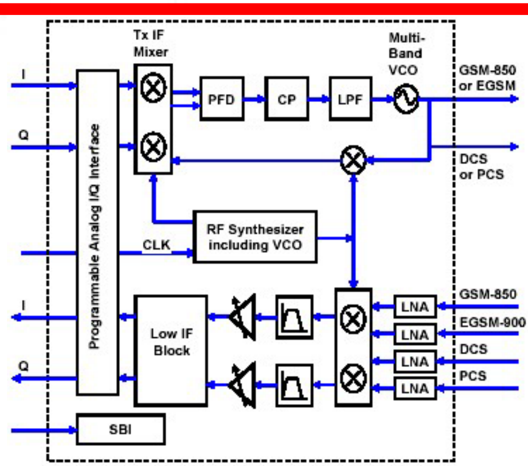


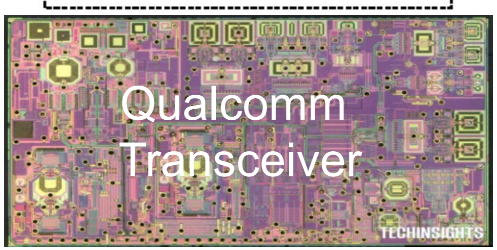


EE2xxx Digital Fundamentals

Transceiver

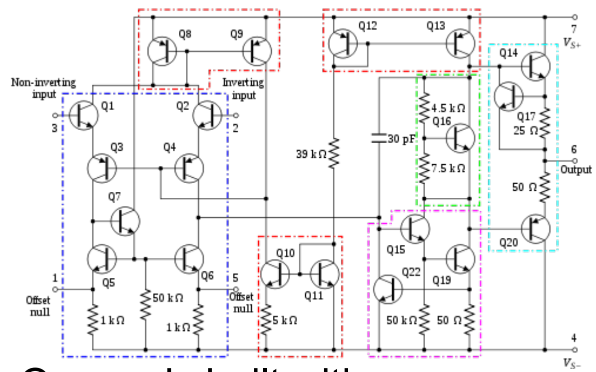




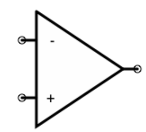


Mixer, Filter, LNA, Oscillator, VGA, ADC, DAC, etc.

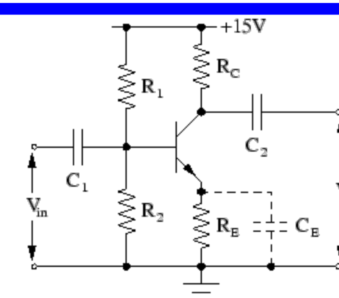
EE3xxx Analog Electronics
EE5xxx Microwave Electronics
EE6xxx RF Transceivers
EE6xxx Advanced IC Design



Opamp is built with current mirror and multistage amplifier

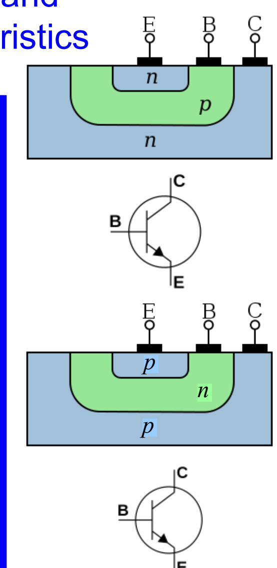


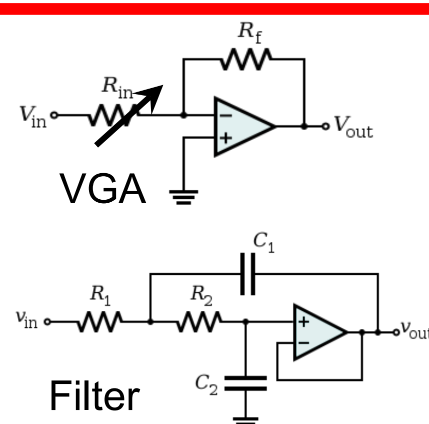
EE3xxx Integrated Analog Design

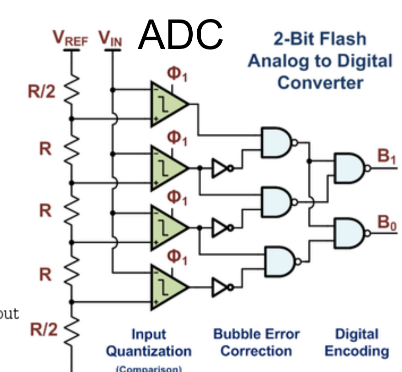


Single Stage Amplifier: CE/CS, CB/CG, CC/CD
Lecture on single stage amp

Transistors
Lecture on BJT and device characteristics

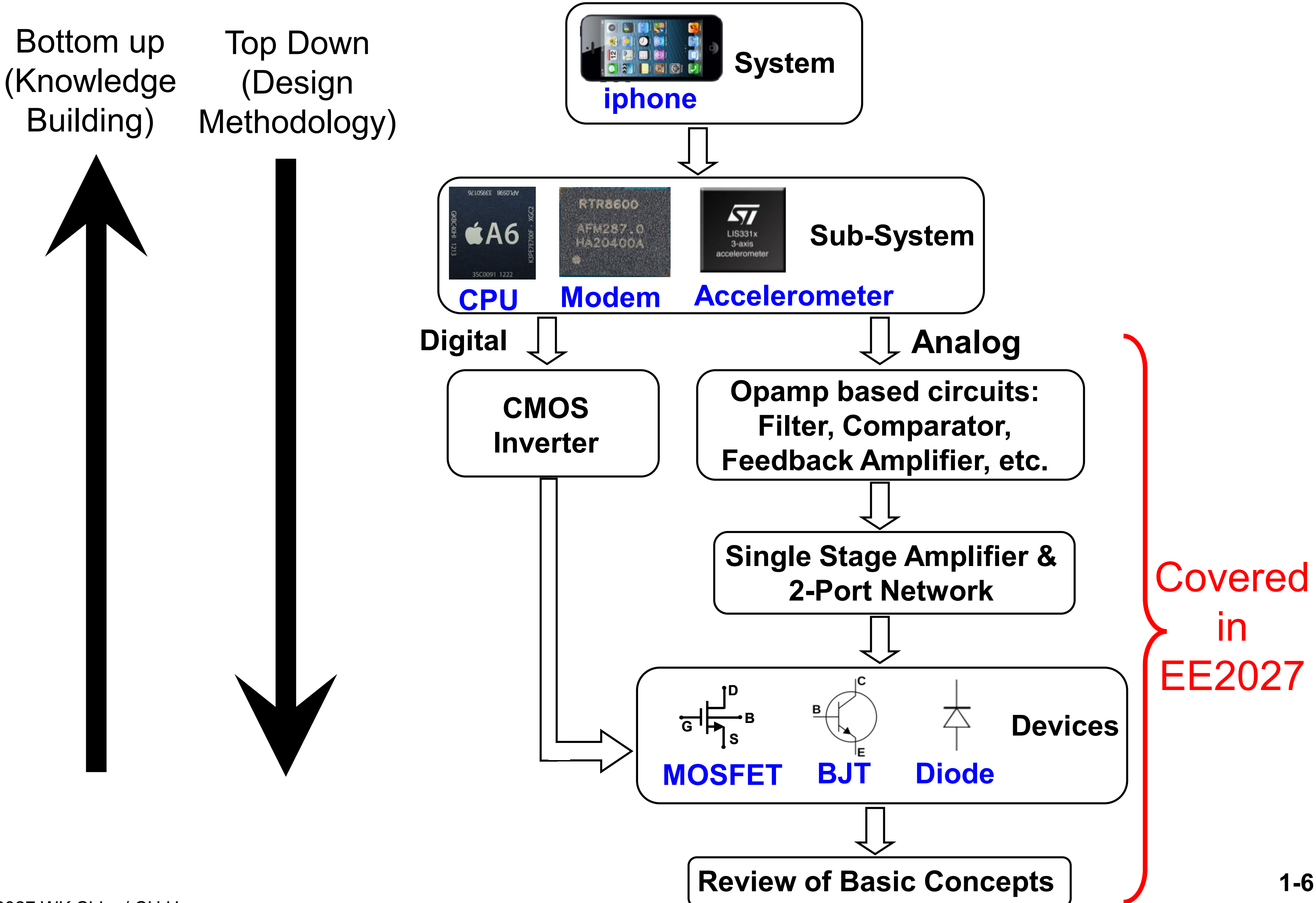






Lecture on Opamp-based circuits

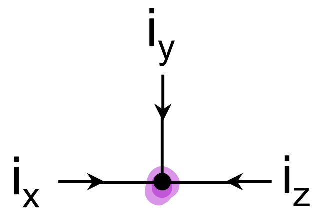
Overview


Bottom up (Knowledge Building)
Top Down (Design Methodology)
System
iphone
Sub-System
CPU
Modem
Accelerometer
Digital
Analog
CMOS Inverter
Opamp based circuits: Filter, Comparator, Feedback Amplifier, etc.
Single Stage Amplifier & 2-Port Network
Devices
MOSFET
BJT
Diode
Review of Basic Concepts
Covered in EE2027

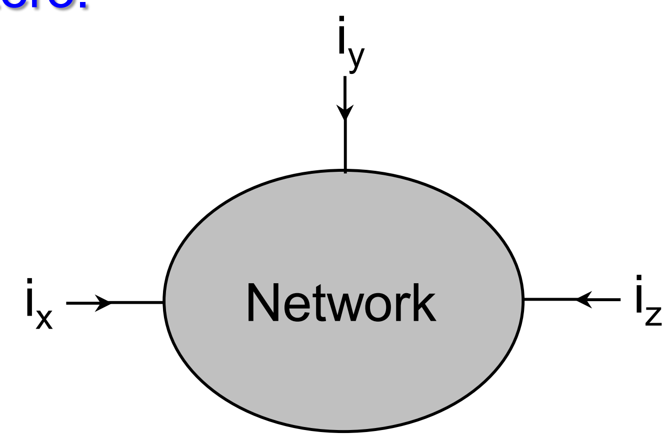
Topics

- Node, Mesh Analysis (KCL and KVL) and Linear Superposition
- Thevenin & Norton Equivalent;
- AC Signal Quantities - Peak, RMS and Average values
- Phasors – Phase and Amplitude
- Impedance of Capacitor and Inductor
- RC Circuit AC Analysis - Passive Filter
- Maximum Power Transfer
- RC Circuit Transients – Charge and Discharge (Self Reading)
- Power and Power Factor (Self Reading)

Node Analysis (KCL)



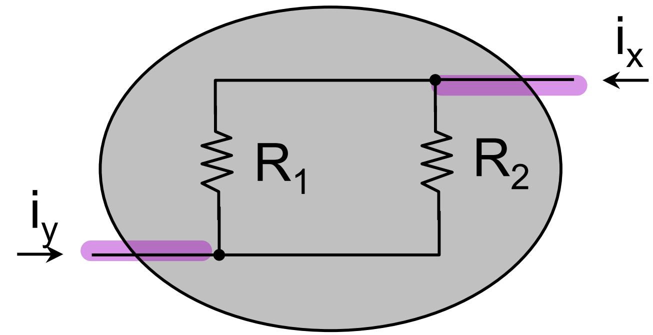
Conventional Kirchhoff's Current Law (KCL):
Currents flowing into a node sum to zero.



Generalized KCL:
Currents flowing into a network sum to zero

$$\sum_k i_k = i_x + i_y + i_z + \cdots = 0$$

Example:



$$i_x + i_y = 0$$

Since current *i* is equal to the rate of flow of charge *q* (i.e., *i* = *dq/dt*), KCL corresponds to the conservation of charge.

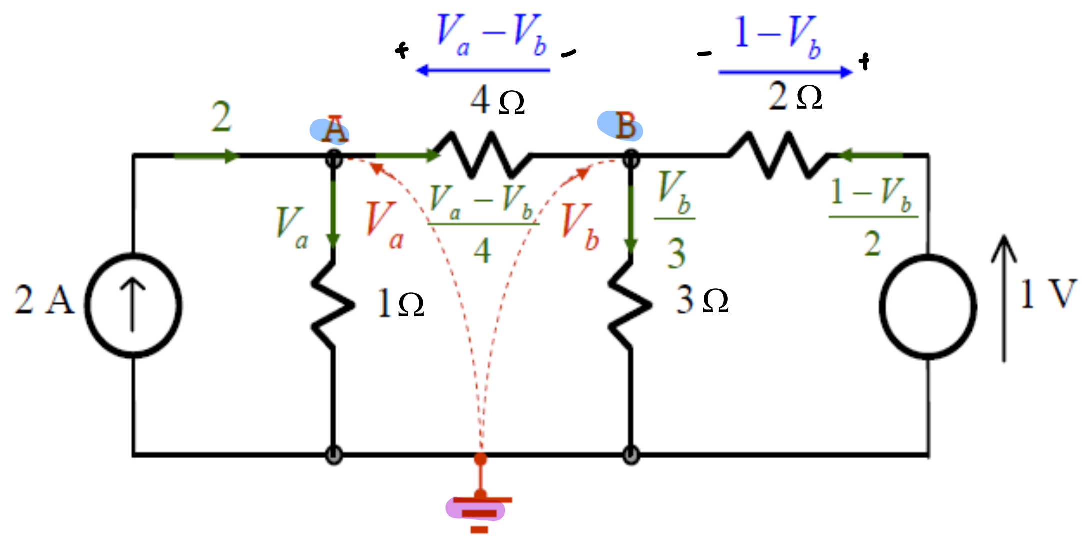
Node Analysis (KCL)

Step 1: Identify nodes & assign nodal voltages, V_a and V_b, w.r.t. reference ground node.

Step 2: Determine branch voltages, e.g., (V_a - V_b) and (1 - V_b).

Note: Potential of node A w.r.t. node B

$$V_{ab} = V_a - V_b$$



Step 3: Find branch currents.

Step 4: Apply KCL to nodes A and B.

Node A: $\boxed{\frac{V_a}{1\Omega} + \frac{V_a - V_b}{4\Omega} = 2A \implies 5V_a - V_b = 8}$ Multiply both sides with 4

Node B: $\boxed{\frac{V_a - V_b}{4\Omega} + \frac{1 - V_b}{2\Omega} + \frac{0 - V_b}{3\Omega} = 0 \implies 3V_a - 13V_b = -6}$ Multiply both sides with 12

Step 5: Solve resulting equations to obtain the nodal voltages.

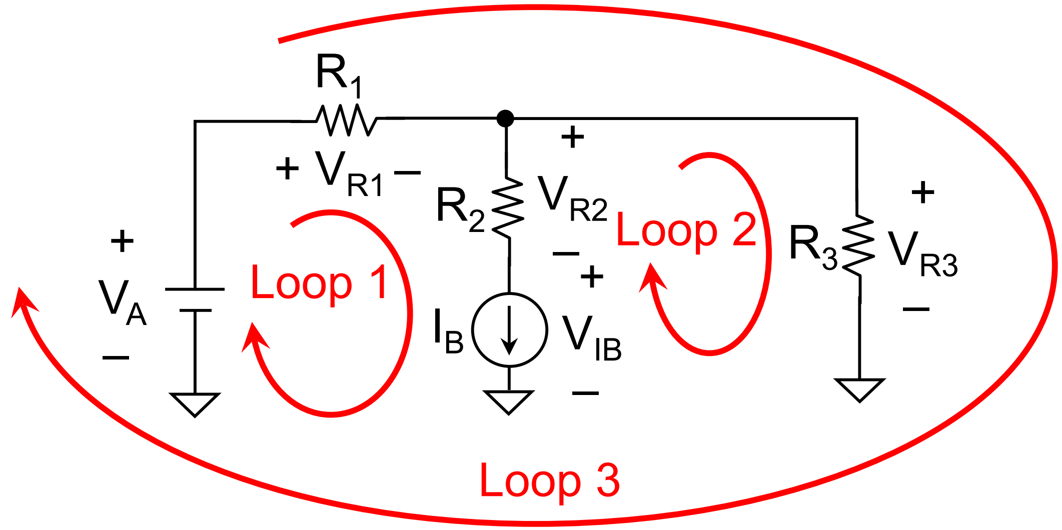
$$V_a = \frac{55}{31}\text{ V},\quad V_b = \frac{27}{31}\text{ V}$$

Mesh Analysis (KVL)

Kirchhoff's Voltage Law (KVL):
The sum of potential differences around any closed-loop is zero. KVL corresponds to the conservation of energy (qV) around any closed loop.

$$\boxed{\sum_k V_k = V_x + V_y + V_z + \cdots = 0.}$$

Example:



Loop 1 **:**

$$V_A + (-V_{R1}) + (-V_{R2}) + (-V_{IB}) = 0$$

Loop 2 **:**

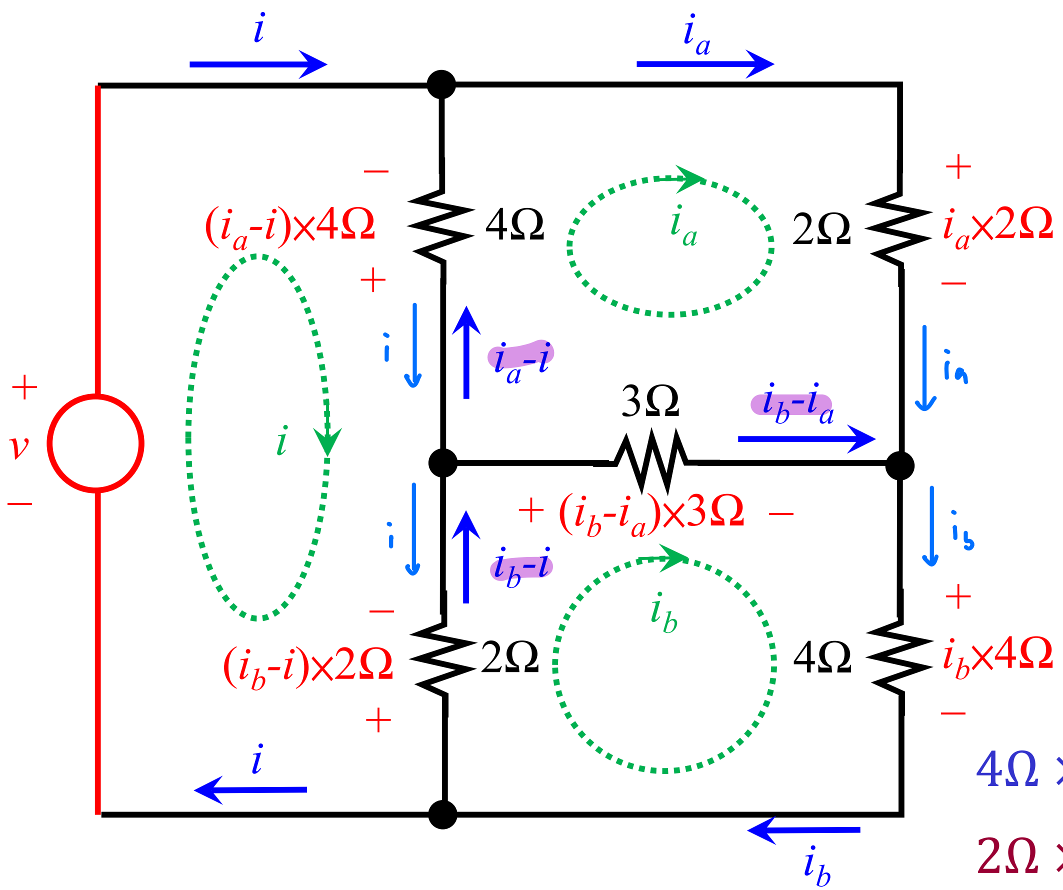
$$V_{IB} + V_{R2} + (-V_{R3}) = 0$$

Loop 3 **:**

$$V_A + (-V_{R1}) + (-V_{R3}) = 0$$

Note: voltage across the current source, I_B, is assumed to be $V_{IB} \neq 0$.

Mesh Analysis (KVL)



Step 1: Identify loops & assign loop currents, i, i_a and i_b.

Step 2: Determine branch currents.

Step 3: Find branch voltages*.

Step 4: Identify independent loops and write the KVL equations as shown below:

$$v = 2\Omega \times i_a + 4\Omega \times i_b$$

$$4\Omega \times (i_a - i) - 3\Omega \times (i_b - i_a) + 2\Omega \times i_a = 0$$

$$2\Omega \times (i_b - i) + 4\Omega \times i_b + 3\Omega \times (i_b - i_a) = 0$$

* + and – of voltage across a resistor follow the defined current flow direction, i.e., high to low

Step 5: Solve resulting equations.

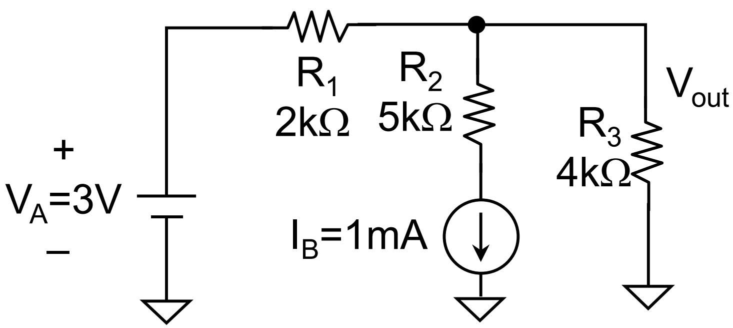
$$v = \frac{17i}{6} \Rightarrow R_{equivalent} = \frac{v}{i} = \frac{17}{6}\ \Omega$$

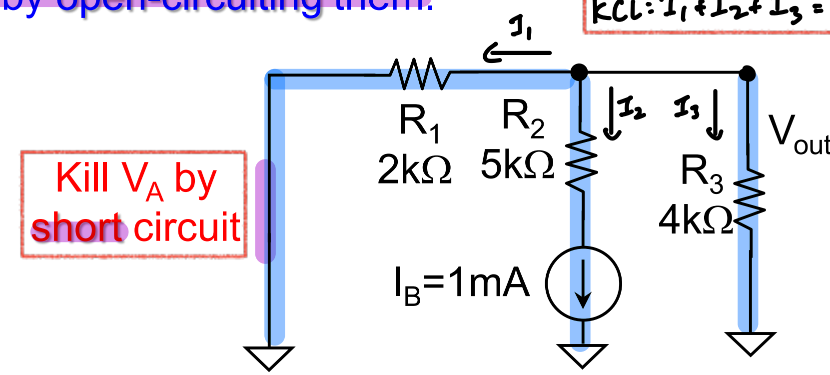
Linear Superposition

- The combined effect of various independent sources can be determined by summing the individual impact from various sources.
- When determining the impact of an individual source, you need to kill all other voltage sources by short-circuiting them, and all other current sources by open-circuiting them.

KCL: $I_1 + I_2 + I_3 = 0$

Example:
Determine I_{R3}





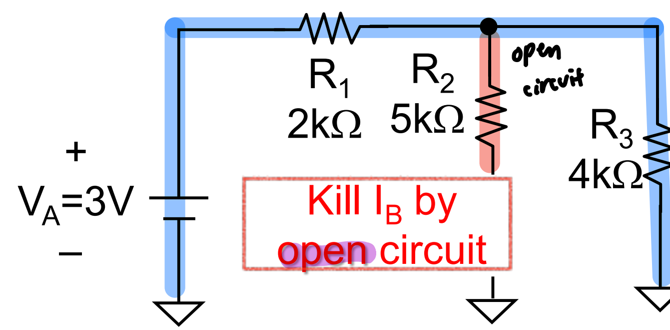
parallel, voltage same

$$\frac{V_{out}}{R_3} + \frac{V_{out}}{R_1} = -I_B = -1mA$$

$$\Rightarrow V_{out} = -1.33V \Rightarrow I_{R3\_IB} = -333\mu A$$

Superposition :

$$I_{R3} = I_{R3\_IB} + I_{R3\_VA} = 167\mu A$$



$$I_{R3\_VA} = \frac{V_A}{R_1 + R_3} = 500\mu A$$

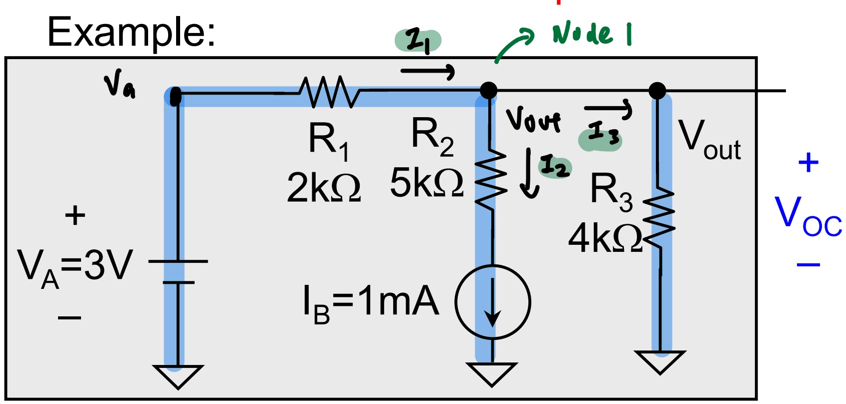
Thevenin & Norton Equivalent

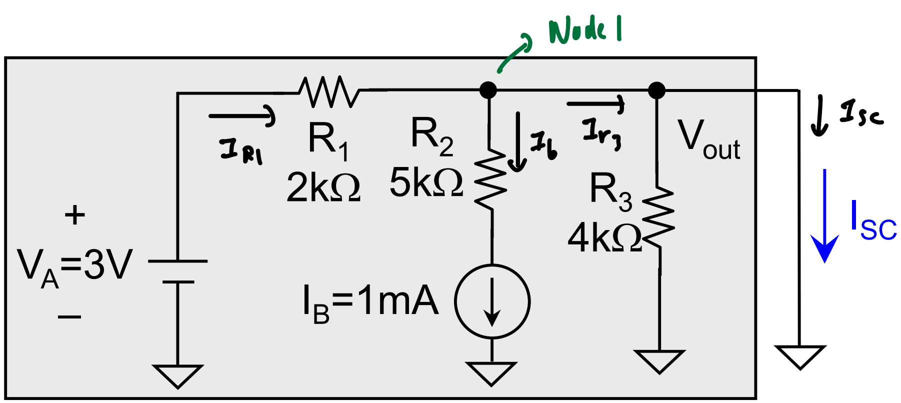
Thevenin:
Any linear network with one port output can be replaced with an equivalent Thevenin voltage source (V_{THV}) in series with a Thevenin resistance (R_{THV}).

Norton:
Any linear network with one port output can be replaced with an equivalent Norton current source (I_{NOR}) in parallel with a Norton resistance (R_{NOR}) .

Notes:
1) The Thevenin voltage source (V_{THV}) is found by evaluating the open-circuit voltage at the port.
2) The Norton current source (I_{NOR}) is found by evaluating the short-circuit current at the port.
3) In finding the equivalent resistance looking into the port, kill the voltage sources with short circuit, and the current sources with open circuit.

Example:





parallel, voltage same

$KCL:$ Node 1

(I1) $\frac{V_A - V_{out}}{R_1} + \frac{0 - V_{out}}{R_3} + (-I_B) = 0$ (I2) (I3)

$\Rightarrow 3V_{out} = 2V_A - I_B \times R_3 = 2V$

$\Rightarrow V_{out} = \frac{2}{3}V = V_{THV}$ (Open-circuit voltage)

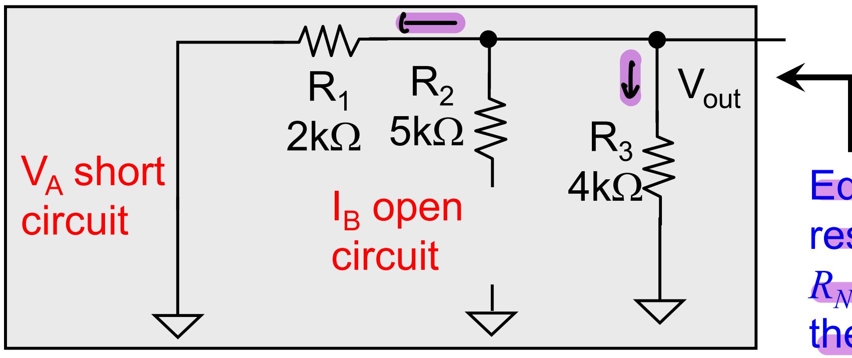
$KCL:$ Node 1

$I_{R1} + (-I_B) + (-I_{R3}) + (-I_{SC}) = 0$

$I_{R1} = \frac{V_A}{R_1} = 1.5mA \quad I_{R3} = 0$

$\Rightarrow I_{SC} = I_{R1} - I_B = 0.5mA = I_{NOR}$ (Short-circuit current)

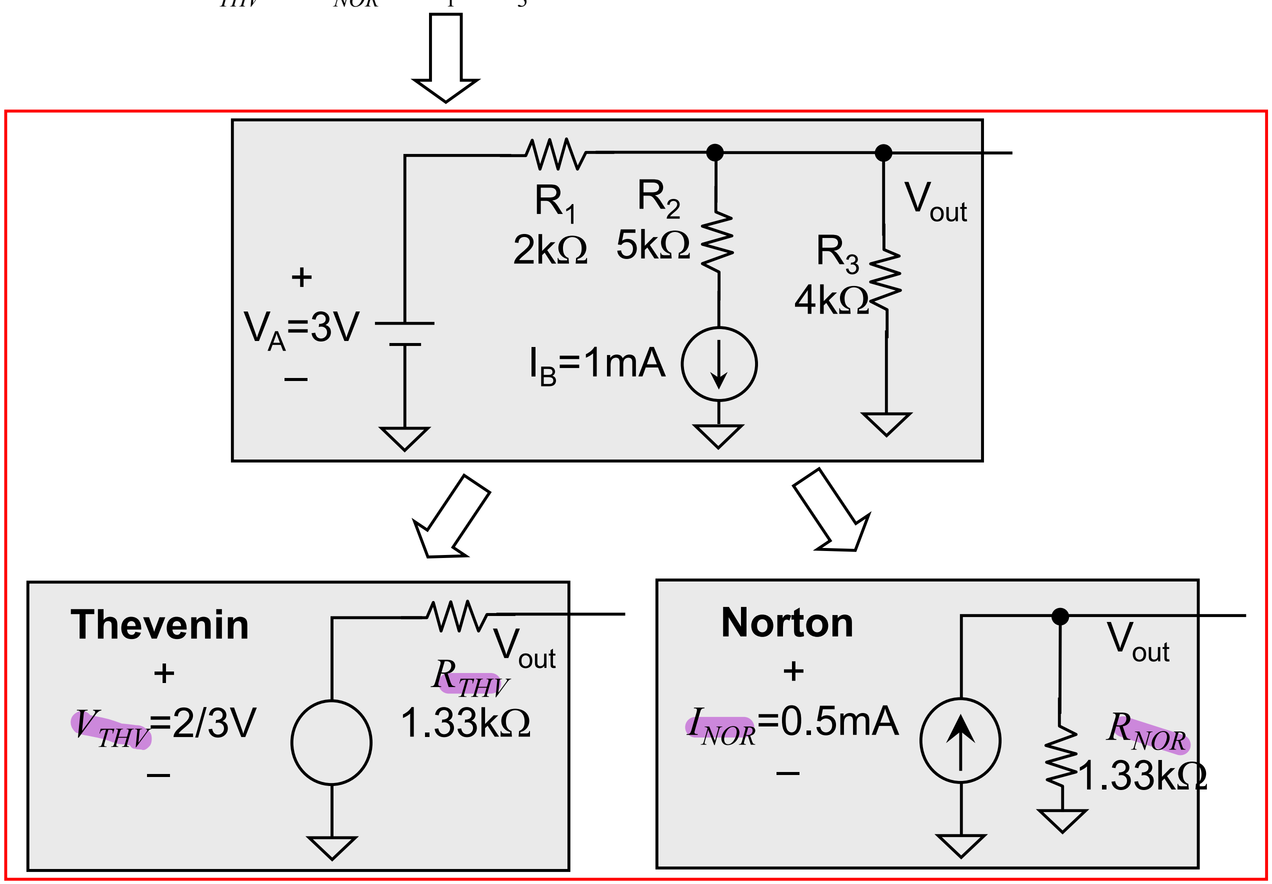
Thevenin & Norton Equivalent



$(\frac{1}{4}+\frac{1}{2})^{-1} = \frac{4}{3}k\Omega$

Equivalent resistance R_{THV} or R_{NOR} looking into the port

$$R_{THV} = R_{NOR} = R_1 // R_3 = 1.33k$$



Significance of Thevenin and Norton Equivalents:

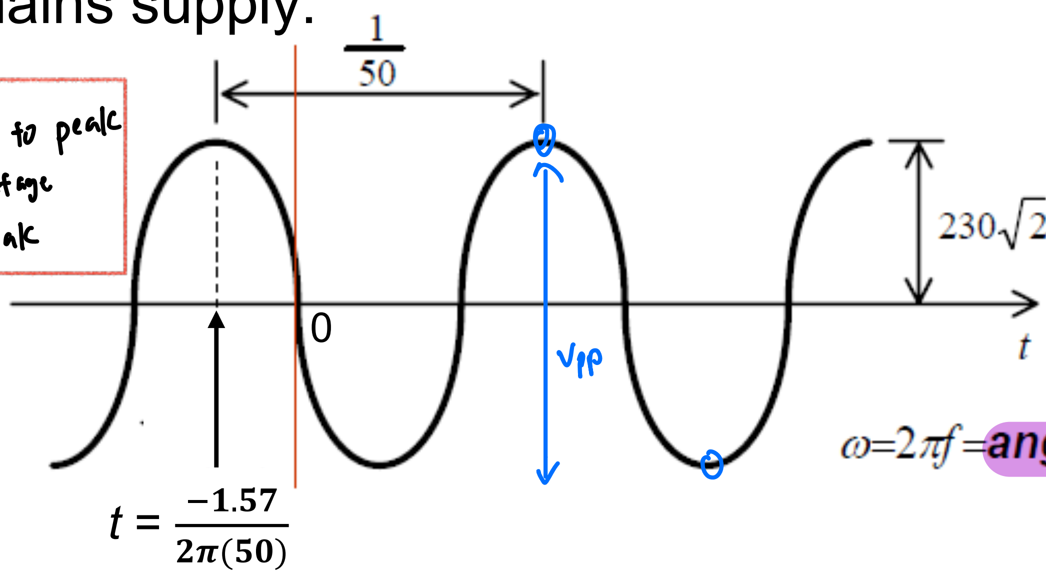
In reality, there is no need for you to open up the black box, determine the components and circuits, and work out the Thevenin or Norton equivalent. You just need a multimeter, and measure open-circuit voltage (When multimeter is used to measure voltage, it actually behaves like open circuit) to get V_{THV}, and measure short-circuit current (When multimeter is used to measure current, it actually behaves like short circuit) to get I_{NOR}.

R_{THV} *and* R_{NOR} can be obtained from the expression V_{THV}/I_{NOR}.

AC Signal Quantities

In **alternating current (AC)** circuits, voltages and currents change with time in a sinusoidal manner. The most common AC voltage source is the mains supply:

Amplitude (Vp): DC to peak voltage
Vpp: Peak to peak



θ = **phase** = 1.57 rad

f = **frequency** $= 50\text{Hz}$

$\omega = 2\pi f =$ **angular frequency** $= 100\pi = 314\text{rad/s}$

$T = \frac{1}{f} =$ **period** $= \frac{1}{50} = 0.02\text{s}$

$$v(t) = \sqrt{2}r\cos(2\pi ft+\theta) = \sqrt{2}r\cos(\omega t+\theta) = \sqrt{2}r\cos\left(\frac{2\pi t}{T}+\theta\right)$$

$$= 230\sqrt{2}\cos(100\pi t + 1.57)$$

$\boxed{\sqrt{2}r = \textbf{peak value} = 230\sqrt{2} = 324\text{V}}$

$\boxed{r = \textbf{rms (root mean square) value} = 230\text{V}}$

Only applicable to sinusoidal signals

$\boxed{\textbf{Average value}, v_{av} = \frac{1}{T}\int_0^T v(t)\,dt}$

integrate for one cycle

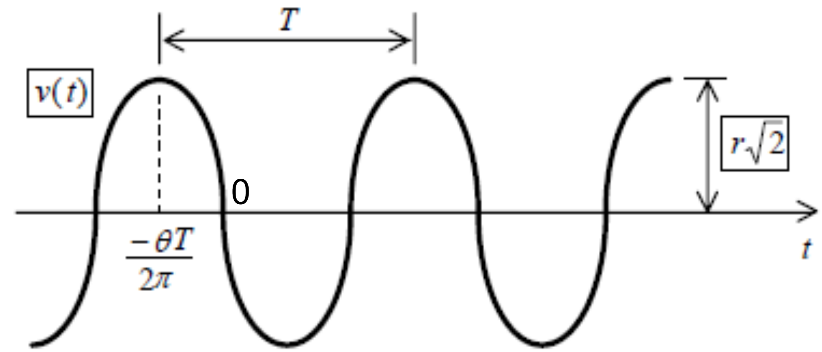
AC Signal Quantities

Instantaneous power: I x V

Avg power: $I_{rms} \times V_{rms}$

Root Mean Square (rms) value (can be defined for any periodic signal with period *T*) as follows:

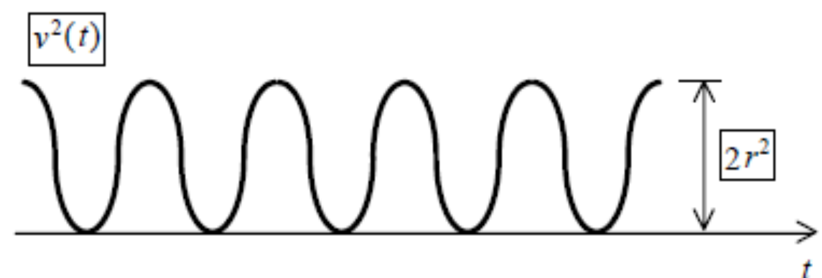
$$v(t) = \sqrt{2}r\cos(2\pi f t + \theta) = \sqrt{2}r\cos\left(\frac{2\pi t}{T} + \theta\right)$$



①

$$v^2(t) = 2r^2\cos^2\left(\frac{2\pi t}{T} + \theta\right) = r^2\left[1 + \cos\left(\frac{4\pi t}{T} + 2\theta\right)\right]$$

since $2\cos^2(x) = 1 + \cos(2x)$



②

Mean (Average) of the square value is:

$$\frac{1}{T}\int_0^T v^2(t)dt = \frac{1}{T}\int_0^T r^2\left[1 + \cos\left(\frac{4\pi t}{T} + 2\theta\right)\right]dt = \frac{1}{T}\int_0^T r^2\,dt = r^2$$

Square root of the mean of the square value is the rms value = $\sqrt{\left[\frac{1}{T}\int_0^T v^2(t)dt\right]} = r$

→ if not periodic need to use original formula and find based on one cycle

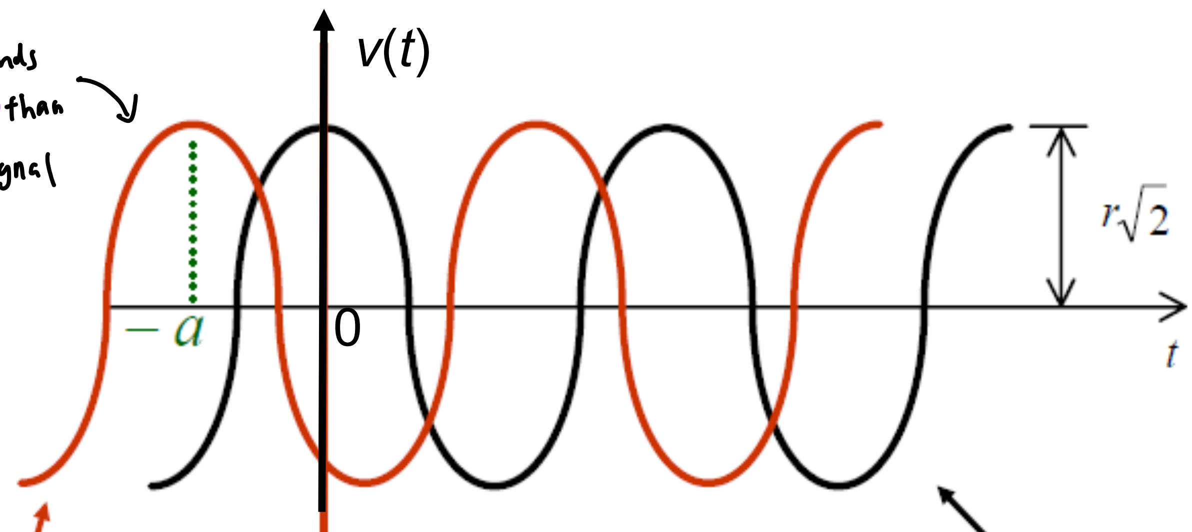
$V_p \longrightarrow \frac{A}{\sqrt{2}} \Rightarrow$ rms only applicable for sin/cos signal (sinusoid)

The expression within the blue box is valid for any periodic signal

Phasors

Determination of Phase for a sinusoidal function:

a seconds earlier than black signal



$$v(t) = r\sqrt{2}\cos(\omega[t + a]) = r\sqrt{2}\cos(\omega t + \omega a)$$

units is in θ

$$v(t) = \boxed{r\sqrt{2}\cos(\omega t)}$$

$$\boxed{\text{Phase } \theta = \omega a, \quad \text{where } -\pi \le \theta \le \pi}$$

Phasors

A sinusoidal signal (voltage or current) is typically represented using complex number format:

$$v(t) = \sqrt{2}\, r\cos(\omega t + \theta) = \sqrt{2}\, r\, Re\left[e^{j(\omega t+\theta)}\right] = Re\Big[\overbrace{(re^{j\theta})}^{\text{impt}}\underbrace{(\sqrt{2}e^{j\omega t})}_{\text{constant so "redundant"}}\Big]$$

$$\boxed{\text{Euler's Formula:} \quad e^{j\omega} = \cos(\omega) + j\sin(\omega)}$$

Using Phasors, the above time-varying AC voltage $v(t)$ becomes a complex time-invariant number/voltage:

$$\boxed{V = r\,e^{j\theta} = r\angle\theta}$$

where r = $|V|$ = Magnitude/Modulus of V = ** rms value of $v(t)$

θ = Arg [V] = Phase of $v(t)$ ** w.r.t cos

↓

dealing with 'real' signal

(think abt Euler formula)

Phasors

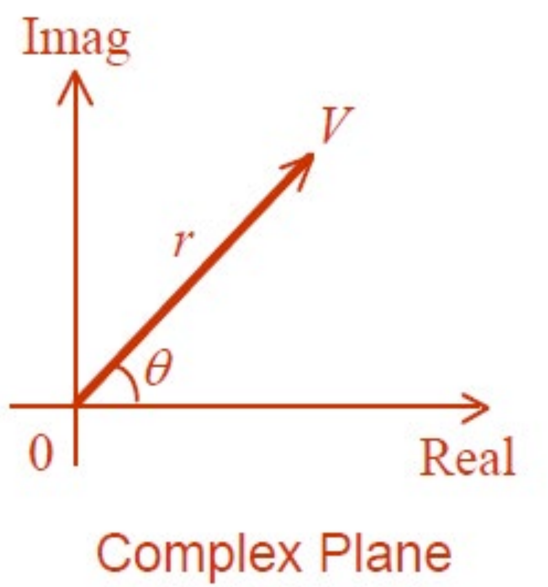
Time-varying sinusoidal voltage:

$$v(t) = \sqrt{2}\, r \cos(wt + \theta) = \sqrt{2}\, r\, Re\left[e^{j(\omega t + \theta)}\right] = Re\left[(re^{j\theta})(\sqrt{2}e^{j\omega t})\right]$$

interchangeable (need to know)

Phasor notation: $\boxed{V = r\, e^{j\theta} = r\angle\theta}$

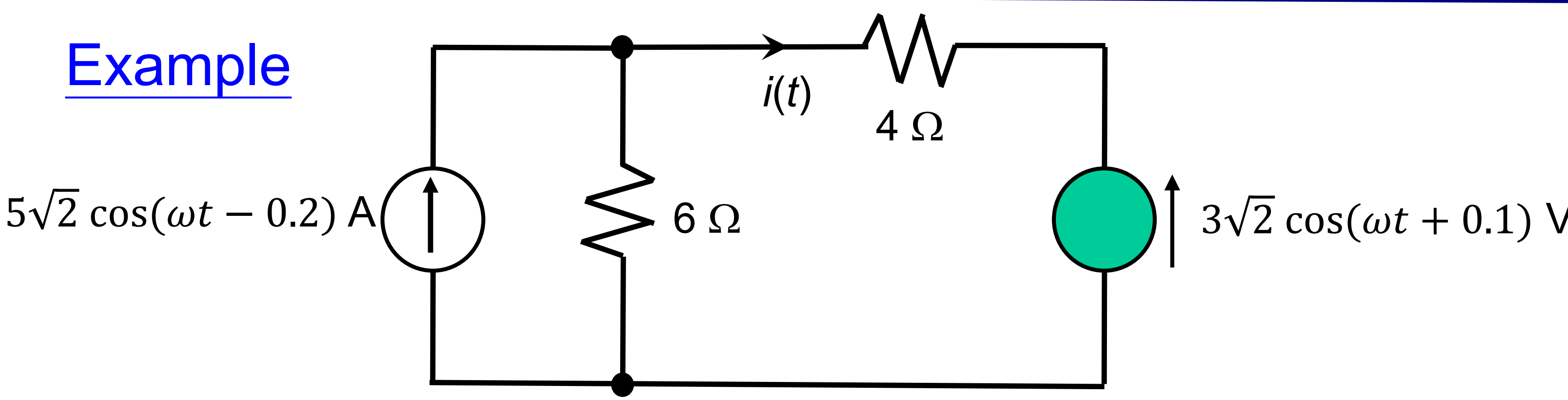
Graphically, on a Phasor diagram:



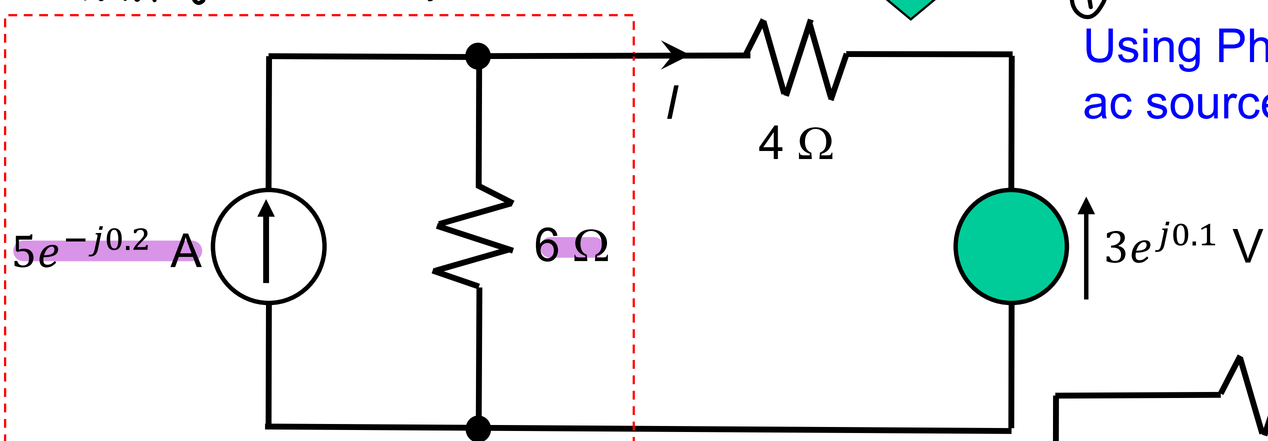
Note: Using Phasors, all time-varying ac quantities become complex DC quantities, and all DC circuit analysis techniques (e.g., Node and Mesh analysis) can be employed for AC circuits (see following example).

Phasors (Circuit Analysis)

Example



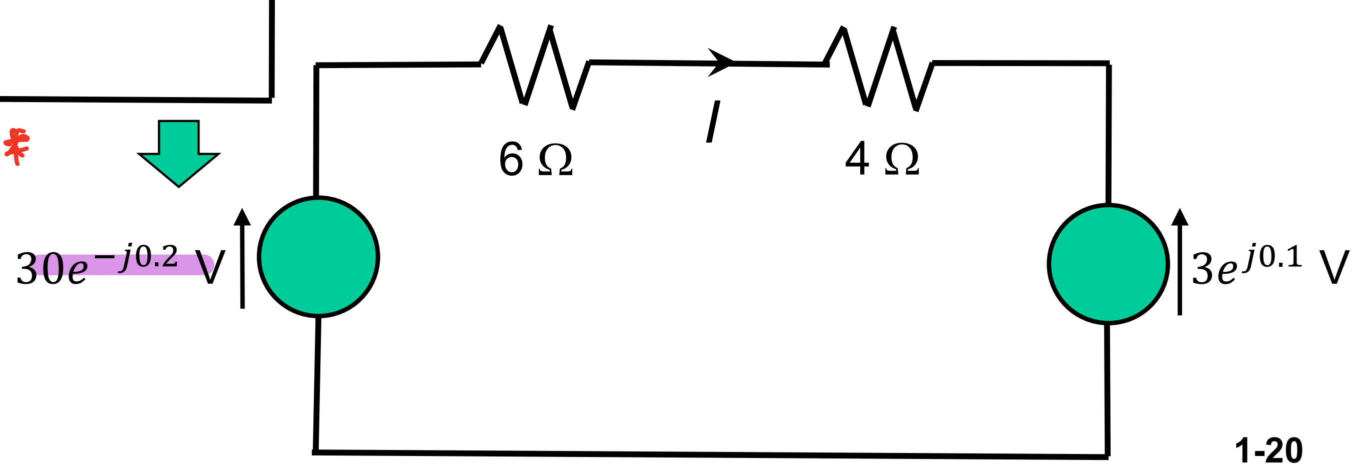
Norton (current // resistor)



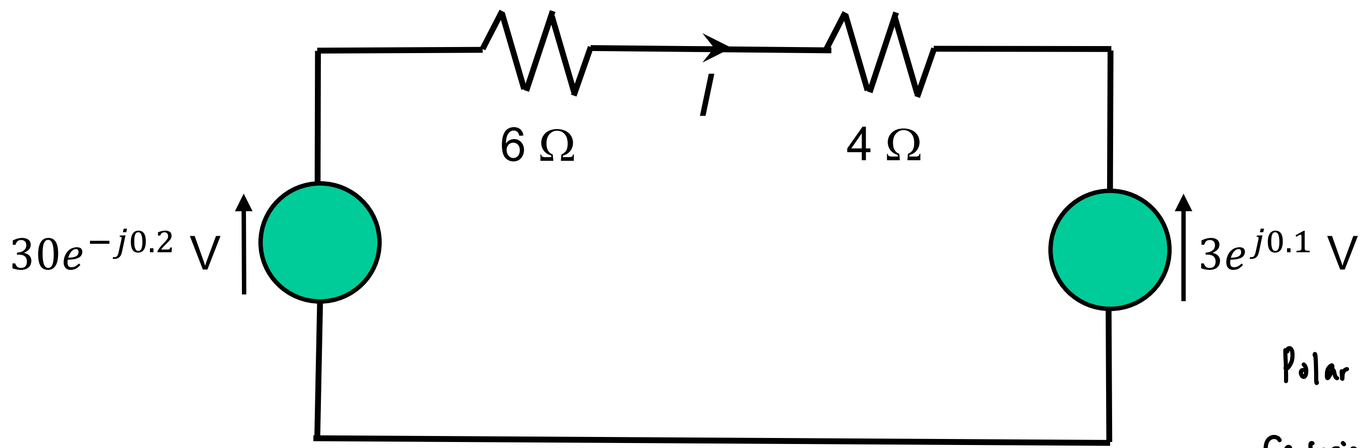
(i) Using Phasor notation for the ac sources

**

Using Thevenin equivalent to replace the ac current source in parallel with 6 Ω resistor by an ac voltage source in series with resistor



Phasors (Circuit Analysis)



Polar form :

Cartesian form :

Applying KVL: $I = \dfrac{30e^{-j0.2} - 3e^{j0.1}}{10}$

$$= 3[\cos(-0.2) + j\sin(-0.2)] - 0.3[\cos(0.1) + j\sin(0.1)]$$

$$= (2.940 - j0.596) - (0.299 + j0.030) = 2.641 - j\,0.626$$

convert back to polar form

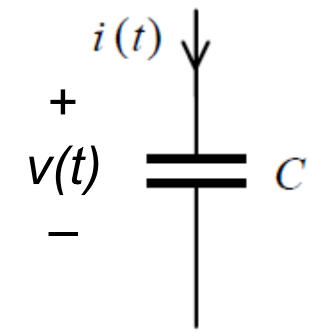
$$= \sqrt{(2.641)^2 + (0.626)^2}\; e^{j\,tan^{-1}\left(\frac{-0.626}{2.641}\right)} = 2.714\, e^{-j0.233}\ \text{A}$$

$$\therefore\ i(t) = 2.714\sqrt{2}\cos(\omega t - 0.233)\ \text{A}$$

need to multiply back $\sqrt{2}$ to get Vp

Capacitor

Circuit symbol for a capacitor with capacitance C is:



Current-voltage (i-v) relationship of a capacitor (with voltage polarity and current direction as indicated) is:

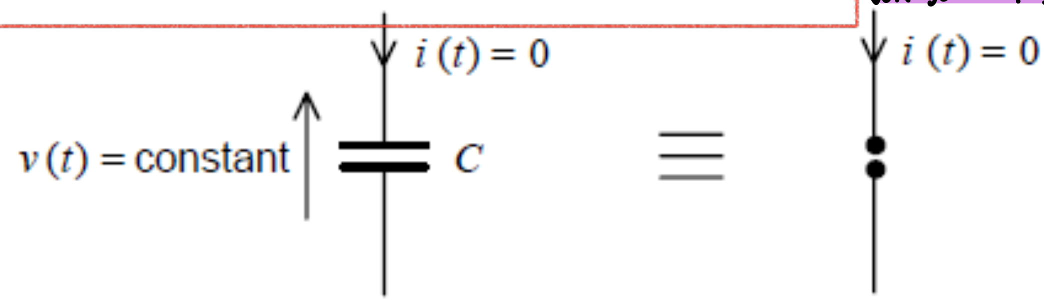
$$i(t) = C\frac{dv(t)}{dt}$$

Note: For a finite current $i(t)$, voltage $v(t)$ across a capacitor cannot change abruptly.

capacitor voltage is continuous, hence it cannot change voltage abruptly

For DC circuits, $v(t)$ = constant.

$$\Rightarrow \frac{dv(t)}{dt} = 0 \Rightarrow i(t) = 0$$



That is why we treat the capacitor as an open circuit in DC circuit analysis. (DC means angular frequency $\omega = 0$)

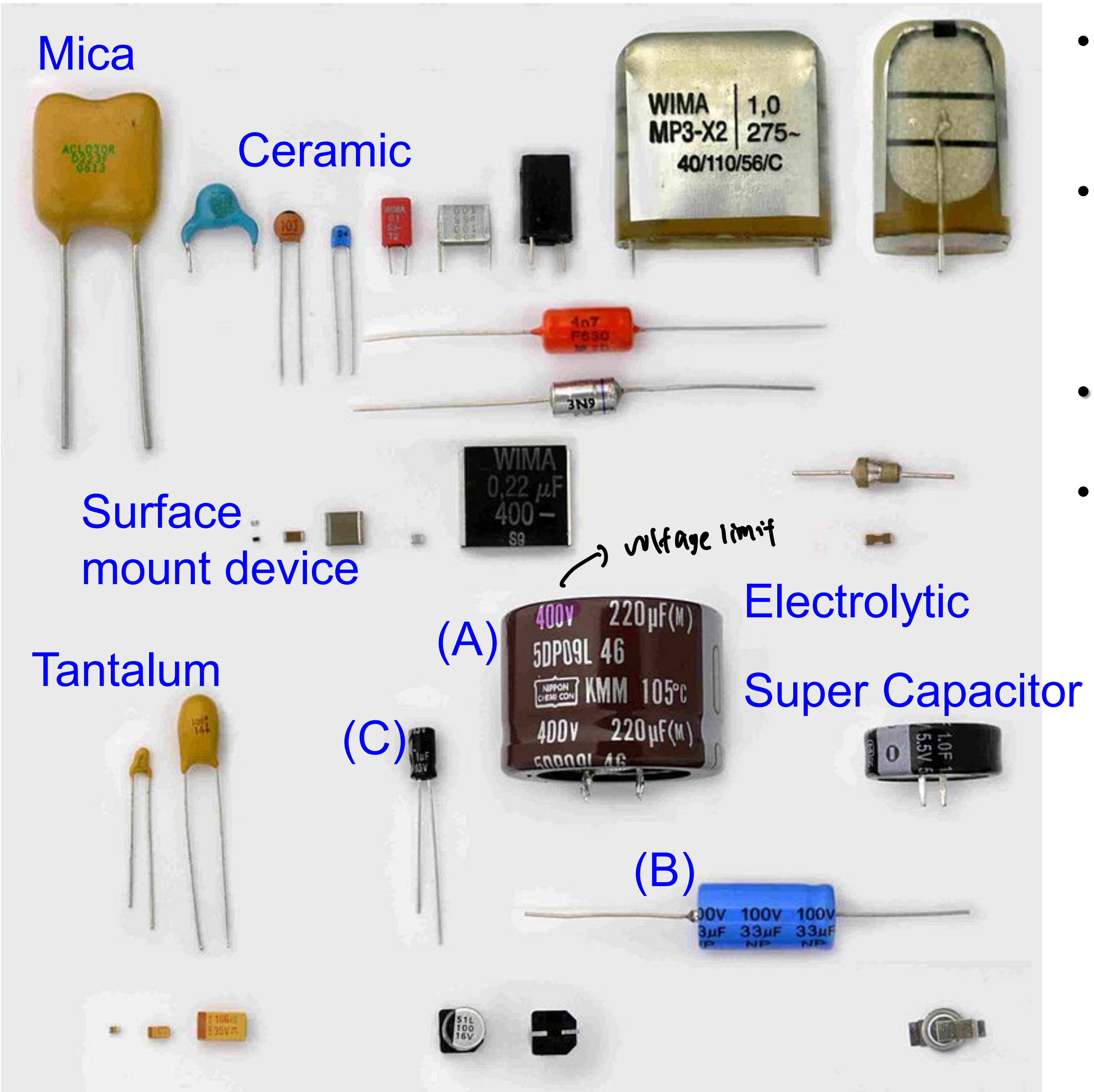
Unit for capacitance: Farad (F). Practical capacitors in electronic circuits (excluding power electronics) typically have values of micro-Farad (μF or 10^{-6} F) or pico-Farad (pF or 10^{-12} F).

Types of Capacitors

Type	Capacitance range	Maximum voltage	Accuracy	Temperature stability	Leakage	Comments
Mica	1pF–0.01μF	100–600	Good		Good	Excellent; good at RF
Tubular ceramic	0.5pF–100pF	100–600		Selectable		Several tempcos (including zero)
Ceramic	10pF–1μF	50–30,000	Poor	Poor	Moderate	Small, inexpensive, very popular
Polyester (Mylar)	0.001μF–50μF	50–600	Good	Poor	Good	Inexpensive, good, popular
Polystyrene	10pF–2.7μF	100–600	Excellent	Good	Excellent	High quality, large; signal filters
Polycarbonate	100pF–30μF	50–800	Excellent	Excellent	Good	High quality, small
Polypropylene	100pF–50μF	100–800	Excellent	Good	Excellent	High quality, low dielectric absorption
Teflon	1000pF–2μF	50–200	Excellent	Best	Best	High quality, lowest dielectric absorption
Glass	10pF–1000pF	100–600	Good		Excellent	Long-term stability
Porcelain	100pF–0.1μF	50–400	Good	Good	Good	Good long-term stability
Tantalum	0.1μF–500μF	6–100	Poor	Poor		High capacitance; polarized, small; low inductance → have polarity (-ve/+ve)
Electrolytic	0.1μF–1.6F	3–600	Terrible	Ghastly	Awful	Power-supply filters; polarized; short life
Double layer	0.1F–10F	1.5–6	Poor	Poor	Good	Memory backup; high series resistance
Oil	0.1μF–20μF	200–10,000			Good	High-voltage filters; large, long life
Vacuum	1pF–5000pF	2000–36,000			Excellent	Transmitters

Source: P. Horowitz, *The Art of Electronics*, Cambridge University Press.

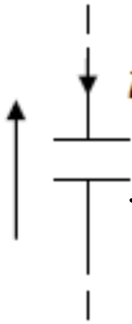
Types of Capacitors



- Size (Area) of a capacitor is directly proportional to its capacitance value.
- For example, note the relative size difference of the (A) 220 μF, (B) 3.3 μF and (C) 1 μF electrolytic capacitors.
- Super capacitor: Large capacitance (1F) with small dimension
- Some capacitors have polarity. For example, note the polarity (negative or minus terminal) of electrolytic capacitors.

Capacitor

Consider the operation of a capacitor in an AC circuit:



$$i(t) = C\frac{dv(t)}{dt} = -\omega C r_v \sqrt{2}\sin(\omega t + \theta_v)$$

$$v(t) = r_v\sqrt{2}\cos(\omega t + \theta_v)$$

↓ polar convert

$r_v \angle \theta_v$

$C = \frac{1}{j\omega C}$

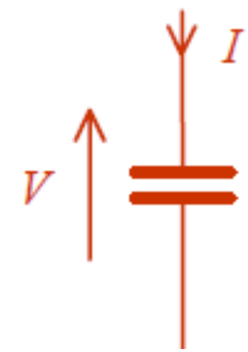
$$= \omega C r_v \sqrt{2}\cos(\omega t + \theta_v + \frac{\pi}{2})$$

Using Phasor representation (Refer to slides 1-17 ~ 1-21):

$$I = \omega C r_v e^{j\theta_v} \underbrace{e^{j\frac{\pi}{2}}}_{j} = j\omega C r_v e^{j\theta_v} = j\omega C V$$

$$V = r_v e^{j\theta_v}$$

Z $\Rightarrow$ $\frac{V}{I} = \frac{1}{j\omega\ C}$ $\Rightarrow$



$\frac{1}{j\omega C}$

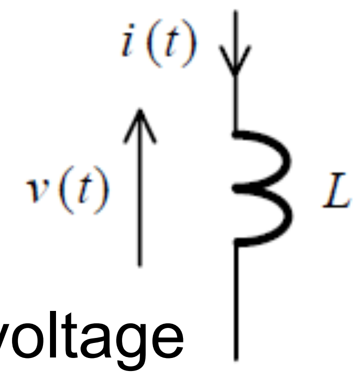
With Phasor representation, the capacitor behaves as if it is a resistor with a "complex resistance" or reactance of

$$\boxed{X_C = \frac{1}{j\omega C}}$$

Inductor

An inductor consists of a coil of wire that establishes a magnetic field when current flows through it.

Circuit symbol for an inductor with inductance L:



Current-voltage (i-v) relationship of an inductor (with voltage polarity and current direction as indicated) is:

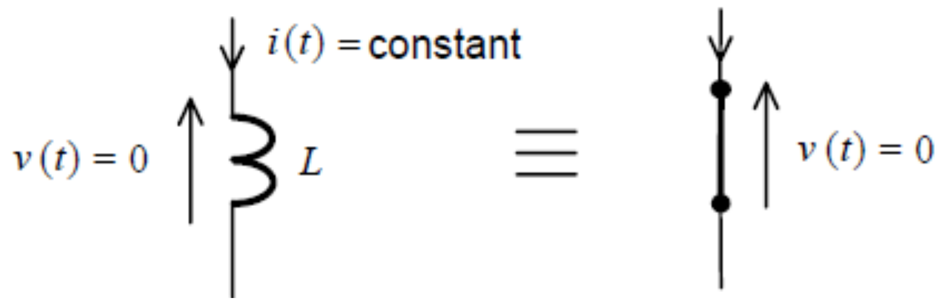
$$v(t) = L\,\frac{di(t)}{dt}$$

Note: For a finite voltage $v(t)$, current $i(t)$ through an inductor cannot change abruptly.

inductor current is continous, hence it cannot change current abruptly

For dc circuits, $i(t)$ = constant.

$$\Rightarrow \frac{di(t)}{dt} = 0 \quad \Rightarrow \quad v(t) = 0$$

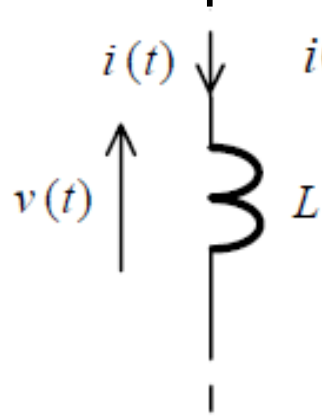


That is why we treat the inductor as a short circuit in DC circuit analysis.

Unit for inductance: Henry (H). Practical inductors typically have values of milli-Henry (mH or 10^{-3} H) or micro-Henry (μH or 10^{-6} H).

Inductor

Consider the operation of an inductor in an AC circuit:

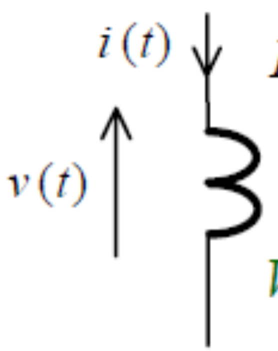


$$i(t) = r_i\sqrt{2}\cos(\omega t + \theta_i)$$

$$v(t) = L\frac{di(t)}{dt} = -\omega L r_i\sqrt{2}\sin(\omega t + \theta_i)$$

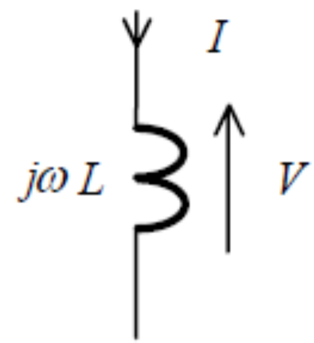
$$= \omega L r_i\sqrt{2}\cos(\omega t + \theta_i + \frac{\pi}{2})$$

Using Phasor representation (Refer to slides 1-17 ~ 1-21):



$$I = r_i e^{j\theta_i}$$

$$V = \omega L r_i e^{j\theta_i}\underbrace{e^{j\pi/2}}_{j} = j\omega L\overbrace{r_i e^{j\theta_i}}^{I} = (j\omega L)I$$



$$\boxed{\frac{V}{I} = j\omega L}$$

With Phasor representation, the inductor behaves as if it is a resistor with a “complex resistance” or reactance of

$$\boxed{X_L = j\omega L}$$

Impedance and Admittance

Impedance (*Z*): Resistance (*R*) + Reactance (*X*)

$$\boxed{Z = R + jX}$$

Admittance (*Y*): Conductance (*G*) + Susceptance (*B*)

$$\boxed{Y = \frac{1}{Z} = G + jB}$$

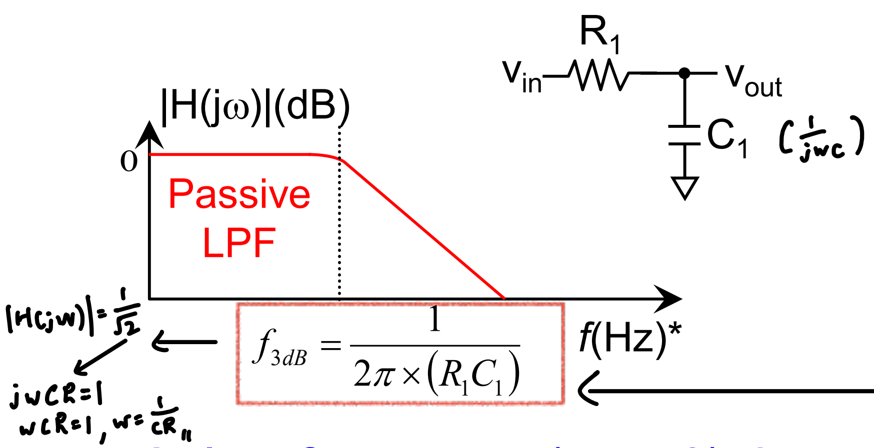
$$Y = \frac{1}{Z} = \frac{1}{R + jX} = \frac{R - jX}{(R + jX)(R - jX)} = \frac{R}{R^2 + X^2} - \frac{jX}{R^2 + X^2}$$

$$\Rightarrow \boxed{G = \frac{R}{R^2 + X^2}} \quad \text{and} \quad \boxed{B = \frac{-X}{R^2 + X^2}}$$

RC Circuit AC Analysis – Passive Filter

- The RC circuit shown below acts as a 1st order passive low pass filter.

Passive 1st Order Low Pass Filter



$$f_{3dB} = \frac{1}{2\pi \times (R_1 C_1)}$$

Simple resistor divider principle:

$$H(j\omega) = \frac{v_{out}}{v_{in}} = \frac{\frac{1}{j\omega C_1}}{\frac{1}{j\omega C_1} + R_1} = \frac{1}{j\omega C_1 R_1 + 1}$$

(1 + 1)

- At low frequency ($\omega \to 0$), ignore 1st term of denominator ($j\omega C_1 R_1$) $\Rightarrow H(j\omega) \approx 1$ $\Rightarrow$ Unity gain
- At high frequency ($\omega \to \infty$), ignore 2nd term of denominator $\Rightarrow H(j\omega) \approx 1/(j\omega C_1 R_1)$ $\Rightarrow$ Gain reduces with increasing frequency
- To appreciate the frequency response of the passive low pass filter (i.e., plot of gain $|H(j\omega)|$ in decibels (dB) versus frequency f*, where $\omega = 2\pi f$), we first need to understand the definition of decibel first.

* frequency, *f*, is plotted on a log-scale. Frequency response is known as **Bode plot**.

RC Circuit AC Analysis – Passive Filter

Definition of Decibel:

$$\textit{Decibel (dB)}[\textit{Power Ratio}]$$

$$Y(dB) = 10 \times \log\left(\left|\frac{P_{out}}{P_{in}}\right|\right) = 10 \times \log(|\textit{Power Gain}|)$$

$$\textit{Power (dBm)}$$

$$P(dBm) = 10 \times \log\left(\frac{P_{out}}{1mW}\right)$$

Example :

Typical WLAN / Bluetooth output power is $100mW$

$$\Rightarrow 10\log\left(\frac{100mW}{1mW}\right) = 20dBm$$

Typical mobile phone output power is $2W$

$$\Rightarrow 10\log\left(\frac{2W}{1mW}\right) = 33dBm$$

$$\textit{Decibel (dB)}[\textit{Voltage Ratio}]$$

$$Y(dB) = 20 \times \log\left(\left|\frac{V_{out}}{V_{in}}\right|\right) = 20 \times \log(|Gain|)$$

Example :

$$\textit{If } Gain = 1 \Rightarrow 20 \times \log(|Gain|) = 0dB$$

$$\textit{If } Gain = -100 \Rightarrow 20 \times \log(|Gain|) = 40dB$$

$$\textit{If } Gain = 0.001 \Rightarrow 20 \times \log(Gain) = -60dB$$

A negative gain in linear scale does not imply loss.
But a negative gain in dB scale means loss.

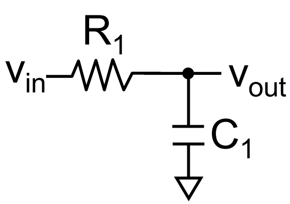
RC Circuit AC Analysis – Passive Filter

- **Half-power point** of a filter or amplifier is the **frequency** at which the output power has decreased to half of its peak value. In decibels, this corresponds to a decrease of 3 dB from the peak gain (in dB).

$$\boxed{10\ log_{10}(0.5) = -3\ \text{dB}}$$

- The half-power point is a commonly used definition of the **3-dB frequency** (f_{3dB}).

Passive 1st Order Low Pass Filter (LPF)



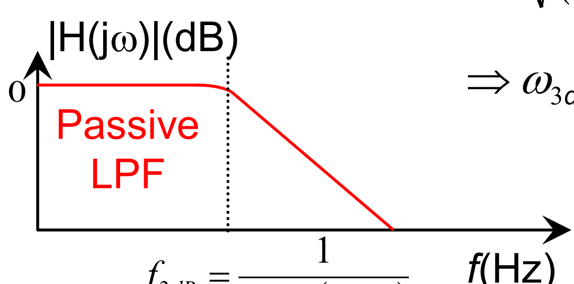
refer to slide 29

$$\boxed{\left|H\left(j\omega_{3dB}\right)\right| = \frac{1}{\sqrt{2}}}$$

$$\Rightarrow \frac{1}{\sqrt{\left(\omega_{3dB}R_1C_1\right)^2+1}} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \omega_{3dB} = \frac{1}{R_1C_1} \quad or \quad f_{3dB} = \frac{1}{2\pi R_1C_1}$$

Interested in finding the frequency (f_{3dB}) where the output power decreases by half. Power is square of voltage, output voltage drops by $1/\sqrt{2}$ at f_{3dB}.

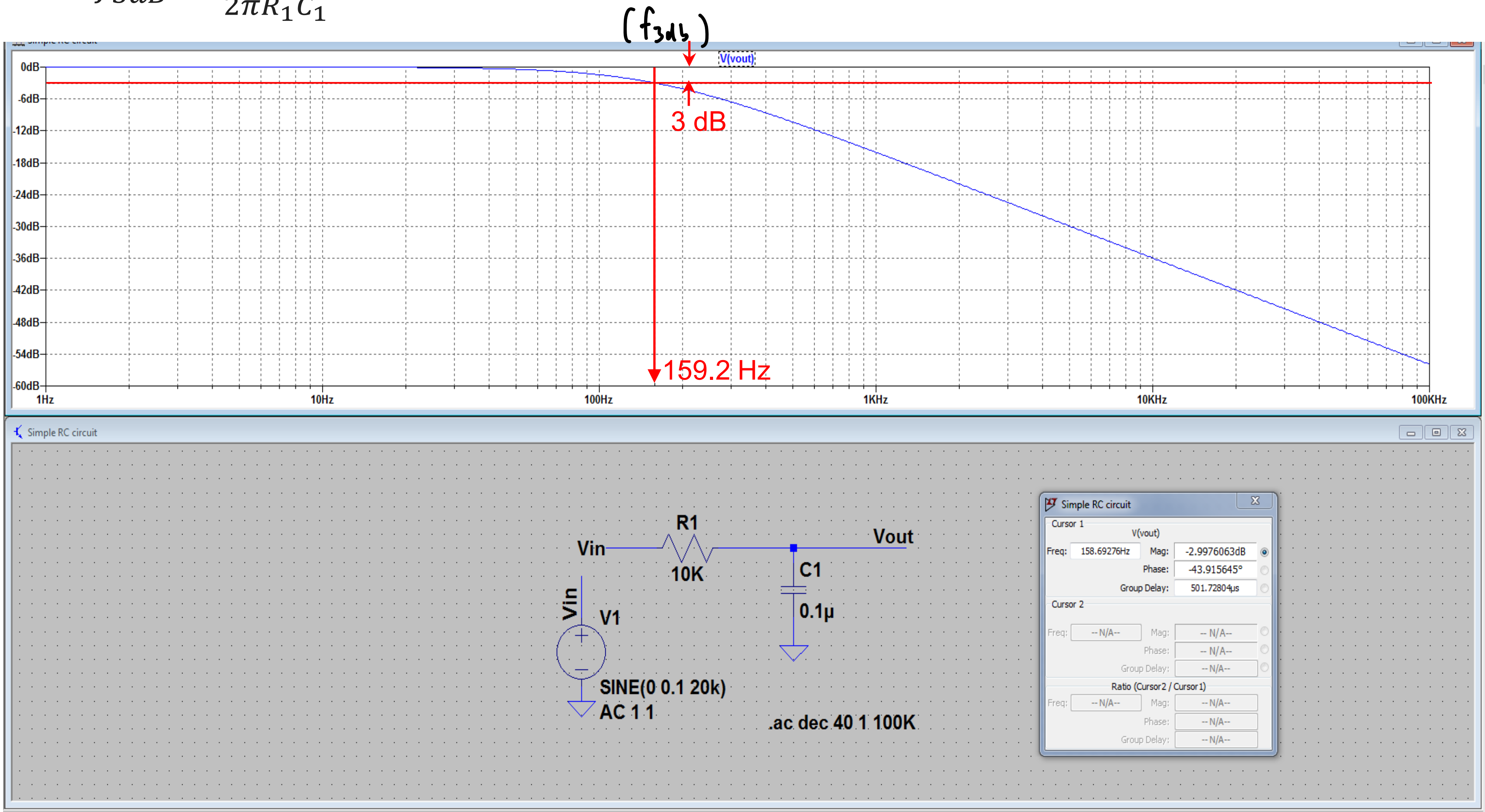


$$f_{3dB} = \frac{1}{2\pi\times\left(R_1C_1\right)}$$

When the gain, $|H(j\omega)|$, is reduced by $\sqrt{2}$, this is equivalent to a reduction by 3 dB.

Simulation of Passive 1st Order LPF

- R_1 = 10 kΩ, C_1 = 0.1 μF
- $f_{3dB} = \frac{1}{2\pi R_1 C_1} = 159.2\ \text{Hz}$

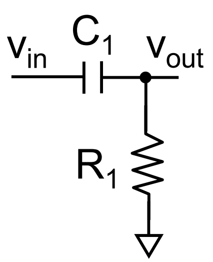


Note: Above frequency response curve is known as Bode plot, where frequency, *f*, is plotted on a log-scale.

RC Circuit AC Analysis – Passive Filter

Passive 1st Order High Pass Filter can be obtained by interchanging the positions of R_1 and C_1 in the passive Low Pass Filter circuit.

Passive 1st Order High Pass Filter (HPF)



$$\boxed{H(j\omega)=\frac{v_{out}}{v_{in}}=\frac{R_1}{\frac{1}{j\omega C_1}+R_1}=\frac{j\omega C_1R_1}{j\omega C_1R_1+1}=\frac{j\omega}{j\omega+\frac{1}{C_1R_1}}}$$

- At low frequency ($\omega \rightarrow 0$), ignore 1st term of denominator ($j\omega$) $\Rightarrow H(j\omega) \approx j\omega C_1R_1 \Rightarrow$ Gain increases with increasing frequency
- At high frequency ($\omega \rightarrow \infty$), ignore 2nd term of denominator ($1/(C_1R_1)$) $\Rightarrow H(j\omega) \approx 1 \Rightarrow$ Unity gain

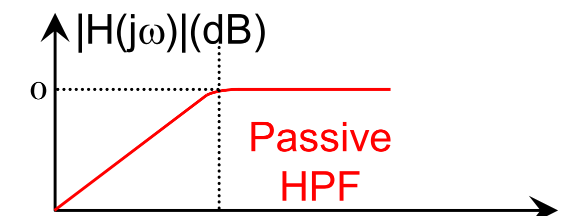
$$\boxed{\left|H(j\omega_{3dB})\right|=\frac{1}{\sqrt{2}}}$$

Interested in finding frequency (f_{3dB}) where the power decreases by half (Power is square of voltage).

$$\Rightarrow \frac{1}{\sqrt{\left(\frac{1}{\omega_{3dB}R_1C_1}\right)^2+1}}=\frac{1}{\sqrt{2}}$$

$$\Rightarrow \omega_{3dB}=\frac{1}{R_1C_1} \quad or \quad f_{3dB}=\frac{1}{2\pi R_1C_1}$$

When the gain, $|H(j\omega)|$, is reduced by $\sqrt{2}$, this is equivalent to a reduction by 3 dB.

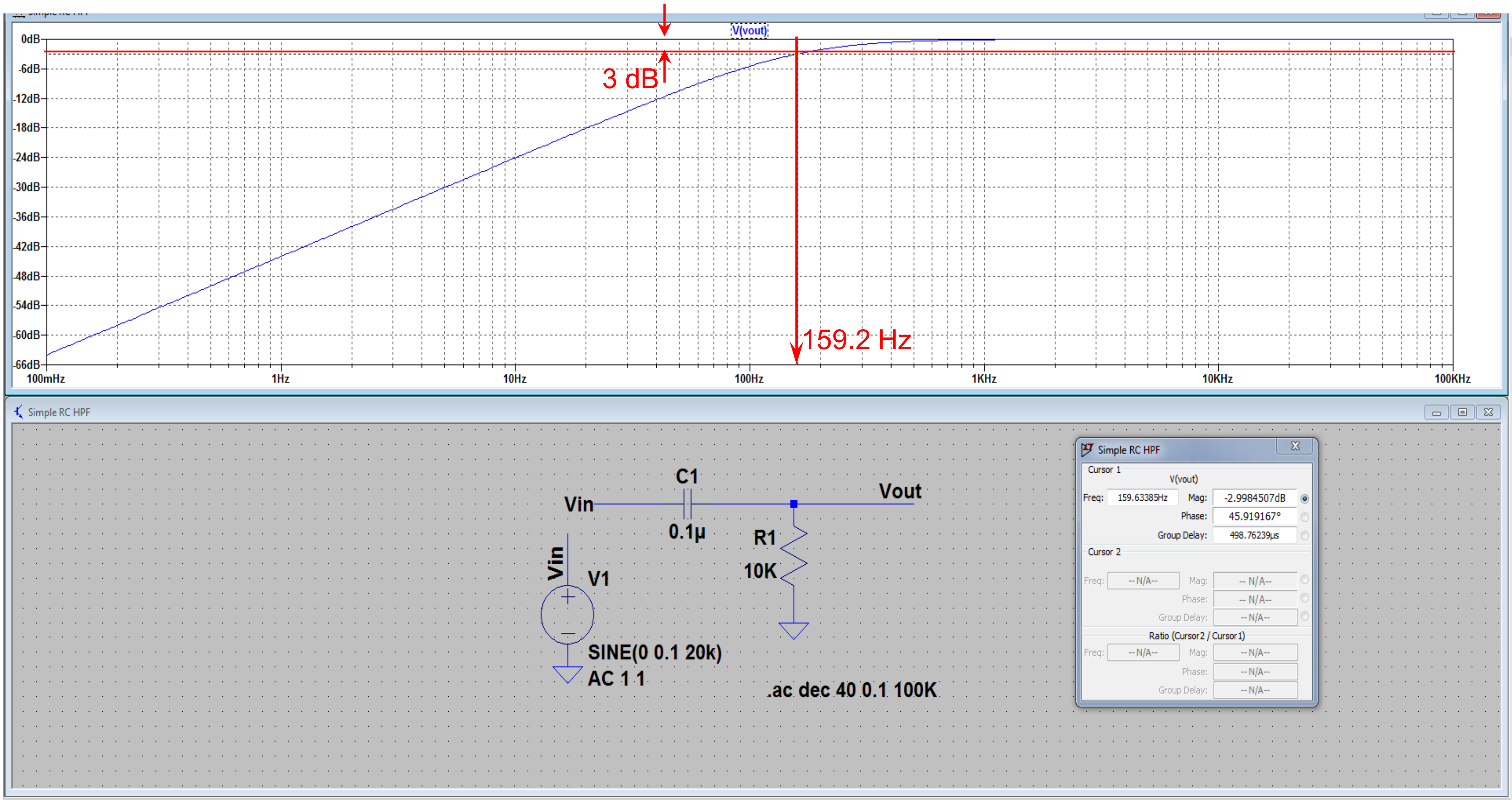


$$f_{3dB}=\frac{1}{2\pi\times(R_1C_1)}$$

f(Hz)

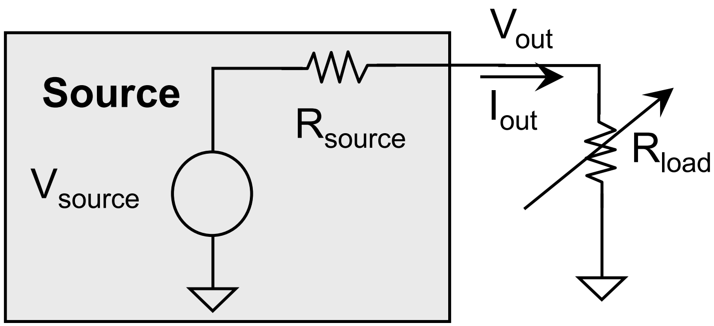
Simulation of Passive 1st Order HPF

- R_1 = 10 kΩ, C_1 = 0.1 μF
- $f_{3dB} = \dfrac{1}{2\pi R_1 C_1} = 159.2\ \text{Hz}$

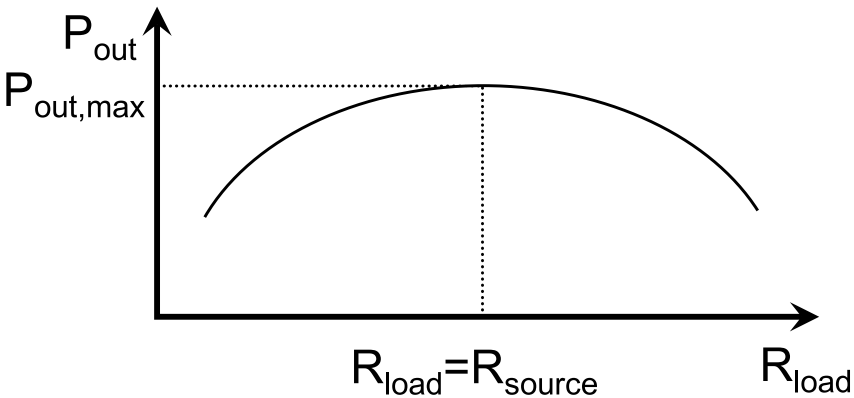


Note: Above frequency response curve is known as Bode plot, where frequency, *f*, is plotted on a log-scale.

Maximum Power Transfer



What is R_{load} that will result in maximum power transfer from source to load?



1) When R_{load} is large, V_{out} is large but I_{out} is small $\Rightarrow P_{out}=V_{out}\times I_{out}$ is small
2) When R_{load} is small, V_{out} is small but I_{out} is large $\Rightarrow P_{out}=V_{out}\times I_{out}$ is small
3) There exists an optimum R_{load} with moderate V_{out} and I_{out} which gives rise to maximum $P_{out}=V_{out}\times I_{out}$

$$P_{out} = \frac{V_{source}}{R_{source}+R_{load}} \times \frac{R_{load}V_{source}}{R_{source}+R_{load}}$$

Use the derivative to find maximum

$$= \frac{R_{load}}{(R_{source}+R_{load})^2} V_{source}^2$$

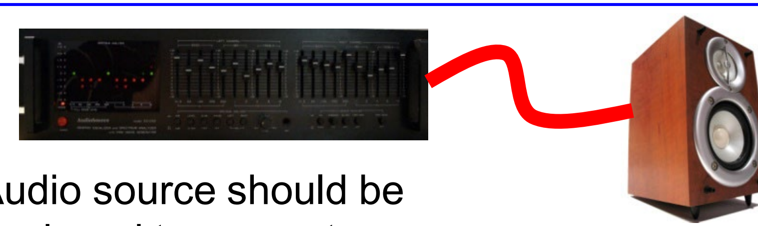
$$\frac{dP_{out}}{dR_{load}} = \frac{1}{(R_{source}+R_{load})^2} - \frac{2R_{load}}{(R_{source}+R_{load})^3} = 0$$

$$\Rightarrow R_{load} = R_{source}$$

Implications:

Antenna behave like source with $R_{source}=50\Omega$

Cell phone should be designed to present $R_{load}=50\Omega$ when interface with antenna



Audio source should be designed to present $R_{source}=8\Omega$ when interface with speaker

Speaker behaves as $R_{load}=8\Omega$

Basic Concepts Covered -

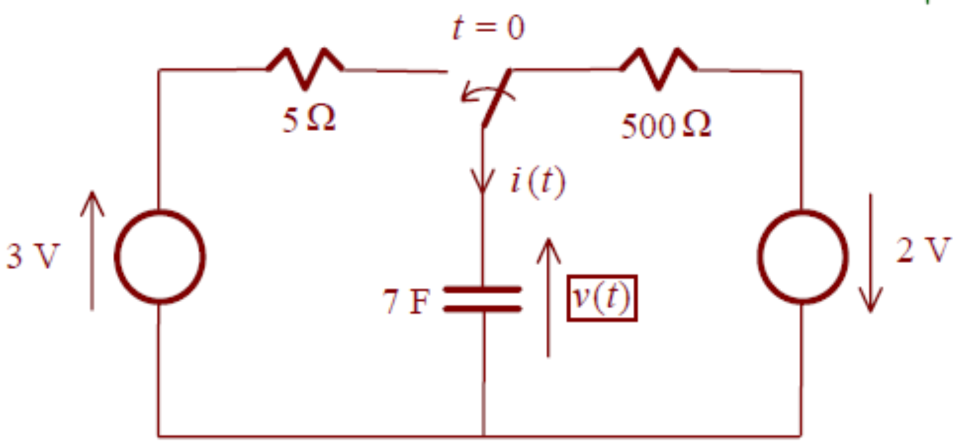
- KCL: Sum of currents into a node, $\sum_k i_k = i_x + i_y + i_z + \cdots = 0$
- KVL: Sum of voltages in a closed loop, $\sum_k V_k = V_x + V_y + V_z + \cdots = 0$
- Linear superposition: while considering effect of one independent source, kill all other independent sources
- Thevenin/Norton Equivalent: V_{THV}/I_{THV} & R_{THV}/I_{THV}
- AC analysis/phasor: $v(t) = \sqrt{2}r\cos(\omega t + \theta)$ vs $\mathrm{V} = r\,e^{j\theta}$
- AC analysis: capacitor: $X_C = \frac{1}{j\omega C}$; Inductor, $X_L = j\omega L$
- Passive 1st order filters: RC low & high pass filters
- Definition of dB (decibel) & f_{3dB} $(\omega_{3dB} = 2\pi f_{3dB})$
- Maximum power transfer: $R_{load} = R_{source}$

Transients (Self Reading)

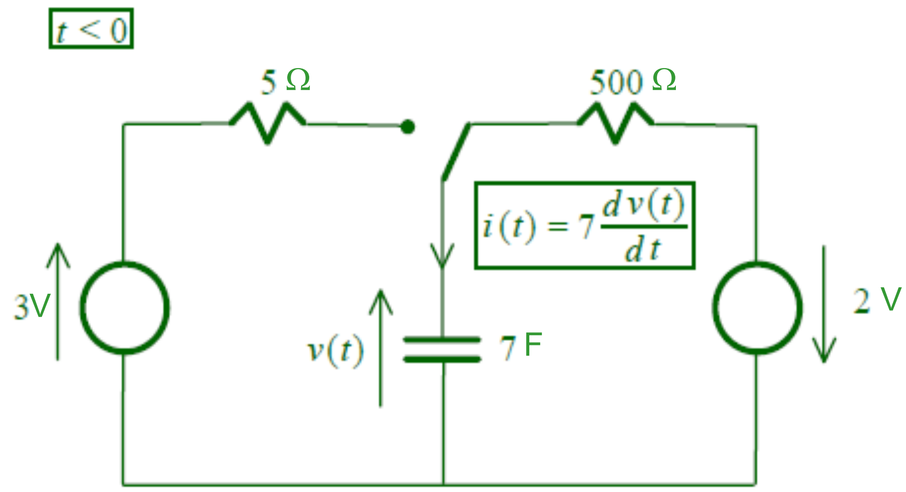
- What is transient analysis?
- DC and AC circuit analysis are typically known as steady state analysis, where transients caused by switching of signal sources are assumed to have died down and all voltages and currents have stabilized.
- On the other hand, when the circuit is first switched on or off, the circuit will not be in the steady state.
- In the non-steady state situation, the determination of the voltages and currents as a function of time is known as transient analysis.

Transients (RC Circuit - Charge)

Consider finding $v(t)$ in the following RC circuit:



where the switch is in the position shown for $t < 0$ and is in the other position for $t \geq 0$.

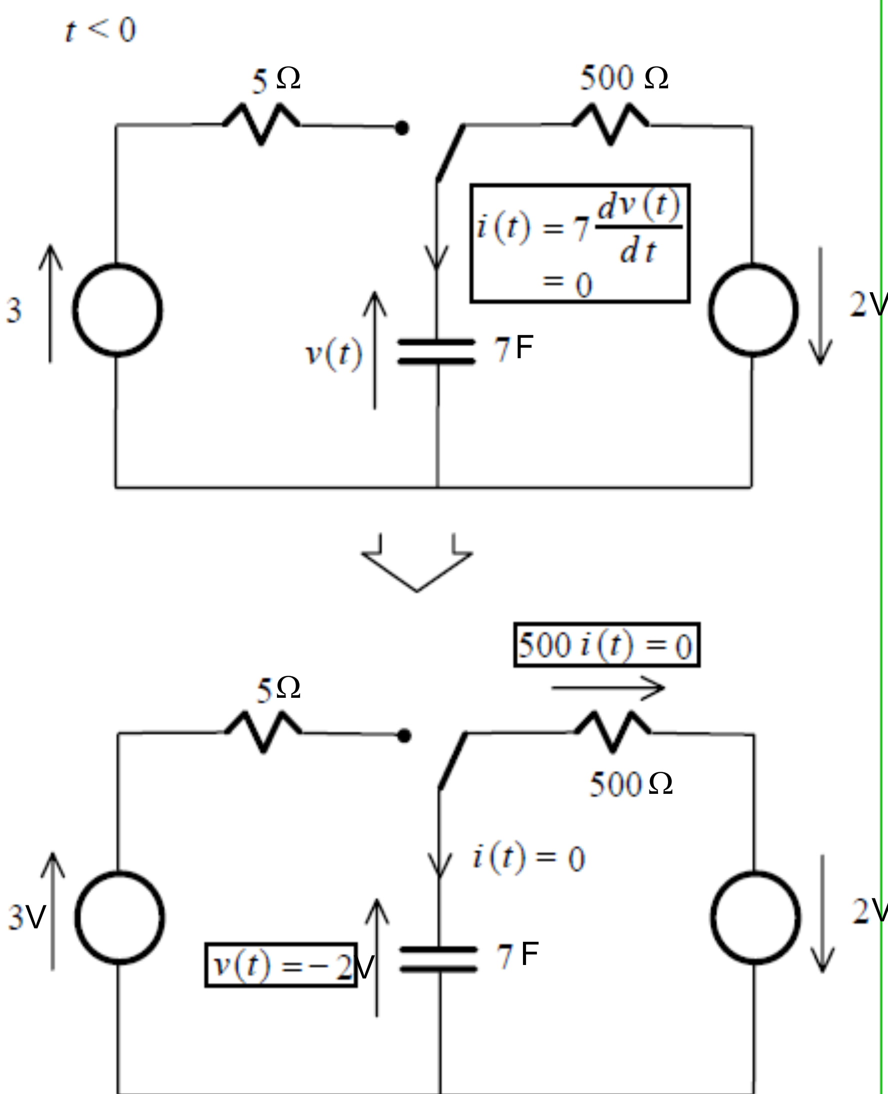


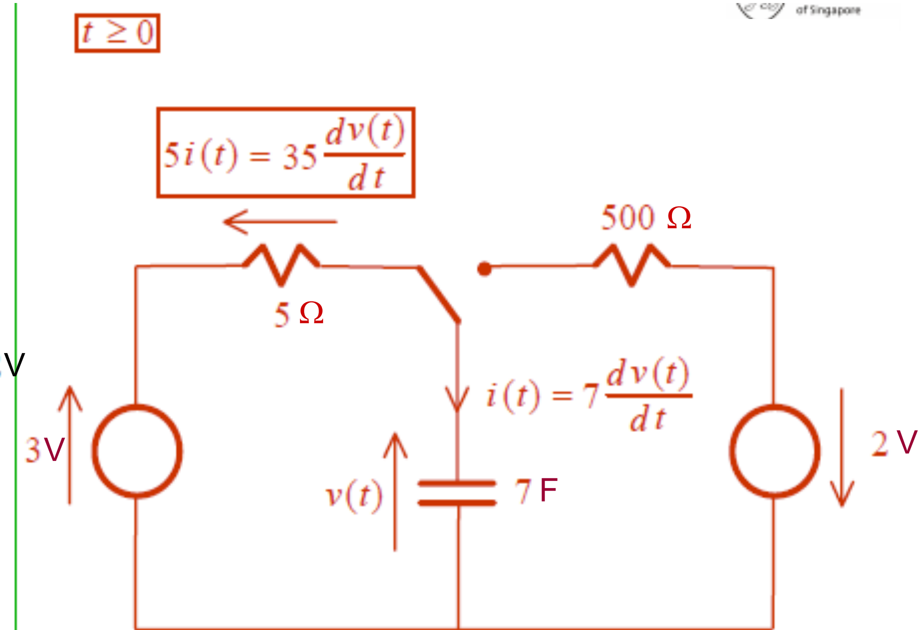
Taking the switch to be in this position starting from $t = -\infty$, the voltages and currents will have settled down to constant values for practically all $t < 0$.

$$i(t) = 7\frac{dv(t)}{dt} = 7\frac{d(\text{constant})}{dt} = 0, \quad t < 0$$

Ref: CC Ko and BM Chen, *Basic Circuit Analysis*.

Transients (RC Circuit - Charge)





Applying KVL:

$$35\frac{dv(t)}{dt} + v(t) = u(t) = 3, \quad t \geq 0$$

which has a solution

$$v(t) = v_{ss}(t) + v_{tr}(t), \quad t \geq 0$$

Transients (RC Circuit - Charge)

$$35\frac{dv(t)}{dt} + v(t) = 3, \qquad t \geq 0$$

Consider first the steady state response: $\frac{dv(t)}{dt} = 0$ and $v(t) = v_{ss}(t)$

$$\therefore v_{ss}(t) = 3, \qquad t \geq 0$$

Next, consider the transient response:

$$35\frac{dv_{tr}(t)}{dt} + v_{tr}(t) = 0\,, \qquad t \geq 0$$

$$\frac{dv_{tr}(t)/_{dt}}{v_{tr}(t)} = -\frac{1}{35}\,, \qquad t \geq 0$$

$$\therefore v_{tr}(t) = ke^{-\frac{t}{35}}, \qquad t \geq 0$$

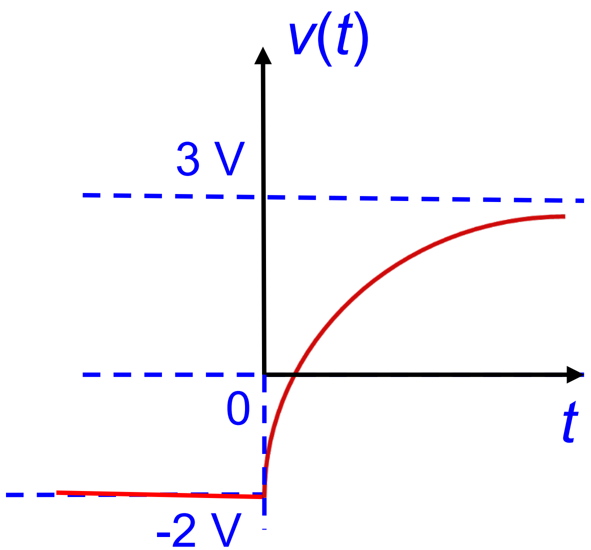
Therefore, the complete response is: $v(t) = 3 + ke^{-\frac{t}{35}}, \qquad t \geq 0$

Transients (RC Circuit - Charge)

- To determine the constant *k* in the transient response of the RC circuit, we use the concept that the voltage *v*(*t*) across a capacitor must be continuous and cannot change abruptly.

- For the RC circuit, the complete solution for *v*(*t*) is:

$$v(t) = \begin{cases} -2\text{ V}, & t < 0 \\ \left(3 + ke^{-\frac{t}{35}}\right)\text{ V}, & t \geq 0 \end{cases}$$



- At $t = 0$ (initial condition),

  $v(0) = -2 = 3 + k$

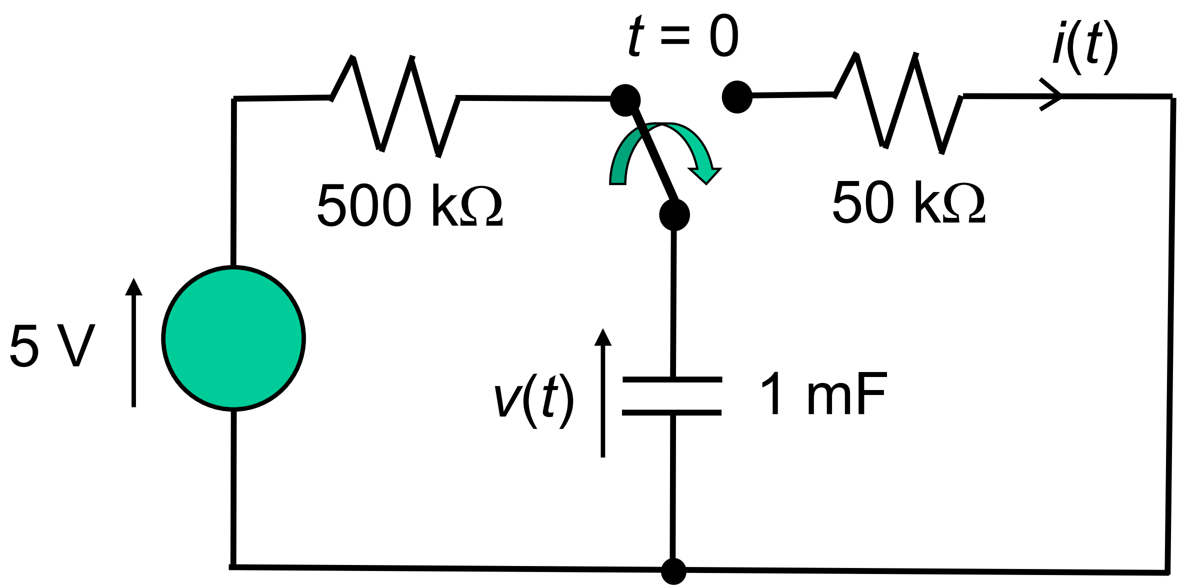
- Thus, $k = -5$.

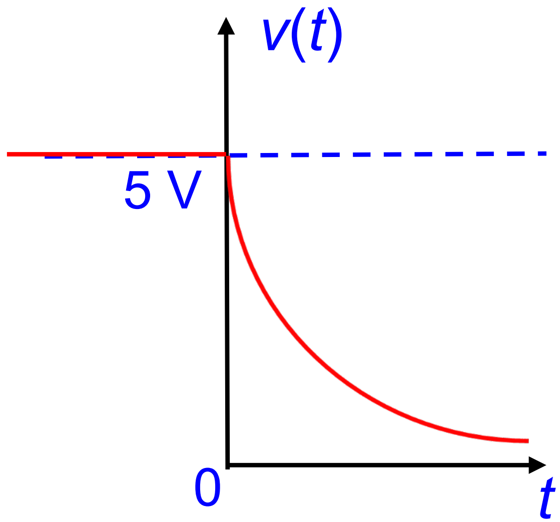
- Hence, the complete solution for *v*(*t*) is:

$$v(t) = \begin{cases} -2\text{ V}, & t < 0 \\ \left(3 - 5e^{-\frac{t}{35}}\right)\text{ V}, & t \geq 0 \end{cases}$$

RC Time Constant = (5 Ω)(7 F) = 35 s

Transients (RC Circuit - Discharge)





$$v(t) = \begin{cases} \mathbf{5\ V,} & \mathbf{t < 0} \\ \mathbf{5e^{-\frac{t}{50}}\ V} & \mathbf{t \geq 0} \end{cases}$$

RC Time Constant =
(50 kΩ)(1 mF) = 50 s

Transients (RL & RLC Circuits)

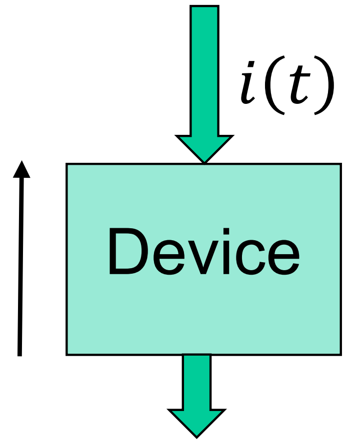
- Similar approach applies to the transient analysis of RL and RLC circuits: Apply KVL to determine the resulting differential equation and then solve it (using appropriate initial condition to determine the constant).

- For RLC circuits, the resulting differential equation is of 2^{nd} order.

Power (Instantaneous & Average) (Self Reading)

$$i(t) = r_i\sqrt{2}\cos(\omega t + \theta_i)$$

$$v(t) = r_v\sqrt{2}\cos(\omega t + \theta_v)$$



Instantaneous power consumed by the device is:

$$p(t) = i(t)v(t) = 2r_i r_v \cos(\omega t + \theta_i)\cos(\omega t + \theta_v)$$
$$= r_i r_v[\cos(\theta_i - \theta_v) + \cos(2\omega t + \theta_i + \theta_v)]$$

Since $2\cos(x_1)\cos(x_2) = \cos(x_1 - x_2) + \cos(x_1 + x_2)$

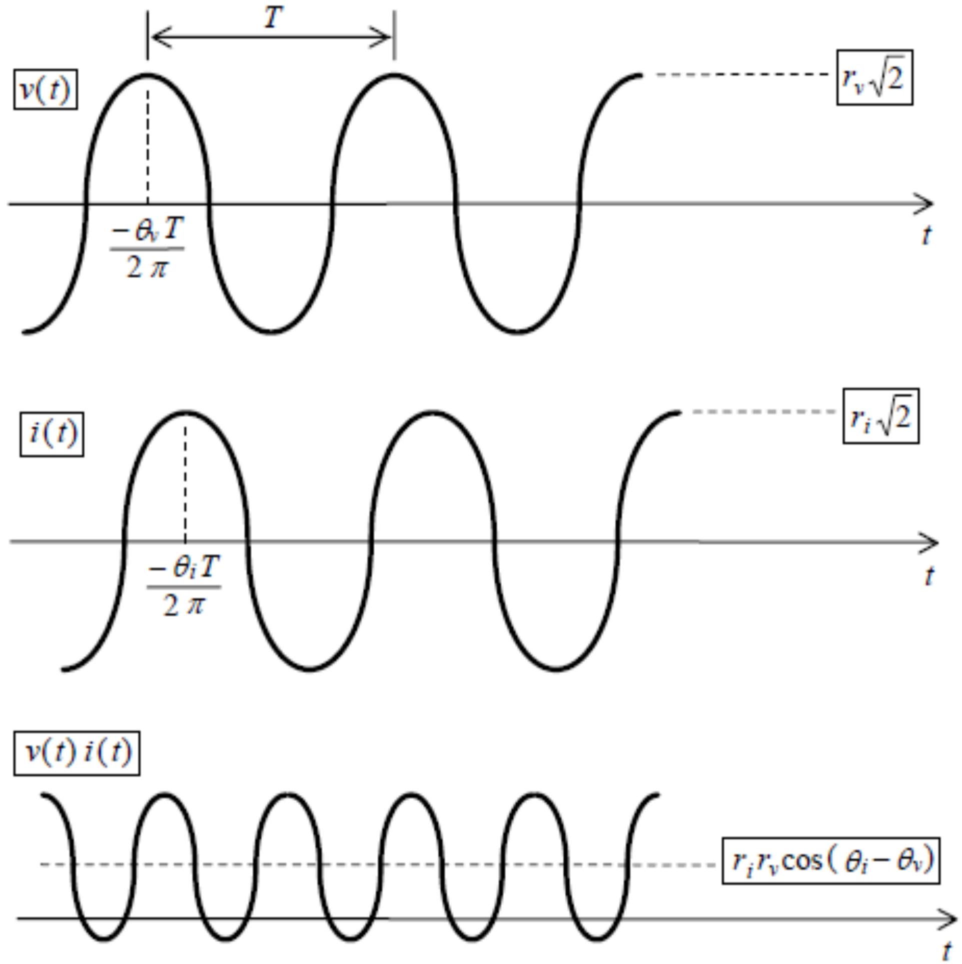
(Take note of the current direction and voltage polarity in the calculation of power consumed)

Average power consumed by the device is:

$$P_{av} = \frac{1}{T}\int_0^T p(t)dt = \frac{r_i r_v}{T}\int_0^T [\cos(\theta_i - \theta_v) + \cos(\frac{4\pi t}{T} + \theta_i + \theta_v)]\,dt$$

$= r_i r_v \cos(\theta_i - \theta_v)$, where T = Period and $\omega = 2\pi / T$

Average Power (P_{av})



Using Phasor notation:

$$V = r_v\, e^{j\theta_v}$$

$$\Rightarrow V^* = r_v\, e^{-j\theta_v}$$

$$I = r_i\, e^{j\theta_i}$$

$$\Rightarrow I^* = r_i\, e^{-j\theta_i}$$

$$V^* I = r_v r_i e^{j(\theta_i - \theta_v)}$$

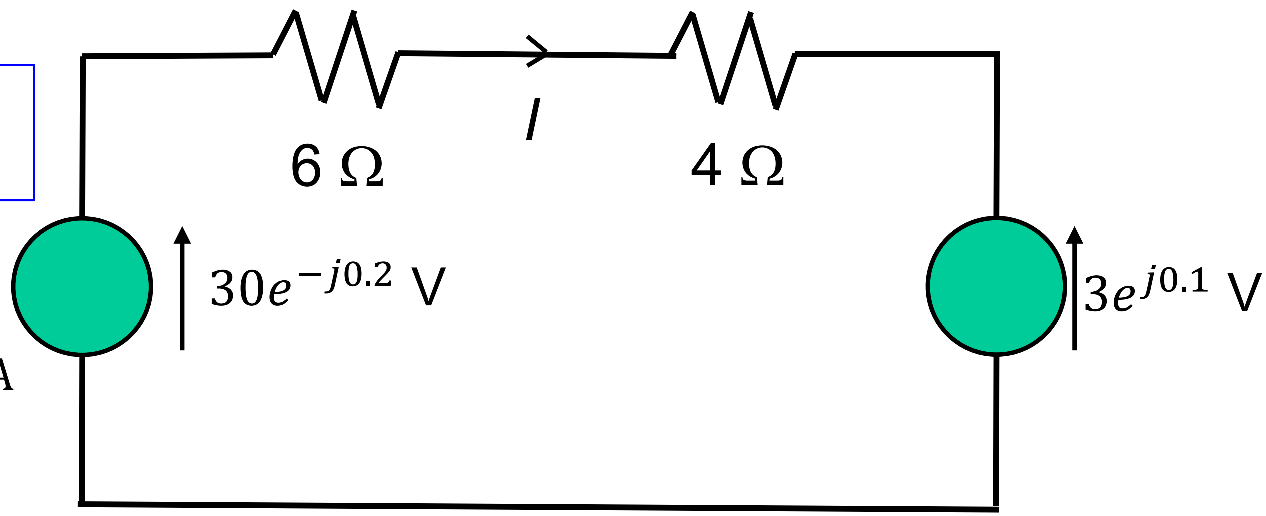
$$V I^* = r_v r_i e^{j(\theta_v - \theta_i)}$$

$$\begin{aligned} P_{av} &= r_v r_i \cos(\theta_v - \theta_i) = r_v r_i \cos(\theta_i - \theta_v) \\ &= Re[r_v r_i e^{j(\theta_v - \theta_i)}] = Re\left[r_v r_i e^{j(\theta_i - \theta_v)}\right] \\ &= Re[V^* I] = Re[V I^*] \end{aligned}$$

Average Power

- The average power equation P_{av} = Re[I^*V] = Re[IV^*] is based on rms current and voltage in ac circuits.
- The average power equation is also valid for dc circuits, which is a special case of ac circuits with f = 0 and V and I having real values.

To show that the Net Average Power (Supplied + Consumed) = 0



Example:

$$I = \frac{30e^{-j0.2} - 3e^{j0.1}}{6+4} = 2.71e^{-j0.23}\ \text{A}$$

$30e^{-j0.2}$ V source: $Re\left[-\left(2.7e^{-j0.23}\right)^*\left(30e^{-j0.2}\right)\right] = -81\cos(0.03) = -80.96$ W

(Negative power indicates power supplied)

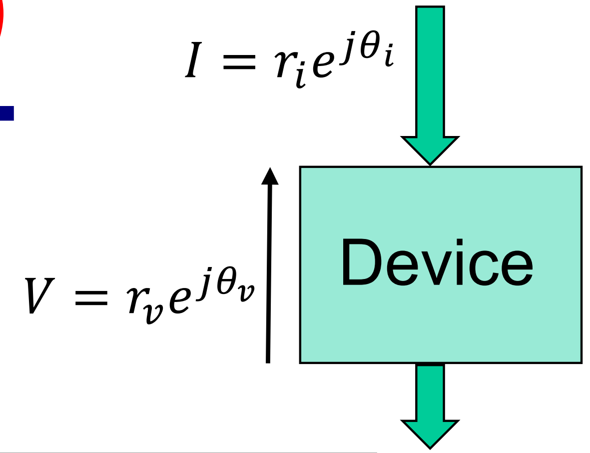
$3e^{j0.1}$ V source: $Re\left[\left(2.7e^{-j0.23}\right)^*\left(3e^{j0.1}\right)\right] = Re\left[8.1e^{j0.33}\right] = 8.1\cos(0.33) = 7.66$ W

6 Ω resistor: $Re\left[\left(2.7e^{-j0.23}\right)^*\left(6 \times 2.7e^{-j0.23}\right)\right] = 6(2.7)^2 = 43.74$ W

4 Ω resistor: $Re\left[\left(2.7e^{-j0.23}\right)^*\left(4 \times 2.7e^{-j0.23}\right)\right] = 4(2.7)^2 = 29.16$ W

Hence, Net Average Power = -80.96 + 7.66 + 43.74 + 29.16 = 0 W

Power Factor (Self Reading)



Consider the previous example of the ac device with current and voltage in Phasor notation:

Real (Average) Power = $\mathrm{Re}[V^*I] = r_v r_i \cos(\theta_i - \theta_v)$ (Unit: W)

Apparent Power (or Voltage-Current Rating) = $|V||I| = r_v r_i$ (Unit: VA)

Power Factor of the device is the ratio of Real Power to Apparent Power:

$$Power\ Factor = \frac{Real\ Power}{Apparent\ Power} = \cos(\theta_i - \theta_v)$$

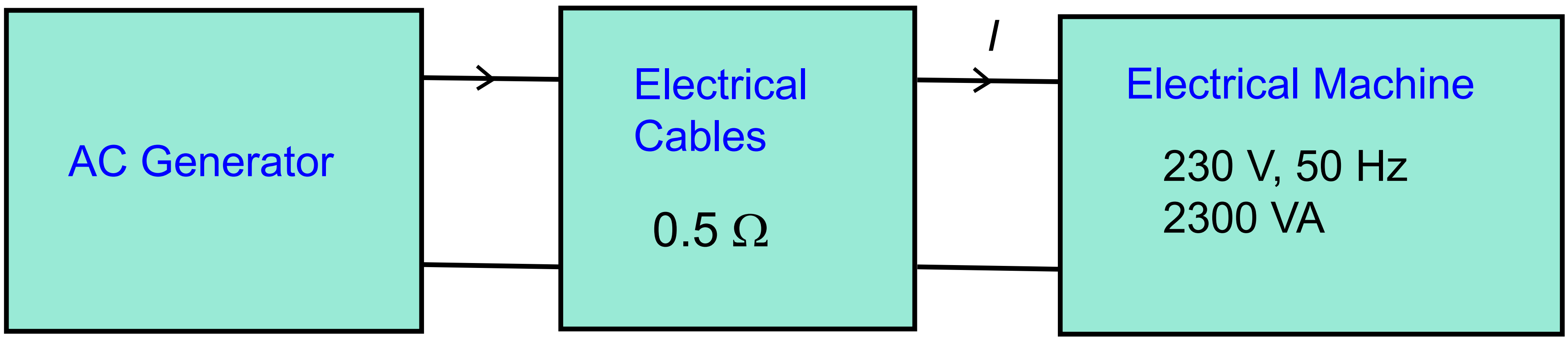
When V and I are in phase => $\theta_i = \theta_v$, and power factor is unity.

Leading Power Factor => I leads V => $\theta_i > \theta_v$

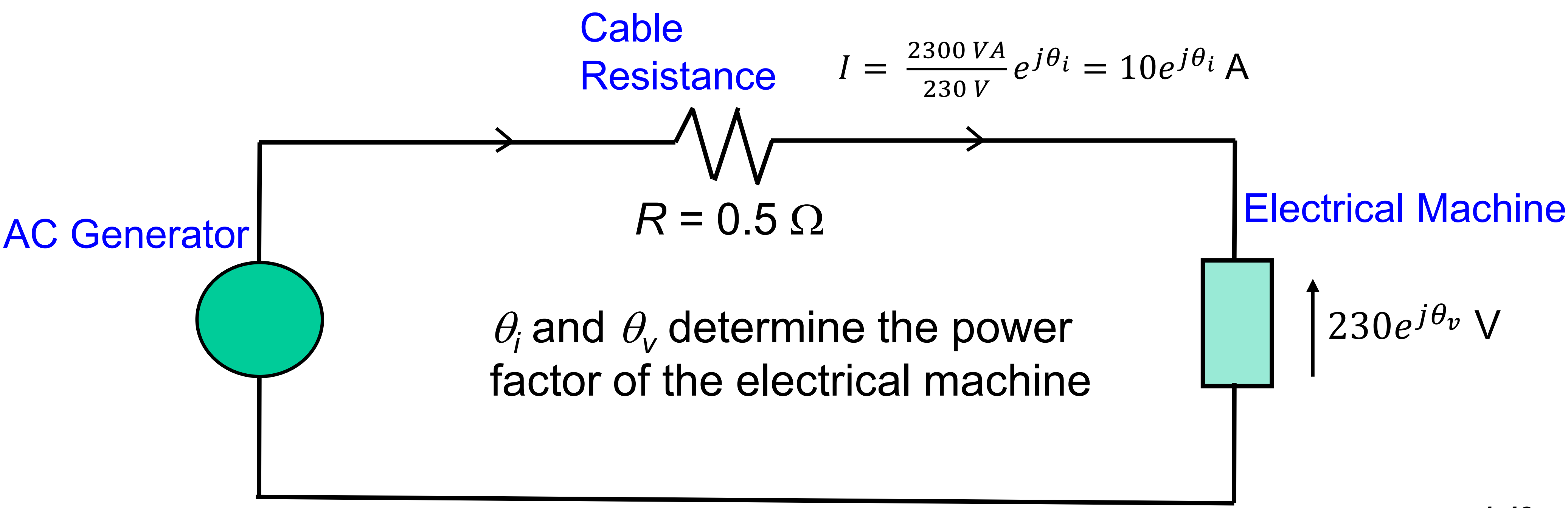
Lagging Power Factor => I lags V => $\theta_i < \theta_v$

Power Factor (Example)

Consider the following example of an AC electrical system:



$$I = \frac{2300\ VA}{230\ V} e^{j\theta_i} = 10e^{j\theta_i}\ \text{A}$$



θ_i and θ_v determine the power factor of the electrical machine

Power Factor (Example)

Power consumed by the electrical machine and power loss at different power factors:

Voltage V (V)	230	230	230
Current I (A)	10	10	10
Voltage-Current Rating (VA) (Apparent Power)	2300	2300	2300
Power Factor $= \cos(\theta_i - \theta_v)$	0.4 leading (I leads V)	1 (I in phase with V)	0.4 lagging (I lags V)
$\theta_i - \theta_v$ (rad)	$\cos^{-1}(0.4) = 1.16$	0	$-\cos^{-1}(0.4) = -1.16$
Power consumed (W) (Real Power)	(2300)(0.4) = 920	2300	(2300)(0.4) = 920
Power loss in cable resistance = I^2R (W)	$(10)^2(0.5) = 50$	$(10)^2(0.5) = 50$	$(10)^2(0.5) = 50$

Power Factor

- At low values of power factor, more apparent power needs to be transferred to obtain the same real power.

- That is, a load with a low power factor draws more current than a load with a high power factor for the same amount of useful (real) power transferred.

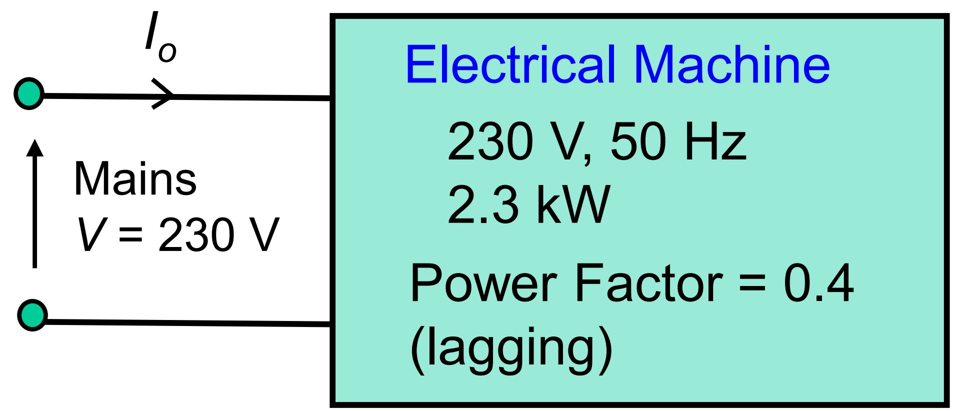
- Higher current drawn increases the energy lost in the distribution system, and requires larger wires and other equipment.

- As a result of the additional cost of larger equipment and wasted energy, electrical utilities will typically charge a higher cost to industrial or commercial customers if the power factor is low.

- A low power factor load will require power factor correction or improvement.

Power Factor Improvement (Example)

- Due to the small power factor, the electrical machine cannot be connected directly to a standard 230 V, 13 A mains outlet even though it consumes only 2.3 kW of power.



- Current I_o can be obtained as follows:

$$|I_o| = \frac{2300\ \text{W}}{(230\ \text{V})(0.4)} = 25\ \text{A}$$

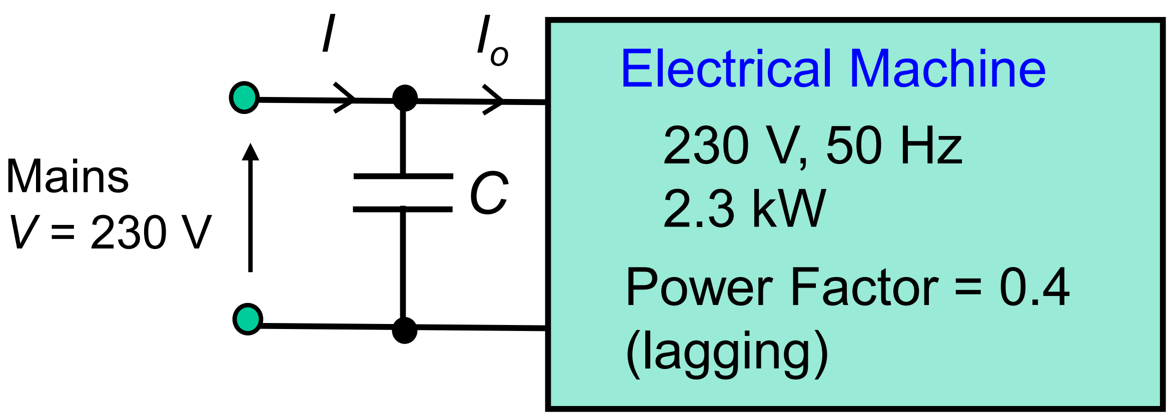
$$\theta_i - \theta_v = -\cos^{-1}(0.4) = -1.16\ \text{rad}$$

$$I_o = 25e^{-j1.16}$$

Question: How can the power factor be improved so that the machine can be connected to a standard 230 V, 13 A mains outlet ?

Power Factor Improvement (Example)

Answer: A capacitor can be connected in parallel across the machine to improve the power factor. We can find the value of the capacitor *C* such that the power factor becomes unity.



Power film capacitor (typically using polypropylene film as dielectric) packaged in a cylindrical metal can for power factor correction

$$I = \frac{V}{Z_C} + 25e^{-j1.16} = (230)(j2\pi)(50)C + 10 - j23 = 10 + j(23000\pi C - 23)$$

- If we choose 23000π*C* = 23, then the imaginary term in *I* will be zero and the overall power factor will be unity.
- Hence *C* = 0.32 mF and *I* = 10 A.

References

- C.C. Ko and B.M. Chen, *Basic Circuit Analysis for Electrical Engineering*, Prentice Hall, 2nd Edition, 1998.
- R.J. Smith and R.C. Dorf, *Circuits, Devices and Systems: A First Course in Electrical Engineering*, John Wiley, 5th Edition, 1992.
- Paul Horowitz, *The Art of Electronics*, Cambridge University Press, 3rd Edition, 2015.