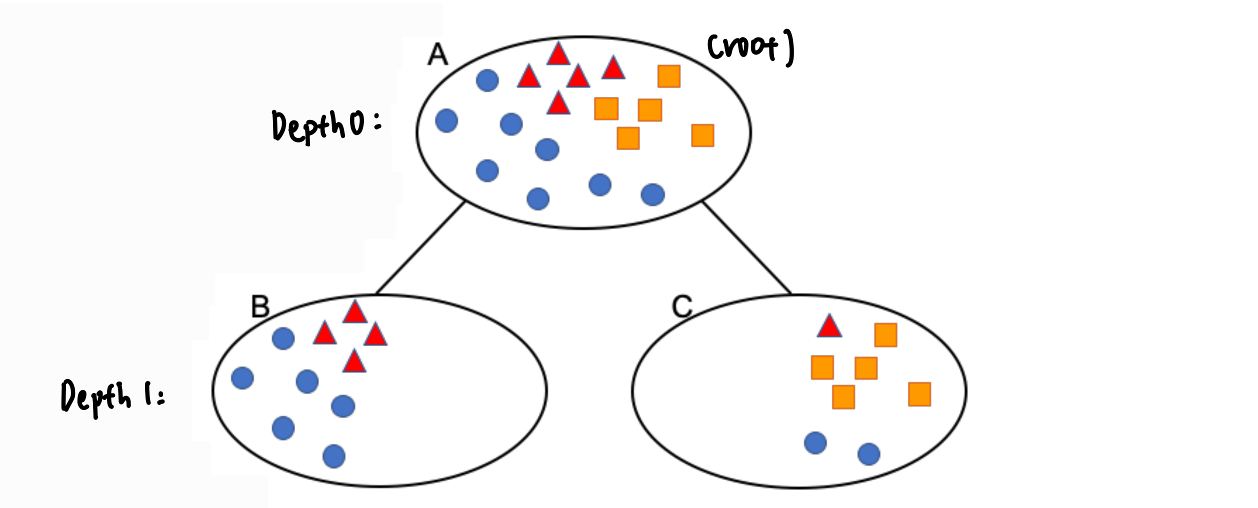
EE2211 Tutorial 9

(Gini impurity, entropy and misclassification rate)

Question 1:

Compute the Gini impurity, entropy, misclassification rate for nodes A, B and C, as well as the overall metrics (Gini impurity, entropy misclassification error) at depth 1 of the decision tree shown below.



Answer:

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Let's assume class 1, class 2 and class 3 correspond to red triangles, orange squares and blue circles respectively.

- For node A, $p_1 = \frac{5}{18}$, $p_2 = \frac{5}{18}$, $p_3 = \frac{8}{18} = \frac{4}{9}$
- For node B, $p_1 = \frac{4}{10} = \frac{2}{5}$, $p_2 = \frac{0}{10} = 0$, $p_3 = \frac{6}{10} = \frac{3}{5}$
- For node C, $p_1 = \frac{1}{8}$, $p_2 = \frac{5}{8}$, $p_3 = \frac{2}{8} = \frac{1}{4}$

For $\boxed{\textbf{Gini impurity}}$ recall formula is $1 - \overset{\text{k= \# of classes: 3}}{\Sigma_{i=1}^{K}}\ p_i^2$

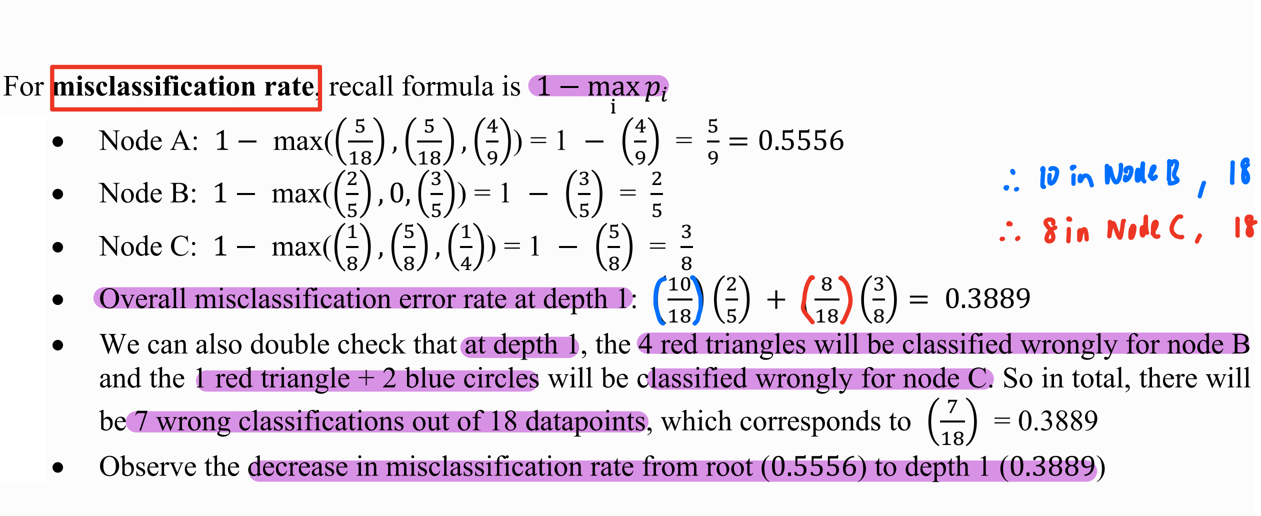
- Node A: $1 - \left(\frac{5}{18}\right)^2 - \left(\frac{5}{18}\right)^2 - \left(\frac{4}{9}\right)^2 = 0.6481$
- Node B: $1 - \left(\frac{2}{5}\right)^2 - (0)^2 - \left(\frac{3}{5}\right)^2 = \boxed{0.48}$
- Node C: $1 - \left(\frac{1}{8}\right)^2 - \left(\frac{5}{8}\right)^2 - \left(\frac{1}{4}\right)^2 = 0.5312$
- Overall Gini at depth 1: $\left(\frac{10}{18}\right)\boxed{0.48} + \left(\frac{8}{18}\right)0.5312 = 0.5028$ [weighted sum, see lect 9 pg 10]

Observe the decrease in Gini impurity from root (0.6481) to depth 1 (0.5028)

For $\boxed{\textbf{entropy}}$, recall formula is $-\overset{\text{3-classes}}{\sum_i} p_i \log_2 p_i$

- Node A: $-\left(\frac{5}{18}\right)\log_2\left(\frac{5}{18}\right) - \left(\frac{5}{18}\right)\log_2\left(\frac{5}{18}\right) - \left(\frac{4}{9}\right)\log_2\left(\frac{4}{9}\right) = 1.5466$
- Node B: $-\left(\frac{2}{5}\right)\log_2\left(\frac{2}{5}\right) - (0)\log_2(0) - \left(\frac{3}{5}\right)\log_2\left(\frac{3}{5}\right) = 0.9710$
- Node C: $-\left(\frac{1}{8}\right)\log_2\left(\frac{1}{8}\right) - \left(\frac{5}{8}\right)\log_2\left(\frac{5}{8}\right) - \left(\frac{1}{4}\right)\log_2\left(\frac{1}{4}\right) = 1.2988$
- Overall entropy at depth 1: $\left(\frac{10}{18}\right)$ 0.9710 + $\left(\frac{8}{18}\right)$ 1.2988 = 1.1167 ∴ 10 in Node B, 18 in depth 1

Observe the decrease in entropy from root (1.5466) to depth 1 (1.1167) ∴ 8 in Node C, 18 in depth 1



For **misclassification rate**, recall formula is $1 - \max_{i} p_i$

- Node A: $1 - \max\left(\left(\frac{5}{18}\right), \left(\frac{5}{18}\right), \left(\frac{4}{9}\right)\right) = 1 - \left(\frac{4}{9}\right) = \frac{5}{9} = 0.5556$
- Node B: $1 - \max\left(\left(\frac{2}{5}\right), 0, \left(\frac{3}{5}\right)\right) = 1 - \left(\frac{3}{5}\right) = \frac{2}{5}$
- Node C: $1 - \max\left(\left(\frac{1}{8}\right), \left(\frac{5}{8}\right), \left(\frac{1}{4}\right)\right) = 1 - \left(\frac{5}{8}\right) = \frac{3}{8}$
- Overall misclassification error rate at depth 1: $\left(\frac{10}{18}\right)\left(\frac{2}{5}\right) + \left(\frac{8}{18}\right)\left(\frac{3}{8}\right) = 0.3889$
- We can also double check that at depth 1, the 4 red triangles will be classified wrongly for node B and the 1 red triangle + 2 blue circles will be classified wrongly for node C. So in total, there will be 7 wrong classifications out of 18 datapoints, which corresponds to $\left(\frac{7}{18}\right) = 0.3889$
- Observe the decrease in misclassification rate from root (0.5556) to depth 1 (0.3889)

∴ 10 in Node B , 18 in depth 1

∴ 8 in Node C, 18 in depth 1

(MSE of regression trees) pg 17 Lect 9

Question 2:

Calculate the overall MSE for the following data at depth 1 of a regression tree assuming a decision threshold is taken at $x = 5.0$. How does it compare with the MSE at the root?

$\{x, y\}$: {1, 2}, {0.8, 3}, {2, 2.5}, {2.5, 1}, {3, 2.3}, {4, 2.8}, {4.2, 1.5}, | {6, 2.6}, {6.3, 3.5}, {7, 4}, {8, 3.5}, {8.2, 5}, {9, 4.5}

$x > 5$

Answer:

$$\bar{y} = \frac{2.6+3.5+4+3.5+5+4.5}{6}$$

o.g. target   predict

MSE at depth 1 : $\frac{1}{J_m}\sum_{j=1}^{J_m}(y_j - \hat{y}_m)$ ∴ J_m : num of training samples in leaf node

At depth 1, when $x > 5$

- $y = \{2.6, 3.5, 4, 3.5, 5, 4.5\}$ => $\bar{y} = 3.85$ mean
- MSE = $\frac{1}{6}\underbrace{((2.6-\bar{y})^2 + (3.5-\bar{y})^2 + (4-\bar{y})^2 + (3.5-\bar{y})^2 + (5-\bar{y})^2 + (4.5-\bar{y})^2)}_{J_m=6} = 0.5958$

At depth 1, when $x \leq 5$

$$\bar{y} = \frac{2+3+2.5+1+2.3+2.8+1.5}{7}$$

- $y = \{2, 3, 2.5, 1, 2.3, 2.8, 1.5\}$ => $\bar{y} = 2.1571$ mean
- MSE = $\frac{1}{7}\underbrace{((2-\bar{y})^2 + (3-\bar{y})^2 + (2.5-\bar{y})^2 + (1-\bar{y})^2 + (2.3-\bar{y})^2 + (2.8-\bar{y})^2 + (1.5-\bar{y})^2)}_{J_m=7} =$ 0.4367

Overall MSE at depth 1: $\frac{6}{13} \times 0.5958 + \frac{7}{13} \times 0.4367 = 0.5102$

total samples: 13

∴ $x > 5$: 6 samples   (weighted sum)
$x \leq 5$: 7 samples
total : 13

At the root:

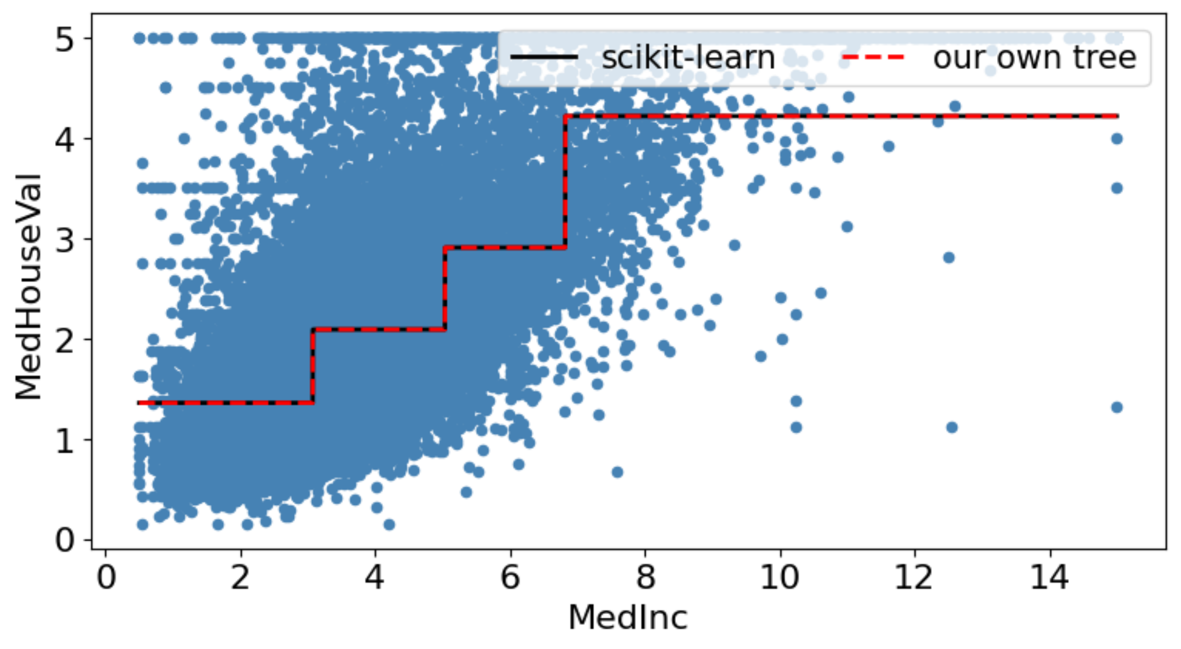
- $y = \underbrace{\{2, 3, 2.5, 1, 2.3, 2.8, 1.5, 2.6, 3.5, 4, 3.5, 5, 4.5\}}$ => $\bar{y} = 2.9385$ mean
- MSE = $\frac{1}{13}((2.6-\bar{y})^2 + (3.5-\bar{y})^2 + (4-\bar{y})^2 + (3.5-\bar{y})^2 + (5-\bar{y})^2 + (4.5-\bar{y})^2 + (2-\bar{y})^2 + (3-\bar{y})^2 + (2.5-\bar{y})^2 + (1-\bar{y})^2 + (2.3-\bar{y})^2 + (2.8-\bar{y})^2 + (1.5-\bar{y})^2) = 1.2224$

Therefore, MSE has decreased from 1.2224 at the root to 0.5102 at depth 1

(Regression tree, Python)
Question 3:
Import the California Housing dataset “from sklearn.datasets import fetch_california_housing” and “housing = fetch_california_housing()”. This data set contains 8 features and 1 target variable listed below. Use “MedInc” as the input feature and “MedHouseVal” as the target output. Fit a regression tree to depth 2 and compare your results with results generated by “from sklearn.tree import DecisionTreeRegressor” using the “squared error” criterion.

```
Target: ['MedHouseVal']
Features:['MedInc', 'HouseAge', 'AveRooms', 'AveBedrms', 'Population', 
'AveOccup', 'Latitude', 'Longitude']
```

Answer:
Please refer to Tut9_Q3_zhou.py. We can exactly replicate the results from scikit-learn. Note that in the plot below, the blue dots are the training datapoints. The curves from scikit-learn (black line) and our own tree (red dashed line) are on top of each other, so they might be hard to tell apart.



(Classification tree, Python)
Question 4:
Get the data set “from sklearn.datasets import load_iris”. Perform the following tasks.

(a) Split the database into two sets: 80% of samples for training, and 20% of samples for testing using random_state=0
(b) Train a decision tree classifier (i.e., “tree.DecisionTreeClassifier” from sklearn) using the training set with a maximum depth of 4 based on the “entropy” criterion.

(c) Compute the training and test accuracies. You can use `accuracy_score` from `sklearn.metrics` for accuracy computation

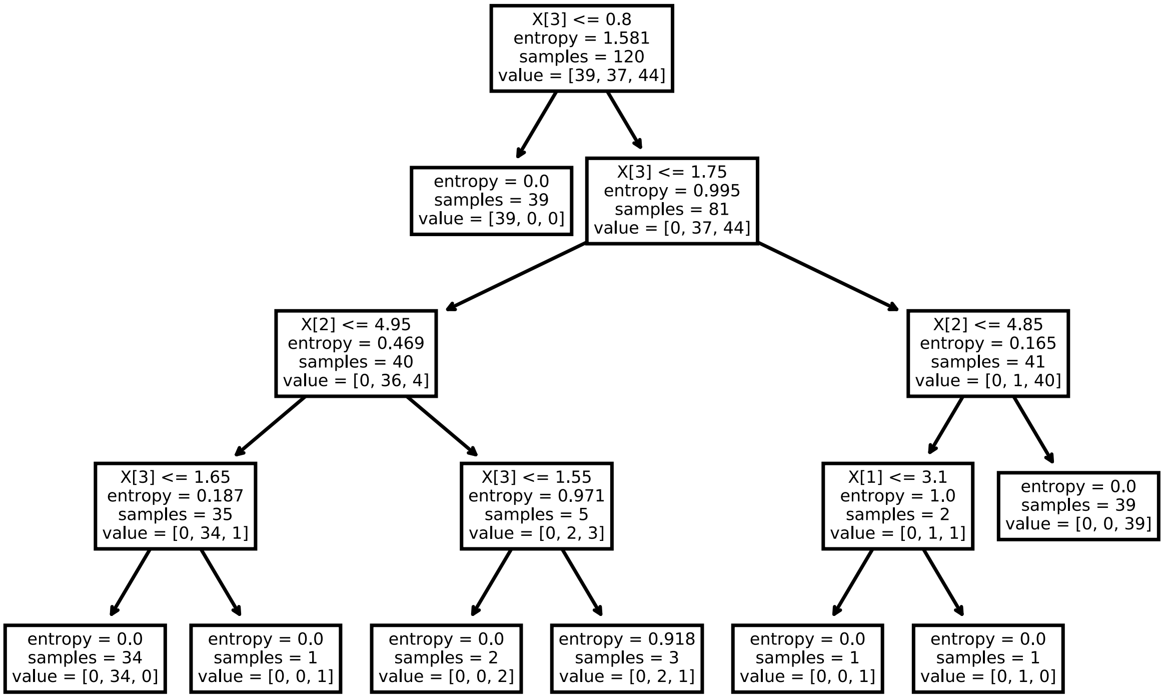
(d) Plot the tree using “`tree.plot_tree`”.

Answer:

Please refer to Tut9_Q4_yeo.py.

Training accuracy: 0.9917
Test accuracy: 1.0

The resulting tree looks like this:


X[3] <= 0.8
entropy = 1.581
samples = 120
value = [39, 37, 44]
entropy = 0.0
samples = 39
value = [39, 0, 0]
X[3] <= 1.75
entropy = 0.995
samples = 81
value = [0, 37, 44]
X[2] <= 4.95
entropy = 0.469
samples = 40
value = [0, 36, 4]
X[2] <= 4.85
entropy = 0.165
samples = 41
value = [0, 1, 40]
X[3] <= 1.65
entropy = 0.187
samples = 35
value = [0, 34, 1]
X[3] <= 1.55
entropy = 0.971
samples = 5
value = [0, 2, 3]
X[1] <= 3.1
entropy = 1.0
samples = 2
value = [0, 1, 1]
entropy = 0.0
samples = 39
value = [0, 0, 39]
entropy = 0.0
samples = 34
value = [0, 34, 0]
entropy = 0.0
samples = 1
value = [0, 0, 1]
entropy = 0.0
samples = 2
value = [0, 0, 2]
entropy = 0.918
samples = 3
value = [0, 2, 1]
entropy = 0.0
samples = 1
value = [0, 0, 1]
entropy = 0.0
samples = 1
value = [0, 1, 0]