EE2211 Tutorial 6

XTXW+LIW = XTY
LW = XTY - XTXW

IA) (XTX+ AI) w= XTy

Lw = xT(y-Xw)

 $W = X^{T} \frac{(y-X_{W})}{\lambda}$

(Ridge Regression in Dual Form)

Question 1:

Derive the solution for linear ridge regression in dual form (see Lecture 6 notes page 16).

$$\therefore W = X^{T} A \qquad A = \frac{(y - Xw)}{\lambda}$$

(Polynomial Regression, 1D data)

Ouestion 2:

Given the following data pairs for training

Да = y-Xw Да = y-Xx^ta

XX^Ta + La = y (XX^Tf L) a = y a = (XX^Tf L) - 1 y

" w=XTa= XTCXXT+A) Ty

- (a) Perform a 3rd-order polynomial regression and sketch the result of line fitting.
- (b) Given a test point $\{x = 9\}$ predict y using the polynomial model.
- (c) Compare this prediction with that of a linear regression.

constant (C(3,1)) (C(3)2)

(Polynomial Regression, 3D data, Python) $+ w_{333} x_3^3 + w_{122} x_1 x_2^2 + w_{133} x_1 x_3^2 + w_{211} x_2 x_3^2 + w_{211} x_2 x_1^2 + w_{213} x_2 x_3^2 + w_{211} x_2 x_1^2 + w_{213} x_2 x_2^2 + w_{211} x_2 x_1^2 + w_{21$

- (a) Write down the expression for a 3rd order polynomial model having a 3-dimensional input.
- (b) Write down the **P** matrix for this polynomial given $\mathbf{X} = \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & -1 & 1 \end{bmatrix}$.
- (c) Given $\mathbf{y} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, can a unique solution be obtained in dual form? If so, proceed to solve it.
- (d) Given $\mathbf{y} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, can the primal ridge regression be applied to obtain a unique solution? If so, proceed to solve it.
- (e)

(Binary Classification, Python)

Question 4:

Given the training data:

$${x = -1} \rightarrow {y = class1}$$

 ${x = 0} \rightarrow {y = class1}$
 ${x = 0.5} \rightarrow {y = class2}$
 ${x = 0.3} \rightarrow {y = class1}$
 ${x = 0.8} \rightarrow {y = class2}$

Predict the class label for $\{x = -0.1\}$ and $\{x = 0.4\}$ using linear regression with signum discrimination.

(Multi-Category Classification, Python)

Question 5:

Given the training data:

Sscape:
$$\begin{cases} \{x = -1\} \to \{y = class1\} \\ \{x = 0\} \to \{y = class1\} \\ \{x = 0.5\} \to \{y = class2\} \\ \{x = 0.3\} \to \{y = class3\} \\ \{x = 0.8\} \to \{y = class2\} \end{cases}$$

(a) Predict the class label for $\{x = -0.1\}$ and $\{x = 0.4\}$ based on linear regression towards a one-hot encoded one input feature, x

(b) Predict the class label for $\{x = -0.1\}$ and $\{x = 0.4\}$ using a polynomial model of 5th order and a one-hot encoded target. f(x)= Wo+ W1x+ W2 x2 + W1x3+ W4x+ W+ x5

(Multi-Category Classification, Python)

Question 6 (continued from Q3 of Tutorial 2):

Get the data set "from sklearn.datasets import load iris". Use Python to perform the following tasks.

- (a) Split the database into two sets: 74% of samples for training, and 26% of samples for testing. Hint: you might want to utilize from sklearn.model selection import train test split for the splitting.
- (b) Construct the target output using one-hot encoding.
- (c) Perform a linear regression for classification (without inclusion of ridge, utilizing one-hot encoding for the learning target) and compute the number of test samples that are classified correctly.
- (d) Using the same training and test sets as in above, perform a 2nd order polynomial regression for classification (again, without inclusion of ridge, utilizing one-hot encoding for the learning target) and compute the number of test samples that are classified correctly. Hint: you might want to use from sklearn.preprocessing import PolynomialFeatures for generation of the polynomial

Question 7

Question 7

Question 7

C) unique solution; matrix must be square, full rank

$$X = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 3 & 3 \end{bmatrix}$$

Consider the square of two-dimensional data points $X = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 3 & 3 \end{bmatrix}$ with cols flam rows

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 3 & 3 \end{bmatrix}$$

Chee 4)

corresponding target vector $\mathbf{y} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$. Suppose you want to use a full third-order polynomial model to fit these data. b) X: [0 1]

Which of the following is/are true?

- b) The polynomial learning system is an under-determined one
- c) The learning of the polynomial model has infinite number of solutions
- d) The input matrix X has linearly dependent samples
- e) None of the above

input features of order

(a)
$$C(2_13) = \omega_1 x_1 + C(2_11) + C(2_12) + C(2_13)$$

(b) $C(2_13) = \omega_1 x_2 + 3 + 4$

(c) $C(2_13) = (1 + 2 + 3 + 4)$

(d) $C(2_13) = (1 + 2 + 3 + 4)$

(e) $C(2_13) = (1 + 2 + 3 + 4)$

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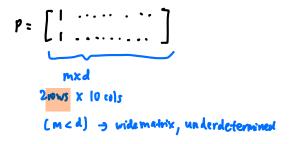
(g) $C(2_13) = (1 + 2 + 3 + 4)$

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(g) $C(2_13) = (1$



Question 8

MCQ: there could be more than one answer. Which of the following is/are true?

- a) The polynomial model can be used to solve problems with nonlinear decision boundary.
- b) The ridge regression cannot be applied to multi-target regression.
- c) The solution for learning feature **X** with target **y** based on linear ridge regression can be written as $\hat{\mathbf{w}} = (\mathbf{X}^T \mathbf{X} + \lambda \mathbf{I})^{-1} \mathbf{X}^T \mathbf{y}$ for $\lambda > 0$. As λ increases, $\hat{\mathbf{w}}^T \hat{\mathbf{w}}$ decreases.
- d) If there are four data samples with two input features each, the full second-order polynomial model is an overdetermined system. $\frac{1}{2}$ ols (x_1, x_2)
- a) True. Polynomial models can capture non-linear decession boundaries by introducing higher degree terms
- b) False. Ridge regression can be applied to multi-farget regression. In multi-target regression, y is a matrix Cinstend of a vector) and ridge regression works by applying L2 regularization to all farget variables.
- c) True.

Using a linear model:

$$\min_{\mathbf{w}} (\mathbf{X}\mathbf{w} - \mathbf{y})^T (\mathbf{X}\mathbf{w} - \mathbf{y}) + \lambda \mathbf{w}^T \mathbf{w}$$

Solution:

$$\frac{\partial}{\partial \mathbf{w}} ((\mathbf{X}\mathbf{w} - \mathbf{y})^T (\mathbf{X}\mathbf{w} - \mathbf{y}) + \lambda \mathbf{w}^T \mathbf{w}) = \mathbf{0}$$

$$\Rightarrow 2\mathbf{X}^T \mathbf{X} \mathbf{w} - 2\mathbf{X}^T \mathbf{y} + 2\lambda \mathbf{w} = \mathbf{0}$$

$$\Rightarrow \mathbf{X}^T \mathbf{X} \mathbf{w} + \lambda \mathbf{w} = \mathbf{X}^T \mathbf{y}$$

$$\Rightarrow (\mathbf{X}^T \mathbf{X} + \lambda \mathbf{I}) \mathbf{w} = \mathbf{X}^T \mathbf{y}$$

where I is the dxd identity matrix

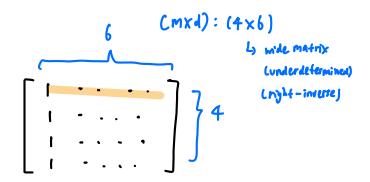
d)
$$X = \begin{bmatrix} x_{11} & x_{11} \\ x_{21} & x_{22} \\ x_{31} & x_{32} \\ x_{41} & x_{41} \end{bmatrix}$$
 4 Samples

2.24

(m) x d)

2nd order

$$P = \left[\left[\begin{array}{cccc} \chi_1 & \chi_2 & \chi_1^2 & \chi_2^2 & \chi_1 \chi_2 \end{array} \right] \rightarrow$$



Xw = y

- $X o ext{Input (feature matrix, shape: } m imes n ext{)}$
- w o Solution (weights/parameters, shape: n imes 1
- y o Target (output vector, shape: m imes 1)

This is a standard **linear regression** equation, where we try to find w that minimizes the error between Xw and u.