EE2211 Tutorial 3

(Probability Mass Function)

Question 1:

The random variable N has probability mass function (PMF)

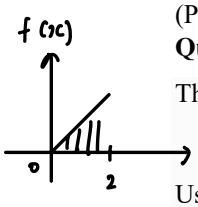
$$P_N(n) = \begin{cases} c\left(1/2\right)^n, & n = 0,1,2 \\ 0, & \text{otherwise} \end{cases}$$

(a) What is the value of the constant c?

(b) What is $Pr[N \leq 1]$?

(Probability Density Function)

Question 2:



The random variable X has probability density function (PDF)

$$f_X(x) = \begin{cases} cx, & 0 \leq x \leq 2 \\ 0, & \text{otherwise.} \end{cases}$$

Use the PDF to find

(a) the constant c,

(b) $Pr[0 \leq X \leq 1]$,

(c) $Pr[-1/2 \leq X \leq 1/2]$.

(Bayes' rule)

Question 3:

Let A = {resistor is within 50Ω of the nominal value}. The probability that a resistor is from machine B is $Pr[B] = 0.3$. The probability that a resistor is acceptable, i.e., within 50 Ω of the nominal value, is $Pr[A] = 0.78$. Given that a resistor is from machine B, the conditional probability that it is acceptable is $Pr[A|B] = 0.6$. What is the probability that an acceptable resistor comes from machine B?

(Discrete random variable in Python)

Question 4:

Consider tossing a fair six-sided die. There are only six outcomes possible, $\Omega = \{1, 2, 3, 4, 5, 6\}$. Suppose we toss two dice and assume that each throw is *independent.*

(a) What is the probability that the sum of the dice equals seven?

i. List out all pairs of possible outcomes together with their sums from the two throws. (hint: enumerate all the items in `range(1,7)`)

ii. Collect all of the *(a, b)* pairs that sum to each of the possible values from two to twelve (including the sum equals seven). (hint: use dictionary `from collections import defaultdict` to collect all of the *(a, b)* pairs that sum to each of the possible values from two to twelve)

(b) What is the probability that half the product of three dice will exceed their sum?

(Continuous random variable in Python)

Question 5:

Assuming a normal (Gaussian) distribution with mean 30 Ω and standard deviation of 1.8 Ω, determine the probability that a resistor coming off the production line will be within the range of 28 Ω to 33 Ω. (Hint: use `stats.norm.cdf` function `from scipy import stats`)

(Correlation versus Causation)

Question 6:

For each of the following graphs,

(i) State what you think the evidence is trying to suggest. Is there correlation or not?

(ii) Give a reason why you agree or disagree with what the evidence is suggesting.

Suggested Discussion:

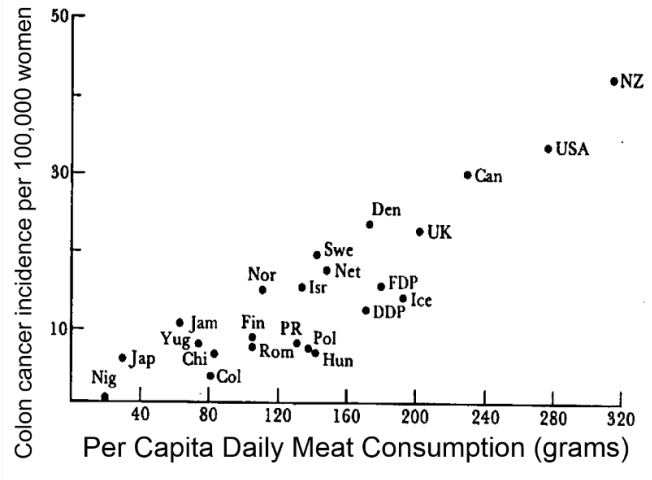
(i) Colon cancer is correlated to the amount of daily meat consumption.

(ii) There is a clear linear trend; countries with the lowest meat consumption have the lowest rates of colon cancer, and the colon cancer rate among these countries progressively increases as meat consumption increases.

(iii) Probably causal.

(iii) Identify whether the variable of the y-axis and the variable of the x-axis are correlated and/or causal?

In the study below investigators used commerce data to compute the overall consumption of meat by various nations. They then calculated the average (per capita) meat consumption per person by dividing total national meat consumption by the number of people in a given country.



http://sphweb.bumc.bu.edu/otlt/MPH-Modules/PH717-QuantCore/PH717-Module1B-DescriptiveStudies_and_Statistics/PH717-Module1B-DescriptiveStudies_and_Statistics6.html

Question 7: (Multiple responses – one or more answers are correct)
If A and B are correlated, but they're actually caused by C, which of the following statements are correct?

a) A and C are correlated
b) B and C are correlated
c) A causes B to happen
d) A causes C to happen

C → A
C → B

Question 8: (Multiple responses – one or more answers are correct)

We toss a coin and observe which side is facing up. Which of the following statements represent valid probability assignments for observing head P['H'] and tail P['T']?

a) P['H']=0.2, P['T']=0.9 → does not sum to 1
b) P['H']=0.0, P['T']=1.0
c) P['H']=-0.1, P['T']=1.1 → -ve probability no such thing
d) P['H']=P['T']=0.5 → fair

Question 9: (True-False)

Are the two vectors $\begin{bmatrix}1\\2\\3\end{bmatrix}$ and $\begin{bmatrix}4\\5\\6\end{bmatrix}$ linearly dependent?

Linear independent because not scalar multiple of each other

Question 10: (Fill-in-blank)

The rank of the matrix $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ is _BLANK_.

rank = 2

Question 11: (Fill-in-blank)

The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ is _BLANK_. 2 x col 2 - col 1

rank = 2