1. Following the in-class example, we need to prove that it is a Gram matrix.

$$\frac{1}{t+s} = \int_0^\infty e^{-(t+s)v}\,dv = \int_0^\infty \underset{\substack{\uparrow \\ x(t)}}{e^{-tv}}\;\underset{\substack{\uparrow \\ x(s)}}{e^{-sv}}\,dv$$

$$(\bar{x}_1, \dots, \bar{x}_n) = \begin{pmatrix} \int_0^\infty e^{-v} e^{-v}\,dv & e^{-v} e^{-2v}\,dv & \int_0^\infty e^{-v} e^{-mv}\,dv \\ \int_0^\infty e^{-2v} e^{-v}\,dv & \int_0^\infty e^{-2v} e^{-2v}\,dv & \int_0^\infty e^{-2v} e^{-mv}\,dv \\ \vdots & \vdots & \vdots \\ \int_0^\infty e^{-nv} e^{-v}\,dv & \int_0^\infty e^{-nv} e^{-2v}\,dv & \int_0^\infty e^{-nv} e^{-mv}\,dv \end{pmatrix}$$

$$= \int_0^\infty \left(x_1 e^{-v} e^{-v} + \dots + x_n e^{-vn} e^{-v}\right)\left(\bar{x}_1 e^{-v} e^{-v} + \dots + e^{-v} e^{-vm}\right) dv$$

$$= \int_0^\infty f(v)\,\overline{f(v)}\,dv \geq 0$$

2. $K(t,s) = e^{ts}$ is positive definite on a real line.

Following the same concept as #1.

$$e^{ts} = \sum_{i=0}^{\infty} \frac{t^i s^i}{i!} \quad \text{(Taylor Expansion)}$$

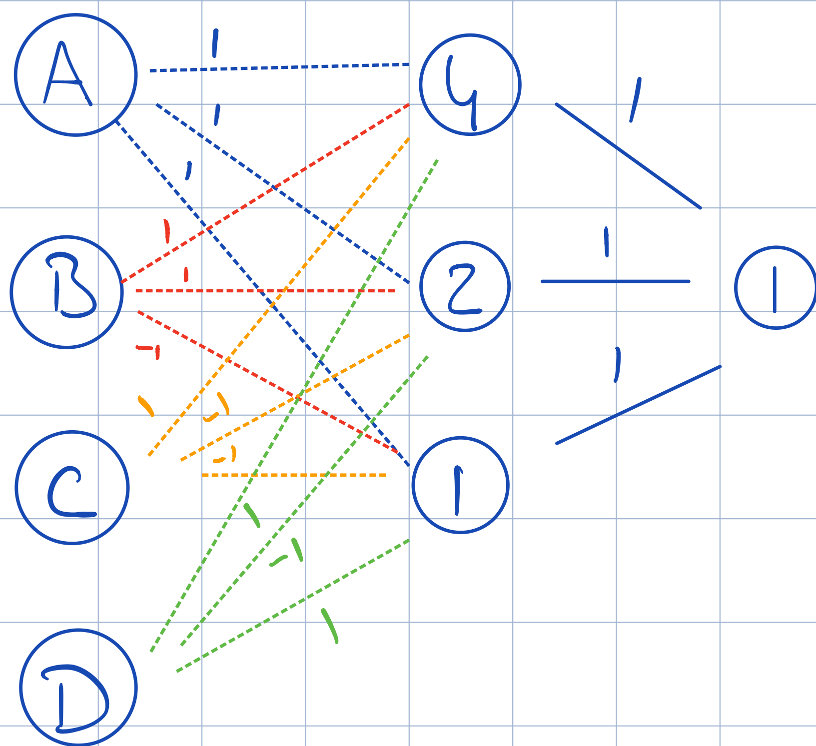
$$e^{x} = \sum \frac{x^i}{i!}$$

$$\sum_{0}^{\infty} (\bar{x}_1, \dots, \bar{x}_n) = \begin{pmatrix} \sum_0^\infty \frac{t_1^i s_1^i}{i!} & \sum_0^\infty \frac{t_1^i s_2^i}{i!} & \cdots & \sum_0^\infty \frac{t_1^i s_m^i}{i!} \\ \sum_0^\infty \frac{t_2^i s_1^i}{i!} & \sum_0^\infty \frac{t_2^i s_2^i}{i!} & & \sum_0^\infty \frac{t_2^i s_m^i}{i!} \\ \vdots & \vdots & & \vdots \\ \sum_0^\infty \frac{t_n^i s_1^i}{i!} & \sum_0^\infty \frac{t_n^i s_2^i}{i!} & & \sum_0^\infty \frac{t_n^i s_m^i}{i!} \end{pmatrix}$$

$$= \sum_{0}^{\infty} \left(\frac{1}{i!} \left(\bar{x}_1 t_1^i s_1^i \cdots \bar{x}_n t_n^i s_n^i \right) \left(x_1 t_1^i s_1^i \cdots \bar{x}_n t_n^i s_n^i \right) \right) \geq 0$$

3. $Y = ABCD \vee AB\bar{C}\bar{D} \vee A\bar{B}\bar{C}D$

A	B	C	D	Y
1	1	1	1	1
1	1	0	0	1
1	0	0	1	1


A
B
C
D
4
2
1
1

4.

5. $h_2(\omega) = 1 \Rightarrow h_1(\omega) = 1$

Compare $\wedge, \vee, \Delta$

<u>Define functions:</u>

$$a = h_1 \vee h_2$$
$$b = h_1 \wedge h_2$$
$$c = h_1 \Delta h_2$$

<u>More general relationship</u>: $h_2 > h_1$, $h_2 > h_3$

<u>$a \geqslant_{lg} b$:</u>

$a(\omega) = 1 \longrightarrow h_2(\omega) = 1,\ h_1(\omega) = 1$

Therefore, $h_2 \geqslant h_1$ or $a \geqslant b$.

$\underline{b \geq c}$: $h_2 \geq_g h_3$

$c(w) = 1$, $h_1(w) = 1$, $h_2(w) = 0$

$h_1(w) = 0$, $h_2(w) = 1$

b will be at least equal to c.

Thus, $b \geq c$.

We can now say that h_1 is more general than or equal.