



	types
•	a
•	b
•	to classify

feature x	feature y	label
1	0	a
1	1	b
-2	4	a
-2	-2	b
1	3	a
2	5	a
-4	-5	a
4	5	b
2	-1	b

K = 3

- $(0,1) \rightarrow \sqrt{(0-0)^2 + (1-0)^2} = \sqrt{1}$
- $(-1,-1) \rightarrow \sqrt{(-1-0)^2 + (-1-0)^2} = \sqrt{2} \Rightarrow$ two labels "b" closer
- $(-2,-2) \rightarrow \sqrt{(-2-0)^2 + (-2-0)^2} = \sqrt{8} = 2\sqrt{2}$ for $K=3$
 \rightarrow label b

K = 2

- $(0,1) \rightarrow \sqrt{(0-(-2))^2 + (1-0)^2} = \sqrt{5} \quad 2 < \sqrt{5}$
- $(-2,-2) \rightarrow \sqrt{(-2-(-2))^2 + (0-(-2))^2} = \sqrt{4} = 2 \rightarrow$ label b

K = 4

- $(5,5) \rightarrow \sqrt{(5-(-2))^2 + (5-4)^2} = \sqrt{50} = 5\sqrt{2}$
 - $(5,5) \rightarrow \sqrt{(5-1)^2 + (5-3)^2} = \sqrt{20} = 2\sqrt{5}$
 - $(5,5) \rightarrow \sqrt{(5-2)^2 + (5-5)^2} = \sqrt{9} = 3$
 - $(5,5) \rightarrow \sqrt{(5-4)^2 + (5-5)^2} = \sqrt{1} = 1$
- \rightarrow 3 "a" neighbors closer to the point \rightarrow a label

Manhattan

$K=3$

• $(0,1) \rightarrow |0-0| + |0-1| = 1$

• $(1,1) \rightarrow |0-1| + |0-1| = 2$ label b

• $(2,-1) \rightarrow |0-2| + |0--1| = 3$

$K=3$

• $(0,1) \rightarrow |1-(-2)| + |1-0| = 4$

• $(1,1) \rightarrow |1-(-2)| + |1-0| = 4$

• $(-2,-2) \rightarrow |-2-(-2)| + |-2-0| = 2$

label b for $K=3$

$K=3$

• $(-2,4) \rightarrow |-2-5| + |4-5| = 8$

• $(1,3) \rightarrow |1-5| + |3-5| = 6$

• $(2,5) \rightarrow |2-5| + |5-5| = 3$

label a for $K=3$

Chessboard

$K=4$

• $(0,1) \rightarrow \max(|0-0|, |1-0|) = 1$

• $(1,1) \rightarrow \max(|0-1|, |0-1|) = 1$

• $(2,-1) \rightarrow \max(|0-2|, |0--1|) = 2$

label b
for $K=4$

$$\bullet (-2, -2) \rightarrow \max(|0 - -2|, |0 - -2|) = 2$$

$K = 3$

$$\bullet (0, 1) \rightarrow \max(|0 - -2|, |1 - 0|) = 2$$

$$\bullet (1, 1) \rightarrow \max(|1 - -2|, |1 - 0|) = 3$$

$$\bullet (-2, -2) \rightarrow \max(|-2 - -2|, |-2 - 0|) = -2$$

label b for $K=3$

$K = 5$

$$\bullet (-2, 4) \rightarrow \max(|-2 - 5|, |4 - 5|) = 7$$

$$\bullet (1, 3) \rightarrow \max(|1 - 5|, |3 - 5|) = 4$$

$$\bullet (2, 5) \rightarrow \max(|2 - 5|, |5 - 5|) = 3$$

$$\bullet (4, 5) \rightarrow \max(|4 - 5|, |5 - 5|) = 1$$

$$\bullet (0, 1) \rightarrow \max(|0 - 5|, |1 - 5|) = 5$$

label a for
 $K=5$

2.4

The separation becomes clearer as noise is reduced.
This means that it will be smoother but may imply more biases as K increases.