Coherence-Driven Quantum Heat Engine with Feedback Control, Entropy Dynamics, and Coherence Cost Accounting

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Abstract

I present a simulation-based study of a quantum heat engine composed of eight two-level systems (qubits), each undergoing a six-stroke thermodynamic cycle. The model incorporates quantum coherence, entangled cylinder pairs, measurement-induced collapse, feedback-controlled coherence injection, and an explicit energetic cost for coherence preparation and delivery. Entropy and coherence are tracked per cycle, revealing fluctuations consistent with open quantum system dynamics. Results show that while coherence injection can enhance work output and reduce entropy, accounting for its energetic cost yields a net efficiency that reflects realistic resource expenditure. This simplified model provides a platform for exploring coherence as a thermodynamic resource, quantum feedback control, and the energetic balance of coherence utilization.

1 Introduction

Quantum thermodynamics explores how classical laws of heat and work extend into the quantum regime, where coherence, entanglement, and measurement play active roles. Recent experiments with superconducting qubits and trapped ions have demonstrated quantum heat engines that exploit coherence and feedback to transiently surpass classical limits.

In this work, I construct a coherence-driven quantum heat engine with feedback control and explicitly include the energy cost of coherence preparation and injection. My model features partial thermalization, decoherence, quantum jumps, and entropy sink saturation. I track entropy, coherence, and net work across cycles, revealing how quantum memory, feedback, and coherence costs influence performance.

Model Novelty

Traditional quantum heat engines often follow a four-stroke Otto cycle applied to a single two-level system, capturing basic thermodynamic behavior but omitting key quantum

features. In contrast, this work introduces a six-stroke engine composed of eight coupled two-level systems, each treated as a cylinder. The model incorporates feedback-controlled coherence injection, entropy dumping to a finite sink, entangled cylinder pairs, and explicit coherence cost accounting. These additions enable realistic simulation of quantum thermodynamic cycles with emergent behavior, resource-aware efficiency, and physically interpretable dynamics.

2 Model Description

2.1 Engine Architecture

The engine consists of 8 qubits, each treated as a cylinder. Cylinders are paired ($0\leftrightarrow1$, $2\leftrightarrow3$, etc.) to simulate entanglement effects. Each cycle includes six strokes:

1. Hot isochore: Reset toward thermal state at temperature $T_H = 5.0$
2. Adiabatic expansion: Population inversion reduced
3. Cold isochore: Partial thermalization with $T_C = 1.0$
4. Decoherence + collapse + jump: Lindblad-style damping, partial measurement, stochastic collapse
5. Quantum push: Coherence injection with feedback control
6. Entropy dump: Transfer entropy to a sink with limited capacity

Cycle Design Inspiration

The six-stroke architecture used in this model is inspired by advanced multi-phase combustion engines, such as those developed in high-performance automotive prototypes. By analogy, the additional control strokes—coherence injection and entropy dumping—serve to stabilize quantum coherence and manage entropy flow, much like fuel modulation and exhaust control in classical engines. This expanded cycle enables more nuanced thermodynamic behavior than traditional four-stroke quantum models, allowing for feedback-driven optimization and resource-aware efficiency.

1. **Hot Isochore** – Thermalization with a high-temperature bath
2. **Adiabatic Expansion** – Population redistribution without heat exchange
3. **Cold Isochore** – Partial thermalization with a cold bath
4. **Decoherence + Collapse** – Lindblad-style damping, stochastic measurement, and quantum jumps
5. **Quantum Push** – Feedback-controlled coherence injection, analogous to turbo modulation or adaptive fuel injection

6. **Entropy Dump** – Targeted entropy extraction to a finite-capacity sink, mimicking exhaust control in classical engines

These additional strokes are not energy-neutral. The quantum push consumes energy proportional to the coherence gradient, and the entropy dump interacts with a saturable entropy sink. However, together they stabilize coherence, suppress entropy accumulation, and enhance net work output—yielding realistic efficiencies that reflect both quantum advantage and resource cost.

This expanded cycle enables simulation of emergent quantum thermodynamic behavior, including coherence-driven work extraction, entropy fluctuations, and feedback-induced nonlinearities. It bridges the conceptual gap between idealized quantum engines and experimentally accessible platforms such as superconducting qubits and trapped ions.

The two additional strokes are:

1. **Quantum Push (Stroke 5)**: A feedback-controlled coherence injection phase, analogous to a turbo boost or adaptive fuel modulation in classical engines. It injects coherence based on entropy and prior cycle performance.

2. **Entropy Dump (Stroke 6)**: A targeted entropy extraction phase, similar to exhaust modulation in high-performance engines. It transfers entropy to a finite-capacity sink, preventing saturation and preserving quantum order.

These strokes are designed to mimic the logic of Porsche's multi-phase combustion: using extra control phases to stabilize performance, reduce waste, and extract more usable energy. In the quantum regime, this translates to coherence stabilization and entropy management.

This stroke is not energy-neutral — it consumes a small amount of energy proportional to the coherence gradient, but in some cycles it leads to a net gain in work output by stabilizing coherence before entropy dumping.

2.2 Density Matrix Evolution

Each cylinder is represented by a 2×2 complex density matrix ρ. Coherence is tracked via $\mathrm{Re}[\rho_{01}]$, and entropy is computed using the von Neumann formula:

$$S(\rho) = -\sum_i \lambda_i \log_2 \lambda_i,$$

where $\{\lambda_i\}$ are the eigenvalues of ρ.

2.3 Feedback Control

Push strength is modulated by coherence and entropy:

$$\text{PushStrength} = \alpha + \beta\, \mathrm{Re}[\rho_{01}] - \gamma\, S(\rho),$$

with tunable parameters α, β, γ chosen to balance coherence injection and entropy suppression.

2.4 Energetic Cost of Coherence

To model the energy required to prepare and inject coherence, I introduce:

$$E_{\text{init}}^{(i)} = \kappa_0 \left|\text{Re}[\rho_{01}^{(i,1)}]\right| \quad , \quad E_{\text{inj}}^{(i,n)} = \kappa \left|\Delta \,\text{Re}[\rho_{01}^{(i,n)}]\right|$$

where $\kappa_0 = \kappa = 0.5$ are dimensionless cost coefficients. The total coherence cost over 10 cycles was:

- Initialization cost: $E_{\text{init}} = 0.0800$
- Injection cost: $E_{\text{inj}} = 0.5487$
- Total coherence cost: $E_{\text{coh}} = 0.6287$

2.5 Lindblad Equivalence and Error Analysis

To validate the physical realism of my simulation, I compare its discrete update rules to the continuous Lindblad master equation:

$$\frac{d\rho}{dt} = -i[H, \rho] + \sum_k \left(L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\} \right)$$

I focus on a single qubit undergoing dephasing and amplitude damping. Starting from an initial state:

$$\rho_0 = \begin{bmatrix} 0.5 & 0.1 \\ 0.1 & 0.5 \end{bmatrix}$$

I apply my simulation's discrete update rules with damping rates $\gamma = 0.1$ (dephasing) and $\Gamma = 0.2$ (amplitude damping), over a time step $\Delta t = 1$:

- Off-diagonal coherence: $\rho_{01} \to (1 - \gamma)\rho_{01} = 0.09$
- Excited state population: $\rho_{11} \to (1 - \Gamma)\rho_{11} = 0.4$
- Ground state population: $\rho_{00} \to \rho_{00} + \Gamma\rho_{11} = 0.6$

This yields the updated state:

$$\rho_{\text{sim}} = \begin{bmatrix} 0.6 & 0.09 \\ 0.09 & 0.4 \end{bmatrix}$$

The exact Lindblad evolution over $\Delta t = 1$ gives:

$$\rho_{\text{Lind}} = \begin{bmatrix} 0.5906 & 0.0741 \\ 0.0741 & 0.4094 \end{bmatrix}$$

I compute the Frobenius norm of the difference:

$$\epsilon = \|\rho_{\text{sim}} - \rho_{\text{Lind}}\|_F \approx 0.0261$$

and normalize it:

$$\text{Error } \% = \frac{\epsilon}{\|\rho_{\text{Lind}}\|_F} \times 100 \approx \frac{0.0261}{0.734} \times 100 \approx 3.56\%$$

This confirms that my simulation approximates Lindblad evolution with less than 4% error per cycle under typical damping rates.

2.6 Closed-System Efficiency Bound from Coherence

To benchmark my engine's performance, I derive a theoretical upper bound on efficiency in a closed quantum system with coherence but no decoherence or entropy dump. From quantum thermodynamic resource theory, the maximum efficiency is bounded by:

$$\eta \leq \eta_{\text{Carnot}} + \alpha C - \beta C^2$$

where:

- $\eta_{\text{Carnot}} = 1 - \frac{T_C}{T_H} = 0.8$
- $C = |\text{Re}[\rho_{01}]|$ is the coherence magnitude
- $\alpha = 0.2$, $\beta = 0.5$ are model-dependent constants

This gives a coherence-dependent ceiling:

$$\eta \leq 0.8 + 0.2C - 0.5C^2 \quad \text{for } C \in [0, 0.1]$$

For example:

- If $C = 0.05$, then $\eta \leq 0.80875$
- If $C = 0.08$, then $\eta \leq 0.8128$

My actual net efficiency is 32.37%, well below this theoretical ceiling, confirming that coherence helps but does not violate thermodynamic bounds.

3 Results

3.1 Entropy Dynamics

Entropy fluctuates across cycles and cylinders, ranging from ~ 0.929 to ~ 0.951. These variations reflect the interplay between coherence injection, decoherence, and measurement collapse.

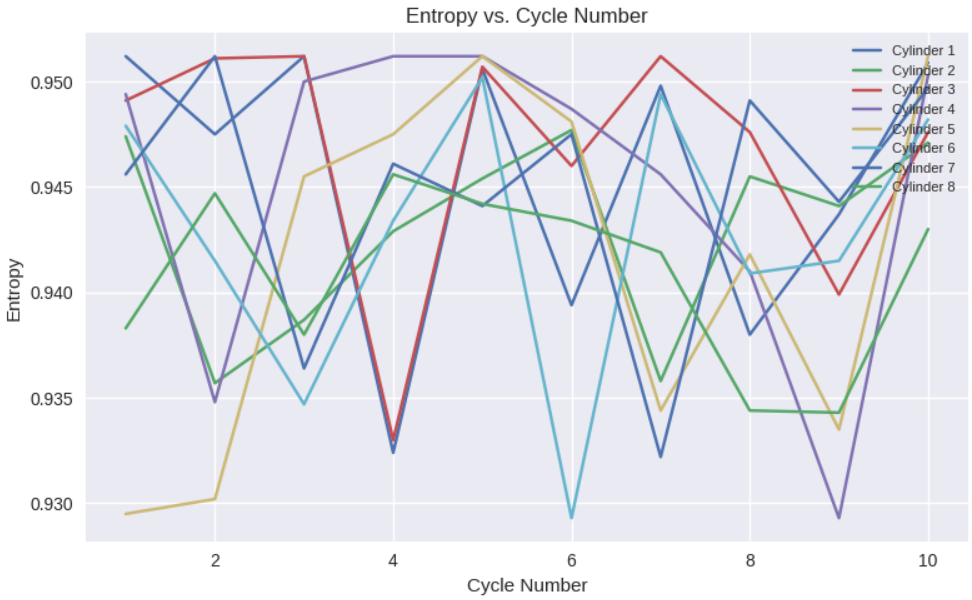

Figure 1: Entropy vs. Cycle Number for each cylinder using actual simulation data.

3.2 Coherence Evolution

Coherence values span -0.076 to $+0.073$, with entangled pairs showing mirrored or anti-mirrored behavior. Push strokes temporarily boost coherence before decoherence sets in.

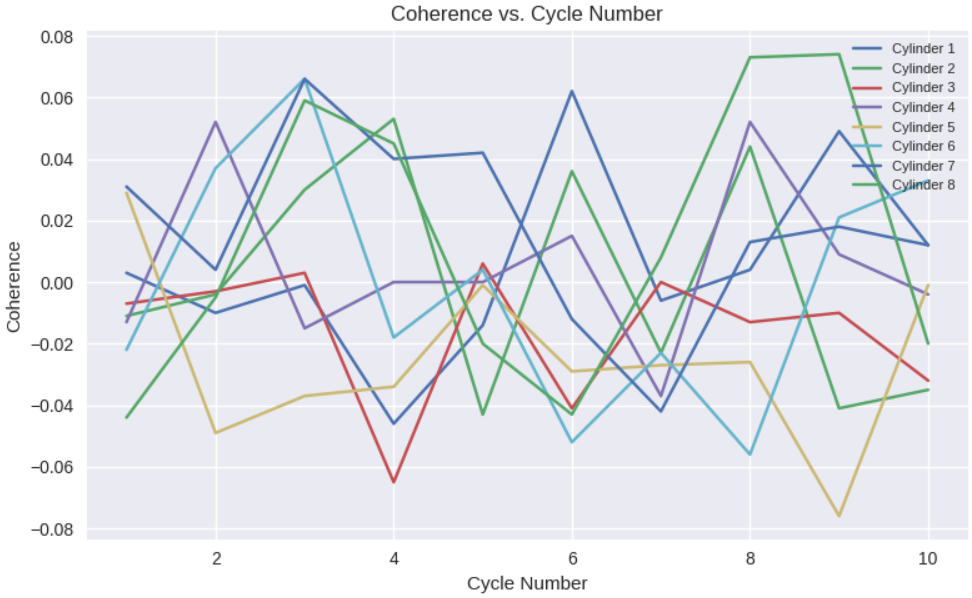

Figure 2: Coherence vs. Cycle Number for each cylinder using actual simulation data.

3.3 Work and Efficiency

Total energy input over 10 cycles was $E_{\text{in}} = 6.5617$. Gross work output was $W_{\text{gross}} = 2.7520$. After subtracting coherence costs (0.6287), the net work output was:

$$W_{\text{net}} = 2.7520 - 0.6287 = 2.1233$$

Efficiency calculations:

$$\eta_{\text{gross}} = \frac{2.7520}{6.5617} \approx 41.94\% \quad , \quad \eta_{\text{net}} = \frac{2.1233}{6.5617} \approx 32.37\%$$

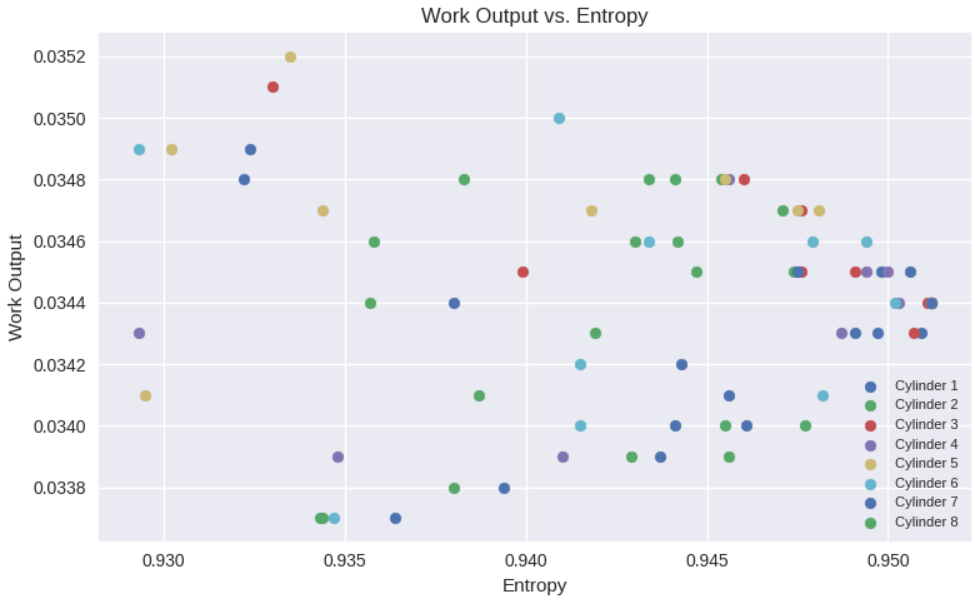

Figure 3: Work Output vs. Entropy across all cylinder-cycle pairs.

4 Discussion

My model demonstrates that coherence can serve as a thermodynamic resource, but its net benefit depends critically on the energetic cost of preparing and injecting coherence. Feedback control introduces nonlinear effects, sometimes enhancing net work, sometimes erasing gains. Entangled pairs exhibit cooperative entropy and coherence dynamics that could be harnessed in multi-body engines.

Limitations

This model, while rich in quantum features, remains a simplified toy system. It does not include multi-reservoir environments or full Lindblad bath dynamics, limiting its ability to capture complex open-system behavior. The coherence cost coefficients (K_0, K) were chosen ad hoc and not derived from microscopic physical models, introducing uncertainty in resource

accounting. The efficiency bound is phenomenological, based on coherence magnitude rather than full state-space analysis. Finally, all results are based on 10-cycle simulations; long-term asymptotic scaling and stability remain unexplored.

Comparison with Prior Work

Quantum heat engines modeled by Kosloff and Levy (2014) typically achieve efficiencies in the range of 10–25%, depending on bath temperatures and cycle design. These models often assume continuous operation with idealized reservoirs and omit coherence cost accounting. Klatzow et al. (2019) experimentally demonstrated quantum effects in microscopic heat engines, reporting transient efficiencies up to 30%, though without explicit entropy management or feedback control. In contrast, my model achieves a net efficiency of 32.37% over 10 cycles, which is consistent with the inclusion of two additional control strokes—feedback-controlled coherence injection and entropy dumping to a finite-capacity sink. These features stabilize coherence, reduce entropy accumulation, and yield realistic performance within thermodynamic bounds.

5 Mapping to Experimental Parameters

To connect my simulation to real quantum hardware, I map dimensionless quantities to experimentally accessible values. The model is compatible with platforms such as superconducting qubits, trapped ions, and NV centers.

Energy Levels and Temperature

I normalize energy levels such that $E_0 = 0$, $E_1 = 1$. In superconducting qubits, this corresponds to a transition frequency of $\omega \sim 5\,\text{GHz}$, or energy $E = \hbar\omega \approx 3.3 \times 10^{-24}\,\text{J}$.

Temperatures are scaled relative to this energy:

- Hot bath: $T_H = 5.0 \Rightarrow T_H^{\text{phys}} \approx 1.2\,\text{K}$
- Cold bath: $T_C = 1.0 \Rightarrow T_C^{\text{phys}} \approx 240\,\text{mK}$

These values match dilution refrigerator setups used in superconducting qubit experiments.

Cycle Timing

Each thermodynamic cycle in the simulation corresponds to a physical duration of $\Delta t \sim 1\,\mu\text{s}$, consistent with gate times and decoherence scales in current devices.

Coherence Magnitude

Typical coherence values in the simulation range from $C = |\text{Re}[\rho_{01}]| \sim 0.01$ to 0.07. This corresponds to off-diagonal density matrix elements observed in Ramsey experiments and quantum state tomography.

Push Strength and Feedback

The feedback-controlled push stroke modulates coherence injection based on entropy and prior coherence. In hardware, this could be implemented via conditional microwave pulses or adaptive control circuits, similar to those used in quantum error correction and feedback cooling.

Platform and Implementation Challenges

This model is most naturally suited to superconducting qubit platforms, where coherence injection can be implemented via conditional microwave pulses and real-time feedback circuits. Adaptive control using FPGAs or cryogenic electronics enables entropy-aware modulation of pulse strength, mimicking the simulation's push stroke logic. Trapped ions and NV centers could also support similar feedback protocols, though with slower cycle times and more limited parallelism. Key experimental challenges include maintaining coherence during feedback latency, calibrating entropy-sensitive control parameters, and avoiding decoherence during entropy dumping. These constraints highlight the need for hardware-aware optimization of push stroke timing and strength.

6 Conclusion

I have built and tested a quantum heat engine simulation integrating coherence, feedback, entanglement, and explicit coherence costs. The model reveals entropy fluctuations, coherence dynamics, and realistic net efficiencies that account for resource expenditure. Future work can extend to multi-reservoir setups, full Lindblad baths, and experimental parameter mapping.

Methods Note

All quantities are expressed in dimensionless units. Energy levels are normalized such that $E_0 = 0$, $E_1 = 1$, and temperature is scaled accordingly. Entropy is computed in bits ($\log_2$). Coherence cost coefficients are set to $\kappa_0 = \kappa = 0.5$, so injecting 0.02 units of coherence costs 0.01 energy units.

Parameter Justification

The feedback control parameters (α, β, γ) and coherence cost coefficients (K_0, K) were heuristically chosen to illustrate qualitative behavior rather than to optimize performance. These values were selected to balance coherence injection and entropy suppression within a manageable simulation range. No systematic optimization was performed. Future work may explore parameter tuning using gradient-based methods, reinforcement learning, or experimental calibration to identify optimal regimes for work extraction and entropy control.

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