Functions

TEAM INFDEV

Hogeschool Rotterdam
Rotterdam, Netherlands

Introduction

Introduction

Lecture topics

- So far we have shown how data representation can be abstracted away
- Building useful containers only once makes it possible to reuse their definition
- Many data structures (tuples, lists, maps, sets, etc.) become thus a new layer of abstraction

Introduction

Lecture topics

- Manipulating these data structures happens in user code
- Often, user code needs to perform operations that are similar to each other
- Similar operations should not require rewriting everything every time

Problem discussion

Problem discussion

Introduction

- Consider many operations on lists
  - finding or removing a specific element in a container
  - computing the length of a list
  - removing all elements that satisfy a condition
  - ...

Lenght of a list

```
1  cnt = 0
2  x = l
3  while not(x.IsEmpty):
4    cnt = cnt + 1
5    x = x.Tail
6  print("List␣l␣contains␣" + str(cnt) + "␣elements.")
```

Introduction

Lenght of a list

```
1  cnt = 0
2  x = l
3  while not(x.IsEmpty):
4    cnt = cnt + 1
5    x = x.Tail
6  print("List␣l␣contains␣" + str(cnt) + "␣elements.")
```

Introduction

- What does `l` contain?
- What do we do with the values of the list?
- Do they even matter?

Problem discussion

Introduction

- Suppose that we now have another list, `k`
- We wish to know its length
- How do we do it?

Lenght of a list

```
1  cnt = 0
2  x = k
3  while not(x.IsEmpty):
4    cnt = cnt + 1
5    x = x.Tail
6  print("List␣k␣contains␣" + str(cnt) + "␣elements.")
```

Introduction

Lenght of a list

```
1  cnt = 0
2  x = k
3  while not(x.IsEmpty):
4    cnt = cnt + 1
5    x = x.Tail
6  print("List␣k␣contains␣" + str(cnt) + "␣elements.")
```

Introduction

- Looks suspiciously like the previous code block
- Why?

General idea

General idea

Adding our own layers

- The goal of this lecture is to add a new layer of abstraction to our programs
- We wish to reuse **implementations**, not only data structures
- This layer of abstraction is called **functions**

Adding our own layers

```
+-------------+
| ...         |
+-------------------+
| Functions         |
+-------------------------+
| data structures         |
+-----------------------------+
| if, for, while, variables   |
+---------------------------------+
| (Python) runtime                |
+---------------------------------+
...
```

General idea

Description

- A function is a collection of instructions and variables
- Some instructions and variables are fixed inside its **body**
- Other instructions and variables come from outside the function, and thus are not fixed; these are called **parameters** of the function
- We try to strike the right balance between flexibility and work done
- The function returns a final result that can be recovered by the code that uses the function

Blueprint of a function (NOT ACTUAL PYTHON CODE!)

```
1  length of a list l:
2    cnt = 0
3    x = l
4    while not(x.IsEmpty):
5      cnt = cnt + 1
6      x = x.Tail
7    return cnt as the final result
```

Description

Blueprint of a function (NOT ACTUAL PYTHON CODE!)

```
1  length of a list l:
2    cnt = 0
3    x = l
4    while not(x.IsEmpty):
5      cnt = cnt + 1
6      x = x.Tail
7    return cnt as the final result
```

Description

- `length` is the **function name**
- `l` is the only **parameter**
- Lines 2 through 6 are **fixed**
- `cnt` is the **final result**

General idea

Using the function

- Code that needs the length of a function can now simply invoke function `length`
- The resulting code will simply be `l_len = length(l)`
- `l_len` will be assigned with the value returned by the function

Technical details

Technical details

Introduction

- A function can be defined in Python quite easily
- The syntax is:
  - `def <<name>>(<<parameters>>):`[a][b]
    - `body`
    - `return <<result>>`
- Inside a function we can put whatever instructions we need
  - `if`
  - `for`
  - ...

[a] Parameters might be none, thus we can write simply `()`

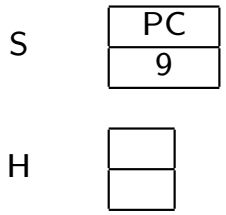
[b] Multiple parameters are separated by a comma, thus `(<<p1>>,<<p2>>,...,<<pn>>)`

Technical details

Using the function

- After we declare a function, we can use it
- The syntax is quite simple
  - `<<name>>(<<parameters>>)` to just call the function and ignore the result
  - `<<v>> = <<name>>(<<parameters>>)` to call the function and assign the result to the `<<v>>` variable
- After calling the function, we enter the local environment of the function
- Variables, the `PC`, etc. are separate from those of the calling site

Runtime example




```
1  def length(l):
2    cnt = 0
3    x = l
4    while not(x.IsEmpty):
5      cnt = cnt + 1
6      x = x.Tail
7    return cnt
8
9  print(length(Node(10, Empty)))
```

Runtime example

S

PC
9

H


```
1  def length(l):
2    cnt = 0
3    x = l
4    while not(x.IsEmpty):
5      cnt = cnt + 1
6      x = x.Tail
7    return cnt
8
9  print(length(Node(10, Empty)))
```

S

PC	length	PC	l
9	nil	2	ref(1)

H

0	1
[I ↦ True]	[I ↦ False; V ↦ 10; T ↦ ref(0)]

Runtime example

S

PC	length	PC	l
9	nil	2	ref(1)

H

0	1
[l ↦ True]	[l ↦ False; V ↦ 10; T ↦ ref(0)]

```
1  def length(l):
2    cnt = 0
3    x = l
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Runtime example

S

PC	length	PC	l
9	nil	2	ref(1)

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0	1
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```
1  def length(l):
2    cnt = 0
3    x = l
4    while not(x.IsEmpty):
5      cnt = cnt + 1
6      x = x.Tail
7    return cnt
8
9  print(length(Node(10, Empty)))
```

S

PC	length	PC	l	cnt
9	nil	3	ref(1)	0

H

0	1
[l ↦ True]	[l ↦ False; V ↦ 10; T ↦ ref(0)]

Runtime example

S

PC	push	PC	l	cnt
9	length	3	ref(1)	0

H

0	1
[l $\mapsto$ True]	[l $\mapsto$ False; V $\mapsto$ 10; T $\mapsto$ ref(0)]

```
1  def length(l):
2    cnt = 0
3    x = l
4    while not(x.IsEmpty):
5      cnt = cnt + 1
6      x = x.Tail
7    return cnt
8
9  print(length(Node(10, Empty)))
```

Runtime example

S

PC	push	PC	l	cnt
9	length	3	ref(1)	0

H

0	1
[l ↦ True]	[l ↦ False; V ↦ 10; T ↦ ref(0)]

```
1  def length(l):
2    cnt = 0
3    x = l
4    while not(x.IsEmpty):
5      cnt = cnt + 1
6      x = x.Tail
7    return cnt
8
9  print(length(Node(10, Empty)))
```

S

PC	length	PC	l	cnt	x
9	nil	4	ref(1)	0	ref(1)

H

0	1
[l ↦ True]	[l ↦ False; V ↦ 10; T ↦ ref(0)]

Runtime example

After a few steps...

S

PC	length	PC	l	cnt	x
9	nil	7	ref(1)	1	ref(0)

H

0	1
[l ↦ True]	[l ↦ False; V ↦ 10; T ↦ ref(0)]

```
1  def length(l):
2    cnt = 0
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4    while not(x.IsEmpty):
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```

Runtime example

After a few steps...

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PC	length	PC	l	cnt	x
9	nil	7	ref(1)	1	ref(0)

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0	1
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```
1  def length(l):
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9  print(length(Node(10, Empty)))
```

Do we still need all the local variables of the function?

Runtime example

After a few steps...

S

PC	length	PC	l	cnt	x
9	nil	7	ref(1)	1	ref(0)

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0	1
[l ↦ True]	[l ↦ False; V ↦ 10; T ↦ ref(0)]

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1  def length(l):
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5      cnt = cnt + 1
6      x = x.Tail
7    return cnt
8
9  print(length(Node(10, Empty)))
```

Do we still need all the local variables of the function?
Where do we put the result?

Runtime example

After a few steps...

S

PC	length	PC	l	cnt	x
9	nil	7	ref(1)	1	ref(0)

H

0	1
[l ↦ True]	[l ↦ False; V ↦ 10; T ↦ ref(0)]

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1  def length(l):
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5      cnt = cnt + 1
6      x = x.Tail
7    return cnt
8
9  print(length(Node(10, Empty)))
```

Do we still need all the local variables of the function? Where do we put the result?

S

PC	length
9	1

H

0	1
[l ↦ True]	[l ↦ False; V ↦ 10; T ↦ ref(0)]

Technical details

Syntax and semantics

- We will now describe how Python functions work precisely
- This is a **fundamental** bit of knowledge that determines if you really do learn how to program or not
- This **absolutely requires** a lot of focus to get
- Please panic a bit on the inside

Technical details

Subtleties that make functions "fun" to use

- About variables
  - Variables and parameters inside a function have precise **scope** (visibility)
  - Primitive values given as parameters can be **changed only locally** to the function
  - References given as parameters can be **permanently changed** from within the function
  - Global variables defined outside the function may be **read but not changed** from within the function[a]
- About behaviour
  - A function may **call itself**, in a process known as **recursion**
  - A function may **get as parameters and return other functions**, in a process known as **higher order functions**

[a] **Unless you use some tricks we strongly discourage**

Technical details

Local and global variables (basics of scope)

- The parameters of a function are added to the list of accessible variables
- They are only visible from inside the function
- Global variables are also visible from inside the function

Technical details

Local and global variables (basics of scope)

- Every call to a function generates a new value of the stack memory `S`
- This contains (private copy of) all local variables
- The heap memory `H` remains the same
- The original stack memory (the **global variables**) remains accessible, just read-only

Technical details

Local and global variables (basics of scope)

- Every call to a function also reserves some special locations in the stack
- The local `PC` of the function
- The local variables of the function
- The returned value when the function is done

Locals and globals

```
1  x = 1
2
3  def f(z):
4    return x * z
5
6  print(f(10))
7  print(f(30))
8  x = 2
9  print(f(10))
```

Local and global variables (basics of scope)

- `x` is a global variable, visible outside and inside the function
- `z` is a local variable, visible only inside the function

Locals and globals

```
1  x = 1
2
3  def f(z):
4    return x * z
5
6  print(f(10))
7  print(f(30))
8  x = 2
9  print(f(10))
```

Local and global variables (basics of scope)

- `x` is a global variable, visible outside and inside the function
- `z` is a local variable, visible only inside the function
- **What does this program print?**

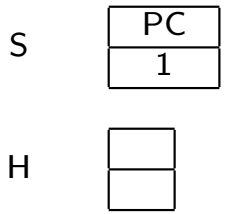
Locals and globals

```
1  x = 1
2
3  def f(z):
4    return x * z
5
6  print(f(10))
7  print(f(30))
8  x = 2
9  print(f(10))
```

Local and global variables (basics of scope)

- `x` is a global variable, visible outside and inside the function
- `z` is a local variable, visible only inside the function
- **What does this program print?**
- 10, 30, 20

Locals and globals




```
1  x = 1
2
3  def f(z):
4    return x * z
5
6  print(f(10))
7  x = 2
8  print(f(10))
```

Locals and globals

S

PC
1

H


```
1  x = 1
2
3  def f(z):
4    return x * z
5
6  print(f(10))
7  x = 2
8  print(f(10))
```

S

PC	x
6	1

H

Locals and globals

S

PC	x
6	1

H

```
1  x = 1
2
3  def f(z):
4    return x * z
5
6  print(f(10))
7  x = 2
8  print(f(10))
```

Locals and globals

S

PC	x
6	1

H

```
1  x = 1
2
3  def f(z):
4    return x * z
5
6  print(f(10))
7  x = 2
8  print(f(10))
```

S

PC	x	f	PC	z
6	1	nil	4	10

H

Locals and globals

S

PC	x	f	PC	z
6	1	nil	4	10

H

```
1  x = 1
2
3  def f(z):
4    return x * z
5
6  print(f(10))
7  x = 2
8  print(f(10))
```

Locals and globals

S

PC	x	f	PC	z
6	1	nil	4	10

H

```
1  x = 1
2
3  def f(z):
4    return x * z
5
6  print(f(10))
7  x = 2
8  print(f(10))
```

S

PC	x	f
7	1	10

H

Locals and globals

S

PC	x	f
7	1	10

H

```
1  x = 1
2
3  def f(z):
4    return x * z
5
6  print(f(10))
7  x = 2
8  print(f(10))
```

Locals and globals

S

PC	x	f
7	1	10

H

```
1  x = 1
2
3  def f(z):
4    return x * z
5
6  print(f(10))
7  x = 2
8  print(f(10))
```

S

PC	x
8	2

H

Locals and globals

S

PC	x	f	PC	z
8	2	nil	4	10

H

```
1  x = 1
2
3  def f(z):
4    return x * z
5
6  print(f(10))
7  x = 2
8  print(f(10))
```

Locals and globals

S

PC	x	f	PC	z
8	2	nil	4	10

H

```
1  x = 1
2
3  def f(z):
4    return x * z
5
6  print(f(10))
7  x = 2
8  print(f(10))
```

S

PC	x	f
8	2	20

H

Locals and globals

```
1  x = 1
2
3  def f(z):
4    return x * z
5
6  print(f(10))
7  x = 2
8  print(f(10))
9  print(z)
```

Local and global variables (basics of scope)

- `x` is a global variable, visible outside and inside the function
- `z` is a local variable, visible only inside the function

Locals and globals

```
1  x = 1
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3  def f(z):
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5
6  print(f(10))
7  x = 2
8  print(f(10))
9  print(z)
```

Local and global variables (basics of scope)

- `x` is a global variable, visible outside and inside the function
- `z` is a local variable, visible only inside the function
- **What does this program do?**

Locals and globals

```
1  x = 1
2
3  def f(z):
4    return x * z
5
6  print(f(10))
7  x = 2
8  print(f(10))
9  print(z)
```

Local and global variables (basics of scope)

- `x` is a global variable, visible outside and inside the function
- `z` is a local variable, visible only inside the function
- **What does this program do?**
- Crash with `NameError: name 'z' is not defined`

Locals and globals

```
1  def f(z):
2    z = z + 1
3    return z * 2
4
5  print(f(10))
6  print(f(30))
```

Local and global variables (basics of scope)

- z is a local variable, visible only inside the function

Locals and globals

```
1  def f(z):
2    z = z + 1
3    return z * 2
4
5  print(f(10))
6  print(f(30))
```

Local and global variables (basics of scope)

- z is a local variable, visible only inside the function
- **What does this program print?**

Locals and globals

```
1  def f(z):
2    z = z + 1
3    return z * 2
4
5  print(f(10))
6  print(f(30))
```

Local and global variables (basics of scope)

- z is a local variable, visible only inside the function
- **What does this program print?**
- 22, 62

Technical details

Shadowing

- The parameters of a function have priority over globals
- They supersede global variables of the same name

Shadowing

```
1  x = 1
2
3  def f(x):
4    return x * 2
5
6  print(f(10))
7  print(f(20))
```

Shadowing

- `x` is a global variable, potentially visible inside the function
- `x` is also a local variable of the function, which has priority over the global `x`

Shadowing

```
1  x = 1
2
3  def f(x):
4    return x * 2
5
6  print(f(10))
7  print(f(20))
```

Shadowing

- `x` is a global variable, potentially visible inside the function
- `x` is also a local variable of the function, which has priority over the global `x`
- **What does this program print?**

Shadowing

```
1  x = 1
2
3  def f(x):
4    return x * 2
5
6  print(f(10))
7  print(f(20))
```

Shadowing

- `x` is a global variable, potentially visible inside the function
- `x` is also a local variable of the function, which has priority over the global `x`
- **What does this program print?**
- 20, 40

Shadowing

S

PC	x
6	1

H

```
1  x = 1
2
3  def f(x):
4    return x * 2
5
6  print(f(10))
7  print(f(20))
```

Shadowing

S

PC	x
6	1

H

```
1  x = 1
2
3  def f(x):
4    return x * 2
5
6  print(f(10))
7  print(f(20))
```

S

PC	x	f	PC	x
6	1	nil	4	10

H

Shadowing

S

PC	x	f	PC	x
6	1	nil	4	10

H

```
1  x = 1
2
3  def f(x):
4    return x * 2
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6  print(f(10))
7  print(f(20))
```

Shadowing

S

PC	x	f	PC	x
6	1	nil	4	10

H

```
1  x = 1
2
3  def f(x):
4    return x * 2
5
6  print(f(10))
7  print(f(20))
```

S

PC	x	f
7	1	20

H

Shadowing

S

PC	x	f
7	1	20

H

```
1  x = 1
2
3  def f(x):
4    return x * 2
5
6  print(f(10))
7  print(f(20))
```

Shadowing

S

PC	x	f
7	1	20

H

```
1  x = 1
2
3  def f(x):
4    return x * 2
5
6  print(f(10))
7  print(f(20))
```

S

PC	x	f	PC	x
7	1	nil	4	20

H

Shadowing

S

PC	x	f	PC	x
7	1	nil	4	20

H

```
1  x = 1
2
3  def f(x):
4    return x * 2
5
6  print(f(10))
7  print(f(20))
```

Shadowing

S

PC	x	f	PC	x
7	1	nil	4	20

H

```
1  x = 1
2
3  def f(x):
4    return x * 2
5
6  print(f(10))
7  print(f(20))
```

S

PC	x	f
8	1	40

H

Technical details

Recursion

- (Recursive) functions are all functions that call themselves in their bodies
- This is based on the principle of induction and in general a very powerful technique
- This leads to a compacter and often more easily correct representation
  - Code is not easier to read, especially to the untrained eye

Technical details

Recursion

- Remember that calling a function creates a new instance of stack memory
- Recursive functions do this a lot
- Each recursive call has its own environment

Recursion

```
1  def length(l):
2    if l.IsEmpty:
3      return 0
4    else:
5      return length(l.Tail) + 1
```

Recursion

- How many `1`'s shall we have?

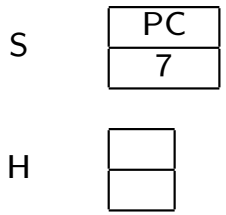
Recursion

```
1  def length(l):
2    if l.IsEmpty:
3      return 0
4    else:
5      return length(l.Tail) + 1
```

Recursion

- How many `l`'s shall we have?
- As many as the nodes of the initial value

Recursion




```
1  def length(l):
2    if l.IsEmpty:
3      return 0
4    else:
5      return length(l.Tail) + 1
6
7  print(length(Node(1,Node(2,Empty))))
```

Recursion

S

PC
7

H


```
1  def length(l):
2    if l.IsEmpty:
3      return 0
4    else:
5      return length(l.Tail) + 1
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7  print(length(Node(1,Node(2,Empty))))
```

S

PC	length	PC	l
7	nil	2	ref(2)

H

0	1	2
[I ↦ True]	[I ↦ False; V ↦ 2; T ↦ ref(0)]	[I ↦ False; V ↦ 1; T ↦ ref(1)]

Recursion

S

PC	length	PC	l
7	nil	2	ref(2)

H

0	1	2
[l ↦ True]	[l ↦ False; V ↦ 2; T ↦ ref(0)]	[l ↦ False; V ↦ 1; T ↦ ref(1)]

```
1  def length(l):
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Recursion

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H

0	1	2
[l ↦ True]	[l ↦ False; V ↦ 2; T ↦ ref(0)]	[l ↦ False; V ↦ 1; T ↦ ref(1)]

```
1  def length(l):
2    if l.IsEmpty:
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4    else:
5      return length(l.Tail) + 1
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7  print(length(Node(1,Node(2,Empty))))
```

S

PC	length	PC	l
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Recursion

S

PC	length	PC	l
7	nil	5	ref(2)

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PC	length	PC	l	length	PC	l	length
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Recursion

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PC	length	PC	l	length	PC	l	length
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```

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PC	length	PC	l	length
7	nil	5	ref(2)	0+1

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Recursion

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PC	length
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Recursion

S

PC	length
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H

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PC	length
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S

PC
8

H

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Assignments

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Build and test, on paper...

- A function add that increments all elements of a list by a fixed value:
  - `add(10, Node(1,Node(2,Node(3,Empty)))) -> Node(11,Node(12,Node(13,Empty)))`
- A function `filterEven` that removes all odd elements from a list:
  - `filterEven(Node(1,Node(2,Node(3,Empty)))) -> Node(2,Empty)`
- A function `sum` that adds all elements of a list:
  - `sum(Node(1,Node(2,Node(3,Empty)))) -> 6`

Conclusion

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Lecture topics

- Often, user code needs to perform operations that are similar to each other
- Through the mechanism of function definition, we can recycle code
- Functions can encode algorithms in many way
  - Simple code abstractions to avoid repetition
  - Recursive problems

This is it!

The best of luck, and thanks for the attention!