$$\lim_{x \to \infty} \left(\left(1+\frac{1}{n}\right)^n \right) = e$$

$$\lim_{x \to 0} \left(1+n\right)^{\frac{1}{n}} = e$$

$$\lim_{x \to 0} \frac{\ln(1+x)}{x} = 1$$

$$\lim_{x \to 0} \frac{\log_a(1+x)}{x} = \frac{1}{\ln a}$$

$$\lim_{x \to 0} \frac{e^x - 1}{x} = 1$$

$$\lim_{x \to 0} \frac{a^x - 1}{x} = \ln a$$

자연상수 e

$$\left(1+\frac{1}{n}\right)^n$$

분수로 나타낼 수 없는 소수
(순환하지 않는 무한소수)

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = \text{무리수} = e$$

e π처럼 표현하는 것

ex) $\frac{1}{x}=t$ $x\to\infty$ 가 되면 $t\to 0$ ($t\to 0+$)

$$\lim_{t\to 0}(1+t)^{\frac{1}{t}} = e \qquad \frac{1}{x}=t \qquad \frac{1}{t}=x$$

$$\lim_{x\to\infty}\left(1+\frac{a}{x}\right)^{\frac{x}{a}} = \lim_{x\to\infty}(1+t)^{\frac{1}{t}} = e$$

$\frac{a}{x}$를 t로 두면

$$\lim_{x\to\infty}\left(1+\frac{1}{ax}\right)^{ax} = e \qquad \lim_{x\to\infty}(1+ax)^{\frac{1}{ax}} = e$$

$$\lim_{x\to-\infty}\left(1-\frac{1}{3x}\right)^{2x} = \lim_{t\to\infty}\left\{\left(1+\frac{1}{3t}\right)^{3t}\right\}^{-\frac{2}{3}} = e^{-\frac{2}{3}}$$

$t=-x$

자연로그 -> 밑이 e인 로그

$\log_e x = \ln x$

$\ln e = 1 \quad \ln e^2 = 2 \quad \log_a b = \frac{\ln b}{\ln a}$

$a>0, \ a \neq 1$

1) $\lim_{x \to 0} \frac{\ln(1+x)}{x} = \lim_{x \to 0} \frac{1}{x} \ln(1+x) = \lim_{x \to 0} \ln \underbrace{(1+x)^{\frac{1}{x}}}_{e}$

$= \ln e = 1$

2) $\lim_{x \to 0} \frac{\log_a(1+x)}{x} = \lim_{x \to 0} \frac{1}{x} \times \frac{\overbrace{\ln(1+x)}^{1}}{\ln a}$

$= \frac{1}{\ln a}$

3) $\lim_{x \to 0} \frac{e^x - 1}{x} = \lim_{t \to 0} \frac{t}{\ln(1+t)} = \lim_{t \to 0} \frac{1}{\underbrace{\frac{1}{t} \times \ln(1+t)}_{e}}$

$e^x - 1 = t$

$e^x = t+1$

$x = \ln(t+1)$

$x = \log_e(t+1)$

$\underline{e^x = t+1}$

$\lim_{t \to 0} \frac{1}{1} = 1$

$\lim_{x \to 0} \frac{e^x - 1}{x} = 1$

4) $\lim_{x \to 0} \frac{a^x - 1}{x} = \ln a$

$a^x - 1 = t$
$a^x = 1 + t$
$x = \log_a(1+t)$

$= \lim_{t \to 0} \frac{t}{\log_a(1+t)}$

$= \lim_{t \to 0} \frac{1}{\frac{1}{t}\log_a(1+t)} = \lim_{t \to 0} \frac{1}{\log_a(1+t)^{\frac{1}{t}}}$

$\hookrightarrow \log_a e = \frac{\ln e}{\ln a}$

$= \frac{1}{\ln a}$

$= \lim_{t \to 0} \frac{1}{\frac{1}{\ln a}} = \ln a$

$\lim_{x \to \infty} \left(1 + \frac{1}{n}\right)^n = e$

$\lim_{x \to 0} (1 + n)^{\frac{1}{n}} = e$

$$y = e^x \to y' = \lim_{h\to 0} \frac{e^{x+h}-e^x}{h} = \lim_{h\to 0} \frac{e^x(e^h-1)}{h}$$

$$= e^x \underbrace{\lim_{h\to 0} \frac{e^h-1}{h}}_{1} \qquad y' = e^x$$

$$y = a^x \to y' = \lim_{h\to 0} \frac{a^{x+h}-a^x}{h} = \lim_{h\to 0} \frac{a^x(a^h-1)}{h}$$

$$= a^x \underbrace{\lim_{h\to 0} \frac{a^h-1}{h}}_{\ln a} = a^x \cdot \ln a$$

$$y' = a^x \log_e a$$

지수함수의 도함수

로그함수의 도함수

$$y = \log_e x \Rightarrow y' = \lim_{h \to 0} \frac{\log_e(x+h) - \log_e x}{h}$$

$$= \lim_{h \to 0} \frac{1}{h} \left(\log_e(x+h) - \log_e x \right)$$

$$= \lim_{h \to 0} \frac{1}{h} \log_e\left(\frac{x+h}{x} \right)$$

$$= \lim_{h \to 0} \frac{1}{h} \log_e\left(1 + \frac{h}{x} \right)$$

$$= \frac{1}{x} \underbrace{\lim_{h \to 0} \frac{x}{h} \log_e\left(1 + \frac{h}{x} \right)}_{e \quad 1} = \frac{1}{x}$$

$$y = \log_e x \rightarrow y' = \frac{1}{x}$$

$$y = \log_a x \rightarrow y = \frac{\log_e x}{\log_e a}$$

$$y' = \frac{1}{\log_e a} \left(\log_e x \right)' = \frac{1}{\log_e a} \times \frac{1}{x}$$

$$y' = \frac{1}{x \cdot \log_e a}$$