δ fuction 특정 위치만 1 반환 else 0

Discrete cdf 계단식

Continuous Rv

· 어떠한 시점의 확률 값은 $\frac{1}{\infty}$ 즉 0에 수렴한다.

ex) $0 \sim 1$ 사이의 값 하나를 뽑은 후 그것의 확률 0.1456897...

It is not possible to define a probability for each x

↳ 확률이 0 이라는 얘기

구간의 확률

$$\lim_{\Delta x \to 0} \frac{P(x < X \le x + \Delta x)}{\Delta x} = \frac{F_X(x+\Delta x) - F_X(x)}{\Delta x} = \frac{0}{0}$$

$$f_X(x) = \lim_{\Delta x \to 0} \frac{F_X(x+\Delta x) - F_X(x)}{\Delta x} = F_X'(x)$$

↳ prob density function (Pdf)

단위길이 당 확률 = 밀도

$$f_X(x) = F_X'(x)$$

$$F_X(x) = \int_{-\infty}^{x} f_X(\tilde{x})\, d\tilde{x}$$

$F(x)$는 감소하지 않는다.

① $F_X'(x) > 0$ 이다.
② $\int_{-\infty}^{\infty} f_X(x)\,dx = 1$
] - pdf의 조건

$$P(a < X \le b)$$
$$= F_X(b) - F_X(a)$$
$$= \int_{-\infty}^{b} f(x)\,dx - \int_{-\infty}^{a} f_X(x)\,dx$$
$$= \int_{a}^{b} f_X(x)\,dx$$

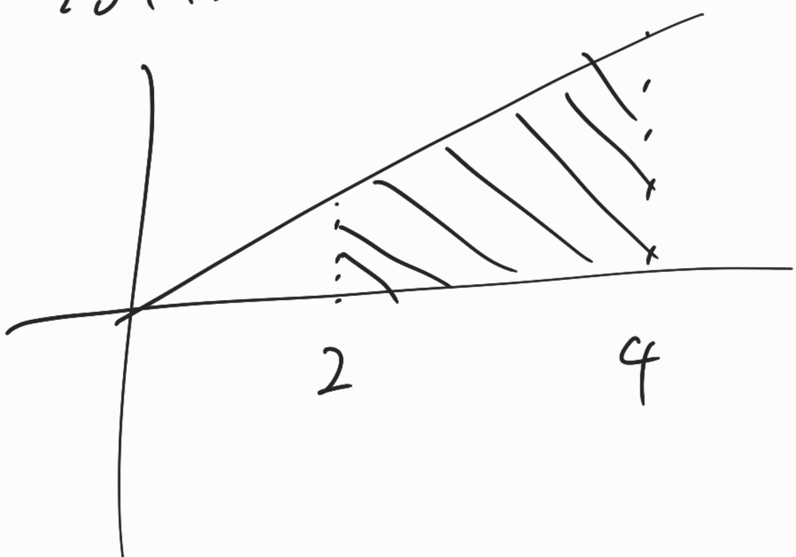
$P(X < a) = P(X \le a)$ why $P(X = a) = 0$

$$= \int_{-\infty}^{a} f_X(x)\,dx = F_X(a)$$

$\dfrac{F(x)}{da}$

uniform distribution

확률이 일정하다.



Expectation (기댓값)

$$\bar{X} = \frac{x_1 + x_2 \cdots\cdots + x_n}{n}$$ => 평균

$$\bar{X} = \frac{w_1 x_1 + w_2 x_2 \cdots\cdots + w_n x_n}{w_1 + w_2 + \cdots\cdots w_n}$$ => freq에서의 평균

발생횟수 event

Expectation (기댓값)

$$E[X] = \sum_i x_i P(x_i) = \int_{-\infty}^{\infty} x f_X(x)\,dx$$

Poison distribution

# of events occurrences in a time interval

x = integer 0, 1, 2, 3 ......

(단위시간 당 발생 횟수)

ex) 은행 직원에게 1시간 당 몇명 사람이 방문하는지

$$P_K(k) = \frac{\lambda^k}{k!} \cdot e^{-\lambda}, \quad k = 0, 1, 2, 3 \ldots$$

$$E[K] = \sum_{k=0}^{\infty} k \cdot \frac{\lambda^k}{k!} e^{-\lambda}$$

$$= \sum_{k=1}^{\infty} \frac{\lambda \cdot \lambda^{k-1}}{(k-1)!} e^{-\lambda}$$

$$= \lambda e^{-\lambda} \sum_{k=1}^{\infty} \frac{\lambda^{k-1}}{(k-1)!}$$

$$= \lambda e^{-\lambda} \underbrace{\sum_{k=0}^{\infty} \frac{\lambda^k}{k!}}_{\text{여수는}} = \lambda$$

$$e^n = \left(1 + \frac{1}{n}\right)^n$$

$$e^{\lambda} = \left(1 + \frac{1}{\lambda}\right)^{\lambda}$$

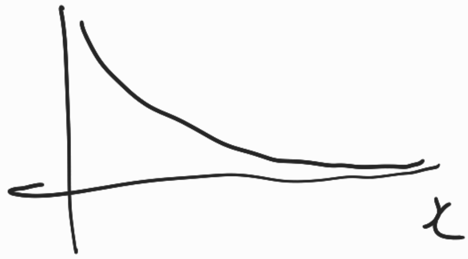
e tailer 급수로

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} \cdots$$

로 표현된다고 함

exponential dist

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$



lifetime 기계의 생명주기를 표현할 때 사용

$$E[X] = \int_{-\infty}^{\infty} x f_X(x)\, dx = \underline{\underline{\int_0^{\infty} x \cdot \lambda e^{-\lambda x} dx}_{\text{6 sum}}}$$

$$= \frac{1}{\lambda}$$

Moments of Rv

- nth - order moments

$$E[X^n] = \sum_i x_i^n P(x_i) = \int_{-\infty}^{\infty} x^n f_X(x) dx$$

$n=1 \Rightarrow$ mean $= E[X] = \mu$

Central Moments (누계중심?)

$$E[(X-\mu)^n] = \begin{cases} \sum_i (x_i-\mu)^n p(x_i) \\ \int_{-\infty}^{\infty} (x_i-\mu)^n f_X(x) dx \end{cases}$$

- $n=1$

$$E[X-\mu] = \int (x-\mu) f_X(x) dx = \int x f_X(x) dx - \mu \int f_X(x) dx$$

$$= \mu - \mu \cdot 1 = 0$$

· $n=2$ (variance) 분산

$E[(X-\mu)^2] = \sigma_X^2$: variance

Proposition

$g_1(X)$, $g_2(X)$ $E[a \cdot g_1(X) + b \cdot g_2(X)]$

└ 임의의 함수 ┘

$$= \int a \cdot g_1(x) \cdot b g_2(x) f_X(x) dx$$

$$= a \int g_1(x) f_X(x) dx + b \int g_1(x) f_X(x) dx$$

$$= a E[g_1(x)] + b E[g_2(x)]$$

Linear 하다!

Linearity

1) homogeniety

$$f(ax) = a \cdot f(x)$$

2) super position

$$f(x_1 + x_2) = f(x_1) + f(x_2)$$

둘 다 만족

$$f(ax_1 + bx_2) = af(x_1) + bf(x_2)$$

=> ex)

1) 원점을 지나는 직선

2) operations

-> 미분, 적분

-> $Ax = b$ (행렬변환)

3) Expectation

$$\sigma_x^2 = E[(x-\mu)^2] = E[x^2 - 2x\mu + \mu^2]$$

$$= E[x^2] - 2\mu \underbrace{E(x)}_{\mu} + \mu^2$$

$$= E(x^2) - 2\mu^2 + \mu^2 = E[x^2] - \mu^2$$