

Time	Group	Submission in Moodle; Mails with subject: [SMD2022]
Th.12:15–13:00	A	lukas.beiske@udo.edu and jean-marco.alameddine@udo.edu
Fr. 8:15–9:00	B	samuel.haefs@udo.edu and stefan.froese@udo.edu
Fr. 10:15–11:00	C	david.venker@udo.edu and lucas.witthaus@udo.edu

Exercise 13 *Kolmogorov–Smirnov-Test*

5 p.

In this task, you investigate the similarity of the Poisson and Gaussian distributions using the Kolmogorov–Smirnov test.

- (a) What values do you have to choose for μ and σ of a Gaussian distribution so that it is as similar as possible to a Poisson distribution with expected value λ ?
- (b) Implement the two-sample Kolmogorov–Smirnov test for binned data.
- (c) The two-sample Kolmogorov–Smirnov test checks the null hypothesis H_0 , whether the two samples stem from the same probability distribution. Investigate at which expected value λ the Poisson and Gaussian distributions are so similar that the Kolmogorov–Smirnov test can no longer distinguish between the two. To do this, draw 10 000 random numbers each from a Poisson distribution and from the corresponding Gaussian distribution for a λ to be tested. Consider the following:
 - Round the values drawn from the Gaussian distribution to whole numbers.
 - Use 100 bins each in the interval $[\mu - 5\sigma, \mu + 5\sigma]$.
 - Determine by iteration the value for λ from which you can no longer reject H_0 on the basis of the Kolmogorov–Smirnov test at a confidence level of $\alpha = 5\%$.
- (d) Determine the value for λ for the confidence levels $\alpha = 2.5\%$ and $\alpha = 0.1\%$ analogously.

Exercise 14 *Balloon Experiment*

5 p.

In an experiment to measure the flux of cosmic rays in the upper atmosphere, protons with an energy between 1 GeV and 100 GeV are counted over a period of one hour from a flying balloon. Over a period of one week, a measurement run of one hour duration is made every day. The measured data are:

Day	1	2	3	4	5	6	7
Counts	4135	4202	4203	4218	4227	4231	4310

- Assume that the cosmic ray flux is constant over the measurement period. Calculate the most probable count rate using the maximum likelihood method.
- Your colleague looks at the readings and hypothesizes that the cosmic ray flux is experiencing a dramatic increase. Assume a linearly increasing flux and calculate numerically the most probable flux parameters using the maximum likelihood method.
- Calculate the significance of his observation using a likelihood ratio test. Evaluate the significance achieved. *Hint:* Assume that Wilks' theorem is valid here. Why can you assume this?
- Your colleague performs another measurement a week later to support his thesis. His measurement results in

Day	14
Counts	4402

Calculate (a) to (c) again for this new data set.

- What is the methodological problem with exercise d)'s approach? Why should you not publish these results, even if the significance is higher than some preset threshold (e.g. 3 or 5σ)?