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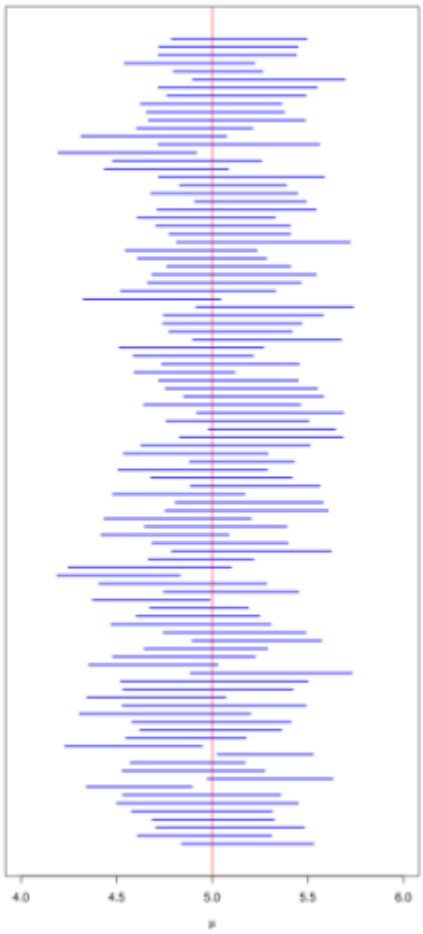
1 Exercise 9 - *Confidence Intervals short questions*

A confidence interval $[x_1, x_2]$ of a parameter x at a confidence level α is given. What is the frequentist, what is the Bayesian interpretation of this interval?

Frequentist:

Primarily: The fraction α does **not** mean that the true value x_{true} lies in the interval $[x_1, x_2]$ with probability α. Instead α represents the fraction of intervals $[x_1, x_2]$ which contain x_{true}. The following picture shows this subtle interpretation:



In this example 100 measurements were taken. For each measurement we can calculate the mean and the variance.

But only 94 of 100 measurements contain the true value $x_{true} = 5$ in their intervals $[x_1, x_2]$, which are consequently dependent on x. In this case $\alpha = 0.95$.

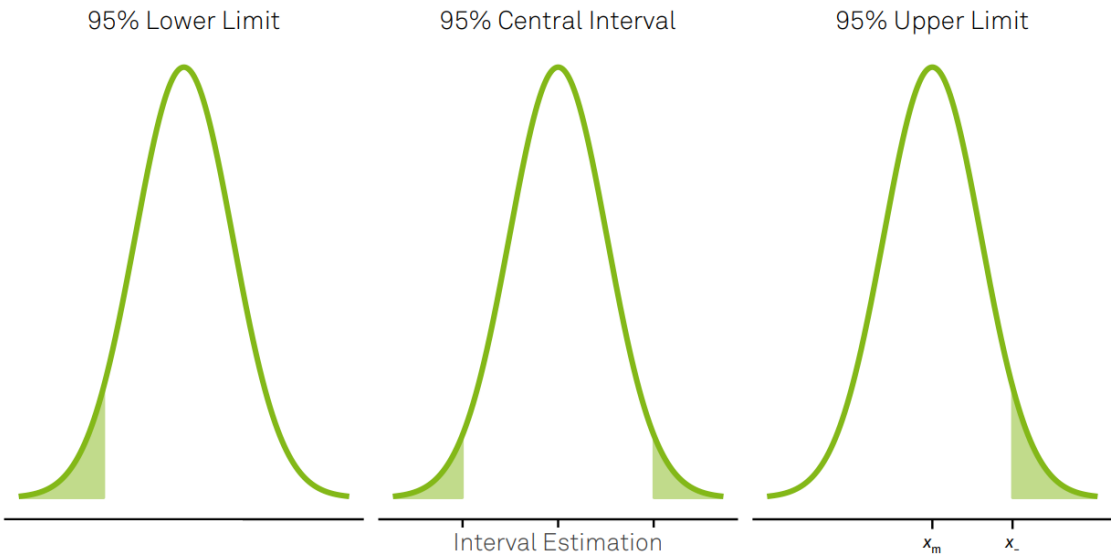
Bayesian:

The statistical approach however defines a more intuitive interpretation of α. In the bayesian interpretation, α represents the probability that for a given PDF, the true value x_{true} lies within the interval $[x_-, x_+]$.

In this case, the confidence level α is given by

$$P\left(x_- \leq x \leq x_+\right) = \int_{x_-}^{x_+} P(x)\mathrm{d}x = \alpha \qquad .$$

This also means that there is some freedom in constructing these intervals. Usually the interval is constructed symmetrically around the expectation value μ or the interval width is minimized, although these constructions being equivalent for symmetric PDFs.



What role does the prior in Bayesian statistics play?

The prior probability distribution expresses one's "believes" about a uncertain quantity **before** evidence is taken into account.

It therefore represents the *unconditional probability*. Respectively the posterior probability distribution is calculated by combining the *prior* with the evidence, following Bayes' theorem.

What freedom is there in choosing these intervals?

As mentioned above, the choice of the confidence interval (bayesian) has a degree of freedom.

For example (see fig. above) the interval can be centralized around the expectation value, or minimized regarding it's width. It can also be arranged in a way that only the most dense regions of the PDF are covered, sometimes resulting in an interval with gaps, or rather multiple intervals.

What happens in the special case of symmetrical PDF?

For symmetric PDFs the constructions mentioned above result in the same intervals. For example the interval symmetrically around the expecation value is equivalent to the shortest interval possible.

What is the difference between intervals and upper/lower limits?

For a given confidence level α, upper/lower limits are half-open intervals $[-\infty, x_-]$ or $[x_+, \infty]$.

The interpretation remains the same and the only difference is the range/width of the intervals, meaning that limits have "infinite width".

The choice and interpretation of an interval/limit depends on the subject/experiment.

See the visualization above.