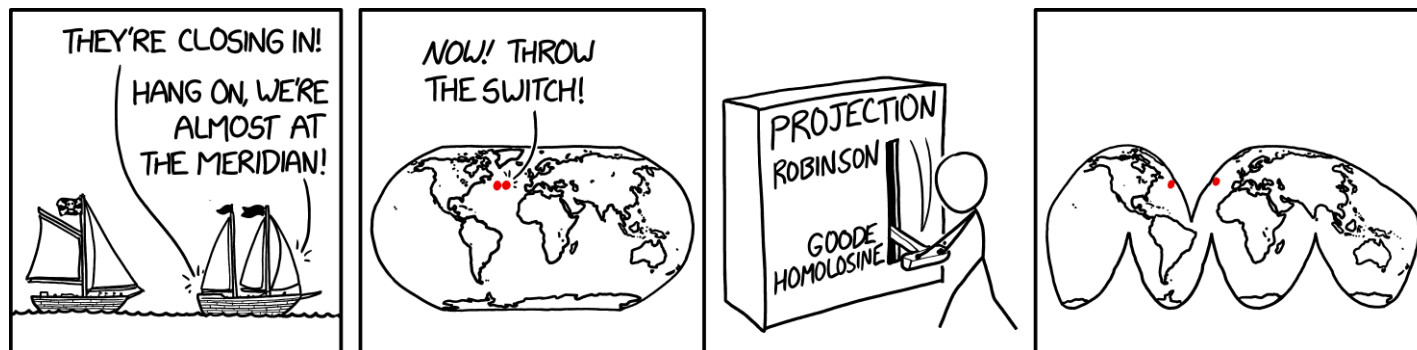


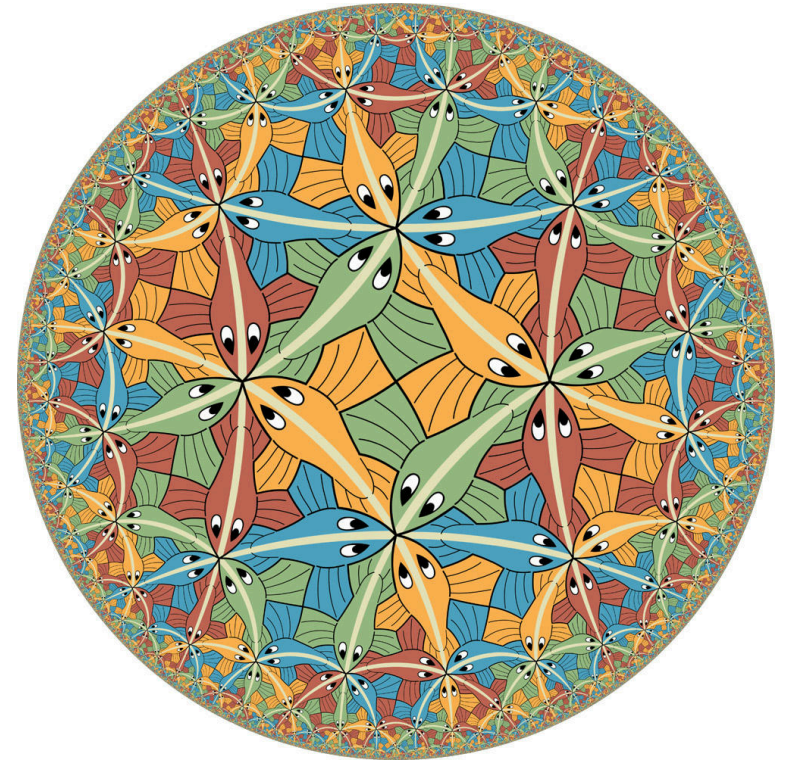
Projection cartographique de la Pseudosphère sur le plan

Gaspar Daguet, n=° XXXXXXXX



Sea Chase - Xkcd

- 1) Introduction & Problématique
- 2) La pseudosphère
- 3) La projection
- 4) projeté des droites et des cercles
- 5) non conservation des longueurs
- 6) conservation des angles



Cercle Limite III — M. C. Escher



1) Introduction & Problématique



1) Introduction & Problématique

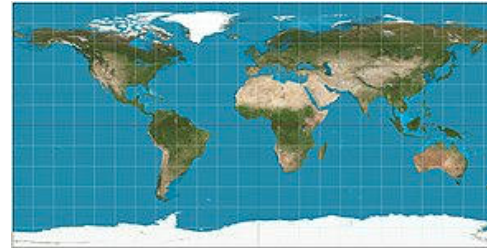
Gaspar Daguet, n=°XXXXXX



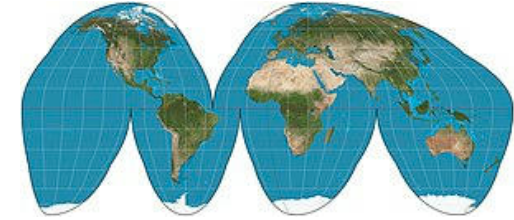
Cahill-Keyes



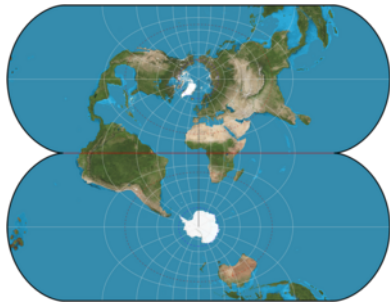
Rétro-azimutale
de Craig



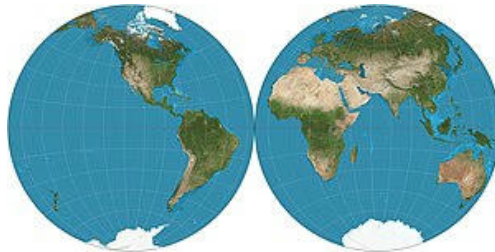
Équirectangulaire



Goode



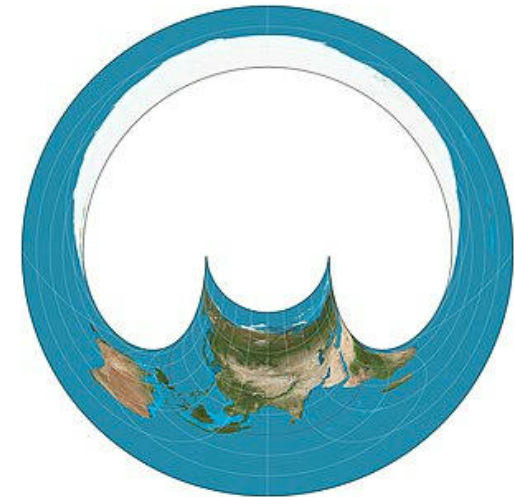
Transverse
Universelle de
Mercator



Globulaire de
Nicolosi



Stéréographique

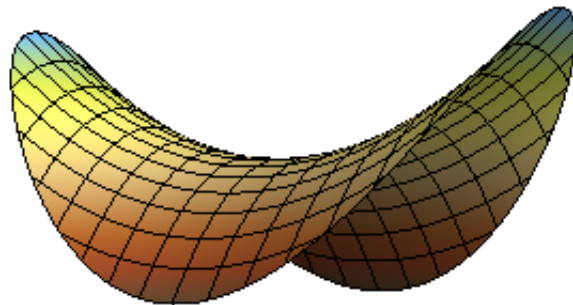


Rétro-azimutale
de Hammer



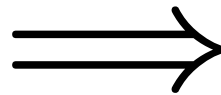
Problématique :

Comment projeter une surface hyperbolique sur le plan



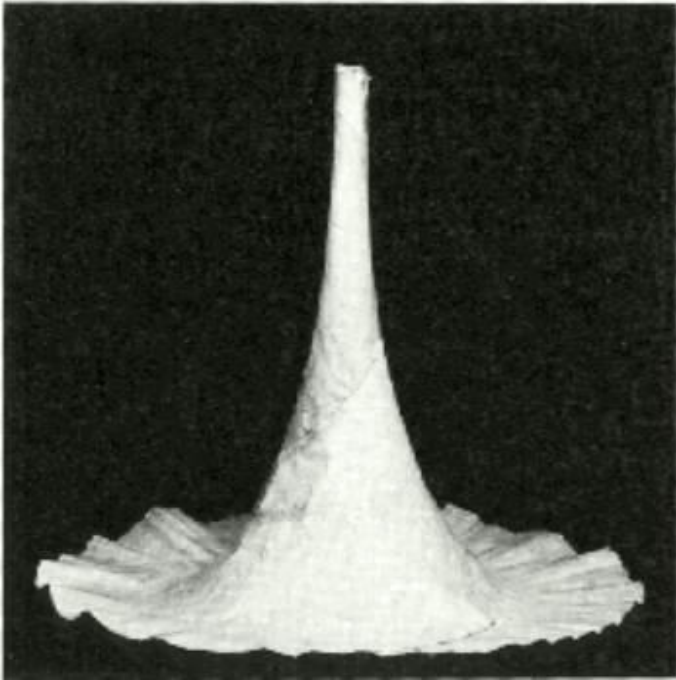
$$z = \frac{x^2}{a^2} - \frac{y^2}{b^2}, \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} < 1$$

un pringle



une carte

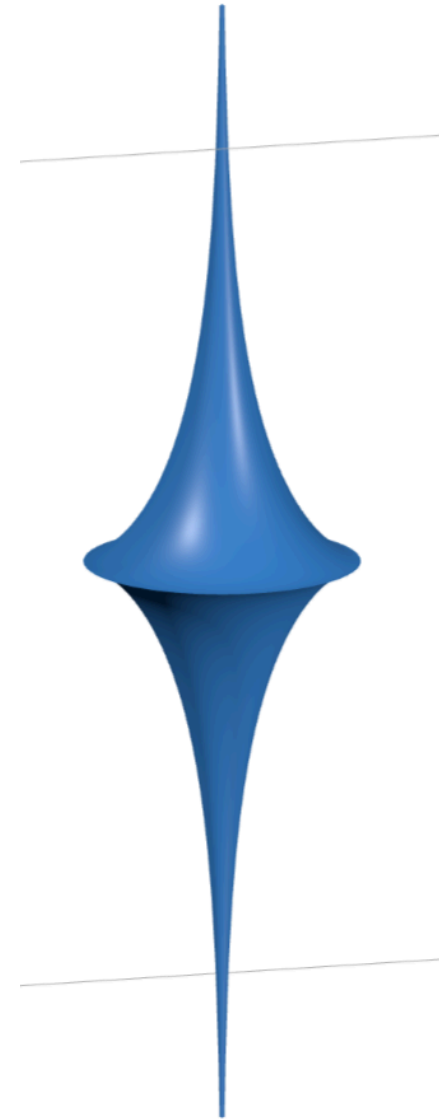




2) La Pseudosphère



$$P : \begin{cases} [0; 2\pi] \times \mathbb{R}_+ \longrightarrow \mathbb{R}^3 \\ (u, v) \longmapsto \begin{pmatrix} \frac{\cos(u)}{\operatorname{ch}(v)} \\ \frac{\sin(u)}{\operatorname{ch}(v)} \\ v - \operatorname{th}(v) \end{pmatrix} \end{cases}$$



On note $\frac{\partial P}{\partial u} = P_u$

$$E(p) = \|P_u\|^2$$

$$F(p) = \langle P_u \mid P_v \rangle$$

$$G(p) = \|P_v\|^2$$

L'application Normale :

$$N : [0; 2\pi] \times \mathbb{R}_+ \longrightarrow \mathbb{R}^3$$

$$p \longmapsto \frac{P_u(p) \wedge P_v(p)}{\|P_u(p) \wedge P_v(p)\|}$$

$$\mathcal{L}(p) = - \langle N_u \mid P_u \rangle$$

$$\mathcal{M}(p) = - \langle N_v \mid P_u \rangle$$

$$\mathcal{N}(p) = - \langle N_v \mid P_v \rangle$$

La courbure en $p \in [0; 2\pi] \times \mathbb{R}_+$

$$K(p) = \frac{\mathcal{L}(p)\mathcal{M}(p) - \mathcal{M}(p)^2}{E(p)G(p) - F^2}$$

Pour la pseudosphère :

$$F = \mathcal{M} = 0 \quad \mathcal{L} = -\mathcal{N} = \frac{\text{sh}(v)}{\text{ch}(v)} \quad \forall p \in [0; 2\pi] \times \mathbb{R}_+, K(p) = -1$$

$$E = \frac{1}{\text{ch}(v)} \quad G = \frac{\text{sh}^2(v)}{\text{ch}^2(v)} \quad \text{Donc surface hyperbolique}$$

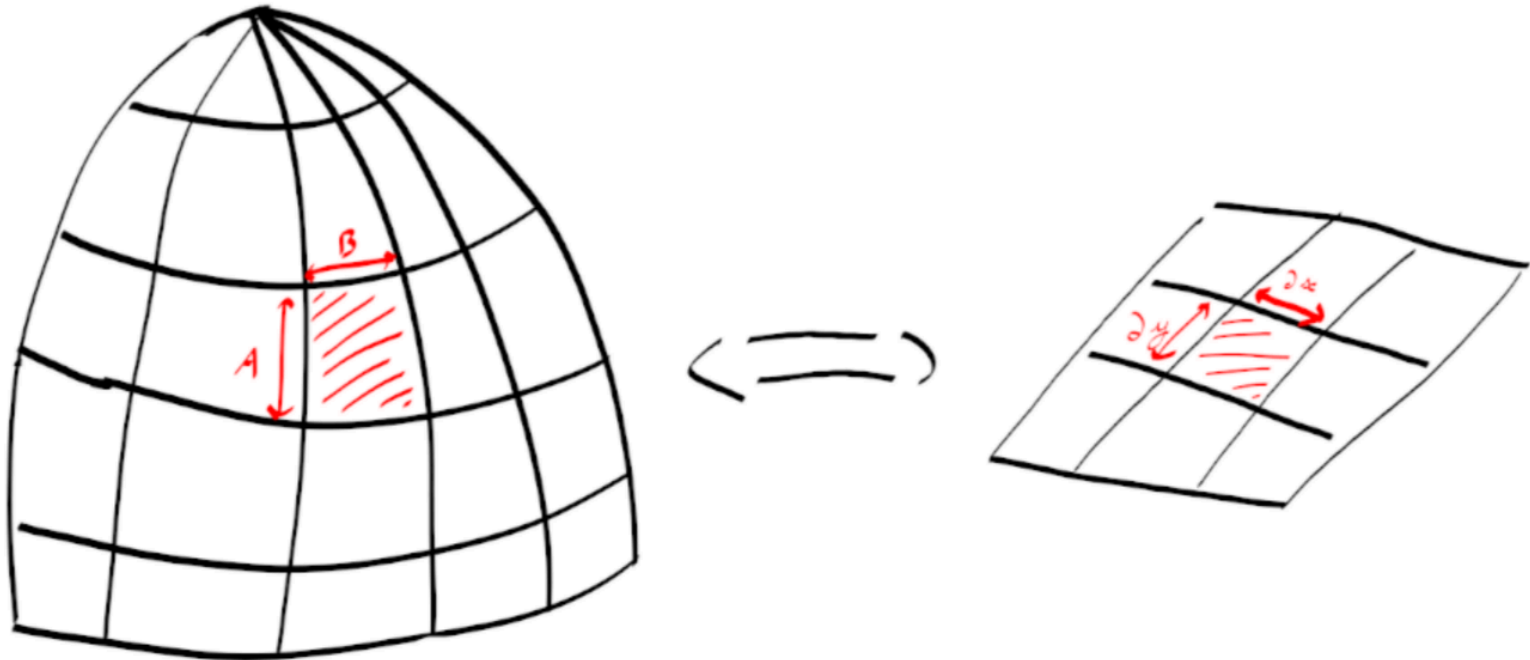


3) La projection



3) La projection

Idée de la projection de Mercator :

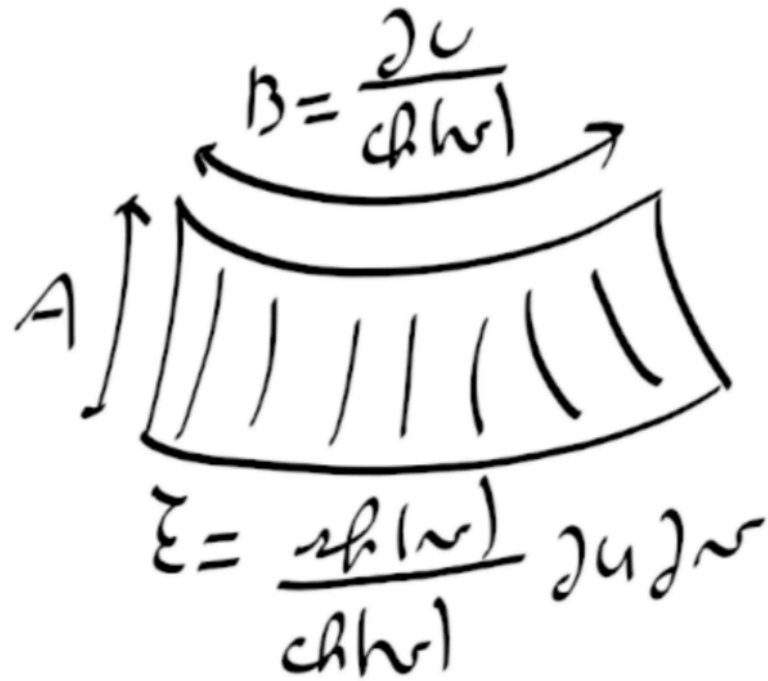


$$\frac{\partial y}{\partial x} = \frac{A}{B}$$



Surface élémentaire :

$$\mathcal{E} = \|f_u \wedge f_v\| \partial u \partial v$$



Pour la pseudosphère :

$$\mathcal{E} = \frac{\text{sh}(v)}{\text{ch}^2(v)} \partial u \partial v$$

Or :

$$B = \frac{\partial u}{\text{ch}(v)}$$

Donc

$$A = \frac{\mathcal{E}}{B} = \text{th}(v) \partial v$$

La projection :

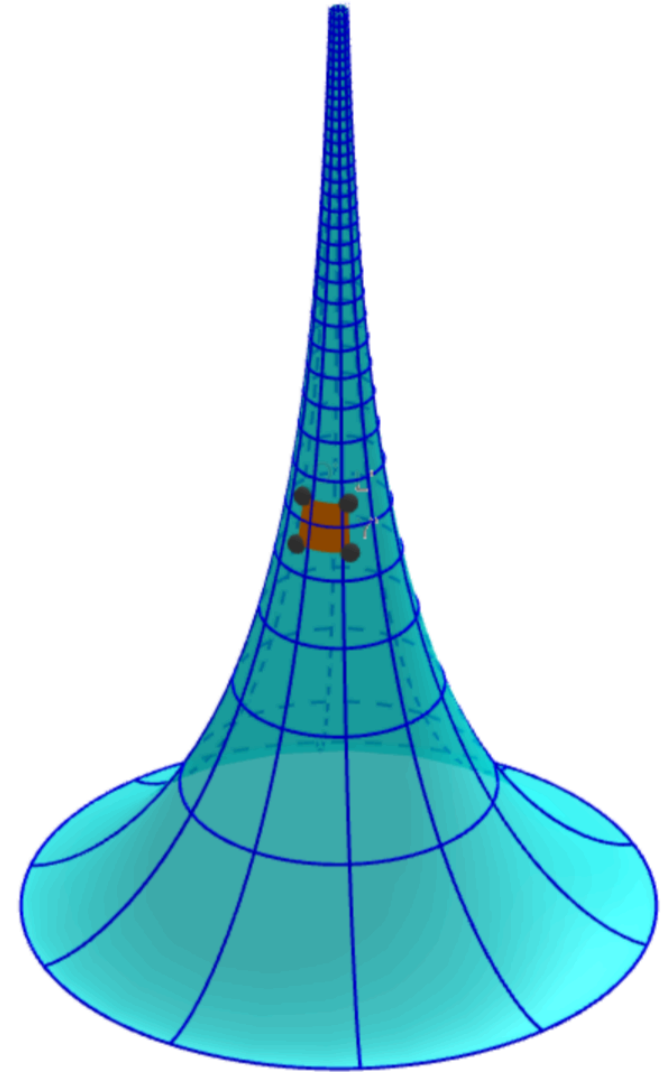
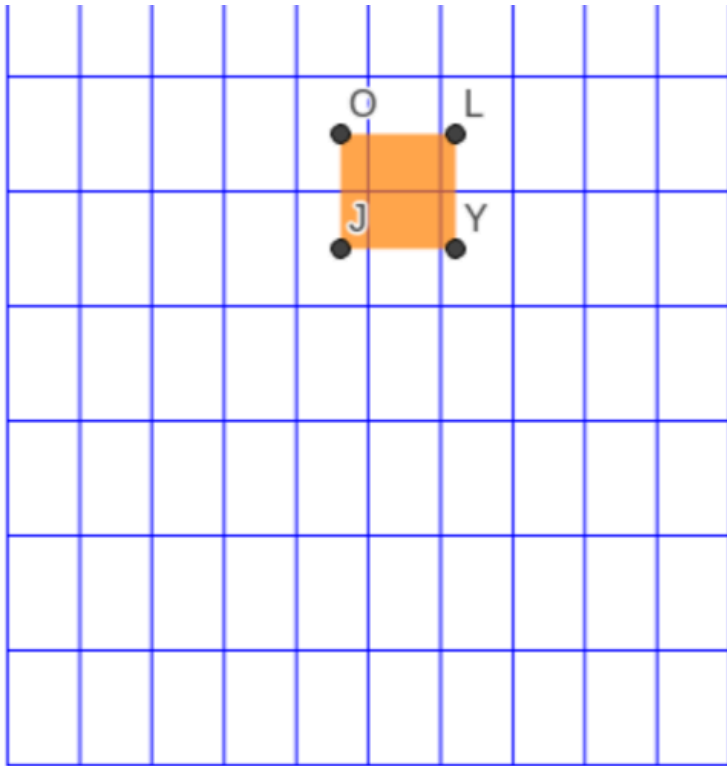
$$C : \begin{cases} [0; 2\pi] \times \mathbb{R}_+ \longrightarrow \mathbb{R}^2 \\ p = (u, v) \longmapsto \begin{pmatrix} u \\ \text{ch}(v) \end{pmatrix} \end{cases}$$



3) La projection

Gaspar Daguet, n=°XXXXXX

Droites Quelconques:



Trouver un moyen pour proj'ter une image



4) Projeté des droites



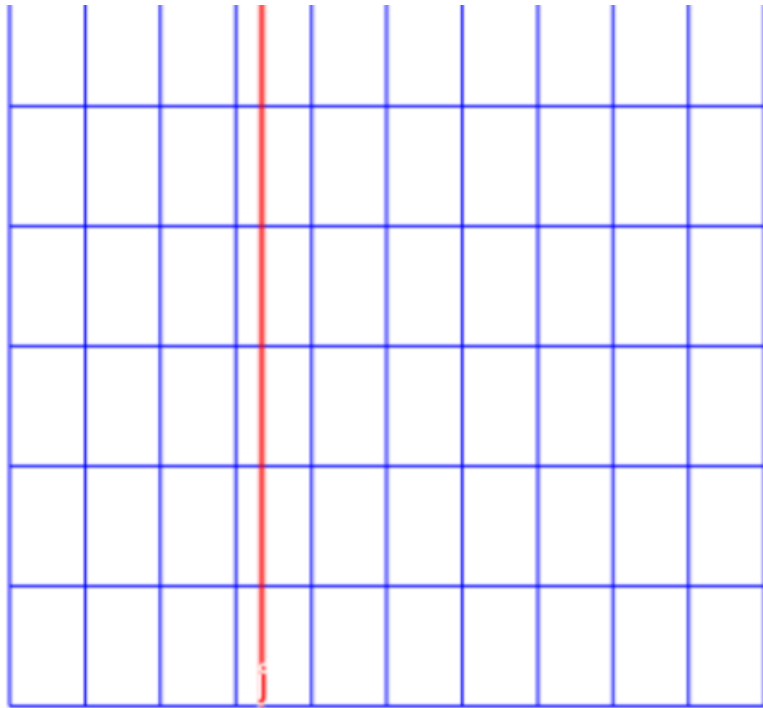
Équation des droites:

- Méridiens: $g : t \mapsto P(u, t)$
- Autres:

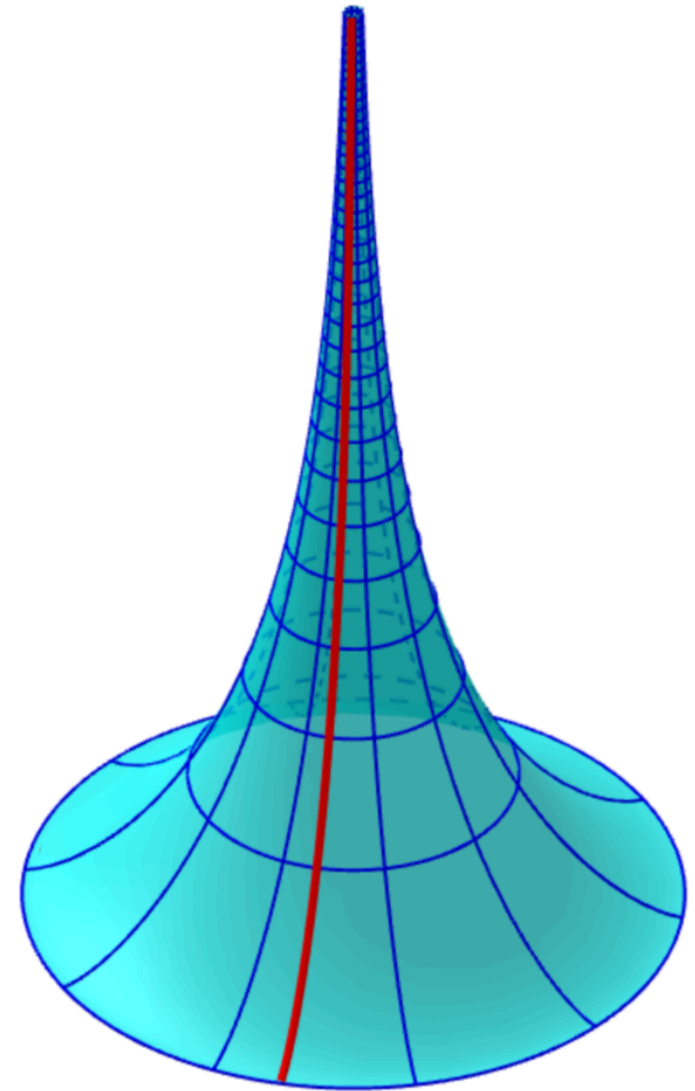
$$g : t \mapsto P\left(t, \text{ch}\left(\sqrt{k^2 - (t + c)^2}\right)\right)$$



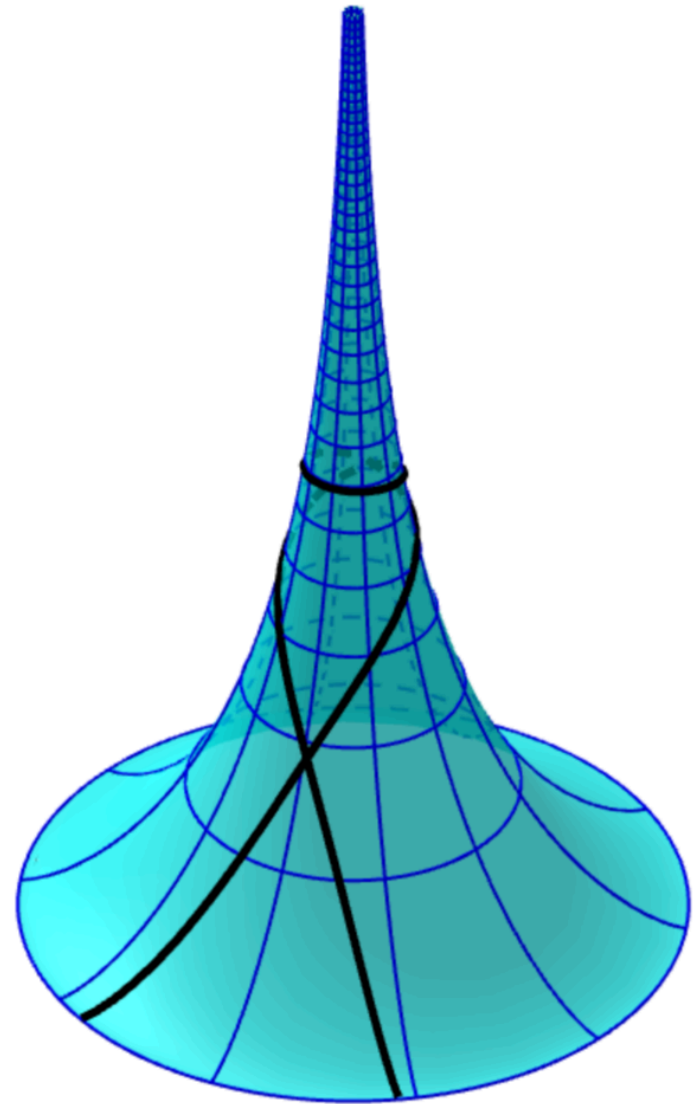
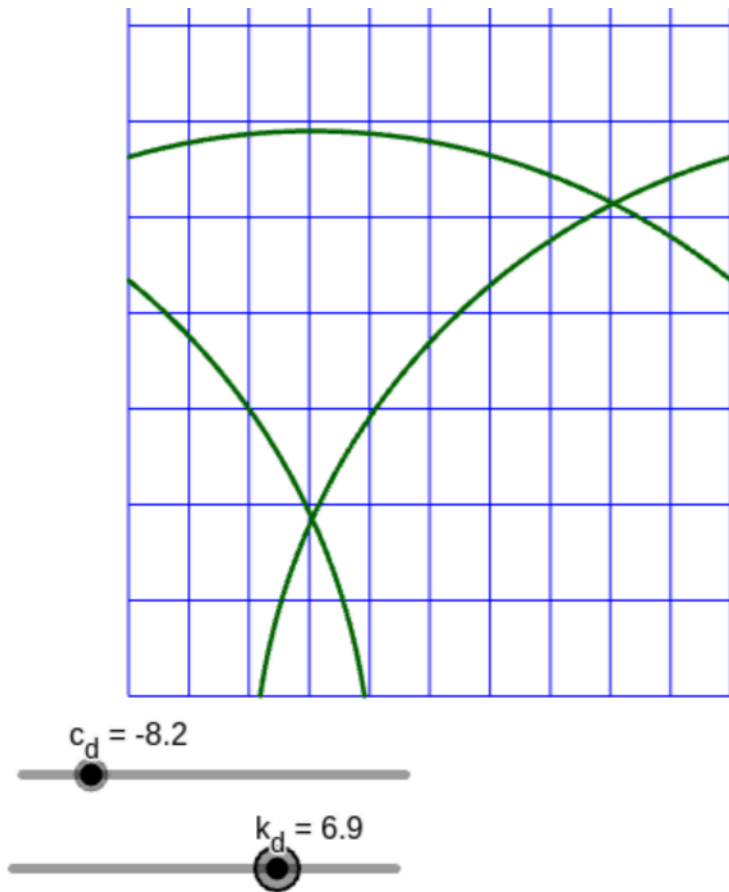
Méridiens:



$u = 2.1$



Droites Quelconques:



Sur la pseudosphère :

$$d(A, B) = \ln \left(\frac{\text{ch}(u_B)}{\text{ch}(u_A)} \right)$$

Longueur d'arc :

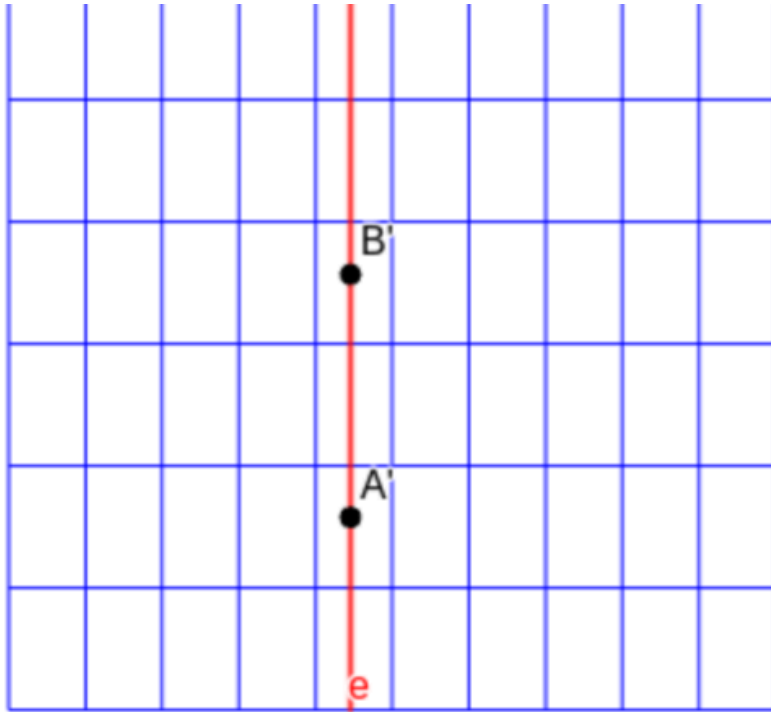
$$d(A, B) = \int_{t_A}^{t_B} \|g'(t)\| dt \quad \neq$$

Sur la carte :

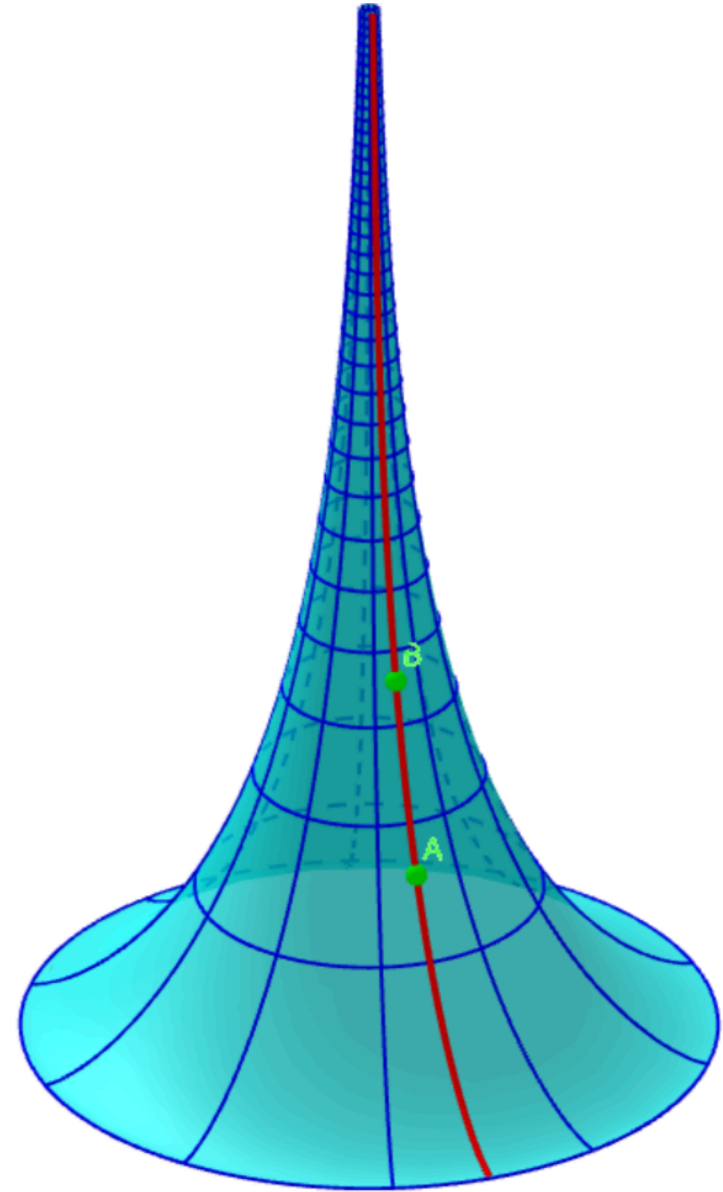
$$d(A, B) = \text{ch}(u_B) - \text{ch}(u_A)$$

Donc la projection n'est pas équivalente





$$d(A, B) = 1,99$$



$$d(A, B) = 0,57$$



5) conservation des angles

Gaspar Daguet, n=°XXXXXXX

faut que je travaille

