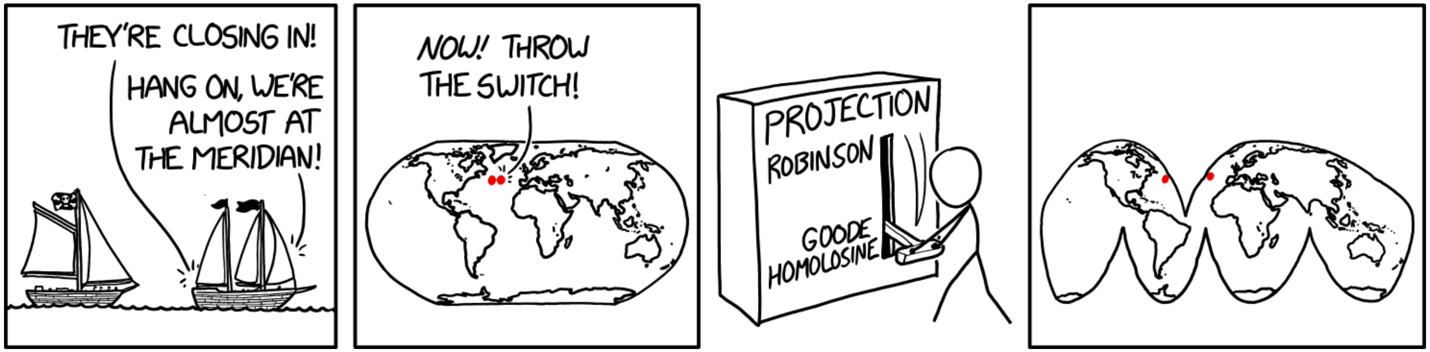
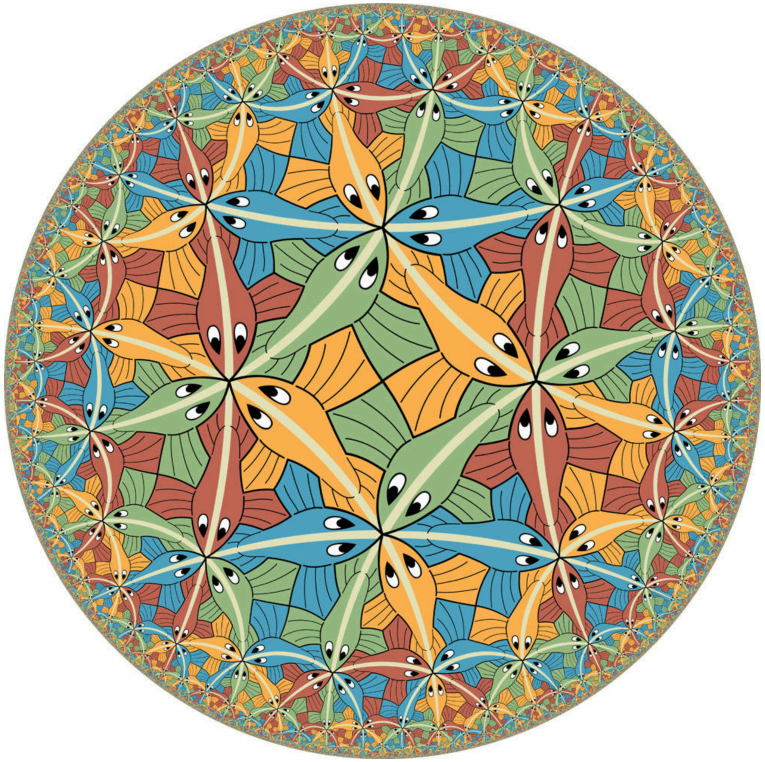
Projection cartographique de la Pseudosphère sur le plan

Gaspar Daguet, n=° XXXXXXX



Sea Chase - Xkcd

Table des Matières



Cercle Limite III — M. C. Escher

1) Introduction & Problématique

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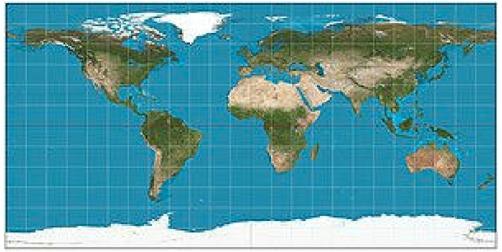
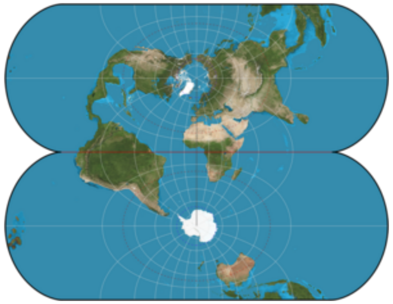
Cahill-Keyes

Rétro-azimutale de Craig



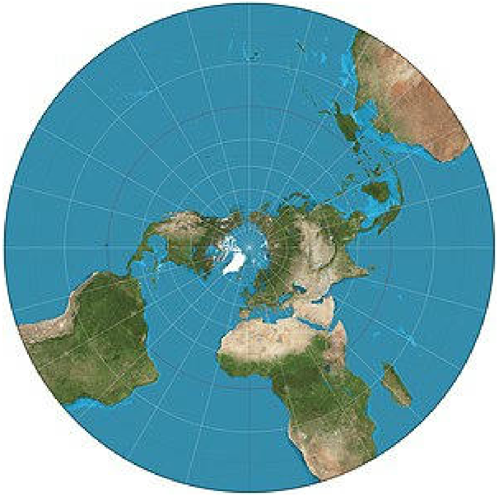
Équirectangulaire

Goode

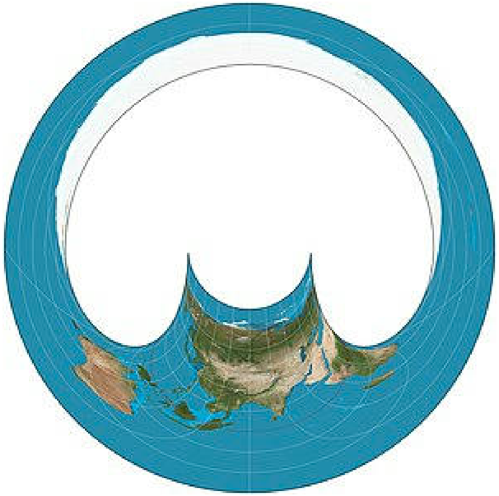


Transverse Universelle de Mercator

Globulaire de Nicolosi



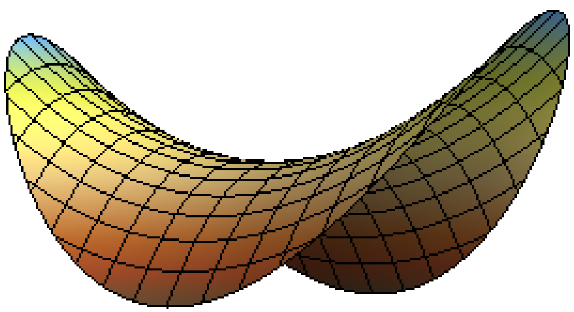
Stéréographique



Rétro-azimutale de Hammer

Problématique :

Comment projeter une surface hyperbolique sur le plan

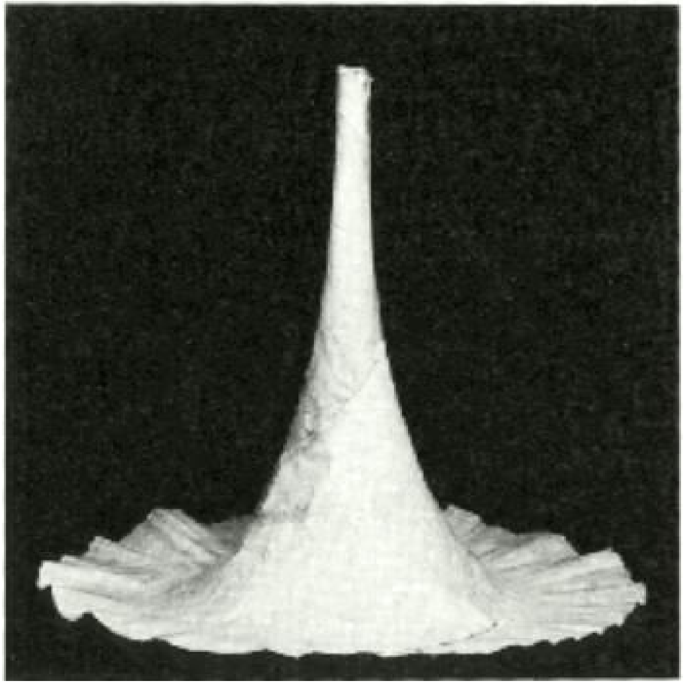


$$z = \frac{x^2}{a^2} - \frac{y^2}{b^2}, \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} < 1$$

un pringle

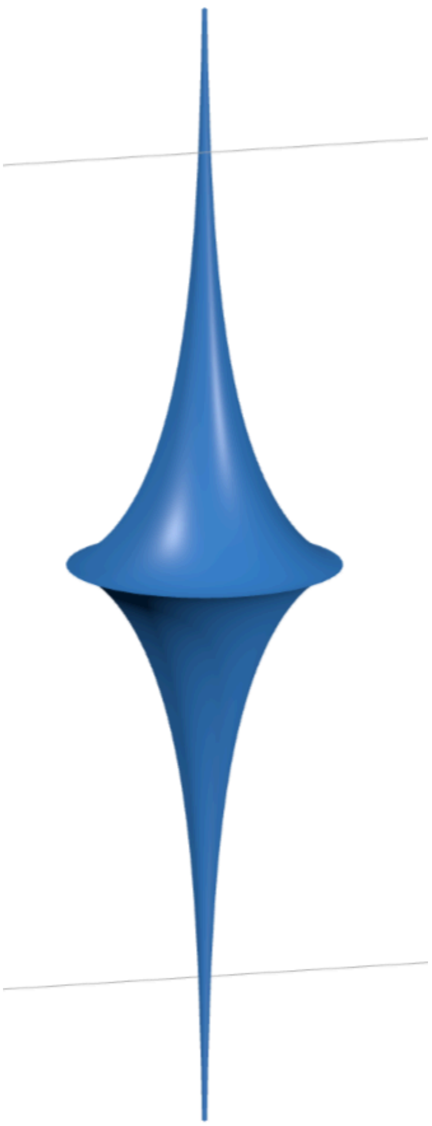
$$\Longrightarrow$$

une carte



2) La Pseudosphère

$$P : \begin{cases} [0; 2\pi] \times \mathbb{R}_+ & \longrightarrow & \mathbb{R}^3 \\ (u, v) & \longmapsto & \begin{pmatrix} \frac{\cos(u)}{\mathrm{ch}(v)} \\ \frac{\sin(u)}{\mathrm{ch}(v)} \\ v - \mathrm{th}(v) \end{pmatrix} \end{cases}$$



2) La Pseudosphère

L'application Normale :

$$N : [0; 2\pi] \times \mathbb{R}_+ \longrightarrow \mathbb{R}^3$$

$$p \longmapsto \frac{P_u(p) \wedge P_v(p)}{\|P_u(p) \wedge P_v(p)\|}$$

$$E(p) = \|P_u\|^2$$

$$F(p) = < P_u \mid P_v >$$

$$G(p) = \|P_v\|^2$$

$$\mathscr{L}(p) = - < N_u \mid P_u >$$

$$\mathscr{M}(p) = - < N_v \mid P_u >$$

$$\mathscr{N}(p) = - < N_v \mid P_v >$$

La courbure en $p \in [0; 2\pi] \times \mathbb{R}_+$

$$K(p) = \frac{\mathscr{L}(p)\mathscr{N}(p) - \mathscr{M}(p)^2}{E(p)G(p) - F^2}$$

Pour la pseudosphère :

$$F = \mathscr{M} = 0 \quad \mathscr{L} = -\mathscr{N} = \frac{\mathrm{sh}(v)}{\mathrm{ch}(v)}$$

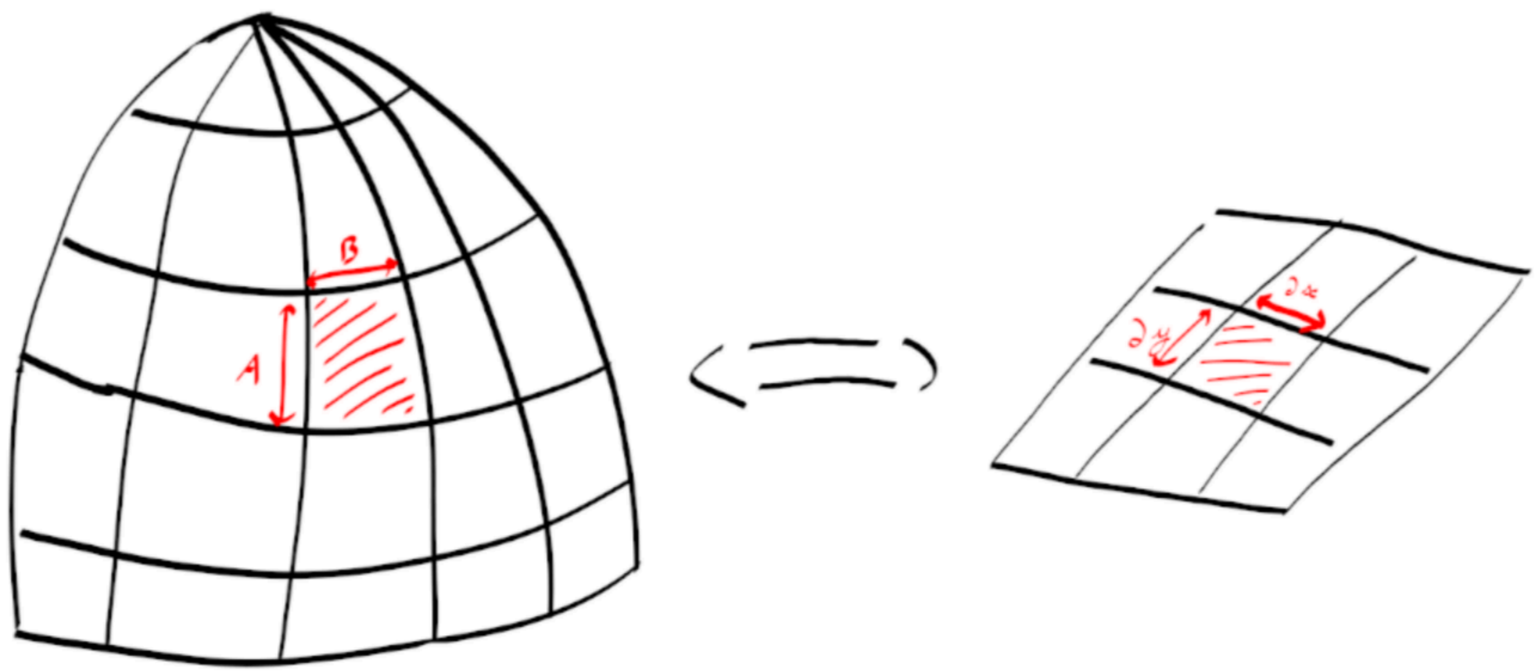
$$E = \frac{1}{\mathrm{ch}(v)} \qquad G = \frac{\mathrm{sh}^2(v)}{\mathrm{ch}^2(v)}$$

$$\forall p \in [0; 2\pi] \times \mathbb{R}_+, K(p) = -1$$

Donc surface hyperbolique

2) La projection

Idée de la projection de Mercator :



$$\frac{\partial y}{\partial x} = \frac{A}{B}$$

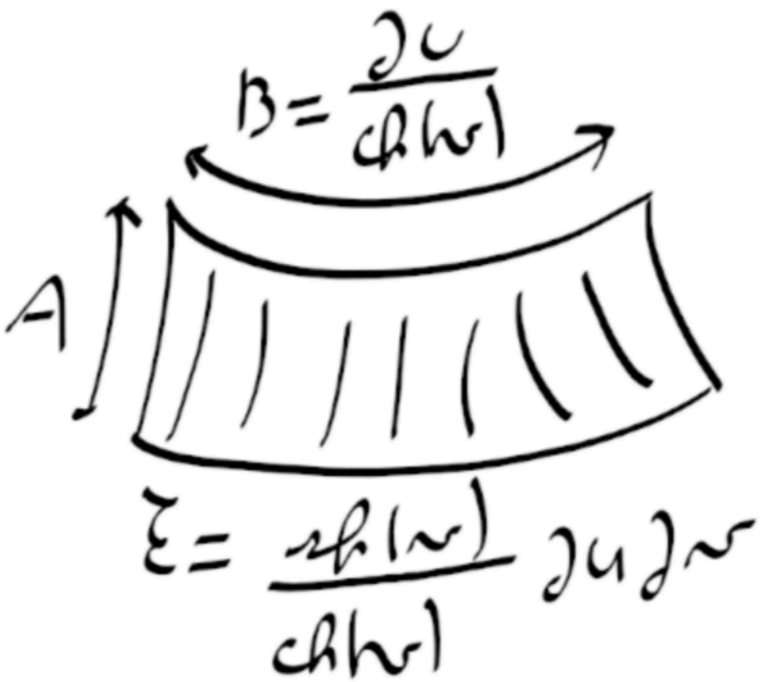
2) La projection

Surface élémentaire :

$$\mathscr{E} = \|f_u \wedge f_v\| \partial u \partial v$$

Pour la pseudosphère :

$$\mathscr{E} = \frac{\mathrm{sh}(v)}{\mathrm{ch}^2(v)} \partial u \partial v$$



Or :

$$B = \frac{\partial u}{\mathrm{ch}(v)}$$

Donc

$$A = \frac{\mathscr{E}}{B} = \mathrm{th}(v) \partial v$$

2) La projection

On pose

$$\frac{\partial x}{\partial u} = 1$$

Et comme

$$\frac{\partial y}{\partial x} = \frac{\mathrm{sh}(v)\partial v}{\partial u}$$

Ainsi :

$$y = \mathrm{ch}(v)$$

La projection :

$$C : \begin{cases} [0; 2\pi] \times \mathbb{R}_+ \longrightarrow \mathbb{R}^2 \\ p = (u, v) \longmapsto \begin{pmatrix} u \\ \mathrm{ch}(v) \end{pmatrix} \end{cases}$$

Plein d'image de projection ‼

3) projeté des droites et des sphères

dèf droites et cercle

calcule par la projection

joli dessin

4) non conservation des longueurs

calcule de la distance sur S et sur P voir que diff

5) conservation des angles

faut que je travaille