Gaspar Daguet

Info : DM2

En logique minimale :

Partie I. Lemme :

Lemme 1

$$\frac{\Gamma, \varphi_1 \vdash \varphi_2 \qquad \Gamma, \varphi_1 \vdash \neg\varphi_2}{\Gamma \vdash \neg\varphi_1}\ \neg_{ei}$$

preuve :

$$\frac{\dfrac{\text{hypp}}{\Gamma, \varphi_1 \vdash \varphi_2} \qquad \dfrac{\text{hypp}}{\Gamma, \varphi_1 \vdash \neg\varphi_2}}{\dfrac{\Gamma, \varphi_1 \vdash \bot}{\Gamma \vdash \neg\varphi_1}\ \neg_i}\ \neg_e$$

Lemme 2

$$\frac{}{\Gamma, \varphi \to \psi, \varphi \vdash \psi}\ \overset{\to}{\text{ax}}$$

preuve :

$$\frac{\dfrac{}{\Gamma, \varphi \to \psi, \varphi \vdash \varphi \to \psi}\ \text{ax} \qquad \dfrac{}{\Gamma, \varphi \to \psi, \varphi \vdash \varphi}\ \text{ax}}{\Gamma, \varphi \to \psi, \varphi \vdash \psi}\ \vec{e}$$

Partie II. Logique minimale :

N=° 1.

$$\frac{\dfrac{}{\Gamma \vdash p \vee (p \wedge q)}\ \text{ax} \qquad \dfrac{}{\Gamma, p \vdash p}\ \text{ax} \qquad \dfrac{\dfrac{}{\Gamma, p \wedge q \vdash p \wedge q}\ \text{ax}}{\Gamma, p \wedge q \vdash p}\ \wedge_e^g}{p \vee (p \wedge q) \vdash p}\ \vee_e$$

Avec $\Gamma = \{p \vee (p \wedge q)\}$

N=° 2.

$$\frac{\dfrac{}{\Gamma \vdash p}\ \text{ax} \qquad \dfrac{\dfrac{}{\Gamma \vdash p \to \neg p}\ \text{ax} \qquad \dfrac{}{\Gamma \vdash p}\ \text{ax}}{\Gamma \vdash \neg p}\ \vec{e}}{p \to \neg p \vdash \neg p}\ \neg_{\text{ei}}$$

Avec $\Gamma = \{p \to \neg p, p\}$

N=° 3.

$$\frac{\dfrac{\dfrac{}{\Gamma, p \vdash p \to q \vee r}\ \text{ax} \qquad \dfrac{}{\Gamma, p \vdash p}\ \text{ax}}{\Gamma, p \vdash q \vee r}\ \vec{e} \qquad \dfrac{}{\Gamma, p, q \vdash s}\ \overset{\to}{\text{ax}} \qquad \dfrac{}{\Gamma, p, r \vdash s}\ \overset{\to}{\text{ax}}}{\dfrac{\Gamma, p \vdash s}{p \to (q \vee r), q \to s, r \to s \vdash p \to s}\ \vec{i}}\ \vee_e$$

Avec $\Gamma = \{p \to (q \vee r), q \to s, r \to s\}$

N=° 4.

$$\frac{\dfrac{\dfrac{}{\Gamma \vdash p \to q}\ \text{ax} \qquad \dfrac{\dfrac{}{\Gamma \vdash p \wedge r}\ \text{ax}}{\Gamma \vdash p}\ \wedge_e^g}{\Gamma \vdash q}\ \vec{e} \qquad \dfrac{\dfrac{}{\Gamma \vdash r \to s}\ \text{ax} \qquad \dfrac{\dfrac{}{\Gamma \vdash p \wedge r}\ \text{ax}}{\Gamma \vdash r}\ \wedge_e^d}{\Gamma \vdash s}\ \vec{e}}{\Gamma \vdash q \wedge s}\ \wedge_i$$

Avec $\Gamma = \{p \to q, r \to s, p \wedge r\}$

N=° 5.

$$\dfrac{\dfrac{}{\Gamma \vdash p \vee r}\,\text{ax} \qquad \dfrac{\dfrac{\dfrac{}{\Gamma, p \vdash q}\,\underset{\text{ax}}{\to}}{\Gamma, p \vdash q \vee s}\,\vee_i^g \qquad \dfrac{\dfrac{}{\Gamma, r \vdash s}\,\underset{\text{ax}}{\to}}{\Gamma, r \vdash q \vee s}\,\vee_i}{\Gamma \vdash q \vee s}\,\vee_e$$

Avec $\Gamma = \{p \to q, r \to s, p \vee r\}$

N=° 6.

$$\dfrac{\dfrac{\dfrac{}{\Gamma_1 \vdash (p \vee q) \to r}\,\text{ax} \qquad \dfrac{\dfrac{}{\Gamma_1 \vdash p}\,\text{ax}}{\Gamma_1 \vdash p \vee q}\,\vee_e^g}{\Gamma_1 \vdash r}\,\underset{e}{\to} \qquad \dfrac{\dfrac{}{\Gamma_2 \vdash (p \vee q) \to r}\,\text{ax} \qquad \dfrac{\dfrac{}{\Gamma_2 \vdash q}\,\text{ax}}{\Gamma_2 \vdash p \vee q}\,\vee_e^d}{\Gamma_2 \vdash r}\,\underset{e}{\to}}{(p \vee q) \to r \vdash (p \to r) \wedge (q \to r)}\,\wedge_i + \underset{i}{\to}$$

Avec $\Gamma_1 = \{(p \vee q) \to r, p\}$ et $\Gamma_2 = \{(p \vee q) \to r, q\}$

Partie III. Lois de De Morgan

N=° 7.

$$\dfrac{\dfrac{\dfrac{}{\Gamma_1 \vdash \neg(p \vee q)}\,\text{ax} \qquad \dfrac{\dfrac{}{\Gamma_1 \vdash p}\,\text{ax}}{\Gamma_1 \vdash p \vee q}\,\vee_i^g}{\neg(p \vee q) \vdash \neg p}\,\neg_{ei} \qquad \dfrac{\dfrac{}{\Gamma_2 \vdash \neg(p \vee q)}\,\text{ax} \qquad \dfrac{\dfrac{}{\Gamma_2 \vdash q}\,\text{ax}}{\Gamma_2 \vdash p \vee q}\,\vee_i^d}{\neg(p \vee q) \vdash \neg q}\,\neg_{ei}}{\neg(p \vee q) \vdash \neg p \wedge \neg q}\,\wedge_i$$

Avec $\Gamma_1 = \{\neg(p \vee q), p\}$ et $\Gamma_2 = \{\neg(p \vee q), q\}$

N=° 8.

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma, q \vdash \neg p \wedge \neg q}\,\text{ax}}{\Gamma, q \vdash \neg q}\,\wedge_e^d \qquad \dfrac{}{\Gamma, q \vdash q}\,\text{ax}}{\Gamma, q \vdash \bot}\,\neg_e \qquad \dfrac{}{\Gamma \vdash p \vee q}\,\text{ax} \qquad \dfrac{\dfrac{}{\Gamma, p \vdash p}\,\text{ax} \qquad \dfrac{\dfrac{\dfrac{}{\Gamma, p \vdash \neg p \wedge \neg q}\,\text{ax}}{\Gamma, p \vdash \neg p}\,\wedge_e^g}{\Gamma, p \vdash \bot}\,\neg_e}{\neg p \wedge \neg q, p \vee q \vdash \bot}\,\vee_e$$

$$\dfrac{\neg p \wedge \neg q, p \vee q \vdash \bot}{\neg p \wedge \neg q \vdash (p \vee q)}\,\neg_i$$

Avec $\Gamma = \{\neg p \wedge \neg q, p \vee q\}$

N=° 9.

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma, \neg q \vdash p \wedge q}\,\text{ax}}{\Gamma, \neg q \vdash q}\,\wedge_e^d \qquad \dfrac{}{\Gamma, \neg q \vdash \neg q}\,\text{ax}}{\Gamma, \neg q \vdash \bot}\,\neg_e \qquad \dfrac{}{\Gamma \vdash \neg p \vee \neg q}\,\text{ax} \qquad \dfrac{\dfrac{}{\Gamma, \neg p \vdash \neg p}\,\text{ax} \qquad \dfrac{\dfrac{\dfrac{}{\Gamma, \neg p \vdash p \wedge q}\,\text{ax}}{\Gamma, \neg p \vdash p}\,\wedge_e^g}{\Gamma, \neg p \vdash \bot}\,\neg_e}{\neg p \vee \neg q, p \wedge q \vdash \bot}\,\vee_e$$

$$\dfrac{\neg p \vee \neg q, p \wedge q \vdash \bot}{\neg p \vee \neg q \vdash \neg(p \wedge q)}\,\neg_i$$

Avec $\Gamma = \{\neg p \vee \neg q, p \wedge q\}$