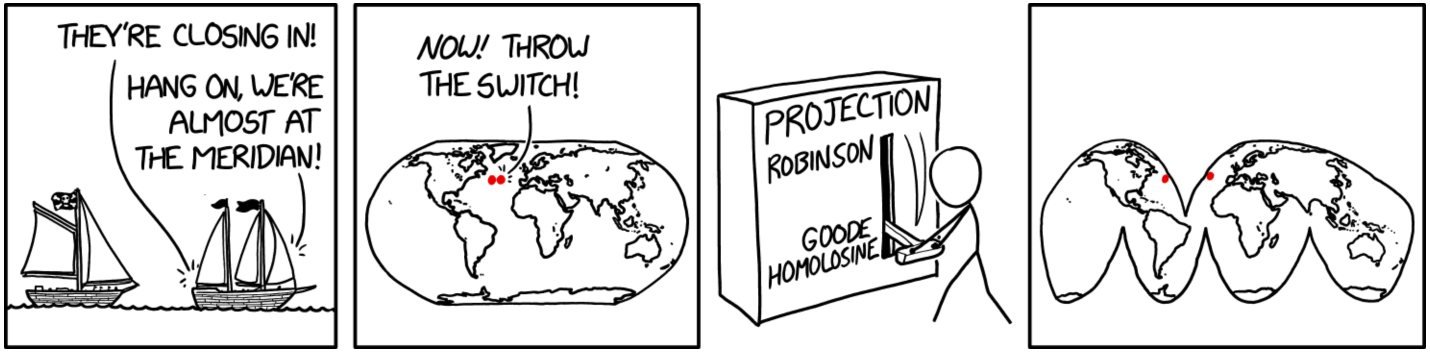
Projection cartographique de la Pseudosphère sur le plan

Gaspar Daguet, n=° 21528



Sea Chase - Xkcd

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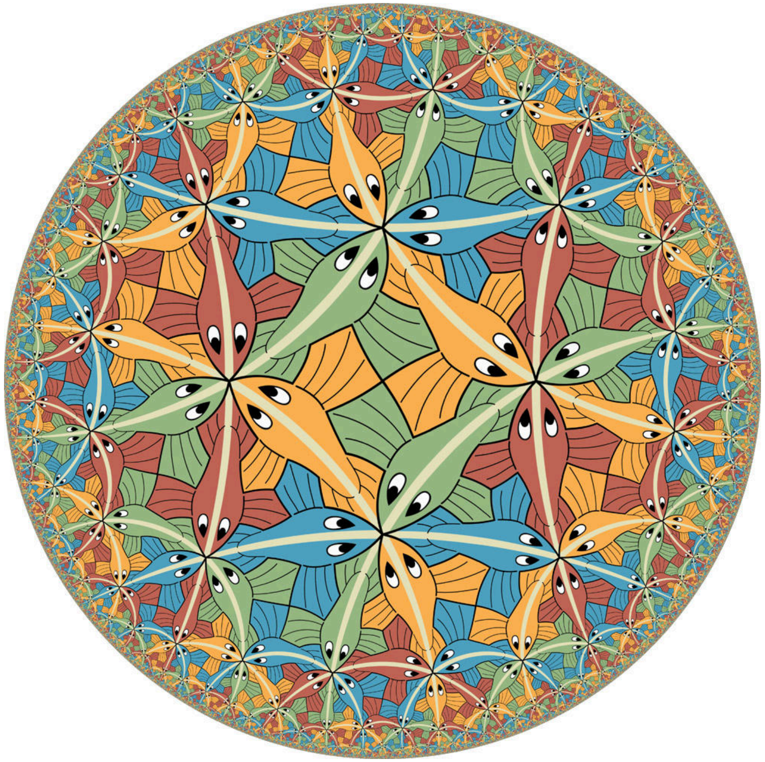

Cercle Limite III — M. C. Escher

1) Introduction & Problématique

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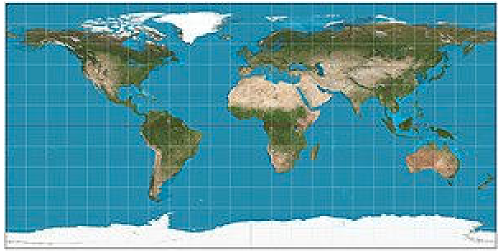
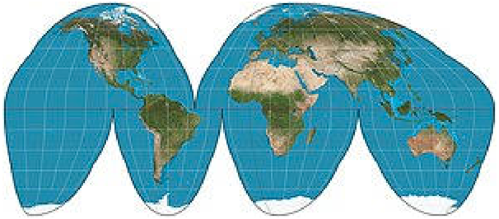
Cahill-Keyes

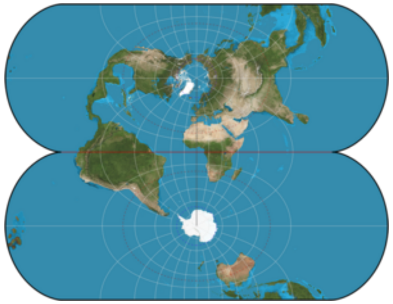
Rétro-azimutale de Craig



Équirectangulaire

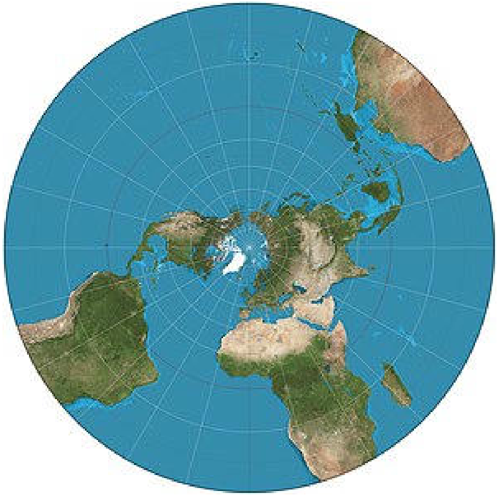


Goode

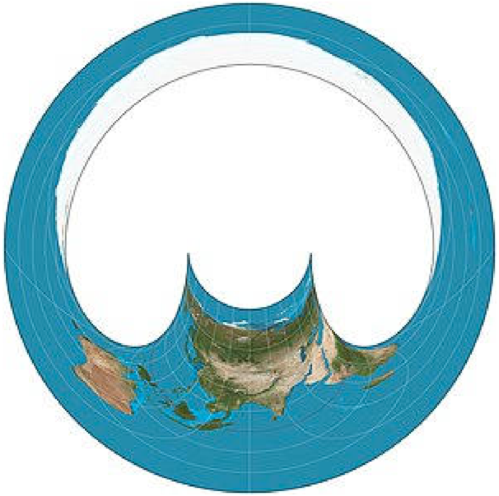


Transverse Universelle de Mercator

Globulaire de Nicolosi



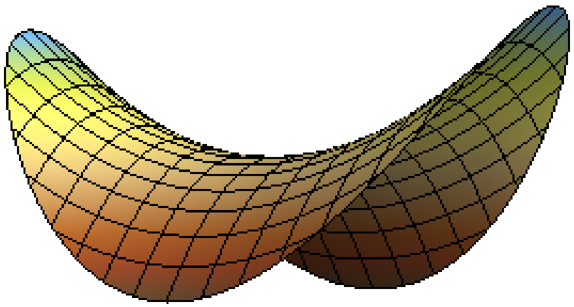
Stéréographique



Rétro-azimutale de Hammer

Problématique :

Comment projeter une surface hyperbolique sur le plan

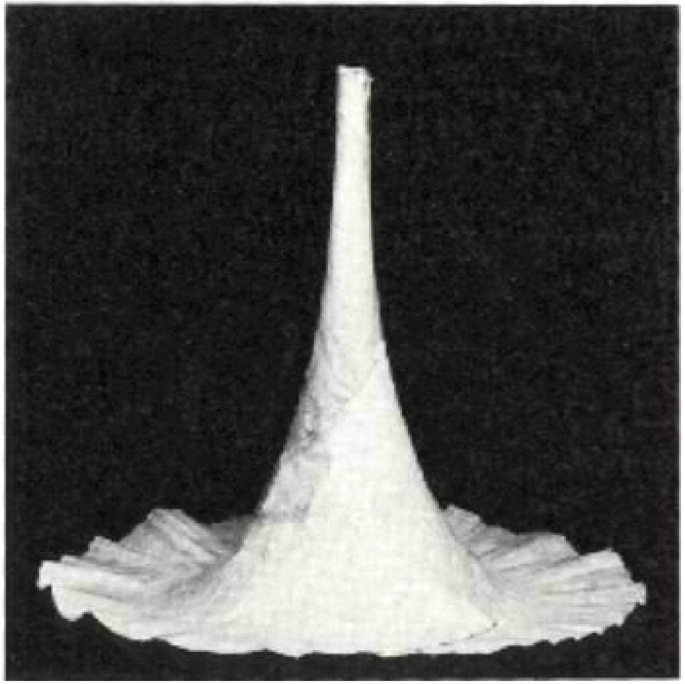


$$z = \frac{x^2}{a^2} - \frac{y^2}{b^2}, \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} < 1$$

une chips

$$\Longrightarrow$$

une carte



2) La Pseudosphère

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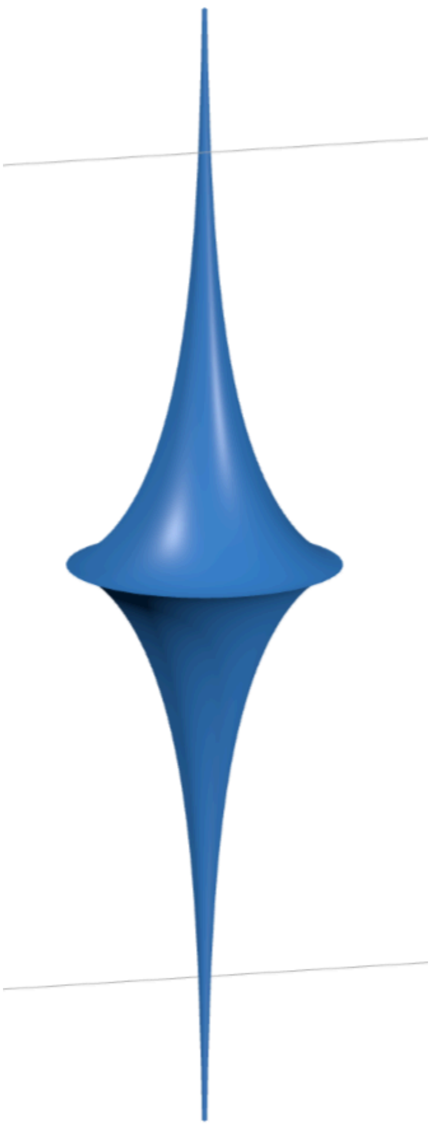
$$P : \begin{cases} [0; 2\pi] \times \mathbb{R}_+ & \longrightarrow & \mathbb{R}^3 \\ (u, v) & \longmapsto & \begin{pmatrix} \frac{\cos(u)}{\mathrm{ch}(v)} \\ \frac{\sin(u)}{\mathrm{ch}(v)} \\ v - \mathrm{th}(v) \end{pmatrix} \end{cases}$$



2) La Pseudosphère

On note $\dfrac{\partial P}{\partial u} = P_u$

L'application Normale :

$$N : [0; 2\pi] \times \mathbb{R}_+ \longrightarrow \mathbb{R}^3$$

$$p \longmapsto \frac{P_u(p) \wedge P_v(p)}{\|P_u(p) \wedge P_v(p)\|}$$

$$E(p) = \|P_u\|^2$$

$$F(p) =< P_u \mid P_v >$$

$$G(p) = \|P_v\|^2$$

$$\mathscr{L}(p) = - < N_u \mid P_u >$$

$$\mathscr{M}(p) = - < N_v \mid P_u >$$

$$\mathscr{N}(p) = - < N_v \mid P_v >$$

2) La Pseudosphère

La courbure en $p \in [0;2\pi] \times \mathbb{R}_+$

$$K(p) = \frac{\mathscr{L}(p)\mathscr{N}(p) - \mathscr{M}(p)^2}{E(p)G(p) - F(p)^2}$$

Pour la pseudosphère :

$$F = \mathscr{M} = 0 \quad \mathscr{L} = -\mathscr{N} = \frac{\mathrm{sh}(v)}{\mathrm{ch}(v)}$$

$$E = \frac{1}{\mathrm{ch}(v)} \qquad G = \frac{\mathrm{sh}^2(v)}{\mathrm{ch}^2(v)}$$

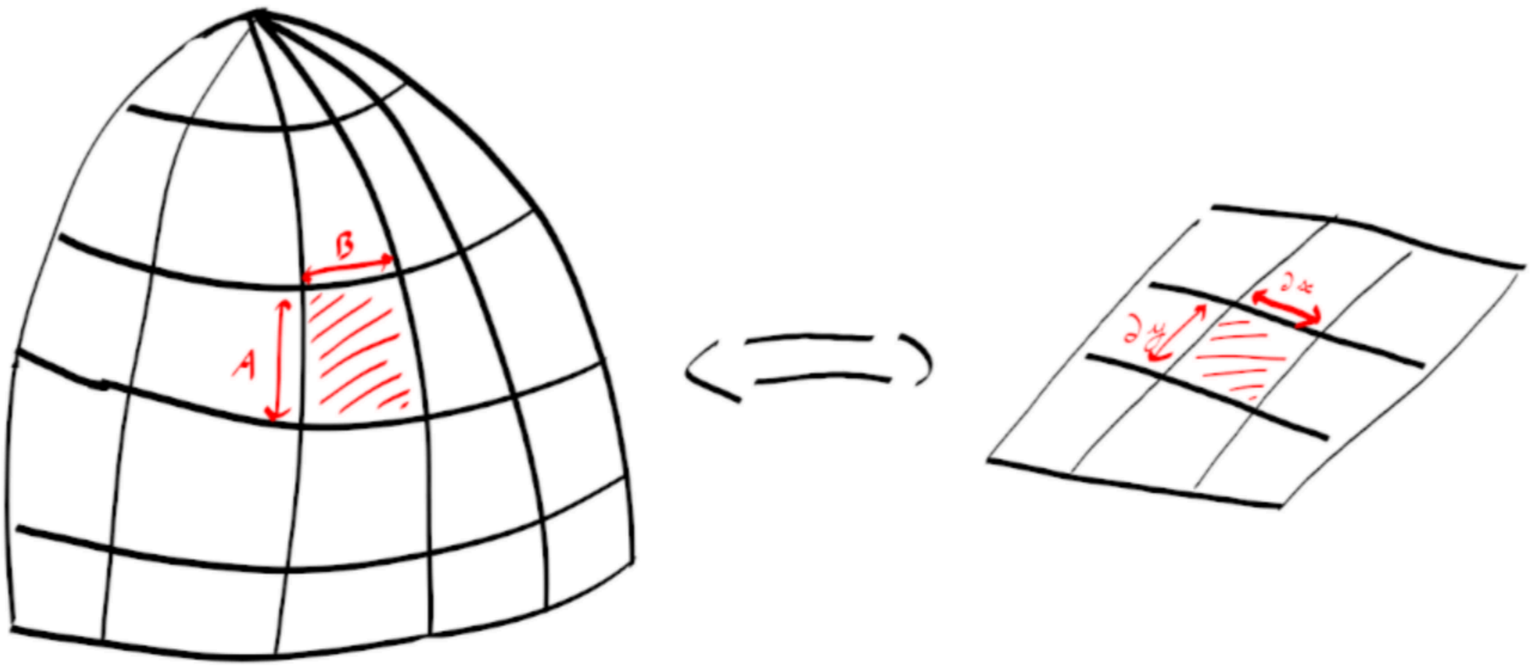
$$\forall p \in [0;2\pi] \times \mathbb{R}_+, K(p) = -1$$

Donc surface hyperbolique

3) La projection

3) La projection

Idée de la projection de Mercator :



$$\frac{\partial y}{\partial x} = \frac{A}{B}$$

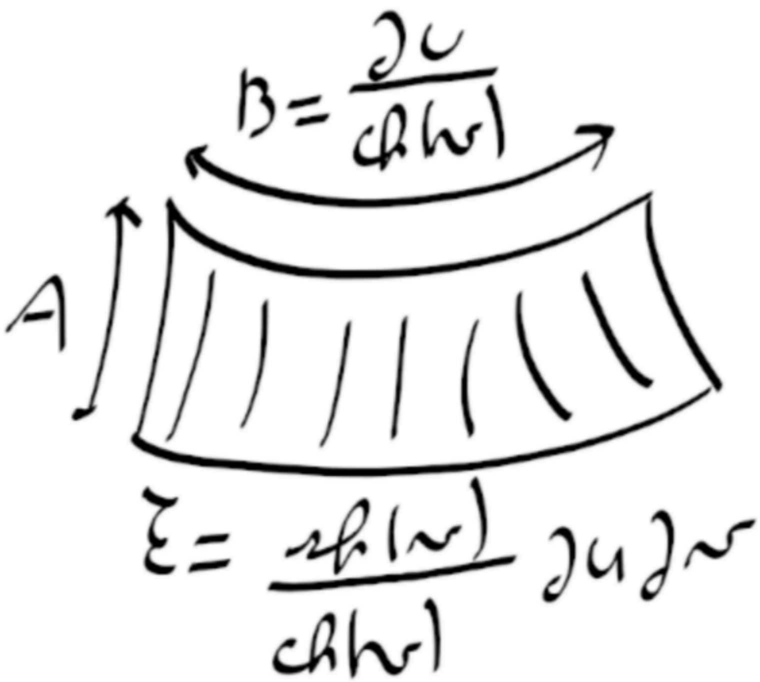
3) La projection

Surface élémentaire :

$$\mathscr{E} = \|P_u \wedge P_v\| \partial u \partial v$$

Pour la pseudosphère :

$$\mathscr{E} = \frac{\mathrm{sh}(v)}{\mathrm{ch}^2(v)} \partial u \partial v$$



Or :

$$B = \frac{\partial u}{\mathrm{ch}(v)}$$

Donc

$$\frac{A}{B} = \frac{\mathscr{E}}{B^2} = \mathrm{sh}(v)\partial v$$

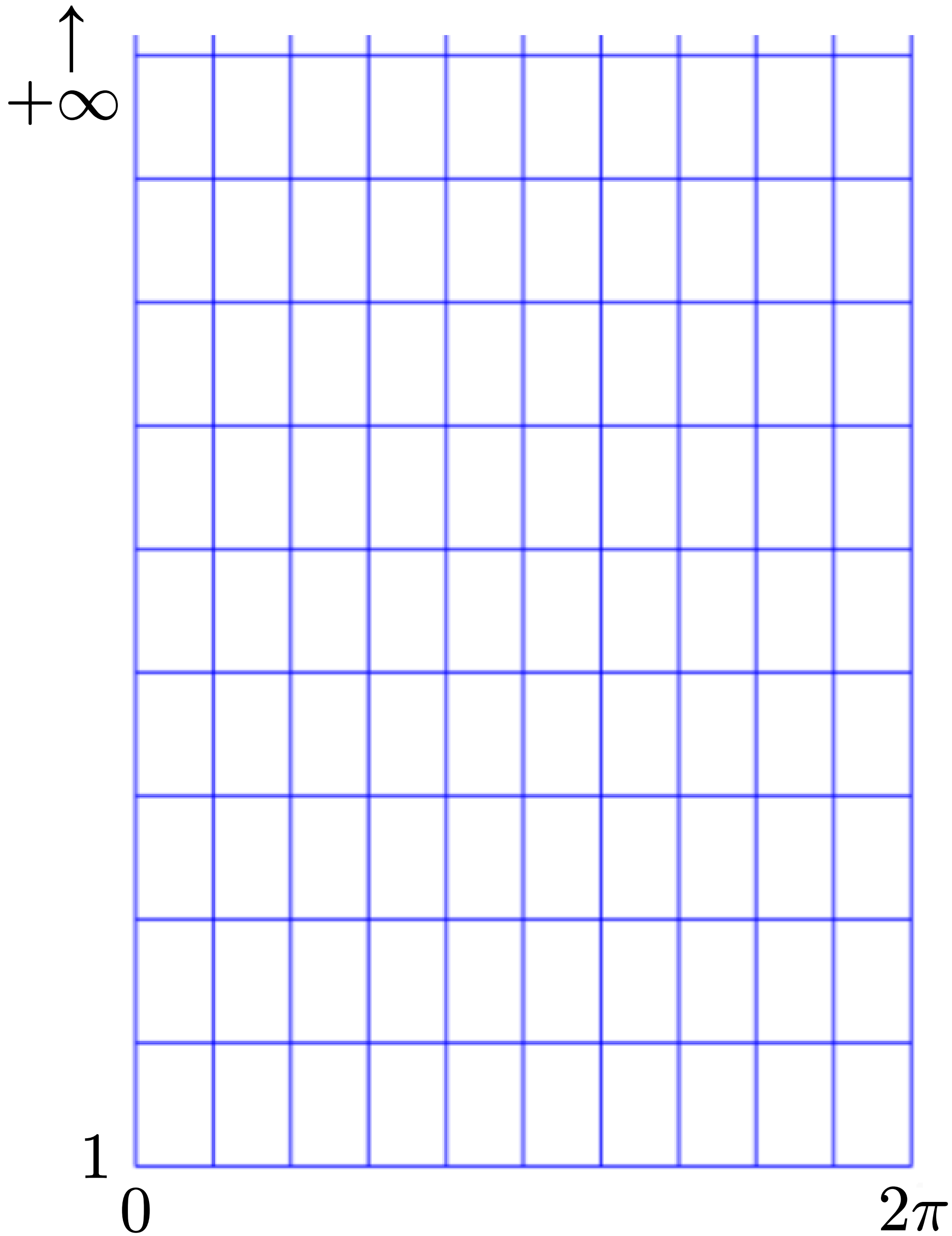
La projection :

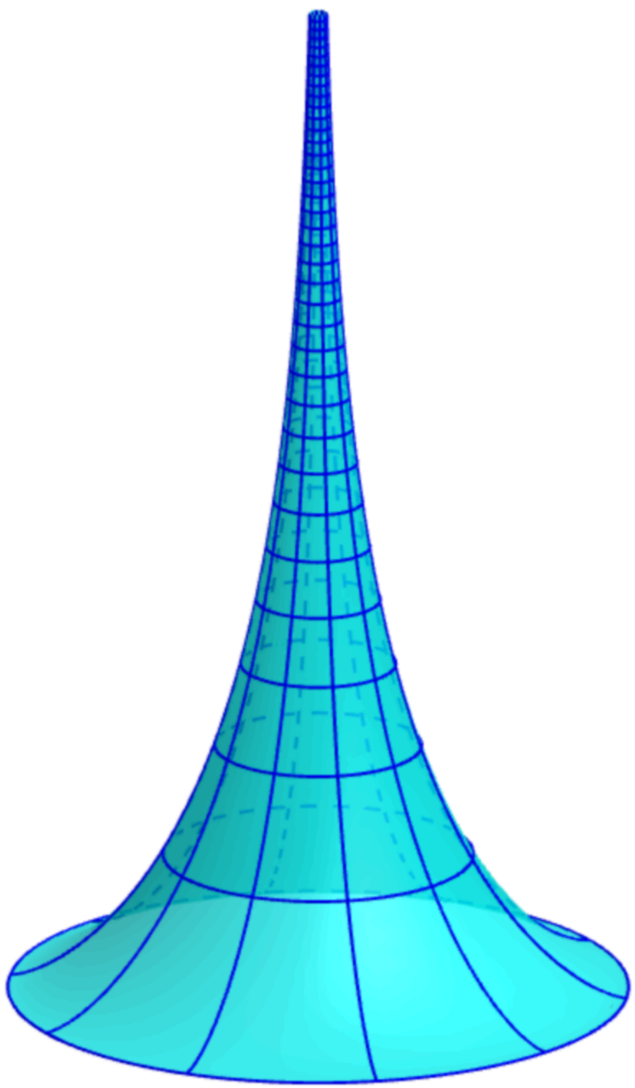
$$C : \begin{cases} [0;2\pi] \times \mathbb{R}_+ \longrightarrow \mathbb{R}^2 \\ p = (u, v) \longmapsto \begin{pmatrix} u \\ \mathrm{ch}(v) \end{pmatrix} \end{cases}$$

Celle de Mercator :

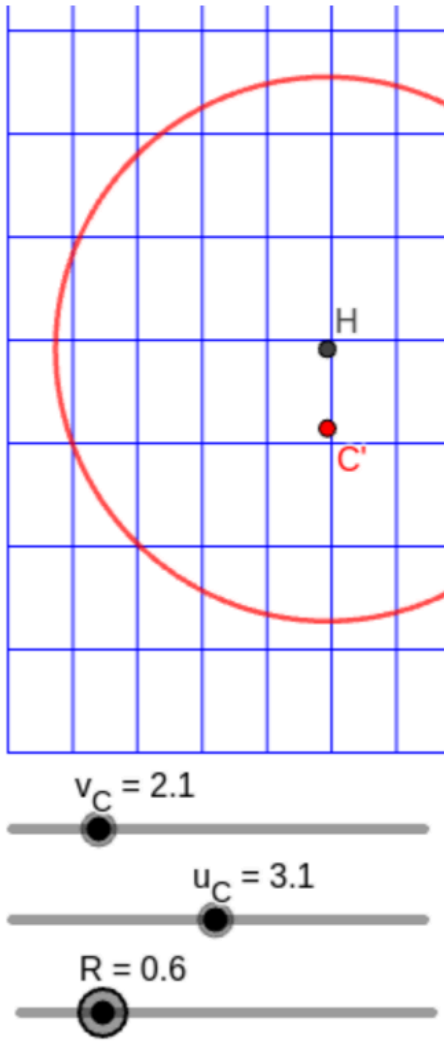
$$M : \begin{cases} [0;2\pi] \times [-\pi;\pi] \longrightarrow \mathbb{R}^2 \\ p = (u, v) \longmapsto \begin{pmatrix} u \\ \ln(\tan(\frac{v}{2}+\frac{\pi}{4})) \end{pmatrix} \end{cases}$$

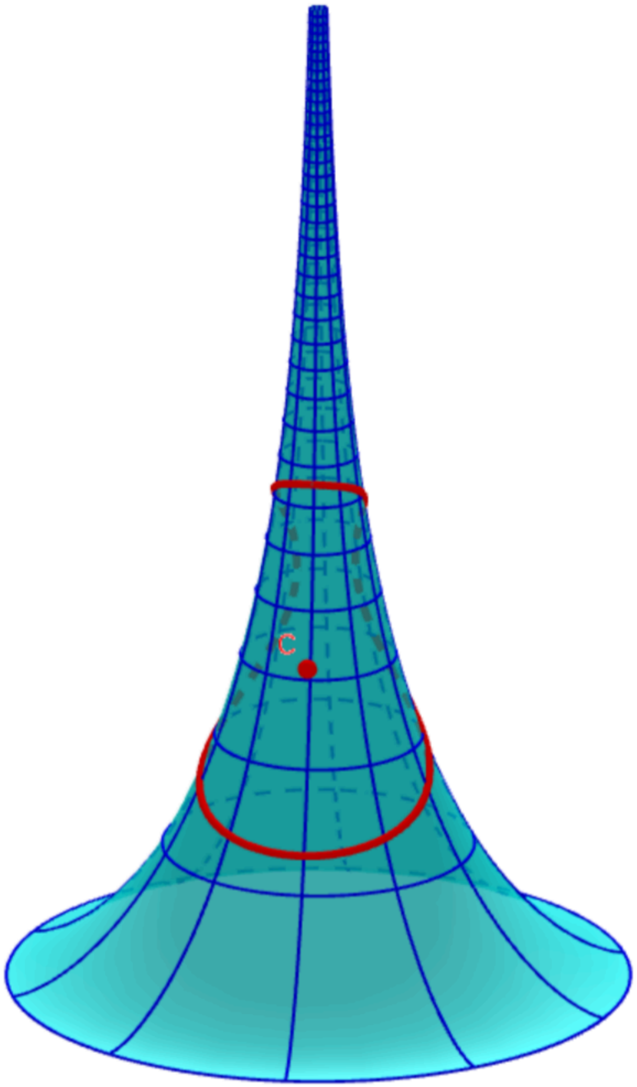
3) La projection





3) La projection





4) Projeté des droites

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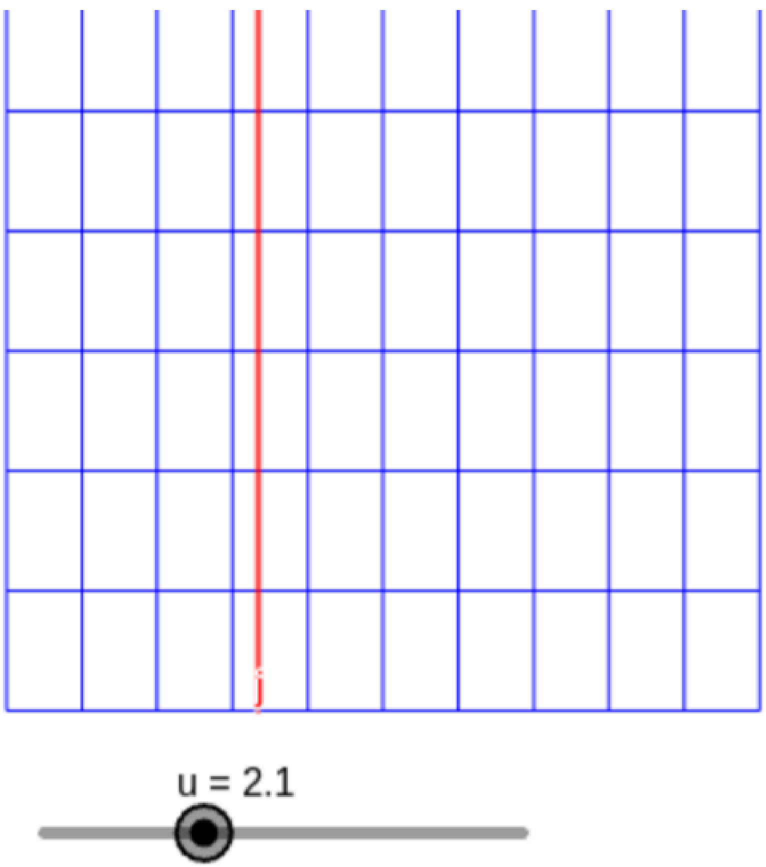
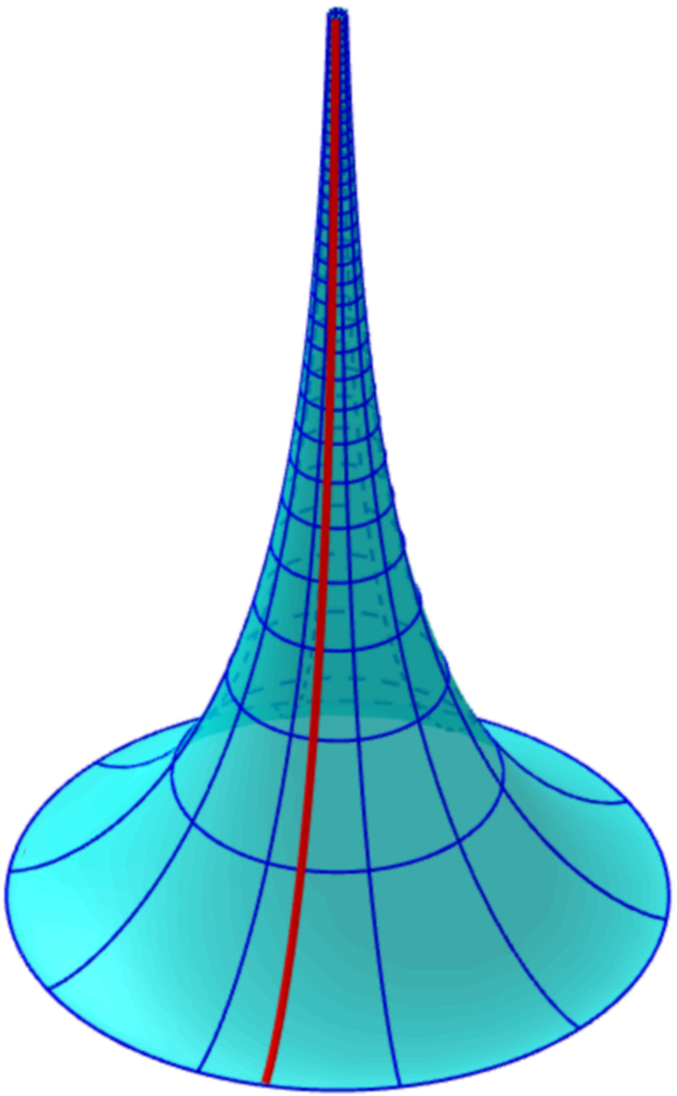
Équation des droites :

- Méridiens : $g : t \mapsto P(u, t)$
- Autres droites :

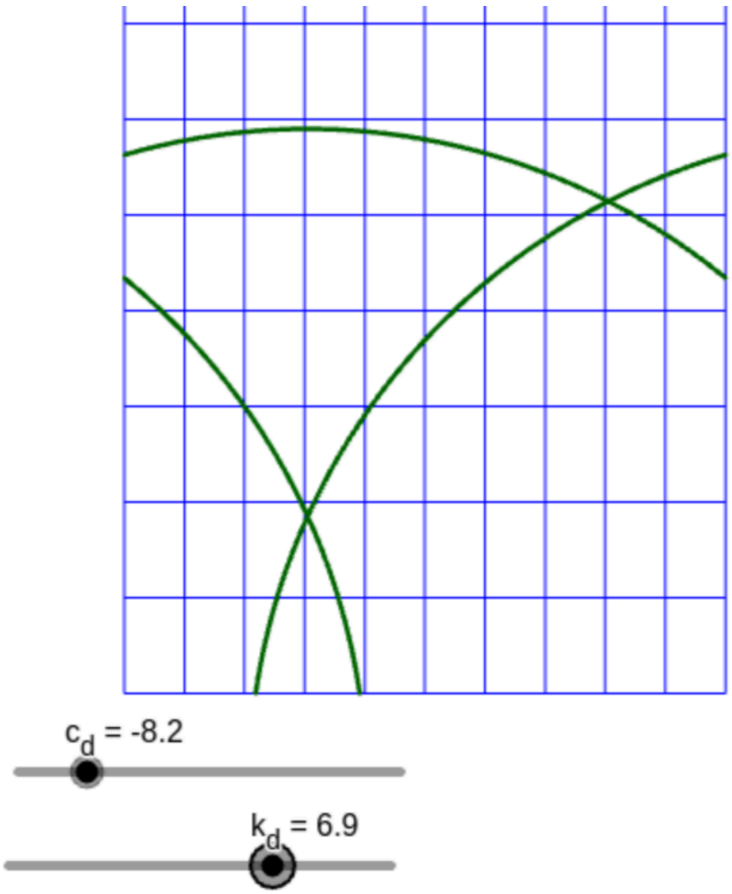
$$g : t \mapsto P\left(t, \mathrm{ch}\left(\sqrt{k^2 - (t+c)^2}\right)\right)$$

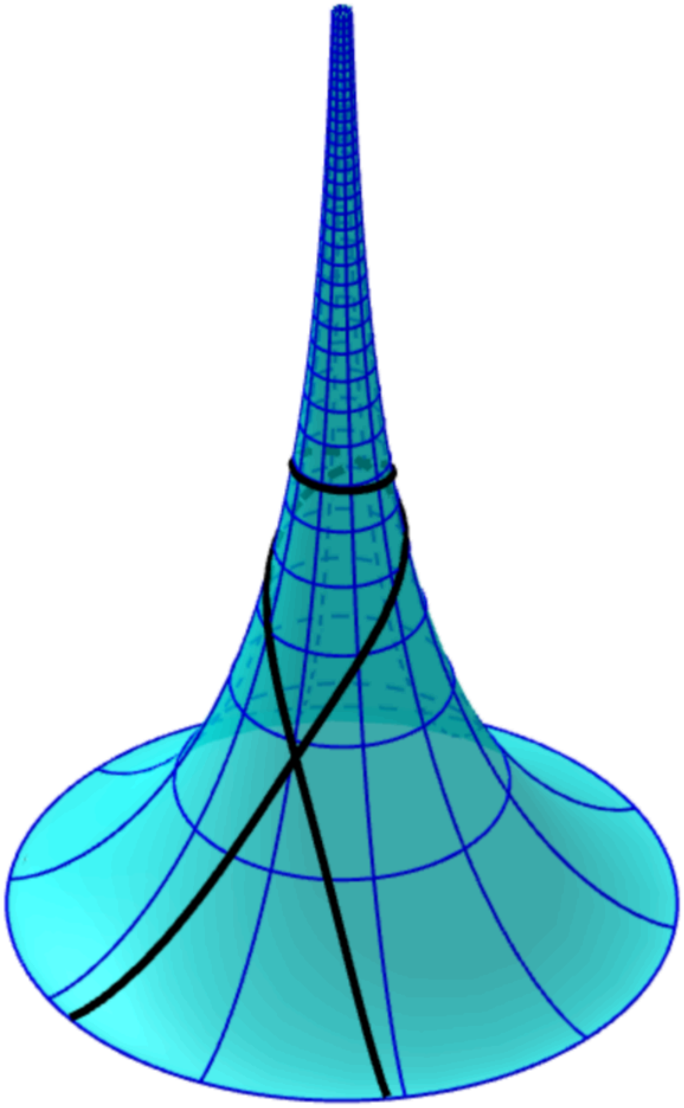
Méridiens :





Droites Quelconques :





Gaspar Daguet, n=°21528

Sur les méridiens ($v_A = v_B$) :

Sur la pseudosphère :

$$d(A,B) = \ln\left(\frac{\mathrm{ch}(u_B)}{\mathrm{ch}(u_A)}\right)$$

Longueur d'arc :

$$d(A,B) = \int_{t_A}^{t_B} \|g'(t)\| \, \mathrm{d}t$$

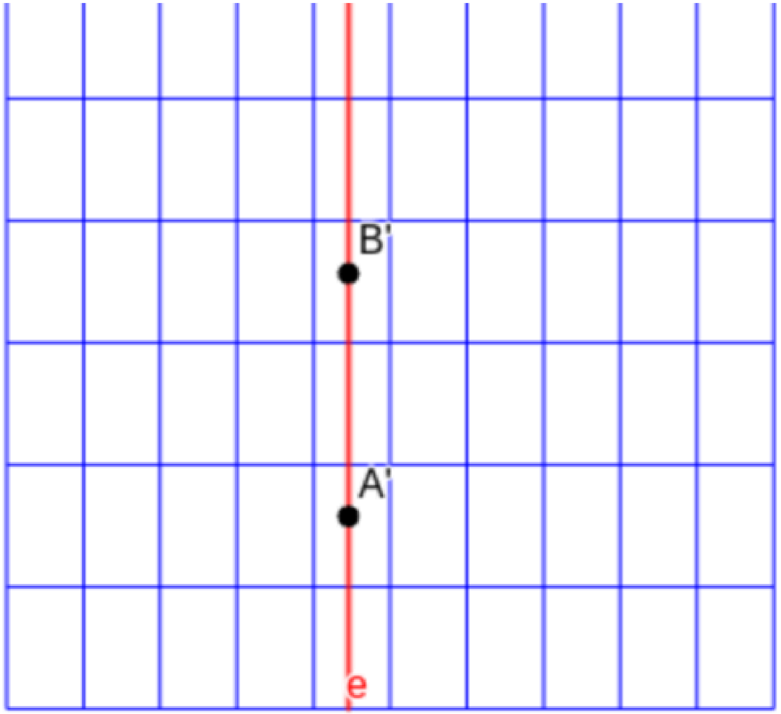
$$\neq$$

Sur la carte :

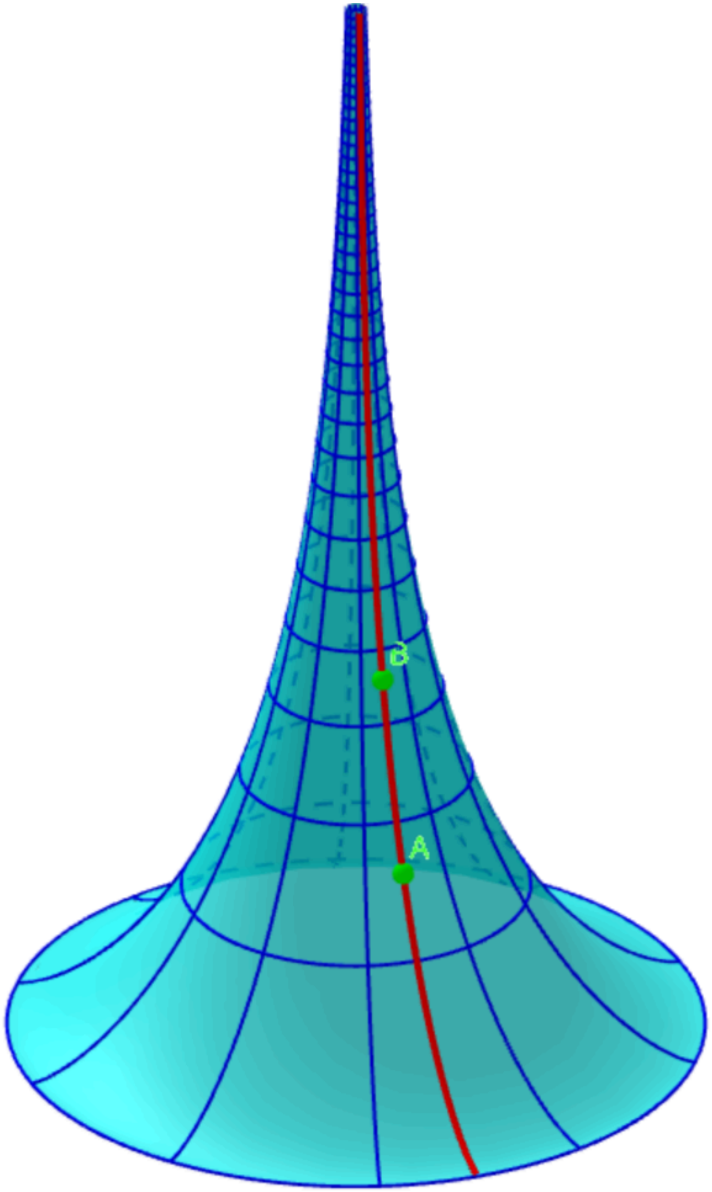
$$d(A,B) = \mathrm{ch}(u_B) - \mathrm{ch}(u_A)$$

Donc la projection n'est pas équivalente

4) projeté des droites

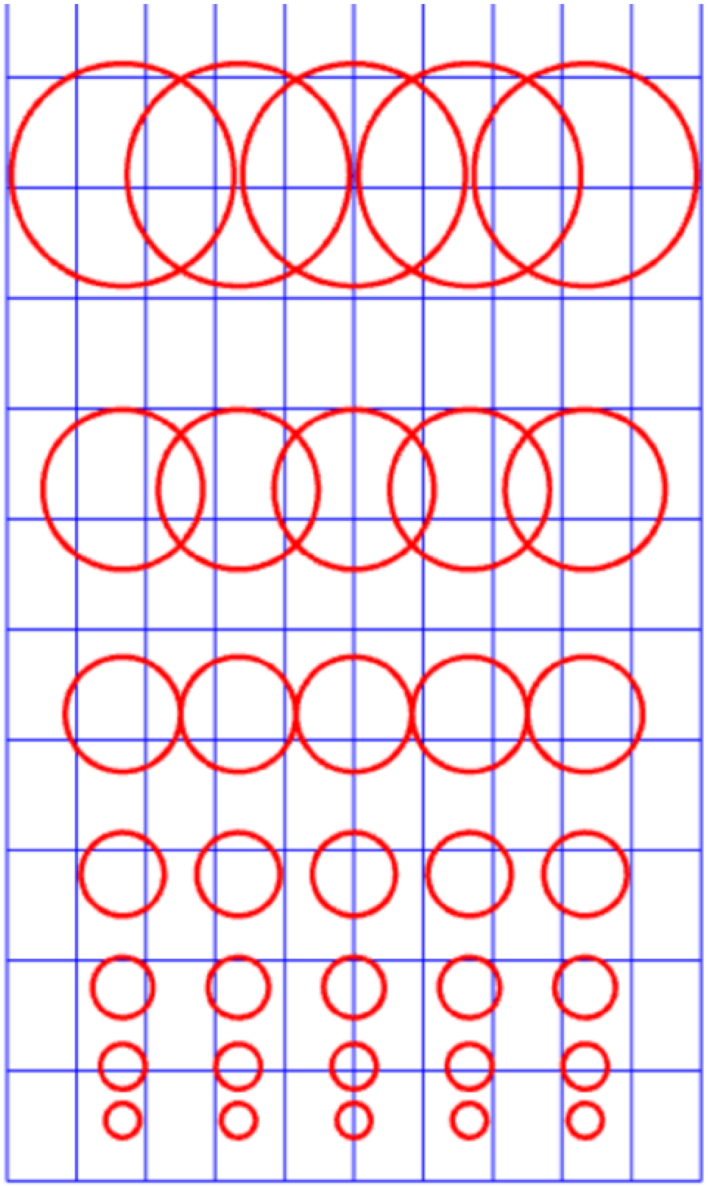


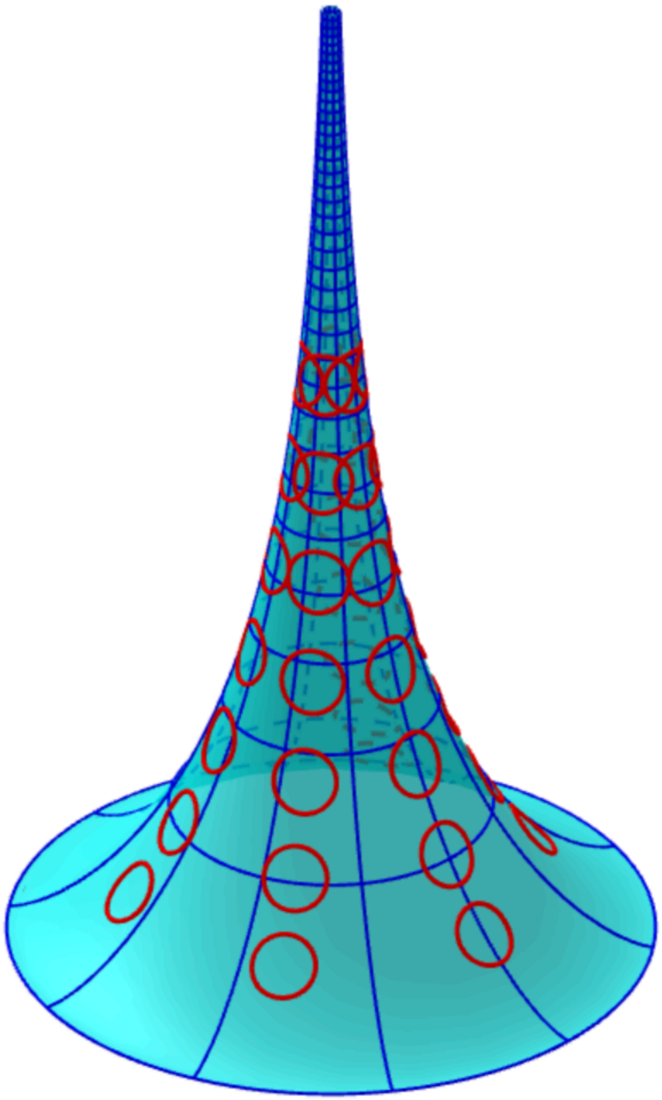
$$d(A, B) = 1,99$$

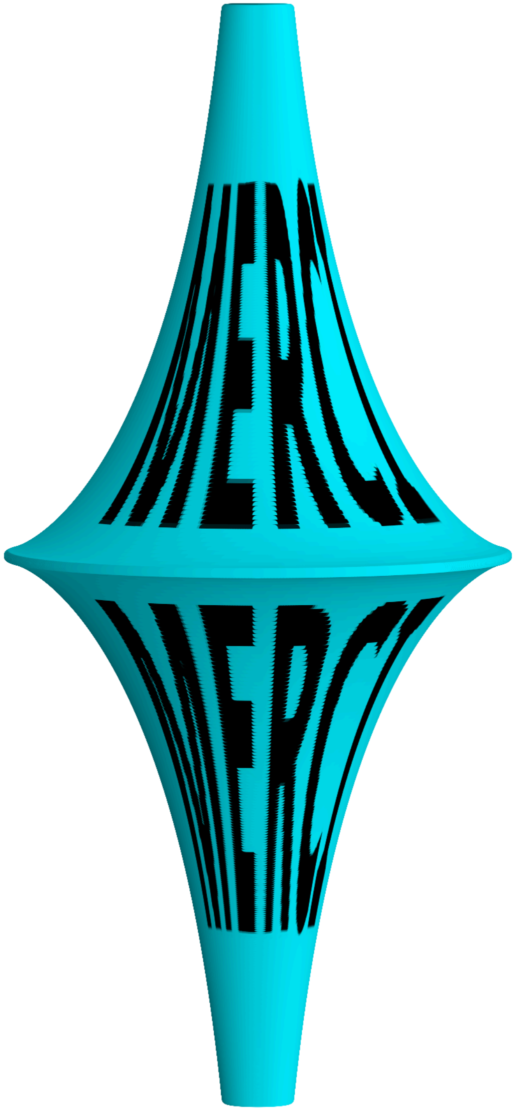


$$d(A, B) = 0,57$$

4) projeté des droites







Annexes