Def. Dimension of affine subspace:

if $X \subseteq \mathbb{R}^d$ is affine subspace, and $x \in X$

then, $X - x$ is linear subspace, called linear subspace parallel to X

$\dim(\mathrm{aff}(X)) = \dim(\mathrm{span}(X - x))$

Dimension of convex set:

for any $C \in \mathbb{R}^d$ is convex set,

$\dim(C) = \dim(\mathrm{aff}(X))$

$*$: $\dim(\emptyset) = -1$

X is full-dimensional 满维, if $\dim(X) = d$

Lemma. X affine indep,

$\dim(\mathrm{aff}(X)) = |X| - 1$

Proof. case 1. X is empty,

$\dim(\mathrm{aff}(X)) = -1$

case 2. X is nonempty, $\forall x \in X$,

$L = \mathrm{aff}(X) - x$ is linear subspace

$\therefore \dim(\mathrm{aff}(X)) = \dim(\mathrm{span}(L))$

$= |X| - 1$