

First model considered for adipocytes “growing”.

L lipides in the blood

$u(t, r)$ adipocytes number at time t of radius r

The link between r and l is the following,

$$V_l l = \frac{4}{3} \pi r^3 - V_{min}$$

with $V_{min} = \frac{4}{3} \pi r_{min}^3$ and $\frac{dl}{dt} = \frac{4\pi}{V_l} \frac{dr}{dt} r^2$.

Model the lipogenesis and the lipolysis (fluxes of triglycerides l) :

$$\frac{dl}{dt} = V_l a r^2 \frac{L}{L + \kappa_L} \frac{\kappa_r^n}{r^n + \kappa_r^n} - V_l (B + b r^2) \frac{l}{l + \kappa_t V_l}$$

we rewrite with the variable r :

$$\frac{dr}{dt} = \frac{a V_l}{4\pi} \frac{L}{L + \kappa_L} \frac{\kappa_r^n}{r^n + \kappa_r^n} - V_l \frac{(B + b r^2)}{4\pi r^2} \frac{\frac{4}{3} \pi r^3 - V_{min}}{\frac{4}{3} \pi r^3 - V_{min} + V_l \kappa_t} = \tau(r, L)$$

From these fluxes, the dynamics of the number of adipocytes u is described by

$$\partial_t u(t, r) + \partial_r (\tau(r, L) u) - D \partial_r^2 u = 0$$

with D a diffusion coefficient. The intracellular quantity of triglycerides is $U(t) = \int l \rho_u dl$ with ρ_u adipocytes density. The total amount of triglycerides is assumed constant over time : $\frac{d}{dt}(L(t) + U(t)) = 0$ and $\frac{dL}{dt} = -\frac{dU}{dt}$

boundary/initial conditions. $u(0, r) = \text{Gaussian density (minimum} = r_{min})$, first test : unimodal (Q : can we recover the bimodal distribution that is observed).

$$L(t) = L_0 + U(0) - U(t)$$

$$(\tau(r, L) u(t, r) - D \partial_r u(t, r))|_{r_{max}} = 0$$

Recruitment of new adipocytes. We assumed that when the level of triglycerides increases too largely, pre-adipocytes differentiate into adipocytes. This recruitment is modeled with the r_{min} BC, as follows,

$$(\tau(r, L) u(t, r) - D \partial_r u(t, r))|_{r_{min}} = f(L)$$

with $f(L) = \alpha L$ or $f(L) = \alpha \frac{L}{(L + \kappa)}$