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Wild Bootstrap Tests for Autocorrelation in Vector Autoregressive Models*

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Abstract

Standard asymptotic and residual-based bootstrap tests for error autocorrelation are unreliable in the presence of conditional heteroskedasticity. In this article we propose wild bootstrap tests for autocorrelation in vector autoregressive models when the errors are conditionally heteroskedastic. In particular, we investigate the properties of Lagrange multiplier tests. Monte Carlo simulations show that the wild bootstrap tests have satisfactory size properties in models with constant conditional correlation generalised autoregressive conditional heteroskedastic (CCC-GARCH) errors, whereas the standard asymptotic and residual-based bootstrap tests are oversized. The tests are applied to credit default swap prices and Euribor interest rates.

1 Introduction

It is good practice to check the adequacy of an estimated time series model by testing for various types of misspecification such as error autocorrelation and autoregressive conditional heteroskedastic (ARCH) errors. Standard tests for error autocorrelation are derived under the assumption of independent and identically distributed (IID) errors, and are unreliable in the presence of conditional heteroskedasticity. Bera et al. (1992) show that the expression for the information matrix in Lagrange multiplier (LM) tests for error autocorrelation depends on the ARCH parameters. Consequently, standard LM tests for autocorrelation will be misleading if the presence of ARCH is neglected. They derive LM tests for autocorrelation based on transformed residuals obtained by dividing the residuals by an estimate of the conditional standard deviation. However, this approach requires the form of the conditional heteroskedasticity to be known. Typically, a GARCH process is assumed, but there is no guarantee that it provides an adequate description of the conditional heteroskedasticity in the data. Further difficulties arise in the multivariate case because it may be difficult to obtain reliable estimates of the GARCH parameters (Gonçalves and Kilian, 2004).

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Conditional heteroskedasticity in the residuals of time series models estimated on macroeconomic and financial data at the monthly, weekly and daily frequencies is so pervasive that it is almost a stylised fact (see e.g. Gonçalves and Kilian , 2004; and Sensier and van Dijk, 2004). Thus there is a need for tests for error autocorrelation that are robust to the presence of conditional heteroskedasticity of unknown form. The theoretical properties of wild bootstrap procedures for autoregressions with conditional heteroskedasticity are investigated by Gonçalves and Kilian. Godfrey and Tremayne (2005) report simulation results on wild bootstrap tests for autocorrelation in dynamic regression models. A treatment of autocorrelation tests in vector autoregressive models when the errors are conditionally heteroskedastic does not seem to be available, however.

In this article we propose wild bootstrap tests for autocorrelation in vector autoregressive (VAR) models when the errors are conditionally heteroskedastic. We use a residual-based recursive wild bootstrap procedure, which extends the results of Gonçalves and Kilian to the multivariate case. In particular, we investigate the properties of Lagrange multiplier (LM) tests for error autocorrelation. Monte Carlo simulations show that the wild bootstrap tests have satisfactory size properties in models with constant conditional correlation generalised autoregressive conditional heteroskedastic (CCC-GARCH) errors. In contrast, standard asymptotic and residual-based bootstrap tests are shown to be oversized. Some simulation evidence on the power of the wild bootstrap tests is given. The tests are applied to credit default swap prices and Euribor interest rates. The results show that there are significant ARCH effects in the residuals of the estimated VAR models. The empirical examples demonstrate that the wild bootstrap tests should be used in place of the standard asymptotic and residual-based bootstrap tests based on the IID error assumption.

The remainder of the paper is organised as follows. In Section 2 LM tests for error autocorrelation are reviewed. In Section 3 we describe the residual-based recursive wild bootstrap algorithm. Section 4 presents simulation evidence on the size and power of the tests. Section 5 contains empirical applications to credit default swap prices and Euribor interest rates. Section 6 concludes.

2 Tests for Error Autocorrelation in Vector Autoregressive Models

Let $\mathbf{X}_t$ be a K-dimensional vector of time series variables assumed to be integrated of order zero or one, denoted $I(0)$ or $I(1)$.

We begin by considering the case where $\mathbf{X}_t$ is $I(0)$. The vector $\mathbf{X}_t$ is assumed to be generated by a vector autoregressive (VAR) model of order p:

$$\mathbf{X}_t = \boldsymbol{\pi} + \mathbf{\Pi}_1 \mathbf{X}_{t-1} + \cdots + \mathbf{\Pi}_p \mathbf{X}_{t-p} + \boldsymbol{\varepsilon}_t, \qquad t = 1, \ldots, T. \tag{1}$$

Here $\mathbf{\Pi}_1, \ldots, \mathbf{\Pi}_p$ are $(K \times K)$ parameter matrices and $\boldsymbol{\pi}$ is a $(K \times 1)$ vector of constants. The error process $\{\boldsymbol{\varepsilon}_t\}$ is assumed to be IID with mean zero, and nonsingular and positive definite covariance matrix $\mathbf{\Omega}$.

Lagrange multiplier (LM) tests for error autocorrelation (Godfrey, 1978, 1991; and Breusch, 1978) are widely used in econometrics. These tests are commonly referred to as Breusch–Godfrey (BG) tests. In the tests a VAR(h) model is considered for the error terms:

$$\boldsymbol{\varepsilon}_t = \boldsymbol{\Psi}_1\boldsymbol{\varepsilon}_{t-1} + \cdots + \boldsymbol{\Psi}_h\boldsymbol{\varepsilon}_{t-h} + \mathbf{e}_t. \tag{2}$$

The hypothesis being tested is

$$H_0 : \boldsymbol{\Psi}_1 = \cdots = \boldsymbol{\Psi}_h = \mathbf{0} \qquad \text{against} \qquad H_1 : \boldsymbol{\Psi}_j \neq \mathbf{0} \text{ for at least one } j,\ 1 \leq j \leq h.$$

The test is implemented by first estimating a VAR model for $\mathbf{X}_t$, and obtaining the residuals $\widehat{\boldsymbol{\varepsilon}}_t$. Then an auxiliary regression of the residuals is run on all the lags of $\mathbf{X}_t$ in the VAR model and the lagged residuals up to order h:

$$\begin{aligned} \widehat{\boldsymbol{\varepsilon}}_t &= \boldsymbol{\pi} + \boldsymbol{\Pi}_1\mathbf{X}_{t-1} + \cdots + \boldsymbol{\Pi}_p\mathbf{X}_{t-p} + \boldsymbol{\Psi}_1\widehat{\boldsymbol{\varepsilon}}_{t-1} + \cdots + \boldsymbol{\Psi}_h\widehat{\boldsymbol{\varepsilon}}_{t-h} + \mathbf{e}_t \\ &= \boldsymbol{\pi} + (\mathbf{Z}_t' \otimes \mathbf{I}_K)\boldsymbol{\phi} + (\widehat{\mathbf{E}}_t' \otimes \mathbf{I}_K)\boldsymbol{\psi} + \mathbf{e}_t, \end{aligned} \tag{3}$$

where $\mathbf{Z}_t' = (\mathbf{X}_{t-1}', \ldots, \mathbf{X}_{t-p}')$, $\boldsymbol{\phi} = \text{vec}(\boldsymbol{\Pi}_1, \ldots, \boldsymbol{\Pi}_p)'$, $\widehat{\mathbf{E}}_t' = (\widehat{\boldsymbol{\varepsilon}}_{t-1}', \ldots, \widehat{\boldsymbol{\varepsilon}}_{t-h}')$ and $\boldsymbol{\psi} = \text{vec}(\boldsymbol{\Psi}_1, \ldots, \boldsymbol{\Psi}_h)'$. Here the symbol $\otimes$ denotes the Kronecker product and the symbol vec denotes the column vectorisation operator. The first h values of the residuals are set to zero in the auxiliary model, so that the series length is equal to the series length in the original VAR model.

The test statistic is a standard LM statistic given by

$$Q_{\text{LM}} = T\widehat{\boldsymbol{\psi}}'(\widehat{\boldsymbol{\Sigma}}^{\psi\psi})^{-1}\widehat{\boldsymbol{\psi}}, \tag{4}$$

where $\widehat{\boldsymbol{\psi}}$ is the generalised least squares (GLS) estimate of $\boldsymbol{\psi}$ and $\widehat{\boldsymbol{\Sigma}}^{\psi\psi}$ is the block of

$$\widehat{\boldsymbol{\Omega}}_{\mathbf{e}}^{-1} = \left(T^{-1} \sum_{t=1}^{T} \begin{bmatrix} \widehat{\mathbf{E}}_t \otimes \mathbf{I}_K \\ \mathbf{Z}_t \otimes \mathbf{I}_K \end{bmatrix} \widehat{\boldsymbol{\Omega}}^{-1} \begin{bmatrix} \widehat{\mathbf{E}}_t' \otimes \mathbf{I}_K & \mathbf{Z}_t' \otimes \mathbf{I}_K \end{bmatrix} \right)^{-1}$$

corresponding to $\boldsymbol{\psi}$.

Bewley (1986) shows that the LM statistic can alternatively be written as

$$Q_{\text{LM}} = T\left(K - \text{tr}(\widehat{\boldsymbol{\Omega}}^{-1}\widehat{\boldsymbol{\Omega}}_e)\right), \tag{5}$$

where $\widehat{\boldsymbol{\Omega}} = T^{-1}\sum_{t=1}^{T}\widehat{\boldsymbol{\varepsilon}}_t\widehat{\boldsymbol{\varepsilon}}_t'$ is the estimator of the error covariance matrix from the VAR model and $\widehat{\boldsymbol{\Omega}}_e$ is the estimator of the error covariance matrix from the auxiliary regression. If the null hypothesis is correct, the two covariance matrix estimators have the same probability limit. The test statistic is asymptotically distributed as a χ^2 random variable with hK^2 degrees of freedom under the null hypothesis. The test reduces to the single equation LM test when $K = 1$.

There are asymptotically equivalent likelihood ratio (LR) and Wald versions of the test statistic, which are given by

$$Q_{\text{LR}} = T\left(\log\det\widehat{\boldsymbol{\Omega}} - \log\det\widehat{\boldsymbol{\Omega}}_e\right) \tag{6}$$

and

$$Q_{\text{W}} = T\left(\text{tr}(\widehat{\boldsymbol{\Omega}}_e^{-1}\widehat{\boldsymbol{\Omega}}) - K\right), \tag{7}$$

respectively.

An alternative to the χ^2 tests is the F approximation to the LR test of Rao (1973, Section 8c.5), given by

$$Q_{\mathrm{F}} = \left[\left(\frac{\det \widehat{\mathbf{\Omega}}}{\det \widehat{\mathbf{\Omega}}_e} \right)^{1/s} - 1 \right] \frac{Ns - \frac{1}{2}K^2 h + 1}{K^2 h}, \tag{8}$$

where

$$s = \left(\frac{K^4 h^2 - 4}{K^2 - K^2 h^2 - 5} \right)^{1/2} \qquad \text{and} \qquad N = T - Kp - 1 - Kh - \frac{1}{2}(K - Kh + 1).$$

The F statistic was proposed by Doornik (1996) in the context of testing for error autocorrelation. The Q_{F} statistic is approximately distributed as $F(hK^2, Ns - \frac{1}{2}K^2 h + 1)$ under the null hypothesis.

We now briefly turn to the case where $\mathbf{X}_t$ is $I(1)$. Following Brüggemann et al. (2006), we consider the cointergated VAR model with $r < K$ cointegrating relations. The model can be written in vector error correction model (VECM) form as

$$\Delta \mathbf{X}_t = \boldsymbol{\pi} + \boldsymbol{\alpha}\boldsymbol{\beta}' \mathbf{X}_{t-1} + \mathbf{\Gamma}_1 \Delta \mathbf{X}_{t-1} + \cdots + \mathbf{\Gamma}_{p-1} \Delta \mathbf{X}_{t-p+1} + \boldsymbol{\varepsilon}_t, \tag{9}$$

where $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are $(K \times r)$ matrices with rank r, and $\mathbf{\Gamma}_1, \ldots, \mathbf{\Gamma}_{p-1}$ are $(K \times K)$ parameter matrices. Brüggemann et al. show that the LM statistic may be calculated from the auxiliary model

$$\begin{aligned} \widehat{\boldsymbol{\varepsilon}}_t &= \boldsymbol{\pi} + \boldsymbol{\alpha}\widehat{\boldsymbol{\beta}}' \mathbf{X}_{t-1} + \mathbf{\Gamma}_1 \Delta \mathbf{X}_{t-1} + \cdots + \mathbf{\Gamma}_{p-1} \Delta \mathbf{X}_{t-p+1} & (10) \\ &\quad + (\mathbf{X}'_{t-1} \widehat{\boldsymbol{\beta}}_{\perp} \otimes \widehat{\boldsymbol{\alpha}}) \boldsymbol{\phi}_1 + \mathbf{\Psi}_1 \widehat{\boldsymbol{\varepsilon}}_{t-1} + \cdots + \mathbf{\Psi}_h \widehat{\boldsymbol{\varepsilon}}_{t-h} + \mathbf{e}_t \\ &= \boldsymbol{\pi} + (\widehat{\mathbf{Z}}'_t \otimes \mathbf{I}_K) \boldsymbol{\phi} + \widehat{\mathbf{Z}}'_{1t} \boldsymbol{\phi}_1 + (\widehat{\mathbf{E}}'_t \otimes \mathbf{I}_K) \boldsymbol{\psi} + \mathbf{e}_t, & (11) \end{aligned}$$

where $\widehat{\mathbf{Z}}'_t = ((\widehat{\boldsymbol{\beta}}' \mathbf{X}_{t-1})', \Delta \mathbf{X}'_{t-1}, \ldots, \Delta \mathbf{X}'_{t-p+1})$, $\boldsymbol{\phi} = \mathrm{vec}(\boldsymbol{\alpha}, \mathbf{\Gamma}_1, \ldots, \mathbf{\Gamma}_{p-1})$ and $\widehat{\mathbf{Z}}'_{1t} = \mathbf{X}'_{t-1} \widehat{\boldsymbol{\beta}}_{\perp} \otimes \widehat{\boldsymbol{\alpha}}$. The terms $\mathbf{X}'_{t-1} \widehat{\boldsymbol{\beta}}_{\perp} \otimes \widehat{\boldsymbol{\alpha}}$ may, in fact, be deleted from the auxiliary model because the estimator of $\boldsymbol{\beta}$ is asymptotically independent of the estimators of $\boldsymbol{\phi}$ and $\boldsymbol{\psi}$ (Brüggemann et al., 2006, Remark 1). The limiting distribution of the LM statistic under the null hypothesis is the same χ^2 distribution as in the $I(0)$ case.

We mention that the unrestricted VAR model (1) may be used when $\mathbf{X}_t$ is $I(1)$. If the cointegration rank is unknown, an unrestricted VAR model is first estimated, and the cointegration rank is determined. We may then proceed as in the $I(0)$ case to test for error autocorrelation (see Brüggemann et al., 2006).

The small sample properties of the tests are examined by Edgerton and Shukur (1999) in the stationary VAR model, and Brüggemann et al. in the cointegrated VAR model. Edgerton and Shukur find that the tests have satisfactory size properties only when K and h are small relative to the number of observations. The LM test is preferable to the LR and Wald versions of the test. The F approximation to the LR test is found to have better size properties than the asymptotic χ^2 tests when the dimensions are large. Brüggemann et al. obtain similar results when the tests are applied to the cointegrated VAR model. If the number of parameters is large relative to the number of observations it is desirable to bootstrap the tests (Davidson and MacKinnon, 2004, Section 13.7).

Godfrey and Tremayne (1995) consider heteroskedasticity-robust versions of the LM test. They employ heteroskedasticity-consistent covariance matrix estimators (HCCME). Notice that the HCCME requires all series to be $I(0)$. In this paper we will not consider unconditional heteroskedasticity but focus on conditional heteroskedasticity.

The tests most commonly used in practice are the Q_{LM} and Q_{F} tests, which are provided in e.g. PcGive and Eviews (in the latter only the Q_{LM} test is available). Other tests for error autocorrelation not considered here include Portmanteau tests (see e.g. Lütkepohl, 2006, Section 4.4) as well as some modifications to the LM tests (see Edgerton and Shukur, 1999).

3 Bootstrap and Wild Bootstrap Tests for Error Autocorrelation

For the bootstrap and wild bootstrap (WB) tests for error autocorrelation we use a non-parametric resampling procedure in which the residuals are resampled. The standard residual-based bootstrap assumes that the errors are IID. The bootstrap tests for error autocorrelation are therefore not valid if the errors are conditionally heteroskedastic. The wild bootstrap is a resampling method for dealing with conditional heteroskedasticity of unknown form (see e.g. Wu, 1986; Beran, 1986; Liu, 1988; and Mammen, 1993). The WB replicates the pattern of conditional heteroskedasticity in the errors.

We use the residual-based recursive wild bootstrap procedure of Gonçalves and Kilian (2004), which is valid for finite-order autoregressions. It is a modification of the recursive-design bootstrap method for autoregressions, which replaces the IID bootstrap by the WB when bootstrapping the residuals of the VAR model. In the WB the bootstrap samples are not constructed from the resampled residuals. Instead the residuals are multiplied by a random draw from an auxiliary distribution. To be specific, the bootstrap errors are generated as $\boldsymbol{\varepsilon}_t^* = w_t\widehat{\boldsymbol{\varepsilon}}_t$, where $\widehat{\boldsymbol{\varepsilon}}_t$ are the residuals from the estimated VAR model, and w_t is an IID sequence with mean zero, variance one and such that $\mathrm{E}|w_t|^4 \leq c < \infty$ (Gonçalves and Kilian, 2004).

Several auxiliary distributions may be considered (see e.g Davidson and Flaichaire, 2008; and Davidson et al., 2007). The Rademacher distribution is given by the simple two-point distribution

$$w_t = \begin{cases} 1, & \text{with probability } \frac{1}{2} \\ -1, & \text{with probability } \frac{1}{2} \end{cases} . \tag{12}$$

It is one of the most commonly used distributions, since it is easy to implement. Davidson and Flachaire recommend it for use in practice.

The asymptotic validity of the WB under conditional heteroskedasticity of unknown form was established by Gonçalves and Kilian. We assume the following conditions adapted from Gonçalves and Kilian to the multivariate case of conditional heteroskedasticity:

(i) $\mathrm{E}(\boldsymbol{\varepsilon}_t|\mathcal{F}_{t-1}) = \mathbf{0}$, almost surely, where $\mathcal{F}_{t-1} = \sigma(\boldsymbol{\varepsilon}_{t-1}, \boldsymbol{\varepsilon}_{t-2}, \ldots)$ is the σ-field generated by $\{\boldsymbol{\varepsilon}_{t-1}, \boldsymbol{\varepsilon}_{t-2}, \ldots\}$.

(ii) $\mathrm{E}(\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t') = \boldsymbol{\Omega} < \infty$ and positive definite.

(iii) $\lim_{n\to\infty} n^{-1}\sum_{t=1}^{n}\mathrm{E}(\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t'|\mathcal{F}_{t-1}) = \boldsymbol{\Omega} > \mathbf{0}$ in probability.

(iv) $\mathrm{E}(\varepsilon_{it}^2 \varepsilon_{j,t-r} \varepsilon_{k,t-s}) = 0$ for all $r \neq s$, for all i, j, k, and all t, $r \geq 1$, $s \geq 1$.

(v) $\lim_{n\to\infty} n^{-1} \sum_{t=1}^{n} \varepsilon_{j,t-r} \varepsilon_{k,t-s} \mathrm{E}(\varepsilon_{it}^2 | \mathcal{F}_{t-1}) = \sigma_i^2 \sigma_j \sigma_k \tau_{r,s,ijk}$, in probability for any $r \geq 1$, $s \geq 1$, where $\tau_{r,s,ijk} = \sigma_i^{-2} \sigma_j^{-1} \sigma_k^{-1} \mathrm{E}(\varepsilon_{it}^2 \varepsilon_{j,t-r} \varepsilon_{k,t-s})$.

(vi) $\mathrm{E}|\varepsilon_{it}|^{4r}$ is uniformly bounded for some $r \geq 2$ and for all t.

The algorithm for the bootstrap and wild bootstrap LM tests for error autocorrelation is given below. We use a '$*$' as a generic notation for bootstrap and WB quantities. The same algorithm is used with the LR, W and F tests.

Algorithm 1 *Bootstrap and wild bootstrap LM tests for error autocorrelation.*

1. Calculate the Q_{LM} statistic from the data, and obtain the parameter estimates and the residuals $\widehat{\boldsymbol{\varepsilon}}_t$ from the VAR model.

2.

   (a) Construct the bootstrap errors $\boldsymbol{\varepsilon}_t^*$ by drawing independently with replacement a sample of size T from $\{\widehat{\boldsymbol{\varepsilon}}_t\}_1^T$
   or

   (b) Draw w_t, $t = 1, ..., T$, independently from a Rademacher distribution (12), and construct the WB errors as $\boldsymbol{\varepsilon}_t^* = w_t \widehat{\boldsymbol{\varepsilon}}_t$.

3. Generate recursively a bootstrap sample $\mathbf{X}_t^*$ using the parameter estimates and the bootstrap errors.

4. Calculate Q_{LM}^* from the bootstrap sample.

5. Repeat steps 3–4 B times to obtain $Q_{\mathrm{LM}_1}^*, \ldots, Q_{\mathrm{LM}_B}^*$.

6. Estimate the bootstrap p-value by

$$\widehat{p}^* = \frac{1}{B} \sum_{i=1}^{B} I(Q_{\mathrm{LM}_i}^* > Q_{\mathrm{LM}}), \tag{13}$$

   where $I(\cdot)$ is the indicator function.

The null hypothesis of no error autocorrelation is rejected at the significance level α if $\widehat{p}^* < \alpha$.

4 Simulations

We investigate the performance of the tests for error autocorrelation when the errors are conditionally heteroskedastic by Monte Carlo simulation experiments. The tests considered are the four versions of the BG test, namely the Q_{LM}, Q_{LR}, Q_{W} and Q_{F} tests.

4.1 Monte Carlo Design

The model for the conditional mean is a stationary VAR(1) model. Data are generated from the following data-generation process (DGP):

$$\mathbf{X}_t = \mathbf{\Pi}_1 \mathbf{X}_{t-1} + \boldsymbol{\varepsilon}_t, \qquad t = 1, \ldots T. \tag{14}$$

The autoregressive structure of the conditional mean is specified as $\mathbf{\Pi}_1 = \phi_1 \mathbf{I}_K$, with $\phi_1 = 0.8$. The dimensions of the system are $K = 2, 5$. We also experimented with a VECM (9) similar to the DGP used in Brüggemann et al. (2006), with cointegration rank $r = 1$ when $K = 2$, and $r = 2$ when $K = 5$. Our simulation results are qualitatively the same for the cointegrated VAR model. Consequently, results are only reported for the stationary VAR model. (The Monte Carlo results for the cointegrated VAR model are available on request.) The cointegrated VAR model is used in the empirical illustrations in Section 5, however.

The model for the errors is a constant conditional correlation generalised autoregressive conditional heteroskedasticity (CCC-GARCH) model (Bollerslev, 1990):

$$\boldsymbol{\varepsilon}_t = \mathbf{D}_t \mathbf{z}_t, \tag{15}$$

where $\mathbf{D}_t = \mathrm{diag}(h_{1t}^{1/2}, \ldots, h_{Kt}^{1/2})$ is a diagonal matrix of conditional standard deviations of $\boldsymbol{\varepsilon}_t$. The sequence $\{\mathbf{z}_t\}$ with the stochastic vector $\mathbf{z}_t = (z_{1t}, \ldots z_{Kt})'$ is a sequence of independent and identically distributed (IID) random variables with mean $\mathbf{0}$ and positive definite covariance matrix $\mathbf{R} = (\rho_{ij})$.

The CCC-GARCH$(1, 1)$ process of $\boldsymbol{\varepsilon}_t$ is defined as follows:

$$\mathbf{h}_t = \mathbf{a}_0 + \mathbf{A}_1 \boldsymbol{\varepsilon}_{t-1}^{(2)} + \mathbf{B}_1 \mathbf{h}_{t-1}, \tag{16}$$

where $\boldsymbol{\varepsilon}_t^{(2)} = (\varepsilon_{1t}^2, \ldots, \varepsilon_{Kt}^2)'$, $\mathbf{h}_t = (h_{1t}, \ldots, h_{Kt})'$ is a $(K \times 1)$ vector of conditional variances of $\boldsymbol{\varepsilon}_t$, $\mathbf{a}_0$ is a $(K \times 1)$ vector of positive constants, and $\mathbf{A}_1$ and $\mathbf{B}_1$ are $(K \times K)$ parameter matrices which are diagonal with positive diagonal elements. Jeantheau (1998) proposes an extended CCC-GARCH model where some of the off-diagonal elements have non-zero values, but this extension is not considered here (see also He and Teräsvirta, 2004).

The simulations are carried out for four different CCC-GARCH$(1, 1)$ DGPs. In DGP 1, $\mathbf{A}_1 = \mathbf{B}_1 = \mathbf{0}$, and then $\boldsymbol{\varepsilon}_t = \mathbf{z}_t$, $\mathbf{z}_t \sim \mathrm{NID}(\mathbf{0}, \mathbf{I}_K)$. DGP 2 has

$$\mathbf{A}_1 = \begin{pmatrix} 0.5 & 0 \\ 0 & 0.5 \end{pmatrix}, \qquad \mathbf{B}_1 = \mathbf{0} \qquad \text{and} \qquad \mathbf{R} = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}, \tag{17}$$

and is characterised by low persistence in the volatility ($a_{ii} = 0.5$). DGP 3 has

$$\mathbf{A}_1 = \begin{pmatrix} 0.8 & 0 \\ 0 & 0.8 \end{pmatrix}, \qquad \mathbf{B}_1 = \mathbf{0} \qquad \text{and} \qquad \mathbf{R} = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}, \tag{18}$$

and is characterised by moderate persistence in the volatility ($a_{ii} = 0.8$). DGP 4 has

$$\mathbf{A}_1 = \begin{pmatrix} 0.08 & 0 \\ 0 & 0.08 \end{pmatrix}, \qquad \mathbf{B}_1 = \begin{pmatrix} 0.9 & 0 \\ 0 & 0.9 \end{pmatrix} \qquad \text{and} \qquad \mathbf{R} = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}, \tag{19}$$

and is characterised by very high persistence in the volatility ($a_{ii} + b_{ii} = 0.98$). Another difference between DGPs 2 and 3 on the one hand, and DGP 4 on the other is that in the former the coefficient on $\varepsilon_{i,t-1}^2$ is large ($a_{ii} = 0.5$ and $a_{ii} = 0.8$, respectively), whereas in the latter the coefficients are spread over an infinite number of lags of ε_{it}^2, as can be seen from the ARCH(∞) representation. DGP 3 is the multivariate generalisation of the DGP used by Godfrey and Tremayne (2005). In the simulations the constant is set to $\mathbf{a}_0 = (0.15, 0, 15)'$. The values for ρ are $\rho = 0, 0.3$ and 0.9, which are the ones used by Nakatani and Teräsvirta (2009). Finally, when $K > 2$ the vector $\mathbf{a}_0$, and the matrices $\mathbf{A}_1$ and $\mathbf{B}_1$ are generalised in the obvious way, and the parameter ρ in $\mathbf{R}$ has the interpretation as the canonical correlation between the first component and the remaining $K - 1$ components.

The CCC-GARCH processes in (17), (18) and (19) satisfy the conditions for weak and strict stationarity (He and Teräsvirta, 2004; and Nakatani and Teräsvirta, 2009). Notice that the validity of the residual-based recursive wild bootstrap procedure requires the existence of at least 8th moments. He and Teräsvirta give a result concerning the existence of the fourth moment matrix of $\boldsymbol{\varepsilon}_t$. The condition is that the largest eigenvalue of a certain matrix is less than 1. DGPs 2 and 4 satisfy this condition (the largest eigenvalue is 0.75 and 0.9732, respectively). The condition is violated by DGP 3 (the largest eigenvalue is 1.92). The conditions for the existence of the 8th moment matrix of $\boldsymbol{\varepsilon}_t$ are not known. We therefore only know that the conditions are not satisfied by DGP 3.

The series lengths are $T = 100, 200, 500$ and 1000. The number of replications is 100000. The computations and simulations are performed in R, version 2.13.2. We use the ccgarch package version 0.2.0 (Nakatani, 2010) for simulating the CCC-GARCH models and checking the fourth moment condition. The size and power of the bootstrap and wild bootstrap tests are simulated using the fast bootstrap method of Davidson and MacKinnon (2006).

4.2 Size

The simulated rejection frequencies of the asymptotic, bootstrap and wild bootstrap tests for error autocorrelation of orders $h = 1, 4$ and 12 at the nominal significance level 5% are reported in Table 1 for $K = 2$ and Table 2 for $K = 5$. The results show that the Q_{LM} and Q_{F} tests are preferable over the Q_{LR} and Q_{W} tests which are oversized in DGP 1 with IID normal errors when $K = 2$. The asymptotic tests are all oversized when the dimensions are large ($K = 5$). The differences in size between the asymptotically equivalent tests disappear in finite samples when they are bootstrapped. This is expected, because the bootstrap provides a size-correction of the asymptotic tests (see Davidson and MacKinnon, 2006). We may therefore compare the size of the tests under conditional heteroskedasticity by comparing the size of the bootstrap tests with the size of the wild bootstrap tests. (To save space we report only the rejection frequencies of the LM version of the bootstrap and WB tests.)

The bootstrap tests are oversized in DGPs 2–4 with conditionally heteroskedastic errors. The rejection frequencies do not approach the nominal level as the number of observations increases. In fact, the rejection frequencies increase with the number of observations, as may be expected since the tests are not valid when the errors are not IID. The size of the bootstrap tests in DGP 2 with low persistence in volatility

($a_{ii} = 0.5$) is at least 35% for $h = 1$ and 4, and 23% for $h = 12$ (these numbers are the rejection frequencies reported for $K = 2$ and $T = 1000$ in Table 1). The size of the bootstrap tests in DGP 4 with very high persistence in volatility ($a_{ii} + b_{ii} = 0.98$) is at least 14% for $h = 1$, 24% for $h = 4$ and 36% for $h = 12$ (these numbers are the rejection frequencies reported for $K = 2$ and $T = 1000$ in Table 1). The size distortion of the tests for autocorrelation at lag $h = 1$ is larger in DGPs 2 and 3 with ARCH errors than in DGP 4 with GARCH errors, which is caused by the interrelationship between autocorrelation and ARCH at lag $h = 1$ (Bera et al., 1992). It can be seen that in DGP 2 the maximal size is obtained for $h = 1$, whereas in DGP 4 it is obtained for $h = 12$. Similar results are obtained for $K = 5$.

The wild bootstrap corrects the size distortion caused by the conditional heteroskedasticity. As the sample size is increased the size of the WB tests approaches the nominal level 5%. The size of the WB tests in DGP 4 with very high persistence in volatility is close to 5% for all values of K and h. It can be noted that the convergence to the nominal level is slower in DGP 3 for which the fourth moment matrix of $\boldsymbol{\varepsilon}_t$ does not exist than in DGPs 2 and 4 for which it does. The size of the WB tests appears to converge to 6% in DGP 3 rather than the nominal level 5%. However, longer series than $T = 1000$ observations may be needed for the size to converge to the nominal level. Overall, the simulation results show that failure of the sufficient condition of the existence of 8th moments to hold has little effect on the performance of the WB tests in finite samples. Gonçalves and Kilian (2004) obtained a similar result in the univariate case. Finally, the correlation coefficient (ρ) has some impact on the size of the bootstrap tests but little impact on the size of the WB tests.

4.3 Power

In the simulations for power the errors $\boldsymbol{\varepsilon}_t$ have an autoregressive structure:

$$\boldsymbol{\varepsilon}_t = \boldsymbol{\Psi}_1 \boldsymbol{\varepsilon}_{t-1} + \mathbf{e}_t,$$

where $\boldsymbol{\Psi}_1 = \varphi_1 \mathbf{I}$, $\varphi_1 = 0, 0.01, \ldots, 0.99$ and $\mathbf{e}_t$ is a K-dimensional vector of errors following DGPs 1–4.

Davidson and MacKinnon (2006) argue that the bootstrap is the best way to do size-adjustment in practice. We may therefore use the bootstrap for doing size-adjustment of the asymptotic test, i.e. the power function of the bootstrap test is used to estimate the power function of the size-corrected asymptotic test.

Figure 1 plots the power functions of the bootstrap and wild bootstrap tests when $K = 2$, $N = 100$, and $h = 1$, 4 and 12. The different versions of the bootstrap and WB tests have identical power functions. Because of this fact we draw the power functions only for the LM version of the tests. The bootstrap and WB tests have identical power functions in DGP 1 with IID normal errors. This suggests that the wild bootstrap can be used in testing for autocorrelation without loss of power. The WB tests have lower power when the errors are conditionally heteroskedastic compared to the case with IID errors. We find that there is only a slight decrease in power in DGP 4, whereas the decrease in power is progressively larger in DGPs 2 and 3. Figure 2 plots the power functions when $K = 5$, $N = 100$, and $h = 1, 4$ and 12. The differences in power between the bootstrap test in DGP 1 and the WB tests in DGPs 2–4 are smaller when the dimensions are large.

Table 1: Simulated size of the asymptotic, bootstrap and WB tests for error autocorrelation. The dimensions are $K = 2$. The nominal significance level is 5%.

ρ	0	0	0.3	0.9	0	0.3	0.9	0	0.3	0.9
	DGP 1	DGP 2			DGP 3			DGP 4		
$T = 100$					$h = 1$					
Q_{LM}	0.053	0.157	0.161	0.201	0.280	0.288	0.368	0.070	0.071	0.079
Q_{F}	0.050	0.151	0.155	0.195	0.273	0.281	0.361	0.066	0.067	0.074
Q_{W}	0.071	0.189	0.193	0.234	0.316	0.324	0.404	0.092	0.092	0.102
Q_{LR}	0.062	0.173	0.177	0.218	0.298	0.306	0.387	0.081	0.081	0.090
$Q_{\mathrm{LM}}^{\mathrm{B}}$	0.050	0.152	0.154	0.197	0.277	0.282	0.360	0.066	0.066	0.074
$Q_{\mathrm{LM}}^{\mathrm{WB}}$	0.050	0.063	0.063	0.070	0.070	0.076	0.085	0.055	0.052	0.054
					$h = 4$					
Q_{LM}	0.048	0.110	0.111	0.141	0.229	0.235	0.317	0.070	0.075	0.088
Q_{F}	0.047	0.108	0.110	0.138	0.227	0.234	0.316	0.068	0.073	0.087
Q_{W}	0.128	0.225	0.227	0.262	0.363	0.370	0.452	0.169	0.175	0.195
Q_{LR}	0.084	0.165	0.167	0.199	0.297	0.304	0.386	0.115	0.121	0.138
$Q_{\mathrm{LM}}^{\mathrm{B}}$	0.050	0.113	0.114	0.145	0.233	0.235	0.312	0.072	0.079	0.088
$Q_{\mathrm{LM}}^{\mathrm{WB}}$	0.052	0.058	0.059	0.061	0.067	0.069	0.074	0.052	0.054	0.054
					$h = 12$					
Q_{LM}	0.030	0.047	0.049	0.061	0.104	0.108	0.157	0.049	0.050	0.064
Q_{F}	0.038	0.057	0.059	0.073	0.122	0.126	0.180	0.060	0.062	0.076
Q_{W}	0.498	0.538	0.538	0.553	0.597	0.600	0.636	0.563	0.568	0.599
Q_{LR}	0.218	0.260	0.263	0.280	0.342	0.347	0.402	0.276	0.280	0.309
$Q_{\mathrm{LM}}^{\mathrm{B}}$	0.052	0.075	0.078	0.088	0.140	0.145	0.194	0.079	0.079	0.096
$Q_{\mathrm{LM}}^{\mathrm{WB}}$	0.052	0.054	0.055	0.057	0.063	0.064	0.068	0.053	0.053	0.054
$T = 200$					$h = 1$					
Q_{LM}	0.052	0.186	0.192	0.247	0.371	0.382	0.486	0.078	0.078	0.093
Q_{F}	0.050	0.183	0.189	0.243	0.368	0.379	0.482	0.076	0.076	0.091
Q_{W}	0.060	0.202	0.209	0.264	0.388	0.399	0.503	0.089	0.089	0.104
Q_{LR}	0.056	0.194	0.200	0.256	0.380	0.391	0.494	0.084	0.083	0.098
$Q_{\mathrm{LM}}^{\mathrm{B}}$	0.052	0.186	0.190	0.245	0.369	0.381	0.481	0.077	0.076	0.093
$Q_{\mathrm{LM}}^{\mathrm{WB}}$	0.052	0.060	0.060	0.062	0.068	0.069	0.076	0.053	0.052	0.054
					$h = 4$					
Q_{LM}	0.048	0.146	0.151	0.202	0.364	0.372	0.484	0.093	0.096	0.123
Q_{F}	0.048	0.145	0.150	0.201	0.362	0.371	0.484	0.091	0.095	0.121
Q_{W}	0.082	0.202	0.209	0.265	0.431	0.441	0.548	0.143	0.145	0.178
Q_{LR}	0.064	0.173	0.180	0.234	0.398	0.408	0.517	0.117	0.120	0.150
$Q_{\mathrm{LM}}^{\mathrm{B}}$	0.049	0.148	0.153	0.202	0.363	0.370	0.478	0.095	0.097	0.123
$Q_{\mathrm{LM}}^{\mathrm{WB}}$	0.050	0.058	0.059	0.060	0.066	0.067	0.070	0.052	0.052	0.052
					$h = 12$					
Q_{LM}	0.039	0.080	0.082	0.110	0.214	0.224	0.314	0.091	0.095	0.135
Q_{F}	0.043	0.085	0.088	0.117	0.223	0.234	0.325	0.097	0.102	0.142
Q_{W}	0.208	0.288	0.291	0.326	0.445	0.457	0.538	0.324	0.331	0.388
Q_{LR}	0.106	0.171	0.175	0.206	0.327	0.337	0.427	0.195	0.201	0.253
$Q_{\mathrm{LM}}^{\mathrm{B}}$	0.050	0.096	0.101	0.129	0.234	0.243	0.327	0.111	0.116	0.155
$Q_{\mathrm{LM}}^{\mathrm{WB}}$	0.051	0.054	0.055	0.056	0.064	0.063	0.066	0.051	0.054	0.055

ρ	0	0	0.3	0.9	0	0.3	0.9	0	0.3	0.9
	DGP 1	DGP 2			DGP 3			DGP 4		
$T = 500$					$h = 1$					
Q_{LM}	0.050	0.222	0.231	0.313	0.490	0.502	0.636	0.092	0.094	0.119
Q_{F}	0.049	0.221	0.230	0.311	0.489	0.501	0.635	0.091	0.094	0.118
Q_{W}	0.053	0.229	0.238	0.320	0.496	0.508	0.642	0.097	0.099	0.124
Q_{LR}	0.051	0.226	0.235	0.316	0.493	0.506	0.639	0.094	0.097	0.121
$Q_{\mathrm{LM}}^{\mathrm{B}}$	0.048	0.222	0.232	0.312	0.487	0.500	0.630	0.091	0.094	0.118
$Q_{\mathrm{LM}}^{\mathrm{WB}}$	0.050	0.055	0.058	0.058	0.063	0.064	0.069	0.051	0.051	0.052
					$h = 4$					
Q_{LM}	0.049	0.197	0.204	0.285	0.550	0.565	0.701	0.130	0.136	0.188
Q_{F}	0.049	0.196	0.204	0.285	0.550	0.565	0.701	0.129	0.135	0.188
Q_{W}	0.061	0.221	0.229	0.311	0.574	0.589	0.720	0.151	0.157	0.213
Q_{LR}	0.055	0.209	0.217	0.298	0.562	0.577	0.711	0.140	0.146	0.201
$Q_{\mathrm{LM}}^{\mathrm{B}}$	0.050	0.199	0.207	0.289	0.544	0.560	0.688	0.132	0.137	0.189
$Q_{\mathrm{LM}}^{\mathrm{WB}}$	0.050	0.054	0.055	0.056	0.062	0.062	0.065	0.050	0.052	0.053
					$h = 12$					
Q_{LM}	0.046	0.123	0.125	0.180	0.399	0.415	0.544	0.164	0.176	0.256
Q_{F}	0.047	0.126	0.128	0.183	0.403	0.418	0.548	0.167	0.179	0.259
Q_{W}	0.096	0.200	0.204	0.267	0.487	0.504	0.626	0.260	0.273	0.363
Q_{LR}	0.069	0.159	0.163	0.222	0.443	0.459	0.586	0.210	0.223	0.310
$Q_{\mathrm{LM}}^{\mathrm{B}}$	0.051	0.132	0.135	0.187	0.400	0.414	0.531	0.173	0.186	0.268
$Q_{\mathrm{LM}}^{\mathrm{WB}}$	0.050	0.055	0.054	0.054	0.059	0.061	0.060	0.054	0.051	0.053
$T = 1000$					$h = 1$					
Q_{LM}	0.051	0.243	0.257	0.356	0.580	0.593	0.730	0.104	0.106	0.139
Q_{F}	0.051	0.242	0.256	0.355	0.580	0.592	0.730	0.103	0.105	0.139
Q_{W}	0.052	0.246	0.260	0.359	0.583	0.595	0.732	0.106	0.108	0.142
Q_{LR}	0.052	0.245	0.259	0.357	0.582	0.594	0.731	0.105	0.107	0.141
$Q_{\mathrm{LM}}^{\mathrm{B}}$	0.051	0.243	0.259	0.354	0.579	0.589	0.726	0.104	0.106	0.139
$Q_{\mathrm{LM}}^{\mathrm{WB}}$	0.052	0.053	0.052	0.055	0.061	0.061	0.066	0.051	0.051	0.051
					$h = 4$					
Q_{LM}	0.048	0.228	0.239	0.345	0.673	0.692	0.827	0.157	0.165	0.239
Q_{F}	0.048	0.227	0.239	0.345	0.672	0.691	0.827	0.157	0.164	0.239
Q_{W}	0.054	0.241	0.252	0.359	0.684	0.702	0.834	0.169	0.177	0.253
Q_{LR}	0.051	0.234	0.245	0.352	0.679	0.697	0.831	0.163	0.170	0.247
$Q_{\mathrm{LM}}^{\mathrm{B}}$	0.049	0.230	0.241	0.345	0.667	0.682	0.817	0.158	0.168	0.241
$Q_{\mathrm{LM}}^{\mathrm{WB}}$	0.049	0.054	0.052	0.055	0.059	0.062	0.062	0.051	0.053	0.055
					$h = 12$					
Q_{LM}	0.047	0.151	0.158	0.233	0.546	0.563	0.709	0.222	0.238	0.356
Q_{F}	0.048	0.153	0.159	0.234	0.548	0.565	0.710	0.223	0.239	0.357
Q_{W}	0.070	0.190	0.198	0.277	0.588	0.604	0.741	0.274	0.290	0.410
Q_{LR}	0.058	0.170	0.177	0.254	0.567	0.584	0.726	0.248	0.263	0.383
$Q_{\mathrm{LM}}^{\mathrm{B}}$	0.050	0.156	0.164	0.234	0.536	0.552	0.688	0.227	0.243	0.364
$Q_{\mathrm{LM}}^{\mathrm{WB}}$	0.050	0.053	0.054	0.053	0.059	0.060	0.061	0.051	0.053	0.054

Table 2: Simulated size of the asymptotic, bootstrap and WB tests for error autocorrelation. The dimensions are $K = 5$. The nominal significance level is 5%.

ρ	0	0	0.3	0.9	0	0.3	0.9	0	0.3	0.9
	DGP 1	DGP 2			DGP 3			DGP 4		
$T = 100$					$h = 1$					
Q_{LM}	0.077	0.191	0.193	0.213	0.360	0.363	0.410	0.095	0.098	0.098
Q_{F}	0.054	0.152	0.154	0.173	0.315	0.318	0.365	0.069	0.072	0.073
Q_{W}	0.208	0.372	0.375	0.399	0.548	0.549	0.591	0.238	0.239	0.243
Q_{LR}	0.136	0.280	0.282	0.307	0.458	0.461	0.505	0.160	0.162	0.165
Q_{LM}^{B}	0.050	0.142	0.144	0.160	0.301	0.297	0.346	0.065	0.067	0.065
$Q_{\text{LM}}^{\text{WB}}$	0.052	0.072	0.073	0.077	0.100	0.102	0.108	0.053	0.057	0.056
					$h = 4$					
Q_{LM}	0.104	0.175	0.175	0.191	0.321	0.323	0.368	0.133	0.136	0.141
Q_{F}	0.062	0.118	0.118	0.131	0.253	0.257	0.303	0.084	0.086	0.089
Q_{W}	0.854	0.896	0.897	0.903	0.936	0.934	0.943	0.877	0.877	0.882
Q_{LR}	0.522	0.624	0.625	0.638	0.742	0.742	0.767	0.575	0.577	0.586
Q_{LM}^{B}	0.054	0.103	0.102	0.112	0.222	0.225	0.266	0.074	0.073	0.076
$Q_{\text{LM}}^{\text{WB}}$	0.055	0.066	0.067	0.067	0.086	0.087	0.091	0.058	0.057	0.057
					$h = 12$					
Q_{LM}	0.112	0.133	0.134	0.137	0.195	0.197	0.218	0.136	0.139	0.142
Q_{F}	0.088	0.107	0.109	0.112	0.175	0.179	0.204	0.107	0.110	0.114
Q_{W}	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Q_{LR}	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Q_{LM}^{B}	0.062	0.078	0.080	0.079	0.123	0.129	0.146	0.075	0.081	0.081
$Q_{\text{LM}}^{\text{WB}}$	0.062	0.066	0.067	0.064	0.080	0.080	0.084	0.061	0.067	0.063
$T = 200$					$h = 1$					
Q_{LM}	0.061	0.215	0.215	0.251	0.481	0.486	0.547	0.086	0.088	0.094
Q_{F}	0.052	0.196	0.196	0.232	0.461	0.466	0.528	0.074	0.075	0.082
Q_{W}	0.107	0.297	0.298	0.336	0.564	0.569	0.625	0.143	0.145	0.153
Q_{LR}	0.082	0.255	0.256	0.293	0.524	0.528	0.587	0.113	0.115	0.122
Q_{LM}^{B}	0.050	0.193	0.193	0.227	0.452	0.459	0.520	0.071	0.073	0.079
$Q_{\text{LM}}^{\text{WB}}$	0.050	0.069	0.066	0.071	0.089	0.089	0.096	0.053	0.053	0.056
					$h = 4$					
Q_{LM}	0.066	0.169	0.173	0.201	0.447	0.454	0.518	0.113	0.115	0.127
Q_{F}	0.052	0.143	0.147	0.174	0.417	0.423	0.491	0.091	0.092	0.104
Q_{W}	0.370	0.549	0.551	0.581	0.776	0.777	0.813	0.472	0.475	0.492
Q_{LR}	0.189	0.350	0.353	0.389	0.629	0.635	0.687	0.274	0.278	0.294
Q_{LM}^{B}	0.051	0.138	0.143	0.167	0.399	0.406	0.471	0.089	0.091	0.099
$Q_{\text{LM}}^{\text{WB}}$	0.050	0.063	0.064	0.065	0.084	0.084	0.084	0.054	0.056	0.054
					$h = 12$					
Q_{LM}	0.062	0.101	0.103	0.114	0.250	0.258	0.304	0.117	0.118	0.134
Q_{F}	0.050	0.086	0.088	0.099	0.238	0.247	0.296	0.099	0.100	0.114
Q_{W}	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	1.000	1.000
Q_{LR}	0.878	0.907	0.906	0.909	0.934	0.936	0.942	0.923	0.925	0.931
Q_{LM}^{B}	0.051	0.085	0.089	0.100	0.223	0.233	0.277	0.100	0.101	0.118
$Q_{\text{LM}}^{\text{WB}}$	0.051	0.059	0.057	0.058	0.071	0.075	0.074	0.054	0.055	0.056

ρ	0	0	0.3	0.9	0	0.3	0.9	0	0.3	0.9
	DGP 1	DGP 2			DGP 3			DGP4		
$T = 500$					$h = 1$					
Q_{LM}	0.054	0.254	0.261	0.312	0.650	0.657	0.726	0.093	0.093	0.105
Q_{F}	0.051	0.246	0.254	0.304	0.643	0.651	0.721	0.088	0.088	0.100
Q_{W}	0.068	0.286	0.294	0.345	0.676	0.684	0.749	0.114	0.114	0.128
Q_{LR}	0.061	0.270	0.278	0.328	0.663	0.671	0.738	0.103	0.103	0.116
$Q_{\mathrm{LM}}^{\mathrm{B}}$	0.050	0.246	0.252	0.300	0.638	0.649	0.711	0.086	0.089	0.098
$Q_{\mathrm{LM}}^{\mathrm{WB}}$	0.051	0.058	0.061	0.062	0.074	0.077	0.079	0.051	0.052	0.053
					$h = 4$					
Q_{LM}	0.056	0.213	0.219	0.263	0.686	0.693	0.763	0.134	0.138	0.158
Q_{F}	0.050	0.202	0.207	0.252	0.677	0.684	0.756	0.125	0.128	0.147
Q_{W}	0.132	0.349	0.356	0.405	0.784	0.790	0.844	0.256	0.258	0.287
Q_{LR}	0.088	0.279	0.285	0.332	0.739	0.744	0.807	0.191	0.193	0.219
$Q_{\mathrm{LM}}^{\mathrm{B}}$	0.050	0.203	0.211	0.252	0.667	0.675	0.741	0.128	0.130	0.149
$Q_{\mathrm{LM}}^{\mathrm{WB}}$	0.051	0.058	0.059	0.059	0.076	0.073	0.076	0.053	0.053	0.052
					$h = 12$					
Q_{LM}	0.052	0.128	0.130	0.152	0.476	0.481	0.561	0.173	0.178	0.213
Q_{F}	0.048	0.121	0.123	0.145	0.471	0.476	0.558	0.164	0.168	0.203
Q_{W}	0.597	0.722	0.725	0.743	0.893	0.895	0.918	0.788	0.793	0.816
Q_{LR}	0.266	0.408	0.414	0.442	0.728	0.732	0.782	0.494	0.499	0.539
$Q_{\mathrm{LM}}^{\mathrm{B}}$	0.050	0.123	0.126	0.148	0.461	0.464	0.539	0.170	0.173	0.205
$Q_{\mathrm{LM}}^{\mathrm{WB}}$	0.049	0.056	0.055	0.056	0.068	0.069	0.071	0.053	0.054	0.055
$T = 1000$					$h = 1$					
Q_{LM}	0.052	0.290	0.294	0.357	0.758	0.766	0.829	0.102	0.105	0.118
Q_{F}	0.051	0.286	0.290	0.353	0.756	0.763	0.827	0.099	0.102	0.116
Q_{W}	0.059	0.306	0.310	0.373	0.768	0.776	0.837	0.113	0.116	0.130
Q_{LR}	0.056	0.298	0.302	0.365	0.764	0.771	0.833	0.107	0.110	0.124
$Q_{\mathrm{LM}}^{\mathrm{B}}$	0.050	0.289	0.291	0.354	0.751	0.758	0.819	0.099	0.102	0.116
$Q_{\mathrm{LM}}^{\mathrm{WB}}$	0.050	0.056	0.055	0.055	0.068	0.070	0.073	0.050	0.052	0.050
					$h = 4$					
Q_{LM}	0.051	0.254	0.257	0.319	0.831	0.838	0.892	0.158	0.164	0.198
Q_{F}	0.048	0.248	0.252	0.313	0.828	0.835	0.889	0.152	0.158	0.192
Q_{W}	0.083	0.321	0.325	0.388	0.863	0.870	0.916	0.217	0.224	0.263
Q_{LR}	0.066	0.287	0.291	0.353	0.848	0.855	0.904	0.186	0.193	0.229
$Q_{\mathrm{LM}}^{\mathrm{B}}$	0.048	0.247	0.253	0.311	0.821	0.828	0.880	0.154	0.158	0.193
$Q_{\mathrm{LM}}^{\mathrm{WB}}$	0.048	0.056	0.056	0.056	0.068	0.068	0.070	0.050	0.051	0.053
					$h = 12$					
Q_{LM}	0.049	0.155	0.157	0.194	0.663	0.673	0.750	0.230	0.234	0.294
Q_{F}	0.047	0.151	0.153	0.190	0.661	0.671	0.749	0.225	0.228	0.288
Q_{W}	0.242	0.428	0.431	0.476	0.843	0.850	0.890	0.546	0.549	0.612
Q_{LR}	0.125	0.277	0.281	0.325	0.763	0.771	0.830	0.383	0.385	0.452
$Q_{\mathrm{LM}}^{\mathrm{B}}$	0.049	0.154	0.155	0.191	0.650	0.653	0.726	0.228	0.234	0.292
$Q_{\mathrm{LM}}^{\mathrm{WB}}$	0.050	0.053	0.055	0.055	0.065	0.065	0.067	0.054	0.053	0.054

Figures 3 and 4 plot the power functions when $K = 2$ and 5, respectively, and $N = 500$. The decrease in power of the WB tests when the errors follow DGP 3 becomes more pronounced when the sample size is large.

5 Empirical Illustrations

In order to illustrate the use of the bootstrap and wild bootstrap tests with real data, we consider two empirical applications to credit default swap prices and Euribor interest rates.

The determination of the lag length in vector autoregressive models is an important area of application for tests for error autocorrelation (see e.g. Lütkepohl, 2006, Chapter 4). The simulation results in the previous section strongly suggest that the asymptotic and bootstrap tests may falsely reject the null hypothesis of no error autocorrelation if the errors are conditionally heteroskedastic. The tests may therefore erroneously indicate that a long lag length is required.

5.1 Credit Default Swap Prices

Our first empirical example deals with credit default swap (CDS) prices. A CDS is a credit derivative which provides a bondholder with protection against the risk of default by the company. If a default occurs, the holder is compensated for the loss by an amount which equals the difference between the par value of the bond and its market value after the default. The CDS price is the annualised fee (expressed as a percentage of the principal) paid by the protection buyer. Duffie (1999) derived the equivalence of the CDS price and credit spread (defined as the bond yield minus the risk-free rate).

We denote by p_{CDS} the CDS price and p_{CS} the credit spread on a risky bond over the risk-free rate. The basis is the difference between the CDS price and the bond spread:

$$s = p_{\text{CDS}} - p_{\text{CS}}.$$

If the two markets price credit risk equally in the long run, then the prices should be equal, so that the basis $s = 0$. Since both p_{CDS} and p_{CS} are assumed to be $I(1)$, the non-arbitrage relation can be tested as an equilibrium relation in the cointegrated VAR model (Blanco et al., 2005). The vector $\mathbf{X}_t$ with the value 1 appended to it is $\mathbf{X}_t = (p_{\text{CDS}}, p_{\text{CS}}, 1)'$. The financial theory posits that $\mathbf{X}_t$ is cointegrated with cointegrating vector $\boldsymbol{\beta} = (1, -1, c)'$, so that $\boldsymbol{\beta}'\mathbf{X}_t = p_{\text{CDS}} - p_{\text{CS}} + c$ is a cointegrating relation. In theory $c = 0$, but in practice it may be different from zero (see Blanco et al., 2005, for details). Many papers have tested the equivalence of CDS prices and credit spreads for US and European investment-grade companies, and found that the parity relation holds for most companies, i.e. the bond and CDS markets price credit risk equally (Blanco et al., 2005; Houweling and Vorst, 2005; Zhu, 2006; and Dötz, 2007).

We take a subsample of the companies in Table 1 of Blanco et al. The companies in our subsample are Bank of America, Citigroup, Goldman Sachs, Barclays Bank and Vodafone, the first three of which are US and the remaining two European companies. We use daily 5-year maturity CDS prices and credit spreads from Datastream. The data are daily observations from 1 January 2009 to 31 January 2012, and the number

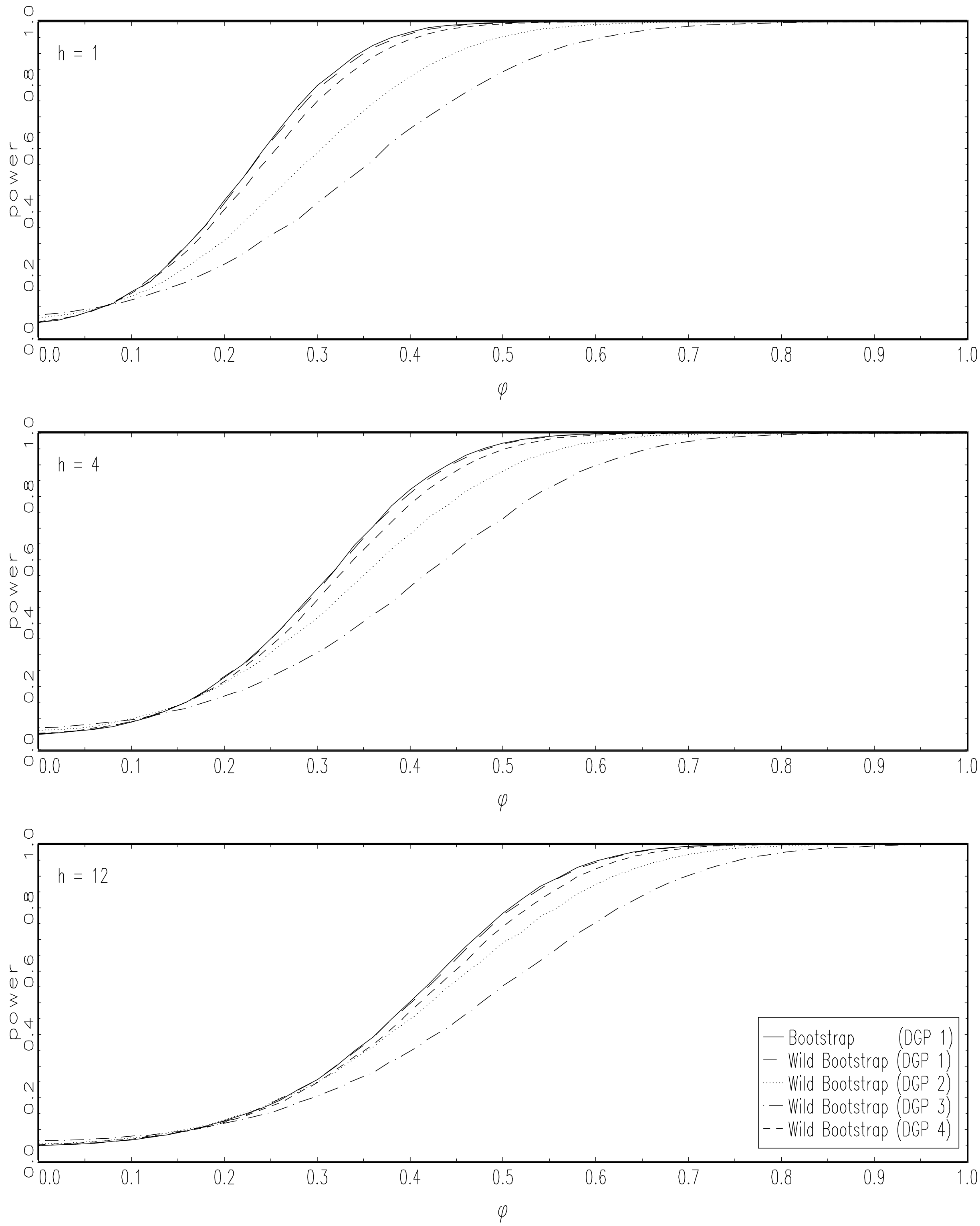

Figure 1: The power functions of the bootstrap and wild bootstrap tests for error autocorrelation when $K = 2$ and $N = 100$.

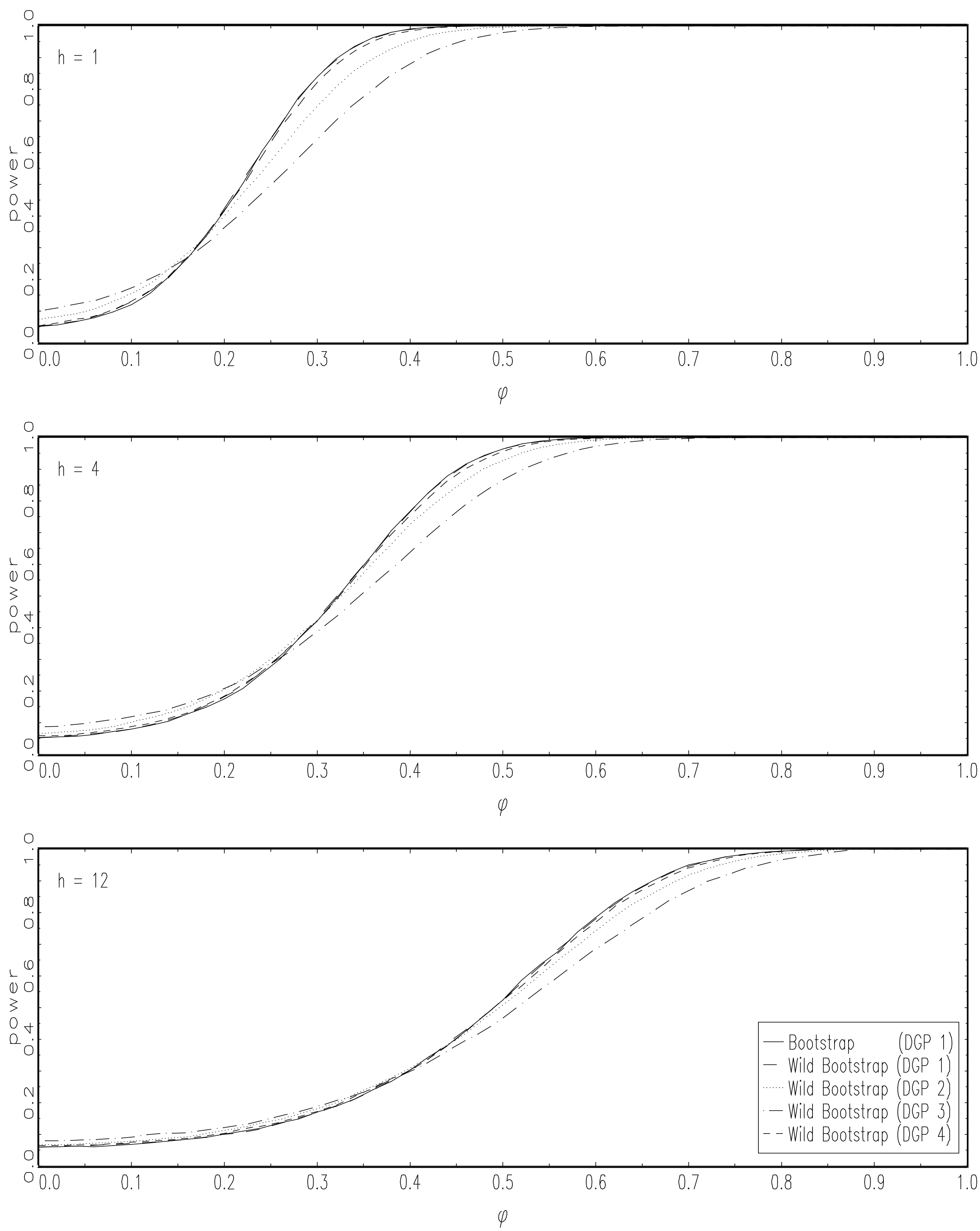

Figure 2: The power functions of the bootstrap and wild bootstrap tests for error autocorrelation when $K = 5$ and $N = 100$.

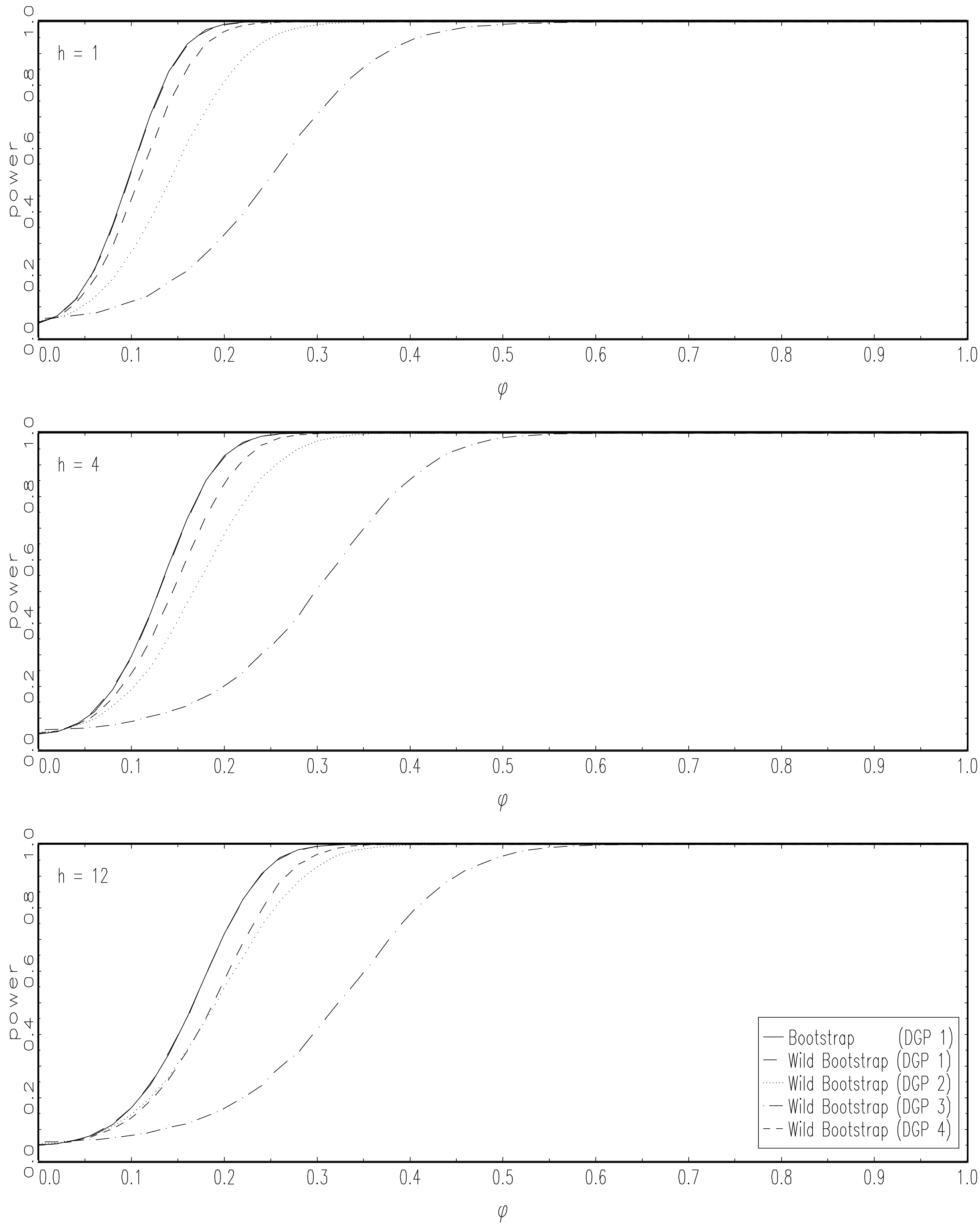

Figure 3: The power functions of the bootstrap and wild bootstrap tests for error autocorrelation when $K = 2$ and $N = 500$.

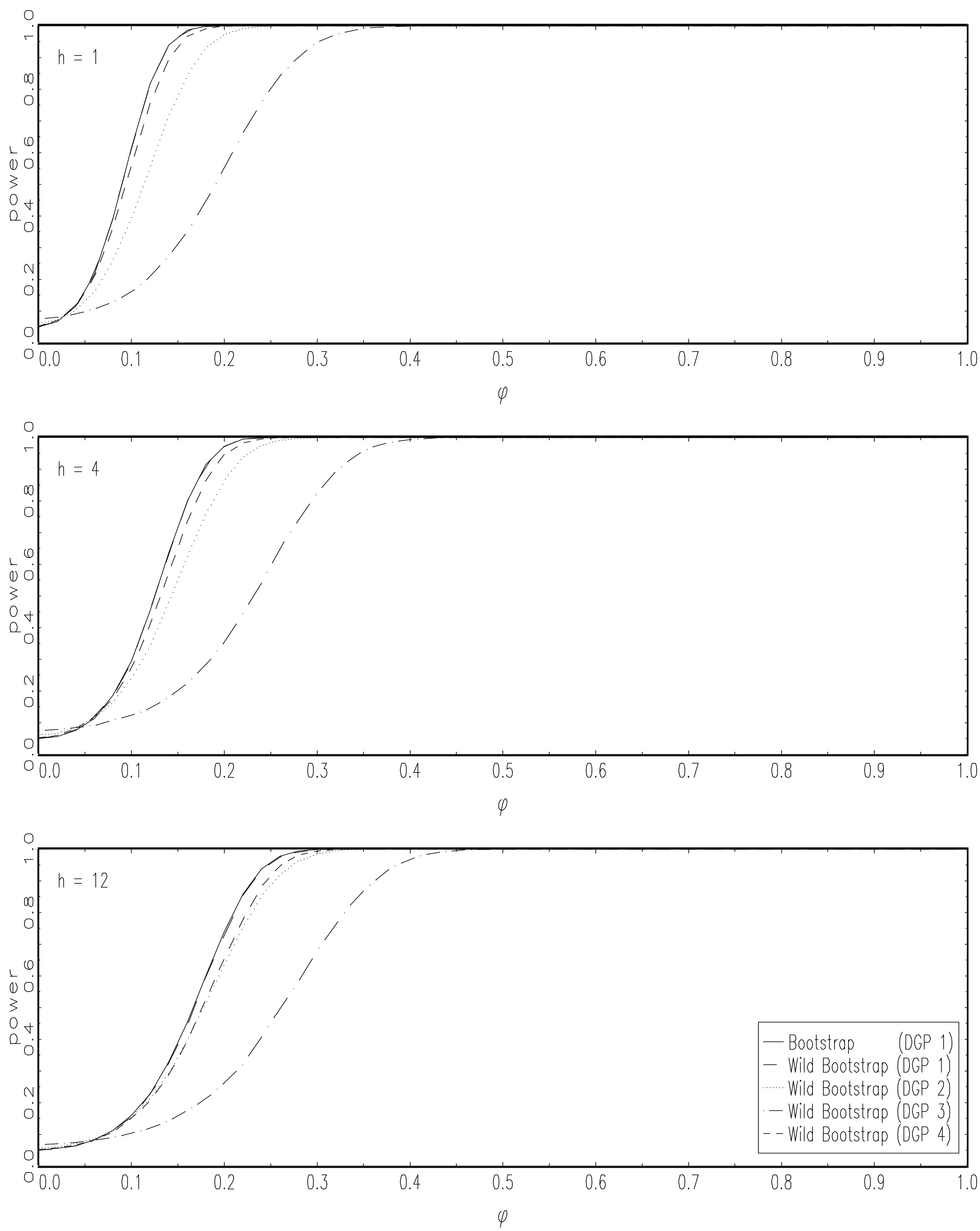

Figure 4: The power functions of the bootstrap and wild bootstrap tests for error autocorrelation when $K = 5$ and $N = 500$.

of daily observations for each company is $T = 804$. Blanco et al. contain a discussion of issues related to the construction of the series.

Figure 5 plots the daily CDS prices and credit spreads for the companies in our subsample. The differences between the two series are small for all companies.

We use information criteria for determining the lag length in the VAR models. For Bank of America the Akaike information criterion (AIC) selects $p = 18$, whereas the Schwarz (SC) and Hannan–Quinn (HQ) information criteria both select $p = 2$. For Barclays Bank the AIC selects $p = 10$, whereas the SC and HQ both select $p = 4$. Similar results are obtained for the other companies. The AIC tends to select a model with a long lag length, which is what we would expect since it is inconsistent. Because the sample size is large, we rely on the SC and HQ information criteria, and select the lag length $p = 2$ for Bank of America, $p = 3$ for Citigroup, Goldman Sachs and Vodafone, and $p = 4$ for Barclays Bank. We estimate VAR(p) models with a restricted constant.

Table 3 shows the tests for error autocorrelation. We discuss the results for Barclays Bank in detail. Similar results are obtained for the other companies, as is readily seen in Table 3. The asymptotic tests reject the null hypothesis of no error autocorrelation at the 5% level. The p-values of the asymptotic tests are about 2.6% for $h = 1$, 0.1% for $h = 4$ and 0 for $h = 12$. (The p-values of the bootstrap tests are 7.2% for $h = 1$, 2.7% for $h = 4$ and 2.8% for $h = 12$.) The wild bootstrap tests do not reject the null hypothesis of no error autocorrelation of orders $h = 1$, 4 and 12. The p-values of the WB tests are 32% for $h = 1$, 44% for $h = 4$ and 61% for $h = 12$. We therefore have a situation where the asymptotic tests reject but the wild bootstrap tests do not reject. Based on the WB tests we conclude that a VAR(4) model provides a good description of the data. Finally, we mention that all VAR models with lag lengths up to $p = 25$ are rejected by the asymptotic tests. The bootstrap tests lead to a VAR(5) model.

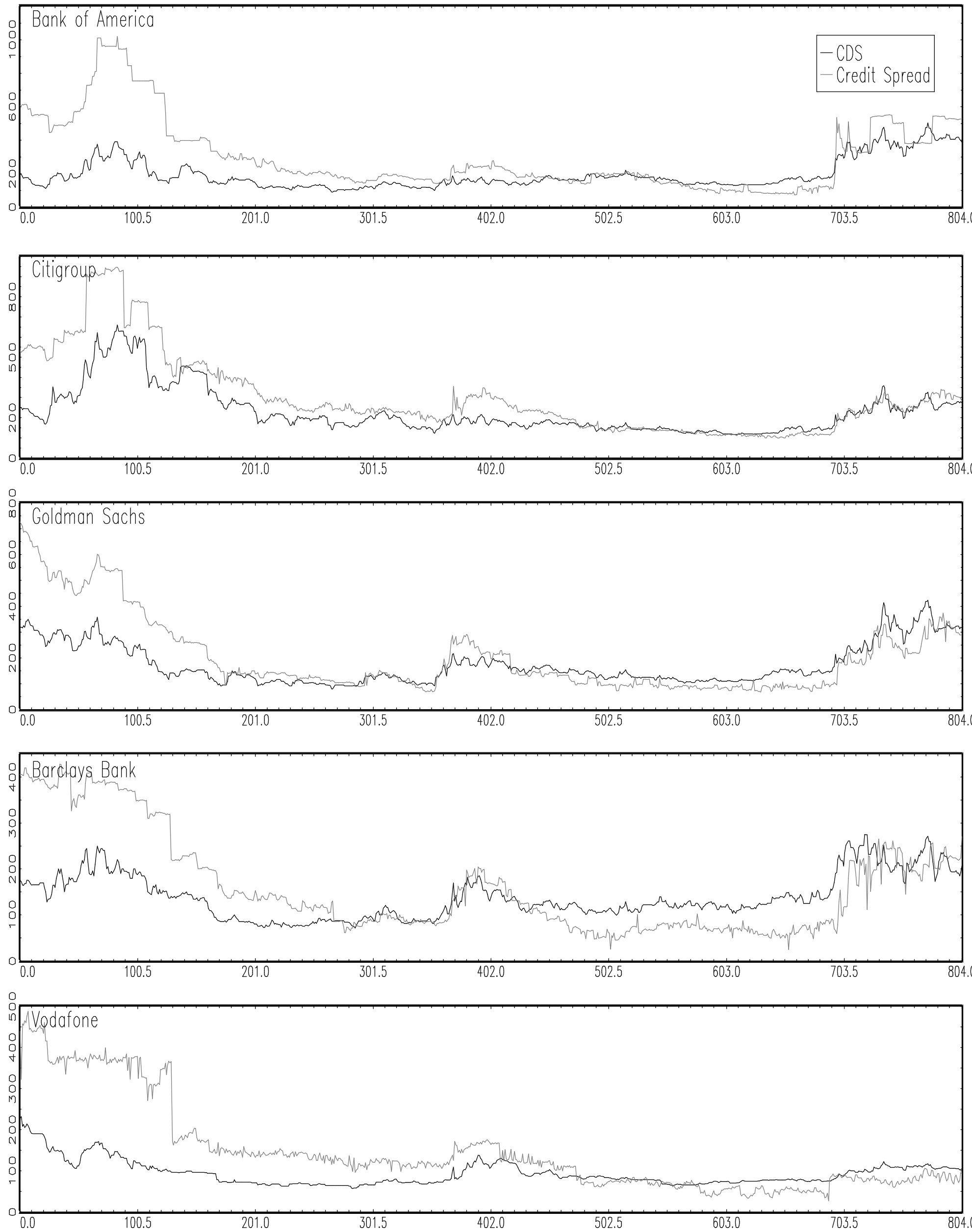

Figure 5: The CDS prices and bond spreads for Bank of America, Citigroup, Goldman Sachs, Barclays Bank and Vodafone. The sample period is 1 January 2009 to 31 January 2012.

Table 3: Asymptotic, bootstrap and WB tests for error autocorrelation in the estimated VAR models for the CDS prices and bond spreads. The table reports the p-values of the tests.

h	Q_{LM}	Q_{LR}	Q_{W}	Q_{F}	$Q^{\mathrm{B}}_{\mathrm{LM}}$	$Q^{\mathrm{B}}_{\mathrm{LR}}$	$Q^{\mathrm{B}}_{\mathrm{W}}$	$Q^{\mathrm{B}}_{\mathrm{F}}$	$Q^{\mathrm{WB}}_{\mathrm{LM}}$	$Q^{\mathrm{WB}}_{\mathrm{LR}}$	$Q^{\mathrm{WB}}_{\mathrm{W}}$	$Q^{\mathrm{WB}}_{\mathrm{F}}$
VAR(2) model for Bank of America												
1	0.194	0.193	0.192	0.197	0.195	0.196	0.196	0.196	0.673	0.673	0.674	0.673
4	0.067	0.063	0.059	0.069	0.117	0.117	0.118	0.117	0.692	0.693	0.694	0.693
12	0.000	0.000	0.000	0.000	0.003	0.004	0.004	0.004	0.145	0.149	0.153	0.149
VAR(3) model for Citigroup												
1	0.065	0.064	0.063	0.067	0.263	0.263	0.264	0.263	0.554	0.554	0.554	0.554
4	0.095	0.090	0.085	0.098	0.247	0.247	0.245	0.247	0.697	0.697	0.697	0.697
12	0.000	0.000	0.000	0.000	0.015	0.016	0.016	0.016	0.443	0.449	0.455	0.449
VAR(3) model for Goldman Sachs												
1	0.001	0.001	0.001	0.001	0.037	0.038	0.038	0.038	0.252	0.254	0.255	0.254
4	0.020	0.019	0.017	0.021	0.139	0.140	0.141	0.140	0.663	0.664	0.666	0.664
12	0.001	0.000	0.000	0.001	0.030	0.031	0.032	0.031	0.732	0.736	0.739	0.736
VAR(4) model for Barclays Bank												
1	0.026	0.026	0.025	0.027	0.072	0.072	0.073	0.072	0.322	0.323	0.324	0.323
4	0.001	0.001	0.001	0.001	0.027	0.027	0.028	0.027	0.443	0.442	0.442	0.442
12	0.000	0.000	0.000	0.000	0.028	0.028	0.028	0.028	0.607	0.610	0.613	0.610
VAR(3) model for Vodafone												
1	0.002	0.002	0.002	0.002	0.799	0.800	0.802	0.800	0.533	0.533	0.533	0.533
4	0.000	0.000	0.000	0.000	0.235	0.245	0.253	0.245	0.504	0.505	0.505	0.505
12	0.000	0.000	0.000	0.000	0.019	0.020	0.021	0.020	0.504	0.504	0.504	0.504

The residuals from the estimated VAR models are plotted in Figure 5. Notice the volatility clustering in the residuals. This is confirmed by Table 4, which reports LM tests for ARCH up to order 2. The asymptotic and bootstrap tests find a strong ARCH effect in the residuals for all companies, except the equation for p_{CS} for Citigroup. The multivariate ARCH tests are significant at the 1% level, with the exception of Vodafone. Because there is very strong evidence of ARCH in the errors, the wild bootstrap tests for error autocorrelation should be preferred over the asymptotic and bootstrap tests which assume IID errors.

Finally, we mention that only for Barclays Bank is the financial theory accepted, since it is the only company for which we accept the cointegration rank $r = 1$. The restrictions on $\boldsymbol{\beta}$ implied by the theory are also accepted.

In order to further investigate the size properties of the tests in the application, we fit CCC-GARCH$(1, 1)$ models to the residuals from the VAR models. Table 5 summarises the parameter estimates. The stationarity condition is $\lambda(\mathbf{\Gamma_C}) < 1$, where λ is the modulus of the largest eigenvalue of a certain matrix $\mathbf{\Gamma_C}$. The fourth moment condition is satisfied when $\lambda(\mathbf{\Gamma_{C\otimes C}}) < 1$, where λ is the modulus of the largest eigenvalue of a certain matrix $\mathbf{\Gamma_{C\otimes C}}$ (He and Teräsvirta, 2004). Notice that the stationarity condition is satisfied only for the residuals from the models for Goldman Sachs and Barclays Bank, indicating that the CCC-GARCH$(1, 1)$ model does not provide a good fit to the residuals of the other companies. The fourth moment condition is not satisfied for any of the models for the errors.

We investigate by simulation the rejection probabilities of the tests for error autocorrelation for Barclays Bank. In each simulation we use the estimated parameters from the VAR model and the CCC-GARCH model for the errors to define the data-generation process. The errors are drawn from a multivariate normal distribution with covariance matrix equal to the estimated covariance matrix. We simulate 100000 time series of length $T = 804$. We test for error autocorrelation in the unrestricted VAR model and the cointegrated VAR model with cointegration rank $r = 1$. The simulated size of the tests are reported in Table 6. The asymptotic and bootstrap tests are severely oversized, whereas the wild bootstrap tests have size close to the nominal level. Focusing on the Q_{LM} test, the size is 13% for $h = 1$, 22% for $h = 4$ and 30% for $h = 12$ in the unrestricted VAR model. In the cointegrated VAR model with $r = 1$ the corresponding sizes are 19%, 25% and 31%. The bootstrap $Q_{\mathrm{LM}}^{\mathrm{B}}$ test is oversized, which is what we would expect since the errors are not IID. The size of the wild bootstrap test $Q_{\mathrm{LM}}^{\mathrm{WB}}$ is close to 5%.

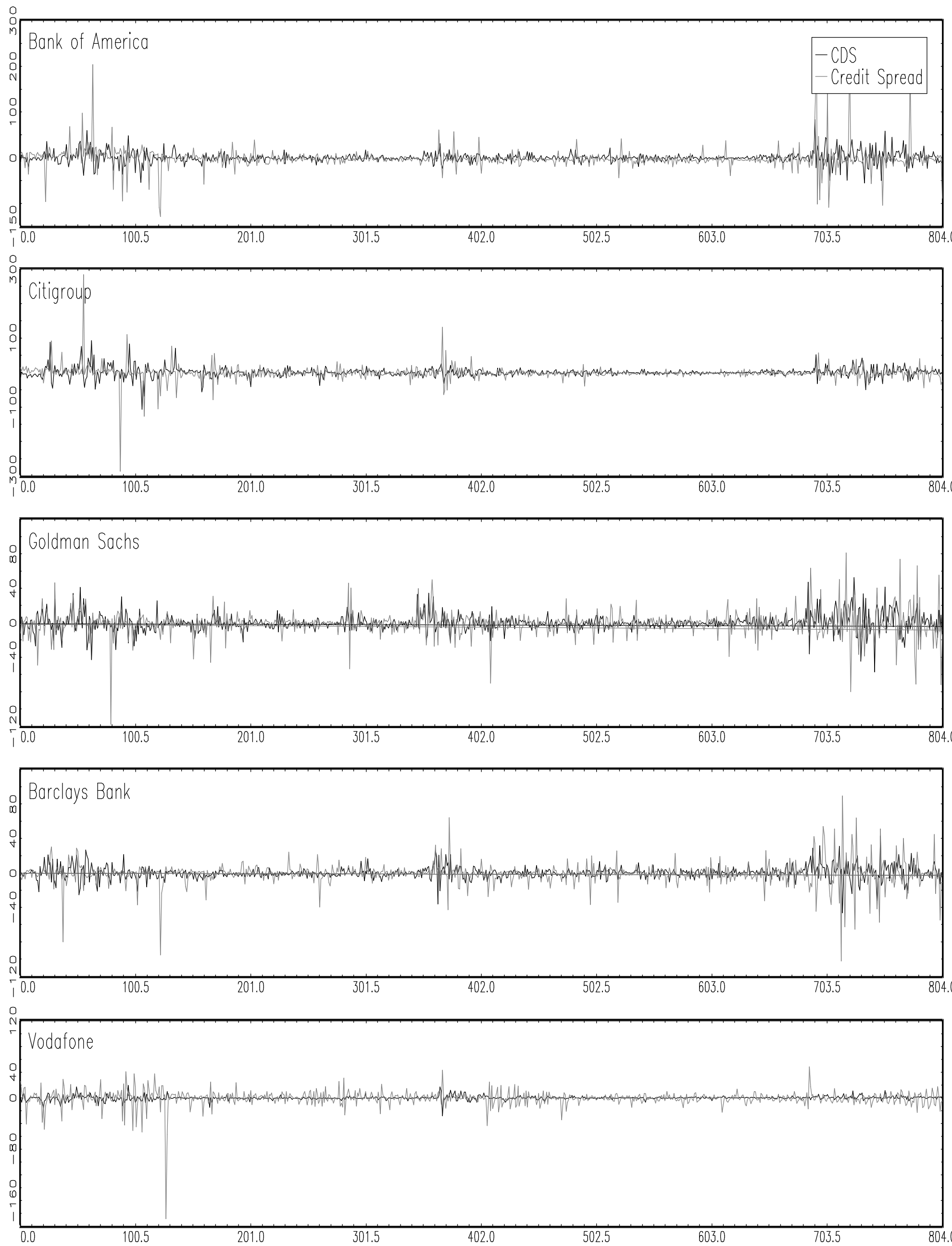

Figure 6: The residuals from the estimated VAR models for the CDS prices and bond spreads.

Table 4: Asymptotic and bootstrap tests for ARCH in the estimated VAR models for the CDS prices and bond spreads. The table reports the p-values of the tests.

	Q_{LM}	$Q_{\mathrm{LM}}^{\mathrm{B}}$	Q_{F}	$Q_{\mathrm{F}}^{\mathrm{B}}$
VAR(2) model for Bank of America				
p_{CDS}	0.000	0.007	0.000	0.007
p_{CS}	0.003	0.024	0.003	0.024
Multivariate	0.000	0.000	–	–
VAR(3) model for Citigroup				
p_{CDS}	0.000	0.002	0.000	0.002
p_{CS}	0.956	0.564	0.957	0.564
Multivariate	0.000	0.028	–	–
VAR(3) model for Goldman Sachs				
p_{CDS}	0.000	0.000	0.000	0.000
p_{CS}	0.008	0.024	0.008	0.024
Multivariate	0.000	0.006	–	–
VAR(4) model for Barclays Bank				
p_{CDS}	0.000	0.000	0.000	0.000
p_{CS}	0.000	0.000	0.000	0.000
Multivariate	0.000	0.000	–	–
VAR(3) model for Vodafone				
p_{CDS}	0.000	0.002	0.000	0.002
p_{CS}	0.017	0.015	0.018	0.015
Multivariate	0.000	0.093	–	–

Table 5: The parameter estimates of the CCC-GARCH$(1,1)$ models fitted to the residuals from the VAR models for the CDS prices and bond spreads. Standard errors are reported in parentheses below the parameter estimates.

		Bank of America	Citigroup	Goldman Sachs	Barclays Banks	Vodafone
p_{CDS}	a_{01}	1.153	0.114	1.355	0.521	0.383
		(1.049)	(0.155)	(1.079)	(0.309)	(0.409)
	a_{11}	0.191	0.138	0.153	0.116	0.141
		(0.124)	(0.096)	(0.110)	(0.033)	(0.156)
	b_{11}	0.832	0.888	0.846	0.878	0.814
		(0.085)	(0.058)	(0.089)	(0.033)	(0.162)
p_{CS}	a_{02}	2.265	0.386	2.559	2.761	2.084
		(1.775)	(1.223)	(2.992)	(2.546)	(2.522)
	a_{22}	0.075	0.071	0.052	0.059	0.100
		(0.046)	(0.037)	(0.036)	(0.033)	(0.050)
	b_{22}	0.936	0.951	0.940	0.927	0.901
		(0.025)	(0.006)	(0.045)	(0.047)	(0.059)
	ρ	0.082	0.046	0.158	0.035	-0.055
		(0.057)	(0.041)	(0.042)	(0.042)	(0.039)
	$\lambda(\boldsymbol{\Gamma}_{\mathbf{C}})$	1.023	1.026	0.999	0.995	1.001
	$\lambda(\boldsymbol{\Gamma}_{\mathbf{C}\otimes\mathbf{C}})$	1.119	1.091	1.045	1.018	1.022

Table 6: Simulated size of the asymptotic, bootstrap and WB tests for error autocorrelation in the estimated VAR model for the CDS price and bond spread for Barclays Bank.

h	Q_{LM}	Q_{LR}	Q_{W}	Q_{F}	$Q^{\mathrm{B}}_{\mathrm{LM}}$	$Q^{\mathrm{B}}_{\mathrm{LR}}$	$Q^{\mathrm{B}}_{\mathrm{W}}$	$Q^{\mathrm{B}}_{\mathrm{F}}$	$Q^{\mathrm{WB}}_{\mathrm{LM}}$	$Q^{\mathrm{WB}}_{\mathrm{LR}}$	$Q^{\mathrm{WB}}_{\mathrm{W}}$	$Q^{\mathrm{WB}}_{\mathrm{F}}$
Unrestricted VAR(4) model												
1	0.134	0.135	0.137	0.130	0.130	0.130	0.130	0.130	0.053	0.053	0.053	0.053
4	0.221	0.229	0.237	0.214	0.209	0.208	0.208	0.208	0.055	0.055	0.055	0.055
12	0.304	0.334	0.365	0.295	0.276	0.275	0.275	0.275	0.057	0.057	0.058	0.057
Cointegrated VAR(4) model with $r = 1$												
1	0.189	0.189	0.193	0.183	0.073	0.073	0.074	0.073	0.055	0.056	0.058	0.056
4	0.249	0.264	0.276	0.240	0.188	0.187	0.189	0.187	0.070	0.067	0.067	0.067
12	0.314	0.343	0.373	0.306	0.245	0.242	0.241	0.242	0.047	0.046	0.046	0.046

5.2 Euribor Interest Rates

In the second empirical example we consider the problem of testing the expectations hypothesis of the term structure of interest rates. The theory makes two predictions, which can be tested in the cointegrated VAR model (Hall et al., 1992). First, if there are K interest rate series in the system, then the cointegration rank is $r = K - 1$. Second, the spreads between the interest rates at different maturities span the cointegration space.

We use monthly data from 1998(12) to 2008(9) on the 1, 3, 6, 9 and 12 month Euribor interest rates. All interest rates are nominal and annualised. The data were retrieved from `www.euribor.org`. Likelihood ratio tests and information criteria lead to the choice of lag length $p = 3$. We estimate a VAR(3) model with a constant and dummy variables to account for interest rate shocks in 2001(4) and 2001(9).

Table 7 shows the tests for error autocorrelation. The asymptotic tests reject the null hypothesis of no error autocorrelation at the 5% level. The p-value of the Q_{LM} test is 1.6% for $h = 1$, and 0 for $h = 4$ and 12. (The p-value of the bootstrap $Q_{\mathrm{LM}}^{\mathrm{B}}$ test is 11% for $h = 1$, 2.6% for $h = 4$ and 0.2% for $h = 12$.) The wild bootstrap tests do not reject the null hypothesis of no error autocorrelation of orders $h = 1$, 4 and 12. The p-value of the WB $Q_{\mathrm{LM}}^{\mathrm{WB}}$ test is 36% for $h = 1$, 38% for $h = 4$ and 19% for $h = 12$. We therefore have a situation where the asymptotic test rejects but the WB test does not reject. Based on the WB test we conclude that a VAR(3) model provides a good description of the data. Finally, we mention that the VAR model with the smallest value of the lag length p which is not rejected by the asymptotic Q_{LM} test is a VAR(10) model. The bootstrap $Q_{\mathrm{LM}}^{\mathrm{B}}$ test leads to a VAR(6) model.

Table 7: Asymptotic, bootstrap and WB tests for error autocorrelation in the estimated VAR(3) model for the Euribor interest rates. The table reports the p-values of the tests.

h	Q_{LM}	Q_{LR}	Q_{W}	Q_{F}	$Q^{\mathrm{B}}_{\mathrm{LM}}$	$Q^{\mathrm{B}}_{\mathrm{LR}}$	$Q^{\mathrm{B}}_{\mathrm{W}}$	$Q^{\mathrm{B}}_{\mathrm{F}}$	$Q^{\mathrm{WB}}_{\mathrm{LM}}$	$Q^{\mathrm{WB}}_{\mathrm{LR}}$	$Q^{\mathrm{WB}}_{\mathrm{W}}$	$Q^{\mathrm{WB}}_{\mathrm{F}}$
1	0.016	0.006	0.002	0.059	0.106	0.100	0.095	0.100	0.364	0.354	0.344	0.354
4	0.000	0.000	0.000	0.014	0.026	0.036	0.049	0.036	0.376	0.440	0.500	0.440
12	0.000	0.000	0.000	0.003	0.002	0.002	0.006	0.002	0.191	0.260	0.382	0.260

Table 8: Asymptotic and bootstrap tests for ARCH in the estimated VAR(3) model for the Euribor interest rates. The table reports the p-values of the tests.

	Q_{LM}	$Q_{\mathrm{LM}}^{\mathrm{B}}$	Q_{F}	$Q_{\mathrm{F}}^{\mathrm{B}}$
1 month	0.000	0.002	0.001	0.002
3 months	0.559	0.466	0.623	0.466
6 months	0.675	0.669	0.727	0.669
9 months	0.342	0.322	0.417	0.322
12 months	0.329	0.306	0.403	0.306
Multivariate	0.000	0.000	–	–

LM tests for ARCH up to order 2 are reported in Table 8. The asymptotic and bootstrap tests find a strong ARCH effect in the residuals from the equation for the 1 month interest rate. The multivariate ARCH tests are significant at the 1% level. Because there is very strong evidence of ARCH in the errors, the wild bootstrap tests for error autocorrelation should be preferred over the asymptotic and bootstrap tests which assume IID errors.

6 Conclusions

In this article we study the performance of tests for error autocorrelation in stationary and cointegrated vector autoregressive models with conditionally heteroskedastic errors. The main results are that the asymptotic and residual-based bootstrap tests based on the IID error assumption are oversized when the errors are conditionally heteroskedastic, whereas the wild bootstrap tests perform well whether the errors are IID or conditionally heteroskedastic. Based on simulation results, the wild bootstrap tests for error autocorrelation perform well when the errors follow a CCC-GARCH process which satisfies the condition for the existence of the fourth moment matrix of the errors. The wild bootstrap tests perform reasonably well even if the condition is not satisfied. The wild bootstrap tests have the same power as the (size-corrected) asymptotic and bootstrap tests in the case of IID errors, but suffer a loss of power if the errors are conditionally heteroskedastic.

The empirical examples demonstrate that the asymptotic and bootstrap tests for error autocorrelation may falsely reject the null hypothesis of no error autocorrelation if the errors are conditionally heteroskedastic. In such cases the tests may erroneously lead to selecting a model with a long lag length. This potential pitfall can be avoided by using the wild bootstrap tests for error autocorrelation. We therefore recommend the WB tests for use in practice if conditional heteroskedasticity is suspected.

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