❺ Exerciții rezolvate

Soluție (Exercițiul 3.3)

$$E\{\varphi[n]\,e[n]\} = \begin{bmatrix} -r_{ey}[1] \\ r_{eu}[1] \end{bmatrix}$$

- Condiții de consistență $\left(\lim_{N\to\infty} \theta_N = \theta\right)$:

$$\begin{cases} a)\ E\{\varphi[n]\varphi^T[n]\} \gneq 0 \quad \text{(inversabilă)} \\ b)\ r_{ey}[1] = 0\ ,\quad r_{eu}[1] = 0 \\ c)\ e = \text{zgomot alb } (0, \lambda^2) \end{cases}$$

- Detalii privind condiția b):

$$r_{ey}[1] = 0 \iff \frac{B(q^{-1})}{A(q^{-1})}\, r_{eu}[1] = 0 \iff$$

$$\iff b\, q^{-1}\left(1 + \alpha_1 q^{-1} + \alpha_2 q^{-2} + \dots\right) r_{eu}[1] = 0 \iff$$

$$\iff b\, r_{eu}[0] + b\,\alpha_1 r_{eu}[-1] + b\,\alpha_2 r_{eu}[-2] + \dots = 0$$

- Condiție suficientă naturală: $\boxed{u = \text{zgomot alb } (0, \sigma_u^2) \text{ necorelat cu } e}$

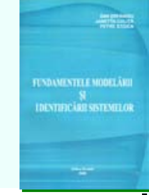
$\Downarrow$

$$r_{ye}[k] = \lambda^2 \delta_0[k]$$

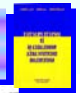
$$r_{e\text{-}y}[0] = r_y[0] - \lambda^2$$

$$r_{yu}[k] = \frac{B(q^{-1})}{A(q^{-1})}\sigma_u^2 \delta_0[k] = b\,\sigma_u^2\left(\delta_0[k-1] + \alpha_1 \delta_0[k-2] + \dots\right)$$

$$\Rightarrow r_{yu}[0] = 0\ ;\quad r_{yu}[1] = b\,\sigma_u^2$$



Soluţie (Exerciţiul 3.3)



$$r_y[k] + a\,r_y[k-1] = b\,r_{uy}[k-1] + r_{ey}[k] + a\,r_{ey}[k-1], \quad \forall k \geq 0$$

$\underline{k=0}$:

$$r_y[0] + a\,r_y[1] = \underbrace{b\,r_{uy}[-1]}_{\substack{b\,r_{yu}[1]= \\ = b^2\sigma_u^2}} + \underbrace{r_{ey}[0]}_{\lambda^2} + \underbrace{a\,r_{ey}[-1]}_{\phi} = $$

$$= b^2\sigma_u^2 + \lambda^2$$

$\underline{k=1}$:

$$r_y[1] + a\,r_y[0] = \underbrace{b\,r_{uy}[0]}_{\phi} + \underbrace{r_{ey}[1]}_{\phi} + \underbrace{a\,r_{ey}[0]}_{a\lambda^2} =$$

$$= a\lambda^2$$

$$(1-a^2)\,r_y[0] = b^2\sigma_u^2 + \lambda^2 - a^2\lambda^2 \iff r_y[0] = \lambda^2 + \frac{b^2}{1-a^2}\sigma_u^2$$

$$r_y[1] = a\left(\lambda^2 - r_y[0]\right) = \frac{ab^2}{a^2-1}\sigma_u^2$$

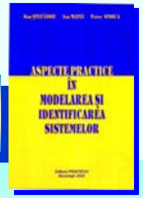
$$E\{\varphi[n]\varphi^T[n]\} = \begin{bmatrix} \frac{b^2}{1-a^2}\sigma_u^2 & 0 \\ 0 & \sigma_u^2 \end{bmatrix}$$

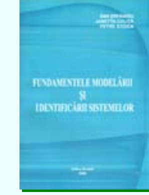
$$E\{\varphi[n]\,y[n]\} = \begin{bmatrix} \frac{ab^2}{a^2-1}\sigma_u^2 \\ b\sigma_u^2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a \\ b \end{bmatrix} = \left[E\{\varphi[n]\varphi^T[n]\}\right]^{-1} \times \left[E\{\varphi[n]\,y[n]\}\right]$$

(se verifică).

❺ Exerciții rezolvate





Exercițiul 3.4

Este posibilă utilizarea MCMMP pentru a determina parametrii necunoscuți ai modelului OE[2,2]? Dacă nu, argumentați răspunsul. Dacă da, determinați ecuațiile de estimare a parametrilor necunoscuți (coeficienți & dispersie zgomot alb), folosind MCMMP.

Soluție

• Forma de regresie liniară:

$$y[n] = \varphi^T[n]\theta + e[n]$$

$$\begin{cases} \varphi^T[n] = \left[e[n-1]-y[n-1] \quad e[n-2]-y[n-2] \;\middle|\; u[n-1] \quad u[n-2] \right] \\ \theta^T = \left[a_1 \quad a_2 \;\middle|\; b_1 \quad b_2 \right] \end{cases}$$

* Ca și în $\boxed{\text{Ex } 3.3}$, se va lucra cu o estimație a z.a., notată prin $\hat{e}$.

Soluție (Exercițiul 3.4)

- Rezultă estimația CMMP:

$$\left[\begin{aligned} \hat{\theta}_N &= R_N^{-1} r_N \\ \hat{\lambda}_N^2 &= \frac{1}{N}\sum_{n=1}^{N}\left(y[n]-\varphi[n]\hat{\theta}_N\right)^2 \end{aligned}\right.,$$

unde:

$$R_N = \left[\begin{array}{cc|cc} r^N_{\hat{e}-y}[0] & r^N_{\hat{e}-y}[1] & r^N_{\hat{e}u}[0]-r^N_{yu}[0] & r^N_{\hat{e}u}[1]-r^N_{yu}[1] \\ r^N_{\hat{e}-y}[1] & r^N_{\hat{e}-y}[0] & r^N_{u\hat{e}}[1]-r^N_{uy}[1] & r^N_{\hat{e}u}[0]-r^N_{yu}[0] \\ \hline r^N_{\hat{e}u}[0]-r^N_{yu}[0] & r^N_{u\hat{e}}[1]-r^N_{uy}[1] & r^N_{u}[0] & r^N_{u}[1] \\ r^N_{\hat{e}u}[1]-r^N_{yu}[1] & r^N_{\hat{e}u}[0]-r^N_{yu}[0] & r^N_{u}[1] & r^N_{u}[0] \end{array}\right]$$

$$r_N = \begin{bmatrix} r^N_{y\hat{e}}[1]-r^N_{y}[1] \\ r^N_{y\hat{e}}[2]-r^N_{y}[2] \\ r^N_{yu}[1] \\ r^N_{yu}[2] \end{bmatrix}$$