

3. (1). 不是分布函数

$\because F_1(x), F_2(x)$ 为分布函数

$\exists X_1$, 当 $x > X_1$, $F_1(x) = 1$

$\exists X_2$, 当 $x > X_2$, $F_2(x) = 1$

$\therefore \exists X_3 = \max(X_1, X_2)$, 当 $x > X_3$

$F_1(x) + F_2(x) = 2 > 1$, 与分布函数定义矛盾

$\therefore F_1(x) + F_2(x)$ 不是分布函数.

(2). $F_1(x), F_2(x)$ 是分布函数

$\therefore a_1 F_1(x) + a_2 F_2(x)$ 始终右连续

$0 \leq F_1(x) \leq 1, 0 \leq F_2(x) \leq 1, a_1 > 0, a_2 > 0$

$\therefore 0 \leq a_1 F_1(x) + a_2 F_2(x) \leq a_1 + a_2 = 1$, 且单调

$\therefore a_1 F_1(x) + a_2 F_2(x)$ 是分布函数

(3). $F_1(x), F_2(x)$ 始终右连续

$$\lim_{\Delta x \rightarrow 0^+} F_1(x + \Delta x) F_2(x + \Delta x) = F_1(x) F_2(x)$$

\therefore 右连续

$0 \leq F_1(x) \leq 1, 0 \leq F_2(x) \leq 1, F_1, F_2$ 单调增

$\therefore 0 \leq F_1(x) F_2(x) \leq 1$

$\therefore F_1(x) F_2(x)$ 是分布函数

b.	X	-1	0	0.5	1
	P	0.125	0.5	0.25	0.125

8. (1)

X	0	1	2	3
P	$\frac{C_{15}^5}{C_{15}^5}$	$\frac{C_{12}^4 \cdot C_3^1}{C_{15}^5}$	$\frac{C_{12}^3 \cdot C_3^2}{C_{15}^5}$	$\frac{C_3^3}{C_{15}^5}$

$$(2) X \sim B(5, 0.2)$$

$$10. (1) P(X=i) = 0.2^{i-1} \cdot 0.8$$

$$(2) P(X=i) = 0.2^{i-r} \cdot 0.8^r \cdot C_i^r \quad (i \geq r)$$

$$11. (1) P(X=6) = \frac{4^6}{6!} e^{-4} = 0.104196$$

$$(2) P(5 \leq X \leq 10) = 0.363031$$

$$14. (1) \begin{cases} P(X=k) \geq P(X=k+1) \\ P(X=k) \leq P(X=k-1) \end{cases} \Rightarrow \lambda - 1 \leq k \leq \lambda$$

①: 当 λ 为整数, $k = \lambda$ 或 $\lambda - 1$

②: 当 λ 为小数, $k = \lfloor \lambda \rfloor$

(2). 由(1)知, 当 $\lambda = 4$ 时, $k = 3$ 或 4 时, 有 $P(X=k)_{\max}$

$$15. (1) P(X=2k-1) = (0.3)^k (0.2)^k \cdot 0.7 \quad (k=1, 2, 3, \dots)$$

$$P(X=2k) = (0.3)^k (0.2)^{k-1} \cdot 0.8 \quad (k=1, 2, 3, \dots)$$

$$(2) P(X=k) = (0.3)^{k-1} (0.2)^{k-1} \cdot 0.7 + (0.3)^k (0.2)^{k-1} \cdot 0.8$$

$$= (0.3)^{k-1} (0.2)^{k-1} \cdot 0.94 \quad (k=1, 2, 3, \dots)$$

$$(3) P(X=k) = (0.3)^k (0.2)^{k-1} \cdot 0.8 + (0.3)^k (0.2)^k \cdot 0.7$$

$$= (0.3)^{k-1} (0.2)^{k-1} \cdot 0.282$$

19. 易知 $P(X < u-t) = P(X > u+t)$

$$F(u-t) = 1 - F(u+t)$$

$$\text{设 } X = u-t$$

$$u+t = 2u-X$$

$$\therefore F(X) = 1 - F(2u-X)$$

$$\therefore \text{当 } X = u$$

$$F(u) = 1 - F(u)$$

$$F(u) = 0.5$$

当为正态分布时, $u=0$

$$\text{则 } \Phi(X) = 1 - \Phi(2 \cdot 0 - X) = 1 - \Phi(-X)$$

$$\Phi(u) = \Phi(0) = 0.5$$

20. (1). $P\{X > -1\}$

$$= P\left\{\frac{X+2}{3} > \frac{-1+2}{3}\right\}$$

$$= 1 - \Phi\left(\frac{1}{3}\right) = 0.3707$$

(2). $P\{-5 \leq X \leq 3\}$

$$= P\left\{\frac{-5+2}{3} \leq \frac{X+2}{3} \leq \frac{3+2}{3}\right\}$$

$$= \Phi\left(\frac{5}{3}\right) - \Phi(-1)$$

$$= 0.7938$$

(3). $P\{0 < X < 5\}$

$$= P\left\{\frac{0+2}{3} < \frac{X+2}{3} < \frac{5+2}{3}\right\}$$

$$= \Phi\left(\frac{7}{3}\right) - \Phi\left(\frac{2}{3}\right)$$

$$= 0.2415$$

$$14). P(|X| > 1)$$

$$= 1 - P(|X| < 1)$$

$$= 1 - P(-1 < X < 1)$$

$$= 1 - P\left(\frac{-1+2}{3} < \frac{X+2}{3} < \frac{1+2}{3}\right)$$

$$= 1 - (\Phi(1) - \Phi(\frac{1}{3}))$$

$$= 0.788$$

$$15). P\{|X+2| < 4\}$$

$$= P\{-4 < X+2 < 4\}$$

$$= P\{-6 < X < 2\}$$

$$= P\left\{-\frac{6+2}{3} < \frac{X+2}{3} < \frac{2+2}{3}\right\}$$

$$= \Phi\left(\frac{4}{3}\right) - \Phi\left(-\frac{4}{3}\right)$$

$$= 0.8164$$

$$16). P\{|X-a| < a\}$$

$$= P\{-a < X-a < a\}$$

$$= P\{0 < X < 2a\}$$

$$= P\left\{\frac{2}{3} < \frac{X+2}{3} < \frac{2a+2}{3}\right\}$$

$$= \Phi\left(\frac{2a+2}{3}\right) - \Phi\left(\frac{2}{3}\right) = 0.01$$

$$a = 0.05$$

22. 每个零件正常工作大于1000的概率

$$P(t > 1000) = 1 - F(1000) = e^{-\lambda x} = e^{-1}$$

大于1500h的概率

$$P(t > 1500) = 1 - F(1500) = e^{-1.5}$$

\therefore 机器工作在1000 ~ 1500h的概率

$$P = (e^{-1})^3 - (e^{-1.5})^3 = 0.038678.$$

$$23. (1) P(X \leq \frac{1}{2}) = \int_{-\infty}^{\frac{1}{2}} f(x) dx = \frac{1}{4}$$

$$\therefore Y \sim B = (3, \frac{1}{4})$$

$$P(Y=2) = C_3^2 \cdot (\frac{1}{4})^2 \cdot (\frac{3}{4}) = \frac{9}{64}$$