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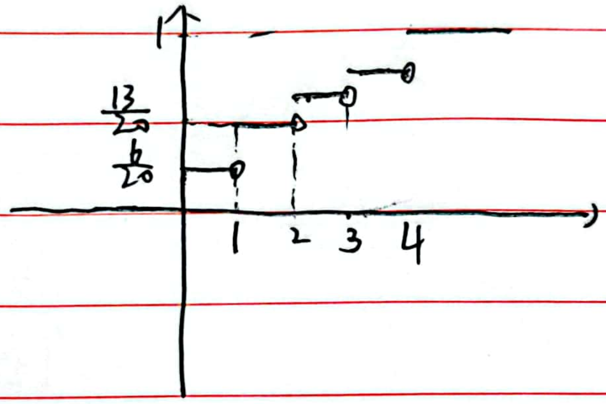
3. $f(x_1,x_2,x_3)=\begin{cases}216x_1x_2x_3(1-x_1)(1-x_2)(1-x_3), & 0<x_1,x_2,x_3<1\\ 0 & \text{其它}\end{cases}$

4. $P\{X_1=x_1, X_2=x_2 \cdots, X_n=x_n\}=\dfrac{\lambda^{\sum_{i=1}^{n}x_i}}{\prod_{i=1}^{n}(x_i!)}\cdot e^{-n\lambda}$

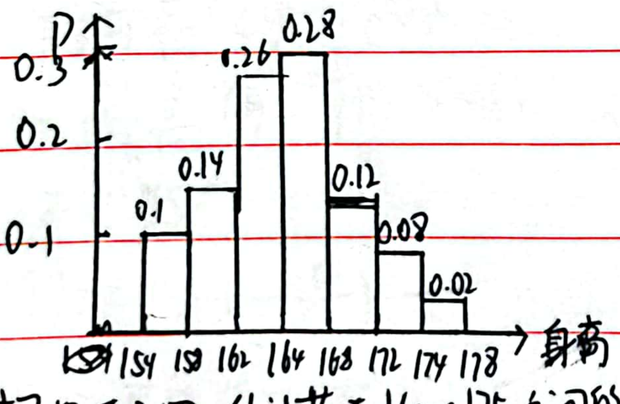
5.

损坏件数k	0	1	2	3	4
损坏k件的概率	$\frac{6}{20}$	$\frac{7}{20}$	$\frac{3}{20}$	$\frac{2}{20}$	$\frac{2}{20}$

$F_{20}(x)=\begin{cases}0, & x<0\\ \frac{6}{20}, & 0\le x<1\\ \frac{13}{20}, & 1\le x<2\\ \frac{16}{20}, & 2\le x<3\\ \frac{18}{20}, & 3\le x<4\\ 1, & x\ge 4\end{cases}$



6.



根据直方图，估计落在160~175之间的概率约为0.75.

8(1) $\bar X=\dfrac{\sum_{i=1}^{l}m_ix_i^*}{\sum_{i=1}^{l}m_i}=\dfrac{\sum_{i=1}^{l}m_ix_i^*}{n}$

$S^2=\frac{1}{n-1}\sum_{i=1}^{l}(x_i^*-\bar X)^2m_i$

(2). $\bar{x}_i=\frac{1}{60}(1\times8+3\times40+6\times10+26\times2)=4$

$S^2=\frac{1}{59}[(1-4)^2\times8+(3-4)^2\times40+(6-4)^2\times10+(26-4)^2\times2]=18.983$

$S=\sqrt{S^2}=4.357.$

9. (1). $Y_i=\frac{X_i-a}{c}$, $X_i=cY_i+a$

$\bar{Y}=\frac{1}{n}\sum_{i=1}^{n}y_i$

$=\frac{1}{n}\sum_{i=1}^{n}\frac{x_i-a}{c}$

$=\frac{1}{cn}\sum_{i=1}^{n}x_i-\frac{a}{c}$

$=\frac{1}{c}\bar{X}-\frac{a}{c}$

$\bar{X}=a+c\bar{Y}$

$S_X^2=\frac{1}{n-1}\sum_{i=1}^{n}(X_i-\bar{X})^2$

$=\frac{1}{n-1}\sum_{i=1}^{n}(cy_i+a-a-c\bar{Y})^2$

$=\frac{1}{n-1}\sum_{i=1}^{n}(cy_i-c\bar{Y})^2$

$=\frac{c^2}{n-1}\sum_{i=1}^{n}(y_i-\bar{Y})^2$

$=c^2S_Y^2$

$E(\bar{Y})=E(\frac{1}{c}\bar{X}-\frac{a}{c})$

$=\frac{1}{c}E(\bar{X})-\frac{a}{c}$

$=\frac{\mu-a}{c}$

$E(S_Y^2)=\frac{1}{c^2}E(S_X^2)=\frac{\sigma^2}{c^2}$

10. 设两次取得的样本均值分别为$\bar{X}_1,\bar{X}_2$

易知 $\bar{X}_1,\bar{X}_2$ 属于独立同分布 $\bar{X}\sim(9,\frac{9}{50})$, $F(X)=\frac{1}{\sqrt{2\pi}\cdot\frac{3\sqrt{2}}{10}}e^{-\frac{(x-9)^2}{2\cdot\frac{9}{50}}}$

$P=\int_{-\infty}^{+\infty} F(x+0.6)-F(x-0.6)\,dx=0.6826.$

11. (1).

$E(\bar{X})=E(\frac{1}{n}\sum_{i=1}^{n}X_i)$

$=\frac{1}{n}\sum_{i=1}^{n}E(X_i)$

$=mp$

$D(\bar{X})=D(\frac{1}{n}\sum_{i=1}^{n}X_i)$

$=\frac{1}{n^2}D(\sum_{i=1}^{n}X_i)$

$=\frac{1}{n^2}\sum_{i=1}^{n}D(X_i)$

$=\frac{1}{n^2}\cdot nmp(1-p)$

$=\frac{mp(1-p)}{n}$

$E(S^2)=E(\frac{1}{n-1}\sum_{i=1}^{n}(X_i-\bar{X})^2)$

$=\frac{1}{n-1}E(\sum_{i=1}^{n}X_i^2-n\bar{X}^2)$

$=\frac{1}{n-1}[\sum_{i=1}^{n}E(X_i^2)-nE(\bar{X}^2)]$

$=\cancel{\frac{1}{n-1}[n(u^2+\sigma^2)-}n$

$=\frac{1}{n-1}[n(m^2p^2+m^2p^2(1-p)^2)-n(m^2p^2+\frac{mp(1-p)}{n})]$

$=\frac{1}{n-1}\cdot(n-1)\cdot mp(1-p)$

$=mp(1-p)$

(2). 同理: $E(\bar{X})=E(X)=\lambda$, $D(\bar{X})=\frac{D(X)}{n}=\frac{\lambda}{n}$, $E(S^2)=D(X)=\lambda$

(3). $E(\bar{X})=E(X)=\frac{a+b}{2}$, $D(\bar{X})=\frac{D(X)}{n}=\frac{(b-a)^2}{12n}$, $E(S^2)=D(X)=\frac{(b-a)^2}{12}$

(4). $E(\bar{X})=E(X)=u$, $D(\bar{X})=\frac{D(X)}{n}=\frac{\sigma^2}{n}$, $E(S^2)=D(X)=\sigma^2$

12. (1) 左 $=\sum_{i=1}^{n}(x_i^2-2ax_i+a^2)$

$=\sum_{i=1}^{n}x_i^2-2a\sum_{i=1}^{n}x_i+na^2$

$=\sum_{i=1}^{n}x_i^2-2an\bar{x}_n+na^2$

右 $=\sum_{i=1}^{n}(x_i-\bar{x}_n)^2+n(\bar{x}_n-a)^2$

$=\sum_{i=1}^{n}x_i^2-n\bar{x}_n^2+n\bar{x}_n^2-2an\bar{x}_n+na^2$

$=\sum_{i=1}^{n}x_i^2-2an\bar{x}_n+na^2$

左=右

(2) 右 $=\bar{x}_n+\frac{1}{n+1}(x_{n+1}-\bar{x}_n)$

$=\bar{x}_n+\frac{1}{n+1}(x_{n+1}+\sum_{i=1}^{n}x_i-\sum_{i=1}^{n}x_i-\frac{1}{n}\sum_{i=1}^{n}x_i)$

$=\bar{x}_n+\frac{1}{n+1}(\sum_{i=1}^{n+1}x_i-\frac{n+1}{n}\sum_{i=1}^{n}x_i)$

$=\bar{x}_n+\bar{x}_{n+1}-\bar{x}_n$

$=\bar{x}_{n+1}=$ 左

易知: $\bar{x}_n=\frac{1}{n}((n+1)\bar{x}_{n+1}-x_{n+1})$ ①

$x_{n+1}-\bar{x}_n=\frac{n+1}{n}(x_{n+1}-\bar{x}_{n+1})$ ②

右 $=\frac{n-1}{n}S_n^2+\frac{1}{n+1}(x_{n+1}-\bar{x}_n)^2$

$=\frac{1}{n}\sum_{i=1}^{n}x_i^2-\bar{x}_n^2+\frac{1}{n+1}(x_{n+1}-\bar{x}_n)^2$

代入 ①.② 得

原式 $=\frac{1}{n}\sum_{i=1}^{n}x_i^2-\frac{1}{n^2}((n+1)\bar{x}_{n+1}-x_{n+1})^2+\frac{1}{n+1}\frac{(n+1)^2}{n^2}(x_{n+1}-\bar{x}_{n+1})^2$

$=\frac{1}{n}\sum_{i=1}^{n}x_i^2-\frac{1}{n^2}[(n+1)^2\bar{x}_{n+1}^2-2(n+1)\bar{x}_{n+1}x_{n+1}+x_{n+1}^2]+\frac{n+1}{n^2}(x_{n+1}^2-2x_{n+1}\bar{x}$

$=\frac{1}{n}\sum_{i=1}^{n}x_i^2-\frac{(n+1)(n+1-1)}{n^2}x_{n+1}^2+\frac{n+1-1}{n^2}x_{n+1}^2$

$=\frac{1}{n}\sum_{i=1}^{n}X_i^2-\frac{n+1}{n}\overline{X_{n+1}}^2+\frac{1}{n}X_{n+1}^2$

$=\frac{1}{n}(\sum_{i=1}^{n+1}X_i^2-n+1\overline{X_{n+1}}^2)$

$=\frac{1}{n}\sum_{i=1}^{n+1}(X_i-\overline{X_{n+1}})^2$

$=S_{n+1}^2$

13. $\overline{Z_{n+m}}=\frac{1}{n+m}\sum_{i=1}^{n+m}Z_i$

$=\frac{1}{n+m}(\sum_{i=1}^{n}X_i+\sum_{i=1}^{m}Y_i)$

$=\frac{n\bar{X}_n+m\bar{Y}_m}{n+m}$

右$=\frac{\sum_{i=1}^{n}X_i^2+\sum_{i=1}^{m}Y_i^2-n\bar{X}_n^2-m\bar{Y}_n^2}{n+m-1}+\frac{nm}{(n+m)(n+m-1)}(\bar{X}_n-\bar{Y}_n)^2$

$=\frac{\sum_{i=1}^{m+n}Z_i^2}{n+m-1}-\frac{(n+m)(n\bar{X}_n^2+m\bar{Y}_n^2)-nm(\bar{X}_n-\bar{Y}_n)^2}{(n+m)(n+m-1)}$

$=\frac{\sum_{i=1}^{n+m}Z_i^2}{n+m+1}-\frac{n^2X_n^2+2mn+m^2\bar{Y}_n^2}{(n+m)(n+m-1)}$

$=\frac{\sum_{i=1}^{n+m}Z_i^2}{n+m+1}-\frac{(n+m)\overline{Z_{n+m}}^2}{n+m-1}$

$=\frac{\sum_{i=1}^{n+m}(Z_i-\overline{Z_{m+n}})^2}{n+m+1}$

$=S_z^2$

14. (-4, -2.1, -2.1, -0.1, -0.1, 0, 0, 1.2, 1.2, 2.01, 2.22, 3.2, 3.21)

1.2, 7.21

15. 易知 $n\bar{X}\sim P(n\lambda)$

则 $P\{\bar{X}=\frac{k}{n}\}=\frac{(n\lambda)^k}{k!}e^{-n\lambda}$, $k=0,1,2\cdots$

16. 易知 $E(\bar{X})=E(X)=\frac{\alpha}{\lambda}$, $D(\bar{X})=\frac{D(X)}{n}=\frac{\alpha}{n\lambda^2}$, 且 $\bar{X}\sim\Gamma(\alpha',\lambda')$ 则 $\bar{X}\sim\Gamma(n\alpha,n\lambda)$

17. $Y=\frac{1}{\sigma^2}\sum_{i=1}^{n}(X_i-\mu)^2$

$=\sum_{i=1}^{n}\frac{(X_i-\mu)^2}{\sigma^2}$

$\frac{X-\mu}{\sigma}\sim N(1,0)$

$\therefore Y\sim\chi^2(n)$

18. $\frac{1}{m}(\sum_{i=1}^{m}X_i)^2\sim\chi^2(1)$

$\frac{1}{n-m}(\sum_{i=m+1}^{n}X_i)^2\sim\chi^2(1)$

根据卡方分布的可加性.

$Y\sim\chi^2(2)$

19. (1). $P\{0.3<\frac{S^2}{\sigma^2}<2.114\}$

$=P\{2.7<\frac{9S^2}{\sigma^2}<19.026\}$

$Y=\frac{(n-1)S^2}{\sigma^2}\sim\chi^2(9)$

$P=P(Y>2.7)-P(Y>19.026)=0.975-0.025=0.95$

(2) $D(Y)=18$

$D(S^2)=D(\frac{\sigma^2}{9}Y)=\frac{\sigma^4}{81}\cdot 18=\frac{2}{9}\sigma^4$

20 (1).

$\sum_{i=0}^{n}X_i\sim N(0,n\sigma^2)$

$\frac{\sum_{i=0}^{n}X_i}{\sqrt{n}\sigma}\sim N(0,1)$

$Y=\frac{1}{n}(\sum_{i=0}^{n}X_i)^2=(\frac{\sum_{i=0}^{n}X_i}{\sqrt{n}\sigma})^2\cdot\sigma^2$

$\therefore\frac{1}{\sigma^2}Y\sim\chi(1)$, $f_Y(y)=\begin{cases}\frac{1}{\sqrt{2\pi}\sigma}y^{-\frac{1}{2}}e^{-\frac{y}{2\sigma^2}}, & y>0\\ 0, & y\leq 0\end{cases}$

20. (2). $\frac{X_i}{\sigma} \sim N(0,1)$

$Y_2 = \frac{1}{n}\sum_{i=1}^{n} X_i^2 = \frac{\sigma^2}{n}\sum_{i=1}^{n}\left(\frac{X_i}{\sigma}\right)^2$

$\frac{nY_2}{\sigma^2} \sim \chi^2(n)$

$$f_Y(y) = \begin{cases} \frac{\sqrt{n}\, y^{-\frac{1}{2}} e^{-\frac{ny}{2\sigma^2}}}{\sqrt{2\pi}\,\sigma}, & y > 0 \\ 0, & \text{其它} \end{cases}$$

21. $X \sim t(n)$，则存在 $Y \sim N(0,1)$，$Z \sim \chi^2(n)$

$X = \frac{Y}{\sqrt{Z/n}}$ $\quad X^2 = \frac{Y^2}{Z/n}$

$Y^2 \sim \chi^2(1)$ $\quad X^2 = \frac{Y^2/1}{Z/n} = F(1,n)$

22. (1). $\sum_{i=1}^{n} X_i \sim N(0, n\sigma^2)$，$\frac{X_i}{\sigma} \sim N(0,1)$

$\sum_{i=1}^{n} \frac{X_i}{\sqrt{n}\sigma} \sim N(0,1)$，设 $Y = \frac{\sum_{i=1}^{n} X_i}{\sqrt{n}\sigma}$，

$\sum_{i=n+1}^{n+m}\left(\frac{X_i}{\sigma}\right)^2 \sim \chi^2(m) = \frac{1}{\sigma^2}\sum_{i=n+1}^{n+m} X_i^2$，$Z = \frac{1}{\sigma^2}\sum_{i=n+1}^{n+m} X_i^2$

$\therefore Y_1 = \frac{\sqrt{m}}{\sqrt{n}} \cdot \frac{\sqrt{n}\,\sigma \cdot Y}{\sqrt{\sigma^2 Z}} = \frac{Y}{\sqrt{Z/m}} \sim t(m)$

(2). $Y_2 = \frac{\sum_{i=1}^{n} X_i^2/(n\sigma^2)}{\sum_{i=n+1}^{n+m} X_i^2/(m\sigma^2)} = \frac{\chi(n)/n}{\chi(m)/m} = F(n,m)$

23. $X_{n+1} - \bar{X} \sim (0, \frac{n+1}{n}\sigma^2)$

$\frac{X_{n+1}-\bar{X}}{\sqrt{\frac{n+1}{n}\sigma^2}} = \sqrt{\frac{n}{n+1}} \cdot (X_{n+1}-\bar{X}) \cdot \frac{1}{\sigma} \sim N(0,1)$

$Y_i \sim \chi^2(2)$

$Y_1, Y_2 \cdots Y_n$ 独立

$\sum_{i=1}^{n} Y_i \sim \chi^2(2n)$

26. $X_1+X_2 \sim (0, 2\sigma^2)$ $\quad \frac{X_1+X_2}{\sqrt{2}\sigma} \sim (0, 1)$

$X_1-X_2 \sim (0, 2\sigma^2)$ $\quad \frac{X_1-X_2}{\sqrt{2}\sigma} \sim (0, 1)$

$\left(\frac{X_1+X_2}{\sqrt{2}\sigma}\right)^2 \sim \chi(1)$

$\left(\frac{X_1-X_2}{\sqrt{2}\sigma}\right)^2 \sim (\chi, 1)$

$$Y = \left(\frac{X_1+X_2}{X_1-X_2}\right)^2 = \frac{\left(\frac{X_1+X_2}{\sqrt{2}\sigma}\right)^2}{\left(\frac{X_1-X_2}{\sqrt{2}\sigma}\right)^2} = \frac{\chi(1)}{\chi(1)} = F(1,1)$$

27. $X \sim U(0,1)$

$$f_X(x) = \begin{cases} 1, & 0<x<1 \\ 0, & 其他 \end{cases} \qquad F_{X_i}(x) = \begin{cases} x, & 0<x<1 \\ 0, & 其他 \end{cases}$$

令 $Y_i = -\ln X_i$

$$F_{Y_i}(y) = P\{Y<y\} = P\{-\ln X_i < y\}$$
$$= P\{X \geq e^{-y}\}$$
$$= 1 - P\{X \leq e^{-y}\}$$
$$= 1 - e^{-y}$$

$f_Y(y) = e^{-y}$, $Y_i \sim exp(1)$

由 25 题结论知

当 $Y \sim Exp(\lambda)$, $Y = 2\sum_{i=1}^{n} Y_i \sim \chi^2(2n)$