8.1: 第一类错误: $P_1=P\{\bar X\geq 0.5|H_0\}$

第二类错误 $P_2=P\{\bar X\leq 0.5|H_1\}$

$4(\bar X-\mu)\sim N(0,1)$

$P_1=P\{\bar X\geq 0.5|\mu=0\}$

$=P\{4\bar X\geq 2\}=\Phi(-2)=0.0228$

$\cancel{=1-\Phi(8)=\Phi(-8)=0.0228}$

$P_2=P\{\bar X\leq 0.5|\mu=1\}$

$=P\{4(\bar X-1)\leq -2\}=\Phi(-2)=0.0228$

$\cancel{=\Phi(-8)=0.0228}$ $\therefore P_1=0.0228,\ P_2=0.0228$

8.2. $H_0:\mu=32.25$, $H_1:\mu\neq 32.25$.

若H_0, $\frac{\bar X-\mu}{\sqrt{\frac{\sigma^2}{n}}}\sim N(0,1)$

$P\left\{\left|\frac{\bar X-\mu}{\sqrt{\frac{\sigma^2}{n}}}\right|<u_{\frac{\alpha}{2}}\right\}=1-\alpha$

$\cancel{-u_{\frac{\alpha}{2}}<\frac{\sqrt n(\bar X-\mu)}{6}<u_{\frac{\alpha}{2}}}$

$\cancel{\bar X\in\left(\frac{-6u_{\frac{\alpha}{2}}}{\sqrt n}+\mu,\ \frac{6u_{\frac{\alpha}{2}}}{\sqrt n}+\mu\right)}$ ~~为接受域~~

$\frac{\bar X-\mu}{\sqrt{\frac{\sigma^2}{n}}}=-2.501$, 当$\alpha=0.05$, $u_{\frac{\alpha}{2}}=1.96$, $|-2.501|>1.96$, 不能

当$\alpha=0.01$, $u_{\frac{\alpha}{2}}=2.57$, $|-2.501|<2.57$, 能

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$H_0: \sigma=0.1$, $H_1: \sigma\neq 0.1$

4. $\frac{\sqrt{n}(\bar{X}-\mu)}{\sigma} \sim N(0,1)$

$\left|\frac{\sqrt{n}(\bar{X}-\mu)}{\sigma}\right| < u_{\frac{\alpha}{2}}$ 即认可 H_0

~~$-u_{\frac{\alpha}{2}} < \frac{\sqrt{n}(\bar{X}-\mu)}{\sigma} < u_{\frac{\alpha}{2}}$~~

~~$\sigma > \frac{\quad}{u_{\frac{\alpha}{2}}}$~~ $\left|\frac{\sqrt{n}(\bar{X}-\mu)}{\sigma}\right| = |0.537| < u_{\frac{\alpha}{2}}$

∴可以认可 H_0，可认为标准差为0.1

5. ~~$\frac{\sqrt{n}(\bar{X}-\mu)}{\sigma} \sim N(0,1)$~~ $n=5$

$\frac{(n-1)S^2}{\sigma^2} \sim t(n-1) \sim t(4)$

$\frac{(n-1)S^2}{\sigma^2} = 13.507 > t_{0.025} = 2.7764$

∴不正常

7. ~~假设~~ H_0: 甲乙无差异 ($\mu_1=\mu_2$)，$H_1: \mu_1\neq\mu_2$

$\frac{(\bar{X}-\bar{Y})-(\mu_1-\mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}} \sim N(0,1)$

$\frac{(\bar{X}-\bar{Y})-(\mu_1-\mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}} = -0.262 > u_{0.975} = -1.96$

∴无显著差异

8. H_0: 无差异，$\sigma_1^2=\sigma_2^2$，H_1: 有差异.

$\frac{\bar{X}-\bar{Y}}{\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}\,S_W} \sim t(n_1+n_2-2)$

$S_W = \sqrt{\frac{(n_1-1)S_{W_1}^2+(n_2-1)S_{W_2}^2}{n_1+n_2-1}}$

$\frac{\sigma_2^2 S_{W_1}^2}{\sigma_1^2 S_{W_2}^2} \sim F(7,8)$

$\frac{S_{W_1}^2}{S_{W_2}^2} = 3.75 < F_{0.025}(7,8) = 4.53$

∴无显著差异

9. ~~$H_0: \sigma_1=\sigma_2=\sigma^2$, $H_1: \sigma_1^2\neq\sigma_2^2$~~

~~$\frac{(\bar{X}_1-\bar{X}_2)-(\mu_1-\mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}}$~~

~~$\frac{S_1^2}{S_2^2} \sim F(7,8)$~~

9. $H_0: \sigma_1^2=\sigma_2^2=\sigma^2$，$H_1: \sigma_1^2\neq\sigma_2^2$

$\frac{S_{W_1}^2}{S_{W_2}^2} \sim F(n_1-1,n_2-1) \sim F(9,9)$

$1.494 < 4.03$

$F_{0.975}(9,9) < \frac{S_{W_1}^2}{S_{W_2}^2} < F_{0.025}(9,9)$

∴无显著差异

10. $H_0: \sigma_1=\sigma_2; H_1: \sigma_1\neq\sigma_2$.

$\frac{\sigma_2^2 S_{W1}^2}{\sigma_1^2 S_{W2}^2} \sim F(n_1-1, n_2-1) \sim F(7,8)$

当 $\sigma_1=\sigma_2$

$\frac{S_{W1}^2}{S_{W2}^2} = 3.474 < F_{0.025}(7,8) = 4.53$

∴服从同一分布.

12. $H_0: u \geq 5.10, H_1: u < 5.10$, 设 $u_0 = 5.10$

$\frac{\bar{X}-u_0}{\sqrt{\frac{\sigma^2}{n}}} \sim N(0,1)$

$\cancel{\frac{\bar{X}-u_0}{\sqrt{\frac{\sigma^2}{n}}}} \frac{\sqrt{n}(\bar{X}-u_0)}{S} \sim t(n-1) \sim t(19)$

$\frac{\sqrt{n}(\bar{X}-u_0)}{S} = 2.233 > t_{0.025}(19) = -2.093$

∴超过5.10

13. H_0: 不显著偏大, H_1: 显著偏大

$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1) \sim \chi^2(8)$

$\frac{(n-1)S^2}{\sigma^2} = 15.68 > \chi^2_{0.8}(8)$

∴显著偏大

14. $\frac{\bar{X}-\bar{Y}-2}{\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} \cdot S_W} \sim t(22)$

$S_W = \sqrt{\frac{(n_1-1)S_{1n}^2+(n_2-1)S_{2n}^2}{n_1+n_2-2}}$

$\frac{\bar{X}-\bar{Y}-2}{\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} \cdot S_W} = 4.362 > t_{0.05}(22) = 0.7171$

∴拒绝 H_0

15. $H_0: \frac{\sigma_1^2}{\sigma_2^2} \leq 1$, $H_1: \frac{\sigma_1^2}{\sigma_2^2} > 1$

$\frac{\sigma_2^2 S_{W1}^2}{\sigma_1^2 S_{W2}^2} \sim F(n_1-1, n_2-1) \sim F(5,8)$

$\frac{S_{W1}^2}{1 \cdot S_{W2}^2} = 0.686 \leq F_{0.05}(5,8) = 3.69$

∴接受 H_0, 甲不比乙高.

18. $H_0: P=0.3, H_1: P\neq 0.3$, $p_0 = 0.3$

$\frac{\bar{P}-P_0}{\sqrt{\frac{P_0(1-P_0)}{n}}} \sim N(0,1)$

$\frac{\bar{P}-\bar{P_0}}{\sqrt{\frac{P_0(1-P_0)}{n}}} = \cancel{0.982}\ -1.597$

$u_{0.975} = 1.96 > |-1.597|$

∴无显著影响

$H_0: u \geq 1500,\ H_1: u < 1500$

19. $\frac{\sqrt{n}(\bar{X}-u)}{S} \sim t(n-1) \sim t(9)$

$t(9)_{1-0.05} = -1.8331$

$\frac{\sqrt{n}(\bar{X}-u)}{S} = -0.439 > t(9)_{0.95}$

∴ 超过1500h

20. $H_0: u_1-u_2 \geq 0,\ H_1: u_1-u_2 < 0$

$\frac{(\bar{X}-\bar{Y})-\cancel{u}}{S_w\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} \sim t(n_1+n_2-2) \sim t(28)$

$S_w = \sqrt{\frac{(n_1-1)S_1^2+(n_2-1)S_2^2}{n_1+n_2-2}} = 1.997$

$t_{0.95}(28) = -1.7011 < 1.997$

∴ 该药有效.

22. $\sum_{i=1}^{11} \frac{(m_i-np_i)^2}{np_i} \sim \chi(10)$

$\hat{\lambda} = E(X) = 3.88$，则 $p_i = \frac{\hat{\lambda}^i}{i!} e^{-\lambda}$

$\sum_{i=1}^{11} \frac{(m_i-np_i)^2}{np_i} \leq \chi(10)_{0.025} = 20.483$

∴ 服从泊松分布

24. $\sum_{i=1}^{4} \frac{(m_i-np_i)^2}{np_i} \sim \chi(3)$

$p_1 = 1-e^{-0.5}$ $\quad p_3 = e^{-1}-e^{-1.5}$

$p_2 = e^{-0.5}-e^{-1}$ $\quad p_4 = e^{-1.5}$

24. $E(X)=u$

$\hat{u}=\bar{X}=10.0024$

$\hat{\sigma}^2=\frac{1}{n}\sum_{i=1}^{100}(X_i-\bar{X})^2$

则 $\sum_{i=1}^{8}\frac{(m_i-np_i)^2}{np_i} \sim \chi^2(7)$

$\cancel{\chi_{0.975}(7) < \sum_{i=1}^{8}\frac{(m_i-np_i)^2}{np_i} < \chi^2_{0.025}(7)}$

$\sum_{i=1}^{8}\frac{(m_i-np_i)^2}{np_i} > \chi^2_{0.025}(7)$

∴不符合正态分布