

$$1-4. (1). 3n^2 + 10n = O(n^2)$$

$$(2). \cancel{n^2} / 10 + 2^n = O(2^n)$$

$$(3). 21 + 1/n = O(1)$$

$$(4). \log n^3 = O(\log n)$$

$$(5). 10 \log 3^n = O(n)$$

1-9.

$$(1). f(n) = \log n^2 = \Theta(\log n + 5)$$

$$f(n) = \log n^2 = 2 \log n$$

$$\exists C_1, n_1, \text{当 } n > n_1,$$

$$\exists C_1 > 2, \text{当 } n > n_1, f(n) < C_1(\log n + 5)$$

$$\exists C_2 < 2, n_2 > 0, \text{当 } n > n_1, C_2(\log n + 5) < f(n) = 2 \log n$$

$$\therefore f(n) = \Theta(g(n))$$

$$(2). f(n) = O(\sqrt{n})$$

$$\exists C_1, n_1, \text{当 } n > n_1, f(n) < C_1 g(n)$$

$$\therefore f(n) = O(\sqrt{n}) = O(g(n))$$

$$(3). f(n) = n = \Omega(\log^2 n)$$

$$\exists C_1, n_1, \text{当 } n > n_1, n > C_1(\log^2 n)$$

$$(4). f(n) = n \log n + n = \Omega(\log n)$$

$$\exists C_1, n_1, \text{当 } n > n_1, n \log n + n > C_1 \log n$$

$$(5). f(n) = 10 = \Theta(\log 10)$$





$$\exists C_1 > \frac{1}{\log 10}, 10 < C_1 \log 10$$

$$\exists C_2 < \log 10, 10 > C_2 \log 10$$

$$\therefore f(n) = \Theta(g(n))$$

$$(b). f(n) = \log^2 n = O(\log n)$$

$$\exists C_1 > 0, n_1, \text{当 } n > n_1, \log^2 n > C_1 \log n.$$

$$(7). f(n) = O(g(n))$$

$$\exists C_1 > 0, n_1, \text{当 } n > n_1, 2^n > C_1 n^2$$

$$(8). f(n) = \Omega(g(n))$$

$$\exists C_1 > 0, n_1, \text{当 } n > n_1, 2^n < C_1 3^n \Rightarrow C_1 > \left(\frac{2}{3}\right)^n$$

$$1 - 10.$$

$$\lim_{n \rightarrow \infty} \frac{n!}{n^n} = \lim_{n \rightarrow \infty} \frac{n}{n} \cdot \frac{n-1}{n} \cdot \frac{n-2}{n} \cdots \frac{1}{n} = 0$$

$$\therefore \exists C_1, n_1, \text{当 } n > n_1, n! < C_1 n^n$$

$$\therefore n! = O(n^n)$$

