NUMERICAL TECHNIQUES

Tutorial 1: **Errors**

QUESTION 1:
Compute the absolute and relative error in approximations of p by p^* in the cases

i. $p = e, \quad p^* = 2.718,$ **ii.** $p = 7^\pi, \quad p^* = 450,$
iii. $p = 9!, \quad p^* = \sqrt{18\pi}\,(9/e)^9$

QUESTION 2:
Suppose p^* must approximate p with relative error at most 10^{-3}. Find the largest interval in which p^* must lie for the following values.

i. $p = -0.5,$ **ii.** $p = 451.$

QUESTION 3:
Write the following numbers in floating point form, use a 6 significant digit mantissa along with rounding.

i. 1234.5678 **ii.** 12345687890
iii. π **iv.** $-\sqrt{3}/3^3$
v. $1/e^7$

QUESTION 4:
Use four significant figure arithmetic with rounding to find the most accurate approximations possible for the roots of the quadratic.

$$\frac{1}{3}x^2 + \frac{123}{4}x + \frac{1}{6} = 0.$$

QUESTION 5:
Given the function

$$f(x) = 1.01e^{4x} - 4.62e^{3x} - 3.11e^{2x} + 12.2e^x - 1.99$$

using 3 significant figure arithmetic with rounding, the assumption $e^{1.53} = 4.62$ and the fact that $e^{nx} = (e^x)^n$.

(i) Evaluate $f(1.53)$ directly from the function.

(ii) Rewrite the function in nested form and recalculate $f(1.53)$.

(iii) Give the relative errors for the solutions from parts 1 and 2 when compared to the true three figure result $f(1.53) = -7.61$.

QUESTION 6:
The two-by-two linear system

$$\begin{aligned} ax &+ by &&= e, \\ cx &+ dy &&= f, \end{aligned}$$

where a, b, c, d, e, f are given, can be solved for x and y as followes

$$\begin{aligned} \text{Set } m &= \frac{c}{a}, \quad \text{provided } a \neq 0; \\ d_1 &= d - mb; \\ f_1 &= f - me; \\ y &= \frac{f_1}{d_1}; \\ x &= \frac{e - by}{a}. \end{aligned}$$

Use this procedure and four significant figure arithmetic with rounding to solve the following linear systems.

(i)

$$\begin{aligned} 1.130x &- 6.990y &&= 14.20, \\ 8.110x &+ 12.20y &&= -0.1370. \end{aligned}$$

(ii)

$$\begin{aligned} 1.013x &- 6.990y &&= 14.22, \\ -18.11x &+ 112.2y &&= -0.1376, \end{aligned}$$

QUESTION 7:
Repeat the above question using four significant figure arithmetic with chopping.