Quantum Computing for Sustainable Investments

1. Context and Relevance

In today's evolving financial landscape, there is a growing need for investors, from individuals to large institutions, to align their strategies with long-term, responsible objectives. Sustainable finance has emerged as a central concept in this shift, aiming to direct global capital toward projects and companies that generate both economic returns and positive environmental or social outcomes. It underpins international goals such as the UN's Sustainable Development Goals (SDGs), helping mobilize the investments needed to address challenges like climate change, biodiversity loss, and social inequality. Achieving these objectives requires integrating non-financial data, such as ESG scores (Environmental, Social, and Governance), into traditional financial decision-making. This is a task that increases the complexity of portfolio analysis and optimization.

Classically, portfolio optimization is approached through mathematical and statistical frameworks such as mean-variance analysis, factor modeling, and stochastic optimization. These methods aim to balance expected return against risk, often by minimizing variance or maximizing a risk-adjusted metric. However, as the number of assets, constraints, and objectives increases, which is particularly the case when incorporating sustainability factors like ESG metrics, the problem becomes computationally demanding and difficult to solve exactly. In addition, classical solvers often rely on approximations that can miss globally optimal solutions. Quantum computing may help overcome some of these challenges and allow for more nuanced optimization that accounts for both financial and sustainability criteria without the exponential growth in computational cost faced by classical methods.

2. Challenge Overview

In this hackathon challenge, participants are tasked with developing a quantum-enhanced portfolio management solution that utilizes ESG scores, with a primary focus on the environmental dimension (E). The developed tool aims at empowering investors from retail investors to institutional bodies like banks, pension funds, family offices, and national funds to make well-informed investment decisions that support sustainable finance and broader ESG goals.

Participants are encouraged to:

- As an initial step, start with a simple portfolio optimization model assuming uncorrelated assets, before moving on to a non-trivial covariance matrix Σ.
- Include a risk-return trade-off in terms of a modified objective function or in terms of an additional constraint.
- Determine a distance measure for ESG scores and include them as a constraint.
- Include a budget constraint, too.
- Analyze the tradeoffs between returns, risk and ESG performance
- Incorporate classical pre- or post-processing steps and justify them, e.g. to reduce the amount of data fed into the quantum subroutines of the algorithm.
- Demonstrate a compelling path toward a potential quantum advantage, with a clear articulation of the challenges that must be overcome.
- (Optional) investigate other relevant statistical parameters using quantum algorithms like correlations between assets etc.

3. Detailed Problem Description

a. Mathematical Problem / Model Description

Let n be the total number of assets to choose from and let r_i be the random variable for the return of asset i and let $\mu_i = E[r_i]$ be the expected return. Furthermore, let Σ be the covariance matrix of the returns, i.e. $\Sigma_{ij} = \mathrm{Cov}(r_i, r_j) = E[(r_i - \mu_i)(r_j - \mu_j)]$. Notice that the diagonal terms of Σ are the variances of each asset's return, i.e. its individual risk.

Now, the parameters we can play with are the weights of each asset, i.e. real numbers $w_i \in [0,1]$ summing up to one. The standard mean–variance optimization problem can then be written as a quadratic program

$$
\begin{aligned}
\min_{\mathbf{w}} \quad & \mathbf{w}^\top \Sigma \mathbf{w} \\
\text{s.t.} \quad & \mu^\top \mathbf{w} \geq R_t, \\
& \sum_{i=1}^{n} w_i = 1, \\
& w_i \geq 0 \quad \forall i,
\end{aligned}
$$

where R_t is the target expected return.

This is just one way to formulate the problem. Alternative formulations exist, e.g. such that include a risk-return trade-off through a multiplicative parameter q in the objective function, rather than requiring a target expected return:

$$\min_{\mathbf{w}} \ \mathbf{w}^\top \Sigma \mathbf{w} - q\, \mu^\top \mathbf{w}$$

So how to include ESG criteria and possibly further data enhancing the simple ESG score? There are multiple ways of doing so. Probably the easiest is a constraint-based formulation, where certain criteria on ESG performance are included in an additional constraint. There is also the option to include the ESG performance of the portfolio in the objective function, then doing a multi-objective optimization. Lastly, we also mention the option of introducing a penalty term in the expected return that accounts for bad ESG performance.

We encourage the participants to think of other approaches or clever combinations of the above ones.

Given daily prices of the assets from the past, one may wonder how to approximate the distribution of the asset random variables r_i. We encourage the participants to find ways of doing so, keeping in mind that the raw data that is given as an input is typically made of historical trading data rather than the distributions directly. The generation of the distribution from historical data will have to be calculated in a classical preprocessing step, if one goes with the above formulation of the portfolio optimization problem.

b. Proposed Quantum Approach

There are multiple quantum approaches that can be applied to solving portfolio optimization. Depending on the approach the problem has to be reformulated, e.g. as a QUBO. We leave it to the teams to reformulate the problem according to the approach they are aiming at.

Quantum computing approaches for this challenge should take ESG data as well as historical data of asset prices and output an optimal allocation of investments w.r.t. an optimization approach of their choice. Suggested approaches include:

- Variational quantum algorithms, such as QAOA or VQE
- Variants of other quantum optimisation algorithms, including quantum-inspired algorithms

Both suggested approaches directly relate to finding extrema of the objective function described above in the mathematical problem description.

A deeper investigation into one quantum computing approach is preferred over a superficial overview of many.

The participants are encouraged to think beyond the optimization parameters listed in the challenge and think about means to compute other statistical relevant factors, like correlation between assets, etc. Lastly, when it comes to the working prototype, a key deliverable of this hackathon challenge, NISQ algorithms will likely be the only viable option.

4. Expected Deliverables

- **Working Prototype:** A functional model that takes data as input and produces a solution to the posed challenge. We recommend a step-by-step approach, beginning with a proof-of-concept for a very simple toy model before expanding to a full model, e.g. tackling larger data sets or problem instances.
- **Technical Explanation:** Participants must be able to explain the quantum algorithms used, the data model, and the rationale for choosing the specific approach. The focus should be on the clear and deliberate use of methodology and the benchmarking process, rather than solely on high-performing results. Participants should benchmark their quantum model against standard classical solutions. If more than one model was developed during the hackathon the team may also benchmark against those.
- **Classical Bottleneck Analysis and Justification for Quantum Computing Approach:** An analysis of standard approaches and their bottlenecks. This is the basis for arguing why a quantum computing approach may be beneficial for the problem at hand.
- **SDG Impact Assessment:** An assessment of the impact of your solution on the main SDG tackled by this challenge. Your arguments should be backed by references to publications and publicly available data. Also: What other SDGs may be impacted by your solution (both to the positive or to the negative)? List them and argue for why your solution is relevant also to those.
- **Business Case:** A concise plan outlining the commercial viability, target users, and market strategy. Think about how your solution could be commercialized, how you would approach commercialization, what would the product be, who would be the customers, how you would make money with it.
  Also: In what other fields / problems could your solution be useful?
- **Pitches & Technical Deep Dive:** At the end of the hackathon you will have the chance to present your solution in a final presentation as a 5 min pitch with slides. Please explain the solution's potential and its future roadmap, including an honest discussion of the challenges that need to be addressed before it can be a fully realized product, all based on the above deliverables. The pitch will be followed by a 3 min Q&A with the jury for the respective challenge, which consists of 50% technical experts and 50% business experts.
  The overall 10 best teams across all challenges get to show the same 5 min pitch in

front of all participants and all jury members again, this time without Q&A.

Importantly: A deeper investigation into one quantum computing approach is preferred over a superficial overview of many. The goal is not to find a single "best" model, but to demonstrate a clear understanding of the chosen approach. The solution should focus on demonstrating how the quantum approach can be applied to the problem, specifically addressing the challenge specified above. The goal is to articulate a compelling case for a future computational speed-up or greater accuracy, rather than proving an immediate advantage.

Target audience: The project should be compelling to technical professionals who are interested in exploring how quantum computing can be applied to their domain.

5. Resources

[Context] SDG 8: https://sdgs.un.org/goals/goal8
[Context, ESG score] https://www.investopedia.com/company-esg-score-7480372
[Literature, Portfolio Optimization] https://www.nature.com/articles/s41598-023-45392-w
[Literature, VQE]
https://quantum.cloud.ibm.com/learning/en/courses/quantum-diagonalization-algorithms/vqe
[Literature, QAOA]
https://quantum.cloud.ibm.com/docs/en/tutorials/quantum-approximate-optimization-algorithm