APPENDIX D: SHORT TABLE OF INTEGRALS

Elementary forms

BASIC RULES

1. **Constant rule*** $\int 0\,du = 0 + C$

2. **Power rule** $\int u^n\,du = \dfrac{u^{n+1}}{n+1}; \quad n \neq -1$

$$\int u^n\,du = \ln|u|; \quad n = -1$$

3. **Natural exponential rule** $\int e^u\,du = e^u$

4. **Logarithmic rule** $\int \ln|u|\,du = u\ln|u| - u$

TRIGONOMETRIC RULES

5. $\int \sin u\,du = -\cos u$

6. $\int \cos u\,du = \sin u$

7. $\int \tan u\,du = -\ln|\cos u| = \ln|\sec u|$

8. $\int \cot u\,du = \ln|\sin u|$

9. $\int \sec u\,du = \ln|\sec u + \tan u|$

10. $\int \csc u\,du = -\ln|\csc u + \cot u|$

11. $\int \sec^2 u\,du = \tan u$

12. $\int \csc^2 u\,du = -\cot u$

13. $\int \sec u\tan u\,du = \sec u$

14. $\int \csc u\cot u\,du = -\csc u$

EXPONENTIAL RULE

15. $\int a^u\,du = \dfrac{a^u}{\ln a} \qquad a > 0, a \neq 1$

HYPERBOLIC RULES

16. $\int \cosh u\,du = \sinh u$

17. $\int \sinh u\,du = \cosh u$

18. $\int \tanh u\,du = \ln\cosh u$

19. $\int \coth u\,du = \ln|\sinh u|$

20. $\int \operatorname{sech} u\,du = \tan^{-1}(\sinh u)$ or $2\tan^{-1} e^u$

21. $\int \operatorname{csch} u\,du = \ln\left|\tanh\dfrac{u}{2}\right|$

INVERSE RULES

22. $\int \dfrac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\dfrac{u}{a}$

23. $\int \dfrac{du}{\sqrt{u^2-a^2}} = \cosh^{-1}\dfrac{u}{a}$

24. $\int \dfrac{du}{a^2+u^2} = \dfrac{1}{a}\tan^{-1}\dfrac{u}{a}$

25. $\int \dfrac{du}{a^2-u^2} = \begin{cases} \frac{1}{a}\tanh^{-1}\frac{u}{a} & \text{if } \left|\frac{u}{a}\right| < 1 \\ \frac{1}{a}\coth^{-1}\frac{u}{a} & \text{if } \left|\frac{u}{a}\right| > 1 \end{cases}$

26. $\int \dfrac{du}{u\sqrt{u^2-a^2}} = \dfrac{1}{a}\sec^{-1}\left|\dfrac{u}{a}\right|$

27. $\int \dfrac{du}{u\sqrt{a^2-u^2}} = -\dfrac{1}{a}\operatorname{sech}^{-1}\left|\dfrac{u}{a}\right| = -\dfrac{1}{a}\ln\left|\dfrac{a+\sqrt{a^2-u^2}}{u}\right|$

*Notice that this formula shows the addition of a constant, C. When using an integral table or technology, the constant will usually not be shown, so it is important that you remember to insert the constant of integration each time you evaluate an integral, even when getting it from a table or from technology.

28. $\displaystyle\int \frac{du}{\sqrt{a^2+u^2}} = \ln\left(u+\sqrt{a^2+u^2}\right) = \sinh^{-1}\frac{u}{a}$

29. $\displaystyle\int \frac{du}{u\sqrt{a^2+u^2}} = -\frac{1}{a}\ln\left|\frac{\sqrt{a^2+u^2}+a}{u}\right| = -\frac{1}{a}\operatorname{csch}^{-1}\left|\frac{u}{a}\right|$

Linear and quadratic forms

INTEGRALS INVOLVING $au+b$

30. $\displaystyle\int (au+b)^n du = \frac{(au+b)^{n+1}}{(n+1)a}$

31. $\displaystyle\int u(au+b)^n du = \frac{(au+b)^{n+2}}{(n+2)a^2} - \frac{b(au+b)^{n+1}}{(n+1)a^2}$

32. $\displaystyle\int u^2(au+b)^n du = \frac{(au+b)^{n+3}}{(n+3)a^3} - \frac{2b(au+b)^{n+2}}{(n+2)a^3} + \frac{b^2(au+b)^{n+1}}{(n+1)a^3}$

33. * $\displaystyle\int u^m(au+b)^n du = \begin{cases} \dfrac{u^{m+1}(au+b)^n}{m+n+1} + \dfrac{nb}{m+n+1}\displaystyle\int u^m(au+b)^{n-1}\,du & \text{or} \\ \dfrac{u^m(au+b)^{n+1}}{(m+n+1)a} - \dfrac{mb}{(m+n+1)a}\displaystyle\int u^{m-1}(au+b)^n\,du & \text{or} \\ \dfrac{-u^{m+1}(au+b)^{n+1}}{(n+1)b} + \dfrac{m+n+2}{(n+1)b}\displaystyle\int u^m(au+b)^{n+1}\,du \end{cases}$

34. $\displaystyle\int \frac{du}{au+b} = \frac{1}{a}\ln|au+b|$

35. $\displaystyle\int \frac{u\,du}{au+b} = \frac{u}{a} - \frac{b}{a^2}\ln|au+b|$

36. $\displaystyle\int \frac{u^2 du}{au+b} = \frac{(au+b)^2}{2a^3} - \frac{2b(au+b)}{a^3} + \frac{b^2}{a^3}\ln|au+b|$

37. $\displaystyle\int \frac{u^3\,du}{au+b} = \frac{(au+b)^3}{3a^4} - \frac{3b(au+b)^2}{2a^4} + \frac{3b^2(au+b)}{a^4} - \frac{b^3}{a^4}\ln|au+b|$

INTEGRALS INVOLVING u^2+a^2

38. $\displaystyle\int \frac{du}{u^2+a^2} = \frac{1}{a}\tan^{-1}\frac{u}{a}$

39. $\displaystyle\int \frac{u\,du}{u^2+a^2} = \frac{1}{2}\ln(u^2+a^2)$

40. $\displaystyle\int \frac{u^2 du}{u^2+a^2} = u - a\tan^{-1}\frac{u}{a}$

41. $\displaystyle\int \frac{u^3\,du}{u^2+a^2} = \frac{u^2}{2} - \frac{a^2}{2}\ln(u^2+a^2)$

42. $\displaystyle\int \frac{du}{u(u^2+a^2)} = \frac{1}{2a^2}\ln\left(\frac{u^2}{u^2+a^2}\right)$

43. $\displaystyle\int \frac{du}{u^2(u^2+a^2)} = -\frac{1}{a^2u} - \frac{1}{a^3}\tan^{-1}\frac{u}{a}$

44. $\displaystyle\int \frac{du}{u^3(u^2+a^2)} = -\frac{1}{2a^2u^2} - \frac{1}{2a^4}\ln\left(\frac{u^2}{u^2+a^2}\right)$

INTEGRALS INVOLVING u^2-a^2, $u^2>a^2$

45. $\displaystyle\int \frac{du}{u^2-a^2} = \frac{1}{2a}\ln\left|\frac{u-a}{u+a}\right|$ or $-\frac{1}{a}\coth^{-1}\frac{u}{a}$

46. $\displaystyle\int \frac{u\,du}{u^2-a^2} = \frac{1}{2}\ln|u^2-a^2|$

*When the integral is given as another integral, then do not add the constant until the form no longer involves an integration.

47. $\displaystyle\int \frac{u^2 du}{u^2 - a^2} = u + \frac{a}{2} \ln \left| \frac{u - a}{u + a} \right|$

48. $\displaystyle\int \frac{u^3 du}{u^2 - a^2} = \frac{u^2}{2} + \frac{a^2}{2} \ln |u^2 - a^2|$

49. $\displaystyle\int \frac{du}{u(u^2 - a^2)} = \frac{1}{2a^2} \ln \left| \frac{u^2 - a^2}{u^2} \right|$

50. $\displaystyle\int \frac{du}{u^2(u^2 - a^2)} = \frac{1}{a^2 u} + \frac{1}{2a^3} \ln \left| \frac{u - a}{u + a} \right|$

51. $\displaystyle\int \frac{du}{u^3(u^2 - a^2)} = \frac{1}{2a^2 u^2} - \frac{1}{2a^4} \ln \left| \frac{u^2}{u^2 - a^2} \right|$

INTEGRALS INVOLVING $a^2 - u^2, u^2 < a^2$

52. $\displaystyle\int \frac{du}{a^2 - u^2} = \frac{1}{2a} \ln \left| \frac{a + u}{a - u} \right|$ or $\displaystyle\frac{1}{a} \tanh^{-1} \frac{u}{a}$

53. $\displaystyle\int \frac{u\, du}{a^2 - u^2} = -\frac{1}{2} \ln |a^2 - u^2|$

54. $\displaystyle\int \frac{u^2 du}{a^2 - u^2} = -u + \frac{a}{2} \ln \left| \frac{a + u}{a - u} \right|$

55. $\displaystyle\int \frac{u^3 du}{a^2 - u^2} = -\frac{u^2}{2} - \frac{a^2}{2} \ln |a^2 - u^2|$

56. $\displaystyle\int \frac{du}{u(a^2 - u^2)} = \frac{1}{2a^2} \ln \left| \frac{u^2}{a^2 - u^2} \right|$

57. $\displaystyle\int \frac{du}{u^2(a^2 - u^2)} = -\frac{1}{a^2 u} + \frac{1}{2a^3} \ln \left| \frac{a + u}{a - u} \right|$

58. $\displaystyle\int \frac{du}{u^3(a^2 - u^2)} = -\frac{1}{2a^2 u^2} + \frac{1}{2a^4} \ln \left| \frac{u^2}{a^2 - u^2} \right|$

59. $\displaystyle\int \frac{du}{(a^2 - u^2)^2} = \frac{u}{2a^2(a^2 - u^2)} + \frac{1}{4a^3} \ln \left| \frac{a + u}{a - u} \right|$

60. $\displaystyle\int \frac{u\, du}{(a^2 - u^2)^2} = \frac{1}{2(a^2 - u^2)}$

61. $\displaystyle\int \frac{u^2 du}{(a^2 - u^2)^2} = \frac{u}{2(a^2 - u^2)} - \frac{1}{4a} \ln \left| \frac{a + u}{a - u} \right|$

62. $\displaystyle\int \frac{u^3 du}{(a^2 - u^2)^2} = \frac{a^2}{2(a^2 - u^2)} + \frac{1}{2} \ln |a^2 - u^2|$

63. $\displaystyle\int \frac{du}{u(a^2 - u^2)^2} = \frac{1}{2a^2(a^2 - u^2)} + \frac{1}{2a^4} \ln \left| \frac{u^2}{a^2 - u^2} \right|$

64. $\displaystyle\int \frac{du}{u^2(a^2 - u^2)^2} = -\frac{1}{a^4 u} + \frac{u}{2a^4(a^2 - u^2)} + \frac{3}{4a^5} \ln \left| \frac{a + u}{a - u} \right|$

65. $\displaystyle\int \frac{du}{u^3(a^2 - u^2)^2} = -\frac{1}{2a^4 u^2} + \frac{1}{2a^4(a^2 - u^2)} + \frac{1}{a^6} \ln \left| \frac{u^2}{a^2 - u^2} \right|$

INTEGRALS INVOLVING $au^2 + bu + c$

66. $\displaystyle\int \frac{du}{au^2 + bu + c} = \begin{cases} \dfrac{2}{\sqrt{4ac - b^2}} \tan^{-1} \dfrac{2au + b}{\sqrt{4ac - b^2}} \\ \dfrac{1}{\sqrt{b^2 - 4ac}} \ln \left| \dfrac{2au + b - \sqrt{b^2 - 4ac}}{2au + b + \sqrt{b^2 - 4ac}} \right| \end{cases}$

67. $\displaystyle\int \frac{u\, du}{au^2 + bu + c} = \frac{1}{2a} \ln |au^2 + bu + c| - \frac{b}{2a} \int \frac{du}{au^2 + bu + c}$

68. $\displaystyle\int \frac{u^2 du}{au^2 + bu + c} = \frac{u}{a} - \frac{b}{2a^2} \ln |au^2 + bu + c| + \frac{b^2 - 2ac}{2a^2} \int \frac{du}{au^2 + bu + c}$

69. $\displaystyle\int \frac{u^m du}{au^2 + bu + c} = \frac{u^{m-1}}{(m-1)a} - \frac{c}{a} \int \frac{u^{m-2} du}{au^2 + bu + c} - \frac{b}{a} \int \frac{u^{m-1} du}{au^2 + bu + c}$

70. $\displaystyle\int \frac{du}{u(au^2+bu+c)} = \frac{1}{2c}\ln\left|\frac{u^2}{au^2+bu+c}\right| - \frac{b}{2c}\int \frac{du}{au^2+bu+c}$

71. $\displaystyle\int \frac{du}{u^2(au^2+bu+c)} = \frac{b}{2c^2}\ln\left|\frac{au^2+bu+c}{u^2}\right| - \frac{1}{cu} + \frac{b^2-2ac}{2c^2}\int \frac{du}{au^2+bu+c}$

72. $\displaystyle\int \frac{du}{u^n(au^2+bu+c)} = -\frac{1}{(n-1)cu^{n-1}} - \frac{b}{c}\int \frac{du}{u^{n-1}(au^2+bu+c)} - \frac{a}{c}\int \frac{du}{u^{n-2}(au^2+bu+c)}$

73. $\displaystyle\int \frac{du}{(au^2+bu+c)^2} = \frac{2au+b}{(4ac-b^2)(au^2+bu+c)} + \frac{2a}{4ac-b^2}\int \frac{du}{au^2+bu+c}$

74. $\displaystyle\int \frac{u\,du}{(au^2+bu+c)^2} = -\frac{bu+2c}{(4ac-b^2)(au^2+bu+c)} - \frac{b}{4ac-b^2}\int \frac{du}{au^2+bu+c}$

75. $\displaystyle\int \frac{u^2\,du}{(au^2+bu+c)^2} = \frac{(b^2-2ac)u+bc}{a(4ac-b^2)(au^2+bu+c)} + \frac{2c}{4ac-b^2}\int \frac{du}{au^2+bu+c}$

76. $\displaystyle\int \frac{u^m\,du}{(au^2+bu+c)^n} = \frac{-u^{m-1}}{(2n-m-1)a(au^2+bu+c)^{n-1}} - \frac{(n-m)b}{(2n-m-1)a}\int \frac{u^{m-1}\,du}{(au^2+bu+c)^n} + \frac{(m-1)c}{(2n-m-1)a}\int \frac{u^{m-2}\,du}{(au^2+bu+c)^n}$

Radical forms

INTEGRALS INVOLVING $\sqrt{au+b}$

77. $\displaystyle\int \frac{du}{\sqrt{au+b}} = \frac{2\sqrt{au+b}}{a}$

78. $\displaystyle\int \frac{u\,du}{\sqrt{au+b}} = \frac{2(au-2b)}{3a^2}\sqrt{au+b}$

79. $\displaystyle\int \frac{u^2\,du}{\sqrt{au+b}} = \frac{2(3a^2u^2-4abu+8b^2)}{15a^3}\sqrt{au+b}$

80. $\displaystyle\int \frac{du}{u\sqrt{au+b}} = \begin{cases} \dfrac{1}{\sqrt{b}}\ln\left|\dfrac{\sqrt{au+b}-\sqrt{b}}{\sqrt{au+b}+\sqrt{b}}\right| \\ \dfrac{2}{\sqrt{-b}}\tan^{-1}\sqrt{\dfrac{au+b}{-b}} \end{cases}$

81. $\displaystyle\int \frac{du}{u^2\sqrt{au+b}} = -\frac{\sqrt{au+b}}{bu} - \frac{a}{2b}\int \frac{du}{u\sqrt{au+b}}$

82. $\displaystyle\int \sqrt{au+b}\,du = \frac{2\sqrt{(au+b)^3}}{3a}$

83. $\displaystyle\int u\sqrt{au+b}\,du = \frac{2(3au-2b)}{15a^2}\sqrt{(au+b)^3}$

84. $\displaystyle\int u^2\sqrt{au+b}\,du = \frac{2(15a^2u^2-12abu+8b^2)}{105a^3}\sqrt{(au+b)^3}$

INTEGRALS INVOLVING $\sqrt{u^2+a^2}$

85. $\displaystyle\int \sqrt{u^2+a^2}\,du = \frac{u\sqrt{u^2+a^2}}{2} + \frac{a^2}{2}\ln\left(u+\sqrt{u^2+a^2}\right)$

86. $\displaystyle\int u\sqrt{u^2+a^2}\,du = \frac{(u^2+a^2)^{3/2}}{3}$

87. $\displaystyle\int u^2\sqrt{u^2+a^2}\,du = \frac{u(u^2+a^2)^{3/2}}{4} - \frac{a^2u\sqrt{u^2+a^2}}{8} - \frac{a^4}{8}\ln\left(u+\sqrt{u^2+a^2}\right)$

88. $\displaystyle\int u^3\sqrt{u^2+a^2}\,du = \frac{(u^2+a^2)^{5/2}}{5} - \frac{a^2(u^2+a^2)^{3/2}}{3}$

89. $\displaystyle\int \frac{du}{\sqrt{u^2+a^2}} = \ln\left(u+\sqrt{u^2+a^2}\right) \text{ or } \sinh^{-1}\frac{u}{a}$

90. $\displaystyle\int \frac{u\,du}{\sqrt{u^2+a^2}} = \sqrt{u^2+a^2}$

91. $\displaystyle\int \frac{u^2\,du}{\sqrt{u^2+a^2}} = \frac{u\sqrt{u^2+a^2}}{2} - \frac{a^2}{2}\ln(u+\sqrt{u^2+a^2})$

92. $\displaystyle\int \frac{u^3\,du}{\sqrt{u^2+a^2}} = \frac{(u^2+a^2)^{3/2}}{3} - a^2\sqrt{u^2+a^2}$

93. $\displaystyle\int \frac{du}{u\sqrt{u^2+a^2}} = -\frac{1}{a}\ln\left|\frac{a+\sqrt{u^2+a^2}}{u}\right|$

94. $\displaystyle\int \frac{du}{u^2\sqrt{u^2+a^2}} = -\frac{\sqrt{u^2+a^2}}{a^2u}$

95. $\displaystyle\int \frac{du}{u^3\sqrt{u^2+a^2}} = -\frac{\sqrt{u^2+a^2}}{2a^2u^2} + \frac{1}{2a^3}\ln\left|\frac{a+\sqrt{u^2+a^2}}{u}\right|$

96. $\displaystyle\int \frac{\sqrt{u^2+a^2}}{u}\,du = \sqrt{u^2+a^2} - a\ln\left|\frac{a+\sqrt{u^2+a^2}}{u}\right|$

97. $\displaystyle\int \frac{\sqrt{u^2+a^2}}{u^2}\,du = -\frac{\sqrt{u^2+a^2}}{u} + \ln\left(u+\sqrt{u^2+a^2}\right)$

INTEGRALS INVOLVING $\sqrt{u^2-a^2}$, $a > 0$

98. $\displaystyle\int \frac{du}{\sqrt{u^2-a^2}} = \ln\left|u+\sqrt{u^2-a^2}\right|$

99. $\displaystyle\int \frac{u\,du}{\sqrt{u^2-a^2}} = \sqrt{u^2-a^2}$

100. $\displaystyle\int \frac{u^2\,du}{\sqrt{u^2-a^2}} = \frac{u\sqrt{u^2-a^2}}{2} + \frac{a^2}{2}\ln\left|u+\sqrt{u^2-a^2}\right|$

101. $\displaystyle\int \frac{u^3\,du}{\sqrt{u^2-a^2}} = \frac{(u^2-a^2)^{3/2}}{3} + a^2\sqrt{u^2-a^2}$

102. $\displaystyle\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right|$

103. $\displaystyle\int \frac{du}{u^2\sqrt{u^2-a^2}} = \frac{\sqrt{u^2-a^2}}{a^2u}$

104. $\displaystyle\int \frac{du}{u^3\sqrt{u^2-a^2}} = \frac{\sqrt{u^2-a^2}}{2a^2u^2} + \frac{1}{2a^3}\sec^{-1}\left|\frac{u}{a}\right|$

105. $\displaystyle\int \sqrt{u^2-a^2}\,du = \frac{u\sqrt{u^2-a^2}}{2} - \frac{a^2}{2}\ln\left|u+\sqrt{u^2-a^2}\right|$

106. $\displaystyle\int u\sqrt{u^2-a^2}\,du = \frac{(u^2-a^2)^{3/2}}{3}$

107. $\displaystyle\int u^2\sqrt{u^2-a^2}\,du = \frac{u(u^2-a^2)^{3/2}}{4} + \frac{a^2u\sqrt{u^2-a^2}}{8} - \frac{a^4}{8}\ln\left|u+\sqrt{u^2-a^2}\right|$

108. $\displaystyle\int u^3\sqrt{u^2-a^2}\,du = \frac{(u^2-a^2)^{5/2}}{5} + \frac{a^2(u^2-a^2)^{3/2}}{3}$

109. $\displaystyle\int \frac{\sqrt{u^2-a^2}}{u}\,du = \sqrt{u^2-a^2} - a\sec^{-1}\left|\frac{u}{a}\right|$

INTEGRALS INVOLVING $\sqrt{a^2-u^2}$, $a > 0$

110. $\displaystyle\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\frac{u}{a}$

111. $\displaystyle\int \frac{u\,du}{\sqrt{a^2-u^2}} = -\sqrt{a^2-u^2}$

112. $\displaystyle\int \frac{u^2\,du}{\sqrt{a^2-u^2}} = -\frac{u\sqrt{a^2-u^2}}{2} + \frac{a^2}{2}\sin^{-1}\frac{u}{a}$

113. $\displaystyle\int \frac{u^3\,du}{\sqrt{a^2-u^2}} = \frac{(a^2-u^2)^{3/2}}{3} - a^2\sqrt{a^2-u^2}$

114. $\displaystyle\int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a}\ln\left|\frac{a+\sqrt{a^2-u^2}}{u}\right|$ or $\displaystyle -\frac{1}{a}\operatorname{sech}^{-1}\left|\frac{u}{a}\right|$

115. $\displaystyle\int \frac{du}{u^2\sqrt{a^2-u^2}} = -\frac{\sqrt{a^2-u^2}}{a^2u}$

116. $\displaystyle\int \frac{du}{u^3\sqrt{a^2-u^2}} = -\frac{\sqrt{a^2-u^2}}{2a^2u^2} - \frac{1}{2a^3}\ln\left|\frac{a+\sqrt{a^2-u^2}}{u}\right|$

117. $\displaystyle\int \sqrt{a^2-u^2}\,du = \frac{u\sqrt{a^2-u^2}}{2} + \frac{a^2}{2}\sin^{-1}\frac{u}{a}$

118. $\displaystyle\int u\sqrt{a^2-u^2}\,du = -\frac{(a^2-u^2)^{3/2}}{3}$

119. $\displaystyle\int u^2\sqrt{a^2-u^2}\,du = -\frac{u(a^2-u^2)^{3/2}}{4} + \frac{a^2u\sqrt{a^2-u^2}}{8} + \frac{a^4}{8}\sin^{-1}\frac{u}{a}$

120. $\displaystyle\int u^3\sqrt{a^2-u^2}\,du = \frac{(a^2-u^2)^{5/2}}{5} - \frac{a^2(a^2-u^2)^{3/2}}{3}$

121. $\displaystyle\int \frac{\sqrt{a^2-u^2}}{u}\,du = \sqrt{a^2-u^2} - a\ln\left|\frac{a+\sqrt{a^2-u^2}}{u}\right|$

Trigonometric forms

INTEGRALS INVOLVING $\cos au$

122. $\displaystyle\int \cos au\,du = \frac{\sin au}{a}$

123. $\displaystyle\int u\cos au\,du = \frac{\cos au}{a^2} + \frac{u\sin au}{a}$

124. $\displaystyle\int u^2\cos au\,du = \frac{2u}{a^2}\cos au + \left(\frac{u^2}{a} - \frac{2}{a^3}\right)\sin au$

125. $\displaystyle\int u^3\cos au\,du = \left(\frac{3u^2}{a^2} - \frac{6}{a^4}\right)\cos au + \left(\frac{u^3}{a} - \frac{6u}{a^3}\right)\sin au$

126. $\displaystyle\int u^n\cos au\,du = \frac{u^n\sin au}{a} - \frac{n}{a}\int u^{n-1}\sin au\,du$

127. $\displaystyle\int \cos^2 au\,du = \frac{u}{2} + \frac{\sin 2au}{4a}$

128. $\displaystyle\int u\cos^2 au\,du = \frac{u^2}{4} + \frac{u\sin 2au}{4a} + \frac{\cos 2au}{8a^2}$

129. $\displaystyle\int \cos^3 au\,du = \frac{\sin au}{a} - \frac{\sin^3 au}{3a}$

130. $\displaystyle\int \cos^4 au\,du = \frac{3u}{8} + \frac{\sin 2au}{4a} + \frac{\sin 4au}{32a}$

INTEGRALS INVOLVING $\sin au$

131. $\displaystyle\int \sin au\,du = -\frac{\cos au}{a}$

132. $\displaystyle\int u\sin au\,du = \frac{\sin au}{a^2} - \frac{u\cos au}{a}$

133. $\displaystyle\int u^2\sin au\,du = \frac{2u}{a^2}\sin au + \left(\frac{2}{a^3} - \frac{u^2}{a}\right)\cos au$

134. $\displaystyle\int u^3\sin au\,du = \left(\frac{3u^2}{a^2} - \frac{6}{a^4}\right)\sin au + \left(\frac{6u}{a^3} - \frac{u^3}{a}\right)\cos au$

135. $\displaystyle\int u^n\sin au\,du = -\frac{u^n\cos au}{a} + \frac{n}{a}\int u^{n-1}\cos au\,du$

136. $\displaystyle\int \sin^2 au\,du = \frac{u}{2} - \frac{\sin 2au}{4a}$

137. $\displaystyle\int u\sin^2 au\,du = \frac{u^2}{4} - \frac{u\sin 2au}{4a} - \frac{\cos 2au}{8a^2}$

138. $\displaystyle\int \sin^3 au\,du = -\frac{\cos au}{a} + \frac{\cos^3 au}{3a}$

139. $\displaystyle\int \sin^4 au\,du = \frac{3u}{8} - \frac{\sin 2au}{4a} + \frac{\sin 4au}{32a}$

INTEGRALS INVOLVING $\sin au$ AND $\cos au$

140. $\displaystyle\int \sin au\cos au\,du = \frac{\sin^2 au}{2a}$

141. $\displaystyle\int \sin au\cos bu\,du = -\frac{\cos(a-b)u}{2(a-b)} - \frac{\cos(a+b)u}{2(a+b)}$

142. $\displaystyle\int \sin^n au\cos au\,du = \frac{\sin^{n+1} au}{(n+1)a}$

143. $\displaystyle\int \cos^n au\sin au\,du = -\frac{\cos^{n+1} au}{(n+1)a}$

144. $\displaystyle\int \sin^2 au\cos^2 au\,du = \frac{u}{8} - \frac{\sin 4au}{32a}$

145. $\displaystyle\int \frac{du}{\sin au\cos au} = \frac{1}{a}\ln|\tan au|$

146. $\displaystyle\int \frac{du}{\sin^2 au\cos au} = \frac{1}{a}\ln\left|\tan\left(\frac{\pi}{4} + \frac{au}{2}\right)\right| - \frac{1}{a\sin au}$

147. $\displaystyle\int \frac{du}{\sin au\cos^2 au} = \frac{1}{a}\ln\left|\tan\frac{au}{2}\right| + \frac{1}{a\cos au}$

INTEGRALS INVOLVING $\tan au$

148. $\displaystyle\int \tan au\,du = -\frac{1}{a}\ln|\cos au| \text{ or } \frac{1}{a}\ln|\sec au|$

149. $\displaystyle\int \tan^2 au\,du = \frac{\tan au}{a} - u$

150. $\displaystyle\int \tan^3 au\,du = \frac{\tan^2 au}{2a} + \frac{1}{a}\ln|\cos au|$

151. $\displaystyle\int \tan^n au\,du = \frac{\tan^{n-1} au}{(n-1)a} - \int \tan^{n-2} au\,du$

152. $\displaystyle\int \tan^n au\sec^2 au\,du = \frac{\tan^{n+1} au}{(n+1)a}$

INTEGRALS INVOLVING cot *au*

153. $\int \cot au\, du = \frac{1}{a} \ln |\sin au|$

154. $\int \cot^2 au\, du = -\frac{\cot au}{a} - u$

155. $\int \cot^3 au\, du = -\frac{\cot^2 au}{2a} - \frac{1}{a} \ln |\sin au|$

156. $\int \cot^n au\, du = -\frac{\cot^{n-1} au}{(n-1)a} - \int \cot^{n-2} au\, du$

157. $\int \cot^n au \csc^2 au\, du = -\frac{\cot^{n+1} au}{(n+1)a}$

INTEGRALS INVOLVING sec *au*

158. $\int \sec au\, du = \frac{1}{a} \ln |\sec au + \tan au| = \frac{1}{a} \ln \left|\tan \left(\frac{au}{2} + \frac{\pi}{4}\right)\right|$

159. $\int \sec^2 au\, du = \frac{\tan au}{a}$

160. $\int \sec^3 au\, du = \frac{\sec au \tan au}{2a} + \frac{1}{2a} \ln |\sec au + \tan au|$

161. $\int \sec^n au\, du = \frac{\sec^{n-2} au \tan au}{a(n-1)} + \frac{n-2}{n-1} \int \sec^{n-2} au\, du$

162. $\int \sec^n au \tan au\, du = \frac{\sec^n au}{na}$

INTEGRALS INVOLVING csc *au*

163. $\int \csc au\, du = \frac{1}{a} \ln |\csc au - \cot au| = \frac{1}{a} \ln \left|\tan \frac{au}{2}\right|$

164. $\int \csc^2 au\, du = -\frac{\cot au}{a}$

165. $\int \csc^3 au\, du = -\frac{\csc au \cot au}{2a} + \frac{1}{2a} \ln \left|\tan \frac{au}{2}\right|$

166. $\int \csc^n au\, du = -\frac{\csc^{n-2} au \cot au}{a(n-1)} + \frac{n-2}{n-1} \int \csc^{n-2} au\, du$

167. $\int \csc^n au \cot au\, du = -\frac{\csc^n au}{na}$

Inverse trigonometric forms

INTEGRALS INVOLVING INVERSE TRIGONOMETRIC FUNCTIONS, $a > 0$

168. $\int \cos^{-1} \frac{u}{a}\, du = u \cos^{-1} \frac{u}{a} - \sqrt{a^2 - u^2}$

169. $\int u \cos^{-1} \frac{u}{a}\, du = \left(\frac{u^2}{2} - \frac{a^2}{4}\right) \cos^{-1} \frac{u}{a} - \frac{u\sqrt{a^2 - u^2}}{4}$

170. $\int u^2 \cos^{-1} \frac{u}{a}\, du = \frac{u^3}{3} \cos^{-1} \frac{u}{a} - \frac{(u^2 + 2a^2)\sqrt{a^2 - u^2}}{9}$

171. $\int \frac{\cos^{-1} \frac{u}{a}}{u}\, du = \frac{\pi}{2} \ln |u| - \int \frac{\sin^{-1} \frac{u}{a}}{u}\, du$

172. $\int \frac{\cos^{-1} \frac{u}{a}}{u^2} du = -\frac{\cos^{-1} \frac{u}{a}}{u} + \frac{1}{a} \ln \left|\frac{a + \sqrt{a^2 - u^2}}{u}\right|$

173. $\int \left(\cos^{-1}\frac{u}{a}\right)^2 du = u\left(\cos^{-1}\frac{u}{a}\right)^2 - 2u - 2\sqrt{a^2-u^2}\cos^{-1}\frac{u}{a}$

174. $\int \sin^{-1}\frac{u}{a}\,du = u\sin^{-1}\frac{u}{a} + \sqrt{a^2-u^2}$

175. $\int u\sin^{-1}\frac{u}{a}\,du = \left(\frac{u^2}{2} - \frac{a^2}{4}\right)\sin^{-1}\frac{u}{a} + \frac{u\sqrt{a^2-u^2}}{4}$

176. $\int u^2\sin^{-1}\frac{u}{a}\,du = \frac{u^3}{3}\sin^{-1}\frac{u}{a} + \frac{(u^2+2a^2)\sqrt{a^2-u^2}}{9}$

177. $\int \frac{\sin^{-1}\frac{u}{a}}{u}\,du = \frac{u}{a} + \frac{\left(\frac{u}{a}\right)^3}{2\cdot 3\cdot 3} + \frac{1\cdot 3\left(\frac{u}{a}\right)^5}{2\cdot 4\cdot 5\cdot 5} + \frac{1\cdot 3\cdot 5\left(\frac{u}{a}\right)^7}{2\cdot 4\cdot 6\cdot 7\cdot 7} + \cdots$

178. $\int \frac{\sin^{-1}\frac{u}{a}}{u^2}\,du = -\frac{\sin^{-1}\frac{u}{a}}{u} - \frac{1}{a}\ln\left|\frac{a+\sqrt{a^2-u^2}}{u}\right|$

179. $\int \left(\sin^{-1}\frac{u}{a}\right)^2 du = u\left(\sin^{-1}\frac{u}{a}\right)^2 - 2u + 2\sqrt{a^2-u^2}\sin^{-1}\frac{u}{a}$

180. $\int \tan^{-1}\frac{u}{a}\,du = u\tan^{-1}\frac{u}{a} - \frac{a}{2}\ln(u^2+a^2)$

181. $\int u\tan^{-1}\frac{u}{a}du = \frac{1}{2}(u^2+a^2)\tan^{-1}\frac{u}{a} - \frac{au}{2}$

182. $\int u^2\tan^{-1}\frac{u}{a}du = \frac{u^3}{3}\tan^{-1}\frac{u}{a} - \frac{au^2}{6} + \frac{a^3}{6}\ln(u^2+a^2)$

Exponential and logarithmic forms

INTEGRALS INVOLVING e^{au}

183. $\int e^{au}du = \frac{e^{au}}{a}$

184. $\int ue^{au}du = \frac{e^{au}}{a}\left(u - \frac{1}{a}\right)$

185. $\int u^2e^{au}du = \frac{e^{au}}{a}\left(u^2 - \frac{2u}{a} + \frac{2}{a^2}\right)$

186. $\int u^ne^{au}du = \frac{u^ne^{au}}{a} - \frac{n}{a}\int u^{n-1}e^{au}du$

187. $\int \frac{e^{au}}{u}du = \ln|u| + \frac{au}{1\cdot 1!} + \frac{(au)^2}{2\cdot 2!} + \frac{(au)^3}{3\cdot 3!} + \cdots$

188. $\int \frac{e^{au}}{u^n}du = \frac{-e^{au}}{(n-1)u^{n-1}} + \frac{a}{n-1}\int \frac{e^{au}}{u^{n-1}}du$

189. $\int \frac{du}{p+qe^{au}} = \frac{u}{p} - \frac{1}{ap}\ln|p+qe^{au}|$

190. $\int \frac{du}{(p+qe^{au})^2} = \frac{u}{p^2} + \frac{1}{ap(p+qe^{au})} - \frac{1}{ap^2}\ln|p+qe^{au}|$

191. $\int \frac{du}{pe^{au}+qe^{-au}} = \begin{cases} \frac{1}{a\sqrt{pq}}\tan^{-1}\left(\sqrt{\frac{p}{q}}e^{au}\right), & p>0,\ q>0 \\ \frac{1}{2a\sqrt{-pq}}\ln\left|\frac{e^{au}-\sqrt{-\frac{q}{p}}}{e^{au}+\sqrt{-\frac{q}{p}}}\right|, & p>0,\ q<0 \end{cases}$

192. $\int e^{au}\sin bu\,du = \frac{e^{au}(a\sin bu - b\cos bu)}{a^2+b^2}$

193. $\int e^{au}\cos bu\,du = \frac{e^{au}(a\cos bu + b\sin bu)}{a^2+b^2}$

194. $\displaystyle\int ue^{au}\sin bu\,du = \frac{ue^{au}(a\sin bu - b\cos bu)}{a^2+b^2} - \frac{e^{au}\left[(a^2-b^2)\sin bu - 2ab\cos bu\right]}{(a^2+b^2)^2}$

195. $\displaystyle\int ue^{au}\cos bu\,du = \frac{ue^{au}(a\cos bu + b\sin bu)}{a^2+b^2} - \frac{e^{au}\left[(a^2-b^2)\cos bu + 2ab\sin bu\right]}{(a^2+b^2)^2}$

INTEGRALS INVOLVING $\ln|u|$

196. $\displaystyle\int \ln|u|\,du = u\ln|u| - u$

197. $\displaystyle\int (\ln|u|)^2\,du = u(\ln|u|)^2 - 2u\ln|u| + 2u$

198. $\displaystyle\int (\ln|u|)^n\,du = u(\ln|u|)^n - n\int (\ln|u|)^{n-1}\,du$

199. $\displaystyle\int u\ln|u|\,du = \frac{u^2}{2}\left(\ln|u| - \frac{1}{2}\right)$

200. $\displaystyle\int u^m\ln|u|\,du = \frac{u^{m+1}}{m+1}\left(\ln|u| - \frac{1}{m+1}\right)$