

NLP Assignment

Assignment 1

Part 1

Problem 1:

a)

$$\text{we have } y_w = \begin{cases} 1 & \text{if } w = o \\ 0 & \text{if } w \neq o \end{cases} \Rightarrow -\sum_{w \in \text{vocab}} y_w \cdot \log(\hat{y}_w) = -y_o \cdot \log(\hat{y}_o) = -\log(\hat{y}_o)$$

b) $\tau_{\text{naive-softmax}}(v_c, o, v) = -\log p(o=o, c=c)$

$$= -\log \frac{\exp(u_o^T \cdot v_c)}{\sum_{w \in \text{vocab}} \exp(u_w^T \cdot v_c)}$$

$$\frac{\partial J}{\partial v_c} = \frac{\partial}{\partial v_c} \left(-\log \frac{\exp(u_o^T \cdot v_c)}{\sum_{w \in \text{vocab}} \exp(u_w^T \cdot v_c)} \right)$$

$$= -\frac{\partial}{\partial v_c} \log(\exp(u_o^T \cdot v_c)) + \frac{\partial}{\partial v_c} \cdot \log\left(\sum_w \exp(u_w^T \cdot v_c)\right)$$

$$= -\frac{\partial}{\partial v_c} u_o^T \cdot v_c + \frac{\partial}{\partial v_c} \log\left(\sum_w \exp(u_w^T \cdot v_c)\right)$$

$$= -u_o + \frac{\partial}{\partial v_c} \log\left(\sum_w \exp(u_w^T \cdot v_c)\right)$$

we have:

$$\frac{\partial}{\partial v_c} \log\left(\sum_w \exp(u_w^T \cdot v_c)\right) = \frac{1}{\sum_w \exp(u_w^T \cdot v_c)} \cdot \frac{\partial}{\partial v_c} \sum_{x \in \text{vocab}} \exp(u_x^T \cdot v_c)$$

$$= \frac{1}{\sum_w \exp(u_w^T \cdot v_c)} \cdot \sum_x \frac{\partial}{\partial v_c} \exp(u_x^T \cdot v_c)$$

$$= \frac{1}{\sum_w \exp(u_w^T \cdot v_c)} \cdot \sum_x \exp(u_x^T \cdot v_c) \cdot \frac{\partial}{\partial v_c} (u_x^T \cdot v_c)$$

$$= \frac{1}{\sum_w \exp(u_w^T \cdot v_c)} \cdot \sum_x \exp(u_x^T \cdot v_c) \cdot u_x$$

$$= \sum_x \frac{\exp(u_x^T \cdot v_c)}{\sum_w \exp(u_w^T \cdot v_c)} \cdot u_x$$

$$= \sum_x p(O=x|C=c) \cdot u_x = \sum_x \hat{y}_x \cdot u_x$$

$$\text{So: } \frac{\partial J}{\partial v_c} = -\mu_0 + \sum_{x \in \text{vocab}} \hat{y}_x \cdot u_x$$

$$c) \frac{\partial J}{\partial u_w} = \frac{\partial}{\partial u_w} \left(-\log \frac{\exp(u_0^T \cdot v_c)}{\sum_{w \in \text{vocab}} \exp(u_w^T \cdot v_c)} \right)$$

$$= -\frac{\partial}{\partial u_w} \log(\exp(u_0^T \cdot v_c)) + \frac{\partial}{\partial u_w} \log(\sum_w \exp(u_w^T \cdot v_c))$$

$$= -\frac{\partial}{\partial u_w} u_0^T \cdot v_c + \frac{\partial}{\partial u_w} \log(\sum_w \exp(u_w^T \cdot v_c))$$

In case $w=0$:

$$\frac{\partial J}{\partial u_0} = -\frac{\partial}{\partial u_0} u_0^T \cdot v_c + \frac{\partial}{\partial u_0} \log(\sum_w \exp(u_w^T \cdot v_c))$$

$$= -v_c + \frac{\partial}{\partial u_0} \log(\sum_w \exp(u_w^T \cdot v_c))$$

we have:

$$\frac{\partial}{\partial u_0} \log(\sum_w \exp(u_w^T \cdot v_c)) = \frac{1}{\sum_w \exp(u_w^T \cdot v_c)} \cdot \frac{\partial}{\partial u_0} \sum_{x \in \text{vocab}} \exp(u_x^T \cdot v_c)$$

$$= \frac{1}{\sum_w \exp(u_w^T \cdot v_c)} \cdot \sum_x \frac{\partial}{\partial u_0} \exp(u_x^T \cdot v_c)$$

$$= \frac{1}{\sum_w \exp(u_w^T \cdot v_c)} \cdot \sum_x \exp(u_x^T \cdot v_c) \cdot \frac{\partial}{\partial u_0} u_x^T \cdot v_c$$

$$= \frac{1}{\sum_w \exp(u_w^T \cdot v_c)} \cdot \left(\exp(u_0^T \cdot v_c) \cdot \frac{\partial}{\partial u_0} u_0^T \cdot v_c + \sum_{x \neq 0} \exp(u_x^T \cdot v_c) \cdot \frac{\partial}{\partial u_0} u_x^T \cdot v_c \right)$$

$$= \frac{1}{\sum_w \exp(u_w^T \cdot v_c)} \cdot (\exp(u_0^T \cdot v_c) \cdot v_c + 0)$$

$$= \frac{\exp(u_0^T \cdot v_c) \cdot v_c}{\sum_w \exp(u_w^T \cdot v_c)} = p(O=0 | C=c) \cdot v_c$$

$$= \hat{y}_0 \cdot v_c$$

$$\text{So: } \frac{\partial J}{\partial u_0} = -v_c + \hat{y}_0 \cdot v_c = v_c (1 - \hat{y}_0)$$

In case $w \neq 0$

$$\frac{\partial J}{\partial u_w} = -\frac{\partial}{\partial u_w} u_0^T \cdot v_c + \frac{\partial}{\partial u_w} \log \left(\sum_w \exp(u_w^T \cdot v_c) \right)$$

$$= 0 + \frac{1}{\sum_w \exp(u_w^T \cdot v_c)} \cdot \frac{\partial}{\partial u_w} \sum_{x \in vocab} \exp(u_x^T \cdot v_c)$$

$$= \frac{1}{\sum_w \exp(u_w^T \cdot v_c)} \cdot \sum_x \frac{\partial}{\partial u_w} \exp(u_x^T \cdot v_c)$$

$$= \frac{1}{\sum_w \exp(u_w^T \cdot v_c)} \cdot \sum_x \exp(u_x^T \cdot v_c) \cdot \frac{\partial}{\partial u_w} (u_x^T \cdot v_c)$$

with each $w \neq 0$: $\frac{\partial J}{\partial u_w} = \frac{1}{\sum_w \exp(u_w^T \cdot v_c)} \cdot \left(\frac{\partial}{\partial u_w} (u_w^T \cdot v_c) \right)$

$$= \frac{1}{\sum_w \exp(u_w^T \cdot v_c)} \cdot \left(\exp(u_w^T \cdot v_c) \cdot \frac{\partial}{\partial u_w} (u_w^T \cdot v_c) + \sum_{x \neq w} \exp(u_x^T \cdot v_c) \cdot \frac{\partial}{\partial u_w} (u_x^T \cdot v_c) \right)$$

$$= \frac{1}{\sum_w \exp(u_w^T \cdot v_c)} \cdot (\exp(u_w^T \cdot v_c) \cdot v_c + 0)$$

$$= \frac{\exp(u_w^T \cdot v_c)}{\sum_w \exp(u_w^T \cdot v_c)} \cdot v_c = p(O=w | C=c) \cdot v_c$$

$$= \hat{y}_w \cdot v_c \quad \text{with } w \neq 0$$

d)

$$\frac{d}{dx} \sigma(x) = \frac{d}{dx} \frac{1}{1+e^{-x}} = \frac{d}{dx} (1+e^{-x})^{-1}$$

$$= -(1+e^{-x})^{-2} \cdot \frac{d}{dx} (1+e^{-x})$$

$$= -(1+e^{-x})^{-2} \cdot \left\{ \frac{d}{dx} e^{-x} = -e^{-x} \right\} = -(1+e^{-x})^{-2} \cdot e^{-x} \cdot \frac{d}{dx} (-x)$$

$$= (1+e^{-x})^{-2} \cdot e^{-x}$$

$$= \frac{1}{1+e^{-x}} \cdot \frac{e^{-x}}{1+e^{-x}}$$

$$= \frac{1}{1+e^{-x}} \cdot \frac{e^{-x} + 1 - 1}{1+e^{-x}}$$

$$= \frac{1}{1+e^{-x}} \cdot \left(1 - \frac{1}{1+e^{-x}} \right)$$

$$= \sigma(x) (1 - \sigma(x))$$

$$c) \text{ neg-sample } (v_c, 0, U) = -\log(\sigma(u_0^T v_c)) - \sum_{h=1}^K \log(\sigma(-u_h^T v_c))$$

$$\frac{\partial J}{\partial v_c} = \underbrace{\frac{\partial}{\partial v_c} \log(\sigma(u_0^T v_c))}_{(1)} - \sum_h \underbrace{\frac{\partial}{\partial v_c} \log(\sigma(-u_h^T v_c))}_{(2)}$$

$$(1) = - \frac{1}{\sigma(u_0^T v_c)} \cdot \frac{\partial}{\partial v_c} \sigma(u_0^T v_c)$$

$$= - \frac{1}{\sigma(u_0^T v_c)} \cdot \sigma(u_0^T v_c) \cdot (1 - \sigma(u_0^T v_c)) \cdot \frac{\partial}{\partial v_c} u_0^T v_c$$

$$= (\sigma(u_0^T v_c) - 1) \cdot u_0$$

$$(2) = \sum_h \frac{\partial}{\partial v_c} \log(\sigma(-u_h^T v_c))$$

$$= \sum_h \frac{1}{\sigma(-u_h^T v_c)} \cdot \frac{\partial}{\partial v_c} \sigma(-u_h^T v_c)$$

$$= \sum_h \frac{1}{\sigma(-u_h^T v_c)} \cdot \sigma(-u_h^T v_c) (1 - \sigma(-u_h^T v_c)) \frac{\partial}{\partial v_c} (-u_h^T v_c)$$

$$= \sum_h (\sigma(-u_h^T v_c) - 1) \cdot u_h$$

$$\text{So: } \frac{\partial J}{\partial v_c} = (\sigma(u_0^T v_c) - 1) \cdot u_0 - \sum_{h=1}^K (\sigma(-u_h^T v_c) - 1) \cdot u_h$$

$$\frac{\partial J}{\partial u_w} = \frac{\partial}{\partial u_w} \log(\sigma(u_0^T \cdot v_c)) - \frac{\partial}{\partial u_w} \sum_k \log(\sigma(-u_k^T \cdot v_c))$$

$$= -(1 - \sigma(u_0^T \cdot v_c)) \cdot \frac{\partial}{\partial u_w} u_0^T \cdot v_c - \sum_k (1 - \sigma(-u_k^T \cdot v_c)) \cdot \frac{\partial}{\partial u_w} (-u_k^T \cdot v_c)$$

In case $w = 0$, we have:

$$\frac{\partial J}{\partial u_0} = (\sigma(u_0^T \cdot v_c) - 1) \cdot v_c = 0 \quad // \text{this is because } 0 \notin \{u_1, \dots, u_k\}$$

$$= (\sigma(u_0^T \cdot v_c) - 1) \cdot v_c$$

so the derivative = 0

In case $w \neq 0$, we have:

$$\frac{\partial J}{\partial u_w} = 0 - \sum_k (1 - \sigma(-u_k^T \cdot v_c)) \cdot \frac{\partial}{\partial u_w} (-u_k^T \cdot v_c)$$

$$= (1 - \sigma(-u_w^T \cdot v_c)) \cdot v_c$$

// this because there is only one ~~u_k~~ $k = w$ so the derivative of other = 0

Conclusion:

- This loss function is much better ~~than~~ to compute more than the naive-softmax loss ~~case~~ because it doesn't go through all the word in the vocabulary ~~so that~~ therefore, the computation is less expensive.

Problem 2:

a)

- Neural window-based models can be parallelized, but RNN models cannot

b)

- Use a network with fewer layers

- Increase L2 regularization weight

c) - $\log(0, 2)$

d)

- True

e)

- True

Problem 4:

i) 1. Number of outputs: n

// because each output will correspond to an input at each time step.

2. $\hat{y}^{(t)}$ is the probability distribution over 4 categories including: person, organization, location, none

3. Each input will be a word in the sentence and will produce an output ~~at each~~ correspond to predicted category at each time step

ii) 1. Number of outputs: arbitrary

// because we don't know how many words the model will generate

2. $\hat{y}^{(t)}$ is the probability distribution over all words in vocabulary

3. Each input will be ~~the output~~ of the previous output and will produce a new output which is the next predicted word for the sentence at each time step

Problem 3.

a) Yes, graph-based dependency can produce non-projective ~~parse trees~~ dependency trees while transition-based can't

b) For example, in case $i = j$, the score will be

$$\rightarrow (i, i) = h_i^T \cdot h_i$$

The result of this product will always be positive and higher than the scores of other edges

\Rightarrow This can cause a problem that words can get linked to themselves

\Rightarrow Incorrectly predict dependencies that involve a word and itself.

c) The drawback is that graph-based is slower and require more computational resources since it has to compute all the scores for all possible dependencies between all pairs of words in the sentence while transition-based ~~only~~ just simply build a tree from left to right or right to left incrementally