General Problem Definition

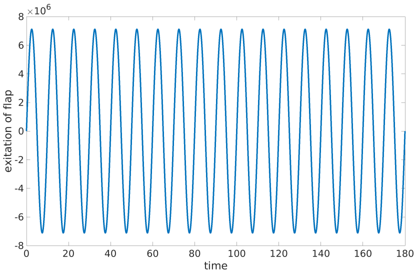
Compare both the OCP and MPC solutions of the wave harvester (Flap model). I will use 2 different waves.
1. The harmonic wave that was first investigated by Matthias and also already covered the the previous paper.
It only consists of one frequency.
2. The stochastic wave. Here the wave consists of multiple frequencies and is much less predictable then the harmonic wave.
Hypothesis:

> The system that is exited with harmonic wave exhibits some form of the turnpike phenomenon that results in unwanted! leaving arcs/solution degeneration.

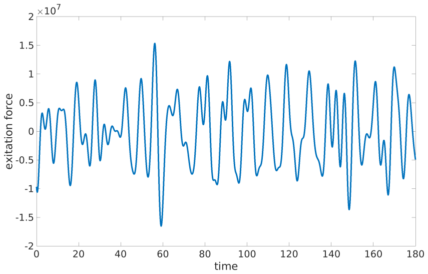
The goal of the simulations is to find a measure for this phenomenon and confirm/disprove the hypothesis.

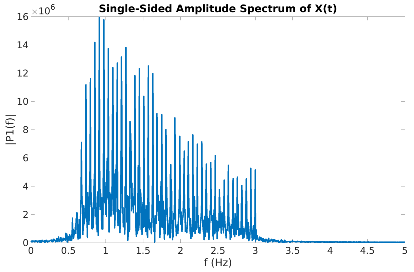
The Waves

The monochromatic wave [MCW] is pretty straightforward
Worth noting is only that wave period is 10s.



Stochastic wave [SW] constructed using multiple smaller waves that are offset by a random amount. This shift is then also changed over time to create a smooth continuous random wave.





Shown above is the spectrum density of the wave. This consists of the wave spectrum function multiplied with the excitation coefficient. 50 individual frequencies were used.

To meaningfully compare both implementations we need some measure to compare both waves. In the first implementation I used a simple average of the amplitude.

Instead of the average Amplitude approach, I used in the first iteration, Giacomo suggested using a more advanced method. By equaling

$$P_w = \frac{1}{T_w}\int_0^{T_w} P_{w,\tau}\mathrm{d}\tau = \frac{\rho_w g^2 H^2 T_w}{32\pi}\left(\tanh(2k_w h_w) + \frac{2k_w h_w}{\cosh(2k_w h_w)}\right),$$

the wave power of the MCW and

$$P_w = \rho_w g \int_0^{\infty} \frac{\omega}{2k_w}\left(1 + \frac{2k_w h_w}{\sinh(2k_w h_w)}\right) S_\omega(\omega)\mathrm{d}\omega$$

the wave power of the SW and solving for the wave hight H. One note: The formulas require a water depth and since currently that is a free parameter we chose 20m.

The parameters are:
H_SW = 1.2
T_Wave_SW = 10s

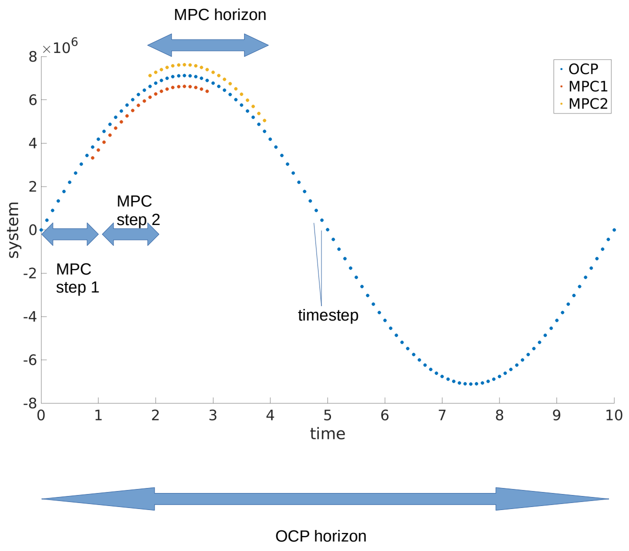
H_MCW = 0.74
T_WAVE_MCW = 10s

Brief explanation of parameters

As briefly explained in the beginning we want to run an OCP simulation over a long time-horizon to get the optimal input U and trajectory x. We then run an MPC algorithm where we use a massively smaller time-horizon to solve the OCP iteratively. There are multiple hyper-parameters to be chosen here:

1) Time horizon and time-step of OCP. Generally and especially for the stochastic wave formulation the longer the time horizon, the better. Since the wave is designed to incorporate random elements it is important to have a long simulation time to be able to reasonably judge the system behavior generally. In other words, to make sure phenomena we observe are not due to some random characteristic that only occurs in a short time. The time-step has to be chosen in such a way that the system behaves reasonable more on that later. In general: the smaller the better.
2) MPC horizon. The time horizon of the MPC is also critical as it defines how much of the problem the MPC „sees". We expect the solution to become closer to the OCP for increased MPC horizons. We also want to see if the „solution degradation" we want to observe actually affects the beginning of the MPC for very small time horizons.
3) MPC step. Once the MPC finds a solution for a certain time-horizon (MPC_horizon) the first step is applied to the system and then the MPC is run again, shifted by one step. One could always chose the smallest available time-step for the step but this is not always favorable and or possible. In a real system depending on how long the MPC takes it might be better to apply several time-steps from the MPC and take larger steps.
4) I will refer to the solution of the individual MPC OCP as the MPC open loop solution and the actual signal consisting of only the first values of each individual MPC's as the closed loop solution. This is the signal that will actually be applied to the system.

This figure is just to illustrate the parameters I explained above. Just so there are no misunderstandings.



The parameters I have chosen.

OCP horizon:

400 seconds. This is mostly just chosen as „good enough" since with a T_wave of 10 we can fit 40 wave periods in the OCP.

OCP timestep:

I solve the ocp and then simulate again using a variable step integrator (ode89). I then check if the resulting trajectory is equal to the one my ocp produced. If so the time-step is small enough. This validation happens every time and is not very time consuming. In the end I landed upon a step-size of 0.2. This means we have 400*5 = 4000 decision variables. This ocp takes pretty long even to initialize so if we want to simulate longer we might have to use larger step-size and sacrifice accuracy

MPC horizon:

Since this parameter has the most influence on the overall behavior I do not choose it but iteratively run the MPCs with MPC time-horizons varying from 10s to 110s.

MPC step:

free parameter. I chose 0.2 so the smallest possible value. This means only one value is chosen from each open loop solution.

Brief explanation of algorithm

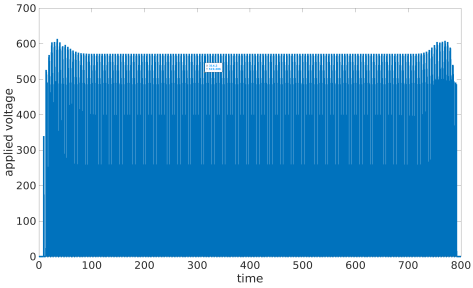
Just to reiterate and since the generation of data is very important here is the algorithm in pseudo-code:

```
- set Wave disturbance either Harmonic/or/Stochastic
-Run OCP with 400 seconds horizon
-Simulate with ode89 to validate time-step is sufficient enough
- do i = 1:20
        MPC Horizon = 10+5*i
        while t<400
                solve OCP from t to t+MPCHorizon
                t = t + 0.2 second
        end
        save Data
end do
```

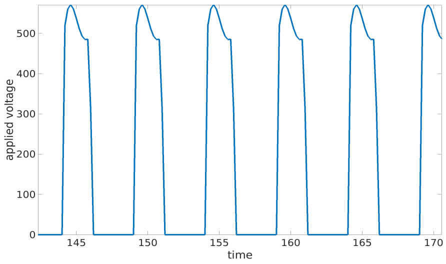
I explain it so thoroughly because i want to make sure any mistakes on my end are caught fast. If there are any mistakes we could wast a lot of time trying to interpret the data and yea you probably get what i mean. But now that we have the data all that follows here is pure speculation on my part. Again we are trying to somehow show the difference in behavior for the different wave models and hopefully extract some insight into the weird solution degradation we see. So the first thing I did was to just visualize what we try to do.

Investigation of the data
MPC vs OCP

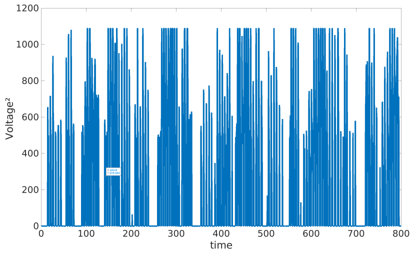
First lets look at the ocp solutions to get a feel for how they look
Left side is harmonic wave and right side is stochastic wave



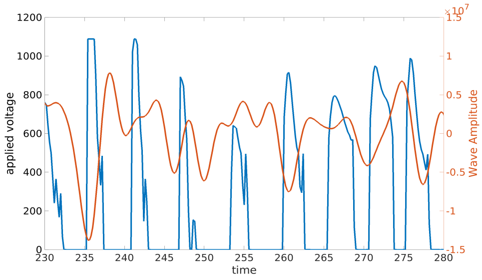
Ok lets zoom in a bit



In the middle of the ocp the behavior is nice and periodic as expected



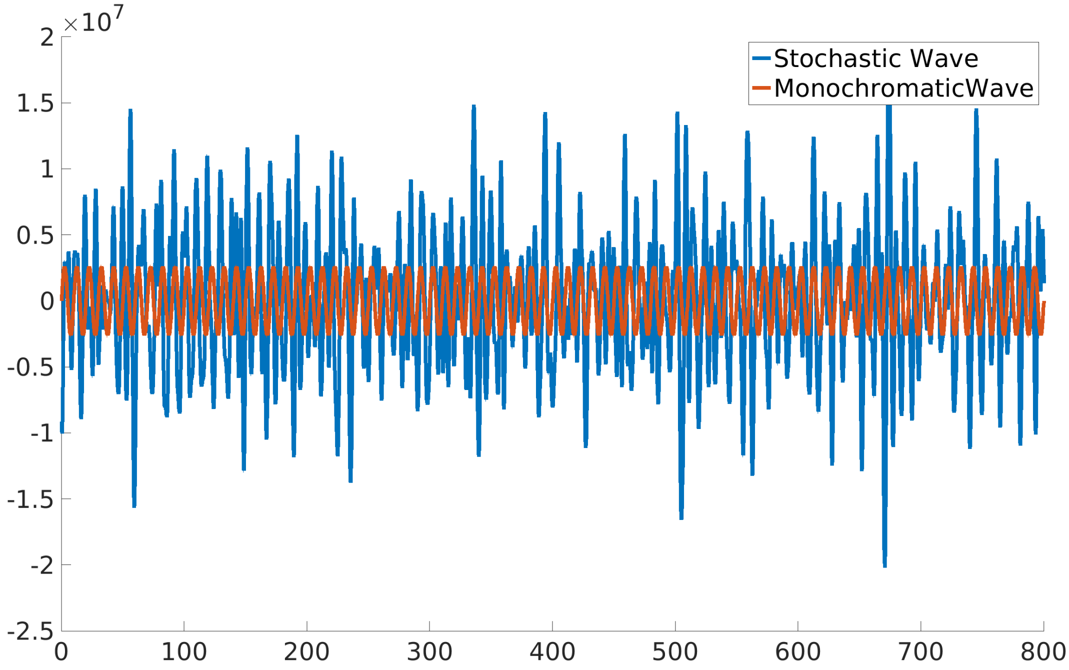
Ok less periodic and less intuitive So lets zoom in and also show the system behavior next to it



Again less periodic but still makes sense. Think of heuristic controller When angle and derivative of angle are in same direction voltage is applied.

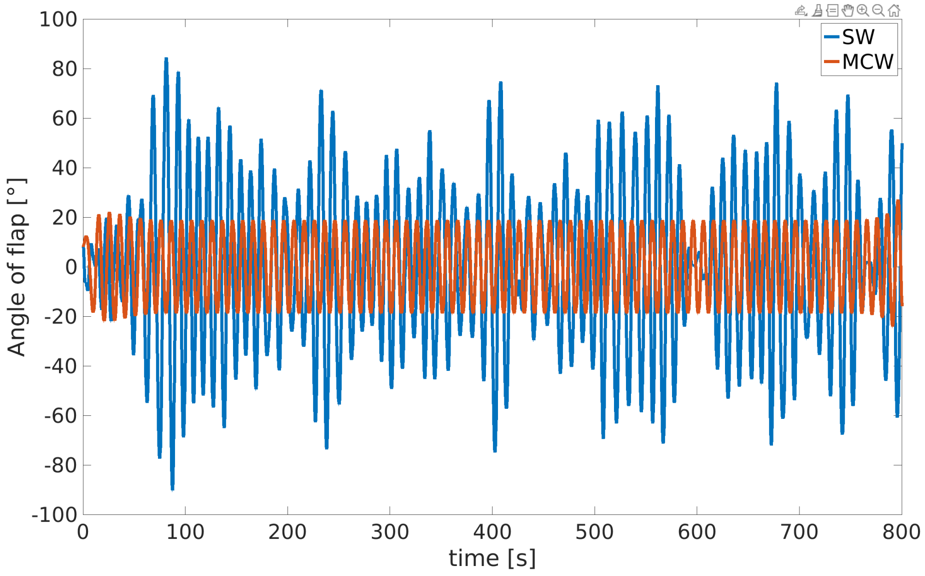
Ok so far so good but lets get into the MPC!

Insert!!
Because the wave scaling is different we need to look at the OCP's again
This part was inserted after changing the scaling according to Giacomos recommendation

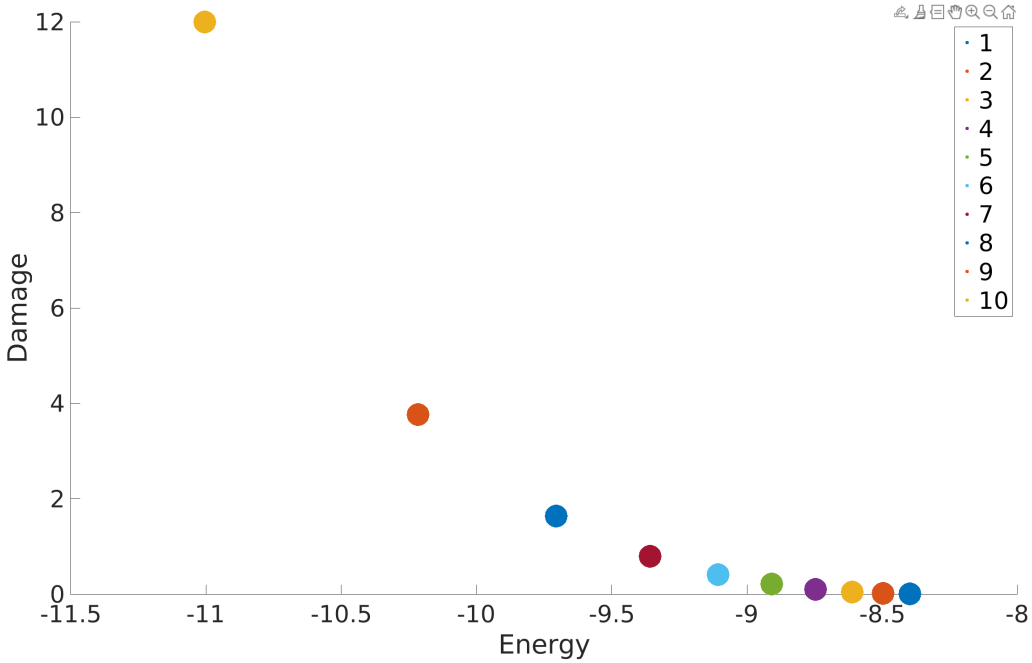


Shown here are the MCW and SW for the simulations and with the new scaling from Giacomos Thesis. We can immediately see that even though the SW and MCW have the same wave power the SW has a lot more pronounces peaks.

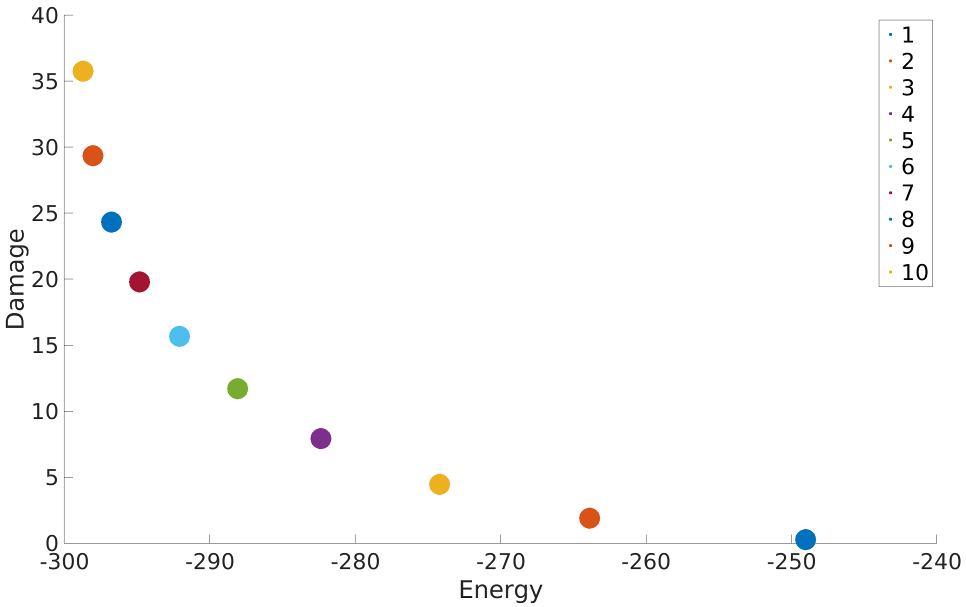
As expected the angle of the flap (with control input) looks similarly different



Which then leads to vastly different energy harvests

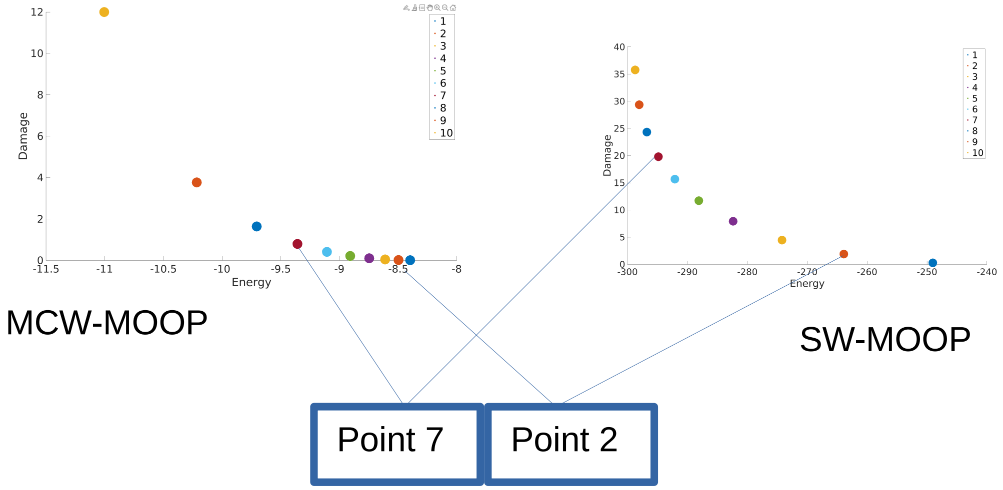


MCW MOOP



SWMOOP

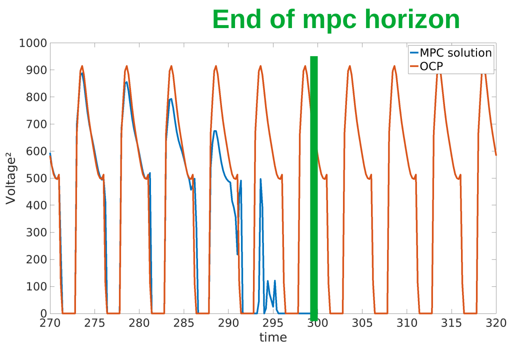
I don’t know if this difference in behavior is a problem but we need to discuss it probably. For now I have redone all the simulations.
Also because I did now want the MOOP to influence the results to much I did all the simulations for 2 Pareto Points. Points 2 and 7



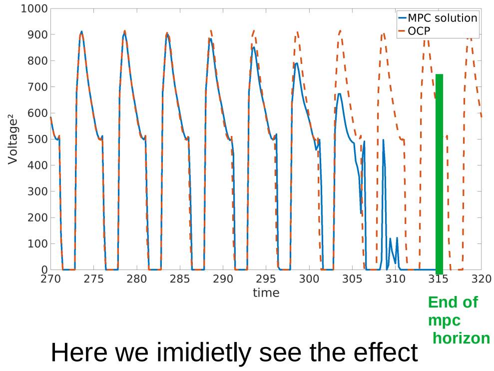
Even though the same weights [Weighted sum method was used for the MOOP] were used the location of the same Points is different. This is to be expected when the Pareto fronts have different shapes

Investigation of the data
MPC vs OCP

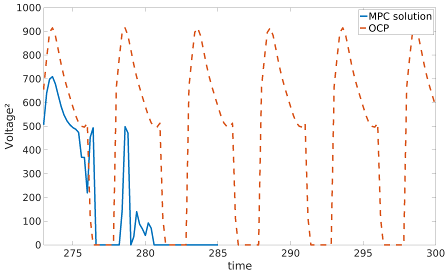
First lets pick some random MPC solutions and compare them to the „correct“ OCP solution.
Left side is harmonic wave and right side is stochastic wave



Here we can immediately see the phenomenon we were trying to capture. The MPC solution „degenerates“ towards the end of it’s time horizon. We can see each successive peak getting lower and lower.

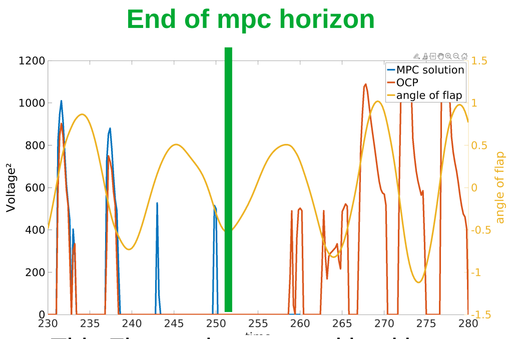


Here we imidietly see the effect again. Shown is the same time interval but with a larger MPC time-horizon The first peak is now identical to the ocp. Because the „solution degeneration“ does not reach as far back.

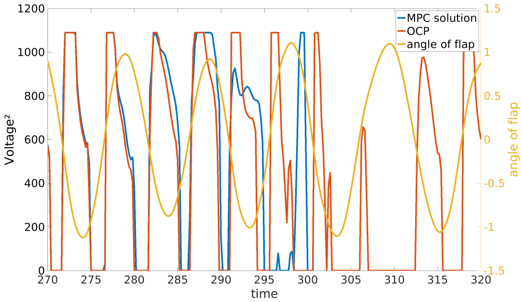


Taken to it’s extreme we see that for a sufficiently small time horizon the solution even degrades at the very start. This is very problematic for the MPC as the actual faulty signal will be applied to the system.

The stochastic wave is not as cut and dry so I plotted the flap angle too so an intuition of the system behavior might be obtained.



This Figure gives a good intuitive feeling for the MPC vs. OCP behavior in the stochastic wave.
The first 2 peaks are pretty similar with ocp and mpc solution. But the next 2 peaks only exist in the mpc solution. Why is that ? Well the mpc is blind after second 250. The ocp knows the waves will increase in strength at the 265 second mark and also knows its beneficial to don't give input between 240 and 255. This is a fundamental property of the MPC because it can only see for a certain horizon. The important part is that the error does not affect the behavior at the beginning of the MPC horizon. Or at least it does not seem that way at first glance.



Similarly here, if we look at a later MPC solution we see a good approximation of the ocp. In the last wave period the MPC chooses a different interval to apply the last voltage peak.

The behavior of the system is definitely more difficult to intuitively understand. In a first heuristic investigation though it does not seem that the horizon brings with it a a degeneration.

After talking to Mathias earlier this week he suggested running another batch of simulations with a smaller MPC step. This means, now only the first value of the MPC solution is applied to the model and not as previously the first 5. This also means we have 5 times as many MPC simulations. This in combination with the fact, that is used a finer grain for the MPC-increment [2 seconds instead of 5] made the simulations a lot more time consuming For this reason I halved the OCP-horizon to 400 seconds. If we deem this not long enough we can run the simulations again but to have data for this Friday it was unfortunately necessary

The investigation on the last page was mostly just „look at the results“ and try to find a pattern. This was a good first step but we have to find more concrete evidence. On thing that is super important for the MPC is the solution at the MPC step. Because if there is a difference in the MPC at the end of the horizon we do not really care. This faulty signal will not be applied to the system. If, however there is an error at the beginning of the MPC we do care because it will be applied. So as the second step I calculated the difference of the applied signal to the ocp solution.

[Side Note:Also as per Giacomos recommendation I used a different scale to measure the error. First the applied signal is hard limited between 0 and 1089 [constraint of the OCP] so as a first step the error is normed to [0 1] from [0 1089].

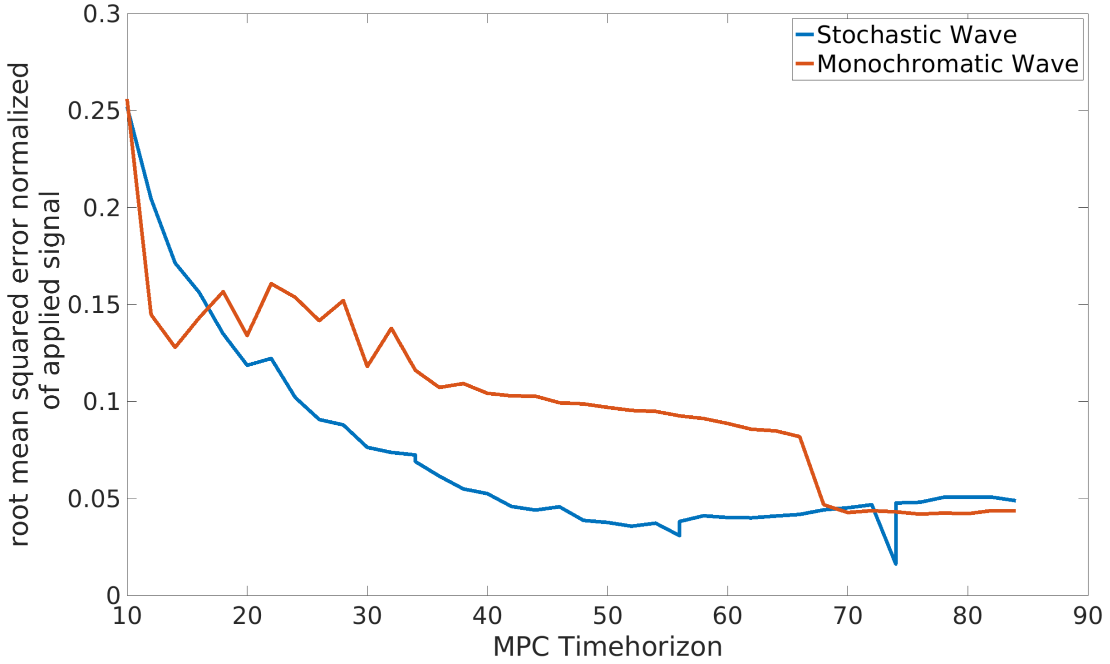
Then a more standard RMSE is calculated.

In pseudo-code:

```
For all MPC Horizons
        For all MPC solutions
                Error  +=(MPC(1) – OCP(MPCtime(1)))/1089).^2
        Total applied signal Error = sqrt(Error / N_increments)
```

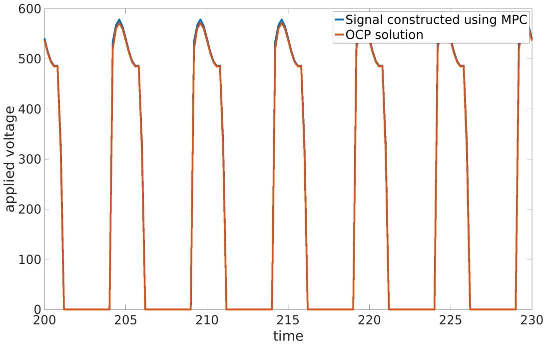
New Simulation
OCP Horizon = 400s
MPC Step = 0.2 [same as timestep now]
MPC increment = 2 s

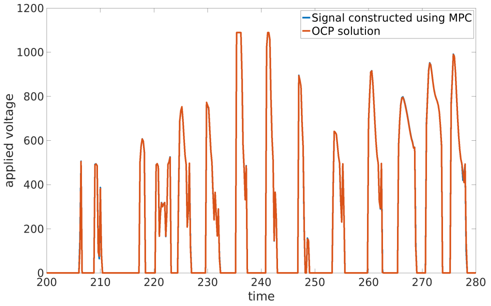
Point 7



I think I wont put too much discussion here but this would probably be the most important piece if we want to compare MCW and SW performance The SW is consistently! Better performing. We can discuss this on Friday in detail. Also really cool is that we can see exactly when the “solution degeneration” reaches the closed loop solution. On the next page I plotted the closed loop MPC and the OCP. If we compare the performance of MPC-Horizon 68s and 66s we see a large difference for the MCW but not for the SW. This corresponds to the kinc in the plot above.

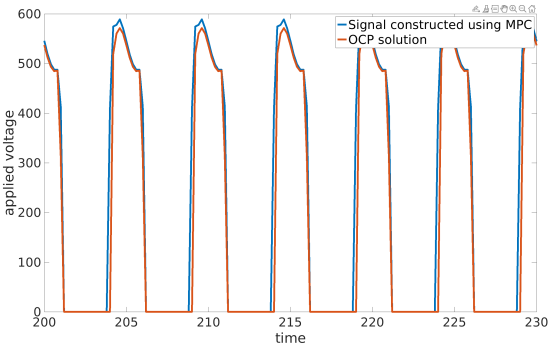
MPC Horizon 68s

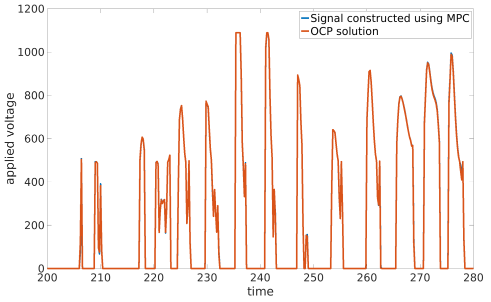




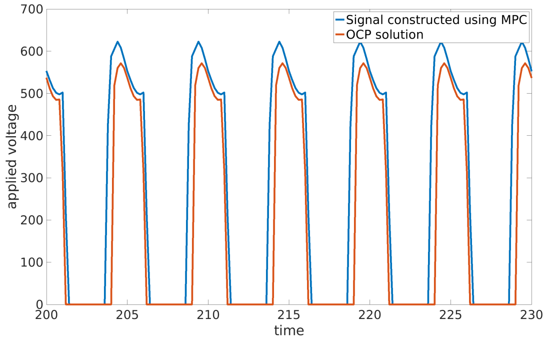
Really nice to see here is that the Kink in the Plot of the previous page at 66-68 corresponds to a significant worsening of the close form MPC solution.

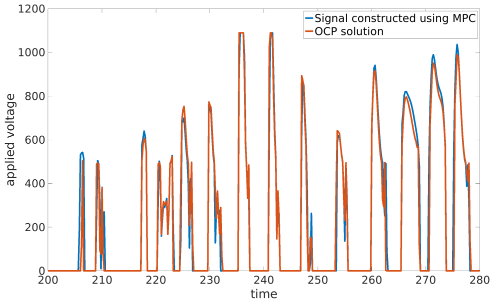
MPC Horizon 66s



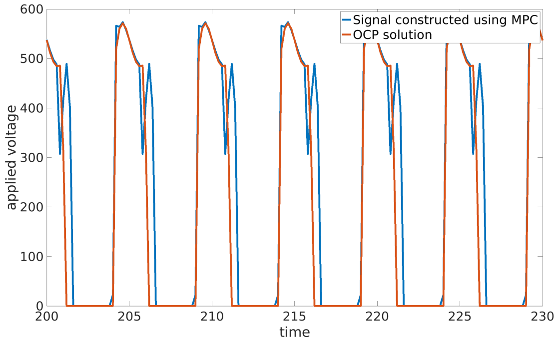


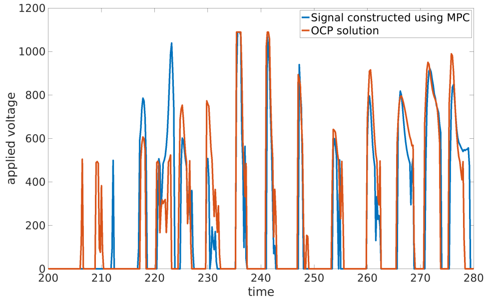
MPC Horizon 32s





MPC Horizon 14s

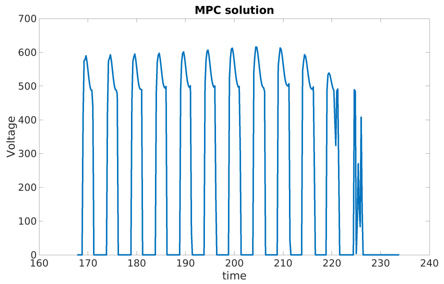




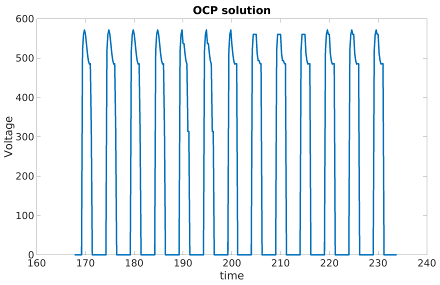
Behavior of the open form solution over the horizon.

In reality we only really care about the applied step, so the first value[s] of the MPC open form solution. But because we want to investigate the behavior of the solution degeneration it might be beneficial to look at the solution accuracy over the MPC horizon.

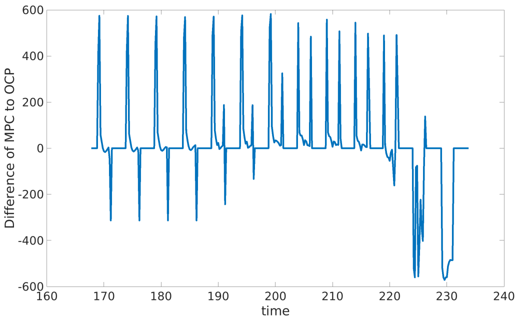
So what I did was:
MPC solution – OCP solution of that segment:



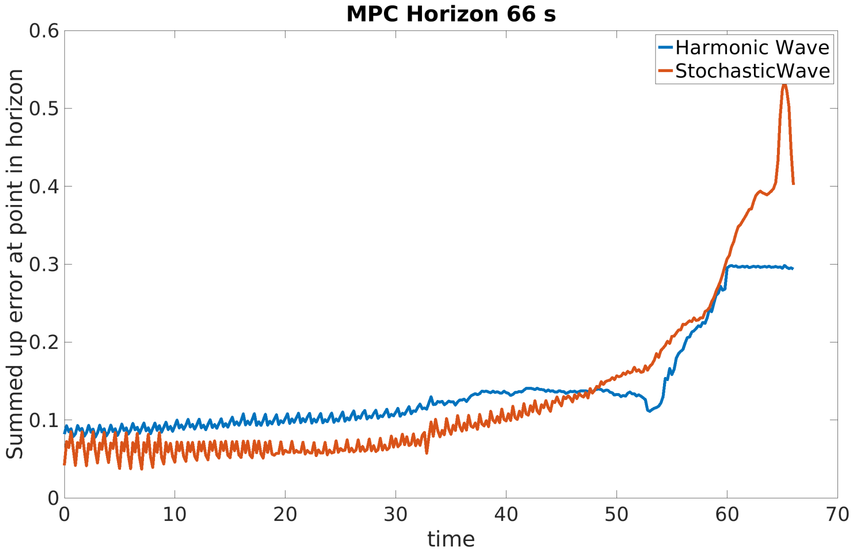
minus



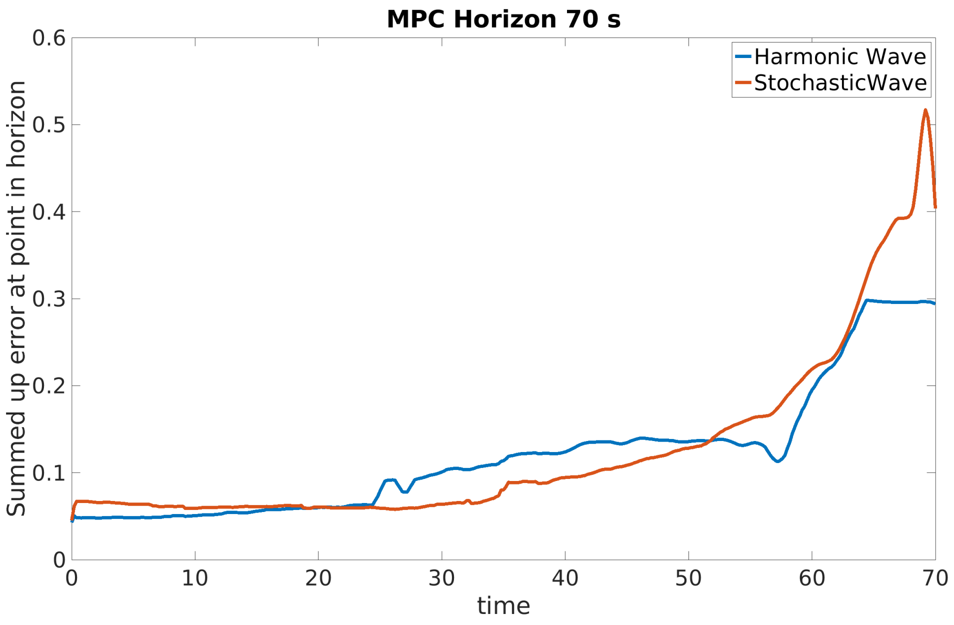
Equal to

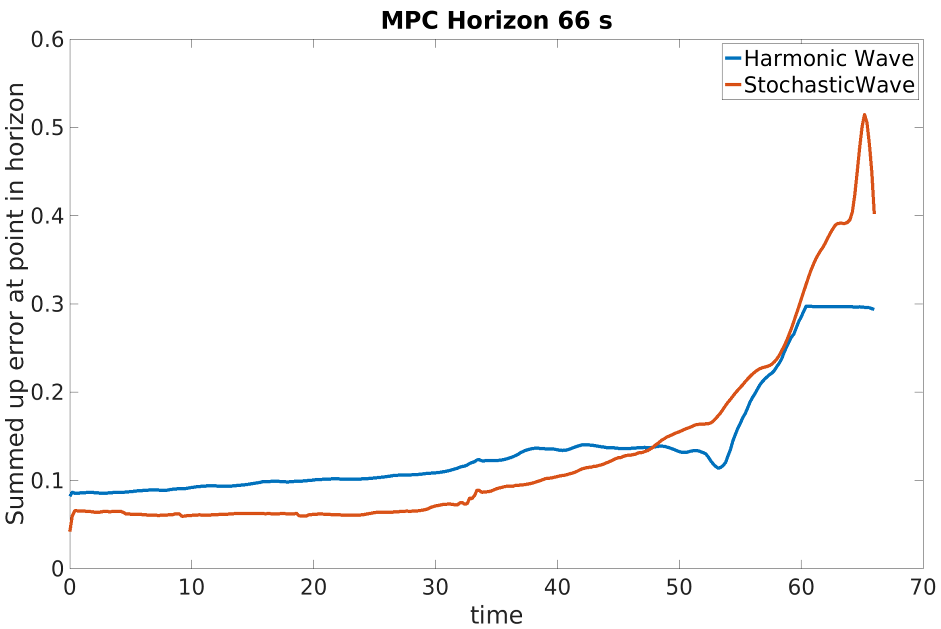


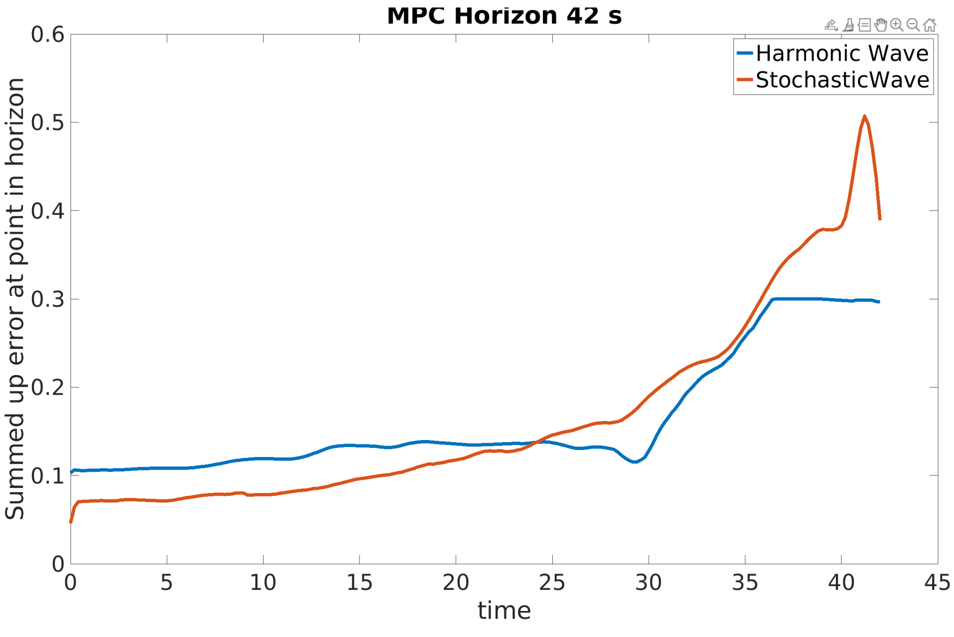
Then I scale from 0 to Max Voltage [1089]. Then I add the squared scaled error of all the MPC solutions together [RMSE].

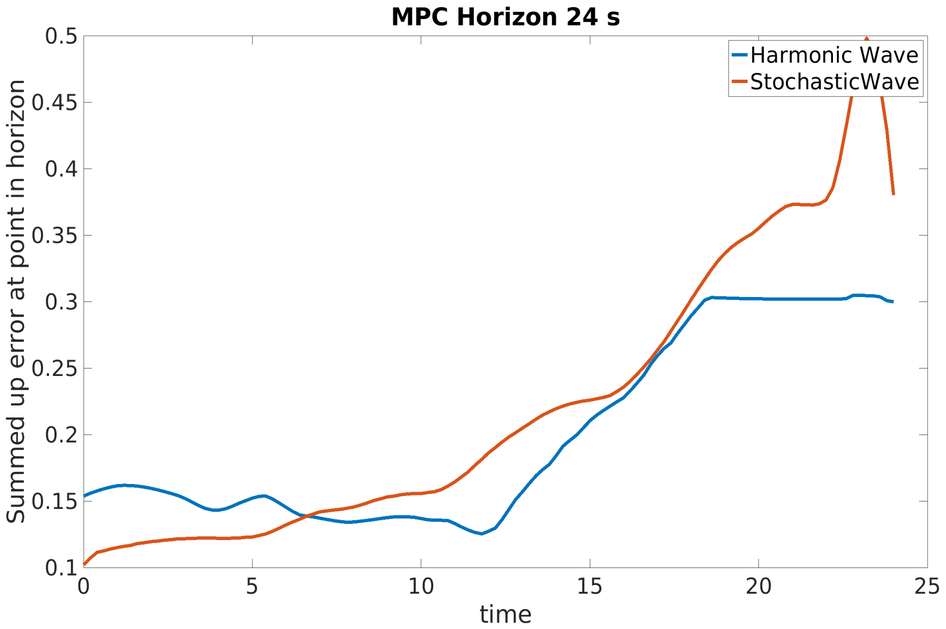


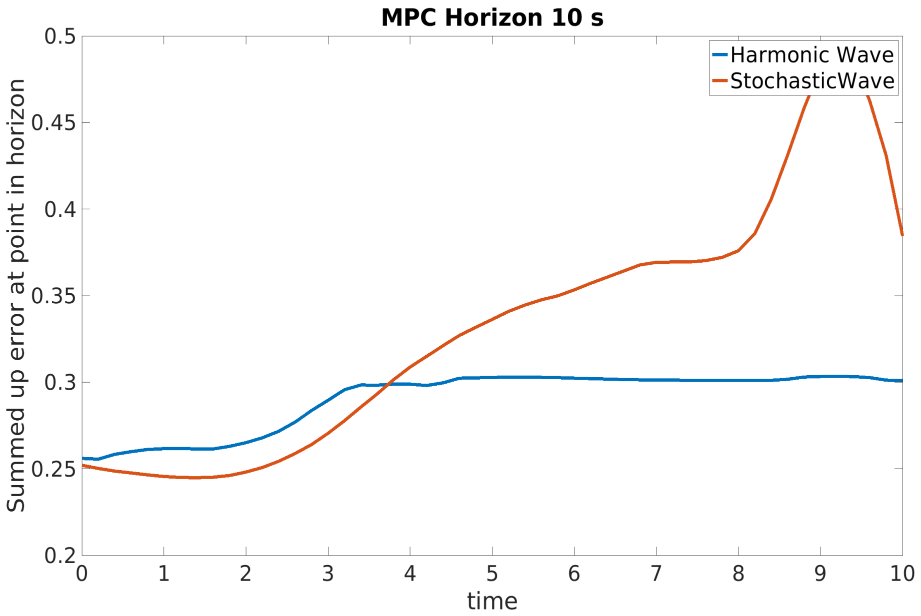
As a last step the plot is smoothed just so an intuitive interpretation is easier.
[This just filters out the oscillations]


MPC Horizon 70 s
Harmonic Wave
StochasticWave
Summed up error at point in horizon
time
0
0.1
0.2
0.3
0.4
0.5
0.6
0
10
20
30
40
50
60
70


MPC Horizon 66 s
Harmonic Wave
StochasticWave
Summed up error at point in horizon
time
0
0.1
0.2
0.3
0.4
0.5
0.6
0
10
20
30
40
50
60
70


MPC Horizon 42 s
Harmonic Wave
StochasticWave
Summed up error at point in horizon
time
0
0.1
0.2
0.3
0.4
0.5
0.6
0
5
10
15
20
25
30
35
40
45

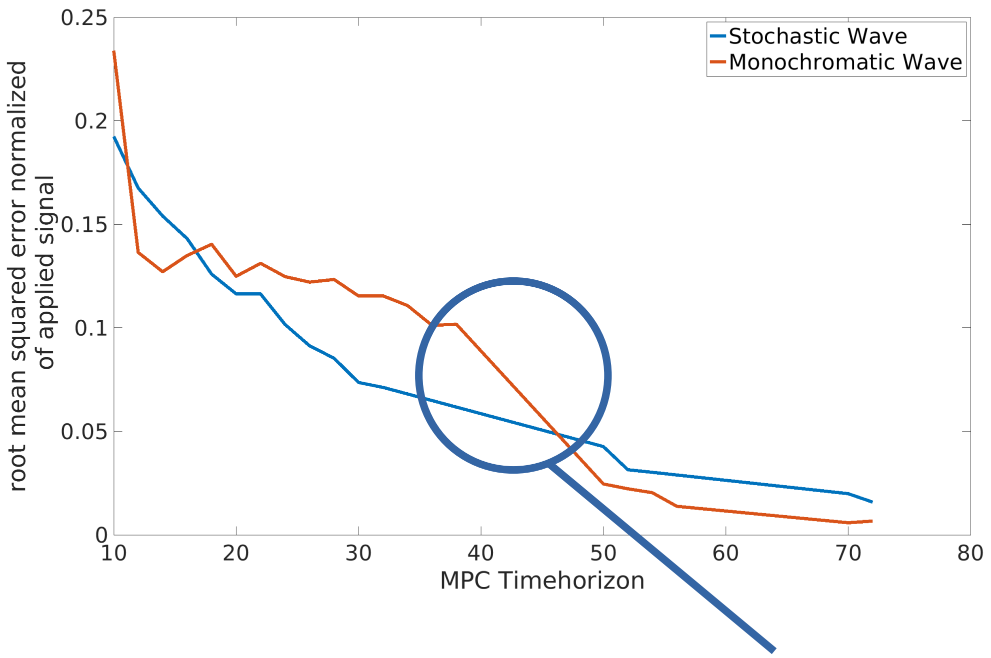

MPC Horizon 24 s
Harmonic Wave
StochasticWave
Summed up error at point in horizon
time
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
0
5
10
15
20
25


MPC Horizon 10 s
Harmonic Wave
StochasticWave
Summed up error at point in horizon
time
0.2
0.25
0.3
0.35
0.4
0.45
0.5
0
1
2
3
4
5
6
7
8
9
10

This is just repetition of the precious section but with a different Pareto point. There are some values missing in the MPC increments because of full storage on the server. [My mistake]. But the missing values are computed right now. Still, we can look at the results for the points that were saved.

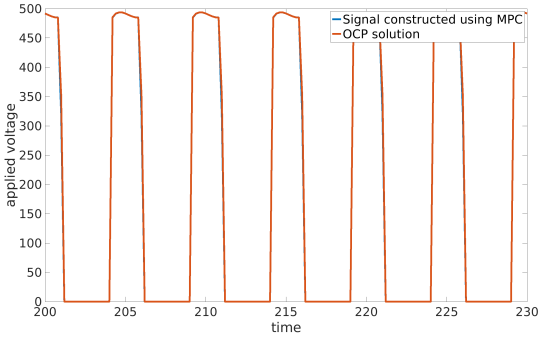
New Simulation
OCP Horizon = 400s
MPC Step = 0.2 [same as timestep now]
MPC increment = 2 s

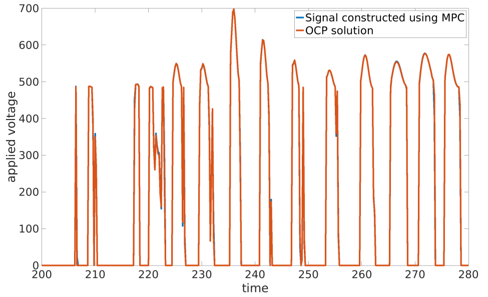
Point 2



Values missing here!

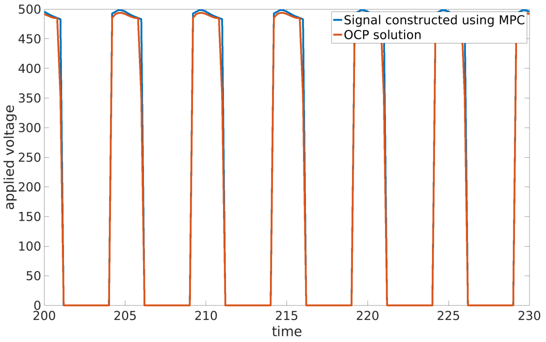
MPC Horizon 70s

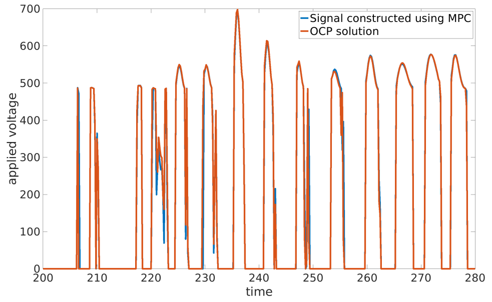




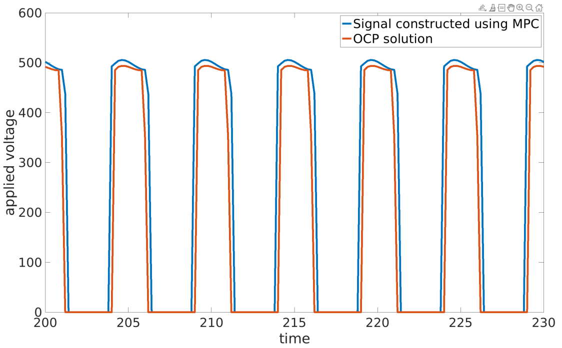
Really nice to see here is that the Kink in the Plot of the previous page at 66-68 corresponds to a significant worsening of the close form MPC solution.

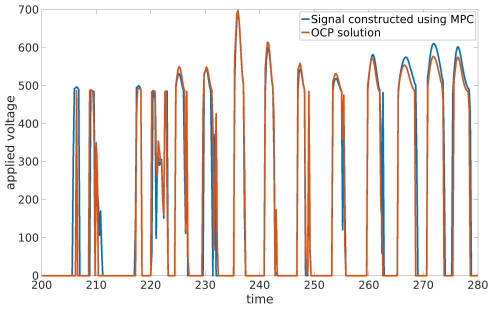
MPC Horizon 50s



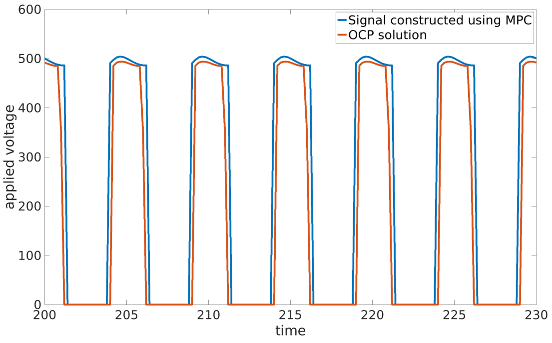


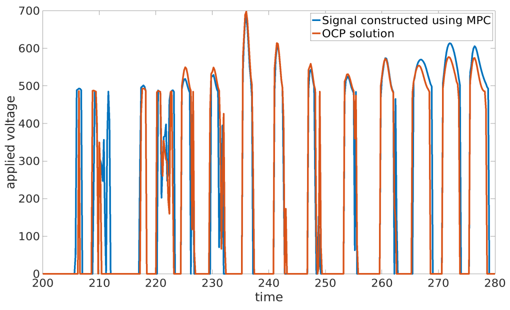
MPC Horizon 30s

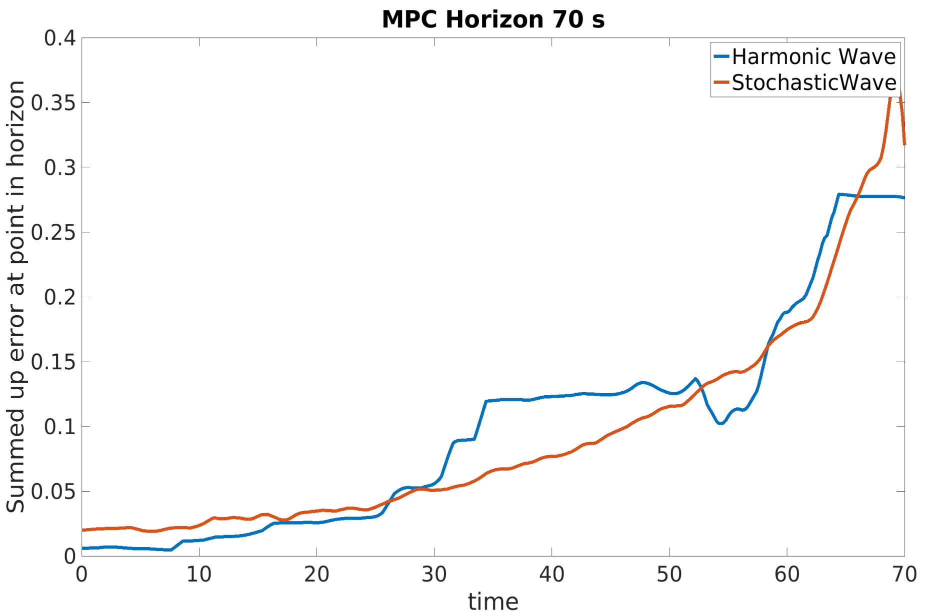


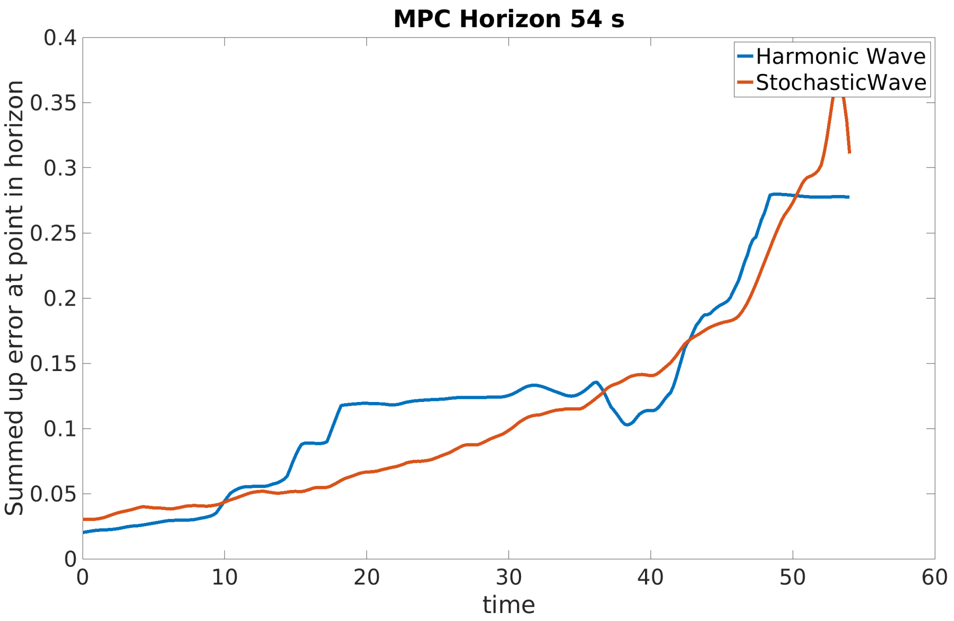


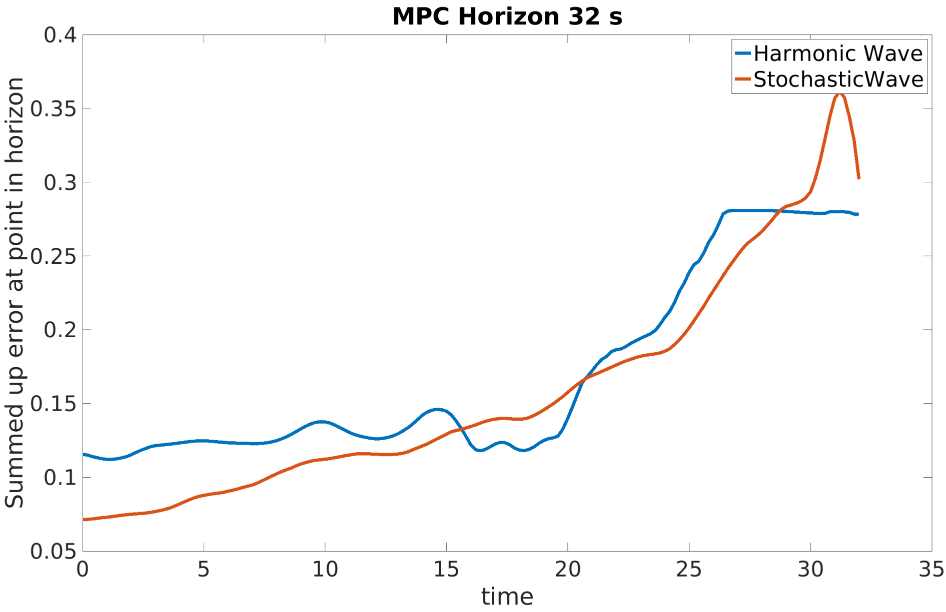
MPC Horizon 26s






MPC Horizon 70 s
Harmonic Wave
StochasticWave
Summed up error at point in horizon
time
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0 10 20 30 40 50 60 70


MPC Horizon 54 s
Harmonic Wave
StochasticWave
Summed up error at point in horizon
time
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0 10 20 30 40 50 60

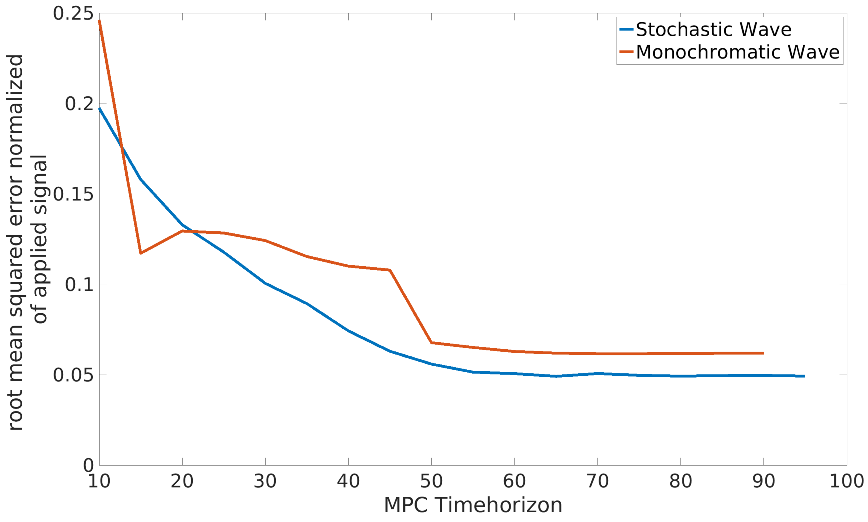

MPC Horizon 32 s
Harmonic Wave
StochasticWave
Summed up error at point in horizon
time
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0 5 10 15 20 25 30 35

Discussion of results

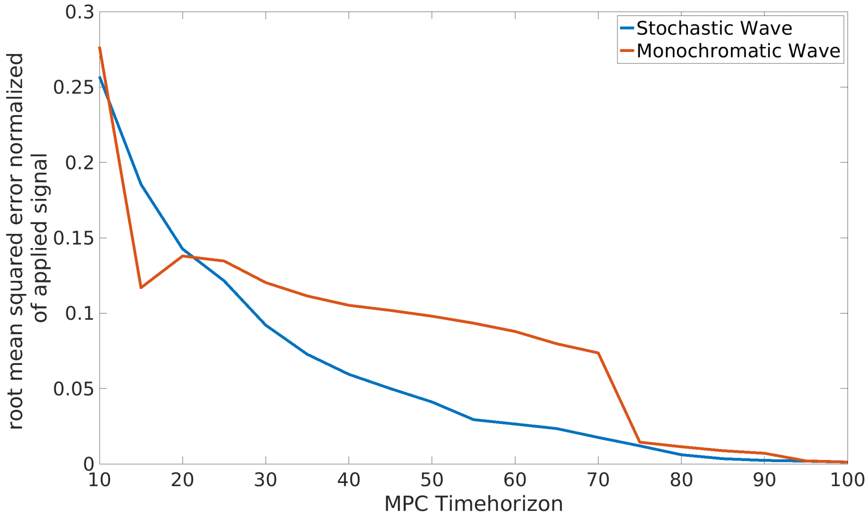
Ok so for both the simulation of Point 2 and Point 7 the performance of the MPC is actually better for the stochastic wave! This is first of all pretty cool but we have to be careful Remember that for a different wave scaling in the first version the MCW-Performance was slightly better. So it seems the 2 are very close together and by the choice of wave parameters we could easily have a system where the MCW-Performance is better. Still, it is really good that the MCW and SW-Performance are at least comparable Without simulating I would have expected the MC-MPC to perform way better. The wave does not randomly change and influence the system but is very predicable in the sine wave actuation of the flap. So the fact that the SW performs equally well is probably something good for us.

Investigation of the data
MPC vs OCP

Here are the results from a previous iteration where the MPC step was 1 second instead of 0.2 seconds. The MPC horizon temporal resolution was lower at 5seconds [instead of now 2 seconds]. The OCP Horizon was larger though at 800 seconds. Discussion and explanation remain the same so I just left the plots here.



Pareto Point 2
[Hight priority on Damage]

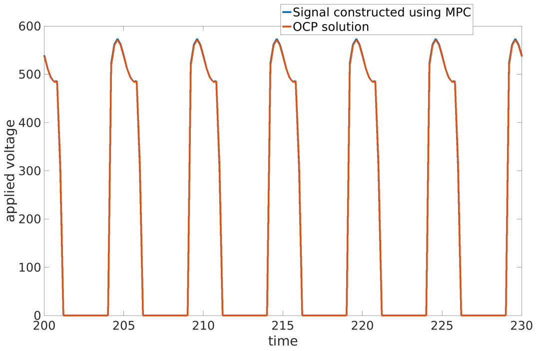


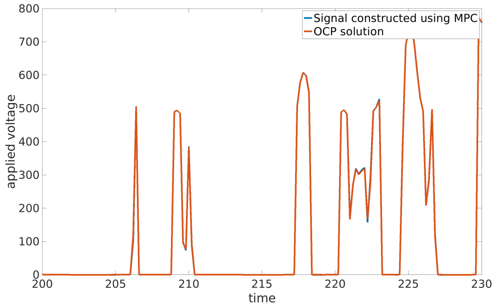
Pareto Point 7
[Hight priority on Energy]

Ok so for both the simulation of Point 2 and Point 7 the poerformance of the MPC is actually better for the stochastic wave! This is first of all pretty cool but we have to be carful. Remeber that for a different wave scaling in the first version the MCW-Performance was slighly better. So it seems the 2 are very close together and by the choice of wave parameters we could easyly have a system where the MCW-Performance is better. Still, it is really good that the MCW and SW-Performance are at least comprable. Without simulating I would have expected the MC-MPC to perform way better. The wave does not randomly change and influence the system but is very predicable in the sine wave actuation of the flap. So the fact that the SW performs equally well is probably something good for us.

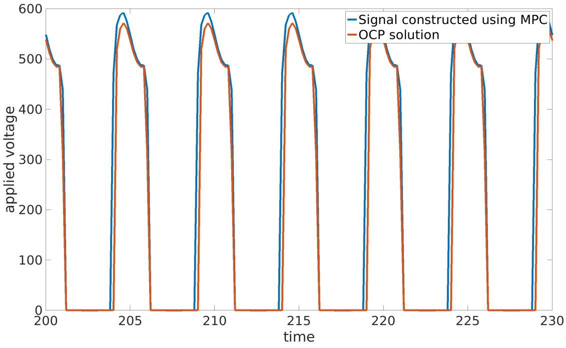
But to get a better look at what the MPC closed form solutions look like here are some comparisons with different Timehorizons [Taken From Point 7]

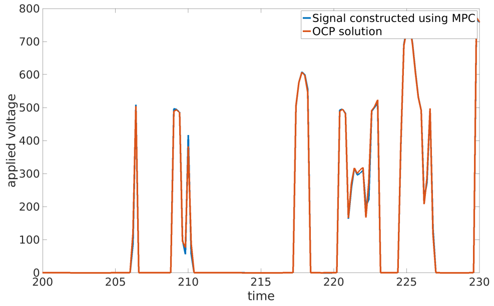
MCW-MPC	SW-MPC



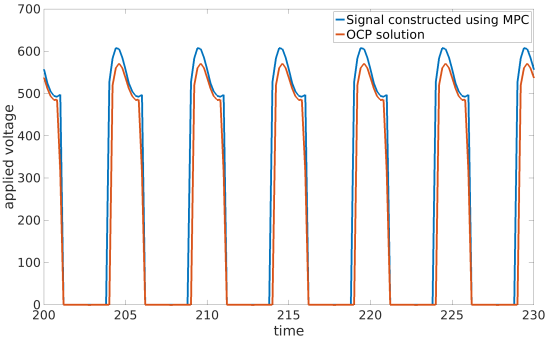


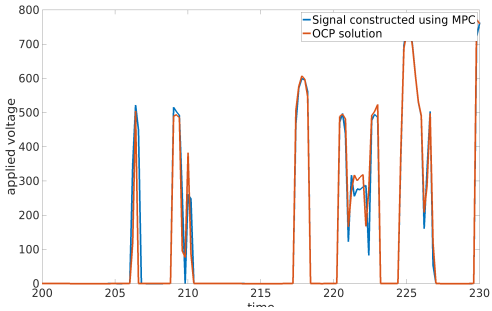
MPC Horizon 75s



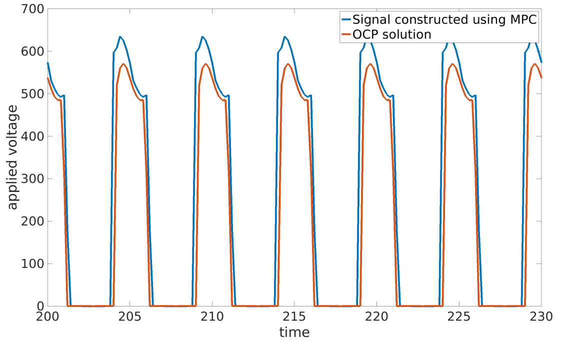


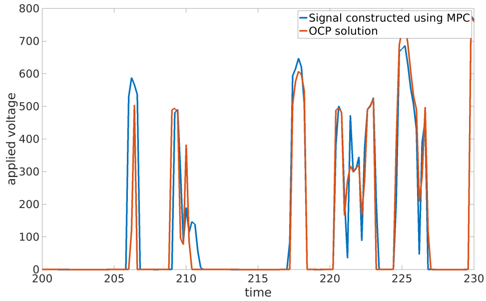
MPC Horizon 75s



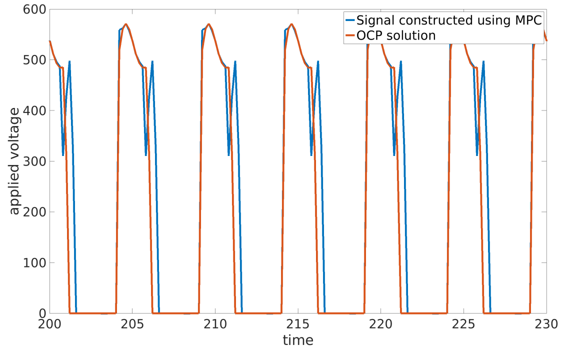


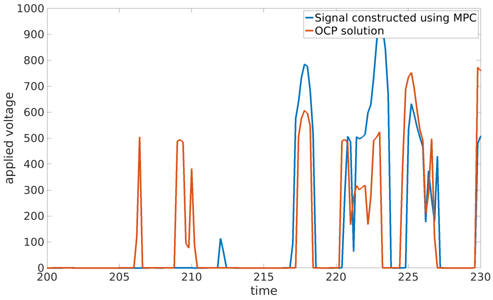
MPC Horizon 45s





MPC Horizon 30s





MPC Horizon 45s