

DATA 621 HW 3 Crime

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1. Data Exploration

```
library(tidyverse)
```

```
## -- Attaching core tidyverse packages ----- tidyverse 2.0.0 --
## v dplyr      1.1.4      v readr      2.1.4
## v forcats    1.0.0      v stringr   1.5.1
## v ggplot2    3.4.4      v tibble    3.2.1
## v lubridate  1.9.3      v tidyr     1.3.0
## v purrr      1.0.2
## -- Conflicts ----- tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()     masks stats::lag()
## i Use the conflicted package (<http://conflicted.r-lib.org/>) to force all conflicts to become errors
```

```
library(psych)
```

```
##
## Attaching package: 'psych'
##
## The following objects are masked from 'package:ggplot2':
##
##      %+%, alpha
```

```
library(corrplot)
```

```
## corrplot 0.92 loaded
```

```
library(MASS)
```

```
##
## Attaching package: 'MASS'
##
## The following object is masked from 'package:dplyr':
##
##      select
```

```
df <- read.csv("https://raw.githubusercontent.com/LeJQC/DATA-621-Group-2/main/HW3/crime-training-data_m
```

Let's get a general sense of the training data set using the `glimpse` function. There are 466 observations and 13 variables in the training data set. Of the 13 variables, 12 are predictor variables and 1 is the target variable. All of the variables seem to be floats or integers.

```
glimpse(df)
```

```
## Rows: 466
## Columns: 13
## $ zn      <dbl> 0, 0, 0, 30, 0, 0, 0, 0, 0, 80, 22, 0, 0, 22, 0, 0, 100, 20, 0~
## $ indus   <dbl> 19.58, 19.58, 18.10, 4.93, 2.46, 8.56, 18.10, 18.10, 5.19, 3.6~
## $ chas    <int> 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, ~
## $ nox     <dbl> 0.605, 0.871, 0.740, 0.428, 0.488, 0.520, 0.693, 0.693, 0.515, ~
## $ rm      <dbl> 7.929, 5.403, 6.485, 6.393, 7.155, 6.781, 5.453, 4.519, 6.316, ~
## $ age     <dbl> 96.2, 100.0, 100.0, 7.8, 92.2, 71.3, 100.0, 100.0, 38.1, 19.1, ~
## $ dis     <dbl> 2.0459, 1.3216, 1.9784, 7.0355, 2.7006, 2.8561, 1.4896, 1.6582~
## $ rad     <int> 5, 5, 24, 6, 3, 5, 24, 24, 5, 1, 7, 5, 24, 7, 3, 3, 5, 5, 24, ~
## $ tax     <int> 403, 403, 666, 300, 193, 384, 666, 666, 224, 315, 330, 398, 66~
## $ ptratio <dbl> 14.7, 14.7, 20.2, 16.6, 17.8, 20.9, 20.2, 20.2, 20.2, 16.4, 19~
## $ lstat   <dbl> 3.70, 26.82, 18.85, 5.19, 4.82, 7.67, 30.59, 36.98, 5.68, 9.25~
## $ medv    <dbl> 50.0, 13.4, 15.4, 23.7, 37.9, 26.5, 5.0, 7.0, 22.2, 20.9, 24.8~
## $ target  <int> 1, 1, 1, 0, 0, 0, 1, 1, 0, 0, 0, 0, 1, 1, 0, 0, 1, 1, 1, 0, ~
```

The variables are:

- **zn**: proportion of residential land zoned for large lots (over 25000 square feet)
- **indus**: proportion of non-retail business acres per suburb
- **chas**: a dummy var. for whether the suburb borders the Charles River (1) or not (0)
- **nox**: nitrogen oxides concentration (parts per 10 million)
- **rm**: average number of rooms per dwelling
- **age**: proportion of owner-occupied units built prior to 1940
- **dis**: weighted mean of distances to five Boston employment centers
- **rad**: index of accessibility to radial highways
- **tax**: full-value property-tax rate per \$10,000
- **ptratio**: pupil-teacher ratio by town
- **lstat**: lower status of the population (percent)
- **medv**: median value of owner-occupied homes in \$1000s
- **target**: whether the crime rate is above the median crime rate (1) or not (0)

Summary Table

Using the `describe` function from the `psych` library, we can get the summary statistics of all the variables as shown in the table below. Something to note here is the large standard deviation with the **zn**, **age** variable compared to its range, which may be because they are proportions. Another thing that stands out is the **tax** variable. The mean, standard deviation, median, and range are much larger than the other predictor variables so we might have to do some transformation on it later on.

```
summary_table <- describe(df)
print(round(summary_table,2))
```

##	vars	n	mean	sd	median	trimmed	mad	min	max	range	skew
## zn	1	466	11.58	23.36	0.00	5.35	0.00	0.00	100.00	100.00	2.18
## indus	2	466	11.11	6.85	9.69	10.91	9.34	0.46	27.74	27.28	0.29
## chas	3	466	0.07	0.26	0.00	0.00	0.00	0.00	1.00	1.00	3.34
## nox	4	466	0.55	0.12	0.54	0.54	0.13	0.39	0.87	0.48	0.75
## rm	5	466	6.29	0.70	6.21	6.26	0.52	3.86	8.78	4.92	0.48
## age	6	466	68.37	28.32	77.15	70.96	30.02	2.90	100.00	97.10	-0.58
## dis	7	466	3.80	2.11	3.19	3.54	1.91	1.13	12.13	11.00	1.00
## rad	8	466	9.53	8.69	5.00	8.70	1.48	1.00	24.00	23.00	1.01
## tax	9	466	409.50	167.90	334.50	401.51	104.52	187.00	711.00	524.00	0.66
## ptratio	10	466	18.40	2.20	18.90	18.60	1.93	12.60	22.00	9.40	-0.75
## lstat	11	466	12.63	7.10	11.35	11.88	7.07	1.73	37.97	36.24	0.91
## medv	12	466	22.59	9.24	21.20	21.63	6.00	5.00	50.00	45.00	1.08
## target	13	466	0.49	0.50	0.00	0.49	0.00	0.00	1.00	1.00	0.03

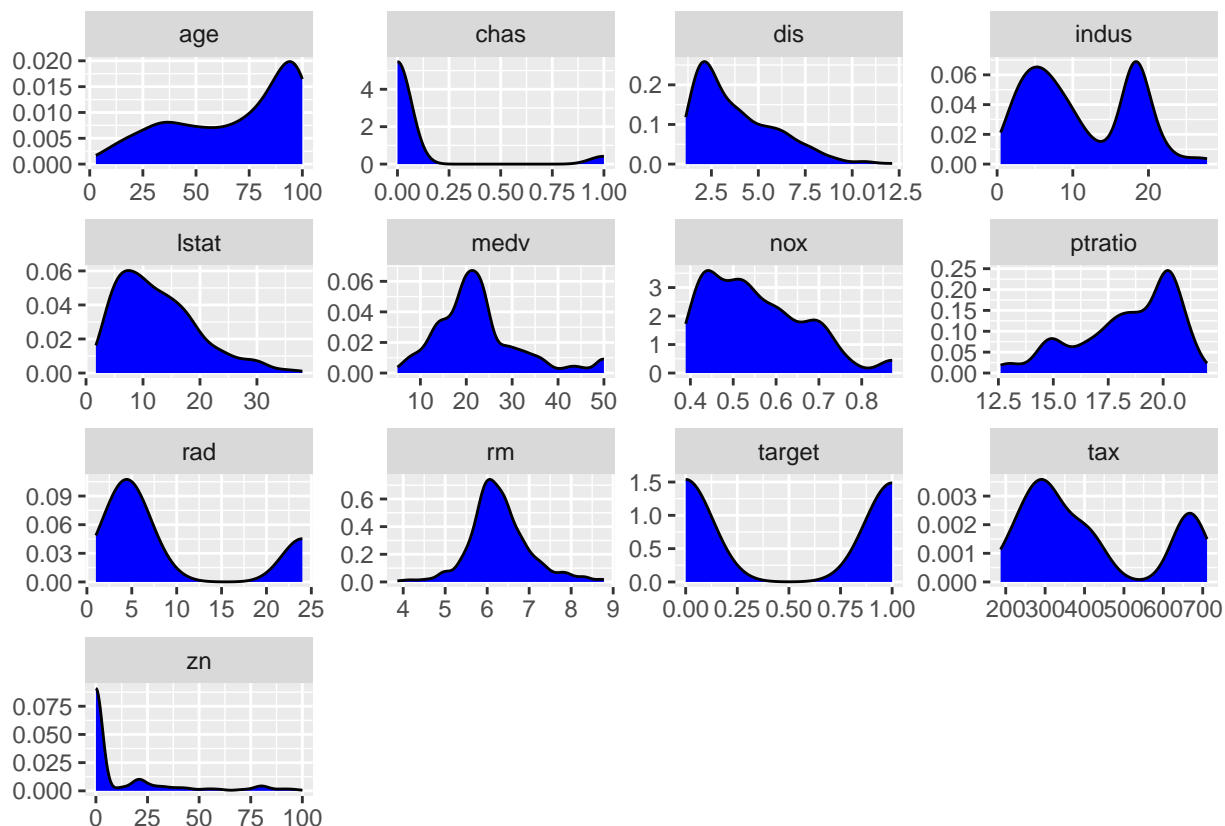
##	kurtosis	se
## zn	3.81	1.08
## indus	-1.24	0.32
## chas	9.15	0.01
## nox	-0.04	0.01
## rm	1.54	0.03
## age	-1.01	1.31
## dis	0.47	0.10
## rad	-0.86	0.40
## tax	-1.15	7.78
## ptratio	-0.40	0.10
## lstat	0.50	0.33
## medv	1.37	0.43
## target	-2.00	0.02

Distrubtion Plot

Next, let's look at the distribution of the 13 variables. As expected the values of the target variable are distributed around 0 and 1. Variables such as age, chas, dis, lstat, nox, ptration, rad, and zn are skewed to the left or right. On the other hand, variables medv and rm are normally distributed. The remaining predictor variables indus and tax have a bimodal distribution curve.

```
df_long <- df %>%
  pivot_longer(
    cols = everything(),
    names_to = "variable",
    values_to = "value"
  )

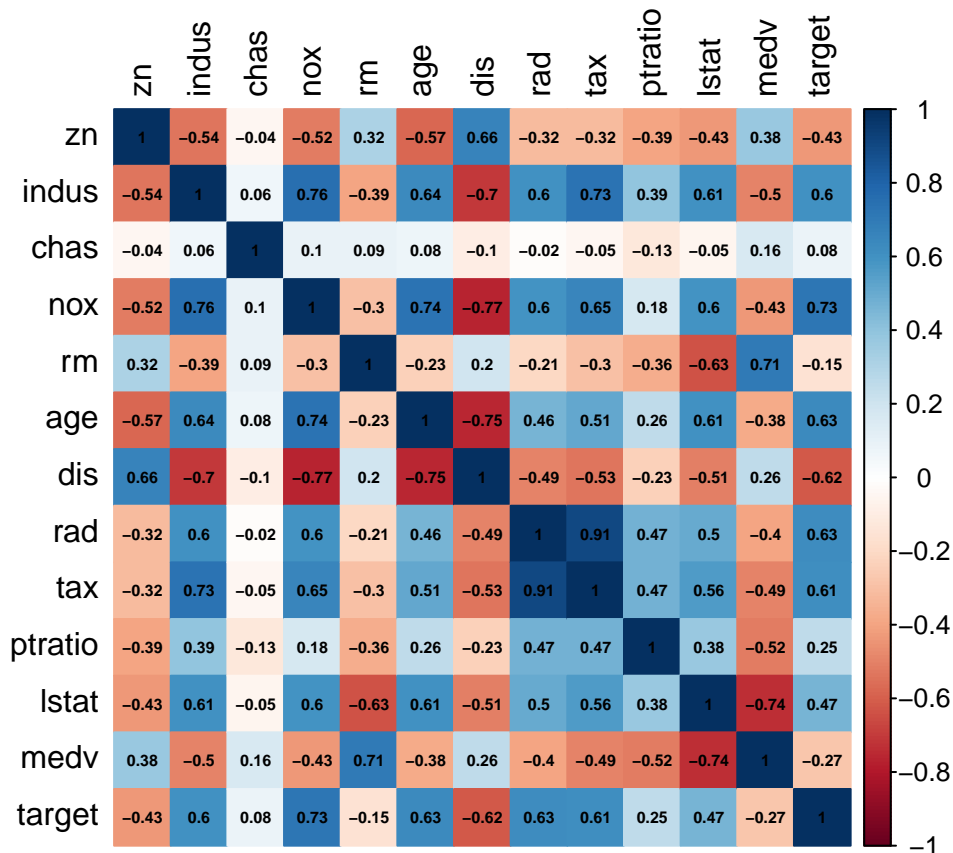
df_long %>%
  ggplot(aes(value)) +
  geom_density(fill = "blue") +
  facet_wrap(~variable, scales = "free", ncol = 4) +
  labs(x = element_blank(), y = element_blank())
```



Correlation Plot

To identify the correlation between each variable we can create a correlation plot by using the `corrplot` library. According to the correlation plot, the variables `nox` (0.73), `age` (0.63), `rad` (0.63), `tax` (0.61), and `indus` (0.60) are most linearly correlated with the `target` variable. In addition, there is a high degree of collinearity with `tax` and `rad` (0.91) and with `nox` and `indus` (0.76). These are variables to keep in mind when we select variables for our models.

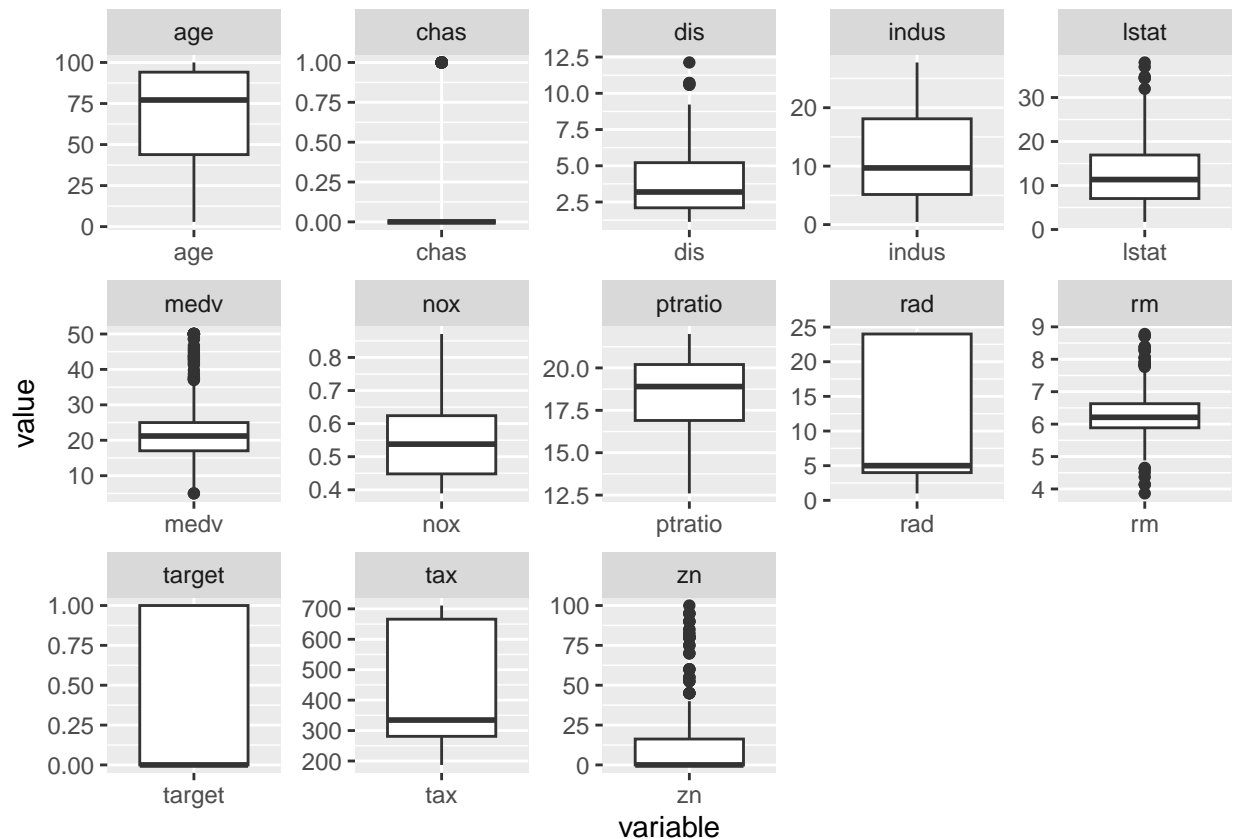
```
df %>%
  cor(use = "complete.obs") %>%
  corrplot(method = "color", tl.col = "black", addCoef.col = "black", number.cex = 0.5)
```



Boxplot

This boxplot shows the distribution and quartile ranges of the variables. We see that there are obvious outliers in chas and dis, which we may have to deal with later on. Other predictor variables such as lstat, medv, rm, and zn also have outliers, which may need to be dealt with later as well.

```
df_long %>%
  ggplot(aes(variable, value)) +
  geom_boxplot() +
  facet_wrap(~variable, scales='free', ncol=5)
```



2. Data Preparation

Fix any missing value in the data

Let's check if there exists a null value or NA in the dataframe. If Null is found it can be fixed using mean or median of the column in which na is found. Attach the data

```
training_set <- df
test_set <- read.csv("https://raw.githubusercontent.com/LeJQC/DATA-621-Group-2/14f8d86b73472ac8f9302415")
```

```
print(sum(is.na(training_set)))
```

```
## [1] 0
```

```
print(sum(is.na(test_set)))
```

```
## [1] 0
```

This shows that there are no missing values in the data and thus no need to bother about the missing values.

Modify an existing variable or create a new variable by combining the two or more variables

It is to be noted that the tax column contain the tax rate per \$10,000 and medv contain the property value in \$1000. The median tax on properties in neighborhood can be calculated by using these two columns.

$$medTax = \frac{medv * 1000}{10000} * Tax$$

or in simple it can be written as $medTax = medv * Tax / 10$

```
training_set<- training_set|>mutate(  
  medTax = medv*tax/10  
)|>  
  relocate(medTax, .before = 10)  
test_set<- training_set|>mutate(  
  medTax = medv*tax/10  
)|>relocate(medTax, .before = 10)  
head(training_set)
```

```
##   zn indus chas   nox   rm   age   dis rad tax  medTax ptratio lstat medv  
## 1  0 19.58    0 0.605 7.929  96.2 2.0459  5 403 2015.00   14.7  3.70 50.0  
## 2  0 19.58    1 0.871 5.403 100.0 1.3216  5 403  540.02   14.7 26.82 13.4  
## 3  0 18.10    0 0.740 6.485 100.0 1.9784 24 666 1025.64   20.2 18.85 15.4  
## 4 30  4.93    0 0.428 6.393   7.8 7.0355  6 300  711.00   16.6  5.19 23.7  
## 5  0  2.46    0 0.488 7.155  92.2 2.7006  3 193  731.47   17.8  4.82 37.9  
## 6  0  8.56    0 0.520 6.781  71.3 2.8561  5 384 1017.60   20.9  7.67 26.5  
##   target  
## 1      1  
## 2      1  
## 3      1  
## 4      0  
## 5      0  
## 6      0
```

Now no need of the variable tax as we have median value of tax per owner occupied home. Drop the variable tax from the data.

```
training_set<- training_set[, -9]  
test_set<- test_set[, -9]  
#head(training_set)  
#head(test_set)
```

Feature scaling

Now the data can be scaled so that the all the predictors have equal effect on the response variable.

It can concluded from the summary table that the range of some variable is high like zn's range is 0-100. This range can be normalize or standardize so that the range is between 0 and 1 or -3 to 3 respectively.

To normalize a number we just divide it by the maximum value in the variable. This can be done conveniently using `scale()` function in R.

Note: No need to standardize the target variable, the predictor variables chas and nox because these already contain 0 and 1 or within the standardized limit.

```
training_set[,1:12][,c(-3, -4)] = scale(training_set[, 1:12][,c(-3, -4)])
test_set = scale(test_set)
head(training_set)
```

```
##          zn          indus chas   nox          rm          age          dis          rad
## 1 -0.4955029  1.2379723    0 0.605  2.3243572  0.9827348 -0.8304864 -0.5215382
## 2 -0.4955029  1.2379723    1 0.871 -1.2593775  1.1169090 -1.1742535 -0.5215382
## 3 -0.4955029  1.0217831    0 0.740  0.2756981  1.1169090 -0.8625232  1.6659082
## 4  0.7884880 -0.9020088    0 0.428  0.1451741 -2.1385822  1.5376766 -0.4064095
## 5 -0.4955029 -1.2628111    0 0.488  1.2262532  0.8414987 -0.5197528 -0.7517957
## 6 -0.4955029 -0.3717609    0 0.520  0.6956449  0.1035403 -0.4459494 -0.5215382
##      medTax    ptratio      lstat      medv target
## 1  3.0265702 -1.6835500 -1.2576171  2.9666315      1
## 2 -0.8025987 -1.6835500  1.9978540 -0.9945441      1
## 3  0.4581106  0.8200407  0.8756176 -0.7780864      1
## 4 -0.3587206 -0.8186732 -1.0478138  0.1202130      0
## 5 -0.3055788 -0.2724352 -1.0999126  1.6570625      0
## 6  0.4372381  1.1386795 -0.6986110  0.4232537      0
```

3. Build Models (Logistic regression binary models)

Model 1

The first task before writing the any model is to select the predictor variables. But the selection of suitable predictor variable is a complex task and if we omit any predictor variable which influence the target variable, it induces a bias in the output of the model which is known as omitted-variable bias. Thus, one needs to be very careful while selecting a suitable predictor variable.

We can use the most correlated variables with the target variable. Using the scatterplot, it can be seen that the target variable is highly correlated with the following variables:

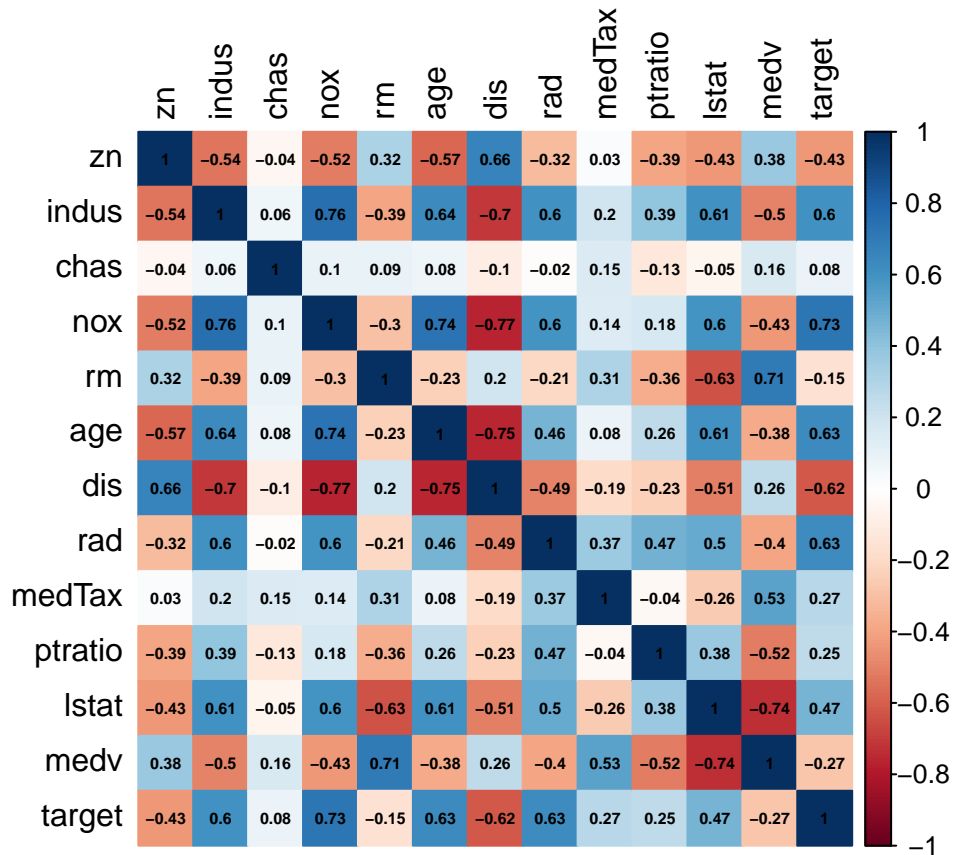
indus, nox, age, rad, lstat

And weakly correlated with medTax, ptratio

And almost neutral with chas and highly negatively correlated with zn, dis, medv, rm.

The correlation plot can be regenerated on the modified or transformed data to see the correlation among the transformed variables.

```
training_set %>%
  cor(use = "complete.obs") %>%
  corrplot(method = "color", tl.col = "black", addCoef.col = "black", number.cex = 0.5)
```

The highly correlated components can be selected for the logistic regression. Therefore, we will use indus, nox, age, rad, lstat, zn, dis. It can be seen that zn is highly correlated with nox, indus, age, dis, ptratio, lstate, medv. Therefore, it will be better to leave zn and no need to include in in the regression.

```
logistic_model <- glm (target ~ indus + nox + age + rad + lstat + dis,data = training_set, family = binomial)
summary(logistic_model)
```

```
##
## Call:
## glm(formula = target ~ indus + nox + age + rad + lstat + dis,
##      family = binomial, data = training_set)
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept) -18.5491     3.6085  -5.140 2.74e-07 ***
## indus       -0.7102     0.3000  -2.367  0.0179 *
## nox         37.6649     6.8269   5.517 3.45e-08 ***
## age          0.6428     0.2906   2.212  0.0270 *
## rad          4.0670     0.9554   4.257 2.07e-05 ***
## lstat       -0.1679     0.2489  -0.675  0.4999
## dis          0.5745     0.3265   1.760  0.0785 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
```

```
## Null deviance: 645.88 on 465 degrees of freedom
## Residual deviance: 226.60 on 459 degrees of freedom
## AIC: 240.6
##
## Number of Fisher Scoring iterations: 8
```

The p-value for the lstat is 0.4999 which is more than the critical value 0.05 thus this variable must be omitted from the regression.

```
logistic_model <- glm(target ~ indus + nox + age + rad + dis, data = training_set, family = binomial)
summary(logistic_model)
```

```
##
## Call:
## glm(formula = target ~ indus + nox + age + rad + dis, family = binomial,
## data = training_set)
##
## Coefficients:
## Estimate Std. Error z value Pr(>|z|)
## (Intercept) -18.3222 3.5812 -5.116 3.12e-07 ***
## indus -0.7445 0.2953 -2.521 0.0117 *
## nox 37.3437 6.7919 5.498 3.83e-08 ***
## age 0.5693 0.2693 2.114 0.0345 *
## rad 4.1268 0.9609 4.295 1.75e-05 ***
## dis 0.5279 0.3210 1.645 0.1000
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
## Null deviance: 645.88 on 465 degrees of freedom
## Residual deviance: 227.06 on 460 degrees of freedom
## AIC: 239.06
##
## Number of Fisher Scoring iterations: 8
```

All the probability of the coefficients are less than 0.05 thus, it can be concluded that all the coefficients are significant for the regression. **Test the hypothesis with chisq-test**

```
anova(logistic_model, test="Chisq")
```

```
## Analysis of Deviance Table
##
## Model: binomial, link: logit
##
## Response: target
##
## Terms added sequentially (first to last)
##
## Df Deviance Resid. Df Resid. Dev Pr(>Chi)
## NULL 465 645.88
```

```
## indus 1 192.641 464 453.23 < 2.2e-16 ***
## nox 1 165.125 463 288.11 < 2.2e-16 ***
## age 1 1.421 462 286.69 0.2333
## rad 1 56.943 461 229.75 4.487e-14 ***
## dis 1 2.686 460 227.06 0.1012
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The probability of dis is more than 0.05 thus the variable dis can be removed from the model and we have the logistic model.

```
logistic_model <- glm(target ~ indus + nox + age + rad, data = training_set, family = binomial)
anova(logistic_model, test="Chisq")
```

```
## Analysis of Deviance Table
##
## Model: binomial, link: logit
##
## Response: target
##
## Terms added sequentially (first to last)
##
##      Df Deviance Resid. Df Resid. Dev Pr(>Chi)
## NULL                                465    645.88
## indus 1 192.641 464 453.23 < 2.2e-16 ***
## nox 1 165.125 463 288.11 < 2.2e-16 ***
## age 1 1.421 462 286.69 0.2333
## rad 1 56.943 461 229.75 4.487e-14 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Now the variable age has $\text{pr}(>\text{Chi})=0.2333>0.05$, thus age seems to be removable, so let's drop it from the logistic model.

```
logistic_model <- glm(target ~ indus + nox + rad, data = training_set, family = binomial)
anova(logistic_model, test="Chisq")
```

```
## Analysis of Deviance Table
##
## Model: binomial, link: logit
##
## Response: target
##
## Terms added sequentially (first to last)
##
##      Df Deviance Resid. Df Resid. Dev Pr(>Chi)
## NULL                                465    645.88
## indus 1 192.641 464 453.23 < 2.2e-16 ***
## nox 1 165.125 463 288.11 < 2.2e-16 ***
## rad 1 55.016 462 233.09 1.196e-13 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Now the all the variables have probability less than 0.05 and thus these three variables are significant for the logistic binary regression. **Prediction of the based on the final model**

```
pred_target <- predict(logistic_model, type="response", data=test_set)
pred_target <- ifelse(pred_target > 0.5, 1, 0)
pred_target[1:10]
```

```
##  1  2  3  4  5  6  7  8  9 10
##  1  1  1  0  0  0  1  1  0  0
```

So the prediction

Model 2

For this binary logistic regression model, we are going to use all the predictor variables from the original training data set. Then we are going to use stepwise selection to remove predictors based on Akaike Information Criterion (AIC) to create an optimal model. This will be done using the `stepAIC` function from the MASS library, which will iterate over the predictors to improve the model's AIC.

```
logistic_model2 <- glm(target ~., family="binomial", data=df)
summary(logistic_model2)
```

```
##
## Call:
## glm(formula = target ~ ., family = "binomial", data = df)
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept) -40.822934   6.632913  -6.155 7.53e-10 ***
## zn          -0.065946   0.034656  -1.903  0.05706 .
## indus       -0.064614   0.047622  -1.357  0.17485
## chas         0.910765   0.755546   1.205  0.22803
## nox         49.122297   7.931706   6.193 5.90e-10 ***
## rm          -0.587488   0.722847  -0.813  0.41637
## age         0.034189   0.013814   2.475  0.01333 *
## dis         0.738660   0.230275   3.208  0.00134 **
## rad         0.666366   0.163152   4.084 4.42e-05 ***
## tax        -0.006171   0.002955  -2.089  0.03674 *
## ptratio     0.402566   0.126627   3.179  0.00148 **
## lstat       0.045869   0.054049   0.849  0.39608
## medv       0.180824   0.068294   2.648  0.00810 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##      Null deviance: 645.88  on 465  degrees of freedom
## Residual deviance: 192.05  on 453  degrees of freedom
## AIC: 218.05
##
## Number of Fisher Scoring iterations: 9
```

From this initial model with all the predictors, we can see that there are several variables that are not statistically significant such as `indus`, `chas`, `rm`, and `lstat`. This model has an AIC of 218.05. Next, let's use the `stepAIC` function to improve this model.

```
step_model <- stepAIC(logistic_model2, direction = "both", trace = FALSE)
summary(step_model)
```

```
##
## Call:
## glm(formula = target ~ zn + nox + age + dis + rad + tax + ptratio +
##      medv, family = "binomial", data = df)
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept) -37.415922   6.035013  -6.200 5.65e-10 ***
## zn          -0.068648   0.032019  -2.144  0.03203 *
## nox         42.807768   6.678692   6.410 1.46e-10 ***
## age          0.032950   0.010951   3.009  0.00262 **
## dis          0.654896   0.214050   3.060  0.00222 **
## rad          0.725109   0.149788   4.841 1.29e-06 ***
## tax         -0.007756   0.002653  -2.924  0.00346 **
## ptratio      0.323628   0.111390   2.905  0.00367 **
## medv         0.110472   0.035445   3.117  0.00183 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##      Null deviance: 645.88  on 465  degrees of freedom
## Residual deviance: 197.32  on 457  degrees of freedom
## AIC: 215.32
##
## Number of Fisher Scoring iterations: 9
```

In this new model, the predictor variables that were not statistically significant were removed. The AIC for this new model is 215.32, which is slightly lower than the initial model (218.05), indicating that removing those predictor variables improved the fit of the model.

The intercept or log-odds when predictors are zero is -37.4. As for coefficients, a one unit increase in `nox` (nitrogen oxide concentration) corresponds to an increase of 42.8 in the log-odds of high crime. A one unit increase in `age`, `dis`, `rad`, `ptratio`, and `medv` corresponds to an increase of 0.03, 0.65, 0.72, 0.32, and 0.11 in the log-odds of high crime, respectively. While a one unit increase in `zn` and `tax` results in a decrease of -0.06 and -0.007 in the log-odds of high crime. Out of all the predictors, it is strange to think that nitrogen oxides concentration had the largest influence on crime rate. How can air pollution be linked to crime? I can see the possibility that an increase in nitrogen oxide concentrations affecting physical and mental health, which can influence criminal behavior. However, this seems like a stretch and it is hard to see a correlation between the nitrogen oxide concentrations and crime rate intuitively. As for `zn` and `tax`, the two variables that corresponded to a decrease in crime, it seems practical that a cramped area with low property value will have a higher rate of crime.

Although there are predictors in the model do not make sense intuitively, all the predictors are statistically significant. Also, based on AIC, this model is a better fit for the target variable than the initial model with all the predictors.