SMU

SINGAPORE MANAGEMENT UNIVERSITY

DSA211 Statistical Learning with R – G1

Group Project Part 2

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1 Project Scope

1.1 Project Objective

The objective of the report is to explore and construct a predictive model for resale *Price* of 5 room HDB flats using the following parameters: *Town, Story, Area, Model and LeaseRem*.

To delve deeper into this study, we explored the training data set, Rm5HDB2023P.csv to derive the best recommended model before testing out its predictive power in the Rm5HDB2023testP.csv dataset.

Figure 1: Best Model selected from all approaches

```
finmodel<- randomForest(Price~.,data=hdb, mtry=4, importance=TRUE)
```

```
Call:
 randomForest(formula = Price ~ ., data = hdb, mtry = 4, importance = TRUE)
               Type of random forest: regression
                     Number of trees: 500
No. of variables tried at each split: 4

          Mean of squared residuals: 2153262009
                    % Var explained: 88.68
```

MSE on Test Rm5HDB2023testP.csv: **2339643383.**

1.2 Model Selection Criteria (Mean Squared Error)

To generalise the model beyond the training data, it is essential that the prediction model does not have high variance or high bias indicating that the training data is either overfitted or underfitted in the model.

To ensure that the model performs well on the test data set, it is necessary to achieve reasonably low variance and low bias due to the variance-bias tradeoff. Therefore, the model with lowest mean-squared error will be selected.

2 Model Selection Methods

Prior to running any models, our team checked the model for NA values which yield 0 fields.

In order to test the significance of the variables and to find the count of all the independent variables (36), our team first decided to run a linear regression of *Price* against all independent variables. All the independent variables were significant at 95% significance level.

To ensure uniformity in testing and enhance the predictability within each model, we have decided to apply cross validation to all models.

The models that we will be comparing are:

1. Multiple Linear Regression Model
2. Best Subset Selection
3. Ridge Regression Approach
4. Lasso Approach
5. Bagging Procedure
6. Random Forest Approach

2.1 Multiple Linear Regression Model

Our team has split our training data into two equal parts: one as training data (“train”), and the other as test data (“test”).

Once the Linear Model was populated on our training set, we used the model to predict the values of Price with our test data. This gave us a Test MSE of **32674663975**.

2.2 Best Subset Selection

To enhance precision accuracy and to control the variance and improve interpretability of our model, we decided to run the Best Subset selection to identify a subset of p predictors.

In order to run the Best Subset Selection, our team first random seed to (6789), followed by calling on the leaps library in order to run subset selection functions. The leaps::regsubsets() function was then run to populate a subset selection on the whole training data set.

Once the subset selection was done, we decided to use cross-validation to make less assumptions about the true underlying model. Cross-Validation was completed in the following steps:

1. Creating a predict.regsubsets function that allowed us to predict subset selection outcomes based on varying data sets and objects.
2. After that, we assigned k = 10 to dictate our number of folds.
3. Created a training set based of the training data to determine the number of folds
4. Created a matrix to store the cross validation error values for each of the 10-fold CV.
5. Once all the vectors were assigned, a nested for loop was run in order to run a best subset selection via 10-fold Cross validation. Each loop will then input a cross validation error value into our cv.errors matrix.
6. Once the cv.errors matrix was filled, the mean cross validation error for each model was calculated using an apply() function on the columns. This provided us with a mean.cv = **3446954895**.
7. Once the means of the cross validation errors was calculated, we had to select the model that provided us with the minimum cross validation error. Based on this particular model (“Best Subset Selection Model”), we assigned the number of regressors within its subset to “bb”.This provided us with a bb = **36**, which indicated that the Best model included 36 regressors.
8. Finally, we regressed the Best Model with the 36 variables against the total training set (regfit.all) to find the coefficient values. This provided us with the model of:

Figure 2: Coefficients of all variables in the Best Model

```
> coef(hdb.all, aa)
          (Intercept)             TownBEDOK            TownBISHAN        TownBUKIT BATOK
          -358109.466            -69783.631             92520.388           -149768.093
      TownBUKIT MERAH      TownBUKIT PANJANG        TownBUKIT TIMAH     TownCHOA CHU KANG
            85147.607           -202439.718            183842.856           -251485.220
       TownCLEMENTI             TownGEYLANG           TownHOUGANG       TownJURONG EAST
            25079.051             59262.830           -155379.588           -138868.042
     TownJURONG WEST    TownKALLANG/WHAMPOA      TownMARINE PARADE         TownPASIR RIS
          -225543.722             28847.425            180995.195           -175969.515
        TownPUNGGOL         TownQUEENSTOWN          TownSEMBAWANG          TownSENGKANG
          -212138.480            123751.140           -225825.085           -218438.116
      TownSERANGOON            TownTAMPINES           TownTOA PAYOH         TownWOODLANDS
           -53649.826           -132530.428             19508.205           -239516.045
         TownYISHUN          Story04 TO 06          Story07 TO 09         Story10 TO 12
          -195853.513             23564.063             41975.236             50199.181
       Story13 TO 15          Story16 TO 18          Story19Above                  Area
            58666.420             77111.692            126983.921              6114.591
       ModelImproved        ModelModel A ModelPremium Apartment         ModelStandard
          -164817.607           -175690.256           -154994.930           -164757.939
             LeaseRem
            7266.997
```

Evaluation

However, since p = 36 is large, the model may be overfitted and have high variance of the coefficient estimates due to enormous search space.

2.2 Regularization Techniques

Besides using Subset Selection approaches like Best Subset Selection, our team checked Regularization techniques such as Ridge Regression where the objective was to shrink the regressors coefficient instead of conducting variable selection. Hence, our team decided to run a model using the ridge regression and lasso method.

Prior to running Regularization techniques, our team has decided to define our x and y variables, while keeping our grid to the default R grid. In order to ease computational inputs, we have also assigned vectors for both x and y in the training and test data.

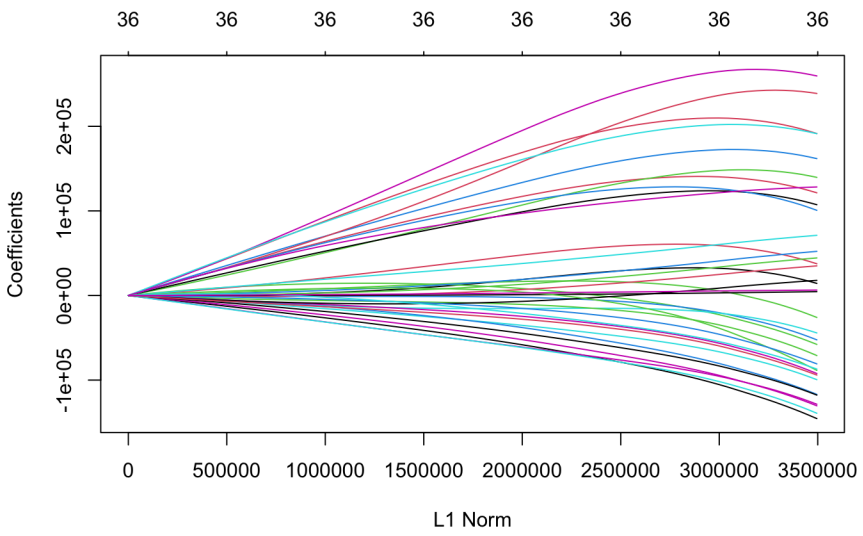
2.3.1 Ridge Regression Approach

As the ridge regression imposes a tuning parameter to control the relative impact of the coefficient regressors in the model by identifying the best lambda as a standardised parameter, allowing the model to have a lowest mse with the most optimal bias-variance tradeoff.

Similar to Best Subset Selection, we first set random seed to (6789), followed by calling the “glmnet” library in order for us to conduct our Regularization techniques. Once all the vectors were assigned properly, we conducted Ridge Regression based on the following steps:

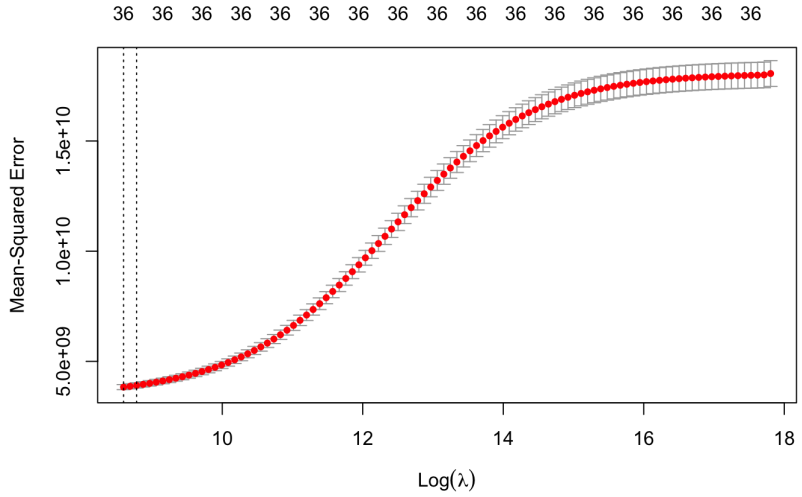
1. We first created our ridge regression model based on the training data.

Figure 3: Plot of Coefficients against L1 Norm for Ridge Regression



2. Next, we conducted cross validation using leaps::cv.glmnet() and identified the optimal tuning parameter (lambda)
   This gave us a value of **5407.103** for our optimal tuning parameter.

Figure 4: Plot of Mean-Squared Error against Log of Lambda for Ridge Regression



3. Next, in order to find out the test MSE, we used our optimal training model to predict the outputs based on the test data set. This provided us with a MSE of **3975723793**.
4. Finally, with our optimal lambda, we predicted the coefficients of the optimal model using our overall training data. This provided us with the model of:

Figure 5: Summary statistics of coefficients in the model incorporating the optimal lambda value

(Intercept)	-253592.624
(Intercept)	.
TownBEDOK	22630.373
TownBISHAN	189995.702
TownBUKIT BATOK	-49483.637
TownBUKIT MERAH	167877.180
TownBUKIT PANJANG	-102804.410
TownBUKIT TIMAH	267288.677
TownCHOA CHU KANG	-144136.321
TownCLEMENTI	118202.320
TownGEYLANG	142049.059
TownHOUGANG	-51114.398
TownJURONG EAST	-45011.945
TownJURONG WEST	-124673.728
TownKALLANG/WHAMPOA	121497.793
TownMARINE PARADE	256030.552
TownPASIR RIS	-66909.483
TownPUNGGOL	-110079.637
TownQUEENSTOWN	204296.709
TownSEMBAWANG	-124281.280
TownQUEENSTOWN	204296.709
TownSEMBAWANG	-124281.280
TownSENGKANG	-117606.423
TownSERANGOON	41782.231
TownTAMPINES	-25563.537
TownTOA PAYOH	121005.538
TownWOODLANDS	-133734.873
TownYISHUN	-91396.551
Story04 TO 06	16613.252
Story07 TO 09	33293.019
Story10 TO 12	41976.477
Story13 TO 15	48941.650
Story16 TO 18	70599.915
Story19Above	131855.340
Area	4395.065
ModelImproved	-92938.882
ModelModel A	-82852.488
ModelPremium Apartment	-83143.371
ModelStandard	-95588.600
LeaseRem	6407.155

Evaluations

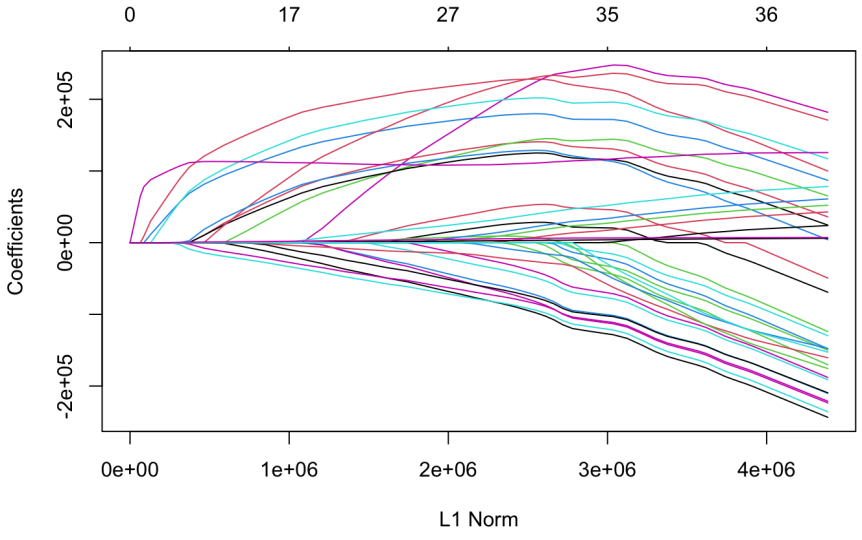
However, we evaluated the limitations of the ridge regression model where the model adds in all p predictors in the final model. Hence, we decided to do a Lasso approach to overcome this disadvantage by minimising the quantity whilst penalising the coefficients of the regressors.

2.3.2 Lasso Approach

Similar to Lasso Approach, we first set random seed to (6789), followed by calling the "glmnet" library in order for us to conduct our Regularization techniques. Once all the vectors were assigned properly, we conducted Lasso based on the following steps:

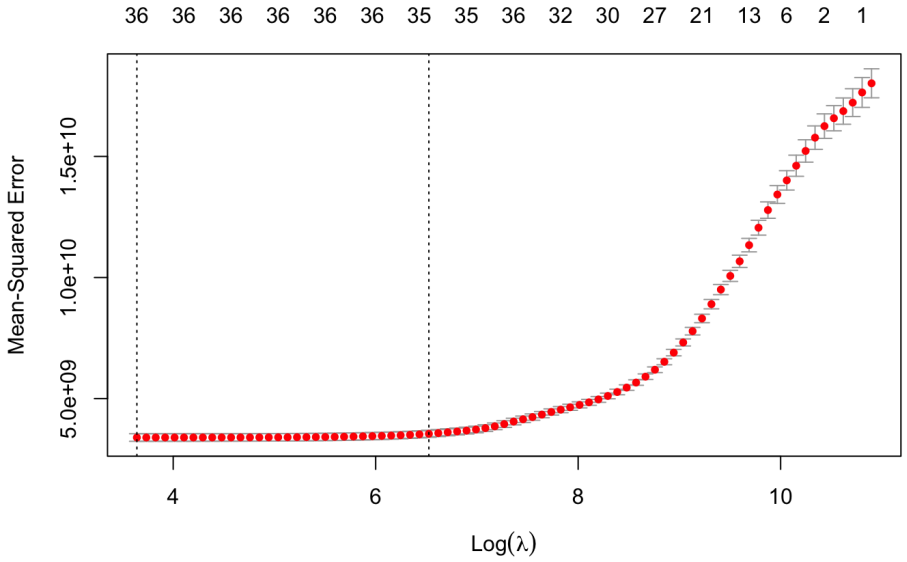
1. We first created our lasso model based on the training data.

Figure 6: Plot of Coefficients against L1 Norm for Lasso



2. Next, we conducted cross validation using leaps::cv.glmnet() and identified the optimal tuning parameter (lambda). This gave us a value of **38.14604** for our optimal tuning parameter.

Figure 7: Plot of Mean-Squared Error against Log of Lambda for Lasso



3. Next, in order to find out the test MSE, we used our optimal training model to predict the outputs based on the test data set. This provided us with a MSE of **3547183929**.
4. Finally, with our optimal lambda, we predicted the coefficients of the optimal model using our overall training data. Additionally, for better visualisation, we decided to exclude all the coefficients that equalled 0. This provided us with the output of:

Figure 8: Summary statistics of coefficients in the model incorporating the optimal lambda value (excluding coefficients equating to 0)

```
  [1] -358794.605  -63594.420   98643.905 -143541.601   90656.224 -196286.258  188996.472
  [8] -245220.045   30746.511   64699.516 -148914.972 -132654.323 -219423.424   34544.615
 [15]  186201.396 -169401.944 -206030.164  129006.614 -219716.310 -212411.384  -47208.264
 [22] -126018.023   25472.612 -233320.298 -189626.283   23058.975   41416.654   49663.217
 [29]   58041.888   76656.411  127190.395    6059.751 -162079.896 -172168.048 -152382.105
 [36] -161825.553    7251.847
```

Evaluation

Lasso and Ridge regression tend to struggle when it comes to capturing non-linear relationships between predictors and response variables. Therefore, if we are uncertain about the linearity of the relationship between these variables, these regression techniques may not be the optimal models for accurately explaining the training set or making predictions for the final test set.

2.4 Tree-Based Methods

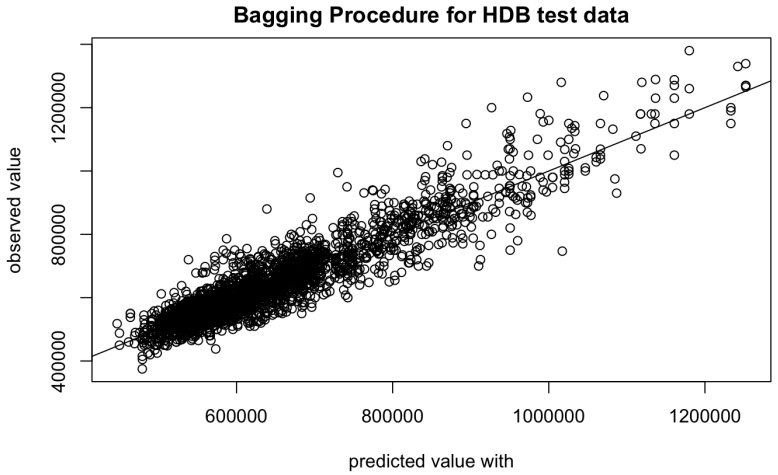
Finally, our group also considered testing for Bagging and Random Forest Approaches as the bagging could solve the issue of having non-linear predictor-response relationships.

2.4.1 Bagging (mtry = 5)

As usual, we set random seed to (6789) to ensure uniformity of results, followed by calling the “randomForest” library. Once it was done, we conducted bagging based on the following steps:

1. With all 5 independent variables, we created a random forest using randomForest::randomForest().
2. We then ran our training model on the test model to find the predicted yhat values with our bagging approach

Figure 9: Plot of observed value against predicted value (Bagging procedure)

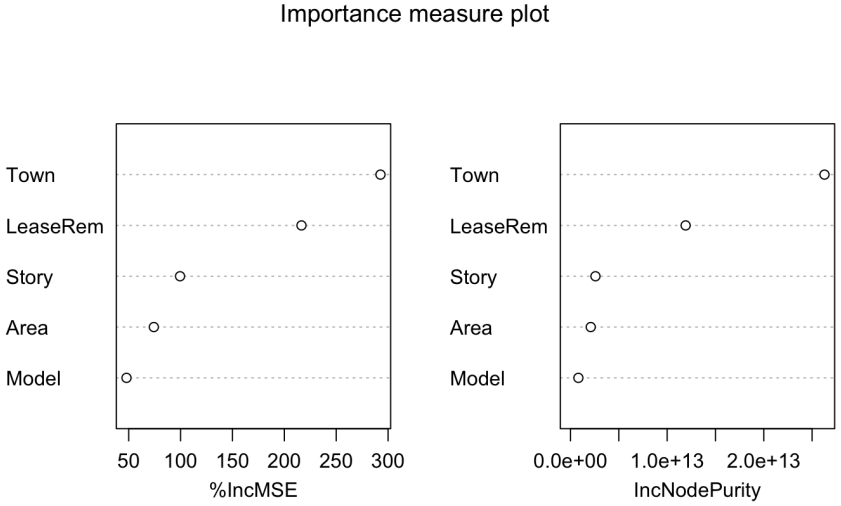


3. Finally, we found the test MSE for the bagging approach. Our test MSE was found to be **2641712577**.

Important Variables

Using randomForest::Importance(), we have identified that the two most important variables are Town (**2.628247e+13**) and LeaseRem (**1.191510e+13**).

Figure 10: Importance measure plot for Bagging Procedure:

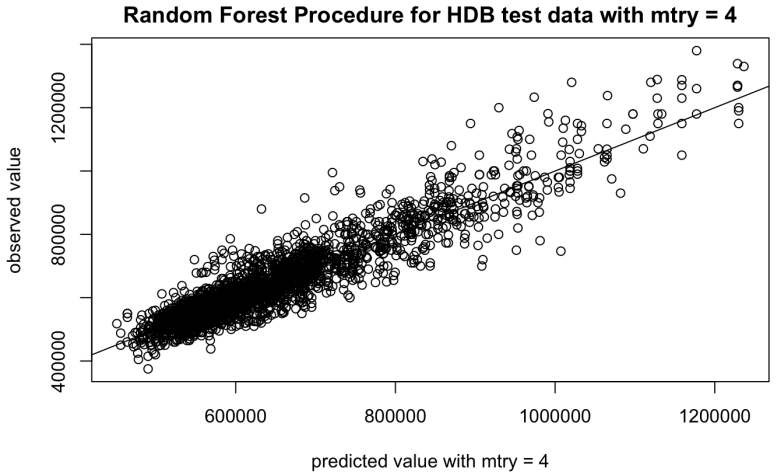


2.4.2 Random Forest Approach (mtry = 4)

Similarly, we set the random seed to (6789) to ensure uniformity of results. Once that was done, we conducted randomForest based on the following steps:

1. With all 5 independent variables, we created a random forest using randomForest::randomForest(), with 4 variables first.
2. We then ran our training model on the test model to find the predicted yhat1 values with our random forest approach

Figure 11: Plot of observed value against predicted value for Random Forest approach

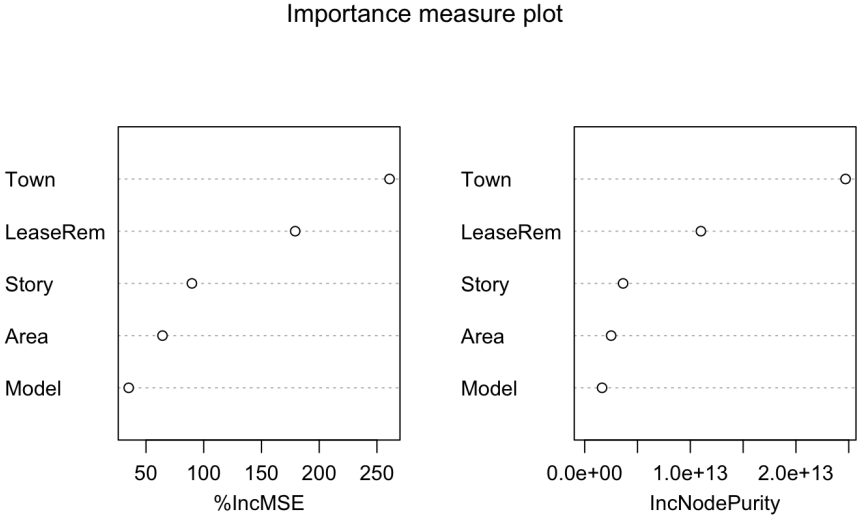


3. Finally, we found the test MSE for the random tree with mtry = 4
4. We then repeat steps 1 to 3 with different quantity of variables, mtry = 1, 2, 3
5. Comparing the different test MSE, our best model will be based on the smallest test MSE. Our smallest test MSE was found to be **2628577302**.

Important Variables

Using randomForest::Importance(), we have identified that the two most important variables are Town (**2.469564e+13**) and LeaseRem (**1.099981e+13**).

Figure 12: Importance measure plot for Random Forest approach



Random Forest (mtry = 4) VS Bagging procedure

Figure 13: Table displaying values of TestMSE associated with increasing the mtry

mtry	Test MSE
5	2641712577
4	2628577302
3	2639320289
2	2736301069
1	3738783894

We can see that mtry = 4 has the lowest test MSE, which implies that within the bagging approach and random forest approaches, mtry = 4 will give the optimal predictive model.

Additionally, the training error of mtry = 4 (**86.47%**) is relatively the same as that of bagging (86.48%). Hence, this means that within the training set itself, mtry = 4 is validated to explain the same amount of variance in the training data as when all independent variables are used.

3 Best Predictive Model

Figure 14: Table displaying values of TestMSE associated with various models

Model	MSE
Multiple Linear Regression Model	32674663975
Best Subset Selection	3446954895
Ridge Regression Approach	3975723793
Lasso Approach	3547183929
Bagging	2641712577
Random Forest Approach (mtry = 4)	2628577302

Based on the training data provided with *Rm5HDB2023P.cv,* we concluded that our best predictive model would be Random Forest (mtry = 4) as it has the lowest MSE. Hence, it would have the strongest predictive power of all models.

4. Predictive Power of Model on Test set *Rm5HDB2023testP.cv*

4.1 Test Error Derivation

Figure 15: Summary statistics of the final model

```
finmodel<- randomForest(Price~.,data=hdb, mtry=4, importance=TRUE)
```

```
Call:
 randomForest(formula = Price ~ ., data = hdb, mtry = 4, importance = TRUE)
               Type of random forest: regression
                     Number of trees: 500
No. of variables tried at each split: 4

          Mean of squared residuals: 2153262009
                    % Var explained: 88.68
```

Using the best predictive model, we predicted the test dataset and concluded the test error of **2339643383.**

4.2 Comparison between Predicted & Observed values

Figure 16: Comparison between the predicted and observed values

	y.1.5. <dbl>	pred <dbl>
1	780000	825482.7
2	630000	672742.1
3	838000	810250.2
4	750000	825482.7
5	800000	873453.0

Upon comparing the Price values predicted by our model against the observed prices in the test dataset, we can observe that the predicted values consistently overestimate the values but not by a very large margin. Furthermore, the differences between both values seem to be consistent throughout the different rows. It is a reasonable indication of the strong predictive power of our model.

4.3 Residual Assumptions

Based on the residual plots that we have indicated in the Annex below, we can observe a recurring trend: the residuals when plotted against the independent variables show a constant variance and no distinct deviation.

This can also be observed in the q-q and p-p plots below, where no distinct deviation can be spotted. We can conclude that our recommended model does not violate any residual assumption, further justifying the predictive power of our model

Hence, **our best predictive model is Random Forest (mtry = 4)** to predict resale prices.

5 Annex

```
> hdb <- read.csv("RmSHDB2023P.csv", stringsAsFactors = TRUE)
> attach(hdb)
The following objects are masked from hdb (pos = 3):

    Area, LeaseRem, Model, Price, Story, Town

The following objects are masked from hdb (pos = 4):

    Area, LeaseRem, Model, Price, Story, Town

The following objects are masked from hdb (pos = 5):

    Area, LeaseRem, Model, Price, Story, Town

The following objects are masked from hdb (pos = 6):

    Area, LeaseRem, Model, Price, Story, Town

The following objects are masked from hdb (pos = 7):

    Area, LeaseRem, Model, Price, Story, Town

The following objects are masked from hdb (pos = 9):

    Area, LeaseRem, Model, Price, Story, Town

The following objects are masked from hdb (pos = 13):

    Area, LeaseRem, Model, Price, Story, Town

> View(hdb)
>
> # for checking
> sum(is.na(hdb)) # 0
[1] 0
```

```
> summary(lm(Price~., data = hdb))

Call:
lm(formula = Price ~ ., data = hdb)

Residuals:
    Min      1Q  Median      3Q     Max
-254770  -36757   -4285   31985  280003

Coefficients:
                          Estimate Std. Error t value Pr(>|t|)
(Intercept)             -358109.5    30675.8 -11.674  < 2e-16 ***
TownBEDOK                -69783.6     7126.7  -9.792  < 2e-16 ***
TownBISHAN                92520.4     8056.6  11.484  < 2e-16 ***
TownBUKIT BATOK         -149768.1     7936.7 -18.870  < 2e-16 ***
TownBUKIT MERAH           85147.6     7188.1  11.846  < 2e-16 ***
TownBUKIT PANJANG       -202439.7     6880.6 -29.422  < 2e-16 ***
TownBUKIT TIMAH          183842.9    19314.9   9.518  < 2e-16 ***
TownCHOA CHU KANG       -251485.2     6441.5 -39.041  < 2e-16 ***
TownCLEMENTI              25079.1     9155.4   2.739 0.006180 **
TownGEYLANG               59262.8     9723.6   6.095 1.18e-09 ***
TownHOUGANG             -155379.6     7094.7 -21.901  < 2e-16 ***
TownJURONG EAST         -138868.0     8464.6 -16.406  < 2e-16 ***
TownJURONG WEST         -225543.7     6379.5 -35.355  < 2e-16 ***
TownKALLANG/WHAMPOA       28847.4     8369.8   3.447 0.000572 ***
TownMARINE PARADE        180995.2    11121.5  16.274  < 2e-16 ***
TownPASIR RIS           -175969.5     7073.6 -24.877  < 2e-16 ***
TownPUNGGOL             -212138.5     6450.3 -32.888  < 2e-16 ***
TownQUEENSTOWN           123751.1     8682.2  14.253  < 2e-16 ***
TownSEMBAWANG           -225825.1     7179.5 -31.454  < 2e-16 ***
TownSENGKANG            -218438.1     6231.2 -35.056  < 2e-16 ***
TownSERANGOON            -53649.8     9231.7  -5.811 6.58e-09 ***
TownTAMPINES            -132530.4     6502.3 -20.382  < 2e-16 ***
TownTOA PAYOH             19508.2     8235.4   2.369 0.017882 *
TownWOODLANDS           -239516.0     6312.4 -37.944  < 2e-16 ***
TownYISHUN              -195853.5     6707.9 -29.197  < 2e-16 ***
Story04 TO 06             23564.1     2805.2   8.400  < 2e-16 ***
Story07 TO 09             41975.2     2772.8  15.138  < 2e-16 ***
Story10 TO 12             50199.2     2898.4  17.320  < 2e-16 ***
Story13 TO 15             58666.4     3196.8  18.351  < 2e-16 ***
Story16 TO 18             77111.7     3978.4  19.383  < 2e-16 ***
Story19Above             126983.9     4147.6  30.616  < 2e-16 ***
Area                       6114.6      212.2  28.811  < 2e-16 ***
ModelImproved           -164817.6     6003.1 -27.455  < 2e-16 ***
ModelModel A            -175690.3     8035.0 -21.866  < 2e-16 ***
ModelPremium Apartment  -154994.9     6361.1 -24.366  < 2e-16 ***
ModelStandard           -164757.9     8005.6 -20.580  < 2e-16 ***
LeaseRem                   7267.0      111.8  65.003  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 58330 on 4958 degrees of freedom
Multiple R-squared:  0.8224,    Adjusted R-squared:  0.8211
F-statistic: 637.8 on 36 and 4958 DF,  p-value: < 2.2e-16
```

5.1 Linear Model

```
> RNGkind(sample.kind = "Rounding")
Warning: non-uniform 'Rounding' sampler used
> set.seed(6789)
>
> train <- sample(1:nrow(hdb),nrow(hdb)/2) # half sample
> test <- -train
>
> lm1 <- lm(Price ~., data = hdb[train,])
> lm.pred <- predict(lm1, hdb[test,])
> mean((hdb$Price-lm.pred)^2) # test MSE 32674663975
Warning: longer object length is not a multiple of shorter object length[1] 32674663975
```

5.2 Best Subset Selection

```
> set.seed(6789)
>
> library(leaps)
>
> # function to predict
> predict.regsubsets <- function(object,newdata,id) {
+       form <- as.formula(object$call[[2]])
+       mat <- model.matrix(form, newdata)
+
+       coefi <- coef(object, id = id)
+       xvars <- names(coefi)
+       mat[,xvars]%*%coefi
+ }
>
> k <- 10 # assign the k-fold CV
> folds <- sample(1:k, nrow(hdb), replace = TRUE) # training set
> cv.errors <- matrix(NA, k, 36, dimnames = list(NULL, paste(1:36))) # 36 independent variables
>
> for (j in 1:k) {
+       best.fit <- regsubsets(Price~., data = hdb[folds!=j,],nvmax = 36) # running the BSS
+       for (i in 1:36) {
+             pred <- predict.regsubsets(best.fit, hdb[folds == j,], id = i) #extracting coefficients and then
 multiplying them to form the prediction
+             cv.errors[j,i] <- mean((hdb$Price[folds ==j]-pred)^2) # calculate test MSE
+       }
+ }
>
> mean.cv <- apply(cv.errors, 2, mean) #average over the column of the matrix to error for each model
> mean.cv
          1           2           3           4           5           6           7           8
15611514414 14916030391 13875880836 12369899127 11474470565 10974061099 10169597851  9599279962
          9          10          11          12          13          14          15          16
 8647693860  8159461064  7503225273  7019126871  6662548326  6475968539  5929992350  5516437205
         17          18          19          20          21          22          23          24
 4929894572  4729360036  4579348822  4516674121  4542593359  4239005796  4127929711  4031496772
         25          26          27          28          29          30          31          32
 3943354527  3920207125  3753072368  3677861554  3696412079  3581142167  3528348667  3463968305
         33          34          35          36
 3450742519  3452416458  3451870783  3446954895
>
> min(mean.cv) # 3446954895
[1] 3446954895
>
```

```
> # finding optimal model
> aa <- which.min(mean.cv)
> aa # 36
36
36
>
> hdb.all <- regsubsets(Price~., data = hdb, nvmax = 36)
> coef(hdb.all, aa)
          (Intercept)             TownBEDOK             TownBISHAN        TownBUKIT BATOK
          -358109.466            -69783.631             92520.388           -149768.093
       TownBUKIT MERAH      TownBUKIT PANJANG        TownBUKIT TIMAH      TownCHOA CHU KANG
            85147.607           -202439.718            183842.856           -251485.220
         TownCLEMENTI            TownGEYLANG            TownHOUGANG       TownJURONG EAST
            25079.051             59262.830           -155379.588           -138868.042
       TownJURONG WEST    TownKALLANG/WHAMPOA      TownMARINE PARADE          TownPASIR RIS
           -225543.722             28847.425            180995.195           -175969.515
          TownPUNGGOL         TownQUEENSTOWN          TownSEMBAWANG          TownSENGKANG
           -212138.480            123751.140           -225825.085           -218438.116
         TownSERANGOON           TownTAMPINES          TownTOA PAYOH          TownWOODLANDS
            -53649.826           -132530.428             19508.205           -239516.045
          TownYISHUN          Story04 TO 06          Story07 TO 09          Story10 TO 12
           -195853.513            23564.063             41975.236            50199.181
         Story13 TO 15          Story16 TO 18          Story19Above                  Area
            58666.420            77111.692            126983.921             6114.591
         ModelImproved          ModelModel A ModelPremium Apartment          ModelStandard
           -164817.607           -175690.256           -154994.930           -164757.939
              LeaseRem
              7266.997
>
> # plot
> plot(1:36, mean.cv, main = "Relationship between Mean.cv and Number of variables", xlab = "number of variable
s", ylab = "Mean Cross Validation error", type = "b")
```

5.3 Ridge Regression

```
> x <- model.matrix(Price~.,hdb)[,-1]
> y <- hdb$Price
> # grid use default
>
> RNGkind(sample.kind = "Rounding")
Warning: non-uniform 'Rounding' sampler used
> set.seed(6789)
>
> library(glmnet)
>
> train <- sample(1:nrow(hdb),nrow(hdb)/2) # half sample
> test <- -train
>
> x.train <- x[train,]
> y.train <- y[train]
> x.test <- x[test,]
> y.test <- y[test]
>
> ridge.mod <- glmnet(x.train,y.train, alpha = 0) # training model; default grid
>
> # plot
> plot(ridge.mod)
>
> cvrr.out <- cv.glmnet(x.train,y.train, alpha = 0)
> bestlam <- cvrr.out$lambda.min # min lambda
> bestlam # 5407.103
[1] 5407.103
>
> # plot
> plot(cvrr.out)
>
> # for test model to test for MSE
> ridge.pred <- predict(ridge.mod, s = bestlam, newx = x.test)
> mean((ridge.pred - y.test)^2) # 3975723793
[1] 3975723793
```

```
> # for general model
> out.rr <- glmnet(x,y, alpha = 0) # general model; default grid
> predict(out.rr, type = "coefficients", s = bestlam)
37 x 1 sparse Matrix of class "dgCMatrix"
                              s1
(Intercept)           -253592.624
TownBEDOK               22630.373
TownBISHAN             189995.702
TownBUKIT BATOK        -49483.637
TownBUKIT MERAH        167877.180
TownBUKIT PANJANG     -102804.410
TownBUKIT TIMAH        267288.677
TownCHOA CHU KANG     -144136.321
TownCLEMENTI           118202.320
TownGEYLANG            142049.059
TownHOUGANG            -51114.398
TownJURONG EAST        -45011.945
TownJURONG WEST       -124673.728
TownKALLANG/WHAMPOA    121497.793
TownMARINE PARADE      256030.552
TownPASIR RIS          -66909.483
TownPUNGGOL           -110079.637
TownQUEENSTOWN         204296.709
TownSEMBAWANG         -124281.280
TownSENGKANG          -117606.423
TownSERANGOON           41782.231
TownTAMPINES           -25563.537
TownTOA PAYOH          121005.538
TownWOODLANDS         -133734.873
TownYISHUN             -91396.551
Story04 TO 06           16613.252
Story07 TO 09           33293.019
Story10 TO 12           41976.477
Story13 TO 15           48941.650
Story16 TO 18           70599.915
Story19Above           131855.340
Area                     4395.065
ModelImproved          -92938.882
ModelModel A           -82852.488
ModelPremium Apartment -83143.371
ModelStandard          -95588.600
LeaseRem                 6407.155
```

5.4 Lasso

```
> RNGkind(sample.kind = "Rounding")
Warning: non-uniform 'Rounding' sampler used
> set.seed(6789)
>
> lasso.mod <- glmnet(x.train,y.train, alpha = 1) # training model; default grid
>
> plot(lasso.mod)
>
> cvlasso.out <- cv.glmnet(x.train,y.train, alpha = 1)
> bestlam1 <- cvlasso.out$lambda.min # min lambda
> bestlam1 # 38.14604
[1] 38.14604
>
> plot(cvlasso.out)
>
> # for test model to test for MSE
> lasso.pred <- predict(lasso.mod, s = bestlam1, newx = x.test)
> mean((lasso.pred - y.test)^2) # 3547183929
[1] 3547183929
>
> # for general model
> out.lasso <- glmnet(x,y, alpha = 1) # general model; default grid
> lasso.coef <- predict(out.lasso, type = "coefficients", s = bestlam1)
> lasso.coef[lasso.coef!=0]
<sparse>[ <logic> ] : .M.sub.i.logical() maybe inefficient
 [1] -358844.573  -63882.473   98365.140 -143835.991   90412.100 -196575.688  188785.434
 [8] -245514.772   30493.296   64461.528 -149222.404 -132947.905 -219709.815   34290.619
[15]  185984.729 -169715.961 -206316.228  128776.824 -220004.788 -212694.172  -47520.188
[22] -126329.359   25200.544 -233614.554 -189924.200   23094.059   41454.443   49700.865
[29]   58084.042   76690.724  127188.365    6063.013 -162231.080 -172370.968 -152528.994
[36] -162001.595    7252.751
```

5.5 Bagging Approach

```
> RNGkind(sample.kind = "Rounding")
Warning: non-uniform 'Rounding' sampler used
> set.seed(6789)
> library(randomForest)
>
> bag.hdb <- randomForest(Price~., data = hdb, subset = train, mtry = 5, importance = TRUE) # mtry is the number
 of predictors
> bag.hdb

Call:
 randomForest(formula = Price ~ ., data = hdb, mtry = 5, importance = TRUE,      subset = train)
               Type of random forest: regression
                     Number of trees: 500
No. of variables tried at each split: 5

          Mean of squared residuals: 2441168359
                    % Var explained: 86.48
>
> yhat.bag <- predict(bag.hdb, newdata = hdb[test,])
> mean((yhat.bag - hdb[test,]$Price)^2) # 2641712577; % Var explained: 86.48
[1] 2641712577
>
> plot(yhat.bag, hdb[test,]$Price, main = "Bagging Procedure for HDB test data", xlab = "predicted value with", y
lab = "observed value") + abline(0,1)
integer(0)
>
> importance(bag.hdb)
           %IncMSE IncNodePurity
Town     292.68174  2.628247e+13
Story     99.48829  2.582049e+12
Area      74.23896  2.094600e+12
Model     47.87751  8.158154e+11
LeaseRem 216.52058  1.191510e+13
>
> varImpPlot(bag.hdb, main = "Importance measure plot")
>
```

5.6 Random Forest

```
> RNGkind(sample.kind = "Rounding")
Warning: non-uniform 'Rounding' sampler used
> set.seed(6789)
> library(randomForest)
>
> rf.hdb <- randomForest(Price~., data = hdb, subset = train, mtry = 4, importance = TRUE) # mtry is the number
f predictors
> rf.hdb

Call:
 randomForest(formula = Price ~ ., data = hdb, mtry = 4, importance = TRUE,      subset = train)
               Type of random forest: regression
                     Number of trees: 500
No. of variables tried at each split: 4

          Mean of squared residuals: 2442691923
                    % Var explained: 86.47
>
> yhat.rf <- predict(rf.hdb, newdata = hdb[test,])
> mean((yhat.rf - hdb[test,]$Price)^2) # 2628577302, % Var explained: 86.47
[1] 2628577302
>
> plot(yhat.rf, hdb[test,]$Price, main = "Random Forest Procedure for HDB test data with mtry = 4", xlab = "pred
cted value with mtry = 4", ylab = "observed value") + abline(0,1)
integer(0)
>
> importance(rf.hdb)
           %IncMSE IncNodePurity
Town     260.91813  2.469564e+13
Story     89.75755  3.632937e+12
Area      64.27566  2.503133e+12
Model     35.02059  1.642799e+12
LeaseRem 179.20508  1.099981e+13
>
> varImpPlot(rf.hdb, main = "Importance measure plot")
```

5.7 Final Model (Random Forests mtry=4)

```
> finmodel<- randomForest(Price~.,data=hdb, mtry=4, importance=TRUE)
> finmodel

Call:
 randomForest(formula = Price ~ ., data = hdb, mtry = 4, importance = TRUE)
               Type of random forest: regression
                     Number of trees: 500
No. of variables tried at each split: 4

          Mean of squared residuals: 2160505873
                    % Var explained: 88.64
>
```

Testing final Model on Test set

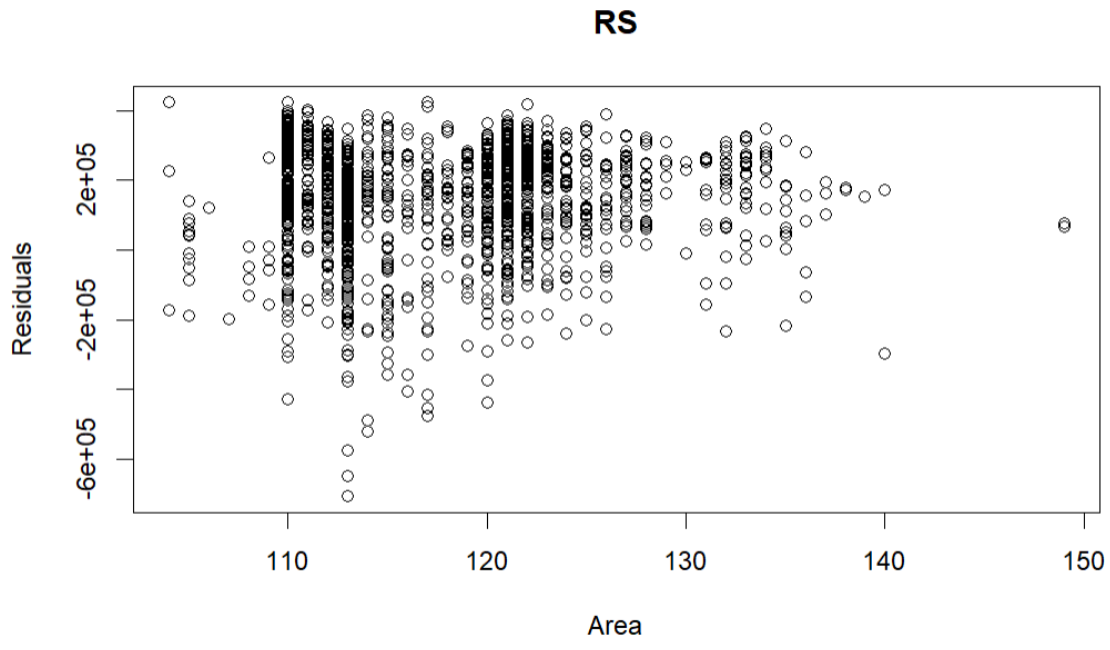
```
> #testing set
> testset <- read.csv("Rm5HDB2023testP.csv", stringsAsFactors = TRUE)
> testset[,1:5]
> finmodel

Call:
 randomForest(formula = Price ~ ., data = hdb, mtry = 4, importance = TRUE)
               Type of random forest: regression
                     Number of trees: 500
No. of variables tried at each split: 4

          Mean of squared residuals: 2153262009
                    % Var explained: 88.68
> test.pred <-predict(finmodel, newdata=testset[,1:5])
> mean((test.pred-testset$Price)^2)
[1] 2339643383
```

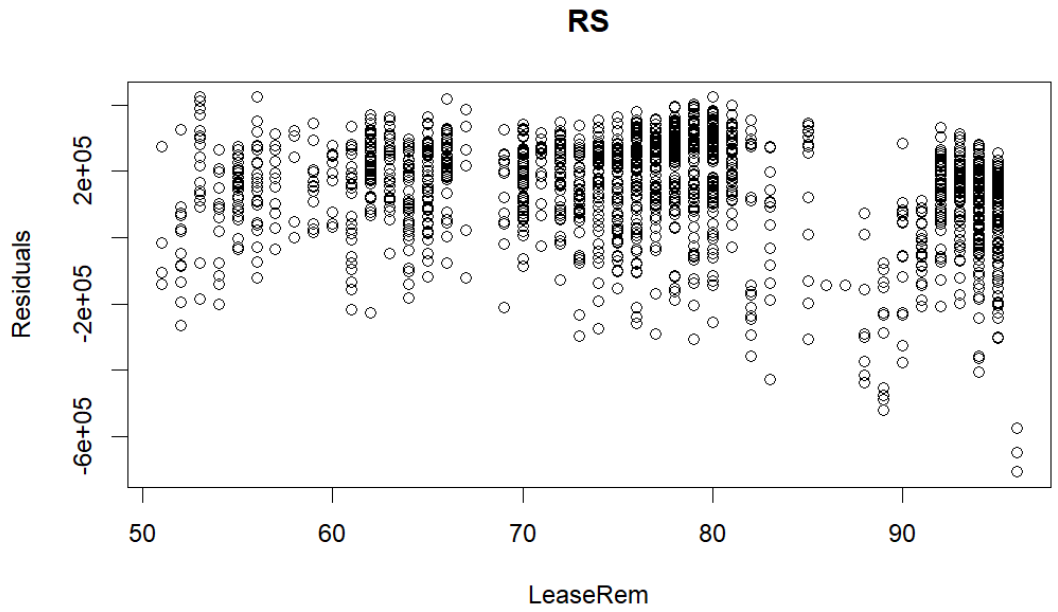
5.8 Comparison between Predicted and Observed Values

```
> y<-testhdb$Price
>
> pred<-predict(finalmodel,newdata=testhdb[1:5,1:5])
> pred
       1        2        3        4        5
825482.7 672742.1 810250.2 825482.7 873453.0
>
> compare<-data.frame(y[1:5],pred)
> compare
>
>
```

5.9 Residual Assumptions

```
> plot(testhdb$Area,pred-y,main="RS",xlab="Area",ylab="Residuals")
Warning: longer object length is not a multiple of shorter object length
> plot(testhdb$LeaseRem,pred-y,main="RS",xlab="LeaseRem",ylab="Residuals")
Warning: longer object length is not a multiple of shorter object length
```






```
> library(fitdistrplus)
> fnorm<-fitdist(pred-y,"norm")
Warning: longer object length is not a multiple of shorter object lengthWarning: NaNs producedWarning: NaNs produced
>
> summary(fnorm)
Fitting of the distribution ' norm ' by maximum likelihood
Parameters :
Loglikelihood:  -21494.63   AIC:  42993.26   BIC:  43004.03
Correlation matrix:
[1] NA

> plot(fnorm)
>
```